A

Project Report on

Economic Design of Control Chart

In partial fulfillment of the requirements of

Bachelor of Technology (Mechanical Engineering)

Submitted By

Debabrata Patel (Roll No.10503031)

Session: 2008-09

Department of Mechanical Engineering

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Under the guidance of

Prof. (Dr.) S. K. Patel

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CERTIFICATE

This is to certify that that the work in this thesis report entitled "**Economic Design of Control Chart**" submitted by Debabrata Patel in partial fulfillment of the requirements for the degree of Bachelor of Technology in Mechanical Engineering Session 2008-2009 in the department of Mechanical Engineering, National Institute of Technology Rourkela, Rourkela is an authentic work carried out by him under my supervision and guidance.

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CONTENTS

Abstract

The major function of control chart is to detect the occurrence of assignable causes so that the necessary corrective action can be taken before a large quantity of nonconforming product is manufactured. The X-bar control chart dominates the use of any other control chart technique if quality is measured on a continuous scale. In the present project, we develop the economic design of the X-bar control chart using Genetic Algorithm to determine the values of the sample size, sampling interval, width of control limits such that the expected total cost per hour is minimized. The genetic algorithms (GA) are applied to search for the optimal values of the three test parameters of the X-bar chart. In genetic algorithm we use mutation and cross-over technique to get the optimal solution. Finally, a sensitivity analysis will be carried out to investigate the effect of model parameters on the solution of the economic design.

Chapter 1

Introduction:

1.1. Control chart:

Control chart is a tool used to monitor processes and to assure that they remain "In Control" or stable.

1.2. Elements Of A Control Chart:

A control chart consists of:
1. a central line,
2. an upper control limit,
3. a lower control limit and
4.process values plotted on the chart.

1.3. Designing a Control Chart:

All the process values are plotted on the chart. If the process values fall within the upper and lower control limits and the process is referred to as "In Control." If the process values plotted fall outside the control limits, the process is referred to as "Out Of Control".

1.4. Economic Design of a Control Chart:

In all production processes, we need to monitor the extent to which our products meet specifications. In the most general terms, there are two "enemies" of product quality:

1. deviations from target specifications, and
2. excessive variability around target specifications.

The economic design of control charts is used to determine various design parameters that minimize total economic costs. The effect of production lot size on the quality of the product may also be significant. If the production process shifts to an out-of-control state at the beginning of the production run, the entire lot will contain more defective items. Hence, it is wiser to reduce the production cycle to decrease the fraction of defective items and, thus, improve output quality. On the other hand, reduction of the production cycle may result in an increase in costs due to frequent setups. A balance must be maintained so that the total cost is minimized. The production of quality goods depends upon the operating condition of the machine tools; however, the performance of machine tools depends upon the maintenance policy. It is assumed that the cost of maintaining the equipment increases with age; therefore, an age replacement strategy is needed to minimize the total cost of the system, which will simultaneously improve quality control and maintenance policy.

1.5. Genetic Algorithm:

Genetic algorithm is a search technique used in computing to find exact or approximate solutions to optimization and search problems. Genetic algorithm is a search technique used in computing to find exact or approximate solutions to optimization and search problems. The basic concept of GAs is designed to simulate processes in natural system necessary for evolution, specifically those that follow the principles first laid down by Charles Darwin of survival of the fittest. As such they represent an intelligent exploitation of a random search within a defined search space to solve a problem.

Not only does GAs provide an alternative methods to solving problem, it consistently outperforms other traditional methods in most of the problems link. Many of the real world problems involved finding optimal parameters, which might prove difficult for traditional methods but ideal for GAs. Genetic algorithms are categorized as global search heuristics. Genetic algorithms are a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover (also called recombination).

1.6. Objective of the Project:

Our goal is to minimize **E(L)** which is loss function per hour. To minimize **E(L)** we apply Genetic algorithm method which apply cross-over and mutation technique to get the optimal solution for **n, h, k** which is a iterative method itself. Where **n** is the sample size, **h** is the sampling frequency or interval between the samples and **k** is the width of the control limits.

Chapter 2

Literature Review:

2.1. General Purpose of control chart:

Production planning, quality control and maintenance policy are three fundamental and interrelated factors in any industrial process. The goal of production planning is to determine the optimal procedure for product manufacturing in order to minimize production cost, holding cost and setup cost and to guarantee that no stock-outs occur during the production cycle.
In all production processes, we need to monitor the extent to which our products meet specifications. In the most general terms, there are two "enemies" of product quality:

(1) deviations from target specifications, and

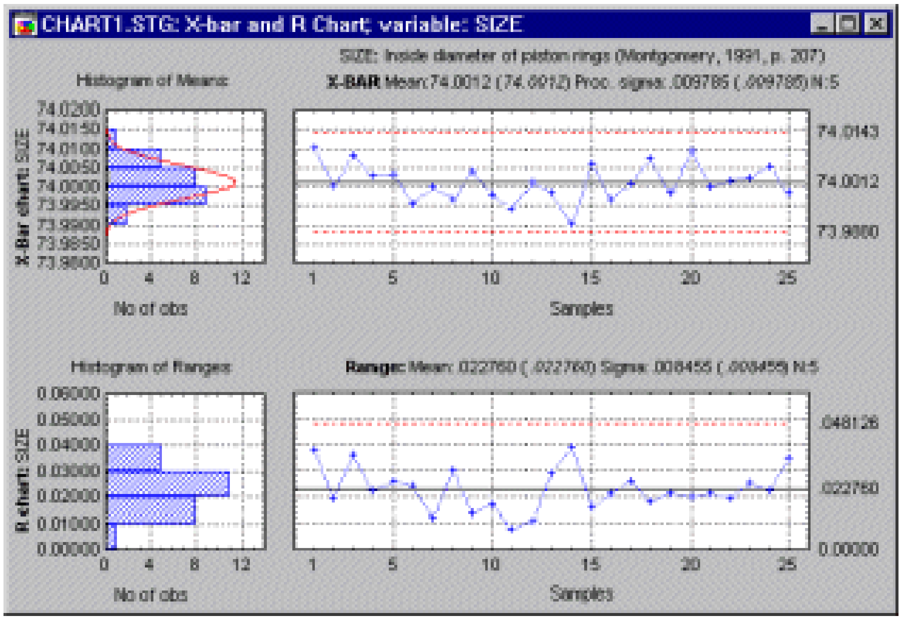
(2) excessive variability around target specifications.

2.2. General Approach:

The general approach to on-line quality control is straightforward: We simply extract samples of a certain size from the ongoing production process. We then produce line charts of the variability in those samples, and consider their closeness to target specifications. If a trend emerges in those lines, or if samples fall outside pre-specified limits, then we declare the process to be out of control and take action to find the cause of the problem. These types of charts are sometimes also referred to as Shewhart control charts.

2.3. Interpreting the chart:

The most standard display actually contains two charts; one is called an *X-bar chart*, the other is called an *R chart*.



In both line charts, the horizontal axis represents the different samples; the vertical axis for the X-bar chart represents the means for the characteristic of interest; the vertical axis for the R chart represents the ranges. For example, suppose we wanted to control the diameter of piston rings that we are producing. The center line in the X-bar chart would represent the desired standard size (e.g., diameter in millimeters) of the rings, while the center line in the R chart would represent the acceptable (within-specification) range of the rings within samples; thus, this latter chart is a chart of the variability of the process (the larger the variability, the larger the range). In addition to the center line, a typical chart includes two additional horizontal lines to represent the upper and lower control limits (*UCL, LCL*, respectively). Typically, the individual points in the chart, representing the samples, are connected by a line. If this line moves outside the upper or lower control limits or exhibits systematic patterns across consecutive samples, then a quality problem may potentially exist.

2.4. Establishing Control Limits:

Even though one could arbitrarily determine when to declare a process out of control (that is, outside the UCL-LCL range), it is common practice to apply statistical principles to do so. Elementary Concepts discusses the concept of the sampling distribution, and the characteristics of the normal distribution.

Example. Suppose we want to control the mean of a variable, such as the size of piston rings. Under the assumption that the mean (and variance) of the process does *not* change, the successive sample means will be distributed normally around the actual mean. Moreover, without going into details regarding the derivation of this formula, we also know (because of the central limit theorem, and thus approximate normal distribution of the means; see, for example, Hoyer and Ellis, 1996) that the distribution of sample means will have a standard deviation of *Sigma* (the standard deviation of individual data points or measurements) over the square root of *n* (the sample size). It follows that approximately 95% of the sample means will fall within the limits μ± *1.96 * Sigma/Square Root(n). Elementary Concepts* for a discussion of the characteristics of the normal distribution and the central limit theorem). In practice, it is common to replace the 1.96 with 3 (so that the interval will include approximately 99% of the sample means), and to define the upper and lower control limits as plus and minus 3 *sigma limits*, respectively.
General case: The general principle for establishing control limits just described applies to all control charts. After deciding on the characteristic we want to control, for example, the standard deviation, we estimate the expected variability of the respective characteristic in samples of the size we are about to take. Those estimates are then used to establish the control limits on the chart.

2.5. Common Types of Charts:

The types of charts are often classified according to the type of quality characteristic that they are supposed to monitor: there are quality control charts for *variables* and control charts for *attributes.* Specifically, the following charts are commonly constructed for controlling variables:
5.1. Charts for controlling variable:

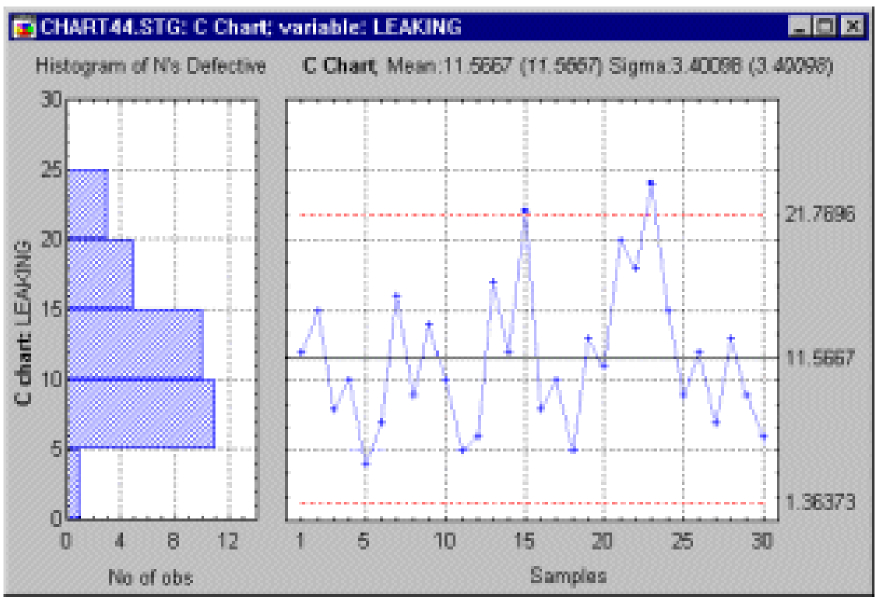
2.5.1. X-bar chart: In this chart the sample *means* are plotted in order to control the mean value of a variable (e.g., size of piston rings, strength of materials, etc.).

2.5.2. R chart: In this chart, the sample *ranges* are plotted in order to control the variability of a variable.

2.5.3. S chart: In this chart, the sample *standard deviations* are plotted in order to control the variability of a variable.

2.5.4. S2 chart:** In this chart, the sample *variances* are plotted in order to control the variability of a variable.
For controlling quality characteristics that represent *attributes* of the product, the following charts are commonly constructed:

2.5.5. C chart: In this chart, we plot the *number of defectives* (per batch, per day, per machine, per 100 feet of pipe, etc.). This chart assumes that defects of the quality attribute are *rare*, and the control limits in this chart are computed based on the Poisson distribution.



2.5.6. U chart: In this chart we plot the *rate of defectives*, that is, the number of defectives divided by the number of units inspected (the *n*; e.g., feet of pipe, number of batches). Unlike the C chart, this chart does not require a constant number of units, and it can be used, for example, when the batches (samples) are of different sizes.
2.5.7. Np chart: In this chart, we plot the number of defectives (per batch, per day, per machine) as in the C chart. However, the control limits in this chart are not based on the distribution of rare events, but rather on the binomial distribution. Therefore, this chart should be used if the occurrence of defectives is not rare (e.g., they occur in more than 5% of the units inspected). For example, we may use this chart to control the number of units produced with minor flaws.

2.5.8. P chart: In this chart, we plot the percent of defectives (per batch, per day, per machine, etc.) as in the U chart. However, the control limits in this chart are not based on the distribution of rare events but rather on the binomial distribution (of proportions). Therefore, this chart is most applicable to situations where the occurrence of defectives is not rare (e.g., we expect the percent of defectives to be more than 5% of the total number of units produced).
All of these charts can be adapted for short production runs (short run charts), and for multiple process streams.

2.6. Short Run Charts:

The short run control chart, or control chart for short production runs, plots observations of variables or attributes for multiple parts on the same chart. Short run control charts were developed to address the requirement that several dozen measurements of a process must be collected before control limits are calculated. Meeting this requirement is often difficult for operations that produce a limited number of a particular part during a production run.
For example, a paper mill may produce only three or four (huge) rolls of a particular kind of paper (i.e., *part*) and then shift production to another kind of paper. But if variables, such as paper thickness, or attributes, such as blemishes, are monitored for several dozen rolls of paper of, say, a dozen different kinds, control limits for thickness and blemishes could be calculated for the *transformed* (within the short production run) variable values of interest. Specifically, these *transformations* will rescale the variable values of interest such that they are of compatible magnitudes across the different short production runs (or parts). The control limits computed for those transformed values could then be applied in monitoring thickness, and blemishes, regardless of the types of paper (parts) being produced. Statistical process control procedures could be used to determine if the production process is in control, to monitor continuing production, and to establish procedures for continuous quality improvement.

2.6.1. Short Run Charts for Variables:

Nominal chart, target chart: There are several different types of short run charts. The most basic are the nominal short run chart, and the target short run chart. In these charts, the measurements for each part are transformed by subtracting a part-specific constant. These constants can either be the nominal values for the respective parts (*nominal* short run chart), or they can be target values computed from the (historical) means for each part (*Target X-bar and R chart*). For example, the diameters of piston bores for different engine blocks produced in a factory can only be meaningfully compared (for determining the consistency of bore sizes) if the mean differences between bore diameters for different sized engines are first removed. The nominal or target short run chart makes such comparisons possible. Note that for the nominal or target chart it is assumed that the variability across parts is identical, so that control limits based on a common estimate of the process sigma are applicable.

Standardized short run chart: If the variability of the process for different parts cannot be assumed to be identical, then a further transformation is necessary before the sample means for different parts can be plotted in the same chart. Specifically, in the standardized short run chart the plot points are further transformed by dividing the deviations of sample means from part means (or nominal or target values for parts) by part-specific constants that are proportional to the variability for the respective parts. For example, for the short run X-bar and R chart, the plot points (that are shown in the X-bar chart) are computed by first subtracting from each sample mean a part specific constant (e.g., the respective part mean, or nominal value for the respective part), and then dividing the difference by another constant, for example, by the average range for the respective chart. These transformations will result in comparable scales for the sample means for different parts.

2.6.2. Short Run Charts for Attributes:

For attribute control charts (C, U, Np or P charts), the estimate of the variability of the process (proportion, rate, etc.) is a function of the process average (average proportion, rate, etc.; for example, the standard deviation of a proportion p is equal to the square root of $p*(1- p)/n)$. Hence, only standardized short run charts are available for attributes. For example, in the short run P chart, the plot points are computed by first subtracting from the respective sample *p* values the average part *p*'s, and then dividing by the standard deviation of the average *p*'s.

Unequal Sample Sizes: When the samples plotted in the control chart are not of equal size, then the control limits around the center line (target specification) cannot be represented by a straight line. For example, to return to the formula *Sigma/Square Root(n)* presented earlier for computing control limits for the X-bar chart, it is obvious that unequal *n*'s will lead to different control limits for different sample sizes. There are three ways of dealing with this situation.

Average sample size: If one wants to maintain the straight-line control limits (e.g., to make the chart easier to read and easier to use in presentations), then one can compute the average *n* per sample across all samples, and establish the control limits based on the average sample size. This procedure is not "exact," however, as long as the sample sizes are reasonably similar to each other, this procedure is quite adequate.

Variable control limits: Alternatively, one may compute different control limits for each sample, based on the respective sample sizes. This procedure will lead to *variable* control limits, and result in step-chart like control lines in the plot. This procedure ensures that the correct control limits are computed for each sample. However, one loses the simplicity of straight-line control limits.

Stabilized (normalized) chart: The best of two worlds (straight line control limits that are accurate) can be accomplished by standardizing the quantity to be controlled (mean, proportion, etc.) according to units of *sigma*. The control limits can then be expressed in straight lines, while the location of the sample points in the plot depend not only on the characteristic to be controlled, but also on the respective sample *n*'s. The disadvantage of this procedure is that the values on the vertical (*Y*) axis in the control chart are in terms of *sigma* rather than the original units of measurement, and therefore, those numbers cannot be taken at face value (e.g., a sample with a value of 3 is 3 times *sigma* away from specifications; in order to express the value of this sample in terms of the original units of measurement, we need to perform some computations to convert this number back).

2.7. Control Charts for Variables vs. Charts for Attributes:

Sometimes, the quality control engineer has a choice between variable control charts and attribute control charts.

2.7.1. Advantages of attribute control charts: Attribute control charts have the advantage of allowing for quick summaries of various aspects of the quality of a product, that is, the engineer may simply classify products as acceptable or unacceptable, based on various quality criteria. Thus, attribute charts sometimes bypass the need for expensive, precise devices and time-consuming measurement procedures. Also, this type of chart tends to be more easily understood by managers unfamiliar with quality control procedures; therefore, it may provide more persuasive (to management) evidence of quality problems.

2.7.2. Advantages of variable control charts: Variable control charts are more sensitive than attribute control charts. Therefore, variable control charts may alert us to quality problems before any actual "unacceptables" (as detected by the attribute chart) will occur. The variable control charts are *leading indicators* of trouble that will sound an alarm before the number of rejects (scrap) increases in the production process.

2.8. Control Chart for Individual Observations:

Variable control charts can by constructed for individual observations taken from the production line, rather than samples of observations. This is sometimes necessary when testing samples of multiple observations would be too expensive, inconvenient, or impossible. For example, the number of customer complaints or product returns may only be available on a monthly basis; yet, one would like to chart those numbers to detect quality problems. Another common application of these charts occurs in cases when automated testing devices inspect every single unit that is produced. In that case, one is often primarily interested in detecting small shifts in the product quality (for example, gradual deterioration of quality due to machine wear). The *CUSUM, MA*, and *EWMA* charts of cumulative sums and weighted averages discussed below may be most applicable in those situations.

2.9. Other Specialized Control Charts:

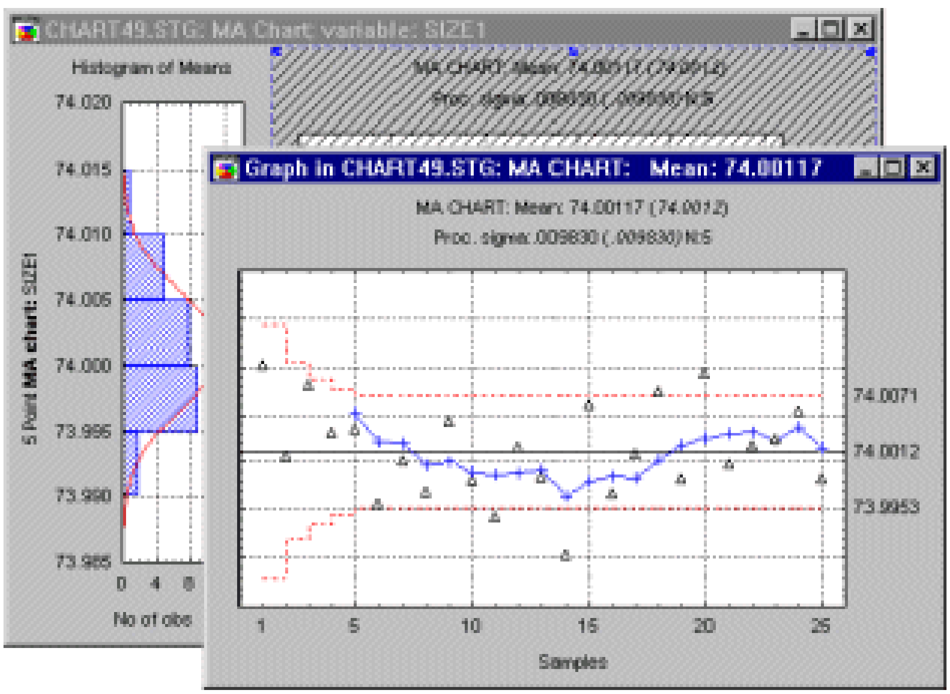
The types of control charts mentioned so far are the "workhorses" of quality control, and they are probably the most widely used methods. However, with the advent of inexpensive desktop computing, procedures requiring more computational effort have become increasingly popular.

2.9.1. X-bar Charts For Non-Normal Data: The control limits for standard X-bar charts are constructed based on the assumption that the sample means are approximately normally distributed. Thus, the underlying individual observations do not have to be normally distributed, since, as the sample size increases, the distribution of the means will become approximately normal; however, note that for *R, S, and S**2* charts, it *is* assumed that the individual observations are normally distributed). The standard normal distribution-based control limits for the means are appropriate, as long as the underlying distribution of observations are approximately normal.
However, when the distribution of observations is highly skewed and the sample sizes are small, then the resulting standard control limits may produce a large number of false alarms (increased alpha error rate), as well as a larger number of false negative ("process-is-in-control") readings (increased beta-error rate). We can compute control limits (as well as process capability indices) for X-bar charts which allows to approximate the skewness and kurtosis for a large range of non-normal distributions. These non- normal X-bar charts are useful when the distribution of means across the samples is clearly skewed, or otherwise non-normal.

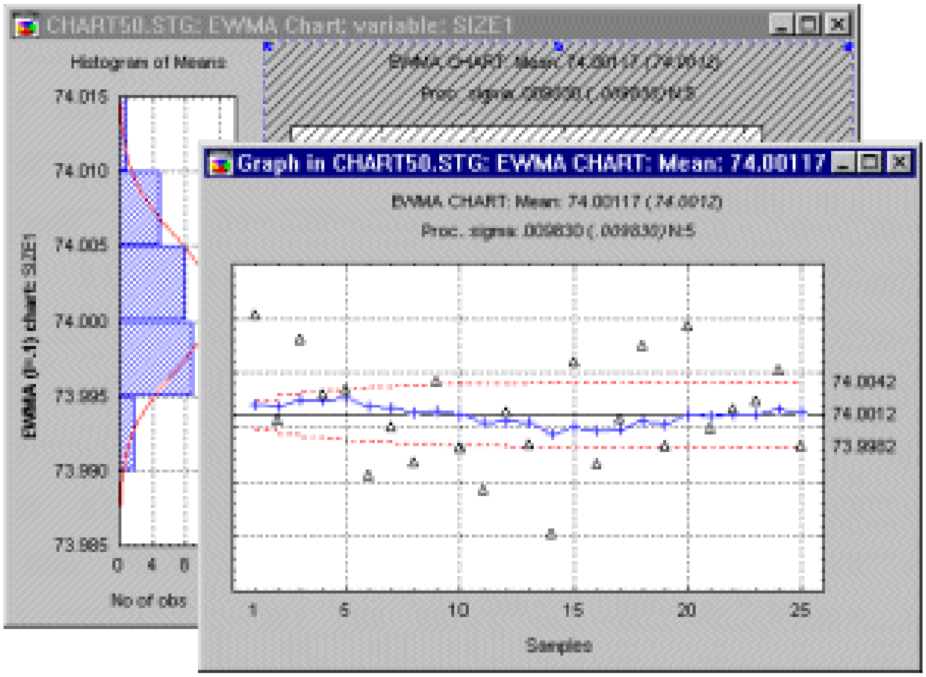
2.9.2. Cumulative Sum (CUSUM) Chart: If one plots the cumulative sum of deviations of successive sample means from a target specification, even minor, permanent shifts in the process mean will eventually lead to a sizable cumulative sum of deviations. Thus, this chart is particularly well-suited for detecting such small permanent shifts that may go undetected when using the X-bar chart. For example, if, due to machine wear, a process slowly "slides" out of control to produce results above target specifications, this plot would show a steadily increasing (or decreasing) cumulative sum of deviations from specification.
To establish control limits in such plots, the *V- mask*, is plotted after the last sample (on the right). The V-mask can be thought of as the upper and lower control limits for the cumulative sums. However, rather than being parallel

to the center line; these lines converge at a particular angle to the right, producing the appearance of a *V* rotated on its side. If the line representing the cumulative sum crosses either one of the two lines, the process is out of control.

2.9.3. Moving Average (MA) Chart: To return to the piston ring example, suppose we are mostly interested in detecting small trends across successive sample means. For example, we may be particularly concerned about machine wear, leading to a slow but constant deterioration of quality (i.e., deviation from specification). The CUSUM chart described above is one way to monitor such trends, and to detect small permanent shifts in the process average. Another way is to use some weighting scheme that summarizes the means of several successive samples; moving such a *weighted mean* across the samples will produce a moving average chart (as shown in the following graph).



2.9.4. Exponentially-weighted Moving Average (EWMA) Chart: The idea of *moving averages* of successive (adjacent) samples can be generalized. In principle, in order to detect a trend we need to weight successive samples to form a moving average; however, instead of a simple arithmetic moving average, we could compute a geometric moving average.

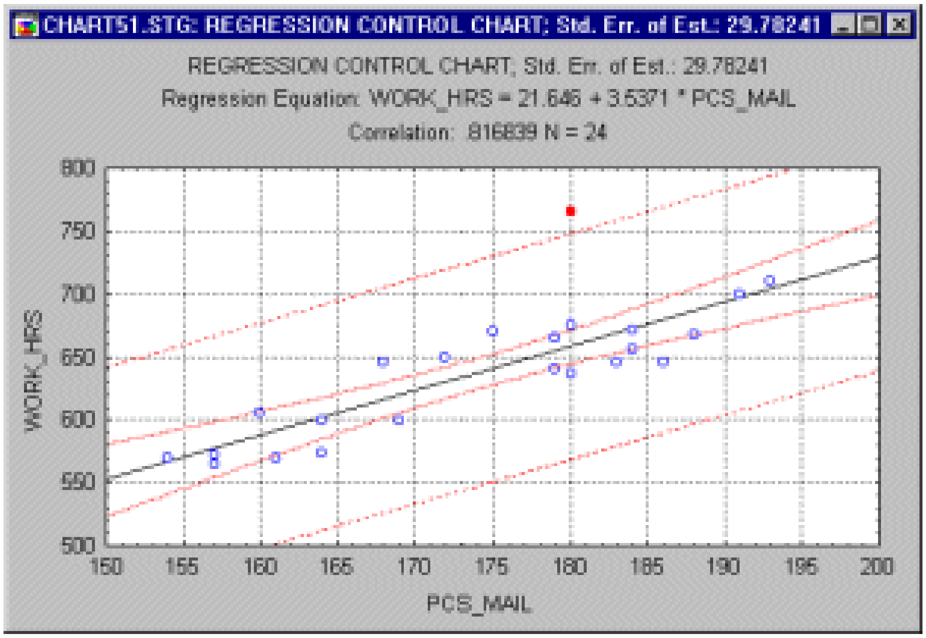


Specifically, we could compute each data point for the plot as:

$z_t = \lambda * \text{x-bar}_t + (1-\lambda) * z_{t-1}$

In this formula, each point z_t is computed as λ (lambda) times the respective mean *x-bar*$_t$, plus one minus λ times the previous (computed) point in the plot. The parameter λ (lambda) here should assume values greater than 0 and less than 1. This method of averaging specifies that the weight of historically "old" sample means decreases geometrically as one continues to draw samples. The interpretation of this chart is much like that of the moving average chart, and it allows us to detect small shifts in the means, and, therefore, in the quality of the production process.

2.9.5. Regression Control Charts: Sometimes we want to monitor the relationship between two aspects of our production process. For example, a post office may want to monitor the number of worker-hours that are spent to process a certain amount of mail. These two variables should roughly be linearly correlated with each other, and the relationship can probably be described in terms of the well-known Pearson product-moment correlation coefficient *r*. The regression control chart contains a regression line that summarizes the linear relationship between the two variables of interest. The individual data points are also shown in the same graph. Around the regression line we establish a confidence interval within which we would expect a certain proportion (e.g., 95%) of samples to fall. Outliers in this plot may indicate samples where, for some reason, the common relationship between the two variables of interest does not hold.



Applications: There are many useful applications for the regression control chart. For example, professional auditors may use this chart to identify retail outlets with a greater than expected number of cash transactions given the overall volume of sales, or grocery stores with a greater than expected number of coupons redeemed, given the total sales. In both instances, outliers in the regression control charts (e.g., too many cash transactions; too many coupons redeemed) may deserve closer scrutiny.
which simply amounts to a histogram showing the distribution of the quality loss (e.g., dollar loss) across some meaningful categories; usually, the categories are sorted into descending order of importance (frequency, dollar amounts, etc.). Very often, this chart provides useful guidance as to where to direct quality improvement efforts.

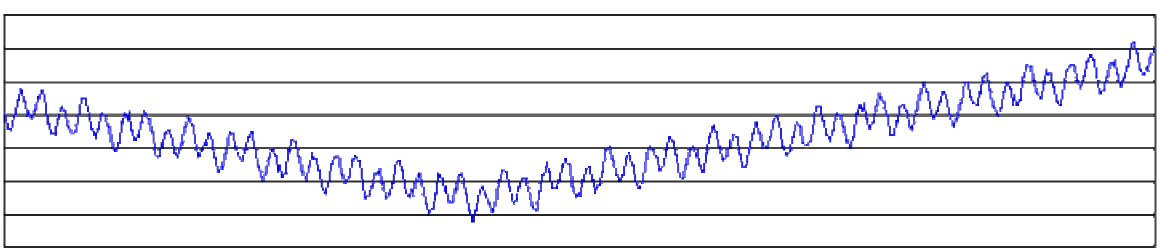
2.10. Genetic Algorithm:

2.10.1. Reproduction:

During reproduction, first occurs **recombination** (or **crossover**). Genes from parents form in some way the whole new chromosome. The new created offspring can then be mutated. **Mutation** means, that the elements of DNA are a bit changed. This changes are mainly caused by errors in copying genes from parents.
The **fitness** of an organism is measured by success of the organism in its life.

2.10.2. Search Space:

If we are solving some problem, we are usually looking for some solution, which will be the best among others. The space of all feasible solutions (it means objects among those the desired solution is) is called **search space** (also state space). Each point in the search space represents one feasible solution. Each feasible solution can be "marked" by its value or fitness for the problem. We are looking for our solution, which is one point (or more) among feasible solutions - that is one point in the search space.
The looking for a solution is then equal to a looking for some extreme (minimum or maximum) in the search space. The search space can be whole known by the time of solving a problem, but usually we know only a few points from it and we are generating other points as the process of finding solution continues.



Example of a search space

The problem is that the search can be very complicated. One does not know where to look for the solution and where to start. There are many methods, how to find some **suitable solution** (ie. not necessarily the **best solution**), for example **hill climbing, simulated annealing** and **genetic algorithm**.

The solution found by this methods is often considered as a good solution, because it is not often possible to prove what is the real optimum.

2.10.3. Basic Description :

Algorithm is started with a **set of solutions** (represented by **chromosomes**) called **population**. Solutions from one population are taken and used to form a new population. This is motivated by a hope, that the new population will be better than the old one. Solutions which are selected to form new solutions (**offspring**) are selected according to their fitness - the more suitable they are the more chances they have to reproduce. This is repeated until some condition (for example number of populations or improvement of the best solution) is satisfied.

2.10.3. Outline of the Basic Genetic Algorithm :

[Start] Generate random population of *n* chromosomes (suitable solutions for the problem)
[Fitness] Evaluate the fitness $f(x)$ of each chromosome x in the population
[New population] Create a new population by repeating following steps until the new population is complete
[Selection] Select two parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to be selected)
[Crossover] With a crossover probability cross over the parents to form a new offspring (children). If no crossover was performed, offspring is an exact copy of parents.
[Mutation] With a mutation probability mutate new offspring at each locus (position in chromosome).
[Accepting] Place new offspring in a new population
[Replace] Use new generated population for a further run of algorithm
[Test] If the end condition is satisfied, **stop**, and return the best solution in current population
[Loop] Go to step **2**

2.11. Operators of GA :

Overview :

As we can see from the genetic algorithm outline above, the crossover and mutation are the most important part of the genetic algorithm. The performance is influenced mainly by these two operators.

Encoding of a Chromosome :

The chromosome should in some way contain information about solution which it represents. The most used way of encoding is a binary string. The chromosome then could look like this:

Chromosome 1	1101100100110110
Chromosome 2	1101111000011110

Each chromosome has one binary string. Each bit in this string can represent some characteristic of the solution. Or the whole string can represent a number.

Of course, there are many other ways of encoding. This depends mainly on the solved problem. For example, one can encode directly integer or real numbers, sometimes it is useful to encode some permutations and so on.

2.11.1. Crossover:

After we have decided what encoding we will use, we can make a step to crossover. Crossover selects genes from parent chromosomes and creates a new offspring. The simplest way how to do this is to choose randomly some crossover point and everything before this point copy from a first parent and then everything after a crossover point copy from the second parent.

Crossover can then look like this (| is the crossover point):

Chromosome 1	11011 | 00100110110
Chromosome 2	11011 | 11000011110
Offspring 1	11011 | 11000011110
Offspring 2	11011 | 00100110110

There are other ways how to make crossover, for example we can choose more crossover points. Crossover can be rather complicated and very depends on encoding of the encoding of chromosome. Specific crossover made for a specific problem can improve performance of the genetic algorithm.

2.11.3 Mutation :

After a crossover is performed, mutation takes place. This is to prevent falling all solutions in population into a local optimum of solved problem. Mutation changes randomly the new offspring. For binary encoding we can switch a few randomly chosen bits from 1 to 0 or from 0 to 1. Mutation can then be following:

Original offspring 1	1101111000011110
Original offspring 2	1101100100110110
Mutated offspring 1	1100111000011110
Mutated offspring 2	1101101100110110

The mutation depends on the encoding as well as the crossover. For example when we are encoding permutations, mutation could be exchanging two genes.

2.12. Parameters of GA :

Crossover and Mutation Probability :

There are two basic parameters of GA - crossover probability and mutation probability.

2.12.1. Crossover probability says how often will be crossover performed. If there is no crossover, offspring is exact copy of parents. If there is a crossover, offspring is made from parts of parents' chromosome. If crossover probability is **100%**, then all offspring is made by crossover. If it is **0%**, whole new generation is made from exact copies of chromosomes from old population (but this does not mean that the new generation is the same!). Crossover is made in hope that new chromosomes will have good parts of old chromosomes and maybe the new chromosomes will be better. However it is good to leave some part of population survive to next generation.

2.12.2. Mutation probability says how often will be parts of chromosome mutated. If there is no mutation, offspring is taken after crossover (or copy) without any change. If mutation is performed, part of chromosome is changed. If mutation probability is **100%**, whole chromosome is changed, if it is **0%**, nothing is changed. Mutation is made to prevent falling GA into local extreme, but it should not occur very often, because then GA will in fact change to **random search**.

2.13. Other Parameters :

There are also some other parameters of GA. One also important parameter is population size.

2.13.1. Population size says how many chromosomes are in population (in one generation). If there are too few chromosomes, GA have a few possibilities to perform crossover and only a small part of search space is explored. On the other hand, if there are too many chromosomes, GA slows down. Research shows that after some limit (which depends mainly on encoding and the problem) it is not useful to increase population size, because it does not make solving the problem faster.
Selection

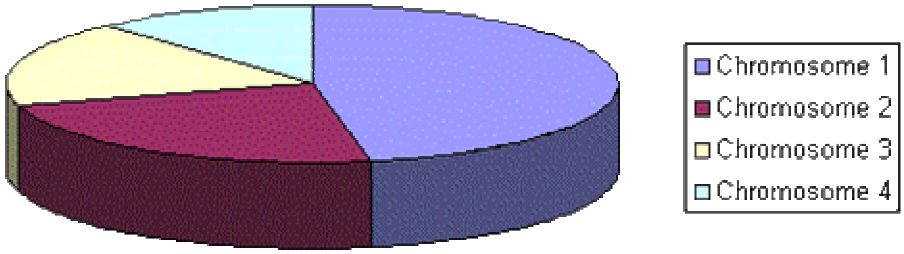
2.14. Parent Selection:

Introduction:

As we already know from the GA outline, chromosomes are selected from the population to be parents to crossover. The problem is how to select these chromosomes. According to Darwin's evolution theory the best ones should survive and create new offspring. There are many methods how to select the best chromosomes, for example roulette wheel selection, Boltzman selection, tournament selection, rank selection, steady state selection and some others.
Some of them are described below.

2.14.1. Roulette Wheel Selection :

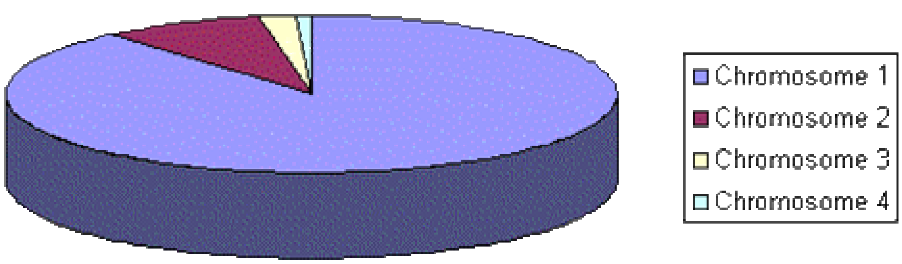
Parents are selected according to their fitness. The better the chromosomes are, the more chances to be selected they have. Imagine a **roulette wheel** where are placed all chromosomes in the population, every has its place big accordingly to its fitness function, like on the following picture.



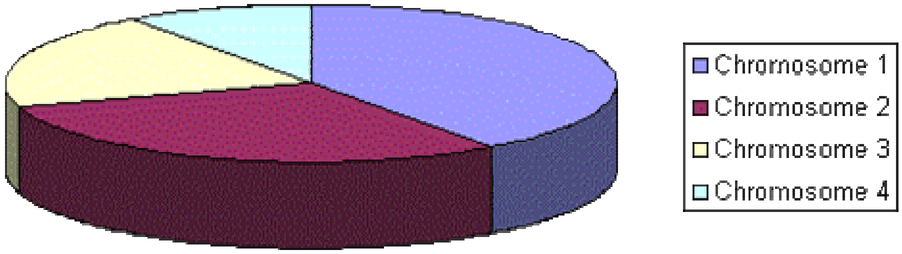
Then a marble is thrown there and selects the chromosome. Chromosome with bigger fitness will be selected more times.
This can be simulated by following algorithm.
[Sum] Calculate sum of all chromosome fitnesses in population - sum ***S***.
[Select] Generate random number from interval ***(0,S)*** - ***r***.
[Loop] Go through the population and sum fitnesses from ***0*** - sum ***s***. When the sum ***s*** is greater then ***r***, stop and return the chromosome where you are.
Of course, step **1** is performed only once for each population.

2.14.2. Rank Selection :

The previous selection will have problems when the fitnesses differs very much. For example, if the best chromosome fitness is 90% of all the roulette wheel then the other chromosomes will have very few chances to be selected.
Rank selection first ranks the population and then every chromosome receives fitness from this ranking. The worst will have fitness ***1***, second worst ***2*** etc. and the best will have fitness ***N*** (number of chromosomes in population).
You can see in following picture, how the situation changes after changing fitness to order number.



Situation before ranking (graph of fitnesses)



Situation after ranking (graph of order numbers)

After this all the chromosomes have a chance to be selected. But this method can lead to slower convergence, because the best chromosomes do not differ so much from other ones.

2.14.3. Steady-State Selection :

This is not particular method of selecting parents. Main idea of this selection is that big part of chromosomes should survive to next generation.

GA then works in a following way. In every generation are selected a few (good - with high fitness) chromosomes for creating a new offspring. Then some (bad - with low fitness) chromosomes are removed and the new offspring is placed in their place. The rest of population survives to new generation.

2.15. Encoding:

Introduction :

Encoding of chromosomes is one of the problems, when we are starting to solve a problem with GA. Encoding very depends on the problem.

In this chapter will be introduced some encodings, which have been already used with some success.

2.15.1. Binary Encoding :

Binary encoding is the most common, mainly because first works about GA used this type of encoding.

In **binary encoding** every chromosome is a string of **bits**, **0** or **1**.

Chromosome A	101100101100101011100101
Chromosome B	111111100000110000011111

Example of chromosomes with binary encoding

Binary encoding gives many possible chromosomes.

2.15.2. Permutation Encoding :

Permutation encoding can be used traveling salesman problem.
In **permutation encoding**, every chromosome is a string of numbers, which represents number in a **sequence**.

Chromosome A	1 5 3 2 6 4 7 9 8
Chromosome B	8 5 6 7 2 3 1 4 9

Example of chromosomes with permutation encoding

Permutation encoding is only useful for . Even for this problems for some types of crossover and mutation corrections must be made to leave the chromosome consistent (i.e. have real sequence in it).

2.15.3. Value Encoding :

Direct value encoding can be used in travelling salesman problem, where some complicated value, such as real numbers, are used. Use of binary encoding for this type of problems would be very difficult.
In **value encoding**, every chromosome is a string of some values. Values can be anything connected to problem, form numbers, real numbers or chars to some complicated objects.

Chromosome A	1.2324 5.3243 0.4556 2.3293 2.4545
Chromosome B	ABDJEIFJDHDIERJFDLDFLFEGT
Chromosome C	(back), (back), (right), (forward), (left)

Example of chromosomes with value encoding

Value encoding is very good for some special problems. On the other hand, for this encoding is often necessary to develop some new crossover and mutation specific for the problem.

2.16. Crossover and Mutation :

Introduction :

Crossover and mutation are two basic operators of GA. Performance of GA very depends on them. Type and implementation of operators depends on encoding and also on a problem.
There are many ways how to do crossover and mutation. Some examples are given below.

2.16.1. Binary Encoding :

1. Crossover:

Single point crossover - one crossover point is selected, binary string from beginning of chromosome to the crossover point is copied from one parent, the rest is copied from the second parent

11001011+11011**111** = **11001111**

Two point crossover - two crossover point are selected, binary string from beginning of chromosome to the first crossover point is copied from one parent, the part from the first to the second crossover point is copied from the second parent and the rest is copied from the first parent

110010**11** + 11**0111**11 = **11011111**

Uniform crossover - bits are randomly copied from the first or from the second parent

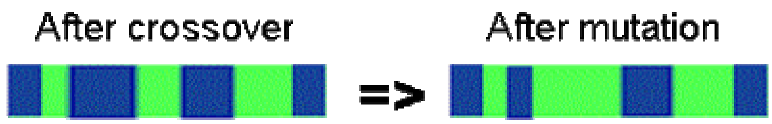
1**100**10**11** + **11**0**111**01 = 11011111

Arithmetic crossover - some arithmetic operation is performed to make a new offspring

11001011 + 11011111 = 11001001 (AND)

2. Mutation :

Bit inversion - selected bits are inverted



1**1**001001 => 1**0**001001

2.16.2. Permutation Encoding :

1. Crossover :

Single point crossover - one crossover point is selected, till this point the permutation is copied from the first parent, then the second parent is scanned and if the number is not yet in the offspring it is added.
(**1 2 3 4 5** 6 7 8 9) + (**4 5 3 6 8** 9 7 2 1) = (**1 2 3 4 5** 6 8 9 7)

2. Mutation :

Order changing - two numbers are selected and exchanged
(1 **2** 3 4 5 6 **8** 9 7) => (1 **8** 3 4 5 6 **2** 9 7)

2.17.3. Value Encoding :

Crossover :
All crossovers from **binary encoding** can be used.

Mutation :
Adding a small number (for real value encoding) - to selected values is added (or subtracted) a small number
(1.29 5.68 **2.86 4.11** 5.55) => (1.29 5.68 **2.73 4.22** 5.55)

2.17. Recommendations:

2.17.1. Parameters of GA :

These recommendations are very general. Probably you will want to experiment with your own GA for specific problem, because today there is no general theory which would describe parameters of GA for *any* problem.
Recommendations are often results of some empiric studies of GAs, which were often performed only on binary encoding.

2.17.2. Crossover rate:

Crossover rate generally should be high, about **80%-95%**. (However some results show that for some problems crossover rate about 60% is the best.)

2.17.3. Mutation rate:

On the other side, mutation rate should be very low. Best rates reported are about **0.5%-1%**.

2.17.4. population size:

It may be surprising, very big population size usually does not improve performance of GA (in meaning of speed of finding solution). Good population size is about **20-30**, however sometimes sizes 50-100 are reported as best. Some research also shows, that best population size depends on encoding, on **size of encoded string**. It means, if you have chromosome with 32 bits, the population should be say 32, but surely two times more than the best population size for chromosome with 16 bits.

2.17.5. Selection:

Basic **roulette wheel selection** can be used many times.

2.17.6. Encoding:

Encoding **depends on the problem** and also on the size of instance of the problem.

2.17.7. Crossover and mutation type:

Operators depend on encoding and on the problem.

2.18. Applications of GA :

Genetic algorithms have been used for machine learning and also for evolving simple programs. They have been also used for some art, for evolving pictures and music.

Advantage of GAs is in their parallelism. GA is travelling in a search space with more individuals so they are less likely to get stuck in a local extreme like some other methods.

They are also easy to implement. Once you have some GA, you just have to write new chromosome (just one object) to solve another problem. With the same encoding you just change the fitness function and it is all.On the other hand, choosing encoding and fitness function can be difficult.

Disadvantage of GAs is in their computational time. They can be slower than some other methods. But with todays computers it is not so big problem.

Chapter 3

3. Literature on Designing of Control Charts:

Park, Lee and Kim (2004) developed an economic model for a Variable Sampling Rate (VSR) control chart and also applied here to evaluate the efficiency of the VSR EWMA chart. The properties of the VSR EWMA chart are obtained by using a Markov chain approach. The model contains cost parameters which allow the specification of the costs associated with sampling, false alarms and operating off target as well as search and repair. This economic model can be used to quantify the cost saving that can be obtained by using a VSR chart instead of fixed Sampling Rate (FSR) chart and can also be used to gain insight into the way that a VSR chart should be designed to achieve optimal economic performance. It is shown that with some design parameter combinations the economically optimal VSR chart has a lower false alarm rate than the FSR chart.

Wang and Chen (1995) Considers the problem of determining economic statistical np -control chart designs under the fuzzy environment of closely satisfying type I and II errors. Goes on to model the problem as fuzzy mathematical programming, and uses a heuristic method to obtaining the solution.

RAHIM and OHTA (2002) opine that the production process is subject to an assignable cause which shifts the process from an in-control state to an out-of-control state. Shifts in both the process mean and the process variance are considered. Under these conditions, a generalized economic model for the joint determination of production quantity, an inspection schedule, and the design of the .*X* and *R* control charts are developed. A direct search optimization method is used to determine the optimal decision variables of the economic model.

Bendaya and Rahim (2000) develop an integrated model for the joint optimization of the economic production quantity, the economic design of (x) bar control chart and the optimal maintenance level. In the proposed model, Preventive Maintenance (PM) activities reduce the shift rate to the out-of-control state proportional to the PM level. Compared to the case with no PM, the extra cost of maintenance results in lower quality control cost which may lead to lower overall expected cost. These issues are illustrated using an example of a Weibull shock model with an increasing hazard rate.

Groenevelt, Pintelon and Seidmann (1992) present two novel extensions to the EMQ model. These extensions are aimed at incorporating stochastic machine breakdowns and deal with analysing the optimal lot size and the associated reorder policy. The purpose is to give bounds to optimal lot sizes for the above two extensions. With these bounds, a simple algorithm to locate the optimal lot sizes will be developed.

Makis and Fung (1998) proposed that in classical Economic Manufacturing Quantity (EMQ) model, all items produced are of perfect quality and the production facility never breaks down. However, in real production, the product quality is usually a function of the state of the production process which may deteriorate over time and the production facility may fail randomly. In this paper, we study the effect of machine failures on the optimal lot size and on the optimal number of inspections in a production cycle. The formula for the long-run expected average cost per unit time is obtained for a generally distributed time to failure. An optimal production/inspection policy is found by minimising the expected average cost.

Kim and Hong (1997) present an EMQ model which determines an optimal lot size in a failure prone machine. It is assumed that time between failures of a machine is generally distributed, and a machine is repaired instantaneously when it fails. Depending on various types of failure rate function of a machine, it is discussed how to determine an EMQ and prove its uniqueness. Variations of an EMQ depending on repair cost are also examined. Through numerical experiments, extensive investigations are carried out on the effects of repair cost and setup cost to an EMQ as well as average cost, and some interesting behaviors are observed.

Makis (1995) analyzed the problem of obtaining the optimal initial level and the optimal resetting time of a “tool-wear” process with a positive shift in the mean value subject to random failure. The cost includes the resetting cost, penalty for failure and a cost due to deviation of the quality characteristic from its target value, which is a quadratic loss function. The failure mechanism is described by the proportional hazards model. A formula for the expected average cost per unit time is derived and it is shown that the optimal solution can be obtained by solving a system of two nonlinear equations.

Sultan and Fawzan (2002) consider the model of Rahim and Banerjee (1988) for a process with random linear drift. We present an extension of their model for a process having both upper and lower specification limits. The model finds the optimal initial setting of the process mean and the optimal cycle length. We use Hooke and Jeeves search algorithm to optimize the model, and provide a numerical example.

Weheba and Nickerson (2004) developed a comprehensive cost model to incorporate two cost functions. A reactive function, which accounts for all quality related costs incurred while maintaining a stable level of the process, and a proactive function, which accounts for the cost of process improvement. Using incremental economics, the two cost functions are assembled to allow an evaluation of process improvement alternatives based on their economic worth. Procedures for obtaining economically optimum designs for controlling the process mean are developed and designed experiments are utilized to investigate model performance over a wide range of input parameters. The results indicate that the model is sensitive to changes in 13 parameters, especially when the magnitude of the process shift is small. Copyright © 2004 John Wiley & Sons, Ltd.

Celano G. and Fichera S (2001) proposed a new approach, based on an evolutionary algorithm, to solve this problem is proposed. The design of the chart has been developed considering the optimisation of the cost of the chart and at the same time the statistical proprieties. The proposed multiobjective approach has been compared to some well-known heuristics; the obtained results show the effectiveness of the evolutionary algorithm.

Chapter 4

4. Model Development and Optimisation for the X-bar Control Chart:

4.1. Introduction:

Control Charts are mainly used to establish and maintain statistical control of a process. It is always required to consider the design of a control chart from an economical point of view. Because the choice of control chart parameters affect the whole cost. The three main control chart parameters are:

(i) sample size(**n**),
(ii) sampling frequency or interval between the samples(**h**),
(iii) width of the control limits(**k**).

Selection of these three parameters is usually called ***the Design of the Control Chart.***
In any production process, regardless of how well designed or carefully maintained it is, a certain amount of inherent or natural variability will always exists. This is called ***chance-causes*** and it is said to be ***statically under control.***
Other kinds of variability which arise mainly from three sources such as: Improperly adjusted or controlled machines, operator errors or defective raw material. This is called ***assignable-causes*** and it is said to be ***out of control***.

The above three parameters(n, h, k) which controls the costs of sampling and testing, cost associated with investigating out-of-control signals and possibly correcting assignable causes and costs of allowing non-conforming units to reach the required quality.

4.2. Process Characteristics:

To formulate an economic model for the design of a control chart, we have to make certain assumptions about the behaviour of the process. The process is assumed to be characterized by a single in-control state. i.e. If the process has one measurable quality characteristics, then the in-control state will correspond to the mean of this quality characteristics when no assignable causes are present.

The process may have greater than one out-of-control states. Where each out-of-control state is usually associated with a particular type of assignable cause. To determine the nature of the transition between the in-control and out-of-control states requires certain assumptions. According to Poisson's process assignable cause occur during an interval of time. It indicates that the length of time the process remains in the in-control state, given that it begins in in-control, is an experimental random variable. This assumption simplifies the development of economical models. The nature in which process shifts occur is sometimes called the process failure mechanism. So it implies that process transitions between states are instantaneous.

Moreover the process is not self-correcting. That is, once a transition to an out-of-control state has occurred, the process can be returned to the in-control condition only by management intervention following the out-of-control signal on the control chart.

4.3. Cost parameters:

In the design of control charts, three categories of costs are considered.

(i) costs of sampling and testing,

(ii) costs associated with investigating an out-of-control signal and with the repair or correction of any assignable causes found,

(iii) costs associated with the production of non-conforming items.

Usually the cost of sampling and testing is assumed to consists of both fixed and variable components, say a_1 & a_2 resptively. Such that the total cost of sampling and testing is:

$$(a_1+a_2*n) \quad \text{where } \mathbf{n}\text{->sample size}$$

The 2nd one is the costs of investigating the assignable cause and possibly correcting an out-of-control state. Cost of investigating an false alarm will differ from the costs of correcting assignable cause. These two costs are represented in the model by two different cost parameters.

The costs associated with producing non-coforming items consist of typical failure costs with external failure. i.e. the costs of rework or scrap for internal failures, or replacement or repair costs for units covered by warranties in the case o external failure.

Economic models are generally formulated using a total cost function, which expresses the relationships between the control chart design parameters and the above three types of costs. The production, monitoring and adjustment process are a series of independent cycles over time. Each cycle begins with the production process in the in-control state and continues until process monitoring via the control chart results in an out-of-control signal. After adjustment it returned to the in-control state to begin another cycle. Then the expected cost per unit time is,

$$E(A) = \frac{E(C)}{E(T)}$$

where **C, T** are dependent random variables.

The above equation is optimized to design economically optimal control chart. The sequence of production-monitoring-adjustment, with accumulation of costs over the cycle, can be represented by a particular type of stochastic process called *Renewal Reward Process.* Where time cost is given by the ratio of the expected reward per cycle to the expected cycle length.

4.5. An Economic Model of the Control Chart:

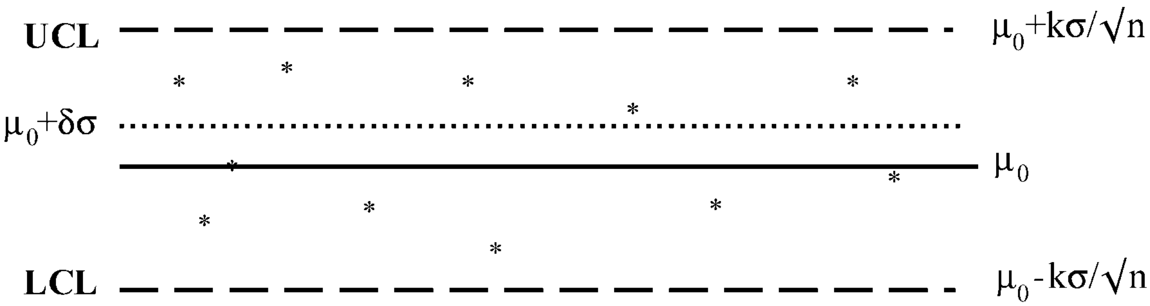
The design of an *x*-control chart requires the determination of various design parameters. These include the size of the sample ***(n)*** drawn in each interval, the sampling interval ***(h)*** and the upper and lower control limits coefficient***(k)***.
So we can conclude that cost is a function of **n, h, k.** i.e. **f(n, h, k).**

Here cost is directly proportional to sample size, no of false alarm and inversely proportional to sampling interval, width of control limits.

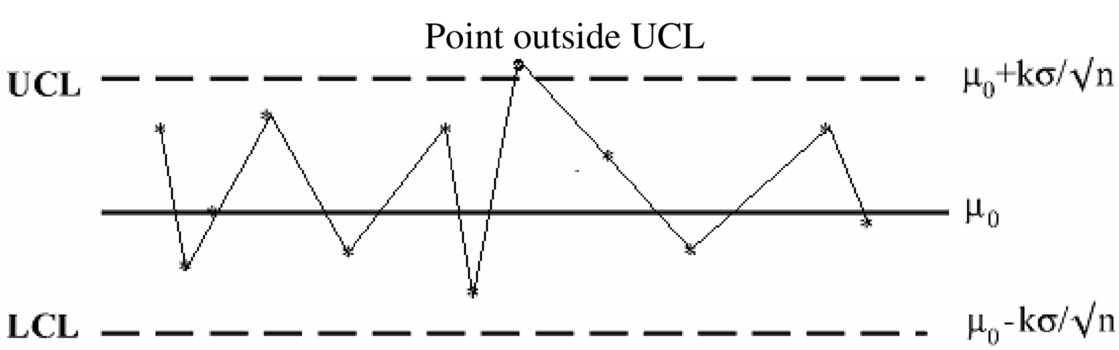
Inspection is necessary to determine the control state of the process in order that the penalties associated with the probabilities of Type I and Type II errors can be minimized. The following costs are important in determining the decision variables in the economic design of *x*-control charts: sampling cost, search cost and the cost of operating both in control and out of control. It is assumed that output quality is measurable on a continuous scale and is normally distributed. When the process is in control, the initial mean is μ_0; however, due to the occurrence of an assignable cause, the initial mean may be shifted from μ_0 to $\mu_0 + \delta\sigma$ or $\mu_0 - \delta\sigma$ (out-of-control state), where δ is the shift parameter and σ is the standard deviation. The control limit of the *x*-control charts is set at $\mu 0 \pm k$ times the standard deviation of the sample means, where *k* is known as the control limit coefficient, such that

$$UCL = \mu 0 + k\sigma/\sqrt{n}$$

$$LCL = \mu 0 - k\sigma/\sqrt{n}$$



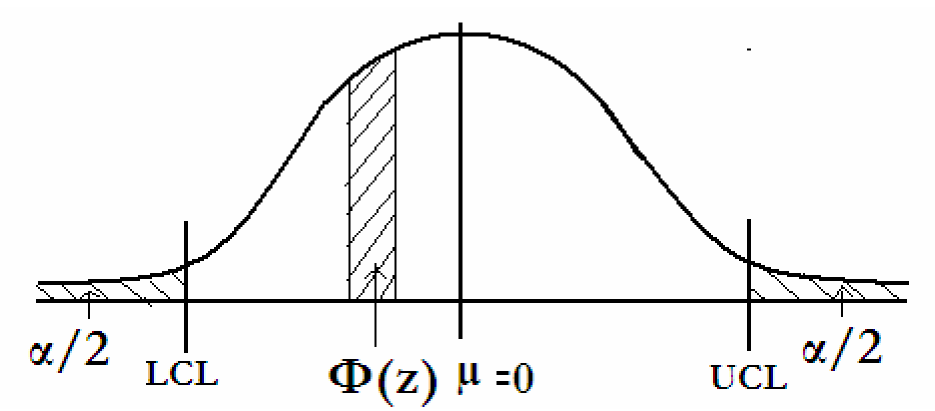
Here a fixed length of sampling and a constant failure rate over each interval were assumed. A fixed sample of size ***n*** is taken from output every ***h*** hours. Whenever the sample mean falls outside the specification limits of the product, the result signals that the process has shifted to an out-of-control state.



Hence, appropriate actions such as identifying the assignable cause and restorative work may be undertaken to bring the process back to an in-control state. Otherwise, the out-of-control state will continue until the end of the production run.
The assignable cause is assumed to according to a Poisson's process with an intensity of **λ** occurrences per hour. That is the process begins in the in-control state, the time interval that the process begins in the in-control state,
The time interval that the process remains in control is an exponential random variable with mean **1/λ h**. Therefore the occurrence of the assignable cause between two consecutive interval (let **jth** & **(j+1)th)** is:

$$\tau = \frac{\int_{jh}^{(j+1)h} \exp(-\lambda t)\,(t-jh)\,dt}{\int_{jh}^{(j+1)h} \exp(-\lambda t)\,\lambda\,dt} = \frac{1-(1+\lambda h)\exp(-\lambda h)}{\lambda\,(1-\exp(-\lambda h))}$$

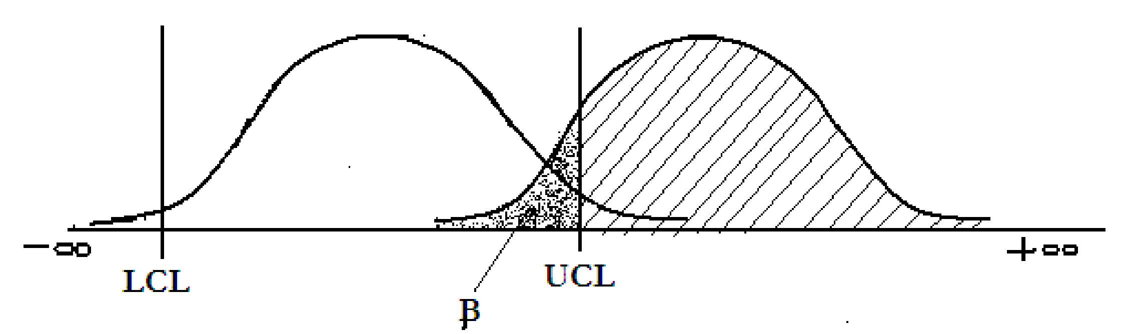
There are two situations which may result in wrong decisions. The first situation is caused by a Type I error, meaning that the process is in control, but an out-of-control signal is reported. Here the symbol α is used to represent the probability of a Type I error. The diagram for this is:



The probability of false alarm is:

$$\alpha = 2 \int_{k}^{\infty} \Phi(z)\, dz$$

The second situation occurs when the process is shifted to an out-of-control state and the control chart fails to report the out-of control condition, this is defined as a Type II error. Here the symbol *β* is used to represent the probability of a Type II error. So *β* is the probability of not detecting the shift. Where as (1- *β*) is the probability of detecting the error, so it is called power of the test.The diagram for this is:

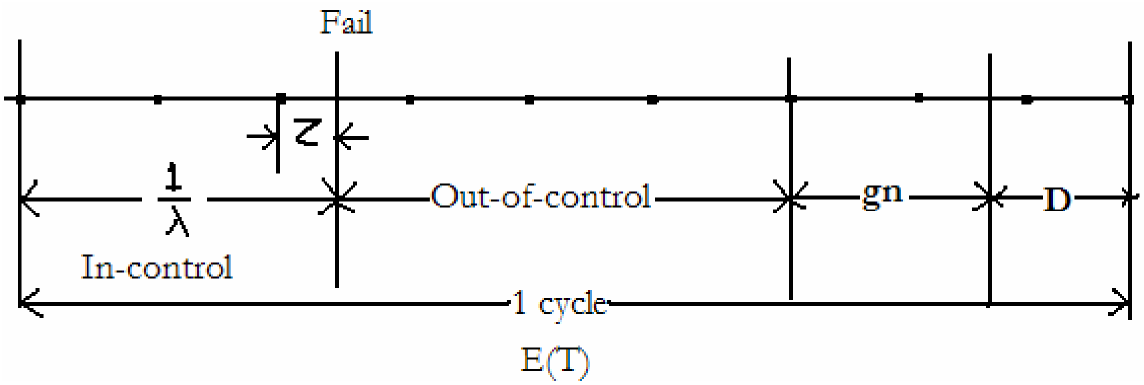


The probability that it will be detected on any subsequent sample is:

$$1-\beta = \int_{(-\infty)}^{(-k-\delta\sqrt{n})} \Phi(z)\, dz + \int_{(k-\delta\sqrt{n})}^{\infty} \Phi(z)\, dz$$

A production cycle is defined as the interval of time from the start of production where the process is assumed to start in in-control state following an adjustment to the detection and elimination of the assignable cause. The cycle consists of four periods:

(i) in-control period
(ii) out-of-control period
(iii) time to take a sample and interpret the result
(iv) time to find the assignable cause



The expected length of in-control period is **1/λ**.Noting the no of samples required to produce an out-of-control signal is a geometric random variable with a mean $1/(1-\beta)$ which is called Average Run Length(**ARL**). Average Run Length is the average no of samples required for the sample to fall out side control limits. we conclude that the expected length of out-of-control is $1/(1-\beta)-\tau$. The time required to take a sample and interpret the result is a constant **g** proportional to the sample size, so that **gn** is the length of the cycle. The time required to find the assignable cause following an action signal is a constant **D.** Therefore the expected length of a cycle is:

$$E(T)= 1/\lambda + (h/(1-\beta) - \tau)+ gn + D$$

Let the net income per hour of operation in the in-control state is $\mathbf{V_0}$ and the net income per hour of operation in the out-of-control state is $\mathbf{V_1}$. The cost of taking a sample of size **n** is assumed to be of the form $(\mathbf{a_1+a_2*n})$. Where **a1** & **a2** represent the fixed and variable components of sampling cost. The expected no of samples taken within a cycle is the expected cycle length divided by the interval between samples i.e. **E(T)/h**. The cost of finding an assignable cause is $\mathbf{a_3}$ and the cost of investigating a false alarm

is $\mathbf{a^1_3}$. The expected no of false alarms generated during a cycle is α times the expected number of samples taken before the shift, so:

$$\alpha \sum_{j=0}^{\infty} \int_{jth}^{(j+1)th} j \exp(-\lambda t)\, dt = \alpha \exp(-\lambda h)/(1- \exp(-\lambda h))$$

Therefore, the expected net income per cycle is:

$$V_0(1/\lambda)+V1(h/(1-\beta)-\tau+gn+D)-a_3- a^1{}_3\alpha\exp(-\lambda h)/(1-\exp(-\lambda h)) -(a_1+a_2*n)E(T)/h$$

The expected net income per hour can be found by dividing **E(C)** by **E(T).** So the result becomes:

$$E(A)=E(C)/E(T)$$

$$E(T)= \frac{V_0(1/\lambda)+V1(h/(1-\beta)-\tau+gn+D)-a_3- a^1{}_3\alpha\exp(-\lambda h)/(1-\exp(-\lambda h)) -(a_1+a_2 * n)E(T)/ h}{1/\lambda + (h/(1-\beta) - \tau)+ gn + D}$$

Let $\mathbf{a_4=V_0- V_1}$: where a4 is the hourly penalty cost associated with production in the out-of-control state.

So **E(A)** can also be written as:

$$E(A)=V_0 - \frac{(a_1+a_2*n)}{h} \frac{-a_4(h/(1- \beta)- \tau+gn+D)+a_3+a^1{}_3\, \alpha \exp(-\lambda/h)/ (1-\exp(-\lambda h))}{1/\lambda + (h/(1-\beta) - \tau)+ gn + D}$$

Or $E(A)=V_0-E(L)$

Where

$$E(L) = \frac{(a_1+a_2*n)}{h} + \frac{a_4(h/(1-\beta)-\tau+gn+D)+a_3+a^1_3\,\alpha\,\exp(-\lambda/h)/(1-\exp(-\lambda h))}{1/\lambda + (h/(1-\beta)-\tau)+gn+D}$$

Here **E(L)** represents the expected loss per hour by the process. **E(L)** is a function of the control parameters n, k, h. So it is clear that by minimizing **E(L)** we can maximize **E(A).**

So to optimize **E(A)** we take the 1st partial derivative of **E(L)** with respect to **n, k, h.** An iterative procedure is applied to solve for the optimal **n, k**. **E(L)** can also be minimized by using an unconstrained optimization or search technique coupled with a digital computer program for repeated evaluations of the cost function.

In this project I have optimized a particular problem where data given are:

1. $a1=1$
2. $a_2=0.01$
3. $a_3=25$
4. $a^1_3=50$
5. $a_4=100$
6. $\lambda=0.05$
7. $g=0.0167$
8. $D=1.0$

Here only unknowns are τ, α & β which can be calculated by the formulae given below.

$$\tau = \frac{\int_{jh}^{(j+1)h} \exp(-\lambda t)\,(t-jh)\,dt}{\int_{jh}^{(j+1)h} \exp(-\lambda t)\,\lambda\,dt} = \frac{1-(1+\lambda h)\exp(-\lambda h)}{\lambda\,(1-\exp(-\lambda h))}$$

Using above formula we can calculate τ.
The probability of false alarm is:

$$\alpha = 2\int_{k}^{\infty} \Phi(z)\,dz$$

The probability that it will be detected on any subsequent sample is:

$$1-\beta = \int_{(-\infty)}^{(-k-\delta\sqrt{n})} \Phi(z)\,dz + \int_{(k-\delta\sqrt{n})}^{\infty} \Phi(z)\,dz$$

So, β can be calculated as,

$$\beta = 1-\int_{(-\infty)}^{(-k-\delta\sqrt{n})} \Phi(z)\,dz + \int_{(k-\delta\sqrt{n})}^{\infty} \Phi(z)\,dz$$

Here α & β are calculated using integration, which are continuous in nature. To simplify the integral I've used the numerical method called Simpson's 1/3 rule which gives the
approximate integrals of the form

$$I = \int_a^b f(x)dx$$

where

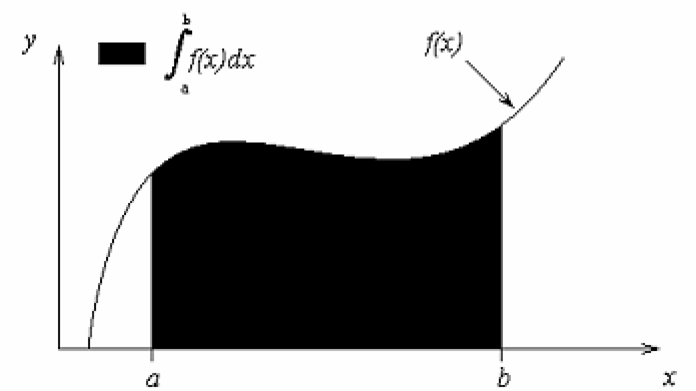
$f(x)$ is called the integrand,

$a =$ lower limit of integration

$b =$ upper limit of integration

Simpson's 1/3 Rule

The trapezoidal rule was based on approximating the integrand by a first order polynomial, and then integrating the polynomial over interval of integration. Simpson's 1/3 rule is an extension of Trapezoidal rule where the integrand is approximated by a second order polynomial.



$$I = \int_a^b f(x)dx$$

$$= \frac{h}{3}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i=even}}^{n-2} f(x_i) + f(x_n) \right]$$

Let $h = \dfrac{b-a}{n}$, where n is the no. of divisions. So we can write

$$\int_a^b f(x)dx \cong \frac{b-a}{3n}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i=even}}^{n-2} f(x_i) + f(x_n) \right]$$

Above is the discredited form of the integral. Using the above rule I've developed a programme to calculate the value of α & β which are unknown for the cost function. Putting the values of τ, α & β in the cost function we can easily get the total cost.

if we've a close look to the formula of cost then we can find that it solely depends upon **n, h & k,** which are the only variables and other data's are constant for a particular process.

So to minimize **E(L)** which is loss function per hour I've developed Genetic algorithm programme to get the optimal solution for **n, h, k** which is a iterative method itself.

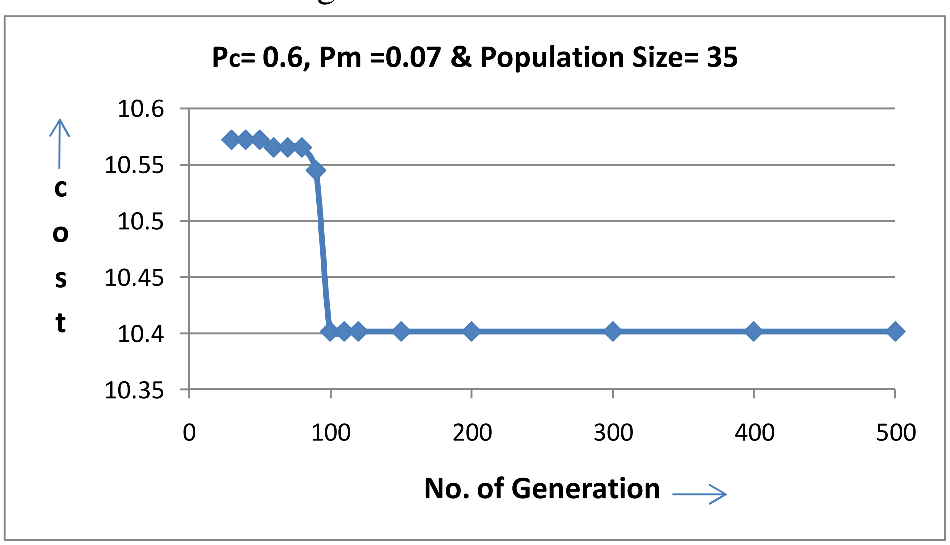
Chapter 5

5.1. Results and Discussion:

Table.1

Sl No.	No. of Generation	Cost
1	20	10.5725
2	30	10.5725
3	40	10.5725
4	50	10.5658
5	60	10.5658
6	70	10.5658
7	80	10.5452
8	90	10.4019
9	100	10.4019
10	110	10.4019
11	120	10.4019
12	150	10.4019
13	200	10.4019
14	300	10.4019
15	400	10.4019
16	500	10.4019

Here is the graph for table.2, which shows that the cost minimizes with the increase of the no. of generation.



So for the minimum cost 10.4019, value of :

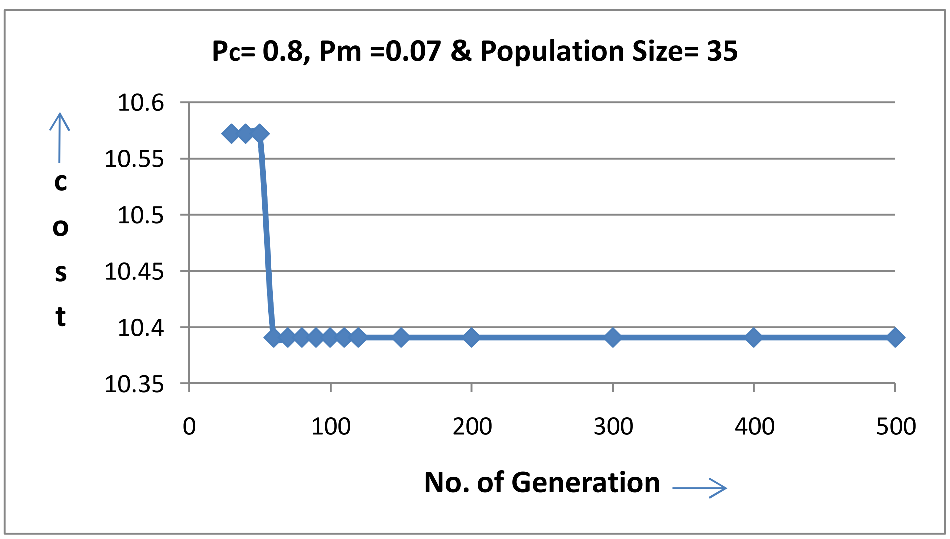
n= 6.046, h= 3.074, k= 0.824

α= 0.002233, power= 0.965623

Table.2

Sl No.	No. of Generation	Cost
1	20	10.5725
2	30	10.5725
3	40	10.5725
4	50	10.5725
5	60	10.3911
6	70	10. 3911
7	80	10. 3911
8	90	10.3911
9	100	10.3911
10	110	10.3911
11	120	10.3911
12	150	10.3911
13	200	10.3911
14	300	10.3911
15	400	10.3911
16	500	10.3911

Here is the graph for table.3, which shows that the cost minimizes with the increase of the no. of generation.



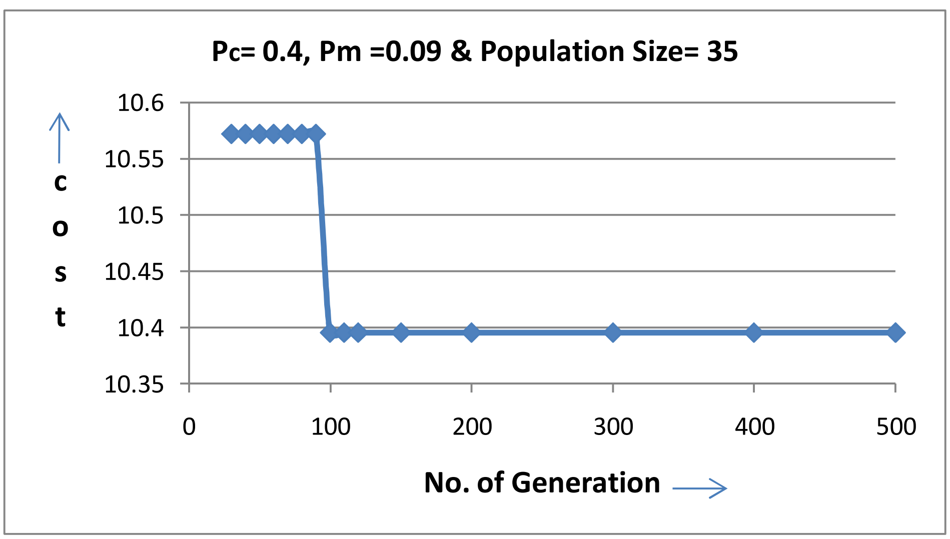
So for the minimum cost 10.3911, value of :

n= 5.089
h= 2.981
k= 0.835
α= 0.002950
power= 0.931452

Table.3

Sl No.	No. of Generation	Cost
1	20	10.5725
2	30	10.5725
3	40	10.5725
4	50	10.5725
5	60	10. 5725
6	70	10. 5725
7	80	10. 5725
8	90	10. 5725
9	100	10. 3957
10	110	10. 3957
11	120	10. 3957
12	150	10. 3957
13	200	10. 3957
14	300	10. 3957
15	400	10. 3957
16	500	10. 3957

Here is the graph for table.4, which shows that the cost minimizes with the increase of the no. of generation.



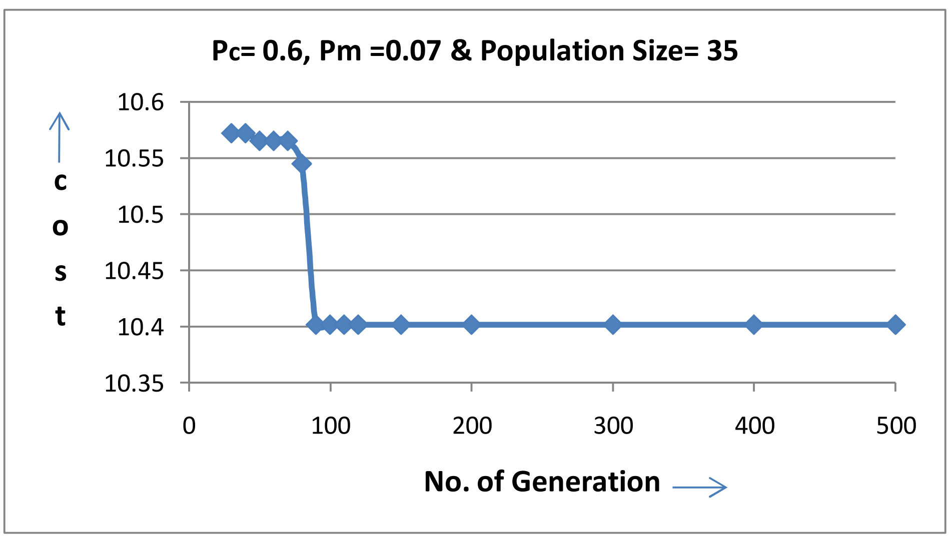
So for the minimum cost 10.3957, value of :

n= 5.089
h= 2.981
k= 0.835
α= 0.002950
power= 0.931452

Table.4

Sl No.	No. of Generation	Cost
1	20	10.5725
2	30	10.5725
3	40	10.5725
4	50	10.5658
5	60	10. 5658
6	70	10. 5658
7	80	10. 5452
8	90	10. 4019
9	100	10. 4019
10	110	10. 4019
11	120	10. 4019
12	150	10. 4019
13	200	10. 4019
14	300	10. 4019
15	400	10. 4019
16	500	10. 4019

Here is the graph for table.5, which shows that the cost minimizes with the increase of the no. of generation.



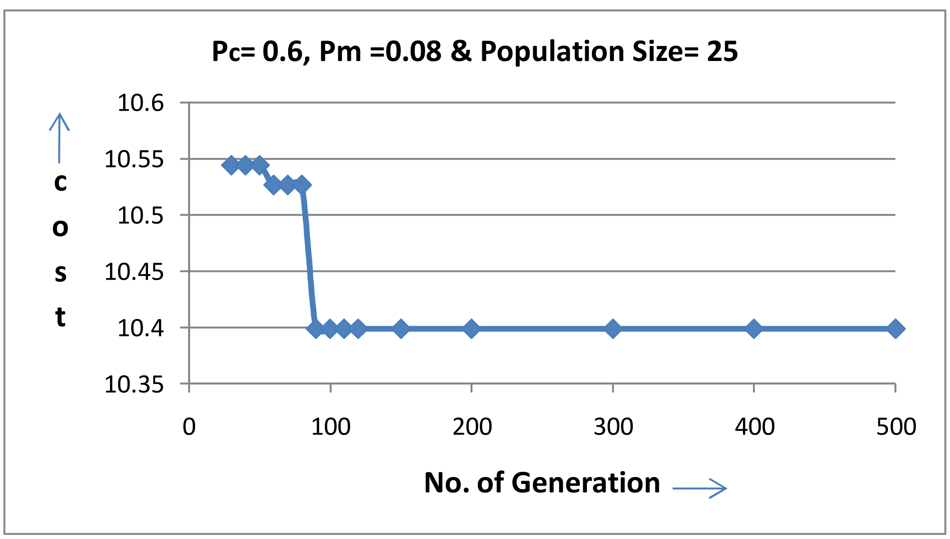
So for the minimum cost 10.4019, value of :

n= 6.046
h= 3.074
k= 0.824
α= 0.002233
power= 0.965623

Table.5

Sl No.	No. of Generation	Cost
1	20	10.5725
2	30	10.5449
3	40	10.5449
4	50	10.5449
5	60	10. 5270
6	70	10. 5270
7	80	10. 5270
8	90	10. 3989
9	100	10. 3989
10	110	10. 3989
11	120	10. 3989
12	150	10. 3989
13	200	10. 3989
14	300	10. 3989
15	400	10. 3989
16	500	10. 3989

Here is the graph for table.6, which shows that the cost minimizes with the increase of the no. of generation.



So for the minimum cost 10.3989, value of :

n= 6.046

h= 3.062

k= 0.872

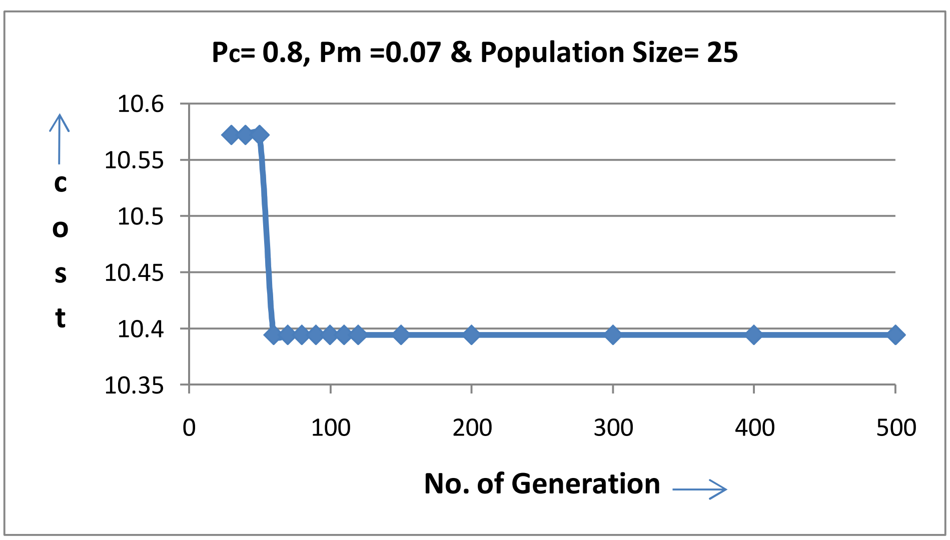
α= 0.002281

power= 0.966126

Table.6

Sl No.	No. of Generation	Cost
1	20	10.5725
2	30	10.5725
3	40	10. 5725
4	50	10. 5725
5	60	10. 3945
6	70	10. 3945
7	80	10. 3945
8	90	10. 3945
9	100	10. 3945
10	110	10. 3945
11	120	10. 3945
12	150	10. 3945
13	200	10. 3945
14	300	10. 3945
15	400	10. 3945
16	500	10. 3945

Here is the graph for table.7, which shows that the cost minimizes with the increase of the no. of generation.



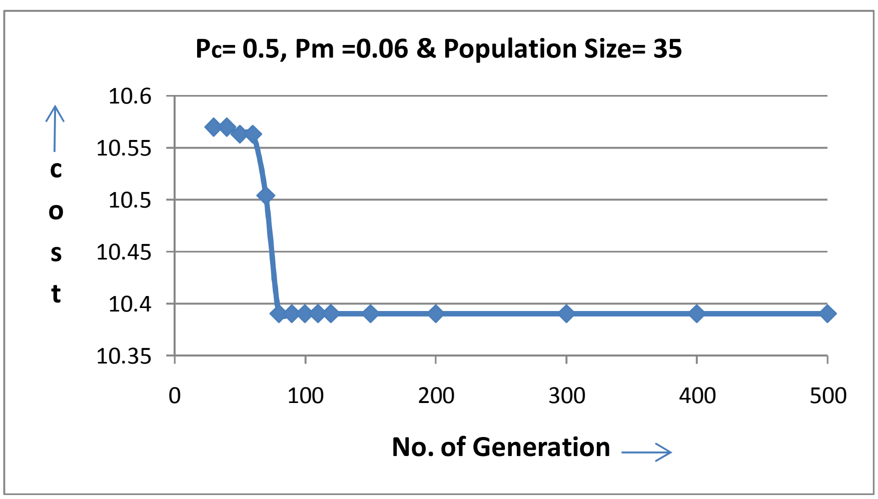
So for the minimum cost 10.3945, value of :

n= 5.060
h= 2.972
k= 0.764
α= 0.003079
power= 0.933198

Table.7

Sl No.	No. of Generation	Cost
1	20	10.5698
2	30	10.5698
3	40	10.5698
4	50	10.5632
5	60	10.5632
6	70	10.5041
7	80	10.3905
8	90	10.3905
9	100	10.3905
10	110	10.3905
11	120	10.3905
12	150	10.3905
13	200	10.3905
14	300	10.3905
15	400	10.3905
16	500	10.3905

Here is the graph for table.1, which shows that the cost minimizes with the increase of the no. of generation.



So for the minimum cost 10.3905, value of :

n=5.002
h=2.909
k=0.883
α=0.003799
power=0.940625

5.2. Conclusion

The obtained for Table.7 is the minimum which is 10.3905. The result obtained is quite appreciable as it is comparable with other optimization methods. So we can conclude that Genetic Algorithm is a good technique for the optimization of cost for X-bar chart.

Chapter 6

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