Dynamic Stability of a Sandwich Plate Under Parametric Excitation

Thesis submitted in partial fulfillment of the requirements
for the degree of

Master of Technology
In
MECHANICAL ENGINEERING
(Specialization: Machine Design and Analysis)

By
RAGHUPATHI AVULA
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DEPARTMENT OF MECHANICAL ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY
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CERTIFICATE

This is to certify that the thesis entitled "**DYNAMIC STABILITY OF A SANDWICH PLATE UNDER PARAMETRIC EXITATION**" submitted by **Raghupathi Avula** in partial fulfillment of the requirements for the award of **Master of Technology** Degree in **Mechanical Engineerin**g at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma

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ABSTRACT

Vibration control of machines and structures incorporating viscoelastic materials in suitable arrangement is an important aspect of investigation. The use of viscoelastic layers constrained between elastic layers is known to be effective for damping of flexural vibrations of structures over a wide range of frequencies. The energy dissipated in these arrangements is due to shear deformation in the viscoelastic layers, which occurs due to flexural vibration of the structures. Sandwich plate like structures can be used in aircrafts and other applications such as robot arms for effective vibration control. These members may experience parametric instability when subjected to time dependant forces. The theory of dynamic stability of elastic systems deals with the study of vibrations induced by pulsating loads that are parametric with respect to certain forms of deformation

The purpose of the present work is to investigate the dynamic stability of a three layered symmetric sandwich plate subjected to an end periodic axial force. Equations of motion are derived using finite element method. The regions of instability for simple and combination resonances are established using modified Hsu's method proposed by Saito and Otomi[74].

It is observed that for plate with simply supported boundary conditions the fundamental frequency, fundamental buckling load increase with increase in core thickness parameter for higher values of core thickness parameter. The fundamental frequency, fundamental buckling load decrease with increase in core thickness parameter for lower values of core thickness parameter. The system fundamental loss factor increases with increase in thickness ratio. The fundamental buckling load and fundamental frequency increase with increase in shear parameter. The fundamental system loss factor has an increasing tendency with increase in shear parameter. The increase in core thickness ratio and shear parameter has stabilizing effect. Whereas increase in static load factor has a destabilizing effect.

Keywords: Parametric excitation, Dynamic stability, Boundary condition, Sandwich plate.

LIST OF FIGURES

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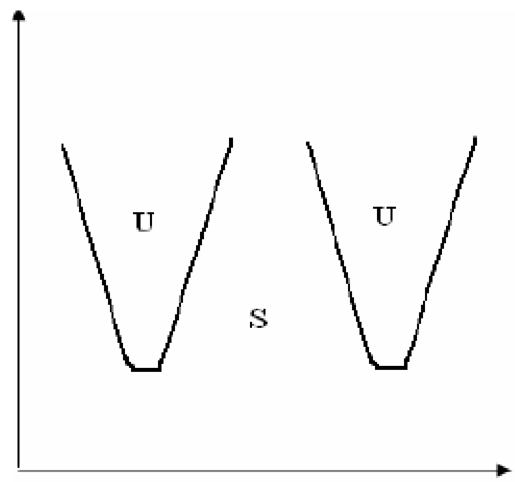
NOMENCLATURE

a	Length of of the sandwich plate.
b	Width of the sandwich plate..
D_1	$E_1h_3/[12(1-\mu^2{}_1)]$ flexural rigidity of the host plate.
E_i	Young's moduli of face layers, i =1,3.
G	Shear modulus of the viscoelastic material.
hi	Thickness of the layer i, i=1,2,3.
h2/h1	Core thickness parameter.
i	Index for layer, i = 1, 2, 3.
K_0	$-P_0h_1b^2/D_1$, non-dimensional static component of the external load.
Kt	$P_th_1b^2/D_1$, non-dimensional dynamic component of the external load.
N_r	Number of elements in the r-direction.
N_z	Number of elements in the z-direction.
P_0	Static component of the external load.
P_t	Dynamic component of the external load.
Ω	Frequency of the external load..
$\{\Gamma\}$	Set of generalized coordinates
η_v	Loss factor of the viscoelastic material.
μ_i	Poisson's ratio of the layer i.
ρ_i	Mass density of the layer i, i =1,2,3.
ω	Natural frequency of the composite plate.
V	Total strain energy of the sandwich plate.
T	Total kinetic energy of the sandwich plate.
W	Work done by the periodic load.
$[K^e]$	Stiffness matrix of the sandwich plate.
[K]	Global Stiffness matrix of the sandwich plate.
$[M^e]$	Element mass matrix of sandwich plate.
[M]	Global mass matrix of sandwich plate.

$[K^e_g]$	Geometric stiffness matrix of plate element.
$[K_g]$	Global stiffness matrix of sandwich plate.
P*	Critical buckling load of the equivalent plate.
$\{\Delta\}$	Global nodal displacement matrix of the plate.
$[\Phi]$	Normalized modal matrix.
S	Stable region
U	Unstable region



CHAPTER-1

INTRODUCTION

1.1 INTRODUCTION

Sandwich plates have been successfully used for many years in the aviation and aerospace industries, as well as in marine, and mechanical and civil engineering applications. This is due to the attendant high stiffness and high strength to weight ratios of sandwich systems. The theory of dynamic stability of elastic systems deals with the study of vibrations induced by pulsating loads that are parametric with respect to certain forms of deformation. A system is said to be parametrically excited if the excitation which is an explicit function of time appears as one of the co-efficients of the homogeneous differential equation describing the system, unlike external excitation which leads to an inhomogeneous differential equation. A well known form of equation describing a parametric system is Hill's equation

$$\ddot{x} + \omega^2 \alpha + \varepsilon x f(t) = 0 \quad \text{……………………….(1)}$$

In the above equation when f(t) = cosΩt, Equation (1) is known as Mathieu's equation .
Equation (1) governs the response of many physical systems to a sinusoidal parametric equation.

In practice parametric excitation can occur in structural systems subjected to vertical ground motion, aircraft structures subjected to turbulent flow, and in machine components and mechanisms. Other examples are longitudinal excitation of rocket tanks and their liquid propellant by the combustion chambers during powered flight, helicopter blades in forward flight in a free-stream that varies periodically and spinning satellites in elliptic orbits passing through a periodically varying gravitational field. In industrial machines and mechanisms, their components and instruments are frequently subjected to periodic or random excitation transmitted through elastic coupling elements, example includes those associated with electromagnetic aeronautical instruments and vibratory conveyers saw blades and belt drives.

In parametric instability the rate of increase in amplitude is generally exponential and thus potentially dangerous while in typical resonance the rate of increase is linear. Moreover damping reduces the severity of typical resonance, but may only reduce the rate of increase during parametric resonance. Moreover parametric instability occurs over a region of parameter space and not at discrete points. The system can experience parametric instability (resonance), when the excitation frequency or any integer multiple of it is twice the natural frequency that is to say $m\Omega=2\omega$, m=1, 2, 3, 4.

The case Ω=2ω is known to be the most important in application and is called main parametric resonance. A vital step in the analysis of parametric dynamic systems is thus establishment of the regions in the parameter space in which the system becomes unstable, these regions are known as regions of dynamic instability or zones of parametric resonance. The unstable regions are separated from the stable ones by the so called stability boundaries and a plot of these boundaries on the parameter space is called a stability diagram. Vibration control of machines and structures incorporating viscoelastic materials in suitable arrangement is an important aspect of investigation. The use of viscoelastic layers constrained between elastic layers is known to be effective for damping of flexural vibrations of structures over a wide range of frequencies. The energy dissipated in these arrangements is due to shear deformation in the viscoelastic layers, which occurs due to flexural vibration of the structures. Multilayered cantilever sandwich beam like structures can be used in aircrafts and other applications such as robot arms for effective vibration control. These members may experience parametric instability when subjected to time dependant forces.

The conventional sandwich construction comprises a relatively thick core of low-density material which separates top and bottom faceplates (or faces or facings) which are relatively thin but stiff. The materials that have been used in sandwich construction have been many and varied but in quite recent times interest in sandwich construction has increased with the introduction of new materials for use in the facings (e.g. fiber- reinforced composite laminated material) and in the core (e.g. solid foams) [96].

1.2 TYPES OF SANDWICH PANELS

Detailed treatment of the behavior of honeycombed and other types of sandwich panels can be found in monographs by Plantema [32] and Allen [37]. These structures are characterized by a common feature of two flat facing sheets, but the core takes many generic forms; continuous corrugated sheet or a number of discrete but aligned longitudinal top-hat, zed or channel sections (see Figures 1.1(a)-(e)). The core and facing plates are joined by spot-welds, rivets or self-tapping screws [57].

1.3 SANDWICH CONSTRUCTION

Sandwich construction is a special kind of laminate consisting of a thick core of weak, lightweight material sandwiched between two thin layers (called "face sheets") of strong material figure (1.2). This is done to improve structural strength without a corresponding increase in weight. The choice of face sheet and core materials depends heavily on the performance of the materials in the intended operational environment.

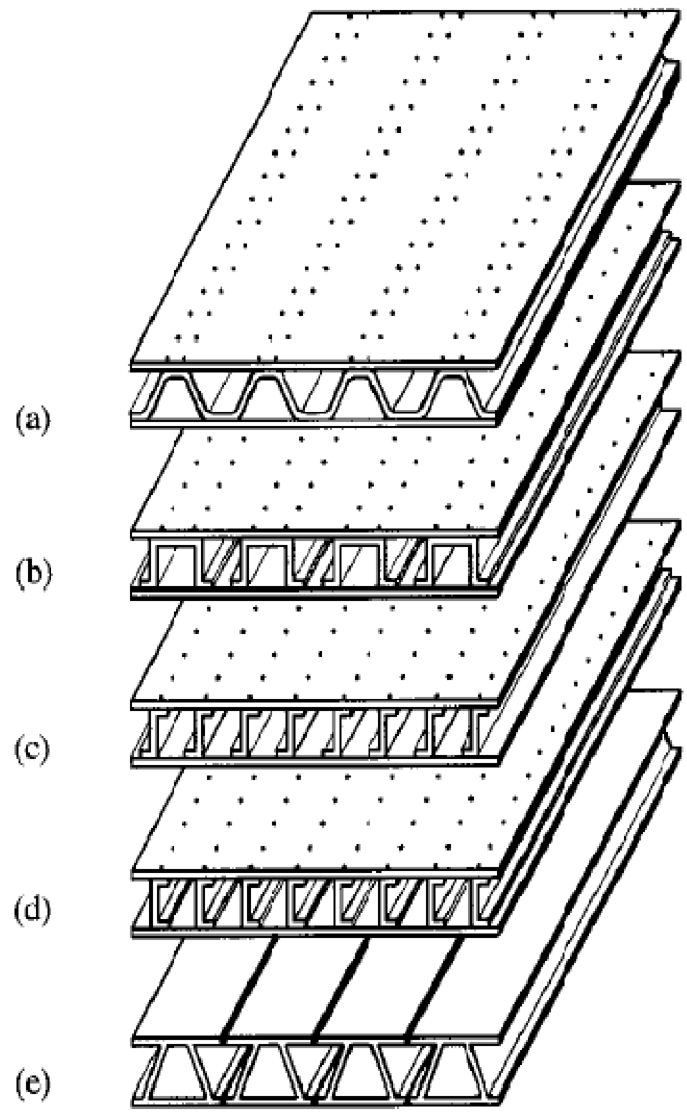


Fig 1.1 Sandwich panel with (a) Continuous corrugated (b) top hat core (c) Zed core (d) Channel core and (e) truss core

Because of the separation of the core, face sheets can develop very high bending stresses. The core stabilizes the face sheets and develops the required shear strength. Like the web of a beam, the core carries shear stresses. Unlike the web, however, the core maintains continuous support for the face sheets. The core must be rigid enough perpendicularly to the face sheets to prevent crushing and its shear rigidity must be sufficient to prevent appreciable shearing deformations. Although a sandwich composite never has a shearing rigidity as great as that of a solid piece of face-sheet material, very stiff and light structures can be made from properly designed sandwich composites.

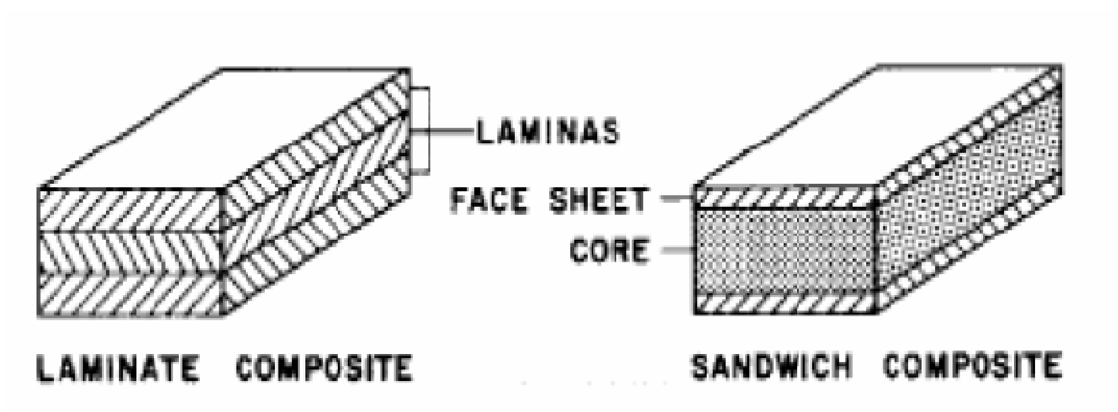


Fig 1.2 Laminate composite and sandwich composite

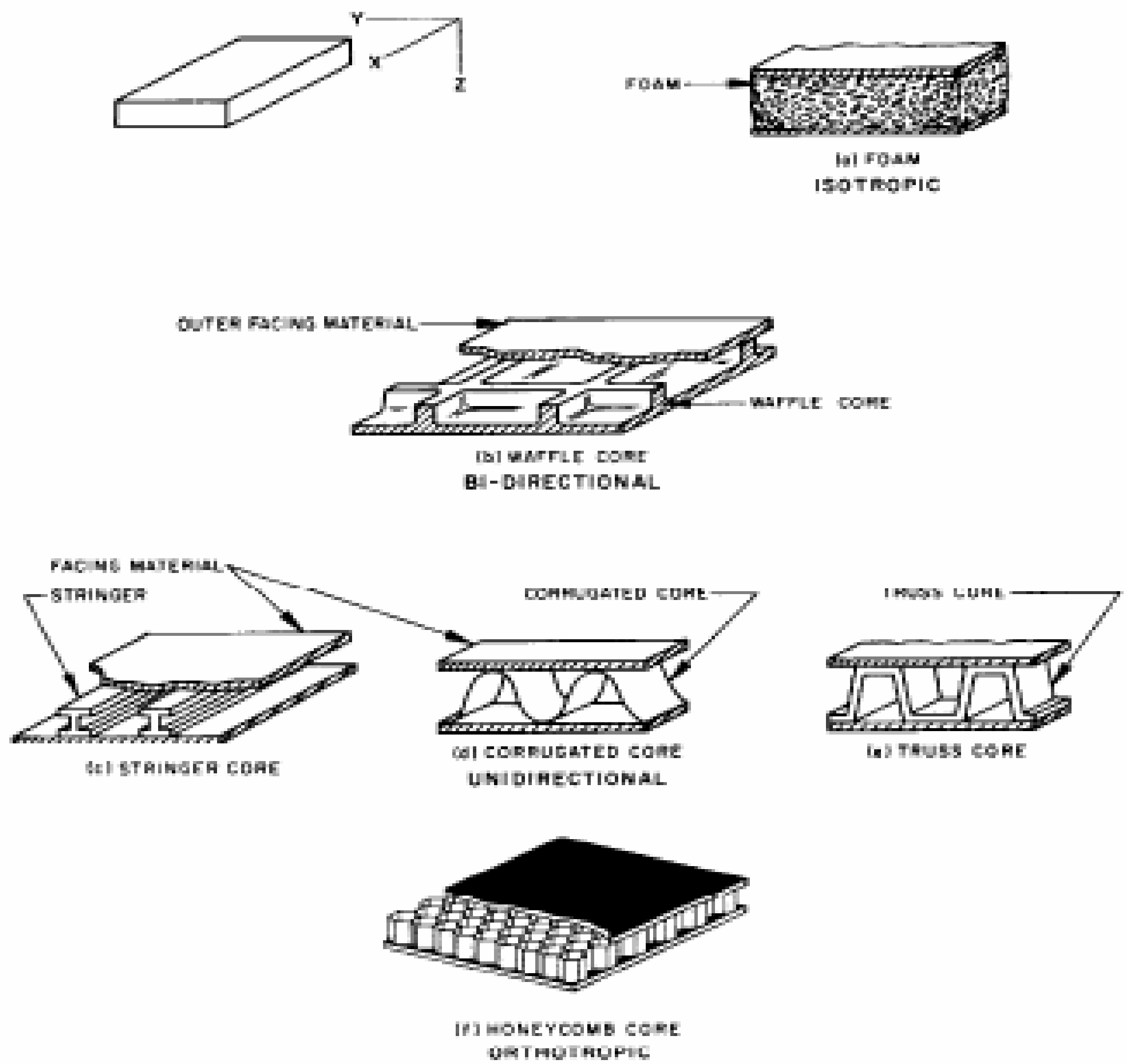


Fig 1.3 Typical sandwich constructions

A useful classification of sandwich composites according to their core properties by respective direction is shown in fig.1.3. To see the core effect upon sandwich strength, let us consider the honeycomb-core and the truss-core sandwich composite.

The honeycomb sandwich has a ratio of shear rigidities in the *xz* and *yz* planes of approximately 2.5 to 1. The face sheets carry in-plane compressive and tensile loads, whereas the core stabilizes the sheets and builds up the sandwich section.

The truss-core sandwich has a shear rigidity ratio of approximately 20 to 1. It can carry axial loads in the direction of the core orientation as well as perform its primary function of stabilizing the face sheets and building up the sandwich section [76].

1.4 PROPERTIES OF MATERIALS USED IN SANDWICH COSNTRUCTION

No single known material or construction can meet all the performance requirements of modern structures. Selection of the optimum structural type and material requires systematic evaluation of several possibilities. The primary objective often is to select the most efficient material and configuration for minimum-weight design [76]

Face Materials

Almost any structural material which is available in the form of thin sheet may be used to form the faces of a sandwich panel. Panels for high-efficiency aircraft structures utilize steel, aluminum or other metals, although reinforced plastics are sometimes adopted in special circumstances. In any efficient sandwich the faces act principally in direct tension and compression. It is therefore appropriate to determine the modulus of elasticity, ultimate strength and yield or proof stress of the face material in a simple tension test. When the material is thick and it is to be used with a weak core it may be desirable to determine its flexural rigidity [37].

Core Materials

A core material is required to perform two essential tasks; it must keep the faces the correct distance apart and it must not allow one face to slide over the other. It must be of low density. Most of the cores have densities in the range 7 to 9.5 lb/ft^3. Balsa wood is one of the original core materials. It is usually used with the grain perpendicular to the faces of the sandwich. The density is rather variable but the transverse strength and stiffness are good and the shear stiffness moderate. Modern expanded plastics are approximately isotropic and their strengths and stiff nesses are very roughly proportional to density. In case of aluminum honeycomb core, all the properties increase progressively with increases in thickness of the foil from which the honeycomb is made [37].

1.5 CURRENT APPLICATIONS

Damped Structures

An increasing number of vibration problems must be controlled by damping resonant response. By using a symmetric sandwich panel with a viscoelastic core, various degrees of damping can be achieved, depending on the core material properties, core thickness, and wavelength of the vibration mode [76]

Aerospace Field

In Aerospace industry various structural designs are accomplished to fulfill the required mission of the aircraft. Since a continually growing list of sandwich applications in aircraft/helicopter (example-Jaguar, Light Combat Aircraft, Advanced Light Helicopter) includes fuselages, wings, ailerons, floor panels and storage and pressure tanks as shown in fig (1.4). Honeycomb sandwich structures have been widely used for load-bearing purposes in the aerospace due to their lightweight, high specific bending stiffness and strength under distributed loads in addition to their good energy-absorbing capacity [8]. In a new space-formed system called "Sunflower," the reflector is of honeycomb construction, having a thin coating of pure aluminum protected by a thin coating of silicon oxide to give the very high reflectivity needed for solar-energy collection. Thirty panels fold together into a nose-cone package in the launch vehicle.

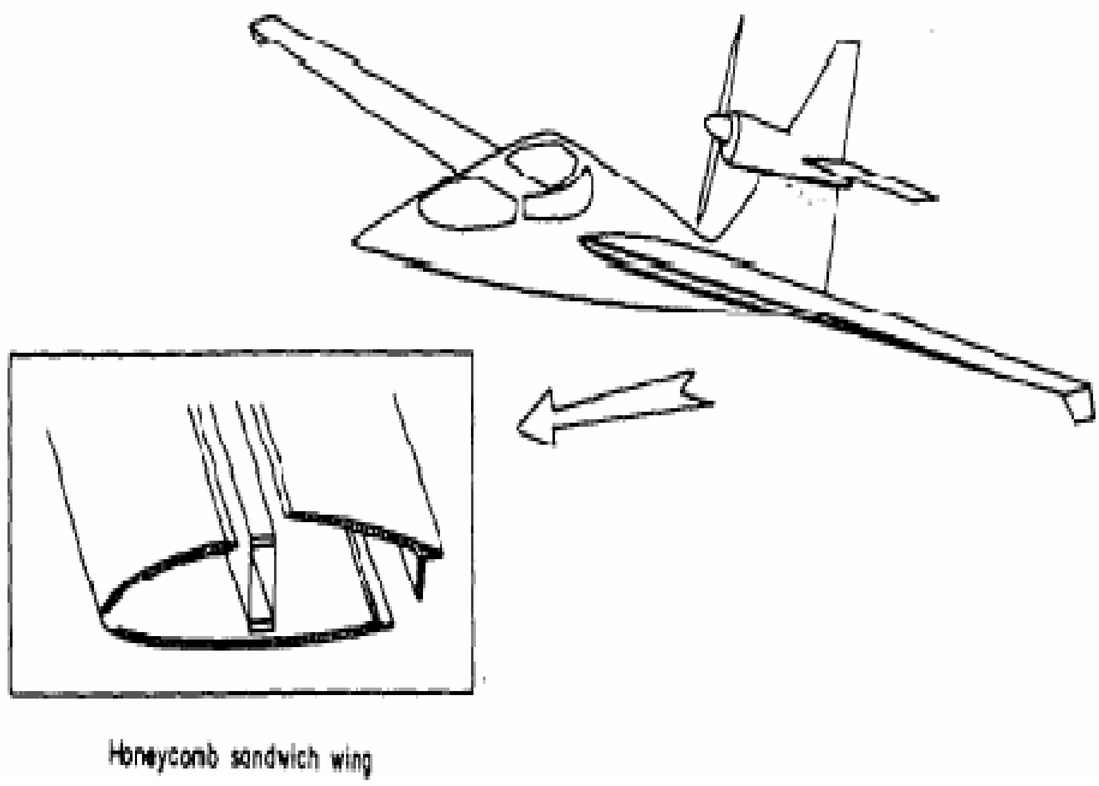


Fig 1.4(a) Application of sandwich structure in aircraft.

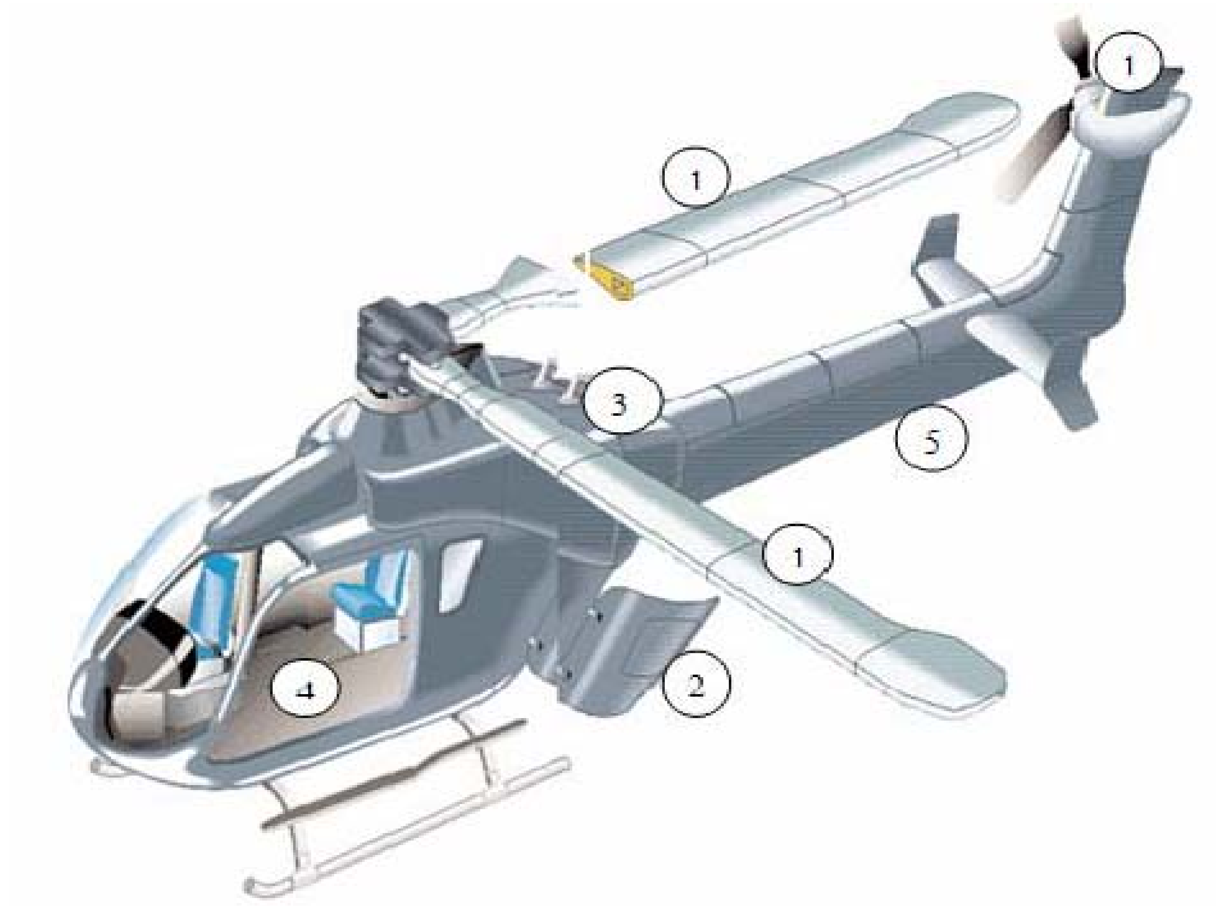


Fig 1.4(b) Application of sandwich structure in helicopter
(1. Rotor Blades, 2. Main and Cargo Doors, 3. Fuselage Panels, 4. Fuselage, 5. Boom and Tail section)

Building Construction

Architects use sandwich construction made of a variety of materials for walls, ceilings, floor panels, and roofing. Cores for building materials include urethane foam (slab or foam-in-place), polystyrene foam (board or mold), phenolic foam, phenolic-impregnated paper honeycomb, woven fabrics (glass, nylon, silk, metal, etc.), balsa wood, plywood, metal honeycomb, aluminum and ethylene copolymer foam. Facing sheets can be made from rigid vinyl sheeting (fiat or corrugated) ; glass-reinforced, acrylic-modified polyester**;** acrylic sheeting; plywood; hardwood; sheet metal (aluminum or steel); glass reinforced epoxy; decorative laminate; gypsum; asbestos; and poured concrete [76].

CHAPTER-2

LITERATURE REVIE

2.1 INTRODUCTION

Discovery of parametric resonance dates back to 1831. The phenomenon of parametric excitation was first observed by Faraday[33], when he noticed that when a fluid filled container vibrates vertically, fluid surface oscillates at half the frequency of the container. Parametric resonance in the case of lateral vibration of a string was reported by Melde[59]. Beliaev [6] was first to provide a theoretical analysis of parametric resonance while dealing with the stability of prismatic rods. These are a few early works.

Several review articles on parametric resonance have also been published. Evan-Iwanowski [31], Ibrahim and coworkers [40-46], Ariarathnam [1] and Simitses [80] gave exhaustive account of literature on vibration and stability of parametrically excited systems. Review article of Habip [38] gives an account of developments in the analysis of sandwich structures. Articles of Nakra [63-65] have extensively treated the aspect of vibration control with viscoelastic materials. Books by Bolotin [10], Schmidt [77] and Nayfeh and Mook [66] deals extensively on the basic theory of dynamic stability of systems under parametric excitations. In this chapter further developments in subsequent years in the field of parametric excitation of system with specific resonance to ordinary and sandwich beams is reported. Reference cited in the above mentioned review works are not repeated except at a few places for the sake of continuity. The reported literature mainly deals with the methods of stability analysis, types of resonance, study of different system parameters on the parametric instability of the system and experimental verification of the theoretical findings.

2.2 METHODES OF SABILITY ANALYSIS OF PARAMETRICALLY EXITED SYSTEMS

There is no exact solution to the governing equations for parametrically excited systems of second order differential equations with periodic coefficients. The researchers for a long time have been interested to explore different solution methods to this class of problem. The two main objectives of this class of researchers are to establish the existence of periodic solutions and their stability. When the governing equation of motion for the system is of Mathieu-Hill type, a few well known methods commonly used are method proposed by Bolotin based on Floquet's theory, perturbation and iteration techniques, the Galerkin's method, the Lyapunov second method and asymptotic technique by Krylov, Bogoliubov and Mitroploskii.

Bolotin's[10] method based on Floquet's theory can be used to get satisfactory results for simple resonance only. Steven [87] later modified the Bolotin's method for system with complex differentials equation of motion. Hsu proposed an approximate method of stability analysis of systems having small parameter excitations . Hsu's method can be used to obtain instability zones of main, combination and difference types. Later Saito and Otomi [74] modified Hsu's method to suit systems with complex differential equation of motion. Takahashi [88] proposed a method free from the limitations of small parameter assumption. This method establishes both the simple and combination type instability zones. Zajaczkowski and Lipinski [97] and Zajaczkowski [98] based on Bolotin's method derived formulae to establish the regions of instability and to calculate the steady state response of systems described by a set of linear differential equations with time dependent parameters represented by a trigonometric series. Lau et al. proposed a variable parameter incrementation method, which is free from limitations of small excitation parameters. It has the advantage of treating non-linear systems. Many investigators to study the dynamic stability of elastic systems have also applied finite element method.

2.3. DYNAMIC STABILITY OF PLATES

General theories involving dynamic stability presented in section 2 can be appropriately recast to study the instability behavior of both isotropic and composite plates.

2.3.1 ISOTROPIC PLATES

The dynamic stability of plates under periodic in-plane loads was considered first by Einaudi [30] in 1936. A comprehensive review of early developments in the parametric instability of structural elements including plates was presented in the review articles. Simons and Leissa [78] explained the stability behavior of homogeneous plates subjected to in-plane acceleration loads. Yamaki and Nagai [94] treated rectangular plates under in-plane periodic compression. The dynamic stability of clamped annular plates is studied theoretically by Tani and Nakamura [91] using the Galerkin procedure. It was found that principal resonance was of most practical importance, but that of combination resonance cannot be neglected when the static compressive force was applied. Dixon and Wright [29] studied experimentally the parametric instability behavior of flat plates by normal or shear periodic in-plane forces. Oscillating tensile in-plane load at the far end causing parametric instability effects around the free edge of the cutout is an interesting phenomenon in structural instability. Carlson [11] conducted experiments on the parametric response characteristics of a tensioned sheet with a crack like opening. Cutouts, cracks and other kinds of discontinuities are inevitable in structures due to practical considerations. Datta [23] investigated experimentally the buckling behavior and parametric resonance behavior of tensioned plates with circular and elliptical openings. Datta [24] later studied the parametric instability of tensioned panels with central openings and edge slot. The parametric resonance experiments for different opening parameters indicate that the dynamic instability effects are more significant for narrow openings than for wider openings. The studies on the dynamic stability of plates by Ostiguy *et al.* [67] showed good agreement between theory and experiment. The emergence of digital computers caused the evolution of various numerical methods besides analytical and experimental procedures. Hutt and Salam [39] used the finite element method for the dynamic stability analysis of homogeneous plates using a thin plate 4-noded finite element model. Extensive results were presented on dynamic stability of rectangular plates subjected to various types of uniform loads with/without consideration of damping. Prabhakara and Datta [69] explained the parametric instability characteristics of rectangular plates subjected to in-plane periodic load using finite element method, considering shear deformation. Plates and shells are seldom subjected to uniform loading at the edges. Cases of practical interest arise when the in-plane stresses are caused by localized or any arbitrary in-plane forces. Deolasi and Datta [25] studied the parametric instability characteristics of rectangular plates subjected to localized tension and compression edge loading using Bolotin's approach. The effect of damping on dynamic stability of plates subjected to non-uniform in-plane loads was investigated by Deolasi and Datta [26] using the Method of Multiple Scales (MMS). They further extended the work [27] to explain the combination resonance characteristics of rectangular plates subjected to non-uniform loading with damping. It was observed that under localized edge loading, combination resonance zones were important as simple resonance zones and the effects of damping on the combination resonances may be destabilizing under certain conditions. Deolasi and Datta [28] verified experimentally the parametric response of plates under tensile loading.

Floquet's theory was used by most of the investigators [25,39,69] to study the dynamic stability of plates. The regions of dynamic instability regions were determined by Bolotin's method. Aboudi *et al.* [2] studied the instability of viscoelastic plates subjected to periodic loads on the basis of Lyapunov exponents. The viscoelastic behavior of the plate was given in terms of the Boltzmann superposition principle, allowing any viscoelastic character. Square and rectangular plates were the subject of research for many investigators

[2,27,39,69, 79]. Shen and Mote [81] discussed the parametric excitation of circular plates subjected to a rotating spring. The analytical works on dynamic stability analysis of annular plates got new direction with the use of finite element method. Chen *et al*. [18] investigated the parametric excitation of thick annular plates subjected to periodic uniform radial loading along the outer edge, using higher order plate theory and axi-symmetric finite element. The dynamic stability of annular plates of variable thickness was studied by Mermertas and Belek [60]. The Mindlin plate finite element model was used to handle both the thin and thick annular plates. Young *et al.* [95] presented results on the dynamic stability of skew plates acted upon simultaneously by an aerodynamic force in a chordwise direction and a random in-plane force in spanwise direction. The dynamic instability of simply supported thick polygonal plates was analyzed by Baldinger *et al.* [5] and the corresponding stability regions of the first and second order are calculated, considering shear and rotatory inertia. Structures consisting of plates are often attached with stiffening ribs for achieving greater strength with relatively less material. Srivastava *et al.* [82] investigated the parametric instability of stiffened plates using the 9-node isoparametric plate element and stiffener element. The results showed that location, size and number of stiffeners have a significant effect on the location of the boundaries of the principal instability regions. As far as loading is concerned, many studies involved dynamic stability of plates subjected to uniform [39, 79] in-plane periodic loading. The dynamic stability of plates subjected to partial edge loading and concentrated in-plane compressive edge loading was considered by Deolasi and Datta [25-27]. Srivastava *et al.* [83-84] investigated the dynamic stability of stiffened plates subjected to non-uniform in-plane periodic loading. Takahashi and Konishi [89] analyzed the parametric resonance as well as combination resonance of rectangular plates subjected to in-plane dynamic force. Takahashi and Konishi [90] further investigated the dynamic stability of rectangular plates subjected to in-plane moments. Langley [53] examined the response of two-dimensional periodic structures to point harmonic loading. The study has extensive application to all types of two-dimensional periodic structures including stiffened plates and shells and it raises the possibility of designing a periodic structure to act as a spatial filter to isolate sensitive equipment from a localized excitation source. Young *et al*. [95] studied the parametric excitation of plates subjected to aerodynamic and random in-plane forces. The numerical studies involving dynamic stability behavior of plates with openings are relatively complex due to non-uniform in-plane load distribution and are relatively new. Prabhakara and Datta [70] investigated the parametric instability behavior of plates with centrally located cutouts subjected to tension or compression in-plane edge loading. Srivastava *et al.* [85]

analyzed the dynamic stability of stiffened plates with cutouts subjected to uniform in-plane periodic loading.

The study considered stiffened plates with holes possessing different boundary conditions, cutout parameters, aspect ratios neglecting the in-plane displacements. The interaction of forced and parametric resonance of imperfect rectangular plates was explained by Sassi *et al.* [75]. In this study, the temporal response and the phase diagram were used besides the frequency response and FFT curves to study the transition zones. The effect of one particular spatial mode of imperfection on a different mode of vibration was investigated for the first time. Cederbaum [12] through a finite element formulation studied the effect of in-plane inertia on the dynamic stability of non-linear plates. Ganapathi *et al.*[34] investigated the non-linear instability behavior of isotropic as well as composite plates, subjected to periodic in-plane load through a finite element formulation. The analysis brought out the existence of beats, their dependency on the forcing frequency, the influence of initial conditions, load amplitude and the typical character of vibrations in different regions. Touati and Cederbaum [92] analyzed the dynamic stability of non-linear visco-elastic plates.

2.3.2 COMPOSITE PLATES

The increasing use of fibre-reinforced composite materials in automotive, marine and especially aerospace structures, has resulted in interest in problems involving dynamic instability of these structures. The effects of number of layers, ply lay-up, orientation and different types of materials introduce material couplings such as stretching-bending and twisting-bending couplings etc. All these factors interact in a complicated manner on the vibration frequency spectrum of the laminates affecting the dynamic instability region. The stability behavior of laminates was essential for assessment of the structural failures and optimal design. As per Evan-Iwanowski , the earliest works on dynamic stability of anisotropic plates were done by Ambartsumian and Khachaturian [3] in 1960. Considerable progress has been made since the survey in this subject. There is a renewed interest on the subject after Birman [9] studied analytically the dynamic stability of rectangular laminated plates, neglecting transverse shear deformation and rotary inertia. The effect of unsymmetrical lamination on the distribution of the instability regions was investigated in the above study. Mond and Cederbaum [61] analyzed the dynamic stability of antisymmetric angle ply and cross ply laminated plates within the classical lamination theory, using the method of multiple scales. It was observed that besides the principal instability regions, other cases could be of importance in some cases. Srinivasan and Chellapandi [86] analyzed thin laminated rectangular plates under uniaxial loading by the finite strip method. The transverse shear deformation and in-plane inertia as well as rotatory inertia were neglected and the region of parametric instability was derived using Bolotin's procedure. Bert and Birman [7] investigated the effect of shear deformation on dynamic stability of simply supported anti-symmetric angle-ply rectangular plates neglecting in-plane and rotary inertia. The parametric studies on the effects of the number of layers, aspect ratio and thickness-to-edge length ratio were investigated. The dynamic instability of composite plates subjected to in-plane loads was investigated by Cederbaum [13] within the shear deformable lamination theory, using the method of multiple scales. Chen and Yang [19] investigated on the dynamic stability of thick anti-symmetric angle-ply laminated composite plates subjected to uniform compressive stress and/or bending stress using Galerkin's finite element. The thick plate model included the effects of transverse shear deformation and rotary inertia. The effects of number of layers, lamination angle, static load factor and boundary conditions were investigated. Moorthy *et al*. [62] considered the dynamic stability of uniformly uniaxially loaded laminated plates without static component of load and the instability regions were obtained using finite element method. Extensive results were presented on the effects of different parameters on dynamic stability of angle-ply plates. Kwon [51] studied the dynamic instability of layered composite plates subjected to biaxial loading using a high order bending theory. Chattopadhyay and Radu [17] used the higher order shear deformation theory to investigate the dynamic instability of composite plates by using the finite element approach. The first two instability regions were determined for various loading conditions using both first and second order approximations. Pavlovic [68] investigated the dynamic stability of anti-symmetrically laminated angle-ply rectangular plates subjected to random excitation using Lyapunov direct method. Tylikowski [93] studied the dynamic stability of non-linear anti-symmetric cross-ply rectangular plates. The parametric results on biaxial loading were compared with those obtained by classical theory. Cederbaum [14] has investigated on the dynamic stability of laminated plates with physical non-linearity. Librescu and Thangjitham [55] analyzed the dynamic stability of simply supported shear deformable composite plates along with a higher order geometrically non-linear theory for symmetrical laminated plates. Gilai and Aboudi [36] obtained results on the dynamic stability of non-linearly elastic composite plates using Lyapunov exponents. The non-linear elastic behavior of the resin matrix was modeled by the generalized Ramberg-Osgood representation. The instability of laminated composite plates considering geometric non-linearity was also reported using C^0 shear flexible QUAD-9 element by Balamurugan *et al*. [4]. The non-linear governing equations were solved using the direct iteration technique. The effect of a large amplitude on the dynamic instability was studied for a simply supported laminated composite plate. The non linear dynamic stability was also carried out using C^1 eight-nodded element by Ganapathi *et al.*[35]. Numerical results were presented to study the influences of ply angle and lay-up of laminate. The parametric resonance characteristics of composite plates for different lamination schemes were also studied. Certain fiber reinforced materials, especially those with soft matrices exhibit quite different elastic behavior depending upon whether the fiber direction strain is tensile or compressive. The dynamic stability of thick annular plates with such materials, called the bimodulus materials was studied by Chen and Chen [20]. The annular element with Lagrangian polynomials and trigonometric functions as shape function was developed. The non-axisymmetric modes were shown to have significant effects in the annular bimodulus plates. The dynamic stability of thick plates with such bimodulus materials were examined by Jzeng *et al.*[49]. The finite element method was used to investigate the stability of bimodulus rectangular plates subjected to periodic in-plane loads. The effects of shear deformation and rotatory inertia were considered using first order shear deformation theory. The dynamic stability of bimodulus thick circular and annular plates was analyzed by Chen and Juang [21]. Chen and Hwang [22] studied the axisymmetric dynamic stability of orthotropic thick circular plates. Cederbaum [15] investigated on the dynamic stability of viscoelastic orthotropic plates. The stability boundaries were determined analytically by using the multiple scale method. Time dependent instability regions and minimum load parameter were derived together with an expression for the critical time at which the system, with a given load amplitude, would turn unstable. Cederbaum *et al*. [16] studied the dynamic instability of shear deformable viscoelastic laminated plates by Lyapunov exponents. Librescu and Chandiramani [56] analyzed the dynamic stability of transversely isotropic viscoelastic plates subjected to in-plane biaxial edge load system. The effects of transverse shear deformation, transverse normal stress and rotatory inertia effects are considered in this study. Sahu and Datta [48] have investigated the dynamic stability of composite plates subjected to non-uniform loads including patch and concentrated loads using finite element method. The dynamic stability of laminated composite stiffened plates or shells due to periodic in-plane forces at boundaries was discussed by Liao and Cheng [54]. The 3-D degenerated shell element and 3-D degenerated curved beam element were used to model plates/shells and stiffeners respectively. The method of multiple scales was used to analyze the dynamic instability regions.

CHAPTER-3

FINITE ELEMENT MODELING

FINITE ELEMENT MODELING

3.1 INTRODUCTION

The Finite Element Method is essentially a product of electronic digital computer age. Though the approach shares many features common to the numerical approximations, it possesses some advantages with the special facilities offered by the high speed computers. In particular, the method can be systematically programmed to accommodate such complex and difficult problems as non homogeneous materials, non linear stress-strain behavior and complicated boundary conditions. It is difficult to accommodate these difficulties in the least square method or Ritz method and etc. an advantage of Finite Element Method is the variety of levels at which we may develop an understanding of technique. The Finite Element Method is applicable to wide range of boundary value problems in engineering. In a boundary value problem, a solution is sought in the region of body, while the boundaries (or edges) of the region the values of the dependant variables (or their derivatives) are prescribed.

Basic ideas of the Finite Element Method were originated from advances in aircraft structural analysis. In 1941 Hrenikoff introduced the so called frame work method, in which a plane elastic medium was represented as collection of bars and beams. The use of piecewise-continuous functions defined over a sub domain to approximate an unknown function can be found in the work of Courant (1943), who used an assemblage of triangular elements and the principle of minimum total potential energy to study the Saint Venant torsion problem. Although certain key features of the Finite Element Method can be found in the work of Hrenikoff (1941) and Courant (1943), its formal presentation was attributed to Argyris and Kelsey (1960) and Turner, Clough, Martin and Topp (1956). The term “Finite Element method” was first used by Clough in 1960.

In early 1960’s, engineers used the method for approximate solution of problems in stress analysis, fluid flow, heat transfer and other areas. A textbook by Argyris in 1955 on Energy Theorems and matrix methods laid a foundation laid a foundation for the development in Finite Element studies. The first book on Finite Element methods by Zienkiewicz and Chung was published in 1967. In the late 1960’s and early 1970’s, Finite Element Analysis (FEA) was applied to non-linear problems and large deformations. Oden’s book on non-linear continua appeared in 1972. Mathematical foundations were laid in the 197**3.**

3.2 BASIC CONCEPT OF FINITE ELEMENT METHOD

The most distinctive feature of the finite element method that separate it from others is the division of a given domain into a set of simple sub domains, called ‘Finite Elements’. Any geometric shape that allows the computation of the solution or its approximation, or provides necessary relations among the values of the solution at selected points called nodes of the sub domain, qualifies as a finite element. Other features of the method include, seeking continuous often polynomial approximations of the solution over each element in terms of solution and balance of inter element forces.

Exact method provides exact solution to the problem, but the limitation of this method is that all practical problems cannot be solved and even if they can be solved, they may have complex solution.

Approximate Analytical Methods are alternative to the exact methods, in which certain functions are assumed to satisfy the geometric boundary conditions, but not necessary the governing equilibrium equation. These assumed functions, which are simpler, are then solved by any conventional method available. The solutions obtained from these methods have limited range of values of variables for which the approximate solution is nearer to the exact solution.

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Finite difference method, the differential equations are approximated by finite difference equation. Thus the given governing equation is converted to a set of algebraic equation. These simultaneous equations can be solved by any simple method such as gauss Elimination, Gauss-Seidel iteration method, Crout's method etc. the method of finite difference yield fairly good results and are relatively easy to program. Hence, they are popular in solving heat transfer and fluid flow problems. However, it is not suitable for problems with awkward irregular geometry and suitable for problems of rapidly changing variables such as stress concentration problems.

Finite element method (F.E.M) has emerged out to be powerful method for all kinds of practical problems. In this method the solution region is considered to be built up of many small-interconnected sub regions, called finite elements. These elements are applied with in interpolation model, which is simplified version of substitute to the governing equation of the material continuum property. The stiffness matrices obtained for these elements are assembled together and boundary conditions of the actual problems are satisfied to obtain the solution all over the body or region FEM is well-suited computer programming.

Boundary element method (B.E.M) like finite element, method is being used in all engineering fields. In this approach, the governing differential equations are transformed into integral identities applicable over the surface or boundary. These integral identities are integrated over the boundary, which is divided into small boundary segments, as infinite element method provided that the boundary conditions are satisfied, set of linear algebraic equations emerges, for which a unique solution is obtained

3.3 FINITE ELEMENT APPROACHES

There are two differential Finite element approaches to analyze structures, namely

Force method

Displacement method

3.3.1 FORCE METHOD

The number of forces (shear forces, axial forces & bending moments) is the basic unknown in the system of equations

3.3.2 DISPLACEMENT METHOD

The nodal displacement is the basic unknown in the system of equations. The analysis of lever arm plate fixture has been using the concept of finite element method (F.E.M). The fundamental concept of finite element method is that is that a discrete model can approximate any continuous quantity such as temperature, pressure and displacement. There are many problems where analytical solutions are difficult or impossible to obtain. In such cases finite element method provides an approximate and a relatively easy solutions. Finite element method becomes more powerful when combined rapid processing capabilities of computers.

The basic idea of finite element method is to discretized the entire structure into small element. Nodes or grids define each element and the nodes serve as a link between the two elements. Then the continuous quantity is approximated over each element by a polynomial equation. This gives a system of equations, which is solved by using matrix techniques to get the values of the desired quantities.

The basic equation for the Static analysis is:

$[K]\ [Q] = [F]$

Where [k] = Structural stiffness matrix

[F] = Loads applied

[Q] = Nodal displacement vector

The global stiffness matrix is assembled from the element stiffness matrices. Using these equations the model displacements, the element stresses and strains can be determined.

3.4 ADVANTAGES OF FEM

The advantages of finite element method are listed below:

1. Finite element method is applicable to any field problem: heat transfer, stress analysis, magnetic field and etc.
2. In finite element method there is no geometric restriction. The body or region analyzed may have any shape.
3. Boundary conditions and loading are not restricted. For example, in a stress analysis any portion of the body may be supported, while distributed or concentrated forces may be applied to any other portion.
4. Material properties are not restricted to isotropy and may change from one element to another or even within an element.
5. Components that have different behavior and different mathematical description can be combined together. Thus single finite element model might contain bar, beam, plate, cable and friction elements.
6. A finite element model closely resembles the actual body or region to be analyzed.
7. The approximation is easily improved by grading the mesh so that more elements appear where field gradients are high and more resolution is required.

3.5 LIMITATIONS OF FEM

The limitations of finite element method are as given below:

1. To some problems the approximations used do not provide accurate results.
2. For vibration and stability problems the cost of analysis by FEA is prohibitive.
3. Stress values may vary from fine mesh to average mesh.

CHAPTER-4

PROBLEM FORMULATION

4.1. PROBLEM FORMULATION

The structure of a sandwich plate with a constraining layer and an viscoelastic core is demonstrated in Fig. 4.1. Layer 3 is a pure elastic, isotropic and homogeneous constraining layer. Layer 2 is an viscoelastic material. The base plate is assumed to be undamped, isotropic and homogeneous and is designated as the layer 1. Before the derivation procedures, the other assumptions used in this study must be mentioned:

1. No slipping between the elastic and viscoelastic layers is assumed.
2. The transverse displacements, w, of all points on any cross-section of the sandwich plate are considered to be equal.
3. There exists no normal stress in the viscoelastic layer, and there exists no shear strain in the elastic layer either.

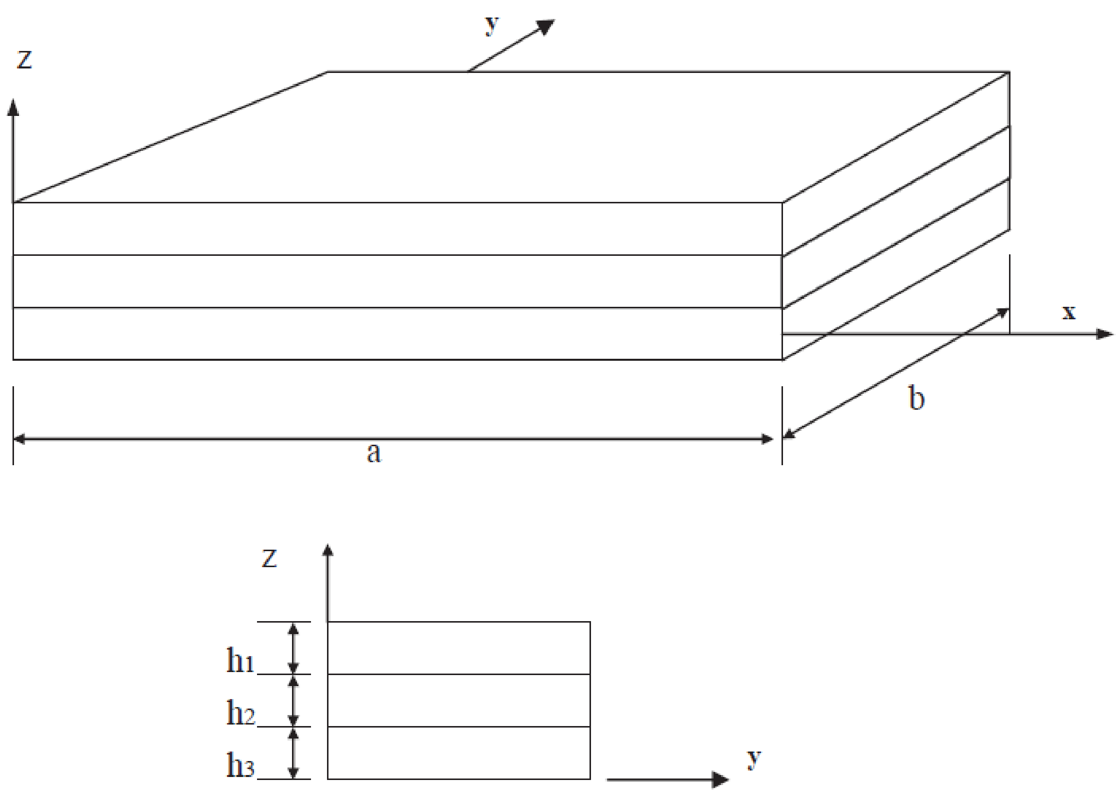

Fig.4. 1. The sandwich plate with viscoelastic core and a constraining layer

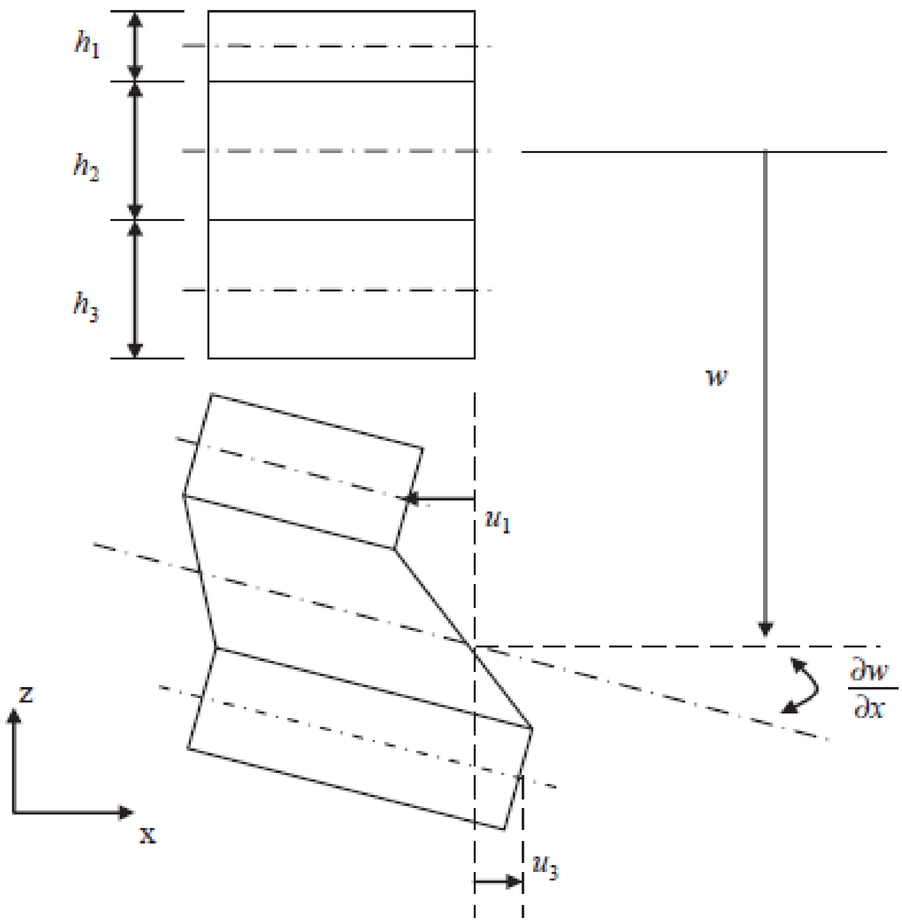

Fig.4. 2. Undeformed and deformed configurations of a sandwich plate

4.2 STRAIN DISPLACEMENT RELATIONS

By referring to Fig. 4.2, the strain–displacement relation of the elastic layer can be expressed as:

$$\varepsilon_{xi} = \frac{\partial u_i}{\partial x} - z_i \frac{\partial^2 w}{\partial x^2},$$

(2)

$$\varepsilon_{yi} = \frac{\partial v_i}{\partial y} - z_i \frac{\partial^2 w}{\partial y^2}, \quad i = 1, 3$$

Where ε_{xi} and ε_{yi} are the bending strains, u_i and v_i are the axial displacements of the mid-plane of layer i at the x and y directions, respectively, and zi is the distance of the mid-height of layer i.

Considering the strain–displacement relation of the viscoelstic layer, the shear deformation can be further expressed as

$$\gamma_{x2} = \frac{\partial w}{\partial x} + \frac{\partial u_2}{\partial z}, \tag{3}$$

$$\gamma_{y2} = \frac{\partial w}{\partial y} + \frac{\partial v_2}{\partial z}, \tag{4}$$

where u_2 and v_2 are the axial displacements in the x and y directions of the viscoelastic layer, respectively. By referring to the geometric relationship between u_1, u_3, v_1, v_3 and ðw/ðz of the face-plate (as shown in Fig. 4.2), it can be obtained that

$$\frac{\partial u_2}{\partial z} = \frac{h_1 + h_3}{2h_2}\frac{\partial w}{\partial x} + \frac{u_1 - u_3}{h_2}, \tag{5}$$

$$\frac{\partial v_2}{\partial z} = \frac{h_1 + h_3}{2h_2}\frac{\partial w}{\partial y} + \frac{v_1 - v_3}{h_2}, \tag{6}$$

where h_1, h_2; and h_3 are the thickness of layers 1, 2, and 3, respectively. Imposing the displacement compatibility (as shown in Fig. 4.3) through the thickness, the following shear strain in the mid-plane can be rewritten as

$$\gamma_{x2} = \frac{d}{h_2}\frac{\partial w}{\partial x} + \frac{u_1 - u_3}{h_2}, \tag{7}$$

$$\gamma_{y2} = \frac{d}{h_2}\frac{\partial w}{\partial y} + \frac{v_1 - v_3}{h_2}, \tag{8}$$

Where

$$d = \frac{h_1}{2} + h_2 + \frac{h_3}{2}.$$

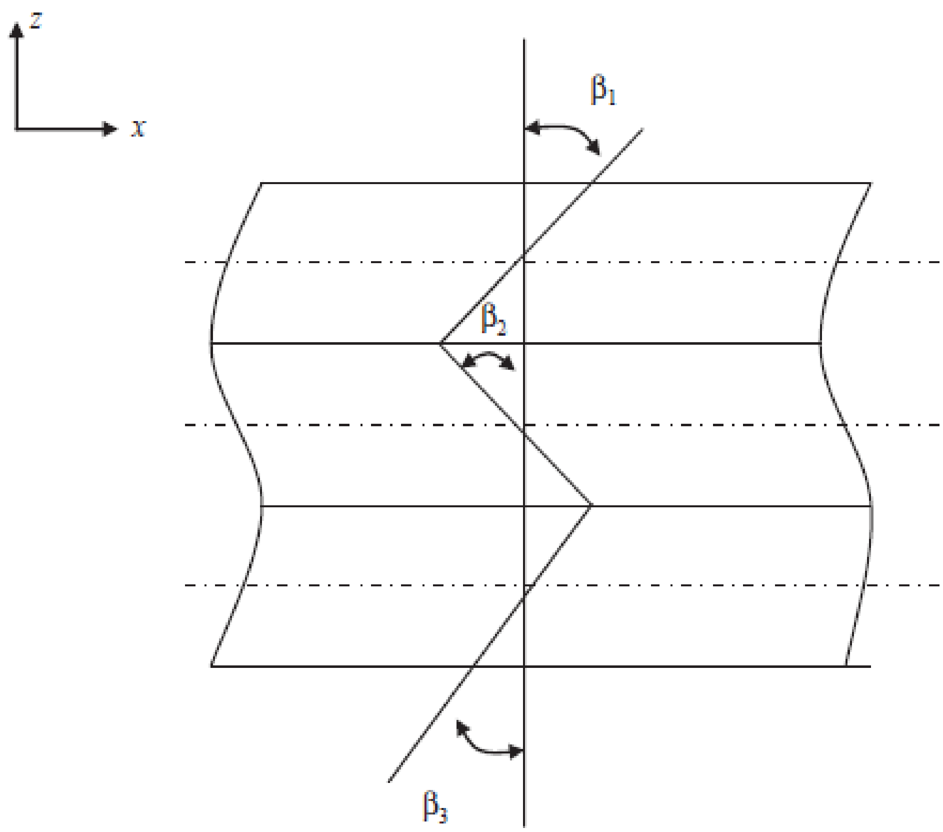

Fig. 4.3. Compatibility relation of systems.

The strain energy associated with the normal strain in the elastic layer can be obtained:

$$V_i = \frac{1}{2}\int_V D_i(\varepsilon_{xi}^2 + \varepsilon_{yi}^2)dv, \quad i = 1,3, \tag{9}$$

Where D_i is the differential operator matrix and listed in the following discussion in detail. Then the strain energy of viscoelastic layer is obtained as follows:

$$V_2 = \int_V G_2(\gamma_{x2}^2 + \gamma_{y2}^2)dv, \tag{10}$$

Where G_2 denotes the shear modulus of the viscoelastic fluid layer.

Let V be the total strain energy of the sandwich plate: then

$$V = V_1 + V_2 + V_3. \tag{11}$$

The kinetic energy of the sandwich plate has the following three parts

1.The kinetic energy associated with the axial displacement

$$T_1 = \frac{1}{2}\iint_A [\rho_1 h_1 (\dot{u}_1^2 + \dot{v}_1^2) + \rho_3 h_3 (\dot{u}_3^2 + \dot{v}_3^2)] dxdy. \tag{12}$$

2.The kinetic energy associated with transverse displacement

$$T_2 = \frac{1}{2}\iint_A (\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3) \dot{w}^2 dxdy. \tag{13}$$

3.The kinetic energy associated with the rotation of the viscoelastic layer:

$$T_3 = \frac{1}{2}\iint_A I_2 (\dot{\gamma}_{x2}^2 + \dot{\gamma}_{y2}^2) dxdy,, \tag{14}$$

Where I_2 is the mass moment of inertia of the viscoelastic layer

Let T be the total kinetic energy of the sandwich plate, then

$$T = T_1 + T_2 + T_3. \tag{15}$$

4.3 FINITE ELEMENT APPROACH

The plate elements used in this study are two-dimensional element bounded by four nodal points. The plate element is shown in Fig.4. 4. Each node has seven degrees of freedom to describe the longitudinal displacements, transverse displacements, and slopes of the sandwich plate. The transverse displacement, longitudinal displacement can be expressed in terms of a nodal displacement vector and a shape function vector:

$$w(x,y,t) = N_w(x,y)\{q(t)\}, \tag{16}$$

$$u_i(x,y,t) = N_{ui}(x,y)\{q(t)\}, \tag{17}$$

$$v_i(x,y,t) = N_{vi}(x,y)\{q(t)\}, \quad i = 1,3, \tag{18}$$

where $q(t) = [u_{1i}, v_{1i}, u_{3i}, v_{3i}, w_i, w_{xi}, w_{yi}]^T, \quad i = 1,2,3,4.$

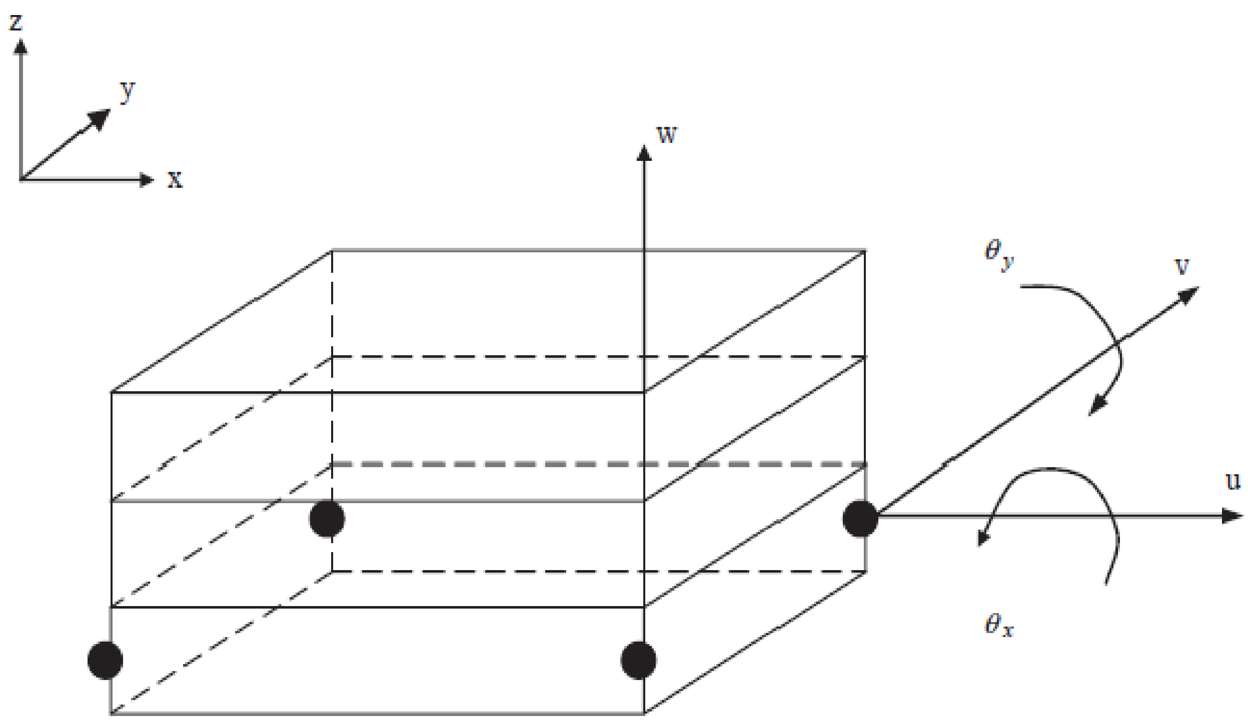

Fig.4. 4. A sandwich plate element with four end nodes and seven DOF per node.

$N_w(x,y),\ N_{ui}(x,y),\ N_{u3}(x,y),\ N_{v1}(x,y),\ N_{v3}(x,y)$ are the shape functions of the plate element.

4.3.1 ELEMENT SHAPE FUNCTIONS

$$N_w(x,y) = [w1, w2, w3, w4], \tag{19}$$

$$w1 = [0,0,0,0,\frac{1}{8}(1-\frac{x}{a})(1-\frac{y}{b})(2-\frac{x}{a}-\frac{x^2}{a^2}-\frac{y}{b}-\frac{y^2}{b^2}),\frac{1}{8}b(1-\frac{x}{a})(1-\frac{y}{b})(1-\frac{y^2}{b^2}),\frac{1}{8}a(1-\frac{x}{a})(1-\frac{y}{b})(1-\frac{x^2}{a^2})],$$

$$w2 = [0,0,0,0,\frac{1}{8}(1+\frac{x}{a})(1-\frac{y}{b})(2+\frac{x}{a}-\frac{x^2}{a^2}-\frac{y}{b}-\frac{y^2}{b^2}),\frac{1}{8}b(1+\frac{x}{a})(1-\frac{y}{b})(1-\frac{y^2}{b^2}),\frac{1}{8}a(1+\frac{x}{a})(1-\frac{y}{b})(1-\frac{x^2}{a^2})],$$

$$w3 = [0,0,0,0,\frac{1}{8}(1+\frac{x}{a})(1+\frac{y}{b})(2+\frac{x}{a}-\frac{x^2}{a^2}+\frac{y}{b}-\frac{y^2}{b^2}),\frac{1}{8}b(1+\frac{x}{a})(1+\frac{y}{b})(1-\frac{y^2}{b^2}),\frac{1}{8}a(1+\frac{x}{a})(1+\frac{y}{b})(1-\frac{x^2}{a^2})],$$

$$w4 = [0,0,0,0,\frac{1}{8}(1-\frac{x}{a})(1+\frac{y}{b})(2-\frac{x}{a}-\frac{x^2}{a^2}+\frac{y}{b}-\frac{y^2}{b^2}),\frac{1}{8}b(1-\frac{x}{a})(1+\frac{y}{b})(1-\frac{y^2}{b^2}),\frac{1}{8}a(1-\frac{x}{a})(1+\frac{y}{b})(1-\frac{x^2}{a^2})],$$

$$N_{u1}(x,y) = [u11, u12, u13, u14], \tag{20}$$

$$N_{u3}(x,y)=[u31,u32,u33,u34], \tag{21}$$

Where

$$u11=[\frac{1}{4}(1-\frac{x}{a})(1-\frac{y}{b}),0,0,0,0,0,0],$$

$$u31=[0,0,\frac{1}{4}(1-\frac{x}{a})(1-\frac{y}{b}),0,0,0,0],$$

$$u12=[\frac{1}{4}(1+\frac{x}{a})(1-\frac{y}{b}),0,0,0,0,0,0],$$

$$u32=[0,0,\frac{1}{4}(1+\frac{x}{a})(1-\frac{y}{b}),0,0,0,0],$$

$$u13=[\frac{1}{4}(1+\frac{x}{a})(1+\frac{y}{b}),0,0,0,0,0,0],$$

$$u33=[0,0,\frac{1}{4}(1+\frac{x}{a})(1+\frac{y}{b}),0,0,0,0],$$

$$u14=[\frac{1}{4}(1-\frac{x}{a})(1+\frac{y}{b}),0,0,0,0,0,0],$$

$$u34=[0,0,\frac{1}{4}(1-\frac{x}{a})(1+\frac{y}{b}),0,0,0,0],$$

$$N_{v1}(x,y)=[v11,v12,v13,v14], \tag{22}$$

$$N_{v3}(x,y)=[v31,v32,v33,v34], \tag{23}$$

Where

$$v11=[0,\frac{1}{4}(1-\frac{x}{a})(1-\frac{y}{b}),0,0,0,0,0],$$

$$v31=[0,0,0,\frac{1}{4}(1-\frac{x}{a})(1-\frac{y}{b}),0,0,0],$$

$$v12=[0,\frac{1}{4}(1+\frac{x}{a})(1-\frac{y}{b}),0,0,0,0,0],$$

$$v32=[0,0,0,\frac{1}{4}(1+\frac{x}{a})(1-\frac{y}{b}),0,0,0],$$

$$v13=[0,\frac{1}{4}(1+\frac{x}{a})(1+\frac{y}{b}),0,0,0,0,0],$$

$$v33=[0,0,0,\frac{1}{4}(1+\frac{x}{a})(1+\frac{y}{b}),0,0,0],$$

$$v14=[0,\frac{1}{4}(1-\frac{x}{a})(1+\frac{y}{b}),0,0,0,0,0],$$

$$v34=[0,0,0,\frac{1}{4}(1-\frac{x}{a})(1+\frac{y}{b}),0,0,0].$$

The strain energy and kinetic energy derived in the above section can be rewritten in terms of nodal displacement variables as follows:

$$V = \frac{1}{2}\{q(t)\}^T ([K_1]+[K_2]+[K_3]+[K_4]+[K_5])\{q(t)\}, \tag{24}$$

Where

$$[K_1] = h_1 \iint_A [N_1]^T [D_{1p}][N_1] dA, \tag{25}$$

$$[K_2] = \iint_A [N_b]^T [D_{1b}][N_b] dA, \tag{26}$$

$$[K_3] = h_3 \iint_A [N_3]^T [D_{3p}][N_3] dA, \tag{27}$$

$$[K_4] = \iint_A [N_b]^T [D_{3b}][N_b] dA, \tag{28}$$

$$[K_5] = G_2 h_2 \iint_A [N_g]^T [N_g] dA, \tag{29}$$

Where

$$[N_i] = \begin{bmatrix} N_{ui,x} \\ N_{vi,y} \\ N_{ui,y} + N_{vi,x} \end{bmatrix},$$

$$[D_{ip}] = \begin{bmatrix} \frac{E_{xi}}{1-\mu_{xi}\mu_{yi}} & \frac{\mu_{xi}E_{yi}}{1-\mu_{xi}\mu_{yi}} & 0 \\ \frac{\mu_{yi}E_{xi}}{1-\mu_{xi}\mu_{yi}} & \frac{E_{yi}}{1-\mu_{xi}\mu_{yi}} & 0 \\ 0 & 0 & \frac{(1-\mu_{yi})E_{xi}}{2(1-\mu_{xi}\mu_{yi})} \end{bmatrix},$$

$$\left[N_b\right] = \begin{bmatrix} N_{w,xx} \\ N_{w,yy} \\ 2N_{w,xy} \end{bmatrix},$$

$$[D_{ib}] = I_i \begin{bmatrix} \frac{E_{xi}}{1-\mu_{xi}\mu_{yi}} & \frac{\mu_{xi}E_{yi}}{1-\mu_{xi}\mu_{yi}} & 0 \\ \frac{\mu_{yi}E_{xi}}{1-\mu_{xi}\mu_{yi}} & \frac{E_{yi}}{1-\mu_{xi}\mu_{yi}} & 0 \\ 0 & 0 & \frac{(1-\mu_{yi})E_{xi}}{2(1-\mu_{xi}\mu_{yi})} \end{bmatrix}, \qquad i = 1,3$$

$$\left[N_g\right] = \frac{d}{h_2}\begin{bmatrix} \frac{N_{u1}-N_{u3}}{d} + N_{w,x} \\ \frac{N_{v1}-N_{v3}}{d} + N_{w,y} \end{bmatrix}$$

Where E_i, v_i, I_i denote Young's modulus, Poisson's ratio, and the area moment of inertia of the i^{th} layer.

In addition, the kinetic energy of the sandwich plate is

$$T = \frac{1}{2}\{\dot{q}(t)\}^T([M_1]+[M_2]+[M_3]+[M_4])\{\dot{q}(t)\}, \tag{30}$$

Where

$$[M_1] = \iint_A (\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3)[N_w]^T[N_w]dxdy, \tag{31}$$

$$[M_2] = \iint_A \rho_1 h_1([N_{u1}]^T[N_{u1}]+[N_{v1}]^T[N_{v1}])dxdy, \tag{32}$$

$$[M_3] = \iint_A \rho_3 h_3([N_{u3}]^T[N_{u3}]+[N_{v3}]^T[N_{v3}])dxdy, \tag{33}$$

$$[M_4] = \iint_A I_2[N_g]^T[N_g])dxdy. \tag{34}$$

Considering the situation of a sandwich plate element with a periodic load. The work done by the periodic load can be expressed as

$$W = \frac{1}{2}\iint_A P(t)\left(\frac{\partial w}{\partial x}\right)^2 dxdy. \tag{35}$$

Substitute the interpolation function into the above equation, and we can obtain that

$$W = \frac{1}{2}\{q(t)^T P(t)[K_g^e]q(t)\}, \tag{36}$$

Where

$$[K_g^e] = \iint_A \left[\frac{\partial N_w}{\partial x}\right]^T \left[\frac{\partial N_w}{\partial x}\right] dxdy.$$

4.4 EQUATION OF MOTION

According to the Hamilton's principle, we have

$$\delta\int_{t1}^{t2}(T - V + W)dt = 0. \tag{37}$$

By substituting the strain energy, kinetic energy, and the work done by the load force into the Hamilton's principle, the governing equation for the sandwich plate element is obtained as Follows

$$\left[M^e\right]\{\ddot{q}(t)\} + ([K^e] - P(t)[K_g^e])\{q(t)\} = 0, \tag{38}$$

Where

$$[K^e] = [K_1] + [K_2] + [K_3] + [K_4] + [K_5], \tag{39}$$

And

$$[M^e] = [M_1] + [M_2] + [M_3] + [M_4]. \tag{40}$$

Assembling mass, elastic stiffness and geometric stiffness matrices of individual element, the equation of motion for the sandwich plate is written as

$$[M]\{\ddot{\Delta}\} + [K]\{\Delta\} - P(t)\left[K_g\right]\{\Delta\} = 0 \tag{41}$$

Where $\{\Delta\}$ is the global displacement matrix.

The static component P_s and dynamic component P_t of the load P(t), can be represented in terms of P^* as $P_s = \alpha P^*$ and $P_t=\beta P^*$ and, where $P^*=D/ba^2$ and $D=\sum_{i=1,3} E_{(2i-1)} I_{(2i-1)} /(1-\upsilon_i^2)$.

Hence substituting $P(t)=\alpha P^*+\beta P^*\cos\Omega t$, where α and β are static and dynamic load factors respectively.

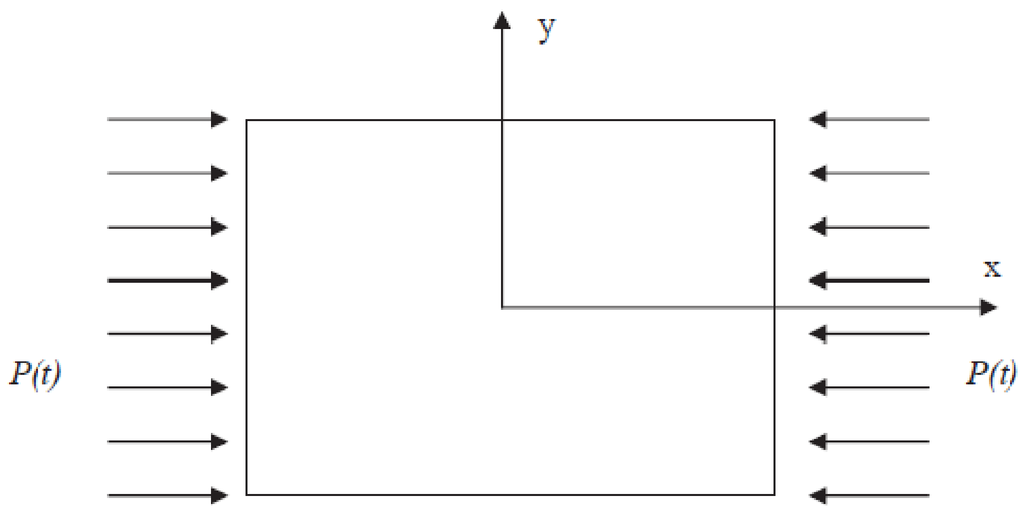

Fig. 4.5. A plate subjected to an in-plane dynamic load.

Substituting P(t), eq.(41) becomes

$$[M]\{\ddot{\Delta}\}+[K]\{\Delta\}-(P_s+P_t\cos\Omega t)[K_g]\{\Delta\}=0 \tag{42}$$

$$[M]\{\ddot{\Delta}\}+([K]-P_s[K_g]_S)\{\Delta\}-P_t\cos\Omega t[K_g]_t\{\Delta\}=0 \tag{43}$$

Where the matrices $[K_g]_s$ and $[K_g]_t$ reflect the influence of P_s and P_t respectively. If the static and time dependent component of loads are applied in the same manner, then $[K_g]_s=[K_g]_t=[K_g]$.

$$[M]\{\ddot{\Delta}\}+[\overline{K}]\{\Delta\}-\beta P^*\cos\Omega t)[K_g]\{\Delta\}=0 \tag{44}$$

Where $\left[\overline{K}\right] = \left[K\right] - \mathrm{P_s}\ \left[K_g\right]$ (45)

The global displacement matrix $\{\Delta\}$ can be assumed as

$$\{\Delta\} = [\Phi]\{\Gamma\} \tag{46}$$

Where $[\Phi]$ is the normalized modal matrix corresponding to

$$[M]\{\ddot{\Delta}\} + \left[\overline{K}\right]\{\Delta\} = \{0\} \tag{47}$$

and $\{\Gamma\}$ is a new set of generalized coordinates .

Substituting eq.46 in eq.44, eq.44 is transformed to the following set of N_c coupled Mathieu equations.

$$\ddot{\Gamma}_m + \left(\omega_m^2\right)\Gamma_m + \beta P^* \cos\Omega t \sum_{n=1}^{N_c} b_{mn}\Gamma_n = 0 \qquad m = 1,2,........N_c, \tag{48}$$

Where $\left(\omega_m^2\right)$ are the distinct eigen values of $[M]^{-1}\left[\overline{K}\right]$ and b_{mn} are the elements of the complex matrix $[B] = -[\Phi]^{-1}[M]^{-1}\left[K_g\right][\Phi]$ and

$\omega_m = \omega_{m.R} + i\,\omega_{m.I}$, $\quad b_{mn} = b_{mn.R} + i\ b_{mn.I}$ and $\mathrm{i} = \sqrt{-1}$

4.5 REGIONS OF INSTABILITY

The boundaries of the regions of instability for simple and combination resonance are obtained by applying the following conditions[78] to the eq.48.

Case (A): Simple resonance

The boundaries of the instability regions are given by

$$\left|\frac{\Omega}{2\omega_0}-\overline{\omega}_{\mu.R}\right| \langle \frac{1}{4}\left[\frac{\beta^2(b^2{}_{\mu\mu R}+b^2{}_{\mu\mu I})}{\overline{\omega}^2{}_{\mu.R}}-16\overline{\omega}^2{}_{\mu.I}\right]^{1/2} \qquad \mu=1,2.......N_c \tag{49}$$

Where $\omega_0=\sqrt{D/ma^4}$, $\overline{\omega}_{\mu.R}=\omega_{\mu.R}/\omega_0$, $\overline{\omega}_{\mu.I}=\omega_{\mu.I}/\omega_0$, m is mass per unit length of the sandwich plate.

When damping is neglected, the regions of instability are given by

$$\left|\frac{\Omega}{2\omega_0}-\overline{\omega}_{\mu.R}\right| \langle \frac{1}{4}\left[\frac{\beta(b_{\mu\mu.R})}{\overline{\omega}_{\mu.R}}\right] \qquad \mu=1,2,......N_c \tag{50}$$

Case (B): Combination resonance of sum type

$$T\left|\frac{\Omega}{2\omega_0}-\frac{1}{2}(\overline{\omega}_{\mu.R}+\overline{\omega}_{\upsilon.R}\right| \langle \frac{1}{8}\frac{(\overline{\omega}_{\mu.I}+\overline{\omega}_{\upsilon.I})}{(\overline{\omega}_{\mu.I}\,\overline{\omega}_{\upsilon.I})^{1/2}}\left[\frac{\beta^2(b_{\mu\upsilon.R}\,b_{\upsilon\mu.R}+b_{\mu\upsilon.I}\,b_{\upsilon\mu.I})}{\overline{\omega}_{\mu.R}\,\overline{\omega}_{\upsilon.R}}-16\overline{\omega}_{\mu.I}\overline{\omega}_{\upsilon.I}\right]^{1/2} \tag{51}$$

$\mu \neq \nu,\ \mu,\nu=1,2,..N_c.$

When damping is neglected

$$\left|\frac{\Omega}{2\omega_0}-\frac{1}{2}(\overline{\omega}_{\mu.R}+\overline{\omega}_{\upsilon.R}\right| \langle \frac{1}{4}\left[\frac{\beta^2(b_{\mu\upsilon.R}\,b_{\upsilon\mu.R})}{\overline{\omega}_{\mu.R}\,\overline{\omega}_{\upsilon.R}}\right]^{1/2} \quad \mu\neq\nu,\ \mu,\nu=1,2,...N_c \tag{52}$$

Case (C): Combination resonance of difference type

The boundaries of the regions of instability of difference type are given by

$$\left|\frac{\Omega}{2\omega_0}-\frac{1}{2}(\overline{\omega}_{\mu.R}-\overline{\omega}_{\upsilon.R})\right| \langle \frac{1}{8}\frac{(\overline{\omega}_{\mu.I}-\overline{\omega}_{\upsilon.I})}{(\overline{\omega}_{\mu.I}\,\overline{\omega}_{\upsilon.I})^{1/2}}\left[\frac{\beta^2(b_{\mu\upsilon I}\,b_{\upsilon\mu I}-b_{\mu\upsilon R}b_{\upsilon\mu R})}{\overline{\omega}_{\mu.R}\,\overline{\omega}_{\upsilon.R}}-16\overline{\omega}_{\mu.I}\overline{\omega}_{\upsilon.I}\right], \tag{53}$$

$\nu \rangle \mu,\ \mu,\nu=1,2,...N_c$

When damping is neglected, the unstable regions are

$$\left|\frac{\Omega}{2\omega_0}-\frac{1}{2}(\bar{\omega}_{\mu.R}-\bar{\omega}_{\upsilon.R})\right|\langle\frac{1}{4}\left[-\frac{\beta^2(b_{\mu\upsilon.R}\,b_{\upsilon\mu.R})}{\bar{\omega}_{\mu.R}\,\bar{\omega}_{\upsilon.R}}\right]^{1/2}\quad \nu\rangle\mu,\mu,\nu=1,2,..N_c. \qquad (54)$$

CHAPTER-5

RESULTS & DISCUSSION

The dynamic stability problems of a sandwich plate with viscoelastic core and constrained layer are studied by finite element method. To validate the proposed algorithm and calculations, comparisons between the present results and the results of existing models are made first. The solutions of natural frequencies and loss factors of a simply supported sandwich plate with a viscoelastic layer are obtained. The numerical results are compared with those obtained by Lall et al. [52] and Jia-Yi Yeh, Lien-Wen Chen [47] in Tables 1 and 2, respectively. The solutions solved by present model are shown to have a good accuracy. A good agreement can be observed in the above results with different geometry.

The geometrical and physical parameters of the sandwich plate are as follows:

a=0.3048m; b=0.3480 m; $E_1=E_3=6.89\times10^{10}$ N/m^2; $\rho_1=\rho_3=2740$ kg/m^3;

h1=h3=0.762mm; $h_2=0.254$mm; $\rho_2=999$kg/m^3; $\mu=0.3$;

G2 =0.896×10^6 N/m^2; $\eta=0.5$

Table 1
Comparison of Resonant frequency parameters and Modal loss factors calculated from present analysis with those of reference [52] at h2/h1=0.33, η=0.5

Ref. [52]			present	
Mode	natural Frequency(rad/sec)	loss factor	Natural Frequency(rad/sec)	loss factor
1.	59.05	0.206	58.69	0.201
2.	113.67	0.213	113.75	0.211
3.	128.89	0.207	129.16	0.208
4.	175.76	0.188	175.46	0.189
5.	196.67	0.179	193.79	0.183

Table 2
Comparison of Resonant frequency parameters and Modal loss factors calculated from present analysis with those of reference [47] at h2/h1=10, η=0.5

Ref. [47]			present	
Mode	natural Frequency(rad/sec)	loss factor	natural Frequency(rad/sec)	loss factor
1.	975.17	0.044	972.89	0.044
2.	2350.79	0.019	2346.45	0.019
3.	2350.79	0.019	2346.45	0.019
4.	3725.33	0.012	3711.90	0.012

The variation of the fundamental buckling load parameter (P_b), defined as the ratio of fundamental buckling load to P^*, with core thickness parameter (h_2/h_1) is shown in figure-11. It is seen from the figure that for core thickness parameter 0.5 to 5.0 there is a linear increase in fundamental buckling load parameter with increase in core thickness parameter.

Figure-12 shows the effect of core thickness parameter on fundamental frequency parameter (f). The fundamental frequency parameter is defined as the ratio of fundamental frequency of the sandwich plate to ω_o. The variation of fundamental frequency parameter with core thickness parameter shows the similar trend as those for fundamental buckling load.

The variation of fundamental system loss factor (η) with core thickness parameter is shown in figure-13. The fundamental loss factor increases with increase in core thickness parameter. It is revealed from the figure that the rate of increase of fundamental loss factor with core thickness parameter is very high for low values (0.01 to 0.5) and for higher values of h_2/h_1 though the η increases the rate of increase is comparatively less

Figure-14 shows the effect of shear parameter (g) on fundamental buckling load. The shear parameter is defined as, $g = \frac{G'}{(h_2/h_1)}\left(\frac{a}{h_1}\right)^2\left(\frac{2}{E}\right)$. It can be seen that with increase in shear parameter the fundamental buckling load increases almost linearly for pinned-pinned end condition.

Figure-15 shows the effect of shear parameter on fundamental frequency parameter (f). With increase in g the fundamental frequency parameter increases.

Figure-16 shows the effect of shear parameter on system fundamental loss factor (η). The system loss factor increases with increase in shear parameter. But for higher values of g the effect becomes less dominant.

Figure-17 shows the effect of core thickness parameter on the first two instability regions of simple resonance of the plate. It is seen that increase in core thickness parameter shifts the occurrence of instability regions to higher excitation frequency and their areas also decreases with increase in thickness ratio. So the increase in thickness ratio improves the stability behaviour of the plate.

In figures-18 the effect of static load factor (α) on the stability behavior of the plate is shown. The figure show the first two instability regions of simple resonance for α= 0.0 and 0.5. It is seen that increase in static load factor has destabilizing effect, because the instability regions move to lower frequency of excitation and their areas also increase with increase in static load factor.

The effect of shear parameter on the stability behavior of the plate has been shown in figure-19. It can be seen that in addition to two instability regions of simple resonance, the instability region of combination resonance ($\omega_1 + \omega_2$) type also exist. It can also be seen from the figure that with increase in shear parameter with constant thickness ratio improves the stability of the plate by shifting them to higher frequency of excitation and there is also marginal reduction in their areas.

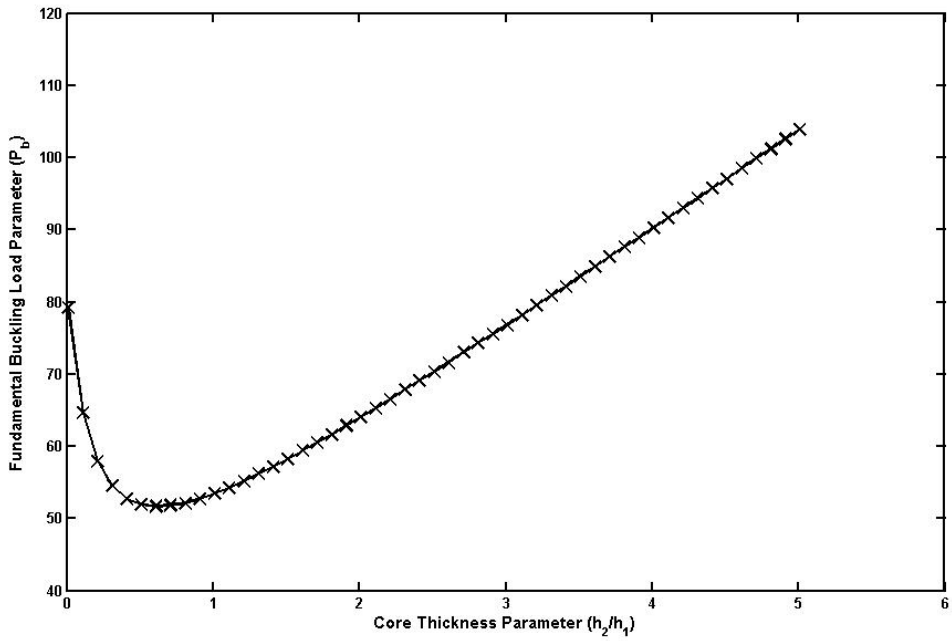

Figure - 11,Effect of Core Thickness Parameter on Fundamental Buckling Load Parameter.

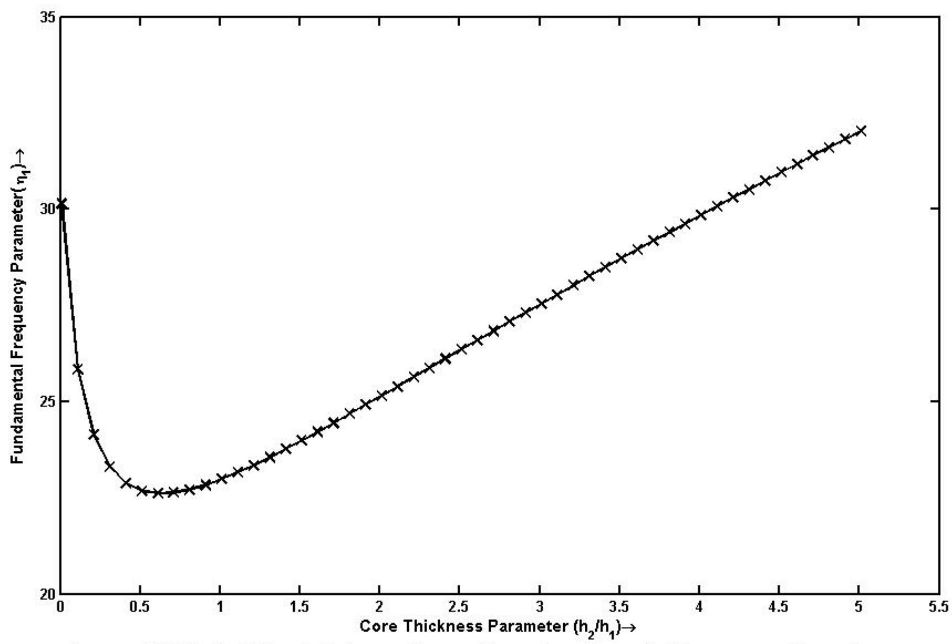

Figure - 12,Effect of Core Thickness Parameter on Fundamental Frequency Prameter,.

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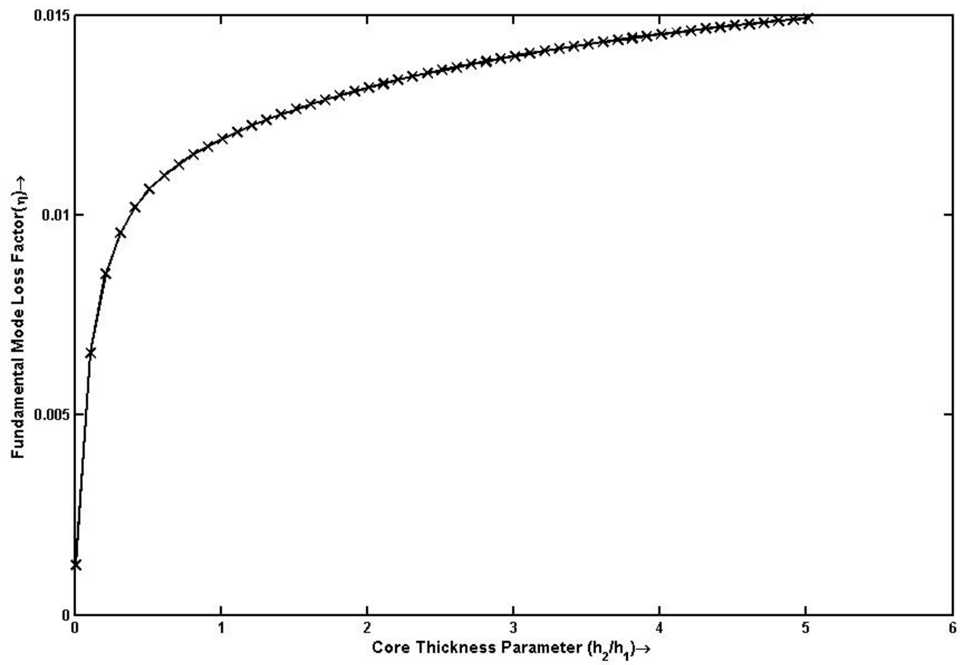

Figure - 13,Effect of Core Thickness Parameter on Fundamental Loss Factor.

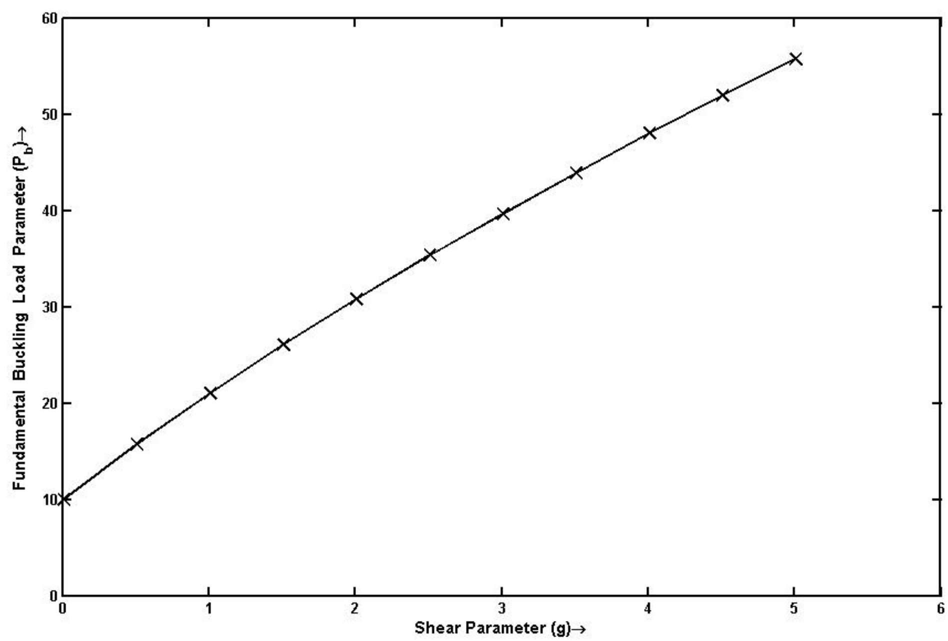

Figure - 14 ,Effect of Shear Parameter on Fundamental Buckling Load Parameter, h_2/h_1= 1.0.

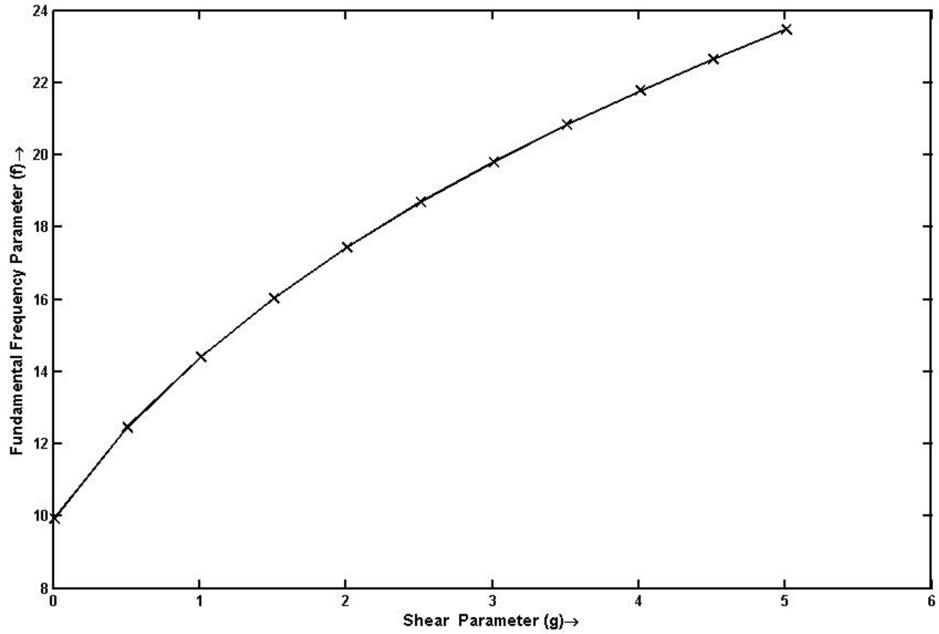

Figure -15 ,Effect of Shear Parameter on Fundamental Frequency Parameter, h_2/h_1= 1.0.

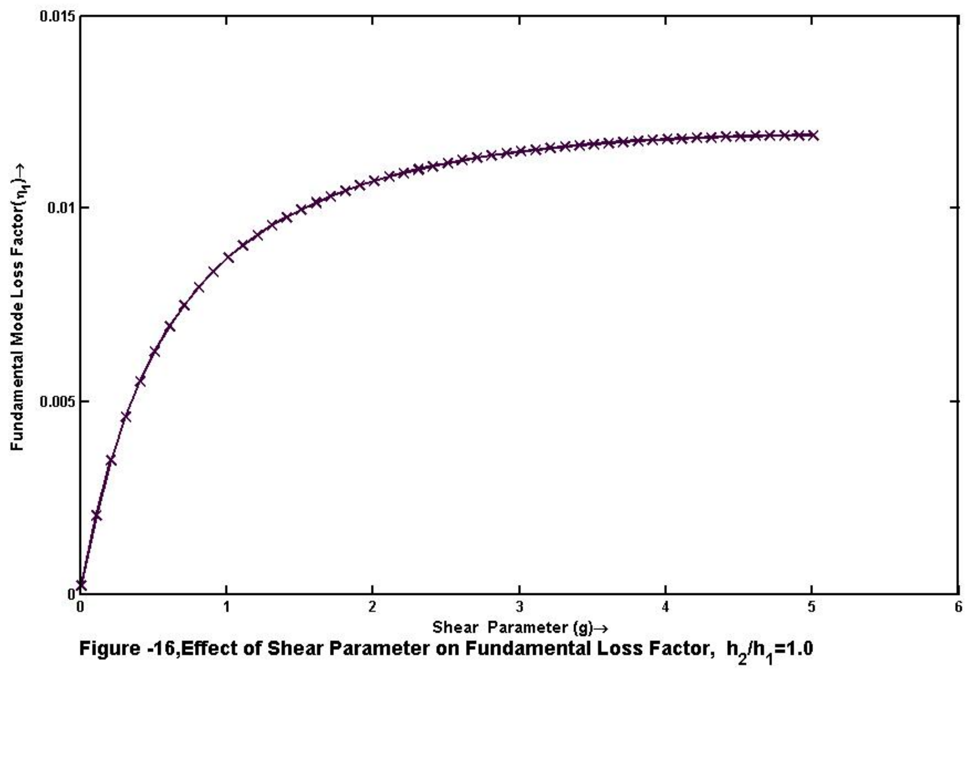

Figure -16,Effect of Shear Parameter on Fundamental Loss Factor, h_2/h_1=1.0

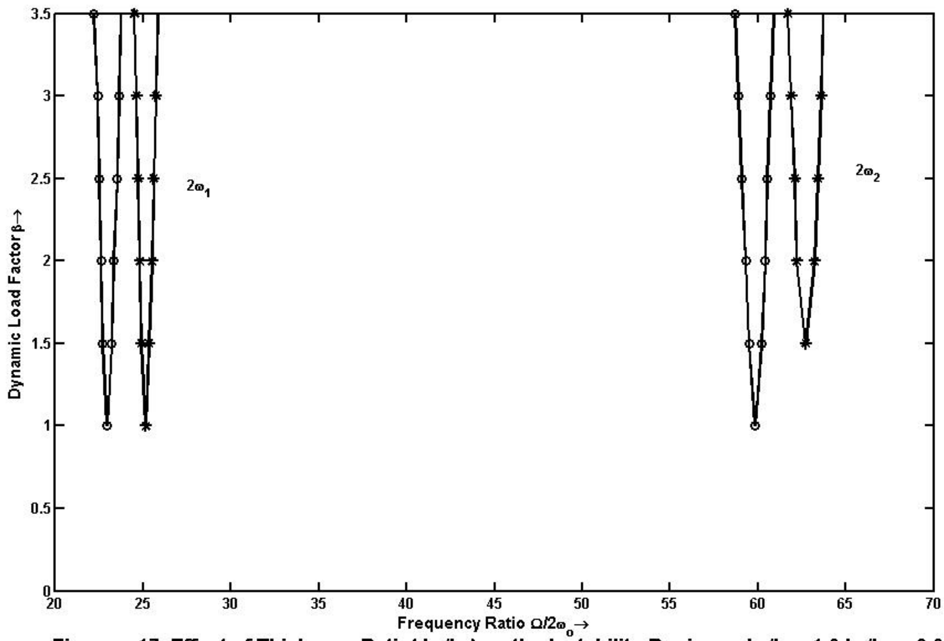

Figure - 17, Effect of Thickness Ratio(h_2/h_1) on the Instability Regions:, h_2/h_1 =1.0,h_2/h_1 = 2.0, *,

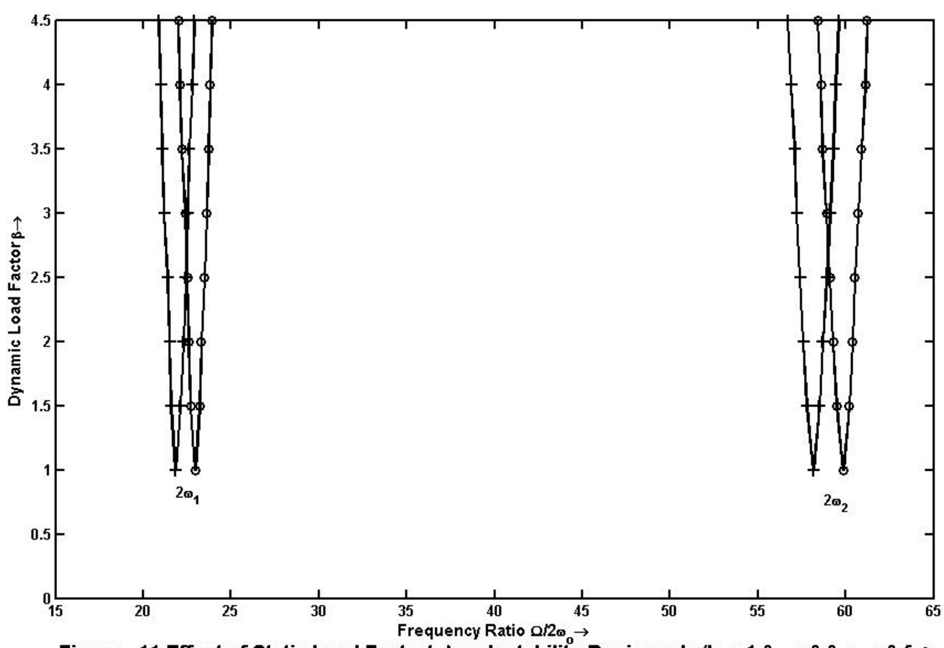

Figure - 11,Effect of Static Load Factor(α) on Instability Regions:,h_2/h_1 =1.0, α=0.0,o,α=0.5,+ ,

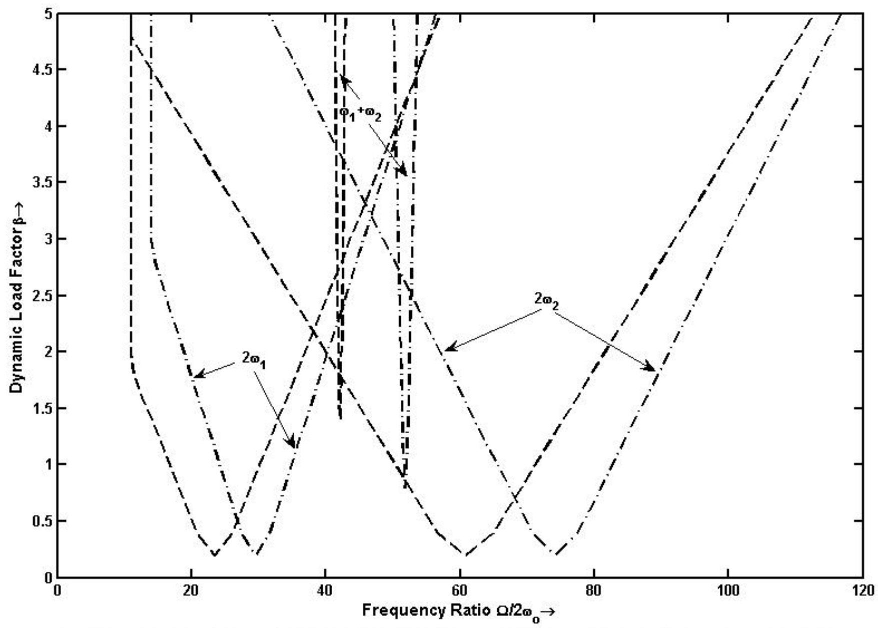

Figure - 19,Effect of Shear Parameter on Instability Regions;h_2/h_1 =1.0, α=0.0, g =5.0,-, g=10.0,-.-,.

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CHAPTER-6

CONCLUSION & SCOPE FOR FUTURE WORK

6.1 CONCLUSION:

The present work investigates the dynamic stability of a sandwich plate with simply supported, end condition. It is found that for plate with simply supported boundary conditions the fundamental frequency, fundamental buckling load increase with increase in core thickness parameter (h_2/h_1) for $h_2/h_1 \geq 0.5$. For $h_2/h_1 < 0.5$, the fundamental frequency, fundamental buckling load decrease with increase in core thickness parameter. The system fundamental loss factor increases with increase in thickness ratio for $5.0 \geq h_2/h_1 \geq 0.01$. The fundamental buckling load and fundamental frequency increase with increase in shear parameter. The fundamental system loss factor has an increasing tendency with increase in shear parameter. The increase in core thickness ratio and shear parameter has stabilizing effect. Whereas increase in static load factor has a destabilizing effect.

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6.2 SCOPE FOR FUTURE WORK

The following works may be carried out as an extension of the present work.

1. Stability of sandwich plates with different boundary conditions.
2. Stability of sandwich plates of different cross sections like I section, trapezoidal section etc.
3. Stability of sandwich plates of different cores like continuous corrugated-core, zed-core, channel-core and truss core
4. Stability of multilayered sandwich plates.

7. REFERENCES

1. 1.Aboudi J, Cederbaum G and Elishakoff I (1990), "Dynamic stability analysis of viscoelastic plates by Lyapunov exponents", Journal of Sound and Vibration, *volume* **139** (3), page no 459-467
2. Ambartsumian SA and Khachaturian AA (1960), "On the stability of vibrations of anisotropic plates", *Izv. .Akad. Nauk*, SSSR No. 1, page no 113-122.
3. Ariarathnam, S.T., "Parametric resonance", proceedings of the tenth U.S. National Congress of applied Mechanics.1986.
4. Balamurugan V, Ganapathi M, and Varadan TK (1996) "Non-linear dynamic stability of laminated composite plates using finite element method", Computers and Structures, volume **60**, page no 125-130.
5. Baldinger M, Beliav AK and Irshik H (2000), "Principal and second instability regions of shear deformable polygonal plates", Computational mechanics, volume **26**, page no 288-294.
6. Beliaev, N.M., "Stability of prismatic rods subjectd to variable longitudinal forces". Collection of papers: Eng. Construct, Struct. Mech., Put', Leningrad, 149 – 167,1924
7. Bert, CW and Birman V (1987), "Dynamic instability of shear deformable anti-symmetric angle-ply plates", International Journal of Solids and Structures, *volume* **23**, page no 1053-1061.
8. Bhavikatti S. S. "Finite Element Analysis". New Delhi: New Age International (P) Limited , 2005
9. Birman, V (1985) "Dynamic stability of un-symmetrically laminated rectangular plates", Mechanics Research Communications, volume **12**, page no 81-86.
10. Bolotin, V.V., "The dynamic stability of elastic Systems". Holden – Day, Inc., san Frasncisco, 1964.
11. Carlson RL (1974), "An experimental study of the parametric excitation of a tensioned sheet with a crack like opening", Experimental Mechanics, *volume* **14**, page no 452-460.
12. Cederbaum G (1994) "On the effect of in-plane inertia on the dynamic stability of non-linear plates", Journal of Sound and Vibration, *volume* **175**(3), page no 428-432.
13. Cederbaum G (1991), "Dynamic stability of shear deformable laminated plates", AIAA Journal, volume **29**, page no 2000-2005.
14. Cederbaum G (1993) "Dynamic stability of laminated plates with physical non-linearity", Composite Structures, volume **23**(2), page no 121-129.

15. Cederbaum G (1991), "Dynamic instability of viscoelastic orthotropic laminated plates", Composite Structures, **volume 19**(2), page no: 131-144.
16. Cederbaum G, Aboudi J and Elishakoff I (1991), "Dynamic instability of shear deformable visco-elastic laminated plates by Lyapunov exponents", International Journal of Solids and Structures, volume 28(3), page no: 317-327.
17. Chattopadhyay A and Radu AG (2000), "Dynamic stability of composite laminates using a higher order theory", Computers and Structures, volume **77**, page no: 453-460.
18. Chen LW, Hwang JR and Doong JL (1989), "Asymmetric dynamic stability of thick annular plates based on a high order plate theory", Journal of Sound and Vibration, volume **130**(3), page no 425-437.
19. Chen CC and Yang JY (1990). "Dynamic stability of laminated composite plates by finite element method, Computers and Structures", volume **36**(5), page no 845-851.
20. Chen LW and Chen CC (1989), "Axisymmetric vibration and dynamic stability of bimodulus thick annular plates", Computers and Structures, volume **31**, page no 1013-1022.
21. Chen LW and Juang DP (1987), "Axisymmetric dynamic stability of a bimodulus thick circular plate", Computers and Structures, volume **26**(6), page no 933-939.
22. Chen LW and Hwang JR(1988), "Axisymmetric dynamic stability of polar orthotropic thick circular plates", Journal of Sound and Vibration, *volume* **125**(3),page no 555-563.
23. Datta PK (1978), "An investigation of the buckling behaviour and parametric resonance phenomenon of a tensioned sheet with a central opening", Journal of Sound and Vibration, *volume* **58** (4), page no 527-534.
24. Datta PK (1983), "Parametric instability of tensioned panels with central opening and edge slot", Proceedings of the 9th CANCAM, University of Saskatchewan Canada, May 30-June 3, page no 99-100
25. Deolasi PJ and Datta PK (1995), "Parametric instability characteristics of rectangular plates subjected to localized edge loading (compression or tension)", Computers and Structures, volume **54**(1), page no 73-82.
26. Deolasi PJ and Datta PK (1995), "Effects of damping on parametric instability characteristics of plates under localized edge loading (compression or tension)",

International Journal of Structural Engineering and Mechanics, *volume* **3**(3), page no 229-244.

27. Deolasi PJ and Datta PK (1997), "Simple and combination resonances of rectangular plates subjected to non-uniform edge loading with damping", Engineering Structures, *volume* **19**(12), page no: 1011-1017.
28. Deolasi PJ and Datta PK (1997), "Experiments on the parametric vibration response of plates under tensile loading", Experimental Mechanics, volume **37**, page no 56-61.
29. Dixon P and Wright J (1971), "Parametric instability of flat plates", A.R.C., 32670.
30. Einaudi, R, (1935/1936), "Sulle configurazioni di equilibrio instabili di una piastra sollecitata da sforgi tangerziale pulsanti", *Atti Accad. Gioenia J. Memoria*, **20**, Nos 1-5.
31. Evan – Iwanowski, R. M. On the "parametric response of structures". Applied Mechanics review, volume 18, page no 699 – 702, 1965.
32. F J Plantema. "Sandwich Construction". New York: Wiley, 1966.
33. Faraday, M., "On a peculiar class of acoustical figures and on certain forms assumedby a group of particles upon vibrating elastic surfaces". Phil. Trans., Roy. Soc.,London, 299 – 318, 1831.
34. Ganapathi M, Patel BP, Boisse P and Touratier M (2000), "Non-linear dynamic stability characteristics of elastic plates subjected to periodic in-plane load", International Journal of Non-linear Mechanics, volume **35**(3), page no 467-480.
35. Ganapathi M, Boisse P and Solaut D (1999), "Non-linear dynamic stability analysis of composite laminates under periodic in-plane loads", International Journal of Numerical Methods in Engineering, volume **46**, page no 943-956.
36. Gilat R and Aboudi J (2000), "Parametric stability of non-linearly elastic composite plates by Lyapunov exponents", Journal of Sound and Vibration, volume **235**, page no 627-637.
37. H G Allen. "Analysis and Design of Structural Sandwich Panels". Pergamon: Oxford, 1969.
38. Habip, L.M., "A survey of modern developments in the analysis of sandwich Structures". Applied Mechanics Reviews, volume 18, page no 93-98, 1965
39. Hutt, JM and Salam AE (1971), "Dynamic Stability of plates by finite element", Journal of Engineering Mechanics, *ASCE*, volume **3**, page no 879-899.

40. Ibrahim, R. A. and Barr, A. D. S., "Parametric vibration", Part I: Mechanics of linear problems. Shock Vib. Dig., volume 10(1), page no 15 – 29, 1978.
41. Ibrahim, R. A. and Barr, A. D. S., "Paranetric vibration, part II": Mechanics of nonlinear problems. Shock Vib. Dig., volume 10(20), page no 9 – 24, 1978.
42. Ibrahim, R. A., "Parametric vibration, Part III: Current problems (1)". Shock vib. Dig., Volume 10 (3), page no 41 –57, 1978.
43. Ibrahim., R. A. , "Parametric vibration, Part IV : Current problems (2)". Shock Vib Dig., Volume 10 (4), page no:19 – 47, 1978.
44. Ibrahim., R. A. , and Roberts, J. W., "parametric vibration, Part V : Stochastic problems." Shock Vib. Dig., volume 10 (5),page no: 17 – 38, 1978.
45. Ibrahim., R. A. , "Parametric vibration, Part Vi : Stochastic problems (2)". Shock Vib. Dig., volume 13 (9), page no 23 – 35, 1981.
46. Ibrahim., R. A. , "Parametric Random Vibration", Research Studies Press Ltd. ,England ,1985.
47. Jia-Yi Yeh, Lien-Wen Chen, "Vibration of a sandwich plate with a constrained layer and electrorheological fluid core", Composite Structures, volume 65, (2004) page no: 251–258.
48. Jia-Yi Yeh, Lien-Wen Chen, "Dynamic stability of a sandwich plate with a constraining layer and electrorheological fluid core", Journal of Sound and Vibration, volume 285, (2005) page no: 637–652.
49. Jzeng BT, Lin PD and Chen LW(1992), "Dynamic stability of bimodulus thick plates", Computers and Structures, volume **45**, page no 745-753.
50. Krishnamoorthy C. S." Finite Element Analysis Theory and Programming". New Delhi: Tata Mc Hill Co., 1988.
51. Kwon YW (1991), "Finite element analysis of dynamic stability of layered composite plates using a high order bending theory", Computers and Structures, volume **38,** page no 57-62.
52. Lall A.K., Asnani N.T., Nakra B.C., "Vibration and damping analysis of reactangular plate with partially covered constrained viscoelastic layer", Journal of vibration , Acoustics, Stress and Reality in Design 109(1987) 241-247..
53. Langley RS (1996), "The response of two-dimensional periodic structures to point harmonic forcing", Journal of Sound and Vibration, *volume* **197**, page no : 447-469

54. Liao CL and Cheng CR, (1994) “Dynamic stability of stiffened laminated composite plates and shells subjected to in-plane pulsating forces”, Journal of Sound and Vibration, volume **174**, page no: 335-351.
55. Librescu L and Thangjitham S (1990), “Parametric instability of laminated composite shear deformable flat panels subjected to in-plane edge loads”, International Journal of Non-linear Mechanics, volume **25**, page no 263-273.
56. Librescu L. and Chandiramani (1989), “Dynamic stability of transversely isotropic viscoelastic plates”, Journal of Sound and Vibration, *volume* **130** (3), page no 467-486.
57. Lok T. S. and Cheng Q. H., “Free vibration of clamped orthotropic sandwich panel.” Journal of Sound and &vibration. Volume 229, No. 2, (2000): p. 311-327.
58. Madhujit Mukhopadhyay, “Matrix and finite element analysis of structures” New Delhi, Gopal Enterprises, 2004.
59. Melde, F., “über erregung stehender wellen eines fadenformigen korpers.” ANN. PHys. Chem., volume 109, page no: 193 – 215, 1859.
60. Mermertas V and Belek HT(1990), “Static and dynamic stability of variable thickness annular plates”, Journal of Sound and Vibration, *volume* **141**(3),page no 435-448.
61. Mond M and Cederbaum G, (1992), “Dynamic instability of antisymmetric laminated plates”, Journal of Sound and Vibration, volume **154**, page no 271-279.
62. Moorthy J, Reddy JN and Plaut RH (1990), “Parametric instability of laminated composite plates with transverse shear deformation”, International Journal of Solids and Structures, *volume* **26**, page no: 801-811.
63. Nakra, B.C., “Vibration control with viscoelastic materials”. The Shock and VibrationDigest, volume 8, page no 3-12, 1976.
64. Nakra, B.C., “Vibration control with viscoelastic materials-II”. The Shock and Vibration Digest, volume 13, page no 17-20, 1981.
65. Nakra, B.C., “Vibration control with viscoelastic materials-III. The Shock and Vibration Digest”, volume 16, page no: 17-22, 1984.
66. Nayfeh, A. H. and Mook, D.T., “Nonlinear Oscillations”. John Willey & Sons, Inc., New York, 1979.
67. Ostiguy GL, Samson LP and Nguyen H (1993), “On the occurrence of simultaneous resonances in parametrically excited rectangular plates”. Journal of Vibration and Acoustics, *Trans. of the ASME,volume* **115**, page no 344-352.

68. Pavlovic RG (1994), "Dynamic stability of anti-symmetrically laminated angle-ply rectangular plates subjected to random excitation", Journal of Sound and Vibration, volume **171**, page no: 87-95.
69. Prabhakara DL and Datta PK (1993), "Parametric instability characteristics of rectangular plates with localized damage subjected to in-plane periodic load", Structural Engineering Review, *volume* **5**(1), page no 71-79.
70. Prabhakara DL and Datta PK (1997), "Vibration, buckling and parametric instability behaviour of plates with centrally located cutouts subjected to in-plane edge loading (Tension or Compression*)",* Thin Walled Structures, volume **27**(4), page no 287-310.
71. Rudra Pratap Singh, "Getting Started with MATLAB7", New Delhi, Narosa Publishing House, 1999.
72. Rao J. S. "Dynamics of Plates." New Delhi :Narosa Publishing House, 1999.
73. Sahu SK and Datta PK (2000), "Dynamic instability of laminated composite rectangular plates subjected to non-uniform harmonic in-plane edge loading", Proc. of Institution of Mech. Engg.*, Part G, U.K.,* **214***,* 295-312.
74. Saito, H. and Otomi, K., "Parametric response of viscoelastically supported beams". Journal of Sound and Vibration*,* volum*e* 63, page no 169 – 178 1979.
75. Sassi S, Smaoui H, Thomas M and Laville F (2001), "More details about the interaction of forced and parametric resonances arising from in-plane excitation of imperfect rectangular plates", Journal of Sound and Vibration, volume **243**(3), page no: 503-524.
76. Scipio L. Albert."Structure Design concepts". Washingoton D. C.: NASA SP-5039, 1967
77. Schmidt, G., "Parameter erregte Schwingungen". VEB Deutscher verlag der Wissenschaften, Berlin, 1975.
78. Simons DA and Leissa AW (1971), "Vibrations of rectangular cantilever plates subjected to in-plane acceleration loads", Journal of Sound and Vibration, volume **17**(3), page no: 407-422.
79. Sinha SK(1989), Comments on ''Dynamic stability of a rectangular plate subjected to distributed in-plane dynamic force'', Journal of Sound and Vibration*, volume* **128** (3), page no 505-507.
80. Simitses, G. J., "Instability of dynamically – loaded structures". Appl. Mech, Rev., 40, page no 1403 – 1408, 1987.

81. Shen IY and Mote CD Jr (1991), “Parametric resonances of a circular plate with inclusions subjected to a rotating spring”, Journal of Sound and Vibration volume , **149** (1), page no 164-169.
82. Srivastava AKL, Datta PK and Sheikh AH (2004), “Parametric instability of stiffened plates”, Int. J. of Applied Mechanics and Engineering, volume **9** (1), page no 169-180.
83. Srivastava AKL, Datta PK and Sheikh AH (2003), “Dynamic instability of stiffened plates subjected to non-uniform harmonic in-plane edge loading”, Journal of Sound and Vibration, volume **262** (5), page no 1171-1189.
84. Srivastava AKL, Datta PK and Sheikh AH (2002), “Vibration and dynamic stability of stiffened plate subjected to non-uniform in-plane edge loading”, Int. J. of Structural stability and dynamics, volume **2**(2), page no 185-206.
85. Srivastava AKL, Datta PK and Sheikh AH (2003), “Dynamic stability of stiffened plate with cutout subjected to in-plane uniform edge loading”, Int. J. of Structural stability and dynamics volume , **3**(3), page no 391-404.
86. Srinivasan RS and Chellapandi P (1986) “Dynamic stability of rectangular laminated composite plates”, Computers and Structures, *volume* **24**, page no 233-238.
87. Stevens, K.K., “On the parametric excitation of a viscoelastic column”. AIAA journal, volume 4, page no 2111-2115,1966.
88. Takahashi, K., “An approach to investigate the instability of the multiple-degree-offreedom nparametric dynamic systems”. Journal of Sound and Vibration, volume 78, page no 519 –529, 1981.
89. Takahashi K and Konishi Y (1988), Dynamic stability of rectangular plate subjected to distributed in-plane dynamic force, Journal of Sound and Vibration, **123**(1), 115-127.
90. Takahashi K, Tagewa M and Ikeda T (1983), “Dynamic stability of a rectangular plate subjected to in-plane moment”, Theoretical and Applied Mechanics, Proc. 33rd Japan National Congress for applied Mechanics, volume **33**, page no 311-318.
91. Tani J and Nakumara T(1978), “Dynamic instability of annular plates under pulsating radial loads”, Journal of Acoustic society of America, volume **64**, page no 827-831.
92. Touati D and Cederbaum G (1994), “Dynamic stability of non-linear visco-elastic plates”, International Journal of Solids and Structures, *volume* **31**, page no 2367-2376.

93. Tylikowski A (1989), "Dynamic stability of non-linear anti-symmetrically laminated cross-ply rectangular plates", ASME Journal of Applied Mechanics, volume **56**, page no 375-381.

94. Yamaki N and Nagai K (1975), "Dynamic stability of rectangular plates under periodic compressive force", Report of the Institute of High speed Mechanics, Tohoku Univ., volume **32**, page no 103-127.

95. Young TH, Lee CW and Chen FY(2002), "Dynamic stability of skew plates subjected to aerodynamic and random in-plane forces", Journal of Sound and Vibration, volume **250** (3), page no 401-414.

96. Yuan W. X. and Dawe D. J., "Free vibration of sandwich plates with laminated faces." Int. J. Numer. Meth. Engng. Volume 54, (2002): p. 195–217.

97. Zajaczkowski, J. and Lipinski, J. "Vibrations of parametrically excited systems". Journal of Sound and Vibration, volume 63, page no 1 – 7, 1979.

98. Zajaczkowski, J., "An approximate method of analysis of parametric vibration" . Journal of Sound and Vibration, 79, 581 – 588, 1981.