SLOPE STABILITY ANALYSIS USING GENETIC ALGORITHM

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

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Civil Engineering

By

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CERTIFICATE

This is to certify that the thesis entitled **"SLOPE STABILTY ANALYSIS USING GENETIC ALGORITHM"** submitted by Sri Anurag Mohanty, Roll No. 10501005 in partial fulfillment of the requirements for the award of Bachelor of Technology degree in Civil Engineering at the National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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Date: **(Anurag Mohanty)**

Abstract

Analysis of stability of slopes is of utmost importance as its failure may lead to loss of lives and great economic losses. Failure of a mass located below the slope is called a slide. It involves downward and outward movement of entire mass of soil that participates in failure. Slides may occur in almost nay conceivable manner slowly or suddenly, with or without apparent provocation.

In the present day lots of methods are available to the modern engineer to obtain the stability of slopes. Some are quite rigorous, while some are expensive.

In this project a comparative of study of such methods has been done with special stress on the application of GA in the analysis of slope stability.

CONTENTS

CHAPTER 1

WHAT IS SLOPE?

INTRODUCTION

A slope may be an unsupported or supported, inclined surface of some mass like soil mass. Slopes can be natural or man made. These may be above ground level as embankments or below ground level as cuttings.

SLOPE STABILITY

In naturally occurring slopes like along hill slopes and river sides, the forces of gravity tends to move soil from high levels to low levels and the forces that resist this action are on account of the shear strength of the soil.

- Presence of water increases weight and reduces shear strength and hence decreases stability.
- Weights of man made structures constructed on or near slopes tend to increase the destabilizing forces and slope instability.

Causes of failure of Slopes:

The important factors that cause instability in slope and lead to failure are

1. Gravitational force.
2. Force due to seepage of water.
3. Erosion of the surface of slope due to flowing water.
4. The sudden lowering of water adjacent to the slope.
5. Forces due to earthquakes.

TYPES OF SLOPE FAILURES

1. Rotational Failure

This type failure occurs by rotation along a slip surface by downward and outward movement of the soil mass.

- **Slope circle failure:**

In this case the failure circle intercepts the surface of the slope itself above the toe.

- **Toe circle failure:**

In this case the failure circle passes through the toe of the slope. This occurs in steep slopes of homogenous soils.

- **Base circle failure:**

In this case the failure circle passes below the toe at a depth n_dH from top of the slope of height H. Such cases occur when slopes are flat with weak soil and a steep stratum occurs below the toe.

Translational Failure

Translational failure occurs in an infinite slope along a long failure surface parallel to the slope. The shape of the failure surface is influenced by the presence of any hard stratum at a shallow depth below the slope surface. These failures may also occur along slopes of layered materials.

Compound Failure

A compound failure is a combination of the rotational slips and the translational slips. A compound failure surface is curved at the two ends plane in the middle portion. A compound failure generally occurs when a hard stratum exists at considerable depth below the toe.

Wedge Failure

A failure along an inclined plane is known as plane failure or wedge failure or block failure. This failure may occur both in infinite and finite slope consisting of two different materials or in a homogeneous slope having cracks, fissures, joints or any other specific plane of weakness.

Miscellaneous Failure

In addition to above four types of failures, some complex type of failures in the form of spreads and flows may also occur.

Various types of failures are shown in **FIG 1.1, 1.2** and **1.3.**

Failure of a road embankment

Failure of a hill slope

Landslide

In determining the stability of slope, first a potential failure surface is assumed and the shearing resistance mobilized along the surface is determined. This is the force that resists the movement of soil along the assumed failure surface and is known as **resisting force**. The forces acting on the segment of the soil bounded by the failure surface and the ground level are also determined and these forces attempt to move the soil segment along the failure surface. This is known as the **activating force** .The factor of safety of the segment is as follows

$$\textbf{Factor of safety for rotation} = \frac{\textbf{Moment of the resisting force}}{\textbf{Moment of the activating force}}$$

$$\textbf{Factor of safety for translation} = \frac{\textbf{Resisting force}}{\textbf{Activating force}}$$

SLOPES ARE USED FOR:-

- Railway formations
- Highway embankments
- Earth dams
- Canal banks
- River training works
- Levees

CHAPTER 2

ANALYSIS OF SLOPE

METHODS OF ANALYSIS

The analysis of stability of soil consists of two parts:

- The determination of the most severely stressed internal surface and the magnitude of the shearing stress to which it is subjected.
- The determination of the shearing strength along this surface.

General Consideration and Assumptions in the Analysis

The general assumptions in the analysis are:

1. The stress system is assumed to be two-dimensional. The stresses in direction which is perpendicular to the section of soil mass are taken as zero.
2. It is assumed that the coulomb equation for shear strength is applicable and the strength parameters c and ø are known.
3. It is assumed that the seepage conditions and water levels are known, and the corresponding pore water pressure can be estimated.
4. The conditions of plastic failure are assumed to be satisfied along the critical surface. In other words, the shearing strains at all points of the critical surface are large enough to mobilize all the available strength.
5. Depending upon the method of analysis, some additional are assumptions are made regarding the magnitude and distribution of forces along various planes.

METHODS OF ANALYSIS OF STABILITY OF SLOPE

The following methods are used for the analysis of stability of slopes.

1. Fellenius method
2. Swedish slip circle method
3. Bishop's method
4. Janbu's method
5. Friction circle method
6. Taylor's stability number method
7. Culmann's method
8. Spencer's method
9. Morgenstern and price method
10. Bell's method

Fellenius Method (The Ordinary Method of Slices)

The Ordinary Method of Slices (OMS) was developed by Fellenius (1936) and is sometimes referred to as "Fellenius Method." This method is applicable to soil slopes with both friction and cohesion. In this method, the forces on the sides of the slice are neglected. The normal force on the base of the slice is calculated by summing forces in a direction perpendicular to the bottom of the slice. Once the normal force is calculated, moments are summed about the center of the circle to compute the factor of safety.

$$\textbf{Factor of safety} = \frac{\sum [c' \Delta\ell + (W \cos\alpha - u\Delta\ell \cos^2 \alpha) \tan\acute{\phi}]}{\sum W \sin \alpha}$$

Where

c' and ǿ = shear strength parameters for the center of the base of the slice

W = weight of the slice

α = inclination of the bottom of the slice

u = pore water pressure at the center of the base of the slice

$\Delta\ell$ = length of the bottom of the slice

Swedish Slip Circle Method

This method is also known as method of slices. This method was proposed by Petterson . It assumes a circular surface of failure and that the resistance is the total cohesion developed along the circle of failure. This method is applicable to purely cohesive soil and soil possessing both cohesion and friction.

- Purely cohesive soil ($\O_u = 0$)

Factor of safety = $(c_u L_a r) / Wx$

- Soil possessing both cohesion and friction (c – Ø analysis)

Factor of safety = $(c \sum \Delta L + \tan \O \sum N) / \sum T$

Where

c_u = unit cohesion

L_a = Length of the slip arc

r = Radius of the slip circle

W = Weight of the soil of the wedge

x = Distance of line of action of W from vertical line passing through the centre of rotation

$\sum T$ = algebraic sum of all tangential components

$\sum N$ = sum of all normal component

$\sum \Delta L$ = length of slip circle

Bishop's Method

Bishop (1955) took into consideration the forces acting on the sides of the slices, which were neglected in the Swedish method. The slip surface is assumed to be an arc of a circle and the factor of safety against sliding is defined as the ratio of the actual shear strength of soil to that required to maintain limiting equilibrium (i.e. mobilized shear strength)

Factor of safety = (τ_f / τ)

Where

τ_f = shear strength

τ = mobilized shear

TOOLS AND PACKAGES AVAILABLE

The following software tools and packages are available for analysis of stability of slopes:

SLOPE/W

SLOPE/W is a software product that uses limit equilibrium theory to compute the factor of safety of earth and rock slopes. The comprehensive formulation of SLOPE/W makes it possible to easily analyze both simple and complex slope stability problems using a variety of methods to calculate the factor of safety. SLOPE/W has application in the analysis and design for geotechnical, civil, and mining engineering projects.

GALENA

GALENA is a powerful and easy to use slope stability analysis system developed for engineers to solve geotechnical problems which offers clear graphical images for a clear understanding of the situation being modeled. GALENA's unique features are designed to provide users with the tools needed to take much of the guesswork out of the natural variability of geological materials.GALENA provides the ability to use the Mohr-Coulomb and Hoek-Brown material strength criteria, and shear/normal stress relationships, for assessing stability of both soil and rock slopes. GALENA can also calculate and use increasing cohesion with depth according to Skempton's relationship for cohesive soils. GALENA has been adopted by the US Government's Office of Surface Mining.

GEO5

GEO5 has a completely revised results presentation system. As in GEO4, the program builds on the generation of static protocol in tree like forms for selecting individual options – the novelty of the system is the option to incorporate graphical results directly into the protocol in a rather simple way. In each regime, input or analysis, the program allows for the addition of one or more graphical results (figures) directly into the list of figures. Each figure can be further edited, zoomed, changed, or modified in terms of color. The figures are automatically included in the static protocol. An arbitrary modification in input prompts an automatic regeneration of figures, so the user does not have to keep track of them. Thus the result is a lucid and comfortable output, which is continuously updated.

SAGE CRISP for Win95

SAGE CRISP for Win95 comprises Pre- and Post-Processing Graphical User Interfaces (GUIs), the finite element analysis program and a dedicated spreadsheet utility for printing data. SAGE CRISP combines the impressive analysis capabilities of CRISP with a modern, user-friendly graphical interface. CRISP has been extensively used many geotechnical problems, including retaining structures, embankments, tunnels and foundations. It has also been used in the analysis of footings, pile foundations, geotextile reinforcement, soil nailing, effect of anisotropy, **slope stability**, borehole stability and construction sequence studies.

OASYS

OASYS Geotechnical Application, i.e. OASYS GEO 18.1,

comprises the following programs:

1. **FREW** - Analysis of the soil structure interaction behavior of flexible retaining walls.
2. **STAWAL** - Sheet pile and diaphragm wall program
3. **SLOPE** - Two dimensional slope stability analysis.
4. **SAFE** - Two dimensional finite element computations.
5. **SEEP** - **2-D** finite element program for analyzing steady state flow of groundwater.

6. Other programs are **GRETA, PILE, TUNSET** etc.

Other popular software in slope stability analysis

	Analysis Method	Optimization tool	Reference
STABR	Bishop’s	Pattern search	Lefebvre(1971)
SSTAB	Spencer	Grid	Wright(1974)
SLOPE	General procedure of slice	Grid	Fredlund et al.(1980)
STABL	Bishop’s	Grid & Random	Seigel (1975)
STABL4	Janbu’s	Grid & Random	Lovell et al.(1984)
STABL5	Spenecer’s	Grid & Random	Carpenter (1986)
GEOMIN	Bishop’s simplified	Nealder-Mead	De Natale(1991)
GEO4	Bishops’s simplified and Sarma’s method	****	

TOOL ADOPTED

Slope program of **OASYS GEO 18.1** has been designed primarily to analyse the stability of slopes, with an option to include soil reinforcement. It can also be used to analyse earth pressure and bearing capacity problems. The program can check circular and non-circular failures, thereby allowing calculations to be carried out for both soil and rock slopes.

The main features of **Slope** are summarised below:

Slope provides the following methods of analysis:

- Swedish circle (Fellenius) method
- Bishop's methods
- Janbu's methods

- The use of these methods allows analysis of both circular and noncircular slip surfaces to be carried out. The location of **circular surfaces** is defined using a rectangular grid of centres and then a number of different radii, a common point through which all circles must pass or a tangential surface which the circle almost touches. **Non-circular** slip surfaces are defined individually as a series of x and y coordinates.

- The ground section is built up by specifying each layer of material, from the surface downwards, as a series of x and y co-ordinates.
- The strength of the materials is represented by specifying cohesion and an angle of shearing resistance. Linear variations of cohesion with depth can also be entered.
- The ground water profile and pore water pressure distribution can be set individually for each soil stratum, using either:
    - A phreatic surface with hydrostatic pore pressure distribution.
    - A phreatic surface with a user-defined "piezometric" pore pressure distribution.
    - An overall value of the pore pressure coefficient Ru.
    - A maximum soil suction can also be specified for each stratum.
- Any combination of reinforcement, consisting of horizontal geotextiles or inclined soil nails, rock bolts or ground anchors, can be specified. The restoring moment contributed by the reinforcement is calculated according to BS8006: 1995.
- Slope which are submerged or partially submerged can be analysed.
- External forces can be applied to the ground surface to represents buildings loads or strut forces in excavations.
- Horizontal acceleration of the slip mass can be included to represent earthquake loading.
- The calculated factor of safety can be applied to:

Soil strength or the magnitude of the applied loads, either

a) causing failure – to represent bearing capacity problems, or

b) preventing failure – for anchor forces.

The methods of analysis available in **Slope** are as follows:

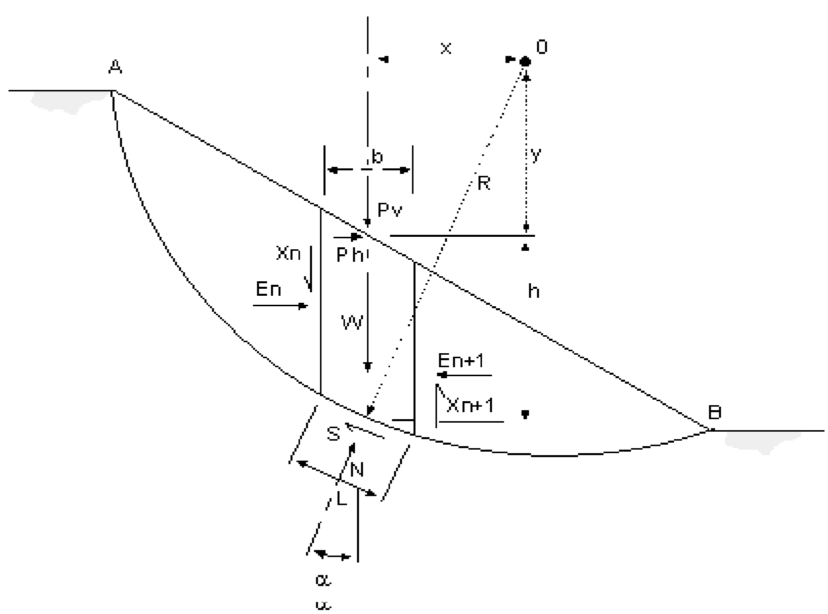
- Swedish circle (Fellenius) method.

- Bishop's methods:
    a) Horizontal interslice forces.
    b) Parallel inclined intersliced forces (spencer's method).
    c) Variably inclined intersliced forces
- Janbu methods:
    a) Horizontal interslice forces
    b) Parallel inclined intersliced forces (spencer's method).
    c) Variably inclined intersliced forces

All these methods of analysis use the method of slices to determine the factors of safety for slope stability.

THEORY OF SLICES

The following provides details of the basic annotation and sign convention for the method of slices,



All forces are given as total forces
(i.e. including water pressure)

F - Factor of Safety

Ph - Horizontal component of external load

Pv - Vertical component of external loads

E - Horizontal Interslice Force

X - Vertical Interslice Force

W - Total weight of soil = γbh

N - Total normal force acting along slice base

R - Distance from slice base to moment centre

S - Shear force acting along slice base

h - Mean height of slice

b - Width of slice

L - Slice base length = $b/\cos\alpha$

u - Pore pressure at slice base

α - Slice base angle to horizontal x - Horizontal distance of slice from moment centre

y - Vertical distance of slice surface from moment centre

γ - Unit weight of soil

c - Cohesion at base

φ - Angle of friction at base

The general expression to calculate the average overall factor of safety for a **circular slip circle** is:

$$\mathbf{F = (\sum S.R) / \sum[(W + P_v)x + P_h.y]}$$

$$\mathbf{= (Restoring\ moment) / (Disturbing\ moment).}$$

Where,

$$S = cL + (N - uL) \tan \varphi,$$

$$N = (W + P_v + X_n - X_{n+1}) \cos \alpha - (E_n - E_{n-1} + P_h) \sin \alpha$$

As the factor of safety (F) is directly related to c and tan φ, it is a factor of safety on material shear strength.

For models which include soil reinforcement, the additional restoring moment contributed by the reinforcement is added to the soil strength restoring moment.

In addition other expressions for equilibrium are as follows:

For vertical equilibrium:

$$N \cos \alpha = W + P_v + (X_n - X_{n+1}) - (S \sin \alpha) / F$$

For horizontal equilibrium:

$$N \sin \alpha = (E_{n+1} - E_n) – P_h + (S \cos \alpha) / F$$

For **non-circular slip circles** the equations for moment equilibrium change to:

$$\sum S\{(h + y)\cos \alpha + x \sin \alpha\} = \textbf{Restoring Moment}$$

$$\sum\{(W + P_v – N\cos \alpha)x + (P_h + N \sin \alpha)y\} = \textbf{Disturbing Moment}$$

METHOD OF ITERATION

Slope uses iteration to reach convergence for each of the Bishop and Janbu methods as follows:

Factors of safety:

For each iteration i, **Slope** calculates a new factor of safety F_i using the ratio of restoring moment to disturbing moment (which is a function of F_{i-1}). when the difference between F_i and F_{i-1} is within the specified tolerance, the calculation is complete. The factor of safety, F, is the ratio of restoring moment to disturbing moment. However, this ratio is itself a function of F, (except in the Swedish circle method) so an iterative solution is necessary.

Horizontal interslice forces:

- **Slope** starts at slice 1 (Slices are numbered from left to right) and, by maintaining vertical equilibrium it calculates the resultant horizontal force.

- The program then uses this as the interslice force with slice 2. The process continues until the last slice which ends up with a resultant horizontal force.

In this method each slice and the slope as a whole is in vertical equilibrium, with zero vertical interslice forces. Horizontal equilibrium is not achieved within each slice or the slope as a whole. Therefore the only force check within each slice is for vertical equilibrium.

Constant inclined interslice forces:

In this method **Slope** varies the ratio (which is constant), between the vertical and horizontal interslice forces, until the resultant of each is reduced to zero. For this method each slice is not in equilibrium, only the slope as a whole. In the calculation equilibrium is effectively maintained for each slice in the direction normal to the interslice forces.

Variably inclined interslice forces:

The variably inclined method is superior as it keeps every slice in horizontal and vertical equilibrium at all times. However, it can exceed the soil strength along the slice interface as it does not check the vertical interslice forces against the shear strength of the material. The results should therefore be checked for this criterion.

The interslice force is adjusted separately, for both the vertical and horizontal direction, by adding the fraction of the residual values from the previous iteration. The fraction is determined by the horizontal length of the slip surface represented by that slice. The interslice force direction can vary by this method, but each slice is in equilibrium at all times as is the slope as a whole

POSITIONING OF SLICES

Slope divides each slip mass into a number of slices. The resulting slice boundaries are located at the following points:

- at the left and right hand extent of the slip surface.
- at the change in gradient of a stratum.
- at each slip surface/stratum intersection
- at each slip surface/phreatic surface intersection
- at the mid point of a slice whose width is greater than the average slice width given by:

$(X_{right} - X_{left})$ / Minimum number of slices

METHOD ADOPTED

BISHOP'S METHODS

Bishop's methods (Bishop AW, 1955) are applicable to **circular** slip surfaces. One of the Bishop methods must be used if reinforcement is specified.

Three methods of solution are available. These are:

a) Horizontal Interslice Forces

b) Parallel Interslice Forces

c) Variably Inclined Interslice Forces

BISHOP'S SIMPLIFIED METHOD - HORIZONTAL INTERSLICE FORCES

This method is applicable to all **circular** slip surfaces.

Assumptions:

1. The interslice **shear** forces are assumed to sum to zero. This satisfies vertical equilibrium, but not horizontal equilibrium,

Where,

$$\{X_n - X_{n+1}\} = 0$$

This leads to errors in the calculated factors of safety, but these are usually small and on the safe side.

2. The method satisfies overall moment equilibrium.

BISHOP'S METHOD - PARALLEL INCLINED INTERSLICE FORCES

This method (also known as **Spencer's Method**) is applicable to **circular** slip surfaces. It is a refinement of Bishop's Simplified Method and satisfies conditions of horizontal, vertical and moment equilibrium for the slip as a whole.

Assumptions:

1. The program assumes that all the interslice forces are parallel, but not necessarily horizontal, i.e. at a constant inclination throughout the slope. Where,

$$\tan\theta = X_n / E_n = X_{n+1} / E_{n+1}$$

θ = angle of resultant of the interslice forces from the horizontal.

2. This satisfies the condition of overall horizontal and vertical equilibrium.

3. The method also satisfies overall moment equilibrium.

The differences between the two methods increase with slope angle. For steep slopes Spencer's method is more accurate and is therefore recommended. This method can have problems of interlock. If it is suspected that this may be a problem the method of variably inclined interslice forces should be used.

BISHOP'S METHOD- VARIABLY INCLINED INTERSLICE FORCES:

This method is applicable to **circular** slip surfaces. It is a further refinement of Bishop's method designed to over-come the problems of interlock.

Assumptions:

1. In this method the program calculates the interslice forces to maintain horizontal and vertical equilibrium **of each slice** .
2. The inclinations of the interslice forces are then varied in each iteration until overall horizontal, vertical and moment equilibrium is also achieved.

Analysis By Slope

Bishop's Method has been adopted for analyzing the stability of slope with variably inclined interslice forces to calculate the factor of safety on shear strength.

INPUT DATA

1. **UNITS AND PREFERENCES**

2. **GENERAL PARAMETERS**
3. **ANALYSIS OPTIONS**
4. **METHOD PARTIAL FACTORS**
5. **TITLES**
6. **MATERIALS**
7. **GROUND WATER**
8. **STRATA**
9. **SLIP SURFACES**
10. **REINFORCEMENT**

GENERAL PARAMETERS

Direction of slip: DOWNHILL

Minimum slip weight [kN/m] : 100

Type of analysis :

- STATIC
- PSEUDOSTATIC (g % = 10%,20%,30%)

Where, g = Horizontal acceleration

ANALYSIS OPTIONS

Factor of safety on : SHEAR STRENGTH

Minimum number of slices : 10

Method: Bishop (Variably inclined interslice forces)

Maximum number of iterations: 100

Reinforcement: NOT ACTIVE

ACTIVE

METHOD PARTIAL FACTORS

Current selection: SLS

Factor on DEAD LOAD: 1.0

Factor on LIVE LOAD: 1.0

Factor on SOIL UNIT WEIGHT: 1.0

Factor on DRAINED SOIL COHESION: 1.0

Factor on UNDRAINED SOIL COHESION: 1.0

Factor on SOIL FRICTION ANGLE: 1.0

Moment correction factor: 1.00

Factor on reinforcement pullout: 1.00

Economic ramification of failure: 1.00

Sliding along reinforcement: 1.00

MATERIAL PROPERTIES

No	Description	Unit Weight		Shear Strength Parameters		
		Above GWL [kN/m3]	Below GWL [kN/m3]	Condition	Phi or Phi0 [°]	c or c0' [kN/m²]
1	made ground	17.90	17.90	Drained - linear	19.00	30.00
2	Sand	18	18	Drained - linear	30	0.00
3	Clay	21	21	Undrained	0	75
4	Sand Lens	18	18	Drained - linear	30	0

No.	STRATUM	DEPTH OF STRATUM (m)
1.	Made ground	4.1
2.	Clay	12.7

SLIP SURFACE SPECIFICATION

Circle centre specification: GRID

Bottom left of grid : x = -15.00 m

y = 10.00 m

Centres on grid : 10 in x direction at 1.00m spacing

10 in y direction at 1.00m spacing

Grid extended to find minimum F.O.S

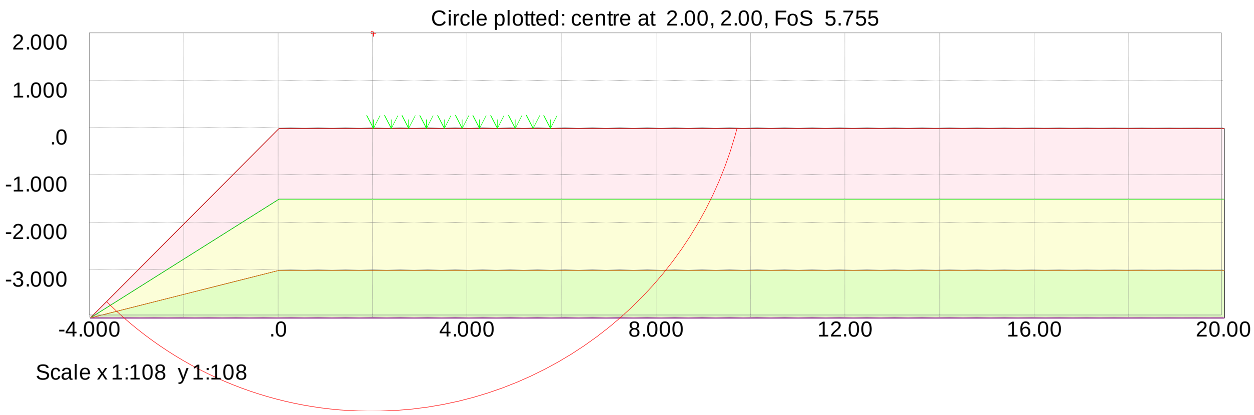
Initial radius of circle 9.00 m

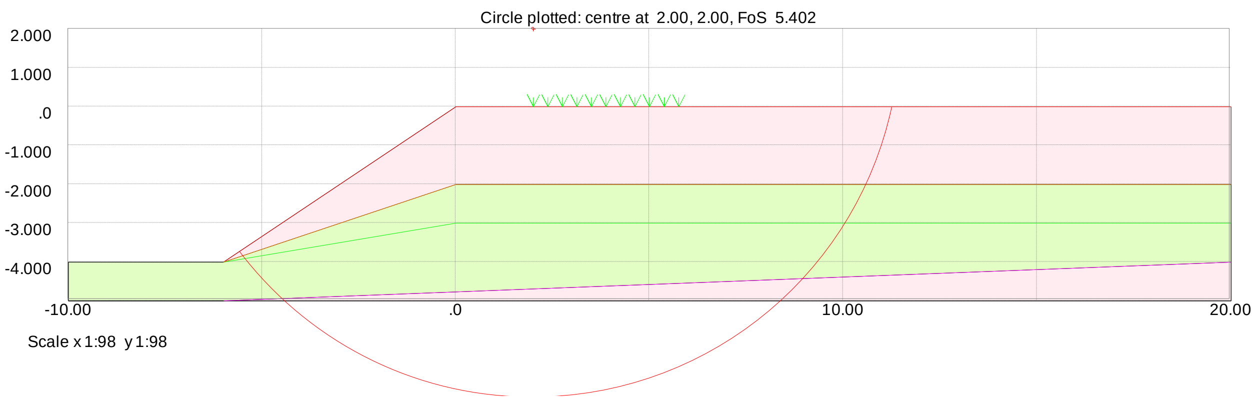
Incremented by 1.00 m until all possible circles considered

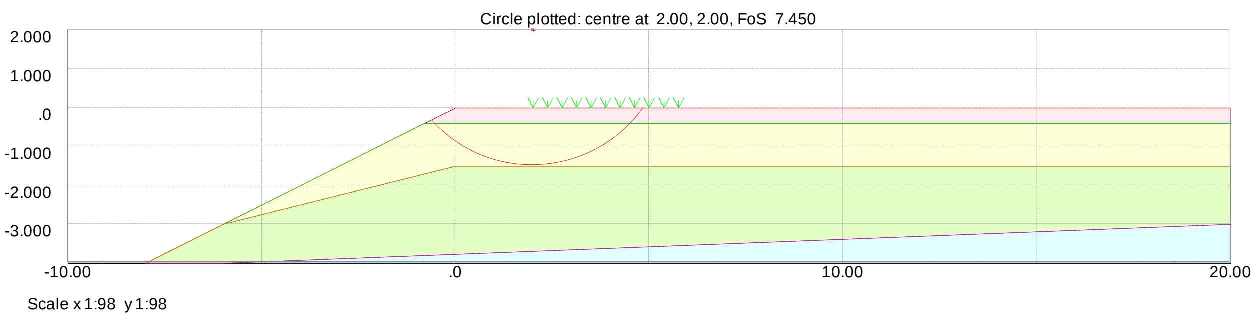
REINFORCEMENT

No reinforcement was used.

RESULTS OF ANALYSIS







CHAPTER 3
GENETIC ALGORITHM

GENETIC ALGORITHM

The GA is a random search algorithm based on the concept of natural selection inherent in natural genetics, presents a robust method for search for the optimum solution to the complex problems. Genetic algorithms are typically implemented as a computer simulation, in which a population of abstract representations (called
Chromosomes) of candidate solutions (called individuals) to an optimization problem, evolves toward better solutions

The algorithms are mathematically simple yet powerful in their search for improvement after each generation (Goldberg, 1989). The artificial survival of better solution in GA search technique is achieved with genetic operators: selection, crossover and mutation, borrowed from natural genetics. The major difference between GA and the other classical optimization search techniques is that the GA works with a population of possible solutions; whereas the classical optimization techniques (Linear Programming, Integer Programming) work with a single solution. Another difference is that the GA uses probabilistic transition rules instead of deterministic rules.

The GA that employs binary strings to represent the variables (chromosomes) is called *binary-coded* GA. The binary-coded GA consists of three basic operators, selection, crossover or mating, and mutation, which are discussed as follow. In the selection procedure, the chromosomes compete for survival in a tournament selection, where the chromosomes with high fitness values enter the mating population and the remaining ones die off. The selection probability (P_s) determines the number of chromosomes to take part in tournament selection process. The selected chromosomes form an intermediate population known as the mating population, on which crossover and mutation operator is applied. The selected chromosomes are randomly assigned a mating partner from within the mating population. Then, a random crossover location is selected in any two parent chromosomes and the genetic information is exchanged between the two mating parent chromosomes with a certain mating probability (P_c), giving birth to a child (new variable) or the next generation. In binary-coded GA, mutation is achieved by replacing 0 with 1 or vice versa in the binary strings, with a probability of P_m. This process of selection, crossover, and mutation is repeated for many generations (iterations) with the objective of reaching the global optimal solution. The flow chart of the general solution procedure of GA is depicted in Fig. 2.

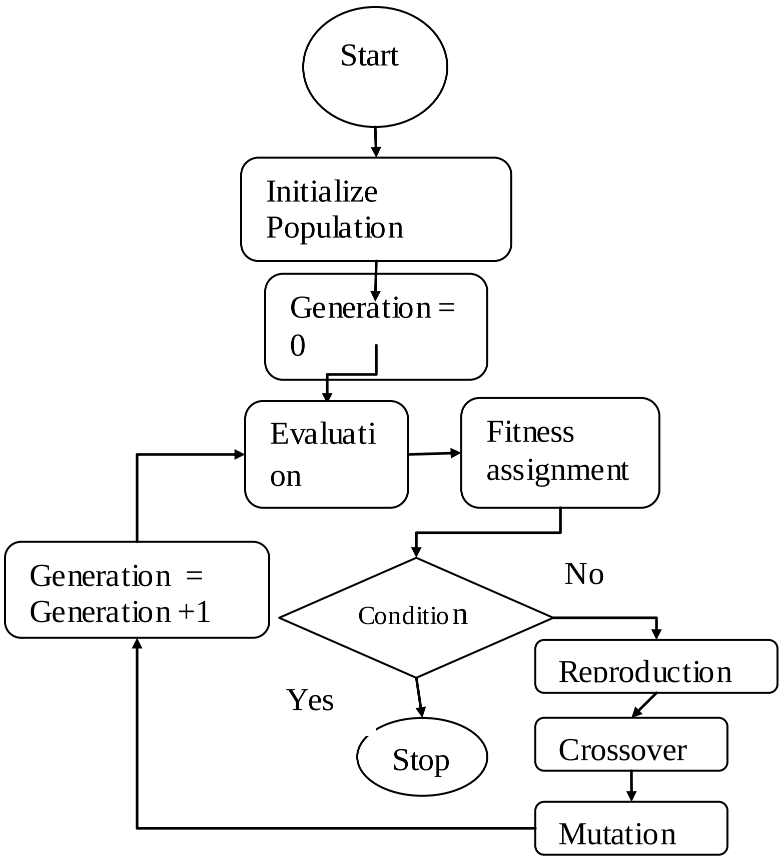


Fig. 2. Flow chart for working principles of genetic algorithm

In the present analysis, a *real- coded* GA has been used, in which there is no need of coding and decoding the design variables. The *real- coded* GA with simulated binary crossovers (SBX), polynomial mutations and a tournament selection type of selection procedure have been used, details of which are available in Deb (2001). The GA was implemented using pseudo code available as freeware at http://www.iitk.ac.in/mech/research_labs.htm.

The GA has an inherent limitation of not being able to handle the equality constraints. The equality constraints need to be converted to inequality constraint using a dummy variable ([illegible] is considered for the present study) in the GA formulation and the revised constraint is written as in Eq.7-11. So the total objective function and the corresponding constraints can be written as

$$\text{Min: } F^2 + \{(P_1 - P_2)\cos\phi_d - N_3 \sin\theta - r_u W_3 \cos\theta \sin\theta + T_3 \cos\theta_3 - C_s W_3\}^2 \quad (7)$$

Subjected to:

$$\varepsilon - \{l_1 \times \cos\theta_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 - H / \tan\beta\}^2 \geq 0.0 \quad (8)$$

$$\varepsilon - \{l_1 \times \sin\theta_1 + l_2 \sin\theta_2 + l_3 \sin\theta_3 - H\}^2 \geq 0.0 \quad (9)$$

$$\square \geq \square_1 \quad (10)$$

$$T_i \geq 0.0; \; i = 1,2,3 \quad (11)$$

The common method of handling the constraints is by penalty function method. However, in the present study the following method (Deb, 2001) is used for constraint handling.

(i) The method uses tournament selection as the selection operator and two solutions are compared at a time.

(ii) Any feasible solution is preferred to any infeasible solution;

(iii) Among two feasible solutions the one having better objective function is preferred and among two infeasible solutions, the one having smaller constraint violation is preferred.

Thus, at any iteration, the infeasible solutions are not computed for objective function if some feasible solutions are present, which helps in reducing the computational effort.

The stability of slope is one of the most important problems in stability analysis of geo-mechanics. Out of various methods (finite element analysis, limit analysis), limit equilibrium method is widely used for its simplicity form and the results found to be close to that rigorous methods. The limit equilibrium method is taken as 2-D plane strain problem with no variation in geometry, material and surcharge in direction parallel to the crest of the slope. The problem lies in finding out the critical failure surface and its corresponding factor of safety (FOS). The above concept has given rise to consider it as an optimization problem (Basudhar, 1976; Baker, 1980).

The development of limit equilibrium as optimization is straight forward, consisting of (i) development of objective function and (ii) selection of optimization technique. Development of objective function is based on different stability analysis method for the sliding mass of the slope. The different methods in use for this are Bishop, Janbu, Spencer, Morgenstern & Price, Chen & Morgenstern, Sharma etc. (Abramson et al., 2002). The stability analysis methods basically differ from one another in the hypothesis assumed in order to satisfy the equilibrium conditions of the potential sliding mass. It has been proved that all these methods, if used respecting the basic hypothesis, gives satisfactory results.

Different sophisticated optimization techniques have been used to search for the critical slip surface, are calculus of variation, linear programming, nonlinear programming and dynamic programming. The variational technique cannot be applied to heterogeneous soil, and as the stability analysis equation is nonlinear, linear programming has not been widely accepted. Dynamic programming has the difficulty in dimensionality, so the nonlinear unconstraint optimizations like Nelder Meade, Hookes & Jeeve & Powells Conjugate direction method, steepest descent, Fletcher-Reeve (FR), Davidon,- Fletcher – Powel(DFP), Broydon-Fletcher-Goldfarb-Shanno (BFGS) have been widely used. Many practical slope problems are not convex (De Natale, 1991), there by having multiple optima. All the above optimization techniques are initial point dependant and there is a need to analyze with wide separated points. It is usually not possible to find global minimum except in special cases (De Natale, 1991).

To avoid the difficulty in finding out the global minima, evolutionary methods such as genetic algorithm is being used, which is more robust in finding out the optimal solution in many complex problems (Goldberg, 1989). Goh (1999) has used GA to find out the critical surface and the factor of safety using method of wedges. McCombie and Wilkinson (2002) used Bishop's simplified method and Sabhahit et al. (2002) have used Janbu's method to search for the critical surface using GA. In the above studies GA could find better solution compared to other traditional optimization tools.

With the above in view, in this study a *real-coded* GA has been used to find out the critical failure surface and the corresponding factor of safety for three wedge method. The *real-coded*

GA has several advantages over binary coded GA (Deb, 2001) and three-wedge method is widely used for stability analysis of mine spoils (Huang, 1983).

METHODOLOGY

The analysis of the problem can be considered in two stages: (i) development of objective function and (ii) the application of GA in solving the objective function.

Development of objective function

In the present study, the three-wedge method for stability analysis of slopes (Huang, 1983) is used for the development of objective function. This is a force equilibrium method and development of the equations used for the analysis is described in details in Huang (1983).

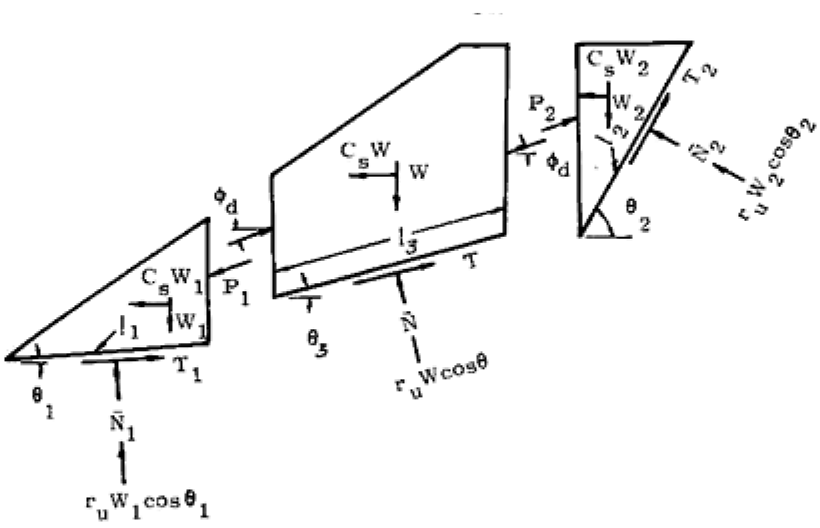


Fig. 1. The free-body diagram for three-wedge method

Fig. 1 shows the free-body diagram showing the forces on each block. There are total six (6) unknowns (*P_1, P_2, N_1, N_2, N_3* and factor of safety, *F*) which can be solved by six equilibrium equations, two for each block.

$$W_1 + P_1 \sin \phi_d - N_1 \cos \theta_1 - r_u W_1 \cos^2 \theta_1 - T_1 \sin \theta_1 = 0 \quad (1.0)$$

$$T_1 = (c_1 l_1 + N_1 \tan\phi_1)/F \qquad (2.0)$$

$$P_1 \cos\phi_d + N_1 \sin\theta_1 + r_u W_1 \sin\theta_1 \cos\theta_1 + C_s W_1 - T_1 \cos\theta 1 = 0 \qquad (3.0)$$

From equation 1.0, 2.0 and 3.0

$$N_1 = \frac{\{W_1[\cos\phi_d - r_u \cos\theta_1 \cos(\phi_d - \theta_1) - C_s \sin\phi_d] + [c_1 l_1 \sin(\phi_d - \theta_1)]/F\}}{\cos(\phi_d - \theta_1) - [\tan\phi_1 \sin(\phi_d - \theta_1)]/F}$$

From equation 3.0

$$P_1 = \frac{(T_1 \cos\theta_1 - N_1 \sin\theta_1 - r_u W_1 \sin\theta_1 \cos\theta_1 - C_s W_1)}{\cos\phi_d}$$

For the top block (block 2)

$$W_2 - P_2 \sin\phi_d - N_2 \cos\theta_2 - r_u W_2 \cos^2\theta_2 - T_2 \sin\theta_2 = 0 \ldots (6.0)$$

$$T_2 = (c_2 l_2 + N_2 \tan\phi_2)/F \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.0)$$

$$P_2 \cos\phi_d - N_2 \sin\theta_2 - r_u W_2 \cos\theta_2 \sin\theta_2 - C_s W_2 + T_2 \cos\theta_2 = 0 .. (8.0)$$

From equation 6.0, 7.0 and 8.0

$$N_2 = \frac{\{W_2[\cos\phi_d - r_u \cos\theta_2 \cos(\phi_d - \theta_2) - C_s \sin\phi_d] + [c_2 l_2 \sin(\phi_d - \theta_2)]/F\}}{\cos(\phi_d - \theta_2) - [\tan\phi_2 \sin(\phi_d - \theta_2)]/F} .. (9.0)$$

From equation 8.0

$$P_2 = \frac{(-T_2 \cos\theta_2 + N_2 \sin\theta_2 - r_u W_2 \sin\theta_2 \cos\theta_2 - C_s W_2)}{\cos\phi_d}$$

For the Middle block

$$W_3 - P_2 \sin\phi_d - P_1 \sin\phi_d - N_3 \cos\theta_3 - r_u W_3 \cos^2\theta_3 - T_3 \sin\theta_3 = 0 \ldots (11.0)$$

$$T_3 = (c_3 l_3 + N_3 \tan\phi_2)/F \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12.0)$$

$$-P_2 \cos\phi_d + P_1 \cos\phi_d - N_3 \sin\theta_3 - r_u W_3 \cos\theta_3 \sin\theta_3 - C_s W_3 + T_3 \cos\theta_3 = 0 .. (13.0)$$

From equation 11.0, 12.0 and 13.0

$$N_3 = \frac{\{W_3(1 - r_u \cos^2\theta_3) + (P_2 - P_1)\sin\phi_d - [c_3 l_3 \sin\theta_3)]/F\}}{\cos\theta_3 + [\tan\phi_2 \sin\theta_3)]/F} \quad (14.0)$$

Equation 13.0 can be written as

Function (F) =

$$(P_1 - P_2)\cos\phi_d - N_3 \sin\theta - r_u W_3 \cos\theta \sin\theta + T_3 \cos\theta_3 - C_s W_3 = 0 \qquad (1)$$

The F can be found out by solving the nonlinear equation as shown in Eq. 15 (Huang, 1983). There are different iterative methods to solve Eq. 1. However, there are some problems in solving such equation using the iterative methods (Bhattacharya and Basudhar, 2001), which is inherent in all numerical techniques. So in the present study the Eq.15 is solved using optimization method.

Where $\phi_d = \tan^{-1}\left(\frac{\tan\phi}{F}\right)$, r_u = pore pressure parameter, W_3 = weight of the 3rd wedge, C_s is horizontal seismic acceleration coefficient and P_1, P_2, N_1, N_2, N_3 are as shown in Fig.1

The problem is formulated in 3 different ways:

Formulation –I: From the equation 15.0 it can be seen that the factor is that value at which this equation is satisfied equals to Zero. We are interested in finding out those variables for which the factor of safety is minimum. We can write

Min F

Subjected to

$$(P_1 - P_2)\cos\phi_d - N_3\sin\theta - r_u W_3\cos\theta\sin\theta + T_3\cos\theta_3 - C_s W_3 = 0.......$$

$\theta_2 > \theta_3$ --- g1

$\theta 3 > \theta_1$ --- g2

$\phi > \phi_d$ --- g3

$\beta > \theta_1$ -----g4

Where the variables are $l_1, l_2, l_3, \theta_1, \theta_2, \theta_3$ and FOS and the application dependant input parameters are β, c, H, ϕ, r_u, C_s at the same time in order to ascertain the shape and location of the slip surface are physically reasonable and kinematically compatible, the following constraints are need to be imposed on the choice of design variable.

Same time as per physically condition it is found that the direction of the T should be +ve.

T_i >0 While formulating this in GA the constraint g3 can be converted to a variable bound

As $\tan\phi_d = (\tan\phi/F)$ which implies that F > 1.0

Formulation :II

Min F

Subject to

$(P_1 - P_2)\cos\phi_d - N_3 \sin\theta - r_u W_3 \cos\theta \sin\theta + T_3 \cos\theta_3 - C_s W_3 = 0$.......

in addition to other constraints. As in GA it is better to consider it as inequality constraint. We take it as

EPSILION –h ≥ 0

$\theta_2 > \theta_3$ --- g1

$\theta 3 > \theta_1$ --- g2

$\phi > \phi_d$ --- g3

$\beta > \theta_1$ -----g4

Formulation _III

Like in the traditional approach they first approach a value of F.S and then go for the calculation and finally to check that equation (15.0) is satisfied i.e some types of iteration process. Now rewriting equation 15.0 we

$$F_{cal} = \frac{\cos\theta_3 (cl_3 + N_3 \tan\phi)}{N3\sin\theta_3 + r_u W_3 \cos\theta_3 \sin\theta_3 \cos\theta_3 - (P_2 - P_1)\cos\phi_d}$$

Then formulating it as

Min

F_{cal} - F_{ini}

Subject To.

$\theta_2 > \theta_3$ --- g1

$\theta 3 > \theta_1$ --- g2

$\phi > \phi_d$ --- g3

$\beta > \theta_1$ -----g4

The expression for objective function in all these cases are not constant, the function changes with different initial point as per the following condition as the weight of the slice changes with position of it i.e initial random numbers.

From The fig

Case I

For $l_1 \cos\theta_1 + l_3 \cos_3 \leq H \cot\beta$

$$W_1 = 0.5\gamma l_1^2 \cos\theta_1 (\tan\beta \cos\theta_1 - \sin\theta_1)$$

$$W_2 = \gamma \begin{bmatrix} \{H + (\tan\beta(l_1 \cos\theta_1 + l_3 \cos\theta_3)/2\} x \{H \cot\beta - l_1 \cos\theta_1 - l3\cos\theta_3\} + \\ \{H * (l2\cos\theta_2 + l1\cos\theta_1 + l3\cos\theta_3 - H \cot\beta)\} - \{(l3\sin\theta_3 + H) * l2\cos\theta_2 / 2\} \end{bmatrix}$$

$$W_3 = 0.5 * \gamma l_3 \cos\theta_3 \left\{\tan\beta(2 * l_1 \cos\theta_1 + l_3 \cos\theta_3) - (2 l_1 \sin\theta_1 + l_3 \sin\theta_3)\right\}$$

Case-II

For $l1\cos\theta 1 < H \cot\beta$ and $l_1 \cos\theta_1 + l_3 \cos_3 > H \cot\beta$

$$W_1 = \gamma l^2 \cos\theta_1 (\tan\beta \cos\theta_1 - \sin\theta_1)$$

$$W_2 = 0.5\gamma l_2^2 \sin\theta_2 \cos\theta_2$$

$$W_3 = \gamma * \begin{bmatrix} (H \cot\beta - l1\cos\theta_1) * (\tan\beta l1\cos\theta_1 + H) * 0.5 + H(l1\cos\theta_1 + l3\cos\theta_3 - H) - \\ l3\cos\theta_3) - l3\cos\theta_3 x (l1\sin\theta_1 + 0.5 l3\sin\theta_3) \end{bmatrix}$$

Case-III

$l1\cos\theta 1 > H\cot\beta$

$$W_1 = \left[(l1\cos\theta_1 - 0.5H\cot\beta)H - 0.5l_1^2\cos\theta_1\sin\theta_1\right]\gamma$$
$$W_2 = \gamma\left[l_2^2\cos\theta_2\sin\theta_2\right]$$
$$W_3 = \gamma\left[l_3\cos\theta_3(H - l_1\sin\theta_1) - 0.5l_3^2\cos\theta_3\sin\theta_3\right]$$

The optimization method may be described as finding out the minimum factor safety which satisfies the Eq. 1 and in mathematical programming form it can be written as:

Min F:

Subjected to

$$(P_1 - P_2)\cos\phi_d - N_3\sin\theta - r_uW_3\cos\theta\sin\theta + T_3\cos\theta_3 - C_sW_3 = 0. \quad (2)$$

The variables (design vectors) are $l_1, l_2, l_3, \theta_1, \theta_2, \theta_3$ and F and the application dependant input parameters are slope angle (β)), cohesion (c_i), height of slope (H), angle of internal friction (ϕ) pore pressure parameter (r_u) and seismic acceleration coefficient C_s

In order to ascertain that the shape and location of the slip surface are physically reasonable and kinematically compatible, the following constraints need to be imposed on the choice of design variable. As per physically condition it is found that the direction of the T_i should be positive (Huang, 1983), and the kinematical conditions are applied for the geometry of the failure surface.

$$l_1 \times \cos\theta_1 + l_2\cos\theta_2 + l_3\cos\theta_3 = H/\tan\beta \quad (3)$$

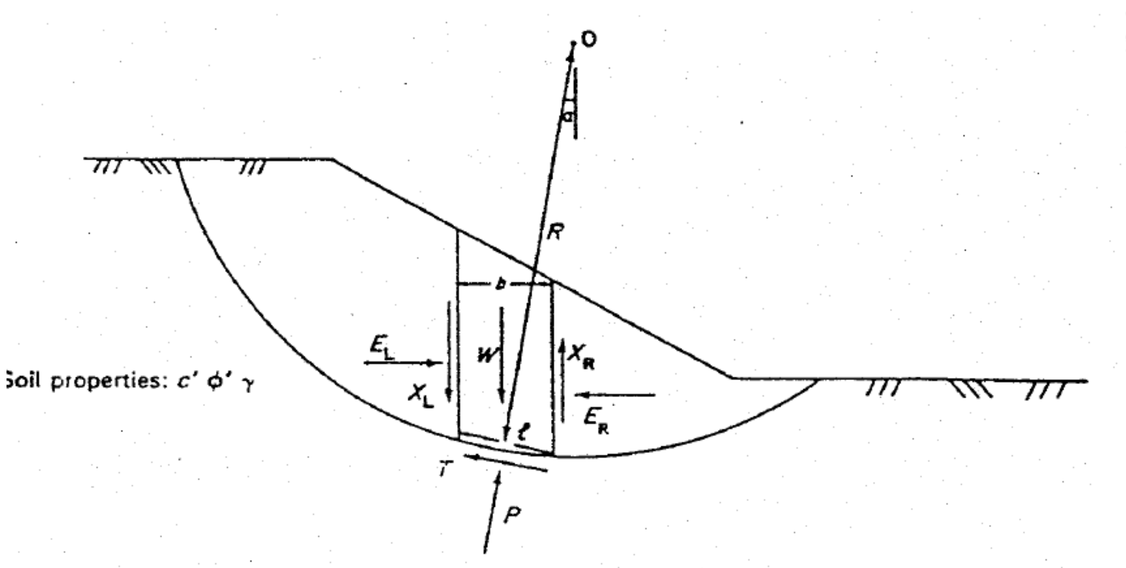
$$l_1 \times \sin\theta_1 + l_2\sin\theta_2 + l_3\sin\theta_3 = H \quad (4)$$

$$\beta \geq \theta_1 \quad (5)$$

$$T_i \geq 0.0;\ i = 1,2,3 \quad (6)$$

Using these 3 objective functions in the genetic algorithm the various parameters were found out.

Result obtained from coding of Bishop's Method:-

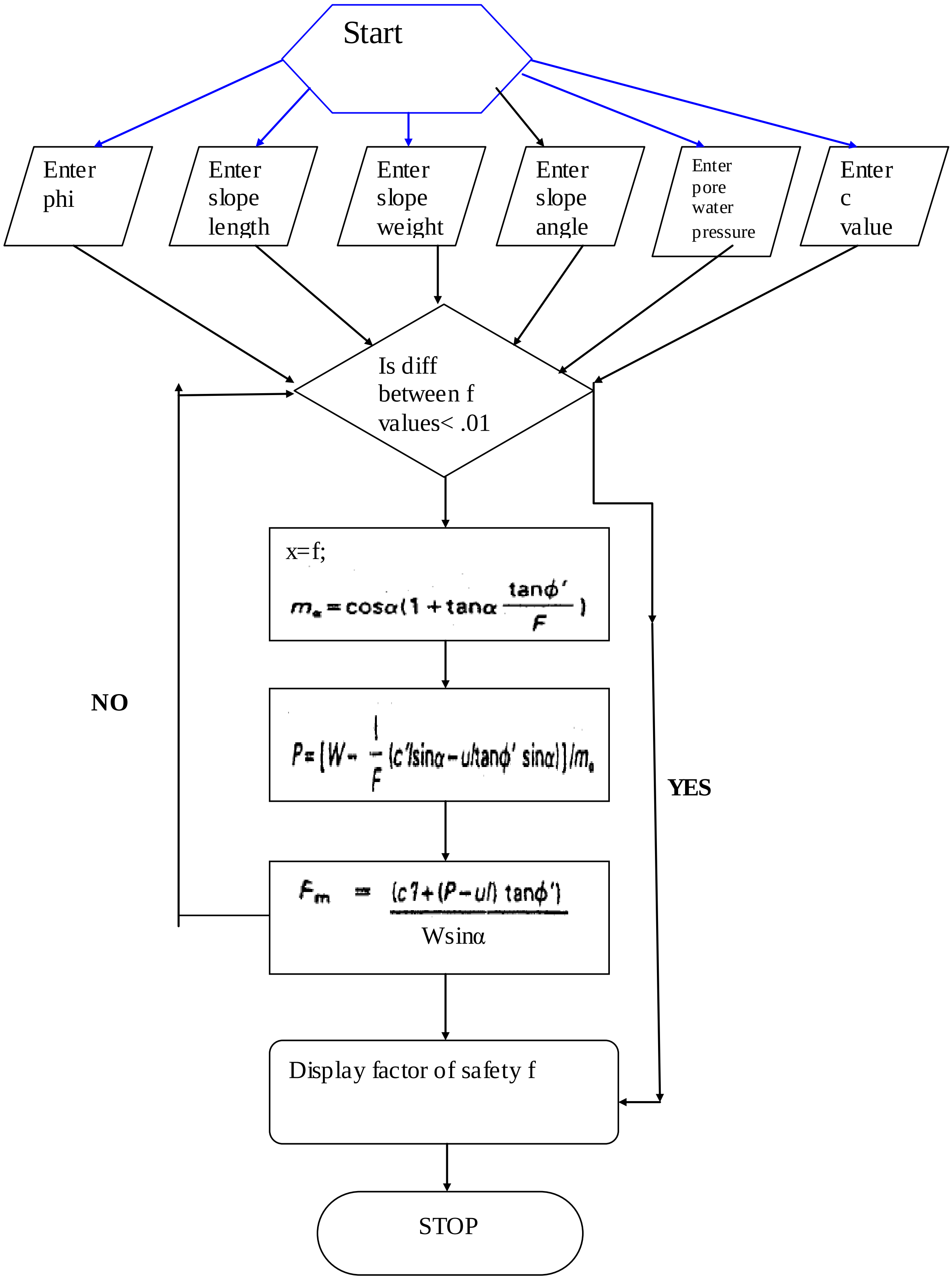


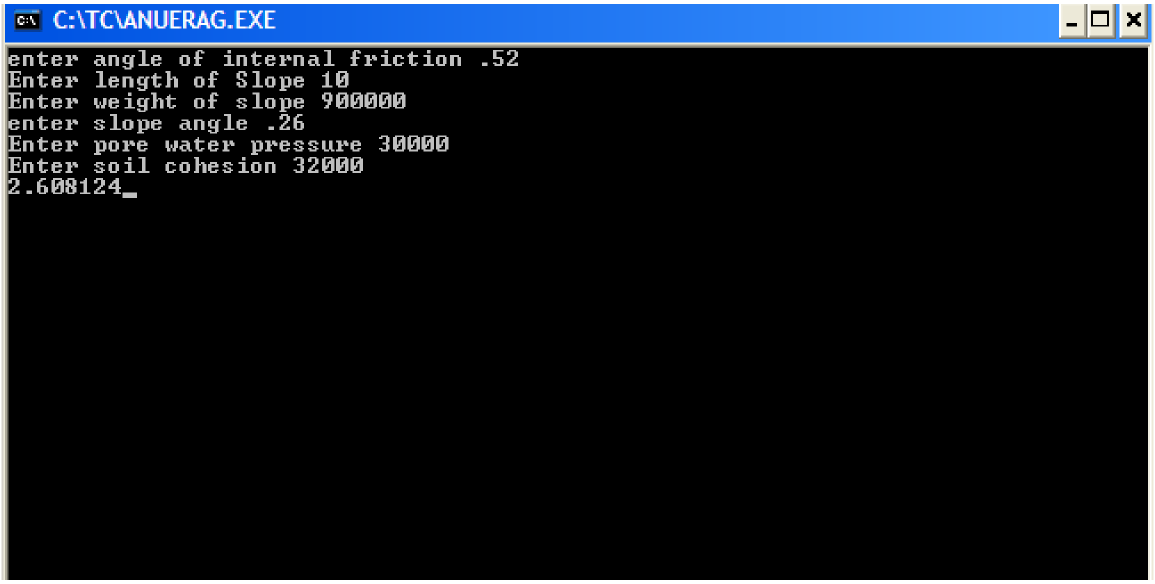
$$P = \left[W - \frac{1}{F}\left(c'l\sin\alpha - ul\tan\phi'\sin\alpha\right)\right]/m_\alpha$$

$$\text{where } m_\alpha = \cos\alpha\left(1 + \tan\alpha\frac{\tan\phi'}{F}\right)$$

$$F_m = \frac{\Sigma\left(c'l + (P - ul)\tan\phi'\right)}{\Sigma W\sin\alpha}$$

Flowchart for the code is as follows:-





Output Obtained

Conclusion

The primary function of an engineer is to design a structure economically without compromising on its strength. In case of design of slopes, steep slopes require less earth work hence, lesser cost. But, the factor of safety is compromised. Factor of safety obtained for 1:1 slope was 5.75 and for 1.5:1 it was found out to be 5.04.

Another, option is to provide reinforced slopes or retaining walls. These slopes have greater factor of safety than corresponding non-reinforced or unsupported slopes. Although, they decrease the amount of earth work involved the cost is significantly increased due to the addition of these structures.

But, the cost of construction of slopes also depends upon the cost of land. Therefore, in urban areas where the cost of land is high steeper slopes may be provided with adequate reinforcement or retaining walls in order to minimize cost.

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