DRILLING IN BONE: MODELING HEAT GENERATION & TEMPERATURE DISTRIBUTION BY USING HBIM.

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

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CERTIFICATE

This is to certify that the thesis entitled "**Drilling in Bone: Modeling Heat Generation & Temperature Distribution using HBIM**" submitted by **Inamdar Anand S**. *(Roll No.* 10503056*)* in partial fulfillment of the requirements for the award of ***Bachelor of Technology*** in the department of Mechanical Engineering, National Institute of Technology, Rourkela is an authentic work carried out under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to elsewhere for the award of any degree.

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ABSTRACT

Drilling of bone is an essential part of internal fixation in orthopedic surgery. In case of fracture of human bones, the best way to better and faster knitting is when it is fixed by drilling and setting the immobilization plates by screws. Because of the drilling process, the surrounding bone tissue is heated and if temperature around the drilled bone hole exceeds the critical limit, this may result in thermal necrosis. This work studies and models heat generation and temperature distribution for the prediction of thermal necrosis and the relationships between different parameters of drilling for the optimization of the drilling process in bone.

Considering the vast no. of variables involved in the process it is almost impossible to conclude the results precisely, but taking into account past researches a primary generalization of the relationship between parameters is presented.

CONTENTS:

Nomenclature:

T - Temperature

t - Time

α - Rake angle, constant

r - Distance in radial direction

θ - Temperature, Rake angle

x,z – Depth of cut

ρ - Density

c - Specific Heat

η - Fraction of drilling heat entering the bone

As - Shear plane area

τs - ultimate shear stress

γAB - shear rate in the shear plane

Vs - shear velocity

HBIM :

U - temperature

S - H

β - Stefan constant

V - approximant

a,b - HBIM parameters

List of Figures:

List of Tables:

CHAPTER 1

INTRODUCTION

The problem of bone fracture in medicine has existed from the times when humans started to treat other people and animals. In case of fracture, it is most important to return the fractured parts into their initial position and to fixate them in a best possible way.

In a recent method the traumatologist drills the bone around the fracture site with classical drills in order to set the immobilisation screws and plates and perform bone fixation. When drilling, the traumatologist has to act by pressure force on the drilling tool in order to insure uniform penetration of the drill through bone. This may result in a temperature increase caused by the plastic deformation of chips and friction between the drilling tool and the bone. The problem in bone drilling can sometimes be the occurrence of bone necrosis, which means the irreversible death of bone cells in the vicinity of hole due to the drilling temperature raised over the critical value of 50° C. Thus, this method is effective only if bone necrosis is avoided.

The severity of the necrosis problem motivates the work in the present thesis. Although the problem has been investigated many times through experiments contradictory results have been concluded. For example, take the parameter of drilling speed which certainly affects the temperature during drilling. But, the effect it has on the temperature is not agreed upon. Some say, low speed drilling would produce optimum results while others suggest high drilling speeds.

An alternate approach to performing physical experiments is to evaluate the temperature distribution surrounding the drill site with a mathematical model. The accuracy of such a model depends on the reliability of input parameters, including the thermal properties. Although most of the relevant physical properties for bone are well established, a search of the literature reveals that the thermal conductivity of cortical bone has been measured only a few times and has produced conflicting outcomes. So value with maximum agreements will be used for conductivity. Although it is well known that properties of bone are anistropic, for this work it is assumed to be isotropic with significant accuracy.

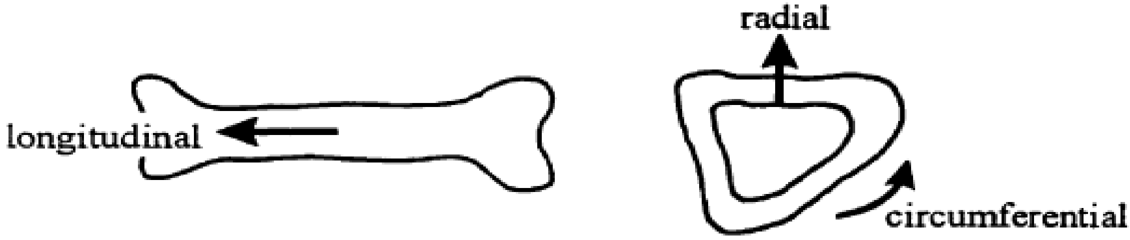

Figure 1.1: Anatomical directions of a long bone. [1]

Many problems are encountered when drilling bone in orthopaedic and trauma surgery such as hole accuracy, drill wander and heat generation. Also, other unpredictable situations can occur due to the non-homogeneous structure of the bone material itself.

Many different drill-bit designs and geometries have been suggested over the years, each with its own claim to success but most of them are based on conventional twist-drill geometry for the drilling of metals. The drills used are: twist-drills, guide wires, and large diameter drills/reamers.

As the application of the knowledge from technical sciences, e.g. mechanical engineering, to medicine is growing daily, the interconnection between the two scientific disciplines is stronger. With the development of mechanical engineering technologies, we believe that we can help to solve a lot of problems in medicine. Thus, the study of the drilling parameters and the dependence of the axial drilling force and the bone drilling temperature on these parameters can significantly contribute to the reduction of the bone necrosis occurrence.

CHAPTER 2

LITERATURE SURVEY

2.1 Effects of drill speed, feed rate and applied force

The drilling parameters investigated most often have been the drill speed and the applied load or feed rate. The influence of these parameters has been measured in two ways: histological examination of the bone tissue, and measurement of the temperature rise at various distances from the drill site. But, temperature measurements are of greater interest in the current research.

Thompson measured the temperature rise in bone in vivo during skeletal pin insertion. He found that the temperature rise at 2.5 mm and 5.0 mm from the drill site increased with drill speed from 125 rpm to 2000 rpm. These results agree with those of his histological study, and were later confirmed by Pallan .

Four years after Thompson, Rafel published a similar study with opposite results. Rafel drilled into human cadaveric mandibles with a variety of surgical drills without irrigation and found that the maximum temperature rise occurred at the lower drill speed (10,000 rpm vs. 350,000 rpm).

The study performed by Matthews and Hirsch in 1972 was the most rigorous and expansive at the time. They measured the temperature rise in human cadaveric femora over ranges of loads (19.6 N to L 17.6 N) and drill speeds (345 rpm to 2900 rpm) using surgical twist drills. The specimens were kept moist during the operation, but no irrigation was applied at the drill site. They found that both the maximum temperature and temperature duration over 50°C decreased with increased loading. This was the first study to look at the influence of applied load. The effect of increased drill speed was less pronounced but followed a similar trend as the effect of increased force. These results contradict those Thompson achieved 14 years earlier.

Vaughn and Peyton tried to extracted molars without irrigation. They found that the temperature rise increased both with increased drill speed (from 1155 rpm to 11,300 rpm) and with increased load (from 1lb to 2 lb.). These results agree with Thompson and conflict with Mathews and Hirsch, a fact that is particularly significant given that the speed and load ranges overlap. Given that the different materials enamel, and dentin are cornmon constituents and similar patterns of organization, it is doubtful that the contradiction in these findings is due solely to differences in material behavior.

Sorenson used a calometric approach. He drilled, without irrigation, into blocks of dentin and found that the amount of heat transferred to the specimen increased with increasing load from 0.3 N to 0.5 N (30 grams to 50 grams), then decreased as the load increased. The heat versus load behavior is similar to the maximum temperature versus load behavior measured by Abouzgia and James . Although free-running speed was held constant at 250,000 rpm throughout the experiment, the rotational speed was measured during drilling and found to decrease dramatically as the load increased. This decrease in rotational speed was also noted by Abouzgia and James .

In 1982, Krause cut troughs, without irrigation, in cadaveric bovine femora using rotating burs. The feed rate and depth of cut were varied. Two rotational speeds (20,000 and 100,000 rpm) and two bur types were used. Although cutting troughs is not the same as drilling, Krause noted that the maximum temperature decreased with increasing feed rate (from 1.80 to 6.35 d s) . The effect of increasing the rotational speed was ambiguous. The temperature dropped significantly at 10000 rpm for one bur type, but not for the other- Forces in the cutting direction were measured, but no correlation with temperature rise was found. It should be noted however that a higher feed rate requires a higher applied force, and thus the results of Krause are similar to those obtained by Matthews and Hirsch-

More than a decade after Matthews and Hirsch examined temperatures during bone drilling, Matthews measured temperatures during skeletal pin insertion into human cadaveric long bones. No irrigation was used. Manual drilling, which produced rotational speeds between 60 and 120 rpm, was compared to the use of electric drills at 300 rpm and 700 rpm. The maximum temperature rise was recorded at 700 rpm, followed by hand drilling, followed by drilling at 300 rpm. Conversely, the maximum temperature duration above 50°C occurred during hand drilling, followed by drilling at 300 rpm and then 700 rpm-

The results of the temperature measurement studies are divided into two groups: those that indicate an increase in temperature with drill speed and applied load and those that indicate the opposite. Furthermore, although most histological studies were performed at ultra-high drill speeds, only Abouzgia and James measured temperatures at these speeds.

2.2 Effects of irrigation

Several investigators studied the effect of irrigation on the maximum temperature rise in bone and, not surprisingly, found significant decreases in temperature were achieved when irrigation was used. Other studies compared the effect of internal irrigation, where the coolant is fed to the tip of the drill through channels in the drill shaft, to that of external irrigation, where the coolant is applied to the surface of the drill at the point of entry. Laveile and Wedgwood compared the effectiveness of both irrigation methods under low-speed (350 rpm) and moderate force (19N), and found that internal irrigation decreased significantly the peak temperatures adjacent to the cavity.

Because irrigation has been proven to reduce the heat impact from drilling, one might be tempted to ignore other aspects of drilling technique and rely solely on irrigation to prevent thermal necrosis. There are still, however, good reasons to continue the investigation of other drilling parameters, First, internal irrigation is not always possible, especially for smaller tools. Furthermore, a histological comparison of externally and internally cooled implants, performed by Haider ,produced conflicting results. Near the surface of the bone, external cooling proved more effective. There was less damage at deeper levels when internal irrigation was used. However, Haider indicated that the reduced amount of damage could be due to better thermal properties of the cancellous bone at those levels. Second, external irrigation is not always effective, especially when the ratio of hole depth to drill diameter becomes large. Eriksson recorded temperatures as high as 96°C in clinical drilling studies where irrigation was employed and Tetsch noted that the temperature in feline jaws increased above 100°C despite the use of coolant. Third, when skeletal pins are inserted, the tight fit required leaves little or no room for irrigation fluid to flow into the hole.

2.3 Previous Investigations of the Effect of Heat Created by Drilling

Abouzgia and James conducted drilling experiments on sections of bovine femora, examining the temperature rise in the region surrounding the drill site under a range of loads (1-5 to 9.0 N) and with various rotational speeds (27,000 to 97,000 rpm, free-running). They did not use any irrigation. They found that the maximum temperature rise decreased with increasing rotational speed, while the maximum temperature rise increased with force up to 3.6 N, then decreased as the force increased above 3.6 N. They explained this behavior as the result of a balance between two effects. As the force increases, the heat generation rate also increases. Conversely, higher force results in a shorter drilling time therefore, in total, less heat is produced.

While Abouzgia and James were not the first to conduct such experiments, they made a significant contribution by measuring the drilling speed throughout the course of the operation, which had been done by only a small number of researchers before them. Abouzgia and James found that the rotational speed during the operation was lower than the free-running speed, sometimes by as much as 50% . That there could be such a dramatic decrease in the drill speed indicates that most of the work done by previous researchers, in which only free-running drill speeds are reported, needs to be re-evaluated.

In the following brief summary , it will become evident that the subject of bone drilling is a complex one, and the debate continues on how the results of such experiments should be reflected in clinical practice.

2.4 Influence of Drill geometry

Figure 2.1 and 2.2 contain schematic drawings of a typical twist drill. The geometric features of interest in the current work are the point angle (labeled 2p in fig. 2.2) , the helix angle (fig 2.1) and the drill diameter.

Many researches have taken place to decrease the temperature by modifying the standard surgical drill bit. These new drill bits had a larger point angle (118 °) than the standard drill bit and had helix angles from 34° to 36°. The temperatures produced with the new drill bits were 41% lower than those created with standard surgical drills of the same 3.2 mm diameter.The geometry of the new bits is similar to those proposed by researchers who were concerned with only mechanical aspects of drilling bit.

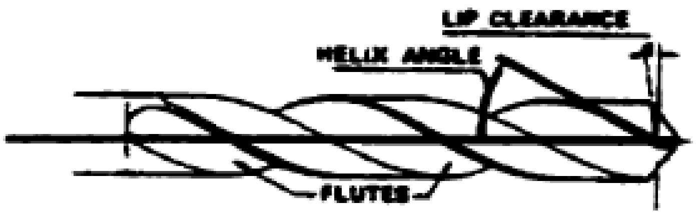

Fig 2.1: geometry of standard drill bit [1]

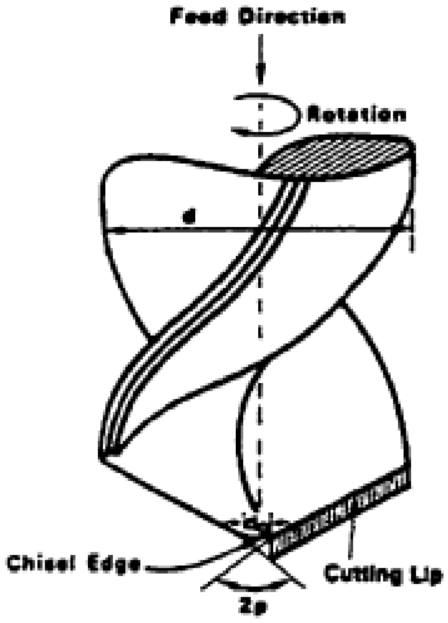

Fig 2.2: geometry of standard drill bit [1]

2.5 Influence of tool wear

Matthews and Hirsch compared the performance of dull (-200 uses) and relatively sharp (-40 uses) surgical twist drills. Not surprisingly, the worn tools produced greater temperatures than new tools.

2.6 Investigations of the Threshold for Thermal Necrosis

One of the most thorough investigations of threshold temperature was done by Moritz and Henriques . They measured the amount of time required to produce damage to the dermal and epidermal layers of both human and porcine skull over a large temperature range (44°C to 100°C). They found that as the temperature increased, the amount of time required to initiate thermal necrosis decreased, resulting in a time-temperature curve similar to that shown in Figure 2.3.

Few studies have looked at threshold temperature behavior for bone, and the data pertain to only a handful of discrete temperatures. Lundskog measured a threshold temperature of 55°C at 30 seconds exposure. Eriksson and Albrektsson established a threshold temperature of 47°C at 1 minute exposure. The results from both studies are consistent with those of Moritz and Henriques .

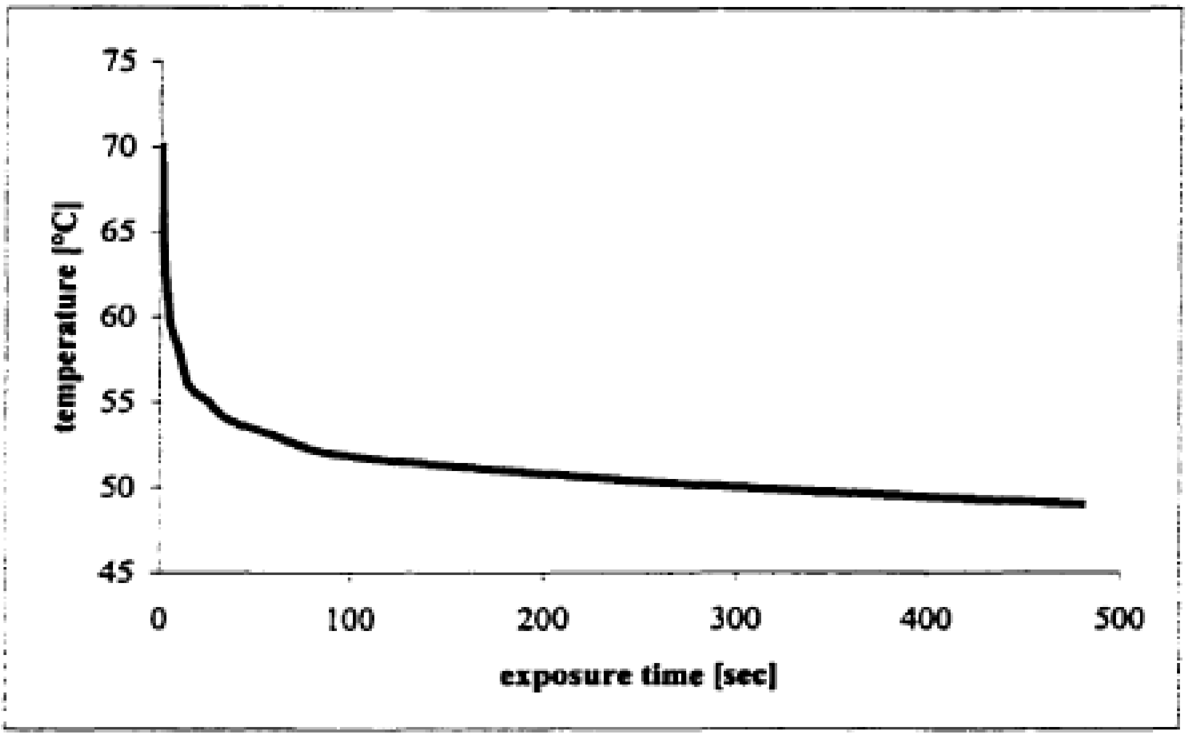

Fig 2.3 Time-temperature curve for thermal necrosis of epithelial cells [1]

2.7 Density

Huiskes summarized the results of three investigators who measured the density of cortical bone. The range was 1.86∗10^3 kg/m^3 to 2.9*10^3 kg/m^3, with an average of 2.2 *10^3 kg/m^3.

2.8 Specific Heat

In the same study, Huiskes summarized the published values for the specific heat of bone. While one researcher measured values in a range from 1.15 to 1.73 KJ/kg K , two researchers reported the same value, 1.26 KJ/kg K.

2.9 Thermal Conductivity

Table 2.1- Thermal Conductivity from different researchers [1]

Investigator(s)	Conductivity [W/mK]	Notes
Biyikli, Modest, Tarr [8]	0.2 0.3	dry samples fresh samples
Zelenov [82]	12.8 9.7 9.9	axial radial tangential†
Lundskog [46]	3.56	dry human samples
Vachon et al. [77]	0.601 2.269	dry ox fresh ox
Kirkland [39]	0.888 to 3.08	bovine and caprine specimens
Chato [33]	0.38	fresh human samples

† Conductivity was measured as a function of temperature. Values presented in the table are for T = 37°C.

2.10 Objective of current work and limitations of the previous works

Even with the limitations on the scope of the survey, the variation amongst drilling experiments is large and thus it is difficult to summarize results in a concise fashion. The main problem in interpreting the results lies in the number of variables. The range of drill speeds used by dental surgeons is different than that used by orthopedic surgeons, and it is difficult to compare results of the experiments that use different drill speeds. The confusion over the role of drill speed might be alleviated if the entire range, from very low to ultra-high, could be investigated under otherwise identical conditions. It is also difficult to compare results from experiments with an applied force to those with an applied feed rate, assuming these parameters are controlled, which occurred only in a minority of the studies examined in this chapter. Some studies used water irrigation, some used forced air, some didn't use any irrigation. There was variation in the design of the different twist drills; burs, reamers and diamond tools were also used. It is not surprising that results from these studies are not in agreement.

The picture might be made clearer if an experiment were designed which used a parametric approach. In a parametric analysis, one of the parameters- drill rotational speed, feed rate, applied force, tool design, or tool geometry - is varied while all others are held constant. The importance of that parameter is determined by measuring the change in thermal impact caused by varying that parameter.

As far as the current work is concerned it is about mathematical modeling of heat generation and temperature distribution. Methods used here may not be accurate but they are supposed to give a primary idea about relationship between certain parameters and hence a gross simplification of working conditions is implemented. The mathematics used here is significantly simplified to accommodate the scope of this work. Basics of any used complicated mathematical method are explained duly.

Chapter 3

Description of the model

The physical situation modeled in the analysis was the drilling of a hole in the mid-diaphysis of a long bone prior to pin insertion. The drill, turning at a constant number of revolutions per minute, starts at the outer surface of the bone and travels inward towards the marrow. Heat is generated where the material is removed, along the cutting edge of the drill bit. A portion of that heat is conducted into the surrounding bone, raising its temperature. Depending on the degree and the duration of the temperature rise, thermal damage may occur.

Thermal damage depends on the history of the temperature distribution around the drill site. The temperature distribution was obtained by solving the Fourier heat conduction equation (without heat generation), given in below equation [1]

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + f_q \qquad R_i \le r \le R_o$$

$$\text{B.C.}: \; T\big|_{d\Omega_D} = T(\bar{s}); \;\; \nabla T\big|_{d\Omega_N} = G(\bar{s}) \qquad \Omega: \; 0 \le \theta \le 2\pi$$

$$\text{I.C.}: \; T\big|_{t=0} = T_0(r, \theta, z) \qquad 0 \le z \le H$$

The computational domain was an annulus with an inner radius Ri, outer radius Ro and height H. The choice of cylindrical co-ordinates to define the domain followed naturally from the cylindrical shape of the drill and the defect it creates.

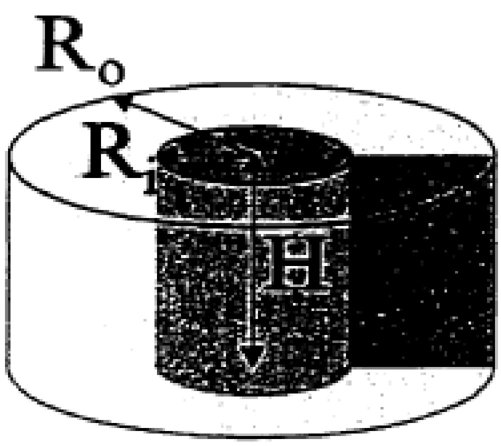

Fig 3.1: Details of the domain [1]

Figure illustrates the domain in more detail. The inner radius, Ri, represented the margin of the hole created by the drill and it is across this boundary that the heat from the drilling operation was transferred- The choice of the outer radius, Ro, will be explained later. In a mathematical model, it is possible to have an outer radius at infinity. It will soon become evident, however, that a numerical approach was required to solve the problem, and thus a finite Ro was necessary. The height of the annulus, H, was chosen to represent the thickness of the cortical bone being drilled, with z=0 representing the surface of the bone exposed during the operation, and z=H the surface of the medullary canal.

The domain represented the drill site after the hole has been drilled. In a real operation, the drill continuously removes material from the bone. A more realistic model would incorporate material removal by continuously changing the size and shape of the domain. Modeling the removal of material was, however, considered beyond the scope of the present study. The choice of domain was appropriate because it was the damage in the material which remains after the operation that was of interest.

To solve equation (3.1), the boundary conditions must be defined . The inner boundary (r = Ri) was considered insulated, with the heat input from the drill modeled as a forcing term applied to a small portion of the boundary. The heat input was evenly distributed around the circumference of the inner boundary and traveled at a constant rate from z = O to z = H. The heat input depended on the rotational speed of the drill, on the rate at which the drill penetrated the bone, and on the geometry of the drill bit. In order to calculate the heat created by the drill, relations designed for orthogonal cutting of metal were adapted for drilling in bone. Details of the forcing term are presented later. The temperature at the boundary r =Ro, was assumed to be equal to normal body temperature, i.e., 37°C.

The boundaries z = O and z = H were assumed to be insulated. The former boundary represented the outer surface of the bone. Although, in reality, there is some convective heat loss, the choice of a heat transfer coefficient would be arbitrary. Similarly, there is some conductive loss where the bone is in contact with marrow, but once again, the amount of heat loss was not known. In both cases, however, the heat loss was not expected to be large, and thus the assumption of an insulated boundary was reasonable.

Since this was a time-Dependant problem, a set of initial conditions was also required. The temperature within the domain was set to 37°C, normal body temperature, at t = 0-

A simplification of the mode1 was made by using the results of the experimental investigation into bone thermal conductivity, presented before. There, it was concluded that cortical bone could be treated as thermally isotropic. Due to the symmetry of the governing equation, the boundary conditions, and the thermal properties, only the temperature in a two-dimensional (r,z) slice of the domain R needed to be calculated to obtain temperature data for the whole volume. This 2-dimensional slice is presented in Figure 3.1 as a darkened rectangle. With this simplification, equation (3.1) reduced to an axisymmetric unsteady heat conduction problem in polar co-ordinates. Equation according to [1] is

$$k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)=\rho c\frac{\partial T}{\partial t}+f_q(z,t)$$

$$\text{B.C.:}\quad \left.\frac{\partial T}{\partial z}\right|_{z=0}=\left.\frac{\partial T}{\partial z}\right|_{z=H}=\left.\frac{\partial T}{\partial r}\right|_{r=R_i}=0,\quad T(R_o,z,t)=37,$$

$$\text{I.C.:}\quad T(r,z,0)=37$$

Chapter 4

Calculation of drilling heat

Equations from the machining theory were used to calculate the heat generated during drilling. In order to apply machining theory to bone it was assumed that bone behaves like metal when it is machined. When metal is machined, a chip is separated from the work piece through shearing. Although it is unclear from previous studies how bone chips are separated from the bone during drilling, it was decided to use machining theory for two reasons. First, no alternative theory was available for machining based on separation of chips by fracture. Second, in studies of orthogonal cutting of bone some aspects of the shear-failure based theory held true despite the fact that the predominant method of chip separation in orthogonal cutting was found to be fracture.

Machining theory applies mainly to orthogonal cutting, which is represented schematically in Fig. 4.1

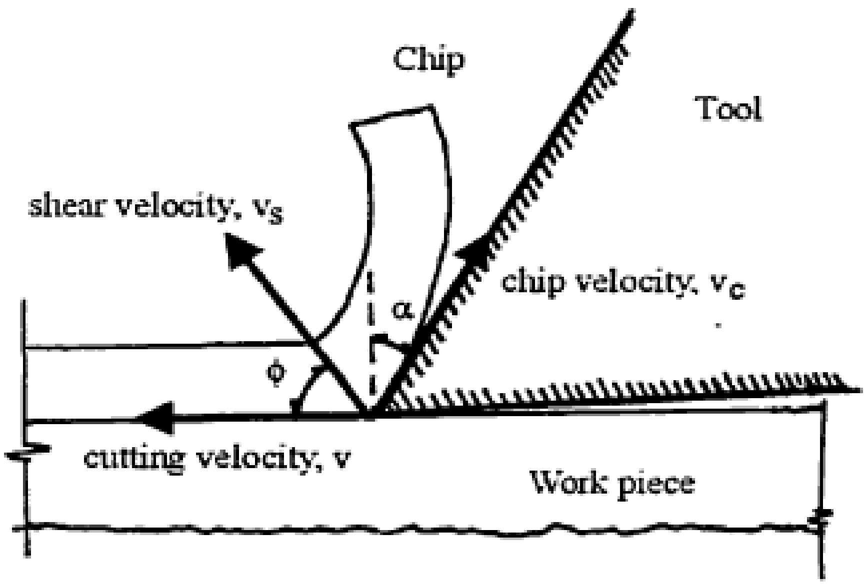

Fig 4.1 Orthogonal Cutting and Material Removal [1]

Table 4.1 Zones of heat generation [1]

Label	Zone	Remark
A	Primary Deformation Zone	Energy involved in shearing the material in this zone is converted to heat.
B	Secondary Zone (1st)	Heat generated in this zone dissipates into the chip and the tool.
C	Secondary Zone (2 nd)	Friction between tool and new surface of the workpiece. Negligible Heat produced (Use of sharp tools)
D	Tertiary Zone	No heat produced (Clearance angle)

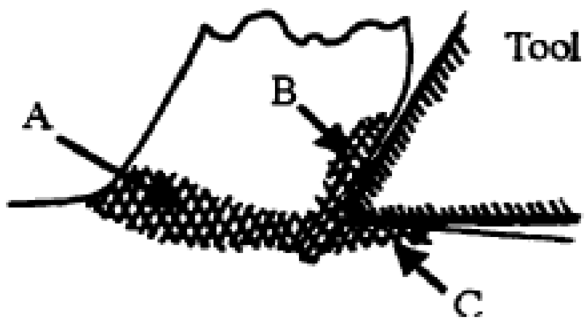

Fig 4.2 Zones of Heat generation in orthogonal cutting [1]

Using the formulas from machining theory viz. shear velocity, Ernst-Merchant relationship, shear force and from previous studies a final expression is achieved.

The final expression for the rate of heat entering the work piece is given by equation

$$Q_w = \eta A_s \tau_s (\dot{\gamma}_{AB}) v_s ,$$

where the brackets denote a functional relationship.

Chapter 5

Application of HBIM

5.1 Heat Balance Integral Method

For solving transport problems (including heat conduction) the basic heat balance integral method introduced by Goodman converts the governing partial differential equation to an ordinary differential equation by : a) assuming a suitable approximating profile, b) satisfying the available boundary conditions, c) integrating the transport equation with respect to the space variable over a suitable interval to create a heat balance integral, and d) solving the ordinary differential equation resulting from step c). The acknowledged sensitivity of method to the form of approximating profile may be alleviated by a combination of domain decomposition and simple piecewise approximants.

There are many ways to solve the governing equation including Finite Element Method or Analytical Method. But the complexity of FEM and limitations of AM force us to use HBIM which is one of many semi-analytical methods. This is analogous to the classical integral technique used for fluid flow and convective heat transfer analysis. This technique is simple yet it gives reasonable accuracy. HBIM has mostly been employed for a variety of Stefan Problems involving one-dimensional conduction.

5.2 Assumptions

Considering the scope of this work it is safely assumed that heat conduction along the radial direction i.e. along the r direction is negligible. That means all the heat generated flows along z direction. This assumption is reasonable as thermal conductivity of bone has been found to be very low (though exact value is disputed). So temperature increase along the drilled hole would be significant than in the transverse direction.

Human body temperature 37˚C is assumed to be base temperature i.e. virtual zero for HBIM calculations/ So at the end of the calculation 37 would be added for the exact value of temperature.

It is assumed that Stefan condition is applicable to this specific problem.

(STEFAN CONDITION)

It is applied to moving boundary problems. From a mathematical point of view, the phases are merely regions in which the co-efficient of underlying PDE are continuous and differentiable up to the order of the PDE. In physical problems such co-efficient represent properties of the medium for each phase. The moving boundaries are infinitesimally thin surfaces that separate adjacent phases; therefore, the co-efficient of underlying PDE and its derivatives may suffer discontinuities across interfaces.

The underlying PDE is not valid at phase change interfaces therefore an additional condition , the Stefan condition is needed for closure. The Stefan condition expresses the local velocity of a moving boundary, as a function of quantities evaluated at both sides of the phase boundary and is usually derived from a physical constraint. In problems of Heat Transfer with phase change, for example, the physical constraint is that of conservation of energy and the local velocity of the interface depends on heat flux discontinuity at surface.

5.3 Application

In this application of HBIM we will use different nomenclature so that confusing clashes of variable names are avoided.

Here U will be temperature, x will be used instead of z (depth of cut), H will be denoted by [7]:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, \quad 0 < x < s, \ \ t > 0,$$

$$\frac{\partial U}{\partial x} = -\beta \frac{\mathrm{d}s}{\mathrm{d}t}, \quad x = s(t), \ \ t \geqslant 0,$$

$$\mathbf{k}\left(\frac{\partial^2 \mathbf{T}}{\partial \mathbf{r}^2} + \frac{1}{\mathbf{r}}\frac{\partial \mathbf{T}}{\partial \mathbf{r}} + \frac{\partial^2 \mathbf{T}}{\partial \mathbf{z}^2}\right) = \rho \mathbf{c}\frac{\partial \mathbf{T}}{\partial \mathbf{t}} + \mathbf{f_q}(\mathbf{z},\mathbf{t})$$

$$\text{B.C.:} \ \left.\frac{\partial \mathbf{T}}{\partial \mathbf{z}}\right|_{z=0} = \left.\frac{\partial \mathbf{T}}{\partial \mathbf{z}}\right|_{z=H} = \left.\frac{\partial \mathbf{T}}{\partial \mathbf{r}}\right|_{r=R_i} = 0, \quad \mathbf{T}(\mathbf{R_o},\mathbf{z},\mathbf{t}) = 37,$$

$$\text{I.C.:} \ \mathbf{T}(\mathbf{r},\mathbf{z},0) = 37$$

Goodman's two-parameter choice for the temperature profile took the form $a(x - s) + b(x - s)^2$ This immediately satisfies boundary condition. However this form results in solution parameters a and b that are functions of time. In the present analysis the following form of the approximant v is used [7]:

$$v(x,t) = a\left(1 - \frac{x}{s}\right) + b\left(1 - \frac{x}{s}\right)^2.$$

The consequence of this is that a and b are now constants which simplifies the analysis. The last boundary condition is immediately satisfied and to satisfy last but one condition requires that 1=a+b, i.e. b=1-a, and hence

$$v(x,t) = a\left(1 - \frac{x}{s}\right) + (1 - a)\left(1 - \frac{x}{s}\right)^2.$$

$$\left[\frac{a+2}{6}\right]\frac{\mathrm{d}s}{\mathrm{d}t} = \left.\frac{\partial v}{\partial x}\right|_{x=s(t)} - \frac{a-2}{s}.$$

The two remaining parameters, a and s, are determined from a combination of Stefan's Condition combining with the heat balance integral equation. The final result is calculated by using Boundary Conditions in conjunction with the above equations.

Hence the value of temperature as a function of depth of cut is found out.

$$T(z) = \{-0.05549z^2 + 2.5761z + 83.4169\}$$

Chapter 6

Results and Discussion

6.1 Result

$$T(z) = \{-0.05549z^2 + 2.5761z + 83.4169\}$$

As this solution shows the general relationship between temperature and the depth of cut, the graph can be plotted. But as the method used is not accurate and gross simplifications are made in this method the graph would only show the basic relationship between the two and is not reliable for practical purpose. The simplifications are reasonable considering the scope of this work.

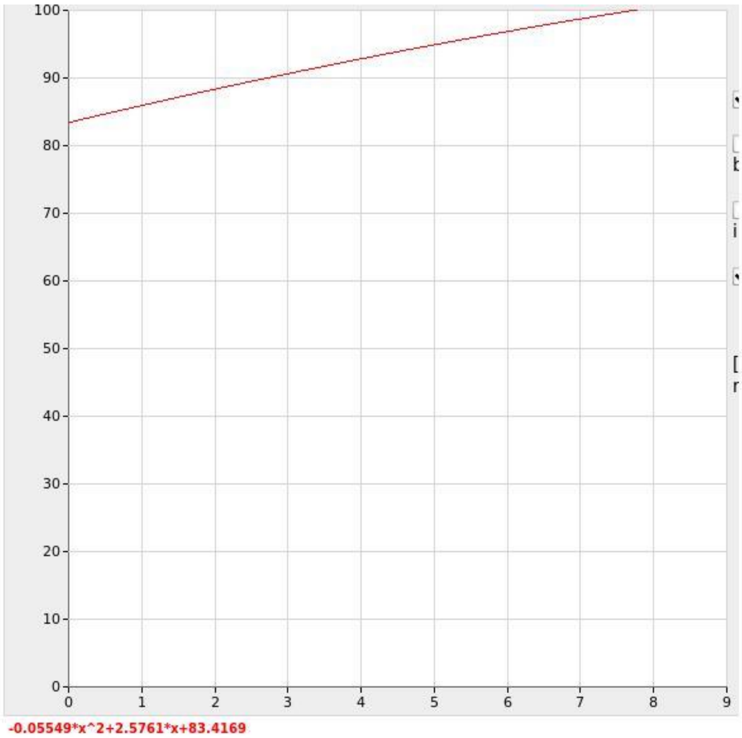

Fig 6.1 Graph between temperature and depth of cut

As can be seen from the graph as the depth of cut increases the temperature increases. The variations as compared to real life relationship are due to gross simplification of the situation.

6.2 Results from previous investigations and discussion

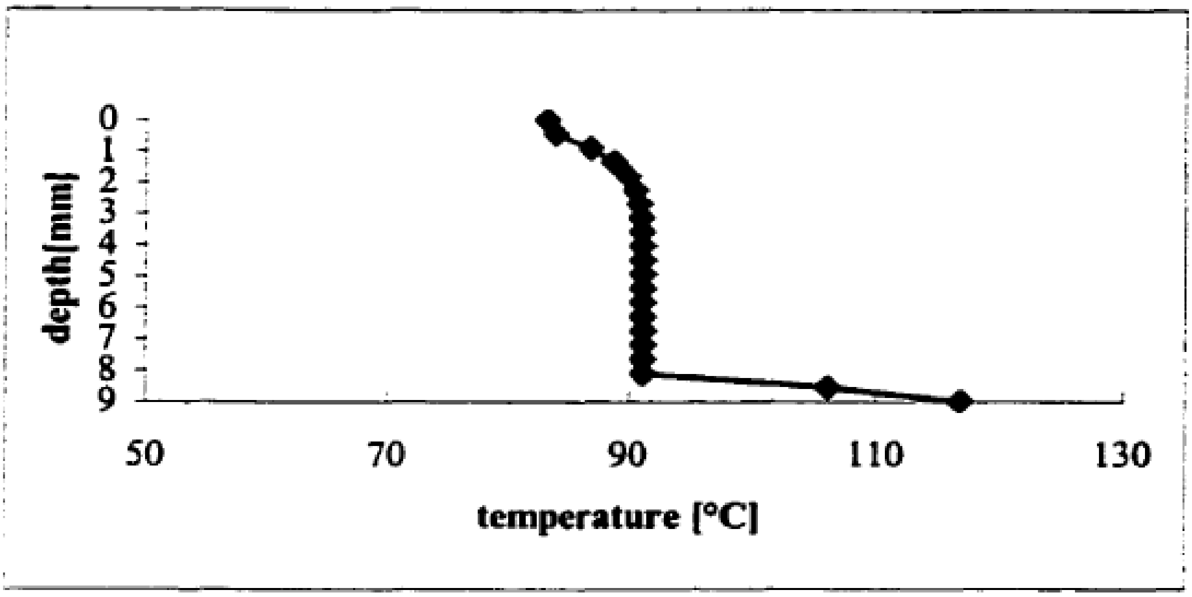

Fig 6.2 Graph between depth of cut and temperature [1]

This graph is taken from the work of Davidson and James. This tendency of graph has been pointed out by many works before this. Thus it can be safely assumed to be reasonably near to ideal conditions. As the comparison between the two graphs shows the simplified solution by HBIM to the heat conduction problem in bone drilling scenario gives a fair idea about the relationship between the two.

The sharp increase in temperature near the bottom of the hole is likely an artifact of the insulating boundary condition at z=H. In the model, the heat becomes trapped at the lower boundary, causing a rapid increase in temperature. In real bone, some heat would diffuse into the bone marrow and the rise in temperature at the interface of the bone and marrow could not be as sharp as indicated.

CONCLUSION

This model of bone drilling predicts an increase in temperature as the depth of cut increases. Compared with the other models it gives a fair idea about the relationship between the two. This model appears to be realistic at low and moderate depths of cut. Even with the shortcomings at high depths of cut, results indicate that drilling operations can be modeled numerically. The results also suggest that the theory of machining for orthogonal cutting of metals can be adapted for drilling in bone, given the proper mechanism of bone failure.

References:

1. Sean R. H. Davidson, David F. James – Drilling in bone: Modeling heat generation and temperature distribution, June 2003, ASME

2. Udiljak T; Ciglar D & Skoric S – Investigation into bone drilling & thermal bone necrosis, Feb 2007, ASME journal

3. Kent N Bachus, Matthew Rondina, Douglar Hutchinson – The effects of drilling force on cortical temperature and their duration: an in vitro study, Feb 2001, Elsevier

4. Sean R.H. Davidson, David F. James – Measurement of thermal conductivity of bovine cortical bone, Dec 2000, Elsevier

5. M.T.Hillary, I.Shuaib – Temperature effects in the drilling of human and bovine bone, Elsevier

6. S.K. Sahu, P.K. Das, S. Bhattacharya – Rewetting analysis of hot surfaces with internal heat source by Heat Balance Integral Method, Dec 2007, Springer

7. A.S. Wood – A new look at the heat balance integral method, Dec 2000, Elsevier

8. Brage Walber F, Mantelli M.B.H., Azavedo Joalo L.F. – A new approach for the heat balance integral method applied to heat conduction problems

9. Eriksson, A. R., and Albrektsson, T., 1983, "Temperature Threshold Levels for Heat-Induced Bone Tissue Injury: A Vital-Microscopic Study in the Rabbit”.