Dynamic Stability of a Sandwich Beam Subjected to Parametric Excitation

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Dynamic Stability of a Sandwich Beam Subjected to Parametric Excitation

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MECHANICAL ENGINEERING

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To my Parents

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Certificate

This is to certify that the thesis entitled "Dynamic Stability of a Sandwich Beam Subjected to Parametric Excitation" by Miss Laxmi Mohanta submitted to the National Institute of Technology, Rourkela for the Degree of Master of Technology by research is a record of bonafide research work, carried out by her in the Department of Mechanical Engineering under my supervision. I believe that the thesis fulfils part of the requirements for the award of master of Technology(Research) . The results embodied in the thesis have not been submitted for the award of any other degree.

S. C. Mohanty

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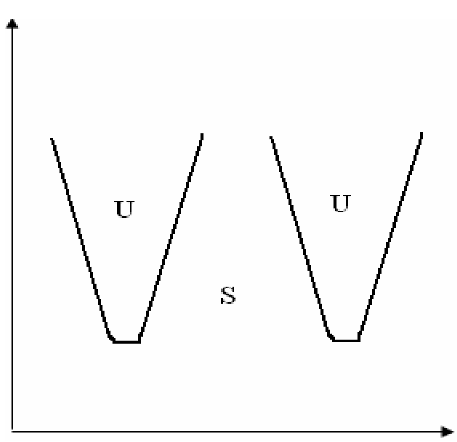
Nomenclature

A_k	:	Cross-sectional area of the k^{th} elastic layer.
A_v	:	Cross-sectional area of the viscoelastic layer.
E_k	:	Young's modulus of the k^{th} elastic layer.
G_v	:	Complex shear modulus of the viscoelastic layer.
h_v	:	Thickness of the viscoelastic layer.
I_k	:	Moment of Inertia of the k^{th} elastic layer.
$[K^{(e)}]$	:	Stiffness matrix of the beam element.
$[K_p^{(e)}]$	:	Stability matrix of the beam element.
$\left[\overline{K}\right]$	:	$[K] - P_0\,[K_p]$.
L	:	Length of the beam.
L_e	:	Length of the beam element.
[M]	:	Global mass matrix.
$[N_k]$	:	Shape function matrix of the k^{th} elastic layer for axial displacement.
$[N]^T$	:	Transpose of shape function matrix.
[Nv]	:	Shape function matrix for viscoelastic layer.
[Nw]	:	Shape function matrix for transverse displacement.
P_0	:	Static component of the load.
P_1	:	Time dependent component of the load.
P(t)	:	Axial periodic force.
P_{cr}	:	Critical buckling load of the equivalent Euler beam.
$T_k^{(e)}$	:	Kinetic energy of the beam element.
$T_v^{(e)}$	:	Kinetic energy of the viscoelastic layer.
u_k	:	Axial displacement of the k^{th} elastic layer.
$U_k^{(e)}$	:	Potential energy of the constraining layer.
$U_v^{(e)}$	:	Potential energy of the viscoelastic layer.
w	:	Transverse displacement.
$W_p^{(e)}$	:	Workdone on the beam element.
x	:	Axial co-ordinate.

α	:	Static load factor.
β	:	Dynamic load factor.
ρ	:	Mass density of the elastic layer.
ρ_v	:	Mass density of viscoelastic layer.
Ω	:	Disturbing frequency.
Φ	:	Rotational angle.
ξ	:	x/L_e.
γ_v	:	Shear strain of viscoelastic layer.
$\{\Delta^e\}$	:	Nodal displacement matrix of the beam element.
$[\Phi]$	:	Normalized modal matrix.
$\{\Gamma\}$	:	New set of generalized co-ordinates.
$'$	:	$\frac{\partial}{\partial x}$
$\cdot$	:	$\frac{\partial}{\partial t}$
$\cdot\cdot$	:	$\frac{\partial^2}{\partial t^2}$
S	:	Stable region
U	:	Unstable region



Abstract

Vibration control of machines and structures incorporating viscoelastic materials in suitable arrangement is an important aspect of investigation. The use of viscoelastic layers constrained between elastic layers is known to be effective for damping of flexural vibrations of structures over a wide range of frequencies. The energy dissipated in these arrangements is due to shear deformation in the viscoelastic layers, which occurs due to flexural vibration of the structures. Multilayered cantilever sandwich beam like structures can be used in aircrafts and other applications such as robot arms for effective vibration control. These members may experience parametric instability when subjected to time dependant forces. The theory of dynamic stability of elastic systems deals with the study of vibrations induced by pulsating loads that are parametric with respect to certain forms of deformation

The purpose of the present work is to investigate the dynamic stability of a three layered symmetric sandwich beam subjected to an end periodic axial force . Equations of motion are derived using finite element method. The regions of instability for simple and combination resonances are established using modified Hsu's method proposed by Saito and Otomi[76].

It is observed that with increase in core thickness parameter fundamental buckling load increases. The fundamental resonant frequency and second mode frequency parameter also increase with increase in core thickness parameter. Fundamental loss factor and second mode loss factor also increase with increase in core thickness parameter.Increase in core thickness parameter enhances the stability of the beam. With increase in core loss factor also the stability of the beam enhances. There is a very good agreement of the experimental results with the theoretical findings.

CHAPTER-1

Introduction

The theory of dynamic stability of elastic systems deals with the study of vibrations induced by pulsating loads that are parametric with respect to certain forms of deformation. A system is said to be parametrically excited if the excitation which is an explicit function of time appears as one of the co-efficients of the homogeneous differential equation describing the system, unlike external excitation which leads to an inhomogeneous differential equation. A well known form of equation describing a parametric system is Hill's equation.

$$\ddot{x} + \omega^2 \alpha + \in x f(t) = 0 \qquad (1.1)$$

When f(t) = cosΩt, Equation (1.1) is known as Mathieu's equation .

Equation (1.1) governs the response of many physical systems to a sinusoidal parametric equation.

In practice parametric excitation can occur in structural systems subjected to vertical ground motion, aircraft structures subjected to turbulent flow, and in machine components and mechanisms. Other examples are longitudinal excitation of rocket tanks and their liquid propellant by the combustion chambers during powered flight, helicopter blades in forward flight in a free-stream that varies periodically and spinning satellites in elliptic orbits passing through a periodically varying gravitational field. In industrial machines and mechanisms, their components and instruments are frequently subjected to periodic or random excitation transmitted through elastic coupling elements, example includes those associated with electromagnetic aeronautical instruments and vibratory conveyers, saw blades and belt drives.

In parametric instability the rate of increase in amplitude is generally exponential and thus potentially dangerous while in typical resonance the rate of increase is linear. Moreover damping reduces the severity of typical resonance, but may only reduce the rate of increase during parametric resonance. Moreover parametric instability occurs over a region of parameter space and not at discrete points. The system can experience parametric instability (resonance), when the excitation frequency or any integer multiple of it is twice the natural frequency that is to say

$m\Omega=2\omega$, m=1,2,3,4

The case $\Omega=2\omega$ is known to be the most important in application and is called main parametric resonance. A vital step in the analysis of parametric dynamic systems is thus establishment of the regions in the parameter space in which the system becomes unstable, these regions are known as regions of dynamic instability or zones of parametric resonance. The unstable regions are separated from the stable ones by the so called stability boundaries and a plot of these boundaries on the parameter space is called a stability diagram.

Vibration control of machines and structures incorporating viscoelastic materials in suitable arrangement is an important aspect of investigation. The use of viscoelastic layers constrained between elastic layers is known to be effective for damping of flexural vibrations of structures over a wide range of frequencies. The energy dissipated in these arrangements is due to shear deformation in the viscoelastic layers, which occurs due to flexural vibration of the structures. Multilayered cantilever sandwich beam like structures can be used in aircrafts and other applications such as robot arms for effective vibration control. These members may experience parametric instability when subjected to time dependant forces.

CHAPTER-2

Literature review

2.1 Introduction

Discovery of parametric resonance dates back to 1831. The phenomenon of parametric excitation was first observed by Faraday[24], when he noticed that when a fluid filled container vibrates vertically, fluid surface oscillates at half the frequency of the container. Parametric resonance in the case of lateral vibration of a string was reported by Melde[57]. Beliaev [10] was first to provide a theoretical analysis of parametric resonance while dealing with the stability of prismatic rods. These are a few early works.

Several review articles on parametric resonance have also been published. Evan-Iwanowski [23], Ibrahim and coworkers [34-40], Ariarathnam [3] and Simitses [84] gave exhaustive account of literature on vibration and stability of parametrically excited systems. Review article of Habip [29] gives an account of developments in the analysis of sandwich structures. Articles of Nakra [60-62] have extensively treated the aspect of vibration control with viscoelastic materials. Books by Bolotin [13], Schmidt [80] and Nayfeh and Mook [63] deals extensively on the basic theory of dynamic stability of systems under parametric excitations. In this chapter further developments in subsequent years in the field of parametric excitation of system with specific resonance to ordinary and sandwich beams is reported. Reference cited in the above mentioned review works are not repeated except at a few places for the sake of continuity. The reported literature mainly deals with the methods of stability analysis, types of resonance, study of different system parameters on the parametric instability of the system and experimental verification of the theoretical findings.

2.2 Methods of stability analysis of parametrically excited system

There is no exact solution to the governing equations for parametrically excited systems of second order differential equations with periodic coefficients. The researchers for a long time have been interested to explore different solution methods to this class of problem. The two main objectives of this class of researchers are to establish the existence of periodic solutions and their stability. When the governing equation of motion for the system is of Mathieu-Hill type, a few well known methods commonly used are method proposed by Bolotin based on Floquet's theory, perturbation and iteration techniques, the Galerkin's method, the Lyapunov second method and asymptotic technique by Krylov, Bogoliubov and Mitroploskii.

Bolotin's[13] method based on Floquet's theory can be used to get satisfactory results for simple resonance only. Steven [85] later modified the Bolotin's method for system with complex differentials equation of motion. Hsu [32-33]proposed an approximate method of stability analysis of systems having small parameter excitations . Hsu's method can be used to obtain instability zones of main, combination and difference types. Later Saito and Otomi [76] modified Hsu's method to suit systems with complex differential equation of motion. Takahashi [88] proposed a method free from the limitations of small parameter assumption. This method establishes both the simple and combination type instability zones. Zajaczkowski and Lipinski [93] and Zajaczkowski [94] based on Bolotin's method derived formulae to establish the regions of instability and to calculate the steady state response of systems described by a set of linear differential equations with time dependent parameters represented by a trigonometric series. Lau et al.[52] proposed a variable parameter incrementation method, which is free from limitations of small excitation parameters. It has the advantage of treating non-linear systems. Many investigators to study the dynamic stability of elastic systems have also applied finite element method. Brown et al [14] studied the dynamic stability of uniform bars by applying this method. Abbas [2] studied the effect of rotational speed and root flexibility on the stability of a rotating Timoshenko beam by finite element method. Abbas and Thomas [1] and Yokoyama [92] used finite element method to study the effect of support condition on the dynamic stability of Timoshenko beams. Shastry and Rao by finite element method obtained critical frequencies [81] and the stability boundaries [82-83] for a cantilever column under an intermediate periodic concentrated load for various load positions. Bauchau and Hong [8] studied the non- linear response and stability of beams using finite element in time. Briseghella et al. [12] studied the dynamic stability problems of beams and frames by using finite element method. Svensson [87] by this method studied the stability properties of a periodically loaded non-linear dynamic system, giving special attention to damping effects.

2.3 Type of parametric resonances

Multidegree freedom systems may exhibit simple resonance, resonance of sum type or resonance of difference type depending upon the type of loading, support conditions and system parameters.

Mettler [58] furnished a classification for various kinds of resonances exhibited by linear periodic system. Iwatsubo and his co-workers [43-44] from their investigation on stability of columns found that uniform columns with simple supported ends do not exhibit combination type resonances. Saito and Otomi [76] on the basis of their investigation of stability of viscoelastic beams with viscoelastic support concluded that combination resonances of difference type do not occur for axial loading, but it exists for tangential type of loading. Celep [15] found that for a simply supported pretwisted column, combination resonances of the sum type may exist or disappear depending on the pretwist angle and rigidity ratio of the cross-section. Ishida et al. [42] showed that an elastic shaft with a disc exhibits only difference type combination resonance. Chen and Ku [17] from their investigations found that for a cantilever shaft disc system, the gyroscopic moment can enlarge the principal regions of dynamic instability.

2.4 Sandwich Beams

The main objectives of the researchers dealing with sandwich beams may be grouped in the following categories.

i) Prediction of resonant frequencies and loss factor

ii) Static and dynamic analysis of sandwich beams

iii) Stability study of sandwich beams and columns

iv) Experimental investigations

2.4.1 Resonant frequencies and loss factor prediction

Kerwin [50] was the first to carry out a quantitative analysis of the damping effectiveness of a constrained viscoelastic layer and he obtained an expression to estimate the loss factor. Ungar [90] derived general expressions for the loss factor of uniform linear composites in terms of the properties of the constituting materials. Di Taranto [22] developed a theory to estimate natural frequencies, loss factors for a finite length sandwich beam. Jones et al.[47] theoretically and experimentally evaluated the damping capacity of a sandwich beam with viscoelastic core. Asnani and Nakra [4] analysed multilayer simply supported sandwich beams and estimated loss factors and displacement response effectiveness for beams of different number of layers. Chatterjee and Baumgarten [16] obtained for a simply supported sandwich beam, the damped natural frequencies and logarithmic decrement for the fundamental mode of vibration. They also conducted experiments to verify their theoretical results, which showed good agreement. Nakra and Grootenhuis [59] studied theoretically as well as experimentally, the vibration characteristics of asymmetric dual core sandwich beams. They did not include the rotary and longitudinal inertia terms in their analysis. Later Rao [70] included both these effects in his analysis. Asnani and Nakra [6] studied the effect of number of layers and thickness ratio on the system loss factors for a simply supported multilayer beam. Rao [66] investigated the influence of pretwist on resonant frequency and loss factor for a symmetric pretwisted simply supported sandwich beam and found that pretwisting reduces loss factor and very soft thick cored beam is especially sensitive to even small changes of pretwist. Rao and Stuhler [67] analysed the damping effectiveness of tapered sandwich beam with simply supported and clamped free end conditions. Rao [70] investigated the free vibration of a short sandwich beam considering the higher order effects such as inertia, extention and shear of all the layers. He found that if these parameters are neglected for short sandwich beam there is an error as high as 45% in estimation of the loss factor and frequencies. Rubayi and Charoenree [75] carried theoretical and experimental investigations to obtain the natural frequencies of cantilever sandwich beams subjected to gravity force only. Rao [69] on another work obtained graphs and equations to estimate frequencies and loss factor for sandwich beam under various boundary conditions. Johnson and his coworkers [45-46] used the finite element method to solve frequencies and loss factors for beams and plates with constrained viscoelastic layer. Vaswani et al.[91] derived equations of motion for a multilayer curved sandwich beam subjected to harmonic excitation. Lall et al.[51] analysed the partially covered sandwich beams using three different methods and found that method by Markus [55] estimates modal loss factors only, whereas Rayleigh-Ritz and classical search method give both loss factor and resonant frequencies. Dewa et al.[21] studied the damping effectiveness of partially covered sandwich beams. They found that partially covered beams have better damping capacity than fully covered beams. Also through experiments he validated his theoretical findings. Imaino and Harrison [41] adopted modal strain energy method and finite element technique to investigate damping of the first and second bending resonance of a sandwich beam with constrained damping layer. He and Rao [30] developed an analytical model to carry out a parameter study of the coupled flexural and longitudinal vibration of a curved sandwich beam. Effects of parameters such as curvature, core thickness and adhesive shear modulus on the system loss factors and resonant frequencies were investigated. Same authors [31] in another work studied the vibration of multispan beams with arbitrary boundary condition. Effects of parameter like location of intermediate supports and adhesive thickness on the resonant frequencies and loss factors were investigated. Bhimaraddi[11] solved both the resonant frequencies and loss factors for a simply supported beam with constrained layer damping using a model which accounted for the continuity of displacements and the transverse shear stresses across the interfaces of the layers. Sakiyama et al.[77] developed an analytical method for free vibration analysis of a three layer continuous sandwich beam and investigated the effect of shear parameter and core thickness on the resonant frequencies and loss factors. Fasana and Marchesiello [25] calculated the mode shapes, frequencies and loss factors for sandwich beams by Rayleigh-Ritz method. They choose polynomials which satisfy the geometric boundary conditions as admissible function. Banerjee[7] studied the free vibration of a three layer sandwich beam using dynamic stiffness matrix method. He calculated the natural frequencies and mode shapes.

2.4.2 Static and dynamic analysis of sandwich beams

The forced vibration analysis of a three layered sandwich beam with viscoelastic core and with arbitrary boundary conditions was carried out by Mead and Markus [56]. They followed the method used by Di Taranto [22] in their analysis. Asnani and Nakra[5] carried out forced vibration analysis of sandwich beams with viscoelastic core and with fixed-fixed and cantilever type end conditions. The forced vibration response obtained by applying Ritz method matched well with the experimental results. Rao [68] studied the forced vibration of a damped sandwich beam subjected to moving forces and found that increasing the shear stiffness of the core materials can reduce the dynamic magnification of the central deflection of the beam. Kapur[48] considered both rotary and longitudinal inertia in his analysis to study the dynamic response of two and three-layered viscoelastically damped beams subjected to half –sine shock excitation .Sharma and Rao [79]determined static deflections and stresses in sandwich beams for both concentrated and distributed loads under various conditions.Frosting and Baruch [26] from their analysis of stresses in a sandwich beam with flexible core under concentrated and distributed loading found that transverse normal stresses at the interface between the skin and core in some cases are significant in determining the sudden failure of the beam. Sun et al.[86] developed a finite element model to study the effect of add-on viscoelastic layer in damping and vibration control of unidirectional composite laminates. Their theoretical results compared well with the exprerimental findings .Qian and Demao[65] carried out modal analysis as well as response calculation in time domain using finite element technique.Salet and Hamelink[78] developed a numerical model based on finite difference method, for non-linear analysis of sandwich beams with simply supported boundary conditions. Ha[28] suggested an exact analysis procedure for bending and buckling analysis of sandwich beam system.

2.4.3 Stability study of sandwich beams and columns

The stability of sandwich columns with simply supported end conditions and subjected to pulsating axial loads was investigated by Bauld[9]. Chonan [19] studied the stability of two layer sandwich cantilever beams with imperfect bonding .They obtained critical loads for divergence and flutter type instabilities and found that these are functions of shear and normal stiffness of the bond .In another work Chonan[20] studied the divergence and flutter type instabilities in symmetric sandwich beams with elastic bonding and found that critical divergence and flutter loads depends on the interface bond stiffness. Kar and Hauger [49] investigated the dynamic stability of a sandwich beam subjected to a direction controlled non-conservative force and determined the critical divergence and flutter loads. Ray and Kar [71] have investigated the dynamic stability of sandwich beams under various boundary Conditions.The same authors[72-74] also investigated the parametric stability of partially convered sandwich beams, dual cored sandwich beams and symmetric sandwich beams with higher order effects. Ray and Kar in these works derived the governing equations of motion by using Hamilton's principle and converted the equation of motion to a set of coupled Hill's equation in the time domain by Galerkin's method.They assumed approximate series solutions , which satisfy majority of the boundary conditions. The effect of rotating speed , setting angle and hub radius on the dynamic stability of a rotating sandwich beam with a constrained damping layer were studied by Lin and Chen [53].

2.4.4 Experimental Investigations

The reported experimental works are mainly related to the experimental validation of theoretically predicted dynamic response,damping values, resonant frequencies and loss factors of sandwich beams.Chatterjee and Baumgarten[16] experimentally determined the logarithmic decrement to validate their theoretically obtained values for damped natural frequencies and damping values for a simply supported sandwich beams.Asnani and Nakra[5] compared their theoretically obtained resonant frequencies by applying Ritz method with experimental result for a three-layer sandwich beam.Trompette et al.[89] carried out experiments to obtain resonant frequencies and damping values and compared with their theoretical results, which showed good agreement .Mace[54] compared the frequency response curve obtained from experiment with his theoretical results and drew the conclusion that his predicted theory is an efficient in predicting the dynamic response of beams that are damped by means of a thin viscoelastic film.Gorrepati and Rao[27] measured from experiment, the natural frequencies and loss factor for a simply supported beam with adhesively bonded double strap joint to validate their results obtained by modal strain energy method.Chen and Chan[18] in order to establish their results obtained from integral finite element method experimentally obtained frequency response functions for elastic-viscoelastric composite structures. In a recent work Nayfeh[64] conducted experiment to obtain resonant frequencies and loss factors and compared with values predicted by his developed model for vibrations parallel to the plane of lamination of a symmetric elastic-viscoelastic sandwich beam.

CHAPTER-3

Theoretical Study

3.1 Introduction

Sandwich structures are getting importance particularly in aerospace and other applications because of their remarkable vibration damping capacity. In one of the earliest work, Mead and Markus[56] investigated the forced vibration characteristics of a three layer damped sandwich beam with arbitrary boundary conditions. Asnani and Nakra[6] studied the vibration damping characteristics of a multilayer sandwich beam. Rubayi and Charoenree [75] calculated the natural frequencies of cantilever sandwich beam for various system parameters. Rao and Stuhler[67] studied the damping effectiveness of tapered symmetric sandwich beams for clamped-free and hinged-hinged boundary conditions. Rao[69] in his latter work obtained frequency and loss factors of sandwich beams with different boundary conditions and presented his findings in the form of graphs and formulae. Rao[66] also investigated the vibration characteristics of pre-twisted sandwich beams. He also studied the forced vibration characteristic of damped sandwich beam subjected to moving forces [68]. Sharma and Rao[79] studied the static deflection and stresses in sandwich beams under various boundary conditions. Bauld[9] determined the instability regions of simply supported sandwich column subjected to pulsating compressive load. Chonan studied the stability of two layered [20] cantilever beam with imperfect elastic bonding and subjected to constant horizontal and tangential compressive forces.

The purpose of the present work is to investigate the dynamic stability of a three layered symmetric sandwich beam subjected to an end periodic axial force . Equations of motion are derived using finite element method. The regions of instability for simple and combination resonances are established using modified Hsu's method proposed by Saito and Otomi[76].

3.2 Formulation of the problem

Figure (3.1) shows a three layered symmetric sandwich beam of length L subjected to a pulsating axial force $P(t) = P_0 + P_1 \cos \Omega t$ acting along its undeformed axis at one end.

The finite element model developed is based on the following assumptions:

(1) The transverse displacement w is same for all the three layers.

(2) The rotary inertia and shear deformation in the constrained layers are negligible.

(3) Linear theories of elasticity and viscoelasticity are used.

(4) No slip occurs between the layers and there is perfect continuity at the interfaces.

(5) Young's modulus of the viscoelastic material is negligible compared to the elastic material.

As shown in figure the element model presented here consists of two nodes and each node has four degrees of freedom. Nodal displacements are given by

$$\{ \Delta^e \} = \{ u_{1i} \; u_{3i} \; w_i \; \Phi_i \; u_{1j} \; u_{3j} \; w_j \; \Phi_j \} \tag{3.1}$$

Where i and j are elemental nodal numbers. The axial displacement of the constraining layer, the transverse displacement and the rotational angle, can be expressed in terms of nodal displacements and finite element shape functions.

$$u_1 = [N_1]\{\Delta^e\},\; u_3 = [N_3]\{\Delta^e\},\; w = [N_w]\{\Delta^e\},\; \Phi = [N_w]'\{\Delta^e\}, \tag{3.2}$$

where the prime denotes differentiation with respect to axial co-ordinate x and the shape functions are given by

$$[N_1] = [1-\xi \;\; 0 \;\; 0 \;\; 0 \;\; \xi \;\; 0 \;\; 0 \;\; 0]$$

$$[N_3] = [0 \;\; 1-\xi \;\; 0 \;\; 0 \;\; 0 \;\; \xi \;\; 0 \;\; 0]$$

and

$$[N_w] = [0 \;\; 0 \;\; (1-3\xi^2+2\xi^3) \;\; (\xi-2\xi^2+\xi^3)L_e \;\; 0 \;\; 0 \;\; 3\xi^2-2\xi^3 \;\; (-\xi^2+\xi^3)L_e] \tag{3.3}$$

Where $\xi = x/L_e$ and L_e is the length of the element.

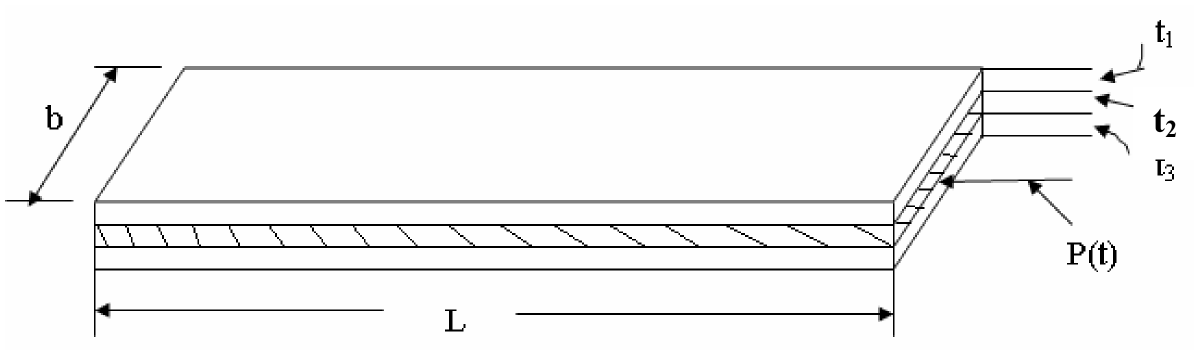


Figure 3.1 Configuration of the beam

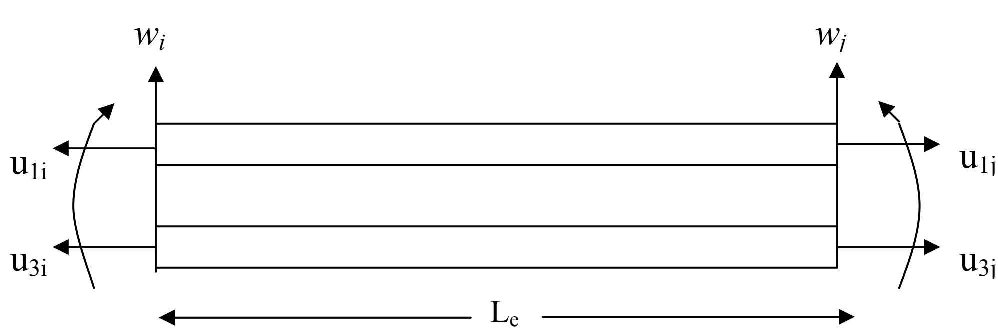


Figure 3.2 Sandwich Beam Element

3.2.1. Constraining Layers

The potential energy of the constraining layers is written as

$$U_{\mathrm{k}}^{(\mathrm{e})} = \frac{1}{2}\int_{0}^{Le} E_k I_k \left(\frac{d^2 w}{d x^2}\right)^2 dx + \frac{1}{2}\int_{0}^{Le} E_k A_k \left(\frac{du}{dx}\right)^2 dx \qquad k = 1,3 \tag{3.4}$$

Where E, A and I are the Young's modulus, cross-sectional area and moment of inertia respectively. The notations 1 and 3 represent the upper and lower constraining layer, respectively.

The kinetic energy of the constraining layers is written as

$$T_{\mathrm{k}}^{(\mathrm{e})} = \frac{1}{2}\int_{0}^{Le} \rho_k A_k \left(\frac{dw}{dt}\right)^2 dx + \frac{1}{2}\int_{0}^{Le} \rho_k A_k \left(\frac{du}{dt}\right)^2 dx \qquad k = 1,3 \tag{3.5}$$

Where ρ is the mass density.

By substituting Eq.(3.2) into Eq.(3.4) and Eq.(3.5), the element potential energy and the kinetic energy of the constraining layers can be written as

$$U_{\mathrm{k}}^{(\mathrm{e})} = \frac{1}{2}\left\{\Delta^{(e)}\right\}\left(\left[K_{ku}^{(e)}\right] + \left[K_{kw}^{(e)}\right]\right)\left\{\Delta^{(e)}\right\} \quad k = 1,3 \tag{3.6}$$

and

$$T_{\mathrm{k}}^{(\mathrm{e})} = \frac{1}{2}\left\{\Delta^{(e)}\right\}\left(\left[M_{ku}^{(e)}\right] + \left[M_{kw}^{(e)}\right]\right)\left\{\Delta^{(e)}\right\} \quad k = 1,3 \tag{3.7}$$

Where

$$\left[K_{ku}^{(e)}\right] = \left[K_{1u}^{(e)}\right] + \left[K_{3u}^{(e)}\right] = E_1 A_1 \int_{0}^{L_e} \left[N_1\right]^T \left[N_1\right] dx + E_3 A_3 \int_{0}^{L_e} \left[N_3\right]^T \left[N_3\right] dx$$

$$\left[K_{kw}^{(e)}\right] = \left[K_{1w}^{(e)}\right] + \left[K_{3w}^{(e)}\right] = E_1 I_1 \int_{0}^{L_e} \left[N_w\right]^T \left[N_w\right] dx + E_3 I_3 \int_{0}^{L_e} \left[N_w\right]^T \left[N_w\right] dx$$

$$\left[M_{ku}^{(e)}\right] = \left[M_{1u}^{(e)}\right] + \left[M_{3u}^{(e)}\right] = \rho_1 A_1 \int_{0}^{L_e} \left[N_1\right]^T \left[N_1\right] dx + \rho_3 A_3 \int_{0}^{L_e} \left[N_3\right]^T \left[N_3\right] dx$$

$$\left[M_{kw}^{(e)}\right] = \left[M_{1w}^{(e)}\right] + \left[M_{3w}^{(e)}\right] = \rho_1 A_1 \int_{0}^{L_e} \left[N_w\right]^T \left[N_w\right] dx + \rho_3 A_3 \int_{0}^{L_e} \left[N_w\right]^T \left[N_3\right] dx$$

3.2.2. Viscoelastic layer

The axial displacement u_v and shear strain γ_v of the viscoelastic layer is derived from kinematic relationships between the constraining layers as given by Mead and Markus [56]. They are expressed as follows:

$$u_v = \frac{u_1 + u_3}{2} + \frac{(t_1 - t_3)}{4}\frac{\partial w}{\partial x} \tag{3.8}$$

$$. \ \gamma_v = \frac{\partial w}{\partial x}\left[\frac{2t_2 + t_1 + t_3}{2t_2}\right] + (u_1 - u_3) \tag{3.9}$$

Substituting Eq. (3.2) into Eq.(3.8) and Eq.(3.9), γ_v and u_v can be expressed in terms of nodal displacements and element shape functions:

$$u_v = (N_v)\{\Delta^{(e)}\}$$

$$\gamma_v = (N_\gamma)\{\Delta^{(e)}\}$$

Where $(N_v) = \frac{1}{2}((N_1) + (N_3))\frac{(t_1 - t_3)}{4}(N_w)$

$$(N_\gamma) = \frac{1}{2}\frac{((N_1) - (N_3))}{t_2} + \frac{(t_1 + 2t_2 + t_3)}{t_2}(N_w)$$

The potential energy of the viscoelastic layer due to shear deformation is written as

$$U_v^{(e)} = \frac{1}{2}\int_0^{L_e} G_v A_v \gamma_v{}^2 dx \tag{3.10}$$

Where A_v is the cross-sectional area and G_v is the complex shear modulus of viscoelastic layer.

The kinetic energy of viscoelastic layer is written as

$$T_v^{(e)} = \frac{1}{2}\int_0^{L_e} \rho_v A_v \left\{\left(\frac{dw}{dt}\right)^2 + \left(\frac{du_v}{dt}\right)^2\right\} dx \tag{3.11}$$

Substituting Eq.(3.2) into Eqs. (3.10) and (3.11), the potential energy and kinetic energy of viscoelastic material layer is given by

$$U_v^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}([K_v^{(e)}])\{\Delta^{(e)}\} \tag{3.12}$$

$$T_v^{(e)} = \frac{1}{2}\left\{\dot{\Delta}^{(e)}\right\}\left(\left[M_v^{(e)}\right]\right)\left\{\dot{\Delta}^{(e)}\right\} \tag{3.13}$$

Where $\left[K_v^{(e)}\right] = G_v A_v \int_0^{L_e} \left[N_\gamma\right]^T \left[N_\gamma\right] dx$

$$\left[M_v^{(e)}\right] = \rho_v A_v \int_0^{L_e} \left[N_v\right]^T \left[N_v\right] dx + \rho_v A_v \int_0^{L_e} \left[N_w\right]^T \left[N_w\right] dx$$

And the dot denotes differentiation with respect to time t.

3.2.3 Work done by axial periodic force

Work done by axial periodic force P(t) is written as

$$W_p^{(e)} = \frac{1}{2}\int_0^{L_e} P(t)\left(\frac{dw}{dx}\right)^2 dx \tag{3.14}$$

Substituting Eq.(3.2) into Eq.(3.14), the work done by the axial periodic load can be rewritten as

$$W_p^{(e)} = \frac{1}{2}\left\{\Delta^{(e)}\right\}^T P(t) k_p^{(e)} \left\{\Delta^{(e)}\right\} \tag{3.15}$$

Where $\left[k_p^{(e)}\right] = \int_0^{L_e} \left[N_w\right]^T \left[N_w\right] dx$

The dynamic load P(t) is periodic and can be expressed in the form $P(t)= P_0 + P_1 \mathrm{Cos}\ \Omega\ t$, where Ω is the disturbing frequency, P_0 the static and P_1 the amplitude of time dependent component of the load, can be represented as the fraction of the fundamental static buckling load $P_{cr} = (\Pi^2 2E_1 I_1)/L^2$ of a reference Euler beam, which is defined as having flexural rigidity $2E_1I_1$ and mass per unit length same as that of the original sandwich beam with pin-pin end conditions. Hence substituting $P(t)= \alpha\ P_{cr} + \beta\ P_{cr}\ \mathrm{Cos}\ \Omega\ t$ with α and β as static and dynamic load factors respectively.

3.2.4 Equation of motion

The element equations of motion for a sandwich beam with constrained damping layer subjected to an axial periodic load is derived by using extended Hamilton's principle.

$$\delta \int_{t_1}^{t_2} \left(T^{(e)} - U^{(e)} + W_p^{(e)}\right) dt = 0 \tag{3.16}$$

Substituting Eqn.(3.6), (3.7), (3.12), (3.13) and (3.15) in to Eq. (3.16) the element equation of motion for the sandwich beam element are obtained as follows:

$$\left[M^{(e)}\right]\left\{\ddot{\Delta}^{(e)}\right\}\left[K^{(e)}\right]\left\{\Delta^{(e)}\right\}-\beta P_{cr}Cos\Omega t\left[K_p{}^{(e)}\right]\left\{\Delta^{(e)}\right\}=0 \tag{3.17}$$

where

$$\left[M^{(e)}\right]=\left[M_{1u}{}^{(e)}\right]+\left[M_{1u}{}^{(e)}\right]+\left[M_{3u}{}^{(e)}\right]+\left[M_{3u}{}^{(e)}\right]+\left[M_{v}{}^{(e)}\right]$$

$$\left[K^{(e)}\right]=\left[K_{1u}{}^{(e)}\right]+\left[K_{1u}{}^{(e)}\right]+\left[K_{3u}{}^{(e)}\right]+\left[K_{3u}{}^{(e)}\right]+\left[K_{v\gamma}{}^{(e)}\right]$$

Assembling individual elements, the equations of motion of the global system can be expressed as

$$\left[M\right]\left\{\ddot{\Delta}\right\}+\left[K\right]\left\{\Delta\right\}-P(t)\left[K_p\right]\left\{\Delta\right\}=0 \tag{3.18}$$

Substituting P(t), Eq.(3.18) becomes

$$\left[M\right]\left\{\ddot{\Delta}\right\}+\left[K\right]\left\{\Delta\right\}-\left(P_0+P_1\cos\Omega t\right)\left[K_p\right]\left\{\Delta\right\}=0 \tag{3.19}$$

$$\left[M\right]\left\{\ddot{\Delta}\right\}+\left(\left[K\right]-P_0\left[K_p\right]\left\{\Delta\right\}-P_1\cos\Omega t\right)\left[K_p\right]\left\{\Delta\right\}=0 \tag{3.20}$$

$$\left[M\right]\left\{\ddot{\Delta}\right\}+\left[\overline{K}\right]\left\{\Delta\right\}-\beta P_{cr}\cos\Omega t\left[K_p\right]\left\{\Delta\right\}=0 \tag{3.21}$$

Where $\left[\overline{K}\right]=\left[K\right]-P_0\left[K_p\right]$ (3.22)

The nodal displacement matrix $\left\{\Delta\right\}$ can be assumed as

$$\left\{\Delta\right\}=\left[\Phi\right]\left\{\Gamma\right\} \tag{3.23}$$

Where $\left[\Phi\right]$ is the normalized modal matrix corresponding to

$$\left[M\right]\left\{\ddot{\Delta}\right\}+\left[\overline{K}\right]\left\{\Delta\right\}=0 \tag{3.24}$$

and $\left\{\Gamma\right\}$ is a new set of generalized coordinates.

Substituting Eq.(3.23) in Eq.(3.21), Eq. (3.21) transforms to the following set of coupled Mathieu equations.

$$\ddot{\Gamma}_m+\left(\dot{w}_m\right)^2\Gamma_m+\beta P_{cr}\cos\Omega t\sum_{n=1}^{N}b_{mn}\Gamma_n=0 \qquad m=1,2,\ldots\ldots\ldots\ldots N, \qquad (3.25)$$

Where $\left(\dot{w}_m\right)^2$ are the distinct eigen values of $\left[M\right]^{-1}\left[\overline{K}\right]$ and b_{mn} are the elements of the complex matrix $\left[B\right]=-\left[\Phi\right]^{-1}\left[M\right]^{-1}\left[K_P\right]\left[\Phi\right]$ and

$\dot{w}_m = w_{m.R}+i\, w_{m.I}, \qquad b_{mn} = b_{mn.R} + I\, b_{mn.I}$

3.2.5 Regions of Instability

The boundaries of the regions of instability for simple and combination resonance are obtained by applying the following conditions [76] to the Eq. (3.25.)

(A) Simple resonance

The boundaries of the instability regions are given by

$$\left|\frac{\Omega}{2\omega_o}-\overline{\omega}_{\mu.R}\right|\langle\frac{1}{4}\left[\frac{\beta^2\left(b^2{}_{\mu\mu.R}+b^2{}_{\mu\mu.I}\right)}{\overline{\omega}^2{}_{\mu.R}}-16\overline{\omega}^2{}_{\mu.I}\right]^{1/2} \qquad \mu=1,2\ldots\ldots\ldots\ldots N \qquad (3.26)$$

Where $\omega_o=\sqrt{2E_1I_1/\rho_1A_1L^4}$, $\overline{\omega}_{\mu.R}=\omega_{\mu.R}/\omega_0$ and $\overline{\omega}_{\mu.I}=\omega_{\mu.I}/\omega_0$

When damping is neglected, the regions of instability are given by

$$\left|\frac{\Omega}{2\omega_o}-\overline{\omega}_{\mu.R}\right|\langle\frac{1}{4}\left[\frac{\beta\left(b_{\mu\mu.R}\right)}{\overline{\omega}_{\mu.R}}\right] \qquad \mu=1,2,\ldots\ldots\ldots\ldots\ldots\ldots N \qquad (3.27)$$

(B) Combination resonance of sum type

The boundaries of the regions of instability of sum type are given by

$$\left|\frac{\Omega}{2\omega_o}-\frac{1}{2}\left(\overline{\omega}_{\mu.R}+\overline{\omega}_{\nu.R}\right)\right|\langle\frac{1}{8}\frac{\left(\overline{\omega}_{\mu.I}+\overline{\omega}_{\nu.I}\right)}{\left(\overline{\omega}_{\mu.I}\overline{\omega}_{\nu.I}\right)^{1/2}}\left[\frac{\beta^2\left(b_{\mu\nu.R}b_{\nu\mu.R}+b_{\mu\nu.I}b_{\nu\mu.I}\right)}{\overline{\omega}_{\mu.R}\overline{\omega}_{\nu.R}}-16\overline{\omega}_{\mu.I}\overline{\omega}_{\nu.I}\right]^{1/2} \qquad (3.28)$$

$\mu\neq\nu,\ \mu,\nu=1,2,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots N$

When damping is neglected,the unstable regions are given by

$$\left|\frac{\Omega}{2\omega_o}-\frac{1}{2}\left(\overline{\omega}_{\mu.R}+\overline{\omega}_{\nu.R}\right)\right|\langle\frac{1}{4}\left[\frac{\beta^2\left(b_{\mu\nu.R}b_{\nu\mu.I}\right)}{\overline{\omega}_{\mu.R}\overline{\omega}_{\nu.R}}\right]^{1/2} \qquad (3.29)$$

$\mu\neq\nu,\ \mu,\nu=1,2,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots N$

(C) Combination resonance of difference type

The boundaries of the regions of instability of difference type are given by

$$\left|\frac{\Omega}{2\omega_o}-\frac{1}{2}\left(\overline{\omega}_{\mu.R}-\overline{\omega}_{v.R}\right)\right| \langle \frac{1}{8}\frac{\left(\overline{\omega}_{\mu.I}+\overline{\omega}_{v.I}\right)}{\left(\overline{\omega}_{\mu.I}\overline{\omega}_{v.I}\right)^{1/2}}\left[\frac{\beta^2\left(b_{\mu v.R}b_{v\mu.R}-b_{\mu v.I}b_{v\mu.I}\right)}{\overline{\omega}_{\mu.R}\overline{\omega}_{v.R}}-16\overline{\omega}_{\mu.I}\overline{\omega}_{v.I}\right]^{1/2} \qquad (3.30)$$

)

$v \rangle \mu,\ \mu, v = 1, 2,$……………………..$N$

When damping is neglected, the unstable regions are given by

$$\left|\frac{\Omega}{2\omega_o}-\frac{1}{2}\left(\overline{\omega}_{\mu.R}-\overline{\omega}_{v.R}\right)\right| \langle \frac{1}{4}\left[\frac{\beta^2\left(b_{\mu v.R}b_{v\mu.R}\right)}{\overline{\omega}_{\mu.R}\overline{\omega}_{v.R}}\right]^{1/2} \qquad (3.31)$$

$v \rangle \mu,\ \mu, v = 1, 2,$……………………..$N$

3.3 Results and discussion

To study the effect of various system parameters, such as core thickness parameter t_{21}, (defined as the ratio of the thickness of the viscoelastic core to the thickness of the elastic layer) and core loss factor, numerical results have been obtained for a three layer symmetric beam with identical elastic layers and having fixed end condition. For calculation purpose the young's modulus E_1 of the elastic layers and the inphase shear modulus of the viscoelastic material layer G_v^* were taken as 70x10^9 N/m^2 and 2.6x10^5 N/m^2 respectively. The ratio of mass density ρ_{21} of the viscoelastic material layer and elastic material layer was taken to be 0.4.With a ten element discretisation of the beam, the resonant frequency parameters and modal system loss factors obtained for a three-layer beam were compared with those of Rao[69] and results were found to be in good agreement. The comparison is shown in Table-3.1. In the following discussion, $g = \frac{G_v^*}{t_{21}}\left(\frac{L}{t_1}\right)^2\left(\frac{2}{E_1}\right)$ as defined in Rao[69], is the shear parameter.

Table-3.1 Comparison of Resonant frequency parameters and Modal loss factors calculated from present analysis with those of reference[69].

$g = 5.0$, $t_{21} = 1.0$, $\eta_c = 0.1, 0.6$

Core loss factor η_c		Fundamental frequency parameter f_1	Fundamental loss factor η_1	Second mode frequency parameter f_2	Second mode loss factor η_2	Third mode frequency parameter f_3	Third mode loss factor η_3
0.1	Present	7.8239	0.0302	33.3295	0.038	77.6149	0.0316
	Reference [69]	7.9213	0.0307	34.0012	0.0391	78.5237	0.03205
0.6	Present	8.0458	0.1557	33.9851	0.2152	78.4573	0.1854
	Reference [69]	8.1932	0.1569	34.0517	0.2192	79.1942	0.1839

Effect of core thickness parameter on fundamental buckling load parameter is shown in figure 3.3 for shear parameter $g = 5.0$ and core loss factor $\eta_c = 0.5$. Fundamental buckling load parameter is defined as the ratio of the buckling load of the sandwich beam and to that of an equivalent Euler beam. It is seen that with increase in core thickness parameter fundamental buckling load parameter increases. The rate of increase is more for higher values of core thickness parameter.

Effect of core thickness parameter on fundamental frequency parameter is shown in figure3.4 for $g = 5.0$ and $\eta_c = 0.5$. Fundamental frequency parameter is defined as the ratio of fundamental resonant frequency of the sandwich beam to that of an equivalent Euler beam. It is seen that fundamental resonant frequency increases almost linearly with increase in core thickness parameter.

Effect of core thickness parameter on second mode frequency parameter is shown in figure3.5 for $g = 5.0$ and $\eta_c = 0.5$. The second mode frequency parameter increases almost linearly with increase in core thickness parameter.

Effect of core thickness parameter on fundamental loss factor of the system is shown in figure 3.6 for $g = 5.0$ and $\eta_c = 0.5$. It is seen that fundamental loss factor increases with increase in core thickness parameter, but the rate of increase is less for higher values of core thickness parameter.

Figure 3.7 shows the effect of core thickness parameter on second mode loss factor for $g = 5.0$ and $\eta_c = 0.5$. In this case also second mode loss factor increases with increase in core thickness parameter.

Figure 3.8 shows the effect of core thickness parameter on the instability regions. Instability regions are shown for two values of core thickness parameter $t_{21} = 2/3$ and 1/3 for $g = 5.0$ and $\eta_c = 0.3$ for both the cases. It is seen that with increase in core thickness parameter the width of the instability regions decreases. The instability regions also shift to higher frequency of excitation along the excitation frequency axis and also shift upward parallel to the dynamic load axis which means that with increase in core thickness parameter the area of the instability regions reduces and instability commences at higher values of excitation frequency and dynamic load component. Thus with increase in core thickness parameter there is improvement of the stability of the beam.

The effect of core loss factor η_c on the instability regions are shown in figure 3.9. Instability regions have been shown for $\eta_c = 0.18$ and 0.3 with $g=5.0$ and $t_{21}=0.67$. It can be seen that with increase in core loss factor the area of the instability region reduces. The instability regions shift vertically upward with increase in core loss factor, but there is little shift of the instability regions parallel to the frequency axis. Thus with increase in core loss factor the instability of the beam enhances with respect to reduction in area of the instability regions and commencement of instability at higher dynamic load component.

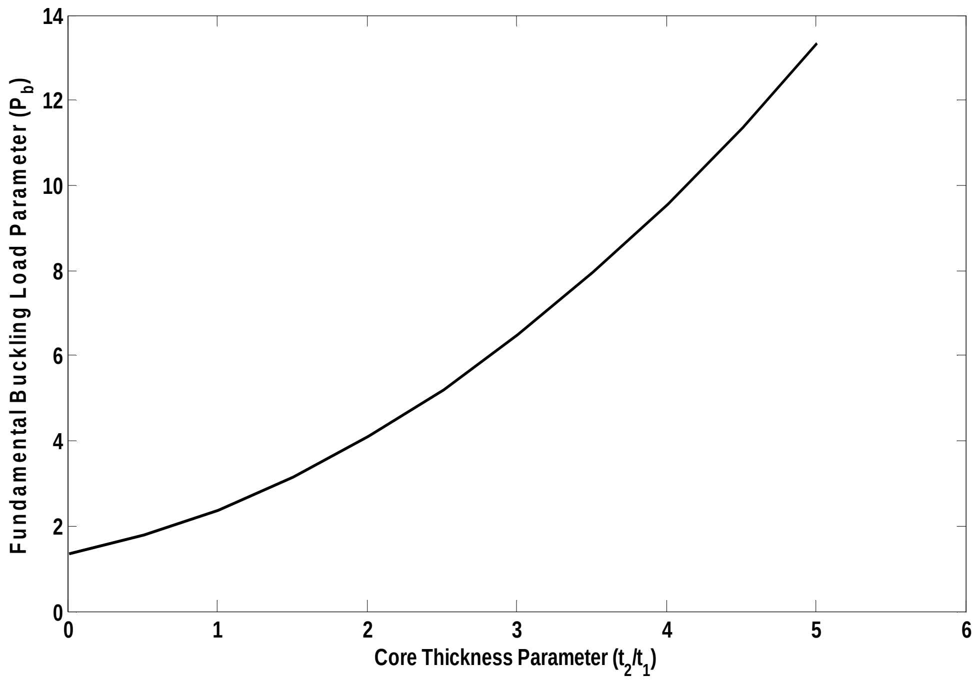


Figure - 3.3 ,Effect of Core Thickness Parameter on Fundamental Buckling Load Parameter,g=5.0, η_c=0.3

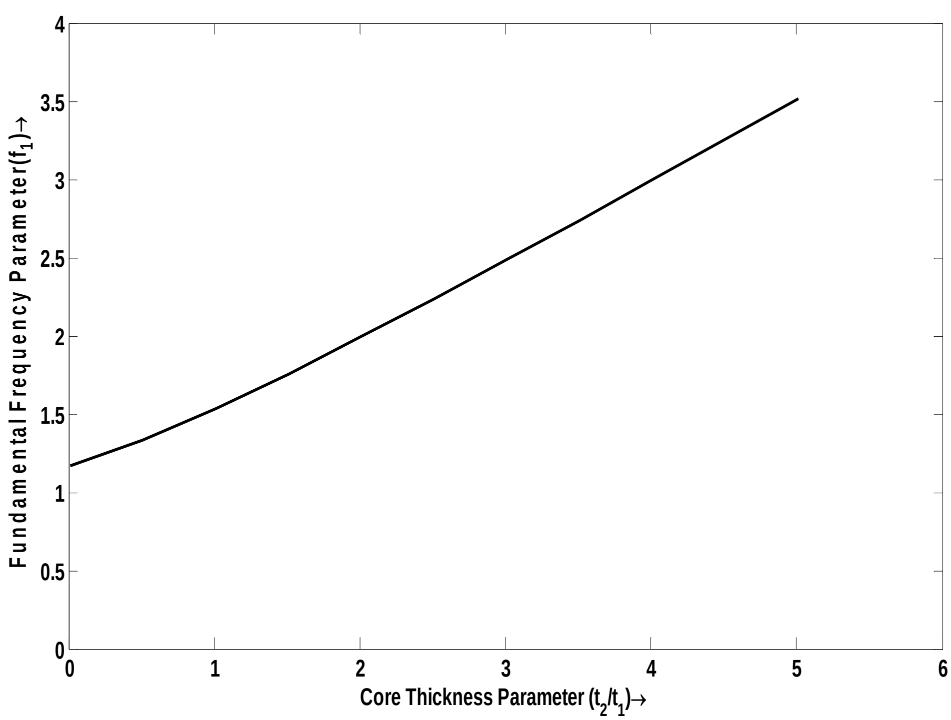


Figure - 3.4 ,Effect of Core Thickness Parameter on Fundamental Frequency Prameter,g=5.0, η_c=0.3

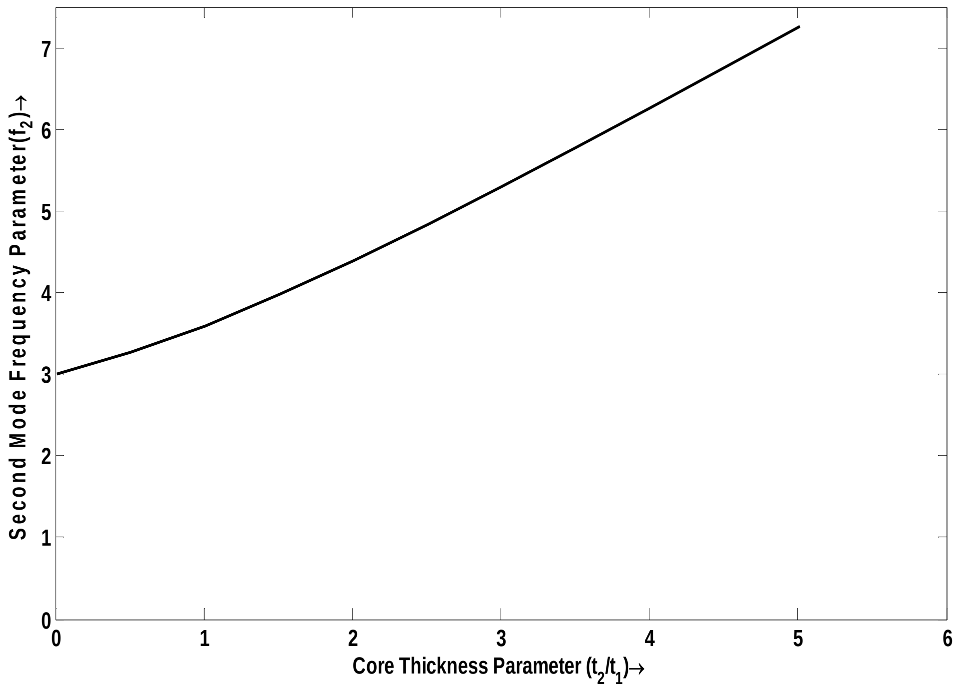


Figure - 3.5 ,Effect of Core Thickness Parameter on Second Mode Frequency Prameter,g_3=5.0, η_c=0.3

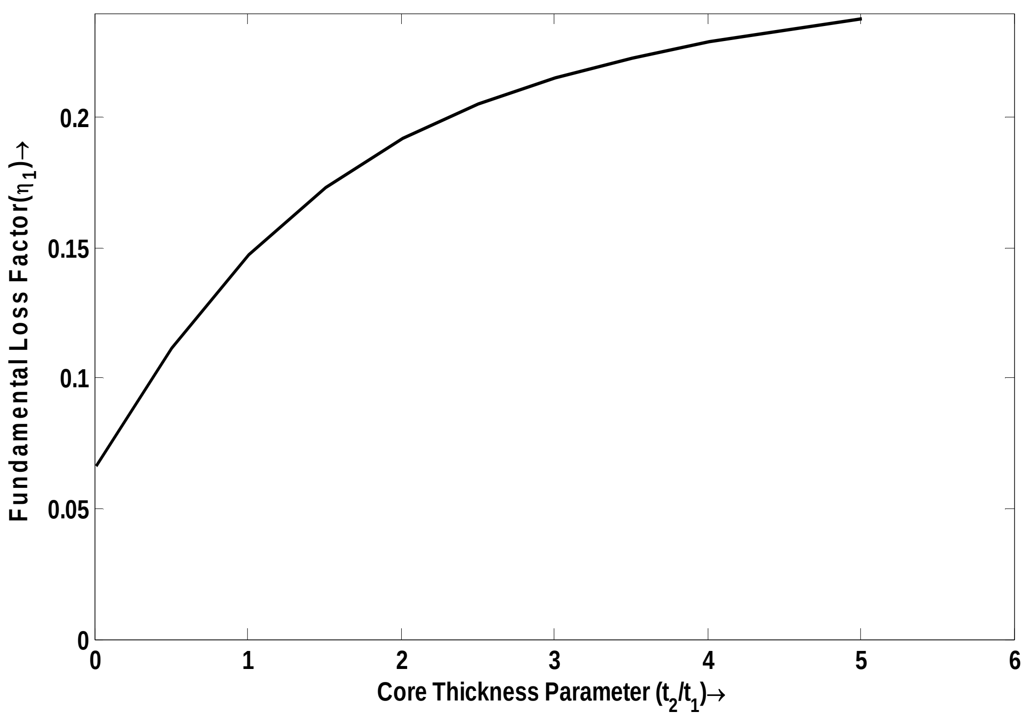


Figure - 3.6,Effect of Core Thickness Parameter on Fundamental Loss Factor,g=5.0, η_c=0.3

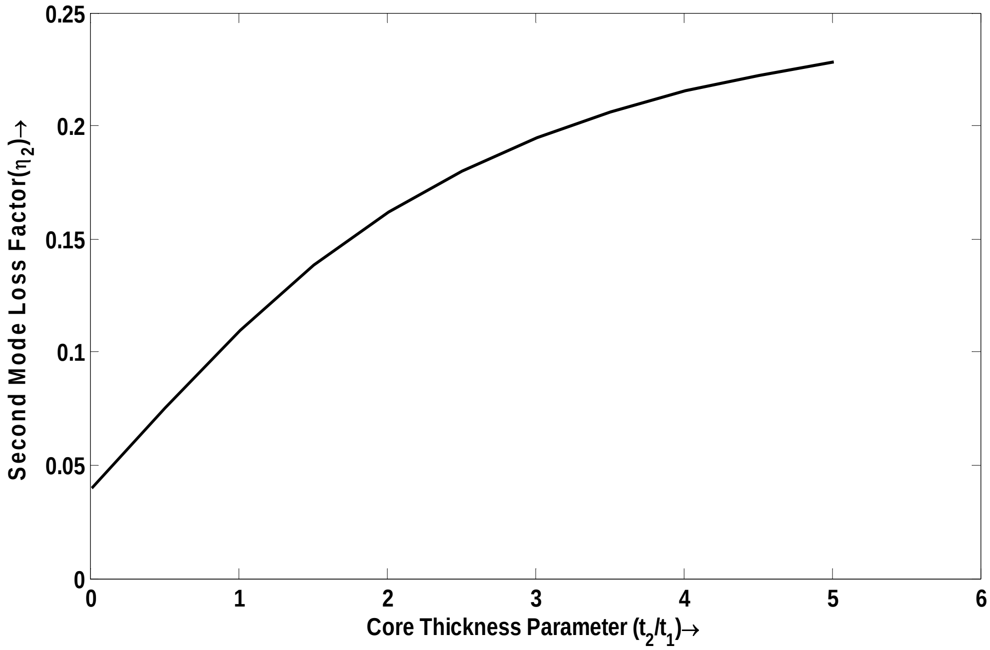


Figure - 3.7 ,Effect of Core Thickness Parameter on Second Mode Loss Factor, g =5.0, η_c=0.3

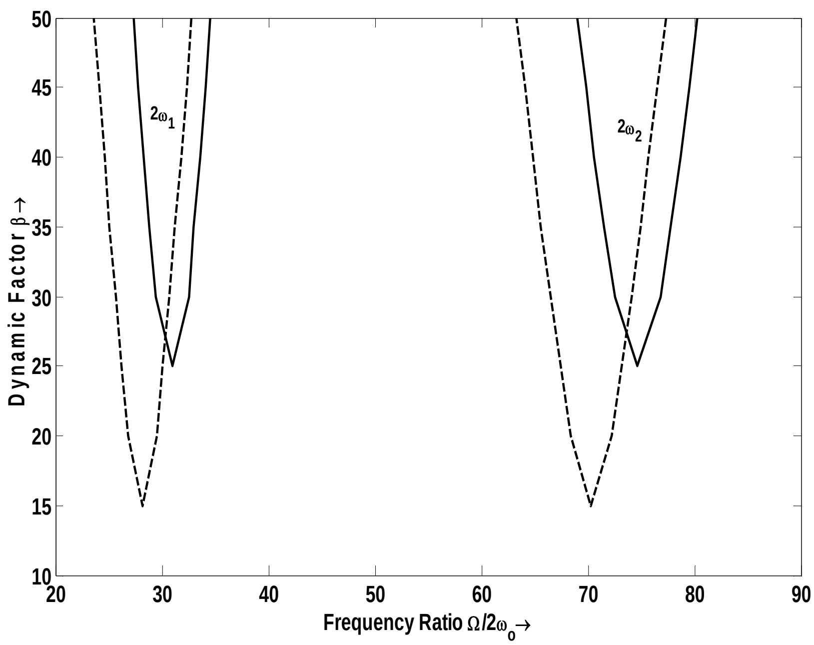


Figure -3.8 ,Instability Regions:g =5.0,η_c=0.3,t_{21} =2/3'--',t_{21}=1/3,'-' ,

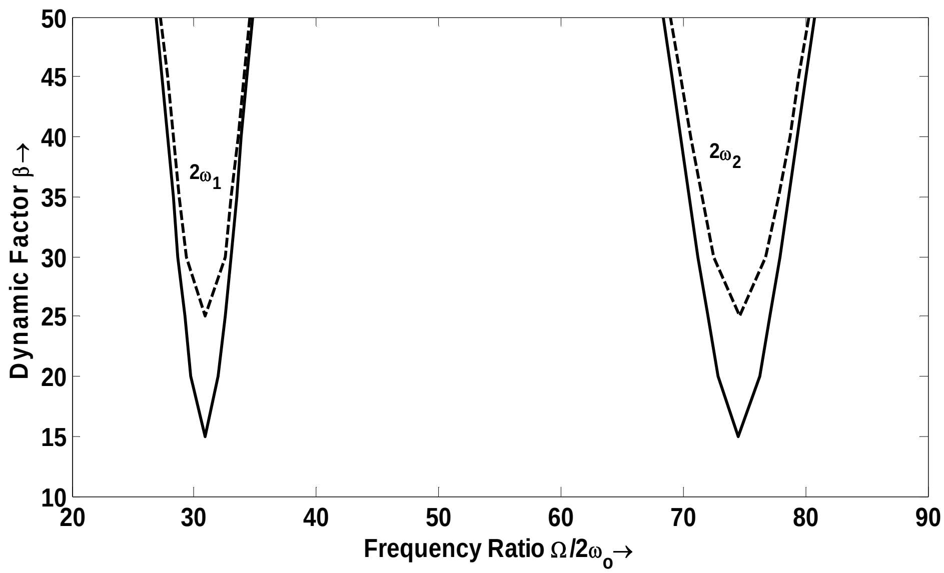


Figure - 3.9,Instability Regions: g =5.0, t_{21} =0.67, η_c=0.18,-, η_c=0.3,--,

3.4 Closure

From the above it can be concluded that with increase in core thickness parameter fundamental buckling load increases. The fundamental resonant frequency and second mode frequency parameter also increase with increase in core thickness parameter. Fundamental loss factor and second mode loss factor also increase with increase in core thickness parameter. Again increase in core thickness parameter enhances the stability of the beam. With increase in core loss factor also the stability of the beam enhances.

CHAPTER-4

Experimental work

4.1 Introduction

The aim of the experimental work is to establish experimentally the stability diagrams for a few typical cases related to sandwich beams .For sandwich beams , the stability diagrams have been experimentally established for three-layered beams.The theoretical and experimental stability diagrams have been compared to assess the accuracy of the theoretical results.

4.2 Description of the experimental set up

The set up consists of 1) Frame 2)Specimen 3)End attachments 4)Vibration Generator/Electrodynamic shaker 5)Vibration Pickup 6)Oscilloscope/Signal Analyzer 7) Loading Device 8)Load Indicator 9)Oscillator & Amplifier

1) FRAME:- It has been fabricated from steel channel sections by welding. The frame is fixed in vertical position to the foundation by means of foundation bolts and it has the provision to accommodate beams of different lengths.

2) END ATTACHMENT:- It is manufactured from steel angles. Holes are drilled on the angle flange. The end of the beam is rigidly fixed by tightening bolts.

3) VIBRATION GENERETOR / ELECTRODYNAMIC SHAKER:- It is used to apply variable loading at different frequencies. The periodic axial load $P_1 \cos \Omega t$ is applied to the specimen by a 500N capacity electrodynamic shaker (Saraswati dynamics ,India, Model no. SEV-005).

4) VIBRATION PICKUP:- It is used to sense the amplitude of vibration of the beam. The vibration response of the test specimen is measured by means of vibration pickups (B & K type.model no. MM-0002).

5) OSCILLOSCOPE: - It is used to observe the response of vibration pick ups and load cell.

6) LOADING DEVICE:- The static load can be applied to the specimen by means of a screw jack fixed to the frame at the upper end.

7) LOAD CELL:- The applied load on the specimen is measured by a piezoelectric load cell (Bruel & Kjaer , model no. 2310-100), which is fixed between the shaker and the specimen.

8) OSCILLATOR & AMPLIFIER:- Oscillator is used to produce the sine wave of required frequency. Amplifier is used for subsequent amplification of the signal generated by the oscillator.

The schematic diagram of the equipments used for the experiment and photographic view of the experimental set up are shown .

4.3. Preparation of specimens

Sandwich beams were fabricated from strips cut from mild steel sheets of suitable thickness. Viscoelastic core of the sandwich beam is P.V.C. In preparing the sandwich beams the face layers were made free from dirt, grease etc. by cleaning their surfaces with acetone and carbon tetrachloride. The adhesive used for bonding the layers is commercially available dendrite. After application of thin layer of adhesive on surfaces, the layers were bonded and the sandwich beams were kept under loads for about six hours. Slipping of the layers were avoided by providing positioning guides at all the edges of the specimens during the setting time .The Young's modulus of the specimen materials were determined by measuring the static deflection of a test specimen under known load. Mass density of the specimen material was measured by measuring the weight and volume of a piece of specimen material.The details of the physical and geometric data of the specimen are given in tables and the photographs of the specimen are shown in plates.

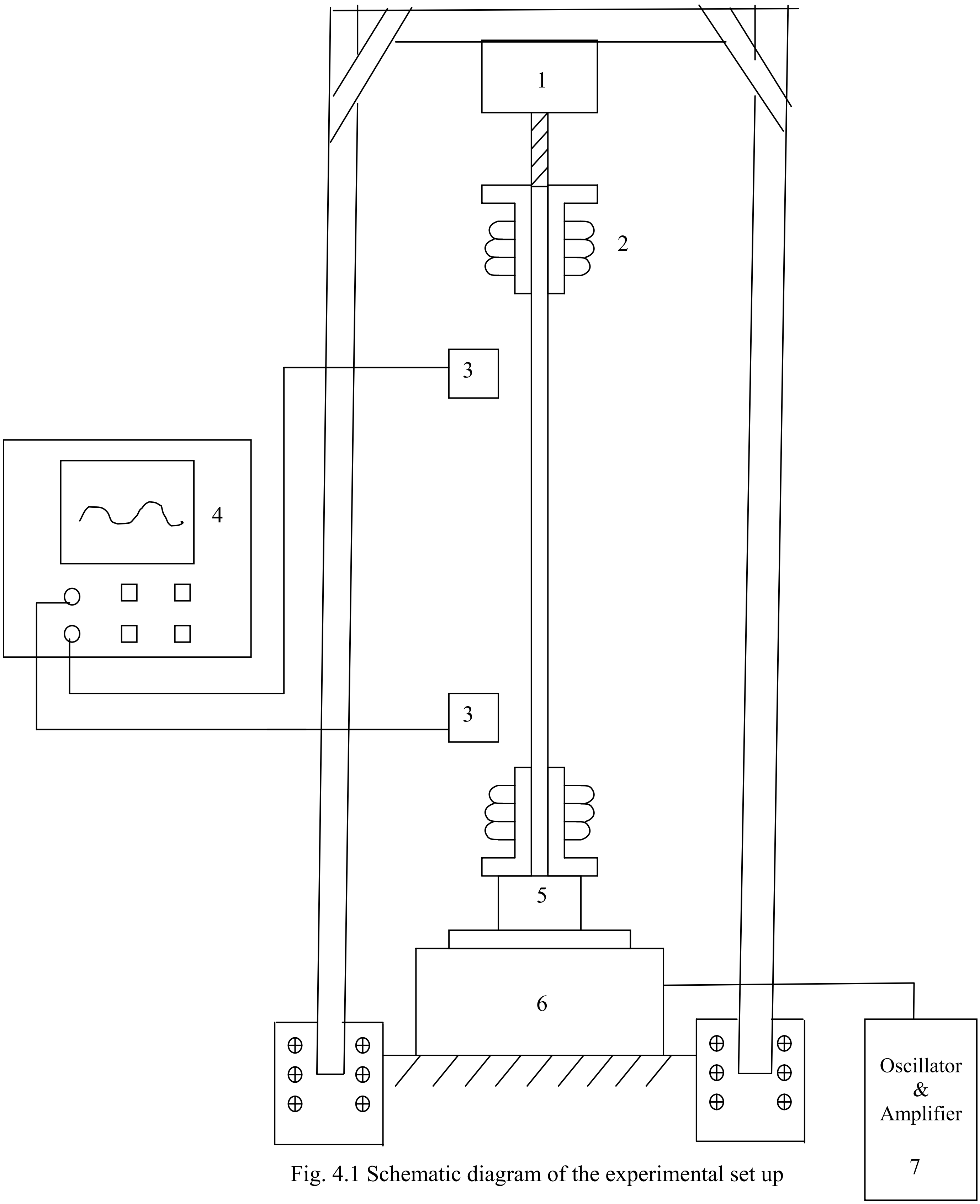
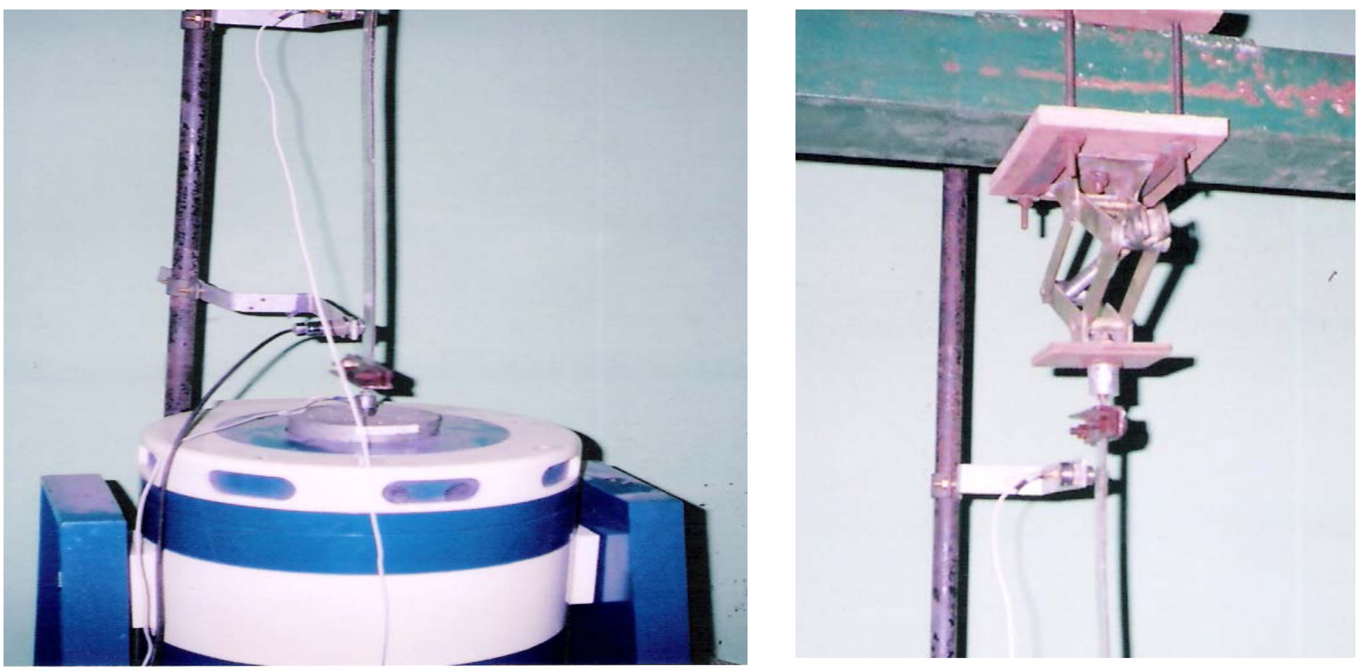


Fig. 4.1 Schematic diagram of the experimental set up

1. Screw jack 2. Fixed end attachment 3. Vibration pick ups 4. Oscilloscope 5. Piezoelectric load cell 6. Electrodynamic shaker 7. Oscillator & amplifier

Test Setup

Specimens



Fixed End Attachment

Table-4.1, Physical and Geometrical parameters of the specimen,

Specimen No.	Length L in m	Breadth B in m	Elastic layer thickness t_1 in m	Viscoelastic layer thickness t_2 in m	Young's modulus E in N/m^2	Inphase shear modulus G_v^* in N/m^2
1	0.5	0.0254	0.001	0.25	2.08×10^{11}	9.33×10^{6}
2	0.5	0.0254	0.001	3.0	2.08×10^{11}	9.33×10^{6}

4.4 Testing Procedure

An oscillator cum power amplifier unit drives the electrodynamic vibration shaker used for providing for dynamic loading.The beam response was recorded by the non-contacting vibration pick-up.Two pick-ups, one at each end of the beam were used to send the vibrartion response to the beam.The air gap between the pick-up and the vibrating surface were so adjusted that the measurements were in the linear range.The amplitude of the signal gives no absolute displacement since it is not calibrated.

Initially the beam was excited at certain frequencies and the amplitude of excitation was increased till the response was observed. Then the amplitude of excitation was kept constant and the frequency of excitation was changed in step by 0.1Hz. The experimental boundary of instability region was marked by the parameters [P_1,Ω], which were measured just before a sudden increase of the amplitude of lateral vibration. For accurate measurement of the excitation frequency an accelerometer was fixed to the moving platform of the exciter .Its response was observed on computer in the frequency domain.The dynamic load component of the applied load was measured from the response curve of the load cell.The excitation frequency was divided by $2\omega_0$ to get the non-dimensional excitation frequency [$\Omega/2\omega_0$].Similarly the dynamic load amplitude was divided by the reference load P_{cr} to get the dynamic load factor β. The details of observations are given in tables.

4.5 Results and discussion

Figures (4.2 & 4.3) show the theoretical and experimental instability diagrams for a three layer sandwich beam with core thickness parameter equal to 0.25 and 3.0 respectively. The measured excitation frequencies and dynamic load are presented in table(4.1 & 4.2). It is seen that the instability regions from theoretical analysis are fairly close to the experimental ones.

4.6 Closure

There is good agreement between the theoretical and experimental results.

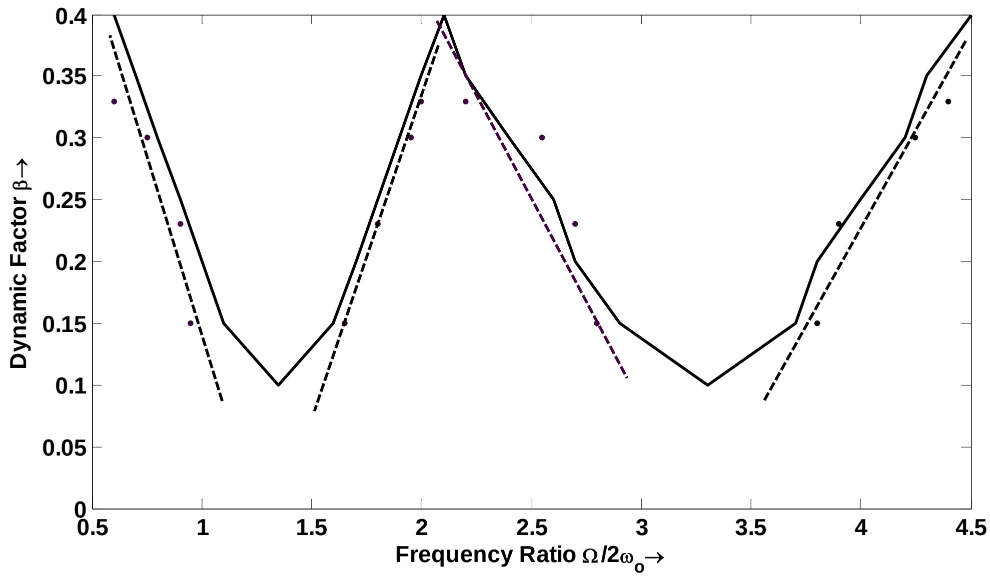


Fig. -4.2,Instability regions for three layer beam:,t_{21} =0.25,η_c=0.55,

Theoretical Boundary from FEM;-, Experimental data;•.

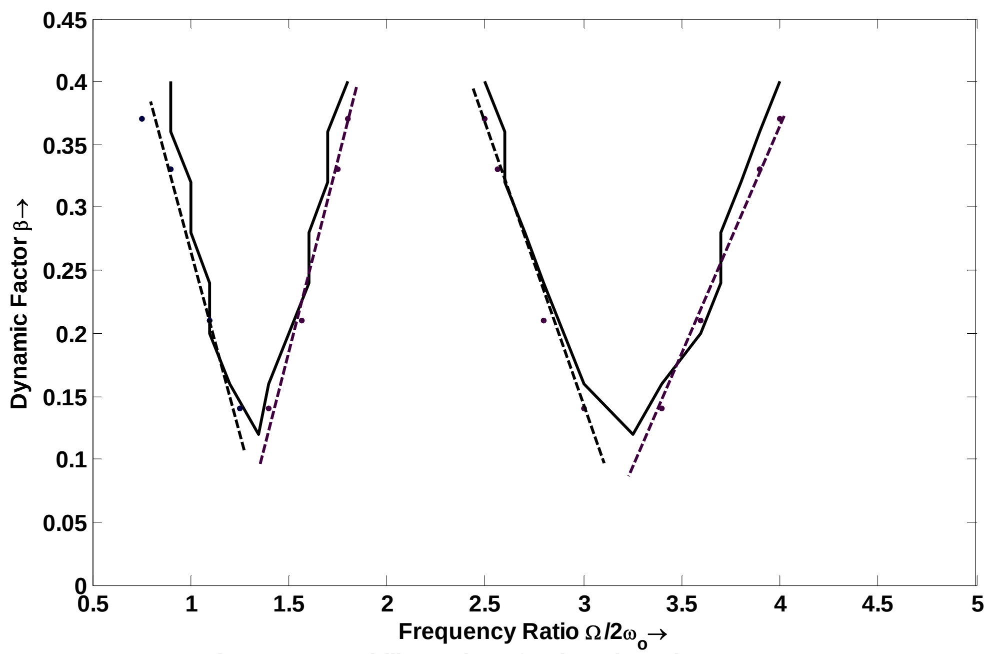


Fig. - 4.3 ,Instability regions for three layer beam:,t_{21} =3.0,η_c=0.55,

Theoretical Boundary from FEM;-, Experimental data;•.

Table-4.2, Experimental boundary frequencies of instability regions for 3- layered sandwich beam,

L= 0.5m, t_1=0.001m, t_{21} = 0.25 , P_{cr}=36.86N, ω_o=20Hz.

Sl No	Dynamic load Amplitude (P_1)	Excitation Frequency (Ω)						Dynamic Load Factor $\beta = P_1 / P_{cr}$	Excitation frequency ratio $\Omega/2\omega_o$					
		1st Zone		2nd Zone		3rd Zone			$2\omega_1$		$\omega_1+\omega_2$		$2\omega_2$	
		Lower limit (Ω_{11})	Upper limit (Ω_{12})	Lower limit (Ω_{21})	Upper limit (Ω_{22})	Lower limit (Ω_{31})	Upper limit (Ω_{32})		Lower limit ($\Omega_{11}/2\omega_o$)	Upper limit ($\Omega_{12}/2\omega_o$)	Lower limit ($\Omega_{21}/2\omega_o$)	Upper limit ($\Omega_{22}/2\omega_o$)	Lower limit ($\Omega_{31}/2\omega_o$)	Upper limit ($\Omega_{32}/2\omega_o$)
1	5.5	19.0	33.0	-	-	56.0	76.0	0.15	0.95	1.65	-	-	2.8	3.8
2	8.5	18.0	36.0	-	-	54.0	78.0	0.23	0.90	1.80	-	-	2.7	3.9
3	11.0	15.0	39.0	-	-	51.0	85.0	0.30	0.75	1.95	-	-	2.55	4.2
4	12.2	12.0	40.0	-	-	44.0	88.0	0.33	0.60	2.0	-	-	2.20	4.4

Table-4.3, Experimental boundary frequencies of instability regions for 3-layered sandwich beam,

L= 0.5m, t_1=0.001m, t_{21} =3 , P_{cr}=36.8N, ω_o= 16.18Hz.

Sl No	Dynamic load Amplitude (P_1)	Excitation Frequency (Ω)						Dynamic Load Factor $\beta = P_1 / P_{cr}$	Excitation frequency ratio $\Omega/2\omega_o$					
		1st Zone		2nd Zone		3rd Zone			$2\omega_1$		$\omega_1+\omega_2$		$2\omega_2$	
		Lower limit (Ω_{11})	Upper limit (Ω_{12})	Lower limit (Ω_{21})	Upper limit (Ω_{22})	Lower limit (Ω_{31})	Upper limit (Ω_{32})		Lower limit ($\Omega_{11}/2\omega_o$)	Upper limit ($\Omega_{12}/2\omega_o$)	Lower limit ($\Omega_{21}/2\omega_o$)	Upper limit ($\Omega_{22}/2\omega_o$)	Lower limit ($\Omega_{31}/2\omega_o$)	Upper limit ($\Omega_{32}/2\omega_o$)
1	5.2	20.2	22.7	-	-	48.5	55.0	0.14	1.25	1.40	-	-	3.0	3.4
2	7.7	17.8	25.4	-	-	45.3	58.2	0.21	1.10	1.57	-	-	2.8	3.6
3	12.1	14.6	28.3	-	-	41.4	63.1	0.33	0.90	1.75	-	-	2.56	3.9
4	13.6	12.1	29.1	-	-	40.5	64.7	0.37	0.75	1.80	-	-	2.5	4.0

CHAPTER-5

5.1 Conclusion:

The following conclusions can be made from the present study.

(i) With increase in core thickness parameter fundamental buckling load increases.

(ii) Fundamental resonant frequency and second mode frequency parameter also increase with increase in core thickness parameter. .

(iii) Fundamental loss factor and second mode loss factor also increase with increase in core thickness parameter.

(iv) Increase in core thickness parameter enhances the stability of the beam.

(v) With increase in core loss factor also the stability of the beam enhances.

(vi) There is a very good agreement of the experimental results with the theoretical findings.

5.2 Scope for Future Work

The following works may be carried out as an extension of the present work.

1. Stability of sandwich beams with different boundary conditions.
2. Stability of sandwich beams of different cross sections like I section, trapezoidal section etc.
3. Stability of multilayered sandwich beams.
4. Stability of sandwich plates.

CHAPTER-6

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