STATIC AND DYNAMIC ANALYSIS OF GRID BEAMS

A PROJECT REPORT SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF BACHELORS OF TECHNOLOGY DEGREE IN CIVIL ENGINEERING

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NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA

2010

National Institute of Technology

Rourkela

CERTIFICATE

This is to certify that the report entitled, "DYNAMIC ANALYSIS OF GRID BEAM STRUCTURES " submitted by Ms. Baishali Das in partial fulfilment of the requirements for the award of Bachelor of Technology Degree in Civil Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in the project report has not been submitted to any other University/Institute for the award of any Degree or Diploma

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ABSTRACT

A grid is a planar structural system composed of continuous members that either intersect or cross each other .Grids are used to cover large column free areas and is subjected to loads applies normally to its plane . It is beneficial over normal beams as it has a better load dispersing mechanism and also this system reduces the normal span to depth ratio which helps in reducing the height of the building .Grid beams are analysed dynamically to determine the natural frequencies of the nodes of the system. Direct stiffness method for finding out the stiffness matrix and Mass matrix of the members of grid system. The lowest natural frequency will determine the minimum dynamic loading that the structure can withstand without resonance.

CHAPTER 1

INTRODUCTION

GENERAL INTRODUCTION:

1.1 DEFINITION:

Interconnected grid systems are being commonly used or supporting building floors bridge decks and overhead water tanks slabs .A grid is a planar structural system composed of continuous members that either intersect or cross each other. Grids are used to cover large column free areas and have been constructed in number of areas in India n abroad. Is subjected to loads applied normally to its plane, the structure is referred as Grid . It is composed of continuous member that either intersect or cross each other .Grids in addition to their aesthetically pleasing appearance, provide a number of advantages over the other types of roofing systems.

1.2 ADVANTAGES OF GRID BEAMS

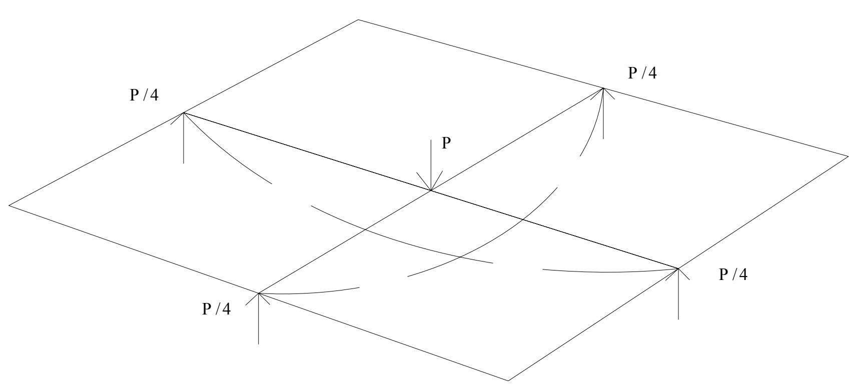
- Grids are very efficient in transferring concentrated loads and in having the entire structure participate in the load carrying action .
- Reduces the depth to span ratio of rectangular grids .
- Reduction in depth ,towers ,structural and other cost by reducing the height of the building .

1.3 LOAD DISTRIBUTION IN GRID BEAMS

A load set on a cable or a beam is channelled to the support along the cable line or the beam axis, an arch ,a frame, and continuous beam produce the same type of “one –directional load dispersal”. These structures are labelled “one – dimensional resisting structures” because they can be described by a straight or curved line , along which the stresses channel the loads.

If they are used to cover a rectangular area ,the system becomes impractical and insufficient. Grid beams structure are two way load dispersal system. When two identical ,simply supported beams at right angles to each other are placed one on the top of other and a concentrated load is applied at their intersection ,the load is transferred to the supports at the ends o both beams and dispersed in two way direction .Being identical they have same deflection and carry half the load .Thus each support reactions equal to one –fourth of load .

In a planar frame structure ,joint displacement are considered as translation in X and Y direction and rotation about Z-axis .However when plane frame is normally loaded ,the displacement of joint are a translation in the Z direction and rotation X and Y axis .In grid connection ,the connection between the members are assumed to be rigid .When beams connected at mid span .It moves downward but remains vertical because of symmetry but when connection is not at mid span the beam rotates and deflects .The continuity introduced by rigid connection transforms the bending rotation of one section into a twisting rotation of other .A grid frame is considered as a special case of a rigid space frame in which the joint rotations may take place about all the three axes and joint displacements may have components along all the three axes.



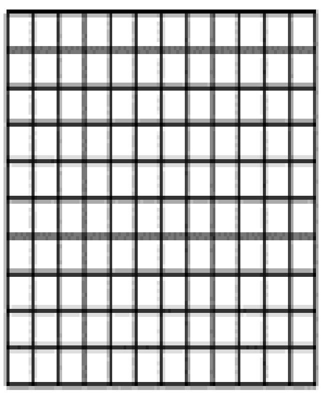
concentrated load on two dimensional system

1.4 TYPES OF GRIDS:

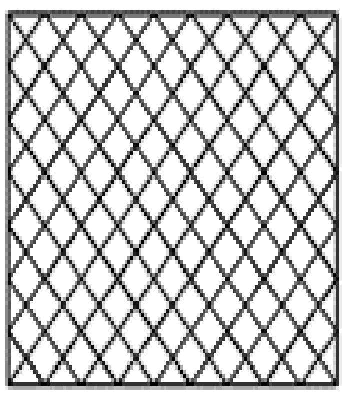
Grids may be divided into two types in structural analysis point of view

1. Supported along all four sides
   These are generally used for buildings roofs and floors .They can be used to cover square rectangular ,triangular ,hexagonal or circular areas .
   - Orthogonal grid
   - Diagonal grid
   - Three way grid
   - Hexagonal grid
   - Skew grid (It is comparatively strong as the stress distribution is uniform)

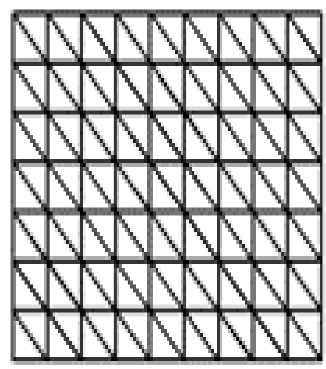
2. Supported along two sides
   These are used rectangular grid is more popular especially for concrete construction .



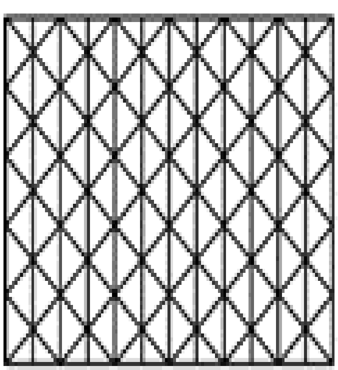
(a) Two-way grid



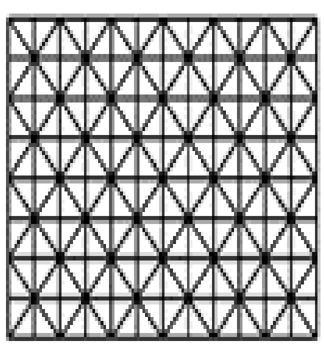
(b) Diagonal grid



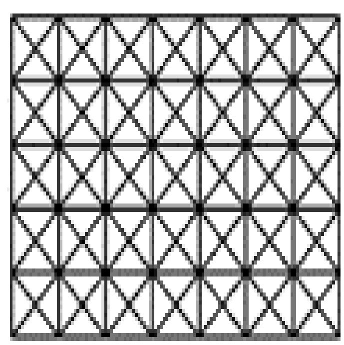
(c) Three-way grid



(d) Three-way grid



(e) Four-way grid



(f) Four-way grid

CHAPTER 2

METHOD USED IN ANALYSIS

STATIC ANALYSIS OF BEAMS:

2.1STIFFNESS METHOD OF STRUCTURAL ANALYSIS:

INTRODUCTION:

Stiffness method or displacement method is an important approach to the analysis of structures .This is an important approach to the analysis of structures. This is used in its basic form for the analysis of structures that are linear and elastic although it can be adapted to non linear analysis. It is generally used for the analysis of statically determinate cases. This method in its basic form considers the nodal displacements of the structures as unknown.

The DIRECT STIFFNESS METHOD is a highly organized, conceptually simple approach for the analysis of all types of structures that is easily implemented in the form of a computer aided analysis procedure using a matrix formulation.

Amongst the most far- reaching developments in structural engineering has been the ability to analyze automatically almost all types of structures with a high degree of accuracy and at reasonable cost. The availability of digital computer has made this development possible Methods of analysis that could easily be computerized were quickly developed.

2.2 STEPS IN DIRECT STIFFNESS METHOD:

STEP 1:

The first step in the analysis is to define the structures coordinate system , the support and loading conditions and the assumptions of analysis . In this stage each of the degree of freedom is numbered and for convenience each of the members is also numbered . The member properties such as M.I. modulus of elasticity , etc. are interested.

STEP2:

With the structures defined , the member coordinate systems are defined and the MEMBER STIFFNESS MATRIX for each of the members is computed in its own coordinate system. The coordinates are defined by arbitrarily choosing one end of the member as origin, and then imposing a coordinate system identical to that used in derivation of member stiffness matrix. With the coordinate system defined member stiffness matrix foe each member is evaluated.

STEP3:

Now we have to assemble the structures stiffness matrix and solve for displacements and internal forces. The problem is to insert the elements from the structure stiffness matrix. For each of the member stiffness matrixes , the corresponding structures degree of freedom should be defined. This information is needed to place the elements in the structures stiffness matrix. After all the elements of the first member have been written into the structure stiffness matrix, those of the second are written .when writing into an already filled a lot , the new contribution is added to the existing value. While this process is tedious for even a small structure using hand calculations.

2.3 STIFFNESS METHODS MERITS AND DEMERITS:

2.3.1 MERITS:

One basic form of the stiffness method could be applied to a wide range of structures, with only minor adjustments to cope with each variant. The advantages of the method can be summarized as:

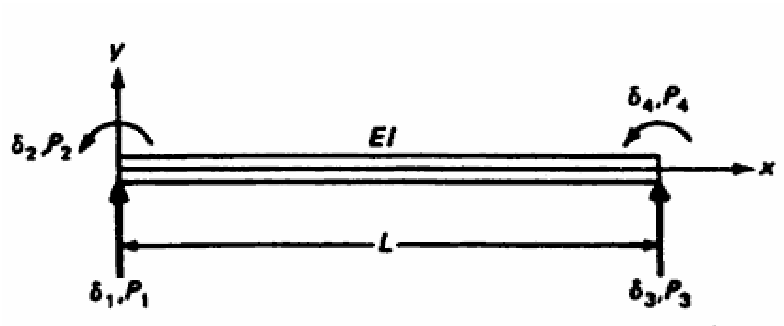
1) A general purpose program is easy to write.
2) It requires a minimum of input data.
3) It can be made entirely automatic .Its use requires no understanding of structural mechanics.

2.3.2 DEMERITS :

The method has a major disadvantage in that no account is taken of the degree of indeterminacy and therefore there is little opportunity to benefit from the structural expertise of the operator. Equally this will be seen as an essential concomitant of the advantage listed in above. The time required to perform an analysis and the amount of computer storage depends almost entirely on the number of degree of freedom involved. Structures having many degrees of freedom but few degree of static indeterminacy should be much more economically analyzed by the flexibility rather than the stiffness method.

2.4 STATIC ANALYSIS OF BEAM SEGMENT :-

Consider a uniform beam segment of cross section with moment of inertia ‘I’ ,length ‘ L ‘ and material modulus of elasticity ‘E’ . P_1 , P_2, P_3 , P_4 are the static forces and moments and δ_1, δ_2, δ_3, δ_4 are corresponding linear and angular displacement .The relationship thus obtained is known as stiffness matrix equation for beam elemnent.



The differential equation for uniform beam element is

$$EI\frac{d^4u}{dx^4} = p(x)$$

$$\frac{dM(x)}{dx} = V(x)$$

$$\frac{dV(x)}{dx} = p(x)$$

Where P(x) is the beam load per unit length and V(x) is shear force.

Considering the beam to be free of load ,p(x)=0. Then

$$\frac{d^4u}{dx^4} = 0$$

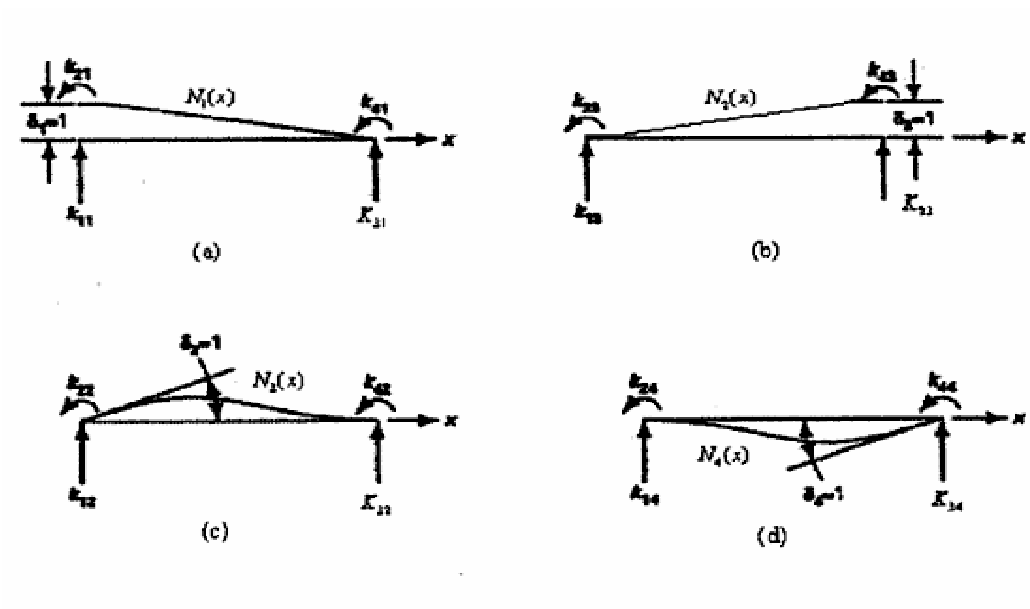
Successive integrations results in,

$$\frac{d^3u}{dx^3} = C_1$$

$$\frac{d^2u}{dx^2} = C_1x + C_2$$

$$\frac{du}{dx} = \frac{1}{2}C_1x^2 + C_2x + C_3$$

$$u = \frac{1}{6}C_1x^3 + \frac{1}{2}C_2x^2 + C_3x + C_4$$



Beam element showing static deflection due to unit displacement at one of the nodal coordinates

Here C1, C2,C3,C4 are constant of integration.

$$\text{at} \quad x = 0 \quad u(0) = 1 \quad \text{and} \quad \frac{du(0)}{dx} = 0$$

$$\text{at} \quad x = L \quad u(L) = 0 \quad \text{and} \quad \frac{du(L)}{dx} = 0$$

Using these conditions in above equation ,the value of C1,C2,C3,C4 are determined. Resulting in the nodal curve equation

$$N_1(x) = 1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3$$

Where N(x) is used instead of u(x), corresponding to the condition where deflection is 1. Similarly,

$$N_2(x) = x\left(1 - \frac{x}{L}\right)^2$$

$$N_3(x) = 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3$$

$$N_4(x) = \frac{x^2}{L}\left(\frac{x}{L} - 1\right)$$

Therefore the total deflection u(x) is

$$u(x) = N_1(x)\delta_1 + N_2(x)\delta_2 + N_3(x)\delta_3 + N_4(x)\delta_4$$

Applying Principle of virtual work

Differential equation for small transverse displacement of a beam is given by

$$W_E = k_{12}\delta_1$$

It is equal to the work performed by the elastic forces during virtual displacements. Considering the work performed by the bending moment we obtain the internal work as,

$$W_1 = \int_0^L M(x)d\theta$$

where M(x) is bending moment at sec X and d0 is the angular displacement. Thus,

$$EI\,N_2''(x) = M(x)$$

$$\frac{d\theta}{dx} = \frac{d^2N_1(x)}{dx^2} = N_1''(x)$$

$$d\theta = N_1''(x)\,dx$$

Equating the external virtual work W_E to the internal virtual work W1,the stiffness coefficient is

$$k_{ij} = \int_0^L E\ I\ N_i''(x)N_j''(x)dx$$

Considering the case of uniform lenghth L and inertia I, the stiffness coefficient is $K_{12,}$

$$N_1''(x) = -\frac{6}{L^2} + \frac{12x}{L^3}$$

$$N_2''(x) = -\frac{4}{L} + \frac{6x}{L^2}$$

$$k_{12} = EI\int_0^L\left(\frac{-6}{L^2} + \frac{12x}{L^3}\right)\left(\frac{-4}{L} + \frac{6x}{L^2}\right)dx$$

$$k_{12} = \frac{6EI}{L^2}$$

Since the stiffness coffcient is defined as the force in the nodal coordinate 1 due to unit displacement, the forces at coordinate 1 due to successive displacement at the 4 coordinates combine to form Pt given by

$$P_1 = k_{11}\delta_1 + k_{12}\delta_2 + k_{13}\delta_3 + k_{14}\delta_4$$

The forces at other nodal coordinates are

$$P_2 = k_{21}\delta_1 + k_{22}\delta_2 + k_{23}\delta_3 + k_{24}\delta_4$$
$$P_3 = k_{31}\delta_1 + k_{32}\delta_2 + k_{33}\delta_3 + k_{34}\delta_4$$
$$P_4 = k_{41}\delta_1 + k_{42}\delta_2 + k_{43}\delta_3 + k_{44}\delta_4$$

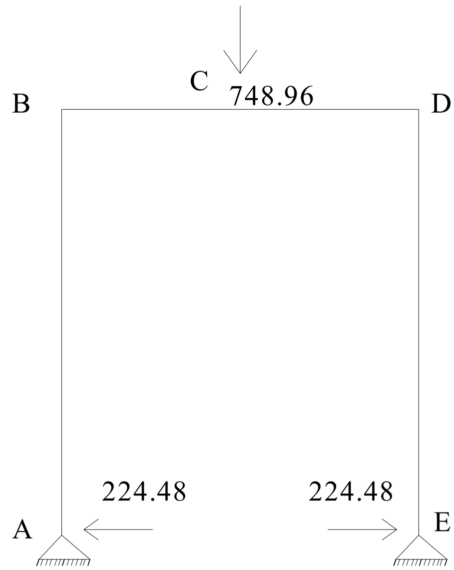
In condensed notation,

$$\{P\} = [k]\{\delta\}$$

Putting the value of all coefficients we get the matrix form

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix}$$

2.5 ILLUSRATIVE EXAMPLE:



2.5.1 MATLAB PROGRAMME FOR STATIC ANALYSIS OF FRAME : (ALGORITHM)

1) The node coordinates of the frames are defined.

2) Characteristics constants of the Beam element like, the Area (A),Modulus of elasticity (E),Moment of inertia(I)etc are defined.

3) For each element the Stiffness matrix is assigned into a 6X6 matrix in local coordinates.

4) Then each of them are transposed to the Global coordinates according to their orientation.

5) The individual Stiffness matrixes are combined to form the Global stiffness matrix of the Frame, according to their degree of freedoms.

6) The degree of freedoms of the fixed nodes are assigned as 0, where as the free nodes have each 3 DOFS.

7)The components of common DOFS are added in the Global stiffness matrix.

8) The Force or load vector is defined .

9) The displacement vector is computed by multiplying the iverse of the stiffness matrix with the load vector

RESULT:

K1

1.0e+006 *

0.3522	0	0.7748	-0.3522	0	0.7748
0	6.8182	0	0	-6.8182	0
0.7748	0	2.2727	-0.7748	0	1.1364
-0.3522	0	-0.7748	0.3522	0	-0.7748
0	-6.8182	0	0	6.8182	0
0.7748	0	1.1364	-0.7748	0	2.2727

K2:

1.0e+007 *

2.1176	0	0	-2.1176	0	0
0	1.0552	-0.8969	0	-1.0552	-0.8969
0	-0.8969	1.0165	0	0.8969	0.5082
-2.1176	0	0	2 .1176	0	0
0	- 1.0552	0.8969	0	1.0552	0.8969
0	-0.8969	0.5082	0	0.8969	1.0165

K3

1.0e+007 *

2.1176	0	0	-2.1176	0	0
0	1.0552	-0.8969	0	-1.0552	-0.8969
0	-0.8969	1.0165	0	0.8969	0.5082
-2.1176	0	0	2.1176	0	0
0	-1.0552	0.8969	0	1.0552	0.8969
0	-0.8969	0.5082	0	0.8969	1.0165

K4

1.0e+006 *

0.3522	0	-0.7748	-0.3522	0	-0.7748

0	6.8182	0	0	-6.8182	0
-0.7748	0	2.2727	0.7748	0	1.1364
-0.3522	0	0.7748	0.3522	0	0.7748
0	-6.8182	0	0	6.8182	0
-0.7748	0	1.1364	0.7748	0	2.2727

THE GLOBAL STRUCTURAL STIFFNESS MATRIX

1.0e+007 *

Columns 1 through 8

2.1529	0	-0.0775	-2.1176	0	0	0	0
0	1.7370	-0.8969	0	-1.0552	-0.8969	0	0
-0.0775	-0.8969	1.2437	0	0.8969	0.5082	0	0
-2.1176	0	0	4.2353	0	0	-2.1176	0
0	-1.0552	0.8969	0	2.1103	0	0	-1.0552
0	-0.8969	0.5082	0	0	2.0329	0	0.8969
0	0	0	-2.1176	0	0	2.1529	0
0	0	0	0	-1.0552	0.8969	0	1.7370
0	0	0	0	-0.8969	0.5082	-0.0775	0.8969

Column 9

0

0

0

0

-0.8969

0.5082

-0.0775

0.8969

1.2437

RANK OF THE MATRIX

9

THE DISPLACEMENT MATRIX

1.0e-003 *

0.0024

-0.0878

0.0665

-0.0000

-0.1799

-0.0000

-0.0024

-0.0878

-0.0665

CHAPTER 3

DYNAMIC ANALYSIS OF GRIDS

3.1 DYNAMIC ANALYSIS OF GRIDS:

In case of grids Torsional effects also come into picture. The dynamic analysis by the stiffness method for grid structures that is for plane frame subjected to normal loads requires the determination of the torsional stiffness and mass coefficients for a typical member of the grid .

The differential Equation for torsional displacement is

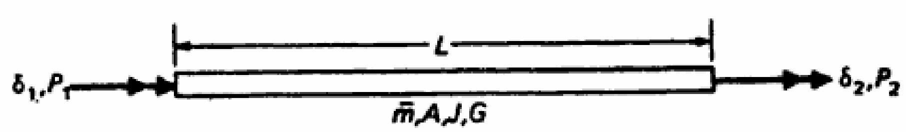
$$\frac{d\theta}{dx} = \frac{T}{JG}$$

In which θ is the angular displacement T is the torsional moment, G is the modulus of elasticity in shear, and J is the constant of cross section(Polar moment of inertia of circular section.)

As a consequence of analogy with previous Equations are obtained,

$$\theta_1(x) = (1 - \frac{x}{L})$$

$$\theta_2(x) = \frac{x}{L}$$



The stiffness influence coefficients for torsional effects may be calculated from

$$k_{ij} = \int_0^L JG\theta_i'(x)\theta_j'(x)dx$$

The consistent mass matrix for torsional effects is given by

$$m_{ij} = \int_0^L I_{\bar{m}}\theta_i(x)\theta_j'(x)dx$$

In which Im is the polar moment of inertia , per unit length along the beam element.

3.2 STIFFNESS MATRIX FOR A GRID ELEMENT:

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} JGL^2/EI & & & & & \\ 0 & 4L^2 & & & & \\ 0 & -6L & 12 & & & \\ -JGL^2/EI & 0 & 0 & JGL^2/EI & & \\ 0 & 2L^2 & -6L & 0 & 4L^2 & \\ 0 & 6L & -12 & 0 & 6L & 12 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{Bmatrix}$$

In condensed form

$$\{P\} = [K]\{\delta\}$$

3.3 CONSISTENT MASS MATRIX OF A GRID ELEMENT:

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{Bmatrix} = \frac{\overline{m}L}{420} \begin{bmatrix} 140I_0/A & & & \text{Symmetric} & & \\ & 4L^2 & & & & \\ 0 & 22L & 156 & & & \\ 70I_0/A & 0 & 0 & 140I_0/A & & \\ 0 & -3L^2 & -13L & 0 & 4L^2 & \\ 0 & 13L & 54 & 0 & -22L & 156 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{Bmatrix}$$

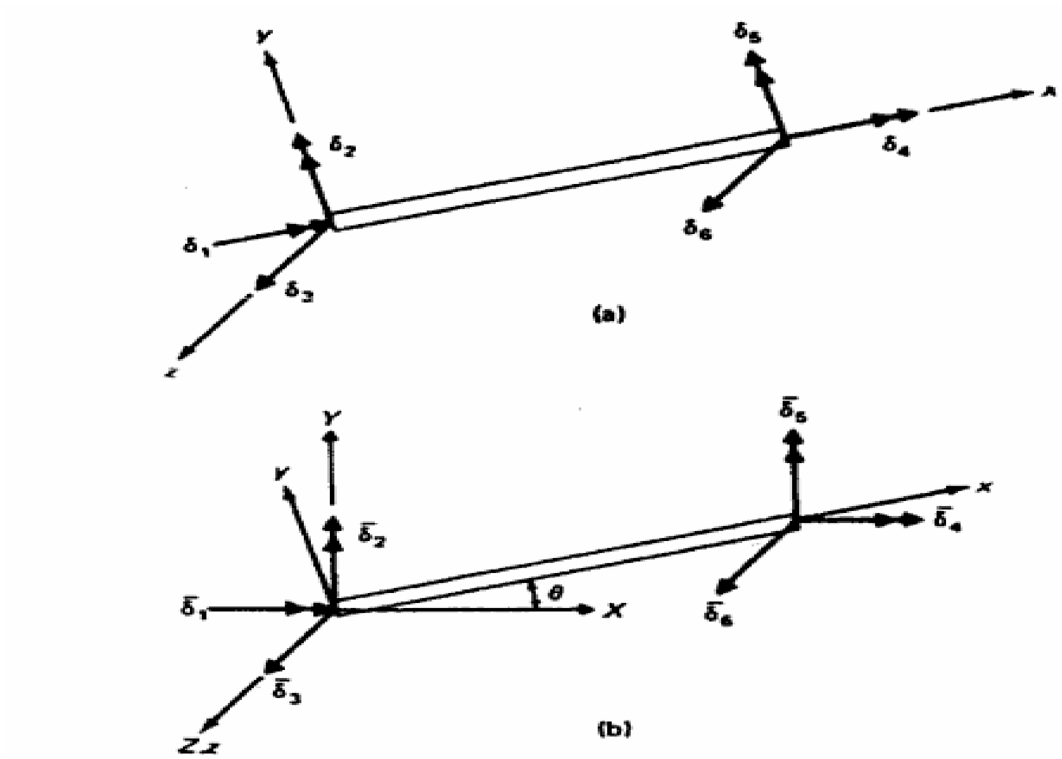
In condensed form

$$\{P\} = [M_c]\{\delta\}$$

where **Mc** is the mass matrix for a typical uniform member of a grid structure.

3.4 LOCAL AND GOBAL COORDINATE SYSTEM:

For a beam element of grid , the local orthogonal axes will be established such that the *x* defines the longitudinal centriodal axis of the member and the *x-y* plane will coincide with the plane of the structural system which will be defined as X-Y Plane. The z axis will be minor principal axis of the cross section while y axis is defines as the major axis of the cross section .The shear centre of the cross section coincides with the centroid of the cross section.



Translatory displacements along the z direction for local axes and along the z direction of the global system are identical. However the rotational components at the nodal coordinates differ .Thus a transformation of coordinates will be required to transform the element matrices from the local to the global coordinates.

$$P_1 = \overline{P}_1 \cos\theta + \overline{P}_2 \sin\theta$$
$$P_2 = -\overline{P}_1 \sin\theta + \overline{P}_2 \sin\theta$$
$$P_3 = \overline{P}_3$$

$$P_4 = \overline{P}_4 \cos\theta + \overline{P}_5 \sin\theta$$
$$P_5 = -\overline{P}_4 \sin\theta + \overline{P}_5 \sin\theta$$
$$P_6 = \overline{P}_6$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \overline{P}_1 \\ \overline{P}_2 \\ \overline{P}_3 \\ \overline{P}_4 \\ \overline{P}_5 \\ \overline{P}_6 \end{Bmatrix}$$

3.5 ASSUMPTION IN ANALYSIS OF A GRID MEMBER:

1) All the members are straight uniform and prismatic.
2) Stress –strain curve is linear.
3) The axis of all the members meeting at a joint intersects without eccentricity.
4) Each member is assumed to have two axes of symmetry in the cross section ,so that bending and torsion occur independently of one another. Thus for symmetrically members ,the shear centre axis must be taken as the axis of the members.
5) The effect of warping is neglected .Since warping is neglected ,the torsional rigidity GJ has been used in the analysis.

3.6 ILLUSTRATIVE EXAMPLE OF GRID BEAM NODE:

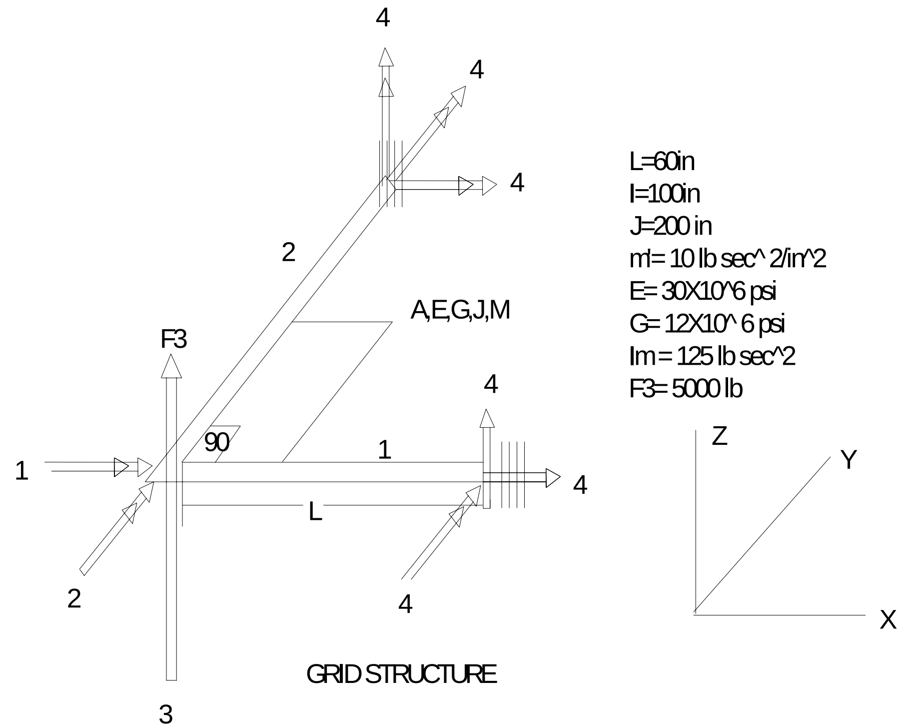
Dynamic analysis of grid beam is of concern to all engineers. Of various methods of analysis the matrix method gives the most general approach to the solution of any arrangement of beams and plan area. MATLAB programme for the analysis of grid structure is developed based of "stiffness method of structural analysis " as explained earlier. The programme can also be used for the analysis of grids of any large size .Following is a sample MATLAB programme of a particular GRID beam node. The respective natural frequencies of the beam is found out by solving the Eigen problem

$$([Ks]- w^2 [Ms]) \{a\} =\{0\}$$

Where Ks= combined global system matrix

Ms= combined global mass matrix

w= natural frequency {a}= modal matrix(Eigen vectors).



GRID STRUCTURE

RESULTS:

1.0e+008 *

0.4000	0	0	-0.4000	0	0
0	2.0000	0.0500	0	1.0000	-0.0500
0	0.0500	0.0017	0	0.0500	-0.0017
-0.4000	0	0	0.4000	0	0
0	1.0000	0.0500	0	2.0000	-0.0500
0	-0.0500	-0.0017	0	-0.0500	0.0017

1.0e+008 *

2.0000	0	-0.0500	1.0000	0	0.0500
0	0.4000	0	0	-0.4000	0
-0.0500	0	0.0017	-0.0500	0	-0.0017

1.0000	0	-0.0500	2.0000	0	0.0500
0	-0.4000	0	0	0.4000	0
0.0500	0	-0.0017	0.0500	0	0.0017

THE GLOBAL STRUCTURAL STIFFNESS MATRIX

1.0e+008 *

2.4000	0	-0.0500
0	2.4000	0.0500
-0.0500	0.0500	0.0033

RANK OF THE MATRIX

3

1.0e+004 *

0.2500	0	0	0.1786	0	0
0	2.0571	0.1886	0	-1.5429	0.1114
0	0.1886	0.0223	0	-0.1114	0.0077
0.1886	0	0	0.2500	0	0
0	-1.5429	-0.1114	0	2.0571	-0.1886
0	0.1114	0.0077	0	-0.1886	0.0223

1.0e+004 *

2.0571	0	-0.1886	-1.5429	0	-0.1114
0	0.2500	0	0	0.1786	0
-0.1886	0	0.0223	0.1114	0	0.0077
-1.5429	0	0.1114	2.0571	0	0.1886
0	0.1886	0	0	0.2500	0
-0.1114	0	0.0077	0.1886	0	0.0223

THE GLOBAL STRUCTURAL MASS MATRIX(Ms)

1.0e+004 *

2.3071	0	-0.1886
0	2.3071	0.1886
-0.1886	0.1886	0.0446

RANK OF THE MATRIX

3

The eigen values are

```
1.0e+004 * 0.0396
           1.0402
           2.3866
```

The frequencies are:

```
 19.9085
101.9925
154.4876
```

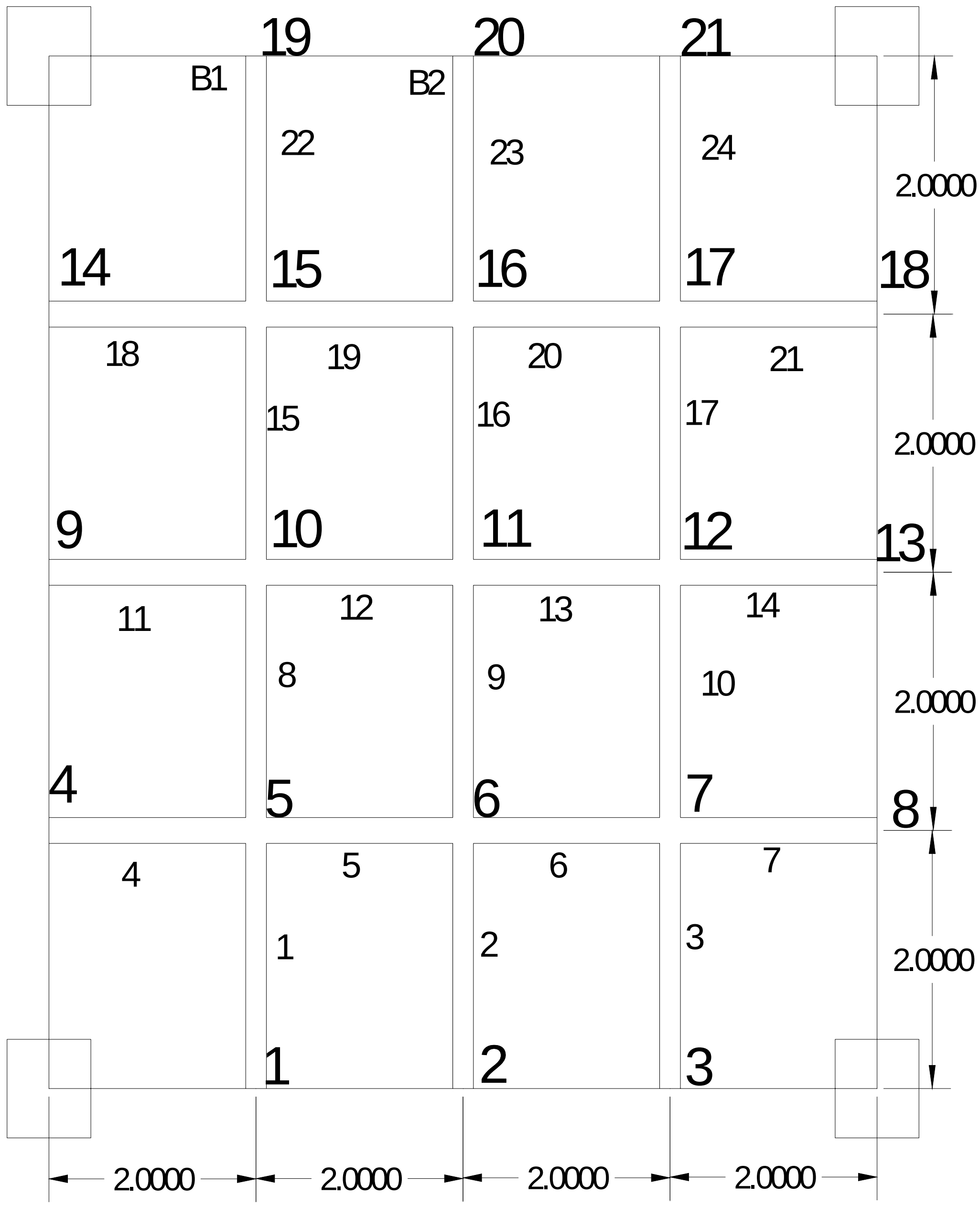
CHAPTER 4

ANALYSIS OF A GRID SYSTEM

4.1PROBLEM DESCRIPTION:

We have to analyze the grid system for a hall measuring 8mX 8m in plan. The roof system is divided into a (4mX4m) square grid pattern as shown in fig .Each element beam of the grid measures 2m. Since the system is a square grid pattern , there is a perfect symmetry with respect to the co ordinate axes.

The different nodes and members are numbered in a regular order . Here we have got 24 members and 21 nodes. Of these nodes the 12 end nodes with 3 members on each side are assumed to be fixed .That is displacements at these joints are specified to be zero. By analyzing these grid system by the methods described in previous chapters , we will get the frequencies of each node.



GRID PATTERN

4.2 LOAD CALCULATION

Assume live load =400 Kg/m^2

Load on one beam from either side due to live load = 400*2=800 Kg/m

Self weight of slab =0.10*2400=240 Kg/m^2

Load due to weight of slab =240*2=480 Kg/m

Total load =800 + 480 =1280 Kg/m

Fixed end moment $=5wl^2/96 = 5*1280*2^2/96$ = 266.70 Kg m

Self weight of rib per metre = 0.20* 0.10*2400= 48 Kg/m

Fixed end moment due to this = $wl^2/12 = 48*2^2/12$ =16 Kg m

Total fixed end moment = 266.7 + 16 =282.7 Kg m = 28270 kg cm

4.2.1 Load at intermediate nodes

Live load = 400*2*2= 1600 Kg

Weight of slab = 2400*2*2 = 960 Kg

Self weight of 4 ribs =4(0.20*0.1*2400*1)= 192 Kg

Total load = 1600*960*192 =2752 Kg

4.2.2 Load at end nodes

Load due to live load and self weight of slab = ½(1600 + 960) =1280 Kg

Self weight of one rib of 1m length =1*0.20*0.10*2400 = 48 Kg

Total load = 1280 + 48 =1328 Kg

4.2.3 Young's Modulus E

Using M15 concrete mix ,According to IS . 456-2000

$E = 5700(f_{ck})^{0.5}$ where f_{ck} =characteristics strength of concrete =15 N/mm^2

$= 5700*15^{0.5} = 22076\ N/mm^2 = 2.2076*10^5\ Kg/cm^2$

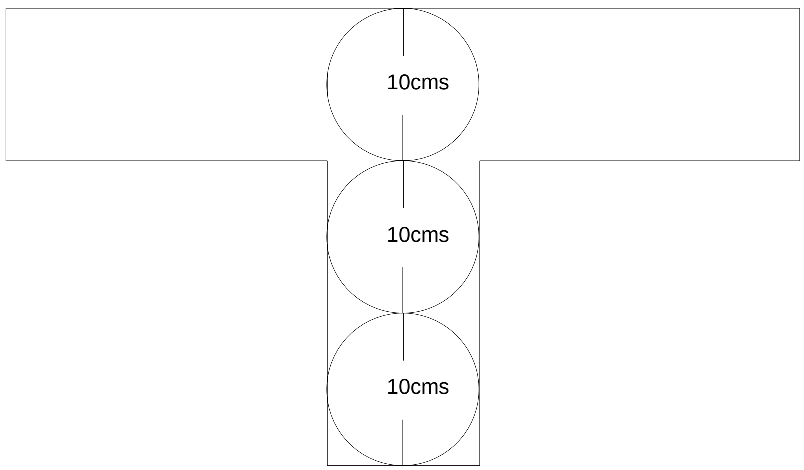
4.2.4 Ratio between E & G

E = 2G(1+1/m) where m = Poisson's ratio

(1/m)= 0.15 for concrete

E/G = 2 (1 +1/m)

=2.3



4.2.5 CALCULATION OF POLAR MOMENT OF INERTIA

Equivalent polar moment of inertia of section

= polar moment of inertia of 3 circles ,each 10 cm diameter

i.e , $J = 3*\Pi d^4/32 = 3*\pi*10^4/32 = 2945.2\ cm^4$

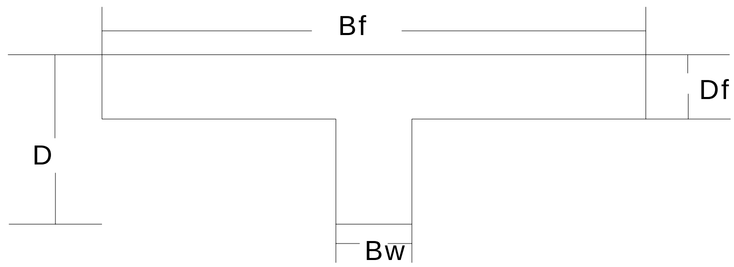
4.2.6 CALCULATION OF POLAR MASS MOMENT OF INERTIA:

Equivalent polar moment of inertia of section

= polar moment of inertia of 3 circles ,each 10 cm diameter

i.e $Im= 3X\ m'\ r^2/2=3X\ 48\ X10^2\ /2= 7200\ cm^4$

4.2.7 CALCULATION OF MOMENT OF INERTIA:



Moment of inertia I of T – section beam is calculated using chart in IS CODE.. 456-2000

$I = K_t \times b_w \times D^3$ bw= 10 cm , Df= 10 cm D= 30 cm bf= 200 cm

bf/bw= 20 Df/D= 0.3333

thus the value of K_t= 2.82

$I = 2.82 \times 110 \times 30^3/12 = 63450\ cm^4$

CHAPTER 5

RESULT , DISCUSSION AND CONCLUSION

RESULT OF THE DYNAMIC ANLYSIS OF THE GRID SYSTEM:

K GLOBAL STIFFNESS MATRIX:

1.0e+016 *

Columns 1 through 12

1.1342	0	0	-0.5671	0	0	0	0	0	0.0000	0	0.0000
0	1.1342	0	0	0.0000	-0.0000	0	0	0	0	-0.5671	0
0	0	0.0000	0	-0.0000	-0.0000	0	0	0	0.0000	0	-0.0000
-0.5671	0	0	1.1342	0	0	-0.5671	0	0	0	0	0
0	0.0000	0.0000	0	1.1342	0	0	0.0000	-0.0000	0	0	0
0	-0.0000	-0.0000	0	0	0.0000	0	-0.0000	-0.0000	0	0	0
0	0	0	-0.5671	0	0	1.1342	0	0	0	0	0
0	0	0	0	0.0000	0.0000	0	1.1342	0	0	0	0
0	0	0	0	-0.0000	-0.0000	0	0	0.0000	0	0	0
0.0000	0	-0.0000	0	0	0	0	0	0	1.1342	0	0
0	-0.5671	0	0	0	0	0	0	0	0	1.1342	0.0000
0.0000	0	-0.0000	0	0	0	0	0	0	0	0.0000	0.0000
0	0	0	0.0000	0	-0.0000	0	0	0	-0.5671	0	0
0	0	0	0	-0.5671	0	0	0	0	0	0.0000	0.0000
0	0	0	0.0000	0	-0.0000	0	0	0	0	-0.0000	-0.0000
0	0	0	0	0	0	0.0000	0	-0.0000	0	0	0
0	0	0	0	0	0	0	-0.5671	0	0	0	0
0	0	0	0	0	0	0.0000	0	-0.0000	0	0	0
0	0	0	0	0	0	0	0	0	0.0000	0	-0.0000
0	0	0	0	0	0	0	0	0	0	-0.5671	0
0	0	0	0	0	0	0	0	0	0.0000	0	-0.0000
0	0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

Columns 13 through 24

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0.0000	0	0.0000	0	0	0	0	0	0	0	0	0
0	-0.5671	0	0	0	0	0	0	0	0	0	0
0.0000	0	-0.0000	0	0	0	0	0	0	0	0	0
0	0	0	0.0000	0	0.0000	0	0	0	0	0	0
0	0	0	0	-0.5671	0	0	0	0	0	0	0
0	0	0	0.0000	0	-0.0000	0	0	0	0	0	0
-0.5671	0	0	0	0	0	0.0000	0	0.0000	0	0	0
0	0.0000	-0.0000	0	0	0	0	-0.5671	0	0	0	0
0	-0.0000	-0.0000	0	0	0	0.0000	0	-0.0000	0	0	0
1.1342	0	0	-0.5671	0	0	0	0	0	0.0000	0	0.0000
0	1.1342	0	0	0.0000	-0.0000	0	0	0	0	-0.5671	0
0	0	0.0000	0	-0.0000	-0.0000	0	0	0	0.0000	0	-0.0000
-0.5671	0	0	1.1342	0	0	0	0	0	0	0	0
0	0.0000	0.0000	0	1.1342	0	0	0	0	0	0	0
0	-0.0000	-0.0000	0	0	0.0000	0	0	0	0	0	0
0	0	0	0	0	0	1.1342	0	0	-0.5671	0	0
0	0	0	0	0	0	0	1.1342	0.0000	0	0.0000	-0.0000
0	0	0	0	0	0	0	0.0000	0.0000	0	-0.0000	-0.0000
0.0000	0	-0.0000	0	0	0	-0.5671	0	0	1.1342	0	0
0	-0.5671	0	0	0	0	0	0.0000	0.0000	0	1.1342	0

0.0000	0	-0.0000	0	0	0	0	-0.0000	-0.0000	0	0	0.0000
0	0	0	0.0000	0	-0.0000	0	0	0	-0.5671	0	0
0	0	0	0	-0.5671	0	0	0	0	0	0.0000	0.0000
0	0	0	0.0000	0	-0.0000	0	0	0	0	-0.0000	-0.0000

Columns 25 through 27

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0.0000	0	0.0000
0	-0.5671	0
0.0000	0	-0.0000
0	0	0
0	0	0
0	0	0
-0.5671	0	0
0	0.0000	-0.0000
0	-0.0000	-0.0000

1.1342	0	0
0	1.1342	0
0	0	0.0000

1.0e+008 *

MASS MATRIX OF THE STRUCTURE

Columns 1 through 12

0.9102	0	0	6.6377	0	0	0	0	0	-0.3017	0	-0.0065
0	0.9102	0	0	-0.3017	0.0065	0	0	0	0	6.6377	0
0	0	0.0016	0	-0.0065	0.0001	0	0	0	0.0065	0	0.0001
0.0111	0	0	0.9102	0	0	6.6377	0	0	0	0	0
0	-0.3017	-0.0065	0	0.9102	0	0	-0.3017	0.0065	0	0	0
0	0.0065	0.0001	0	0	0.0016	0	-0.0065	0.0001	0	0	0
0	0	0	0.0111	0	0	0.9102	0	0	0	0	0
0	0	0	0	-0.3017	-0.0065	0	0.9102	0	0	0	0
0	0	0	0	0.0065	0.0001	0	0	0.0016	0	0	0
-0.3017	0	0.0065	0	0	0	0	0	0	0.9102	0	0
0	0.0111	0	0	0	0	0	0	0	0	0.9102	0.0221
-0.0065	0	0.0001	0	0	0	0	0	0	0	0.0221	0.0016
0	0	0	-0.3017	0	0.0065	0	0	0	0.0111	0	0
0	0	0	0	0.0111	0	0	0	0	0	-0.3017	-0.0065
0	0	0	-0.0065	0	0.0001	0	0	0	0	0.0065	0.0001
0	0	0	0	0	0	-0.3017	0	0.0065	0	0	0
0	0	0	0	0	0	0	0.0111	0	0	0	0
0	0	0	0	0	0	-0.0065	0	0.0001	0	0	0
0	0	0	0	0	0	0	0	0	-0.3017	0	0.0065
0	0	0	0	0	0	0	0	0	0	0.0111	0
0	0	0	0	0	0	0	0	0	-0.0065	0	0.0001
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

Columns 13 through 24

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
-0.3017	0	-0.0065	0	0	0	0	0	0	0	0	0
0	6.6377	0	0	0	0	0	0	0	0	0	0
0.0065	0	0.0001	0	0	0	0	0	0	0	0	0
0	0	0	-0.3017	0	-0.0065	0	0	0	0	0	0
0	0	0	0	6.6377	0	0	0	0	0	0	0
0	0	0	0.0065	0	0.0001	0	0	0	0	0	0
6.6377	0	0	0	0	0	-0.3017	0	-0.0065	0	0	0
0	-0.3017	0.0065	0	0	0	0	6.6377	0	0	0	0
0	-0.0065	0.0001	0	0	0	0.0065	0	0.0001	0	0	0
0.9102	0	0	6.6377	0	0	0	0	0	-0.3017	0	-0.0065
0	0.9102	0	0	-0.3017	0.0065	0	0	0	0	6.6377	0
0	0	0.0016	0	-0.0065	0.0001	0	0	0	0.0065	0	0.0001
0.0111	0	0	0.9102	0	0	0	0	0	0	0	0
0	-0.3017	-0.0065	0	0.9102	0	0	0	0	0	0	0
0	0.0065	0.0001	0	0	0.0016	0	0	0	0	0	0
0	0	0	0	0	0	0.9102	0	0	6.6377	0	0
0	0	0	0	0	0	0	0.9102	0.0221	0	-0.3017	0.0065
0	0	0	0	0	0	0	0.0221	0.0016	0	-0.0065	0.0001
-0.3017	0	0.0065	0	0	0	0.0111	0	0	0.9102	0	0
0	0.0111	0	0	0	0	0	-0.3017	-0.0065	0	0.9102	0
-0.0065	0	0.0001	0	0	0	0	0.0065	0.0001	0	0	0.0016
0	0	0	-0.3017	0	0.0065	0	0	0	0.0111	0	0

0	0	0	0	0.0111	0	0	0	0	0	-0.3017	-0.0065
0	0	0	-0.0065	0	0.0001	0	0	0	0	0.0065	0.0001

Columns 25 through 27

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
-0.3017	0	-0.0065
0	6.6377	0
0.0065	0	0.0001
0	0	0
0	0	0
0	0	0
6.6377	0	0
0	-0.3017	0.0065
0	-0.0065	0.0001
0.9102	0	0
0	0.9102	0

0 0 0.0016

EIGEN VALUES OF K AND M:

1.0e+009 *

1.5352 + 0.2732i

1.5352 - 0.2732i

-0.5418 + 0.6187i

-0.5418 - 0.6187i

0.8307

0.4822

0.1864 + 0.1049i

0.1864 - 0.1049i

0.1434

0.1318

0.0981

0.0797

0.0075

0.0075

0.0070

0.0068

0.0062

0.0063

0.0000

0.0000

0.0000

0.0000

0.0000

0.0000

0.0000

0.0000

0.0000

NATURAL FREQUENCIES OF EACH NODE

1.0e+004 *

3.9335 + 0.3472i

3.9335 - 0.3472i

1.1845 + 2.6117i

1.1845 - 2.6117i

2.8822

2.1960

1.4148 + 0.3707i

1.4148 - 0.3707i

1.1975

1.1481

0.9905

0.8929

0.2746

0.2738

0.2648

0.2602

0.2484

0.2503

0.0001

0.0001

0.0001

0.0000

0.0001

0.0001

0.0001

0.0001

0.0001

DISCUSSION:

The natural frequencies of each node of the grid is computed though the MATLAB programme . The Eigen values of the Stiffness matrix and the mass matrix gives the Natural frequencies and the Eigen vector gives the modal nodes .The lowest of the natural frequency is taken into consideration which is 10 rad/sec. If any dynamic loading is given to the grid system , it should be less than the lowest natural frequency to avoid Resonance.

CONCLUSION:

Thus from the analysis it is concluded that the direct stiffness method is suitable method for analysing structures statically or dynamically. The rectangular grid beam system being intersected in right angles also experiences torsional moment at the nodes. The natural frequencies of the nodes are calculated in the dynamic analysis give a range up to which the dynamic loading should be restricted. If the frequency of the of the load matches the natural frequency then resonance will occur which will increase the damage to the building.

APPENDIX 1

MATLAB PROGRAMME (ALGORITHMS)

1) The coordinates of all 24 nodes are defined.
2) The number of members, free nodes and fixed nodes are assigned.
3) The connection of members in between is described so that the DOFS of each members can be sequentially assigned to the nodes.
4) The length and angle of orientation(α) is calculated of all the members are computed so as to determine the global coordinates of the nodes.
5) The characteristic constants of the structure calculated above such asare given as inputs the Area (A),Modulus of elasticity (E),Moment of inertia(I),Polar moment of inertia (J),Polar mass moment of inertia(Im),m mass per unit length.
6) For each element the Stiffness matrix is assigned into a 6X6 matrix in local coordinates.
7) Then each of them are transposed to the Global coordinates according to their orientation.
8) The individual Stiffness matrixes are combined to form the Global stiffness matrix of the total structure, according to their degree of freedoms.
9) The degree of freedoms of the fixed nodes is assigned as 0, where as the free nodes have each 3 DOFS.
10) The components of common DOFS are added in the Global stiffness matrix.
11) Accordingly the mass matrix is also calculated for each member and combined as global matrix.
12) The Eigen value and Eigen vectors of K and M are calculated. The Eigen values depicts the frequency of the natural vibration of the nodes and the Eigen vectors will give the mode shape of the displacements

REFERENCES:

1) Structural dynamics- theory and computation by Mario paz, 4th edition.
2) Intermediate structural analysis , c.k.wang.
3) **MATLAB Codes for Finite Element Analysis** Solids and Structures, Vol. 157

   Ferreira, A.J.M