

**A THESIS ON**  
**DIGITAL IMAGE COMPRESSION USING DISCRETE**  
**COSINE TRANSFORM & DISCRETE WAVELET**  
**TRANSFORM**

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# **DIGITAL IMAGE COMPRESSION USING DISCRETE COSINE TRANSFORM & DISCRETE WAVELET TRANSFORM**

**A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE  
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By

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### **CERTIFICATE**

This is to certify that the thesis entitled “***IMAGE COMPRESSION USING DCT & DWT***” submitted by Sri Swastik Das(Roll No.10506008) & Sri Rasmi Ranjan Sethy(Roll NO .10506013) in partial fulfillment of the requirements for the award of Bachelor of Technology degree in Computer Science & Engineering at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

Date:

**Prof. R Baliarsingh**

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## **ABSTRACT**

Image Compression addresses the problem of reducing the amount of data required to represent the digital image. Compression is achieved by the removal of one or more of three basic data redundancies: (1) Coding redundancy, which is present when less than optimal (i.e. the smallest length) code words are used; (2) Interpixel redundancy, which results from correlations between the pixels of an image & (3) psycho visual redundancy which is due to data that is ignored by the human visual system (i.e. visually nonessential information). Huffman codes contain the smallest possible number of code symbols (e.g., bits) per source symbol (e.g., grey level value) subject to the constraint that the source symbols are coded one at a time. So, Huffman coding when combined with technique of reducing the image redundancies using Discrete Cosine Transform (DCT) helps in compressing the image data to a very good extent.

The Discrete Cosine Transform (DCT) is an example of transform coding. The current JPEG standard uses the DCT as its basis. The DC relocates the highest energies to the upper left corner of the image. The lesser energy or information is relocated into other areas. The DCT is fast. It can be quickly calculated and is best for images with smooth edges like photos with human subjects. The DCT coefficients are all real numbers unlike the Fourier Transform. The Inverse Discrete Cosine Transform (IDCT) can be used to retrieve the image from its transform representation.

The Discrete wavelet transform (DWT) has gained widespread acceptance in signal processing and image compression. Because of their inherent multi-resolution nature, wavelet-coding schemes are especially suitable for applications where scalability and tolerable degradation are important. Recently the JPEG committee has released its new image coding standard, JPEG-2000, which has been based upon DWT.

## **List of Acronyms**

<b>Acronym</b>	<b>Description</b>
DCT	Discrete Cosine Transformation
DWT	Discrete Wavelet Transformation
DFT	Discrete Fourier Transformation
FFT	Fast Fourier Transformation
FWT	Fast Wavelet Transformation
JPEG	Joint Photographic Expert Group
JPEG-2000	Joint Photographic Expert Group-2000
MPEG	Moving Pictures Experts Group
MSE	Mean Square Error
PSNR	Peak Signal to Noise Ratio
SNR	Signal-to-noise Ratio
ISO	International Standards Organization
LOT	Lapped Orthogonal Transforms
IEC	International Electro-Technical Commission
DPCM	Discrete pulse code modulation
FAX	Facsimile transmission
KLT	Karhunen L��ve Transform
IDCT	Inverse Discrete Cosine Transform
FDCT	A Forward Discrete Cosine Transform

## **List of Symbols**

<b>Symbol</b>	<b>Description</b>
$C_R$	Compression ratio
$R_D$	Relative data redundancy
$SNR_{ms}$	Mean-square Signal-to-noise ratio
$e_{rms}$	Root-mean-square error

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# Chapter :1

## Introduction

### 1.1 Overview

Uncompressed multimedia (graphics, audio and video) data requires considerable storage capacity and transmission bandwidth. Despite rapid progress in mass-storage density, processor speeds, and digital communication system performance, demand for data storage capacity and data-transmission bandwidth continues to outstrip the capabilities of available technologies. The recent growth of data intensive multimedia-based web applications have not only sustained the need for more efficient ways to encode signals and images but have made compression of such signals central to storage and communication technology.

For still image compression, the 'Joint Photographic Experts Group' or JPEG [7] standard has been established by ISO (International Standards Organization) and IEC (International Electro-Technical Commission). The performance of these coders generally degrades at low bit-rates mainly because of the underlying block-based Discrete Cosine Transform (DCT) scheme. More recently, the wavelet transform has emerged as a cutting edge technology, within the field of image compression. Wavelet-based coding provides substantial improvements in picture quality at higher compression ratios.

Over the past few years, a variety of powerful and sophisticated wavelet-based schemes for image compression, as discussed later, have been developed and implemented. Because of the many advantages, the top contenders in the upcoming JPEG-2000 standard are all wavelet-based compression algorithms.

### 1.2 Objectives

Image Compression addresses the problem of reducing the amount of data required to represent the digital image. Compression is achieved by the removal of one or more of three basic data redundancies: (1) Coding redundancy, which is present when less than optimal (i.e. the smallest length) code words are used; (2) Interpixel redundancy, which results from correlations between the pixels of an image; &/or (3) psycho visual redundancy which is due to data that is ignored by the human visual system (i.e. visually nonessential information). Huffman codes contain the smallest possible number of code symbols (e.g., bits) per source symbol (e.g., grey level value) subject to the constraint that the source symbols are coded one at a time. So, Huffman coding when combined with technique of reducing the image redundancies using Discrete Cosine Transform (DCT) helps in compressing the image data to a very good extent.

## **1.3 Organization of the Thesis**

Rest of the thesis is organized into the following chapters :

A discussion on Fundamentals of image compression, Different classes of compression technique ,a typical image coder are given in Chapter 2.

Image compression using discrete cosine transform is proposed in Chapter 3. The objective is to achieve a reasonable compression ratio as well as better quality of reproduction of image with a low power consumption. Simulation result shows that a compression ratio of 2.0026 is achieved.

Image compression using discrete wavelet transform is proposed in Chapter 4.

Parameters associated with the compression process are analysed & the conclusion is given.

# Chapter :2

## IMAGE COMPRESSION

### 2.1 Preview

Image compression addresses the problem of reducing the amount of data required to represent a digital image .The underlying basis of the reduction process is the removal of redundant data. From a mathematical viewpoint, this amounts to transforming a 2-D pixel array into a statistically uncorrelated data set .The transformation is applied prior to storage or transmission of the image. At some later time, the compressed image is decompressed to reconstruct the original image or approximation of it.

Interest in image compression dates back more than 35 years. The initial focus of research efforts in this field was on the development of analog methods for reducing video transmission bandwidth, a process called bandwidth compression. The advent of the digital computer and subsequent development of advanced integrated circuits, however, caused interest to shift from analog to digital compression approaches. with the relatively recent adaption of several key international image compression standards ,the field has undergone significant growth through the practical application of the theoretic work that began in the 1940s, when C.E Shannon and others first formulated the probabilistic view of information and its representation , transmission, and compression[1].

Currently image compression is recognized as an “enabling technology”. In addition to the areas Just mentioned ,image compression is the natural technology for handling the increased spatial resolution of today’s imaging sensors and evolving broadcast television standards. Furthermore image compression plays a major role in many important and diverse applications , including televideo-conferencing ,remote sensing(the use of satellite imagery for weather and other earth –resource applications), document and medical imaging ,facsimile transmission(FAX)[3], and the control of remotely piloted vehicles in military , space and hazardous waste management applications.

## 2.2 Need of compression

The figures in Table 1.1 show the qualitative transition from simple text to full-motion video data and the disk space, transmission bandwidth, and transmission time needed to store and transmit such uncompressed data.

Multimedia Data	Size/Duration	Bits/Pixel or Bits/Sample	Uncompressed Size (B for bytes)	Transmission Bandwidth (b for bits)	Transmission Time (using a 28.8K Modem)
A page of text	11" x 8.5"	Varying resolution	4-8 KB	32-64 Kb/page	1.1 - 2.2 sec
Telephone quality speech	10 sec	8 bps	80 KB	64 Kb/sec	22.2 sec
Grayscale Image	512 x 512	8 bpp	262 KB	2.1 Mb/image	1 min 13 sec
Color Image	512 x 512	24 bpp	786 KB	6.29 Mb/image	3 min 39 sec
Medical Image	2048 x 1680	12 bpp	5.16 MB	41.3 Mb/image	23 min 54 sec
SHD Image	2048 x 2048	24 bpp	12.58 MB	100 Mb/image	58 min 15 sec
Full-motion Video	640 x 480, 1 min (30 frames/sec)	24 bpp	1.66 GB	221 Mb/sec	5 days 8 hrs

**Table 1.1 Multimedia data types and uncompressed storage space, transmission bandwidth, and transmission time required. The prefix kilo- denotes a factor of 1000 rather than 1024.**

The examples above clearly illustrate the need for sufficient storage space, large transmission bandwidth, and long transmission time for image, audio, and video data. At the present state of technology, the only solution is to compress multimedia data before its storage and transmission, and decompress it at the receiver for play back. For example, with a compression ratio of 32:1, the space, bandwidth, and transmission time requirements can be reduced by a factor of 32, with acceptable quality[2].

other hand, first transforms the image from its spatial domain representation to a different type of representation using some well-known transform and then codes the transformed values (coefficients). This method provides greater data compression compared to predictive methods, although at the expense of greater computation.



## 2.3 Fundamentals of image compression

The term data compression refers to the process of reducing the amount of data required to represent a given quantity of information. A common characteristic of most images is that the neighboring pixels are correlated and therefore contain redundant information. The foremost task then is to find less correlated representation of the image. Two fundamental components of compression are redundancy and irrelevancy reduction. Redundancy reduction aims at removing duplication from the signal source (image/video). Irrelevancy reduction omits parts of the signal that will not be noticed by the signal receiver, namely the Human Visual System (HVS).

It is not an abstract concept but a mathematically quantifiable entity. If  $n_1$  and  $n_2$  denote the number of information-carrying units in the two data sets that represent the same information, the relative data redundancy  $R_D$  of the first data set (the one characterized by  $n_1$ ) can be defined as

$$R_D = 1 - 1/C_R$$

Where  $C_R$ , commonly called the compression ratio, is

$$C_R = n_1/n_2.$$

For the case  $n_2 = n_1$  and  $R_D = 0$ , indicating that (relative to the second data set) the first representation of the information contains no redundant data. When  $n_2 \ll n_1$ ,  $C_R \rightarrow \infty$  and  $R_D \rightarrow 1$ , implying significant compression and highly redundant data. When  $n_2 \gg n_1$ ,  $C_R \rightarrow 0$  and  $R_D \rightarrow -\infty$ , indicating that the second data set contains much more data than the original representation. Generally  $C_R = 10(10:1)$  defines that the first data set has 10 information carrying units for every 1 unit in the second or compressed data set. Thus the corresponding redundancy of 0.9 means 90 percent of the data in the first data set is redundant with respect to the second one.[1]

In order to be useful, a compression algorithm has a corresponding decompression algorithm that reproduces the original file once the compressed file is given. There have been many types of compression algorithms developed. These algorithms fall into two broad types, lossless algorithms and lossy algorithms. A lossless algorithm reproduces the data exactly same as the original one. A lossy algorithm, as its name implies, loses some data. Data loss may be unacceptable in

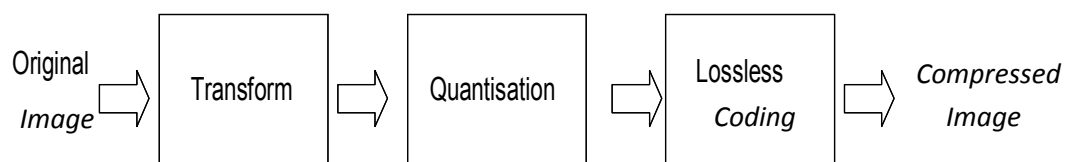
many applications. For example, text compression must be lossless because a very small difference can result in statements with totally different meanings. There are also many situations where loss may be either unnoticeable or acceptable. In image compression, for example, the exact reconstructed value of each sample of the image

## 2.4 Image Compression and Reconstruction

Three basic data redundancies can be categorized in the image compression standard.

1. Spatial redundancy due to the correlation between neighboring pixels.
2. Spectral redundancy due to correlation between the color components.
3. Psycho-visual redundancy due to properties of the human visual system.

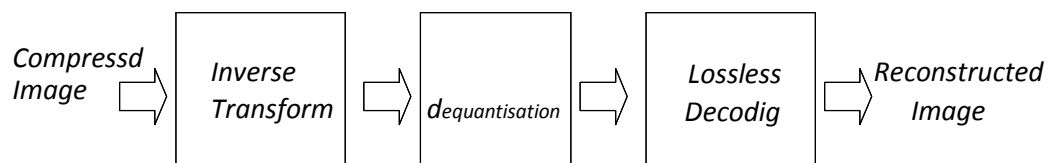
The spatial and spectral redundancies are present because certain spatial and spectral patterns between the pixels and the color components are common to each other, whereas the psycho-visual redundancy originates from the fact that the human eye is insensitive to certain spatial frequencies. The principle of image compression algorithms are (i) reducing the redundancy in the image data and (or) (ii) producing a reconstructed image from the original image with the introduction of error that is insignificant to the intended applications. The aim here is to obtain an acceptable representation of digital image while preserving the essential information contained in that particular data set.[2]



***Figure 1.1 Image compression System***

The problem faced by image compression is very easy to define, as demonstrated in figure 1.1. First the original digital image is usually transformed into another domain, where it is highly de-correlated by using some transform. This de-correlation concentrates the important image information into a more compact form. The compressor then removes the redundancy in the transformed image and stores it into a compressed file or data stream. In the second stage, the quantisation block reduces the accuracy of the transformed output in accordance with some pre- established fidelity criterion. Also this stage reduces the psycho-visual redundancy of the input image. Quantisation operation is a reversible process and thus may be omitted when there is a need of error free or lossless compression. In the final stage of the data compression model the symbol coder creates a fixed or variable-length code to represent the quantiser output and maps the output in accordance with the code. Generally a variable-length code is used to represent the mapped and quantised data set. It assigns the shortest code words to the most frequently occurring output values and thus reduces coding redundancy. The operation in fact is a reversible one.

The decompression reverses the compression process to produce the recovered image as shown in figure 1.2. The recovered image may have lost some information due to the compression, and may have an error or distortion compared to the original image.



**Figure 1.2 Image decompression System**

## 2.5 DIFFERENT CLASSES OF COMPRESSION TECHNIQUES.

Two ways of classifying compression techniques are mentioned here.

**(a) Lossless vs. Lossy compression:** In lossless compression schemes, the reconstructed image, after compression, is numerically identical to the original image. However lossless compression can only achieve a modest amount of compression. An image reconstructed following lossy compression contains degradation relative to the original. Often this is because the compression scheme completely discards redundant information. However, lossy schemes are capable of achieving much higher compression. Under normal viewing conditions, no visible loss is perceived (visually lossless).

**(b) Predictive vs. Transform coding:** In predictive coding, information already sent or available is used to predict future values, and the difference is coded. Since this is done in the image or spatial domain, it is relatively simple to implement and is readily adapted to local image characteristics. Differential Pulse Code Modulation (DPCM) is one particular example of predictive coding [3]. Transform coding, on the other hand, first transforms the image from its spatial domain representation to a different type of representation using some well-known transform and then codes the transformed values (coefficients). This method provides greater data compression compared to predictive methods, although at the expense of greater computation.

## 2.6 A TYPICAL IMAGE CODER .

A typical lossy image compression system is shown in Fig. 1.3. It consists of three closely connected components namely (a) Source Encoder (b) Quantizer, and (c) Entropy Encoder. Compression is accomplished by applying a linear transform to decorrelate the image data, quantizing the resulting transform coefficients, and entropy coding the quantized values.

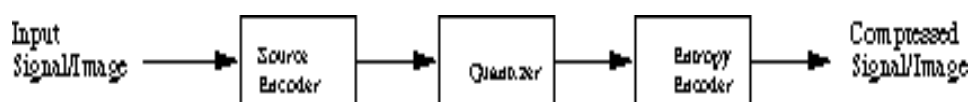


Fig. 1. 3 A Typical Lossy Signal/Image Encoder

### **2.6.1 Source Encoder (or Linear Transformer)**

Over the years, a variety of linear transforms have been developed which include Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT) , Discrete Wavelet Transform (DWT) and many more, each with its own advantages and disadvantages.

### **2.6.2 Quantizer**

A quantizer simply reduces the number of bits needed to store the transformed coefficients by reducing the precision of those values. Since this is a many-to-one mapping, it is a lossy process and is the main source of compression in an encoder. Quantization can be performed on each individual coefficient, which is known as Scalar Quantization (SQ). Quantization can also be performed on a group of coefficients together, and this is known as Vector Quantization (VQ) [6] . Both uniform and non-uniform quantizers can be used depending on the problem at hand. .

### **2.6.3 Entropy Encoder**

An entropy encoder further compresses the quantized values losslessly to give better overall compression. It uses a model to accurately determine the probabilities for each quantized value and produces an appropriate code based on these probabilities so that the resultant output code stream will be smaller than the input stream. The most commonly used entropy encoders are the Huffman encoder and the arithmetic encoder, although for applications requiring fast execution, simple run-length encoding (RLE) has proven very effective [6].

It is important to note that a properly designed quantizer and entropy encoder are absolutely necessary along with optimum signal transformation to get the best possible compression.

## **2.7 Motivation**

Image compression is an important issue in digital image processing and finds extensive applications in many fields. This is the basic operation performed frequently by any digital photography technique to capture an image. For longer use of the portable photography device it should consume less power so that battery life will be more. To improve the Conventional techniques of image compressions using the DCT have already been reported and sufficient literatures are available on this. The JPEG is a lossy compression scheme, which employs the DCT as a tool and used mainly in digital cameras for compression of images. In the recent past the demand for low power image compression is growing. As a result various research workers are actively engaged to evolve efficient methods of image compression using latest digital signal processing techniques. The objective is to achieve a reasonable compression ratio as well as better quality of reproduction of image with a low power consumption. Keeping these objectives in mind the research work in the present thesis has been undertaken. In sequel the following problems have been investigated.

# Chapter :3

## IMAGE COMPRESSION USING DISCRETE COSINE TRANSFORM

### Preview:

Discrete cosine transform (DCT) is widely used in image processing, especially for compression. Some of the applications of two-dimensional DCT involve still image compression and compression of individual video frames, while multidimensional DCT is mostly used for compression of video streams. DCT is also useful for transferring multidimensional data to frequency domain, where different operations, like spread-spectrum, data compression, data watermarking, can be performed in easier and more efficient manner. A number of papers discussing DCT algorithms is available in the literature that signifies its importance and application.[5]

Hardware implementation of parallel DCT transform is possible, that would give higher throughput than software solutions. Special purpose DCT hardware decreases the computational load from the processor and therefore improves the performance of complete multimedia system. The throughput is directly influencing the quality of experience of multimedia content. Another important factor that influences the quality is the finite register length effect that affects the accuracy of the forward-inverse transformation process.

Hence, the motivation for investigating hardware specific DCT algorithms is clear. As 2-D DCT algorithms are the most typical for image compression, the main focus of this chapter will be on the efficient hardware implementations of 2-D DCT based compression by decreasing the number of computations, increasing the accuracy of reconstruction, and reducing the chip area. This in return reduces the power consumption of the compression technique. As the number of applications that require higher-dimensional DCT algorithms are growing, a special attention will be paid to the algorithms that are easily extensible to higher dimensional cases.

The JPEG standard has been around since the late 1980's and has been an effective first solution to the standardization of image compression. Although JPEG has some very useful strategies for DCT quantization and compression, it was only developed for low compressions. The  $8 \times 8$  DCT block size was chosen for speed(which is less of an issue now, with the advent of faster processors) not for performance. The JPEG standard will be briefly explained in this chapter to provide a basis to understand the new DCT related work [7].

### **3.1 The Process:-**

The following is the general overview of the JPEG process. Later we will go through the a detailed tour of JPEG's method so that a more comprehensive understanding of the process may be acquired.

1. The image is broken into  $8 \times 8$  blocks of pixels.
2. Working from left to right, top to bottom, the DCT is applied to each block.
3. Each block is compressed through quantization.
4. The array of compressed blocks that constitute the image is stored in a drastically reduced amount of space.
5. When desired the image is constructed through decompression, a process that uses the Inverse Discrete Cosine Transform(IDCT).



### 3.2 DISCRETE COSINE TRANSFORM (DCT) EQUATION :-

The DCT is a widely used transformation in transformation for data compression. It is an orthogonal transform, which has a fixed set of (image independent) basis functions, an efficient algorithm for computation, and good energy compaction and correlation reduction properties. Ahmed et al found that the Karhunen Løve Transform (KLT) basis function of a first order Markov image closely resemble those of the DCT [7]. They become identical as the correlation between the adjacent pixel approaches to one.

The DCT belongs to the family of discrete trigonometric transform, which has 16 members [44]. The 1D DCT of a  $1 \times N$  vector  $x(n)$  is defined as

$$Y[k] = C[k] \sum_{n=0}^{N-1} x[n] \cos \left[ \frac{(2n+1)k\pi}{2N} \right]$$

where  $k = 0, 1, 2, \dots, N-1$  and

$$C[k] = \begin{cases} \sqrt{\frac{1}{N}} & \text{for } k = 0 \\ \sqrt{\frac{1}{N}} & \text{for } k = 1, 2, \dots, N-1 \end{cases}$$

The original signal vector  $x(n)$  can be reconstructed back from the DCT coefficients  $Y[k]$  using the Inverse DCT (IDCT) operation and can be defined as

$$x[n] = \sum_{k=0}^{N-1} C[k] Y[k] \cos \left[ \frac{(2n+1)k\pi}{2N} \right]$$

where  $n = 0, 1, 2, \dots, N-1$

The DCT can be extended to the transformation of 2D signals or images. This can be achieved in two steps: by computing the 1D DCT of each of the individual rows of the two-dimensional image and then computing the 1D DCT of each column of the image. If  $x(n_1, n_2)$  represents a 2D image of size  $N \times N$ , then the 2D DCT of an image is given by:

$$Y[j, k] = C[j] C[k] \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} x[m, n] \cos\left(\frac{(2m+1)j\pi}{2N}\right) \cos\left(\frac{(2n+1)k\pi}{2N}\right) \quad \dots \text{equ.3.1}$$

where  $j, k, m, n = 0, 1, 2, \dots, N-1$  and

$$C[j] \text{ and } C[k] = \begin{bmatrix} \sqrt{\frac{1}{N}} & \text{for } j, k = 0 \\ \sqrt{\frac{1}{N}} & \text{for } j, k = 1, 2, \dots, N-1 \end{bmatrix}$$

Similarly the 2D IDCT can be defined as

$$x[m, n] = \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} C[j] C[k] Y[j, k] \cos\left(\frac{(2m+1)j\pi}{2N}\right) \cos\left(\frac{(2n+1)k\pi}{2N}\right) \quad \dots \text{equ. 3.2}$$

The DCT is a real valued transform and is closely related to the DFT. In particular, a  $N \times N$  DCT of  $x(n_1, n_2)$  can be expressed in terms of DFT of its even-symmetric extension, which leads to a fast computational algorithm. Because of the even-symmetric extension process, no artificial discontinuities are introduced at the block boundaries. Additionally the computation of the DCT requires only real arithmetic. Because of the above properties the DCT is popular and widely used for data compression operation.

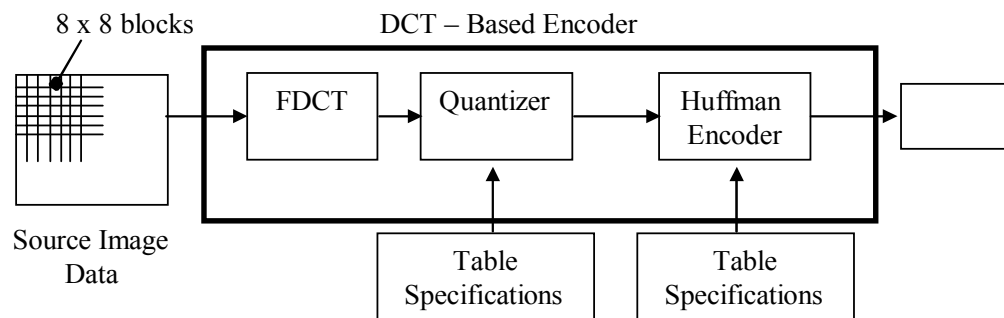
The DCT presented in equations (3.1) and (3.2) is orthonormal and perfectly reconstructing provided the coefficients are represented to an infinite precision. This means that when the coefficients are compressed it is possible to obtain a full range of compressions and image qualities. The coefficients of the DCT are always quantized for high compression, but DCT is very resistant to quantisation errors due to the statistics of the coefficients it produces. The coefficients of a DCT are usually linearly quantised by dividing by a predetermined quantisation step.

The DCT is applied to image blocks  $N \times N$  pixels in size (where  $N$  is usually multiple of 2) over the entire image. The size of the blocks used is an important factor since they determine the effectiveness of the transform over the whole image. If the blocks are too small then the images is not effectively decorrelated but if the blocks are too big then local features are no longer exploited. The tiling of any transform across the image leads to artifacts at the block boundaries. The DCT is associated with blocking artifact since the JPEG standard suffers heavily from this at higher compressions. However the DCT is protected against blocking artifact as effectively as possible, without interconnecting blocks, since the DCT basis functions all have a zero gradient at the edges of their blocks. This means that only the DC level significantly affects the blocking artifact and this can then be targeted. Ringing is a major problem in DCT operation. When edges occur in an image DCT relies on the high frequency components to make the image shaper. However these high frequency components persist across the whole block and although they are effective at improving the edge quality they tend to 'ring' in the flat areas of the block.

This ringing effect increases, when larger blocks are used, but larger blocks are better in compression terms, so a trade off is usually established [1].

### 3.3 JPEG Compression

The JPEG (Joint Photographic Experts Group) standard has been around for some time and is the only standard for lossy still image compression. There are quite a lot of interesting techniques used in the JPEG standard and it is important to give an overview of how JPEG works. There are several variations of JPEG, but only the 'baseline' method is discussed here.



**Figure 1.4: JPEG Encoder**

As shown in the figure 3.1, the image is first partitioned into non-overlapping  $8 \times 8$  blocks. A Forward Discrete Cosine Transform (FDCT) is applied to each block to convert the spatial domain gray levels of pixels into coefficients in frequency domain. To improve the precision of the DCT the image is 'zero shifted', before the DCT is applied. This converts a  $0 \rightarrow 255$  image intensity range to a  $-128 \rightarrow 127$  range, which works more efficiently with the DCT. One of these transformed values is referred to as the DC coefficient and the other 63 as the AC coefficients [4].

After the computation of DCT coefficients, they are normalized with different scales according to a quantization table provided by the JPEG standard conducted by psychovisual evidence. The quantized coefficients are rearranged in a zigzag scan order for further compressed by an efficient lossless coding algorithm such as runlength coding, arithmetic coding, Huffman coding. The decoding process is simply the inverse process of encoding as shown in figure 1.5.

The decoding process is simply the inverse process of encoding as shown in figure 1.5.

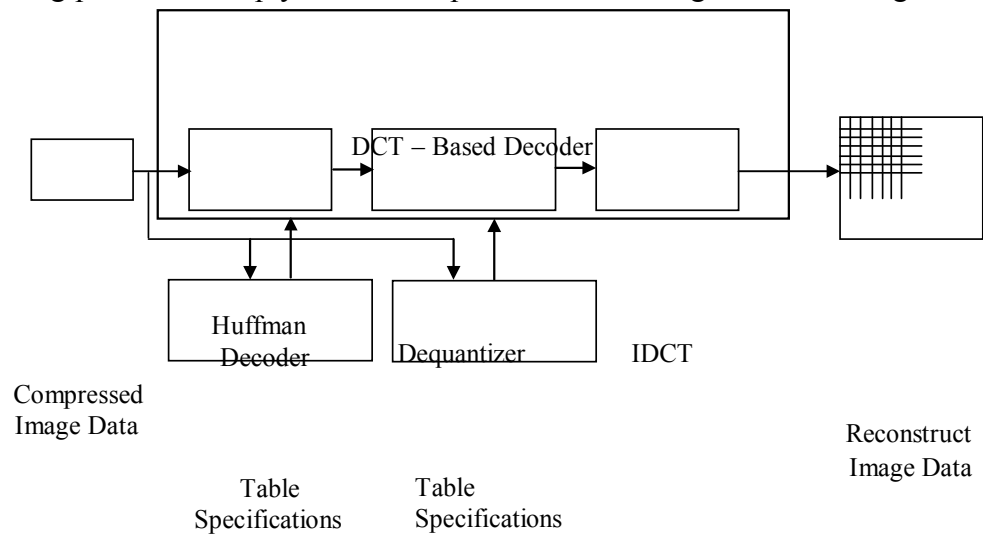


Figure 1.5 : JPEG Decoder

### 3.4 QUANTISATION

DCT-based image compression relies on two techniques to reduce the data required to represent the image. The first is *quantization* of the image's DCT coefficients; the second is entropy coding of the quantized coefficients. Quantization is the process of reducing the number of possible values of a quantity, thereby reducing the number of bits needed to represent it. In lossy image compression the transformation decompose the image into uncorrelated parts projected on orthogonal basis of the transformation. These basis are represented by eigenvectors which are independent and orthogonal in nature. Taking inverse of the transformed values will result in the retrieval of the actual image data. For compression of the image, the independent characteristic of the transformed coefficients are considered, truncating some of these coefficients will not affect the others. This truncation of the transformed coefficients is actually the lossy process involved in compression and known as quantization . So we can say that DCT is perfectly reconstructing, when all the coefficients are calculated and stored to their full resolution. For high compression, the DCT coefficients are normalized by different scales, according to the quantization matrix [6].

Vector quantization, (VQ) mainly used for reducing or compressing the image data . Application VQ on images for compression started from early 1975by Hilbert mainly for the coding of multispectral Landsat imaginary.

### 3.5 CODING

After the DCT coefficients have been quantized, the DC coefficients are DPCM coded and then they are entropy coded along with the AC coefficients. The quantized AC and DC coefficient values are entropy coded in the same way, but because of the long runs in the AC coefficient, an additional run length process is applied to them to reduce their redundancy.

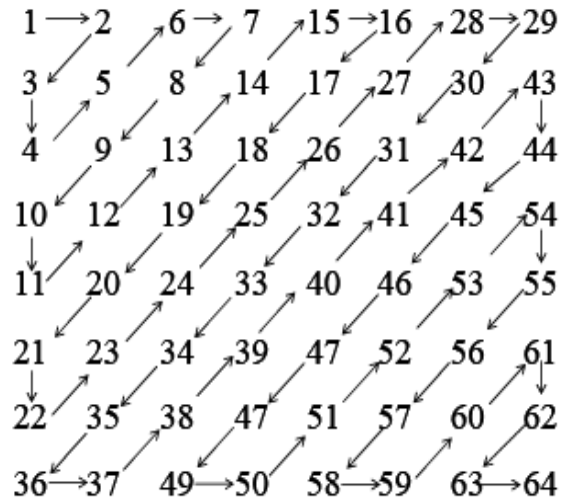


Figure 3.4: Zigzag Scanning

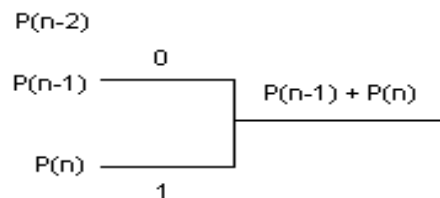
The quantized coefficients are all rearranged in a zigzag order as shown in figure 3.4. The run length in this zigzag order is described by a RUN-SIZE symbol. The RUN is a count of how many zeros occurred before the quantized coefficient and the SIZE symbol is used in the same way as it was for the DC coefficients, but on their AC counter parts. The two symbols are combined to form a RUN-SIZE symbol and this symbol is then entropy coded. Additional bits are also transmitted to specify the exact value of the quantized coefficient. A size of zero in the AC coefficient is used to indicate that the rest of the  $8 \times 8$  block is zeros (End of Block or EOB) [7].

### 3.5.1 HUFFMAN CODING:

Huffman coding is an efficient source coding algorithm for source symbols that are not equally probable. A variable length encoding algorithm was suggested by Huffman in 1952, based on the source symbol probabilities  $P(x_i)$ ,  $i=1,2,\dots,L$ . The algorithm is optimal in the sense that the average number of bits required to represent the source symbols is a minimum provided the prefix condition is met. The steps of Huffman coding algorithm are given below [7]:

1. Arrange the source symbols in increasing order of their probabilities.

2. Take the bottom two symbols & tie them together as shown in Figure 3. Add the probabilities of the two symbols & write it on the combined node. Label the two branches with a '1' & a '0' as depicted in Figure 3



3. Treat this sum of probabilities as a new probability associated with a new symbol. Again pick the two smallest probabilities, tie them together to form a new probability. Each time we perform the combination of two symbols we reduce the total number of symbols by one. Whenever we tie together two probabilities (nodes) we label the two branches with a '0' & a '1'.
4. Continue the procedure until only one procedure is left (& it should be one if your addition is correct). This completes the construction of the Huffman Tree.
5. To find out the prefix codeword for any symbol, follow the branches from the final node back to the symbol. While tracing back the route read out the labels on the branches. This is the codeword for the symbol.

The algorithm can be easily understood using the following example :

**TABLE**

**1.2**

Symbol	Probability	Codeword	Code length
X1	0.46	1	1
X2	0.30	00	2
X3	0.12	010	3
X4	0.06	0110	4
X5	0.03	01110	5
X6	0.02	011110	6
X7	0.01	011111	6



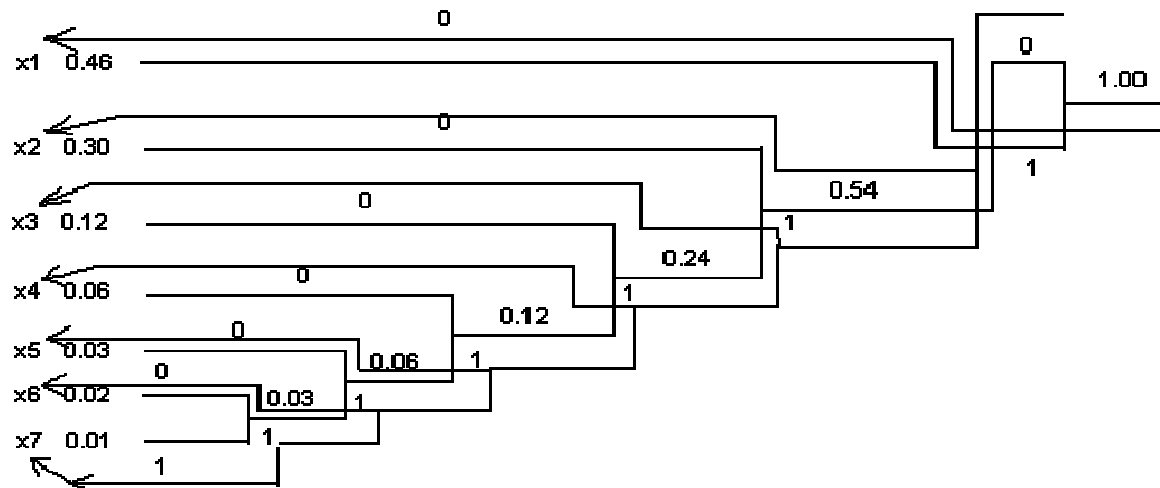


FIGURE: 1.6

Huffman Coding for Table

### 3.5.2 HUFFMAN DECODING

The Huffman Code in Table 1 & FIGURE 4 is an instantaneous uniquely decodable block code. It is a block code because each source symbol is mapped into a fixed sequence of code symbols. It is instantaneous because each codeword in a string of code symbols can be decoded without referencing succeeding symbols. That is, in any given Huffman code, no codeword is a prefix of any other codeword. And it is uniquely decodable because a string of code symbols can be decoded only in one way. Thus any string of Huffman encoded symbols can be decoded by examining the individual symbols of the string in left to right manner. Because we are using an instantaneous uniquely decodable block code, there is no need to insert delimiters between the encoded pixels.

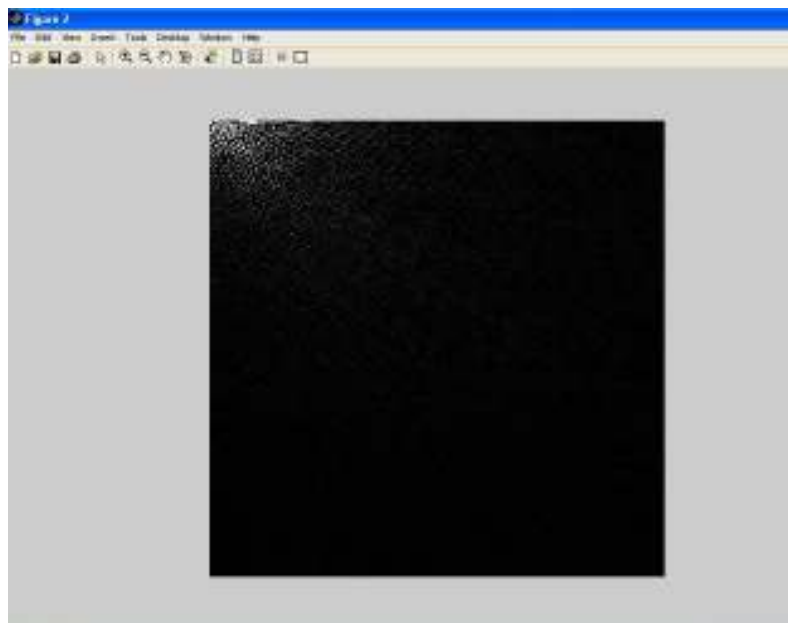
For Example consider a 19 bit string 1010000111011011111 which can be decoded uniquely as x1 x3 x2 x4 x1 x1 x7 [7].

A left to right scan of the resulting string reveals that the first valid code word is 1 which is a code symbol for, next valid code is 010 which corresponds to x1, continuing in this manner, we obtain a completely decoded sequence given by x1 x3 x2 x4 x1 x1 x7.

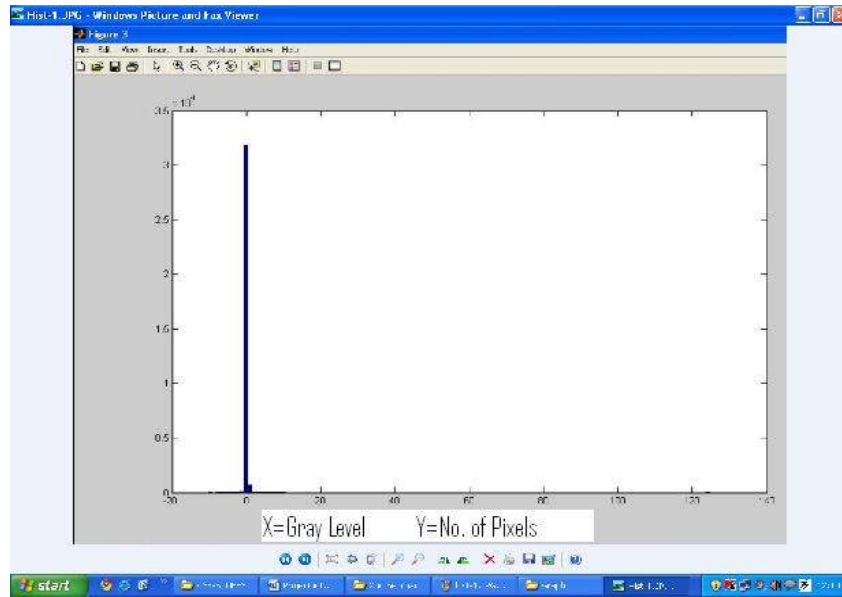
### 3.6 RESULT :-



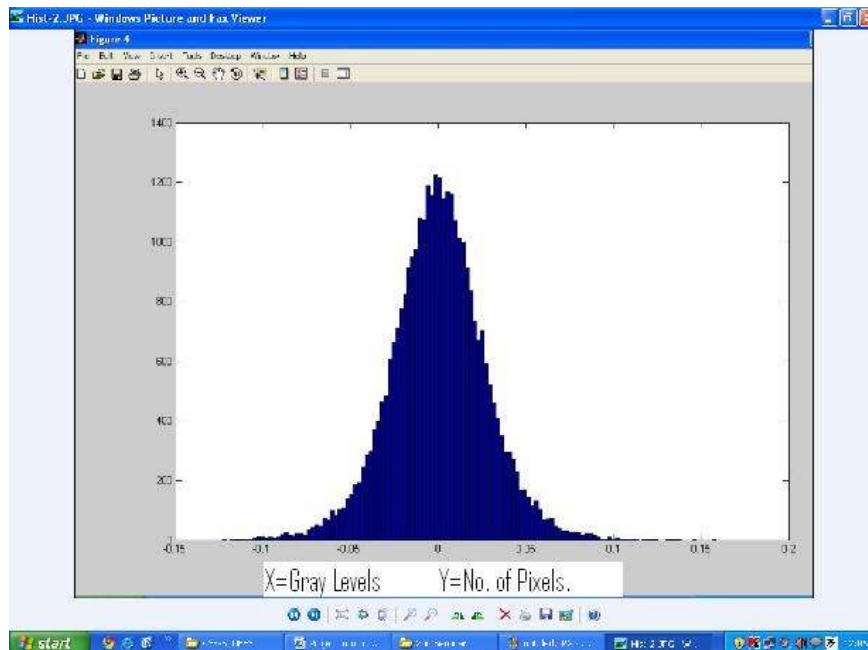
3.6.1 - Original input image(1.7)



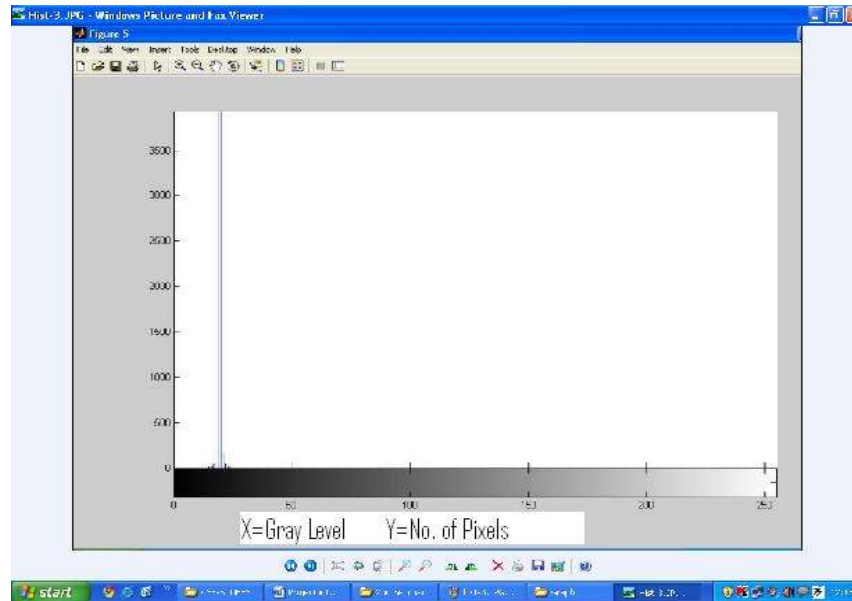
3.6.2 - Image after applying DC (1.8)



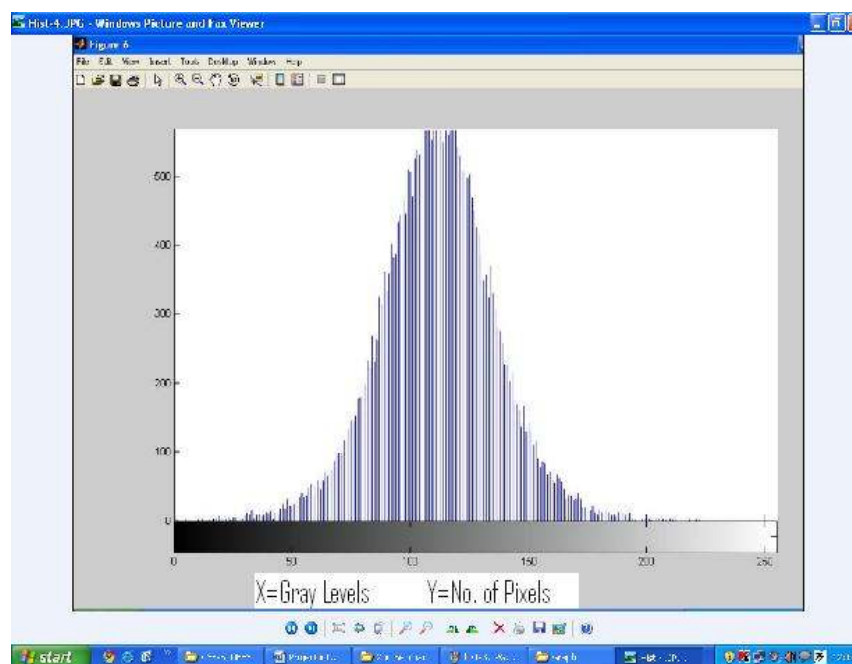
3.6.3 - Histogram of the DCT coefficients of the upper half of the image before quantization(1.9)



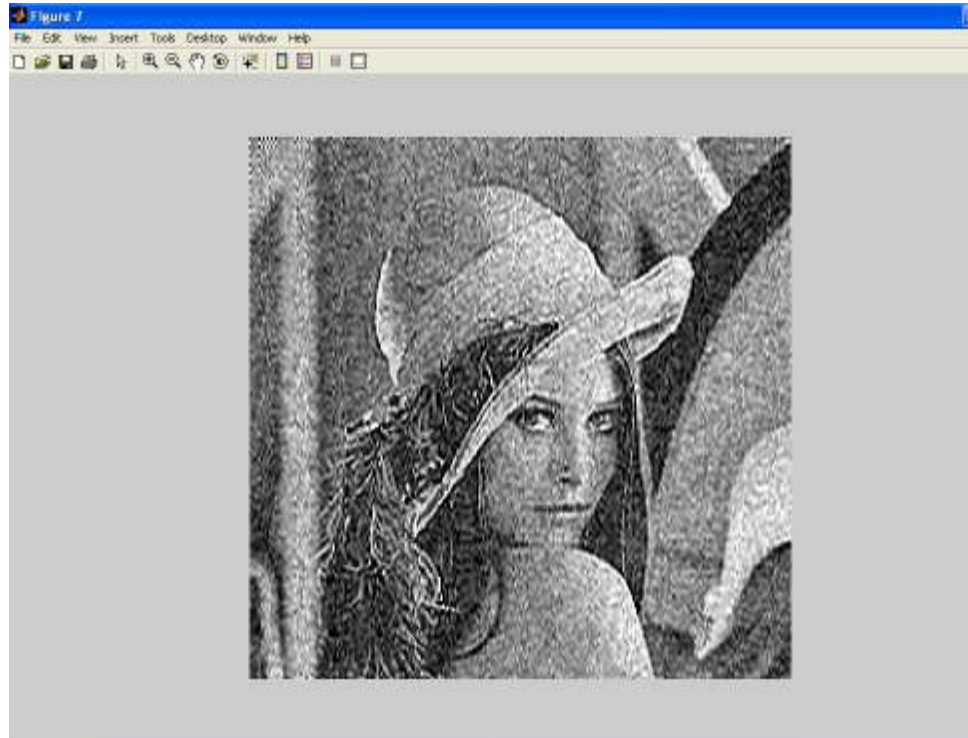
3.6.4 Histogram of the DCT coefficients of the lower half of the image before quantization.(2.0)



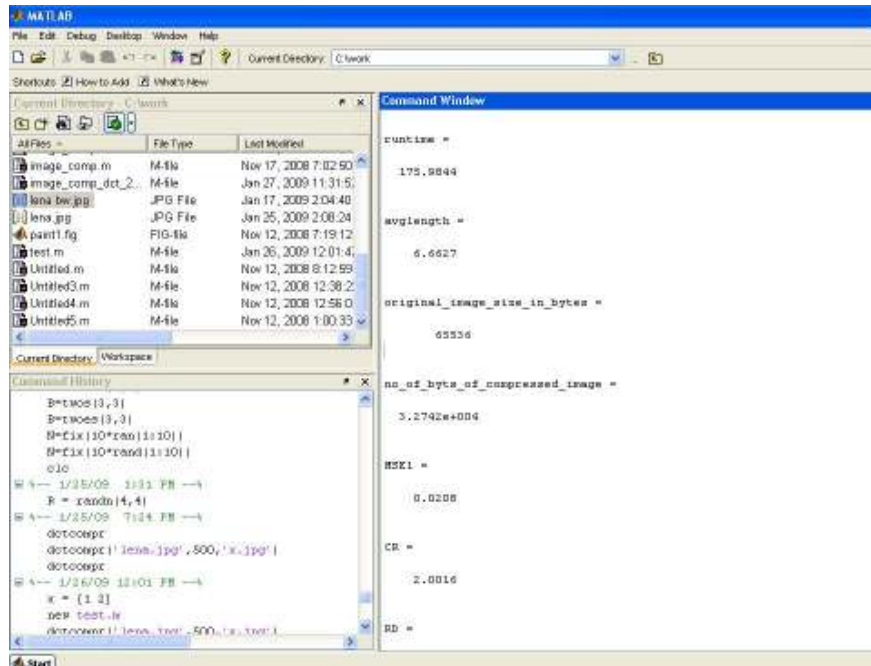
3.6.5 Histogram of the DCT coefficients of the upper half of the image after quantization.(2.1)



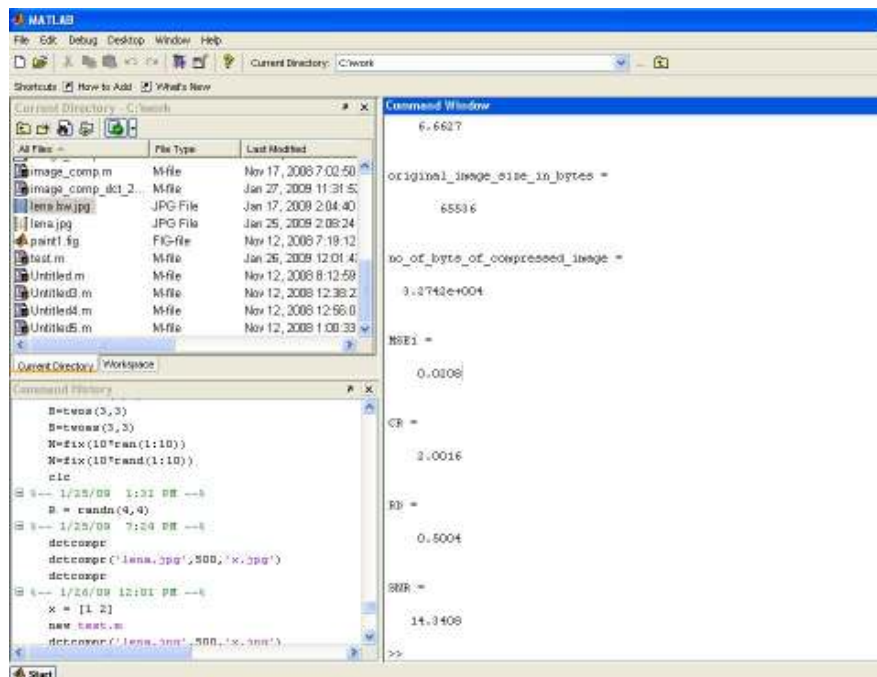
3.6.6 Histogram of the DCT coefficients of the lower half of the image after quantization.(2.2)



3.6.7 - Image after compression(2.3)



3.6.8 - Parameters associated with the output image(2.4)



3.6.9 - Parameters associated with the output image(2.5)

# Chapter :4

## IMAGE COMPRESSION USING DISCRETE WAVELET TRANSFORM :-

The wavelet transform (WT) has gained widespread acceptance in signal processing and image compression. Because of their inherent multi-resolution nature, wavelet-coding schemes are especially suitable for applications where scalability and tolerable degradation are important. Recently the JPEG committee has released its new image coding standard, JPEG-2000, which has been based upon DWT [1].

### 4.1 What is a Wavelet Transform ?

Wavelets are functions defined over a finite interval and having an average value of zero. The basic idea of the wavelet transform is to represent any arbitrary function ( $t$ ) as a superposition of a set of such wavelets or basis functions. These basis functions or baby wavelets are obtained from a single prototype wavelet called the mother wavelet, by dilations or contractions (scaling) and translations (shifts). The Discrete Wavelet Transform of a finite length signal  $x(n)$  having  $N$  components, for example, is expressed by an  $N \times N$  matrix.

### 4.2 Why Wavelet-based Compression?

Despite all the advantages of JPEG compression schemes based on DCT namely simplicity, satisfactory performance, and availability of special purpose hardware for implementation, these are not without their shortcomings. Since the input image needs to be "blocked," correlation across the block boundaries is not eliminated. This results in noticeable and annoying "blocking artifacts" particularly at low bit rates as shown in Fig 2.6 . Lapped Orthogonal Transforms (LOT) [9] attempt to solve this problem by using smoothly overlapping blocks. Although blocking effects are reduced in LOT compressed images, increased computational complexity of such algorithms do not justify wide replacement of DCT by LOT.



(a)



(b)

**Fig. 2.6 (a) Original Lena Image, and (b) Reconstructed Lena with DC component only, to show blocking artifacts**

Over the past several years, the wavelet transform has gained widespread acceptance in signal processing in general, and in image compression research in particular. In many applications wavelet-based schemes (also referred as subband coding) outperform other coding schemes like the one based on DCT. Since there is no need to block the input image and its basis functions have variable length, wavelet coding schemes at higher compression avoid blocking artifacts. Wavelet-based coding is more robust under transmission and decoding errors, and also facilitates progressive transmission of images. In addition, they are better matched to the HVS characteristics. Because of their inherent multiresolution nature [9], wavelet coding schemes are especially suitable for applications where *scalability* and *tolerable degradation* are important.



### 4.3 Subband Coding

The fundamental concept behind Subband Coding (SBC) is to split up the frequency band of a signal (image in our case) and then to code each subband using a coder and bit rate accurately matched to the statistics of the band. SBC has been used extensively first in speech coding and later in image coding [12] because of its inherent advantages namely variable bit assignment among the subbands as well as coding error confinement within the subbands.

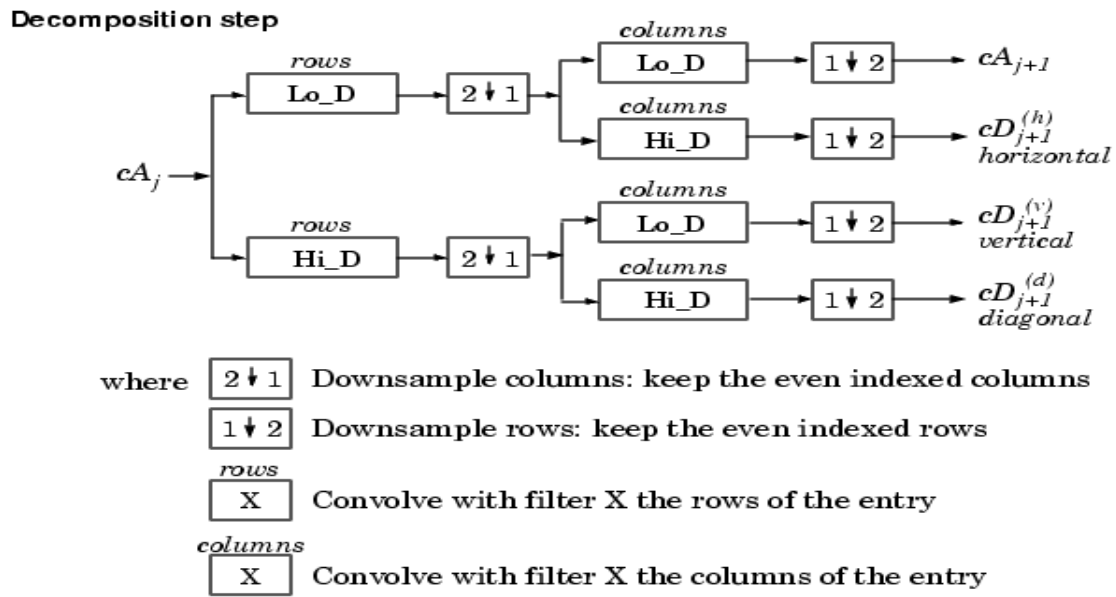


Fig. 2.7 Separable 4-subband Filterbank,

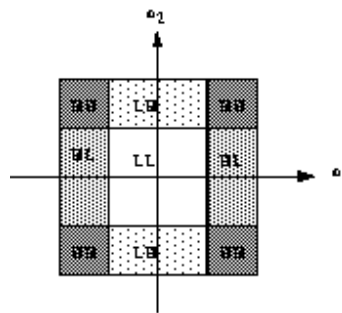


Fig 2.8 . Partition of the Frequency Domain.

ods and O'Neil [12] used a separable combination of one-dimensional Quadrature Mirror Filterbanks (QMF) to perform a 4-band decomposition by the row-column approach as shown in **Fig. 2.7**. Corresponding division of the frequency spectrum is shown in **Fig. 2.8**. The process can be iterated to obtain higher band decomposition filter trees. At the decoder, the subband signals are decoded, upsampled and passed through a bank of synthesis filters and properly summed up to yield the reconstructed image.

#### **4.3.1 From Subband to Wavelet Coding**

Over the years, there have been many efforts leading to improved and efficient design of filterbanks and subband coding techniques. Since 1990, methods very similar and closely related to subband coding have been proposed by various researchers under the name of *Wavelet Coding* (WC) using filters specifically designed for this purpose [12]. Such filters must meet additional and often conflicting requirements. These include short impulse response of the analysis filters to preserve the localization of image features as well as to have fast computation, short impulse response of the synthesis filters to prevent spreading of artifacts (ringing around edges) resulting from quantization errors, and linear phase of both types of filters since nonlinear phase introduce unpleasant waveform distortions around edges. Orthogonality is another useful requirement since orthogonal filters, in addition to preservation of energy, implement a unitary transform between the input and the subbands. But, as in the case of 1-D, in two-band Finite Impulse Response (FIR) systems linear phase and orthogonality are mutually exclusive, and so orthogonality is sacrificed to achieve linear phase.

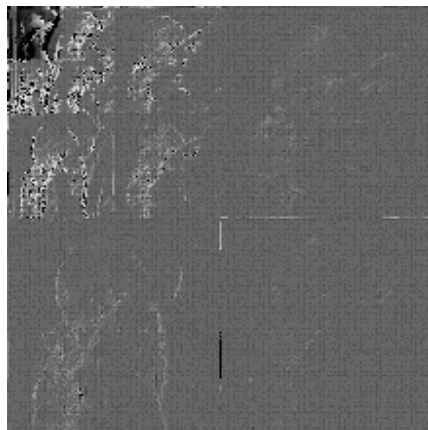
#### **4.4 Link between Wavelet Transform and Filterbank**

Construction of orthonormal families of wavelet basis functions can be carried out in continuous time. However, the same can also be derived by starting from discrete-time filters. Daubechies [9] was the first to discover that the discrete-time filters or QMFs can be iterated and under certain regularity conditions will lead to continuous-time wavelets. This is a very practical and extremely useful wavelet decomposition scheme, since FIR discrete-time filters can be used to implement them. It follows that the orthonormal bases in correspond to a subband coding scheme with exact reconstruction property, using the same FIR filters for reconstruction as for decomposition. So, subband coding developed earlier is in fact a form of wavelet coding in disguise. Wavelets did not gain popularity in image coding until Daubechies established this link

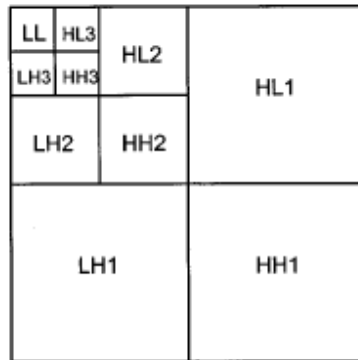
in late 1980s. Later a systematic way of constructing a family of compactly supported biorthogonal wavelets was developed by Cohen, Daubechies, and Feauveau (CDF) . Although the design and choice of various filters and the construction of different wavelets from the iteration of such filters are very important, it is beyond the scope of this article.

#### **4.5 An Example of Wavelet Decomposition**

There are several ways wavelet transforms can decompose a signal into various subbands. These include uniform decomposition, octave-band decomposition, and adaptive or wavelet-packet decomposition [12]. Out of these, octave-band decomposition is the most widely used. This is a non-uniform band splitting method that decomposes the lower frequency part into narrower bands and the high-pass output at each level is left without any further decomposition. **Fig. 2.9** shows the various subband images of a 3-level octave-band decomposed Lena using the popular CDF-9/7 [7] biorthogonal wavelet.



**Fig 2.9 Three level octave-band decomposition of Lena image,**



**Fig 3.0 Spectral decomposition and ordering.**

Most of the subband and wavelet coding schemes can also be described in terms of the general framework depicted as in Fig. 1. The main difference from the JPEG standard is the use of DWT rather than DCT. Also, the image need not be split into 8 x 8 disjoint blocks. Of course, many enhancements have been made to the standard quantization and encoding techniques to take advantage of how the wavelet transforms works on an image and the properties and statistics of transformed coefficients so generated.

#### 4.6 Result :-

Image size	Decomposition Lavel	Threshhold	Compression ratio
256 x 256	3	40	16.1759
256 x 256	3	60	21.6558
256 x 256	3	80	25.9193
256 x 256	4	40	15.7964
256 x 256	4	60	22.4193
256 x 256	4	80	28.9469
256 x 256	5	40	14.9430
256 x 256	5	60	21.3887
256 x 256	5	80	28.1812

**Table 1.3 Parameters associated with the output image at different decomposition level and threshold**

## 4.7 Conclusion :-

Image compression is of prime importance in Real time applications like video conferencing where data are transmitted through a channel.

Using JPEG standard DCT is used for mapping which reduces the interpixel redundancies followed by quantization which reduces the psychovisual redundancies then coding redundancy is reduced by the use of optimal code word having minimum average length.

In JPEG 2000 standard of image compression DWT is used for mapping, all other methods remaining same. DWT is more general and efficient than DCT due to the following result:-

- No need to divide the input coding into non-overlapping 2-D blocks, it has higher compression ratios avoid blocking artifacts.
- Allows good localization both in time and spatial frequency domain.
- Transformation of the whole image → introduces inherent scaling
- Better identification of which data is relevant to human perception → higher compression ratio

## References:

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- [1] R. C. Gonzalez and R. E. Woods, “Digital Image Processing”, Reading. MA: Addison Wesley, 2004.
  - [2] David Salomon, Data Compression, The Complete Reference, 2nd Edition Springer-Verlag 1998.
  - [3] Digital Compression and coding of Continuous-tone still images, part 1, requirements and Guidelines. ISO/IEC JTC1 Draft International Standard 10918-1, Nov. 1991.
  - [4] G. K. Wallace, “The JPEG Still Picture Compression Standard”, IEEE Trans. On Consumer Electronics, vol.38, No.1, pp. xviii – xxxiv, Feb 1992.
  - [5] S. Martucea, “Symmetric convolution and the discrete sine and cosine transform”, IEEE Transaction on Signal Processing, vol. 42, p. 1038-1051, May’ 1994.
  - [6] R. M. Gray, D. L. Neuhoff, “Quantization”, IEEE Trans. Inform. Theory, Vol. 44, No. 6, 1998.
  - [7] N. Ahmed, T. Natrajan, and K. R. Rao, “Discrete Cosine Transform”, IEEE Transactions on Computers, vol. 23, July 1989.
  - [8] Pennebaker, W. B. and Mitchell, J. L. *JPEG - Still Image Data Compression Standards*, Van Nostrand Reinhold, 1993.
  - [9] [http://en.wikipedia.org/wiki/Discrete\\_cosine\\_transform](http://en.wikipedia.org/wiki/Discrete_cosine_transform)
  - [10] Rao, K. R. and Yip, P. *Discrete Cosine Transforms - Algorithms, Advantages, Applications*, Academic Press, 1990.
  - [11] Gersho, A. and Gray, R. M. *Vector Quantization and Signal Compression*, Kluwer Academic publishers
  - [12] Strang, G. and Nguyen, T. *Wavelets and Filter Banks*, Wellesley-Cambridge Press, Wellesley, MA, 1996 <http://www-math.mit.edu/~gs/books/wfb.html>.
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