A BRIEF STUDY ON VARIOUS TIME DOMAIN VISCOELASTICITY MODEL.

A PROJECT REPORT SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

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In

Mechanical Engineering

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CERTIFICATE

This is to certify that the Project Report entitled, "**Various time domain viscoelasticity model and their uses**" submitted by **Nitesh Kumar** for partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at the **National Institute of Technology, Rourkela** (Deemed University) is an authentic work carried out by him under my supervision and guidance.

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CHAPTER 01

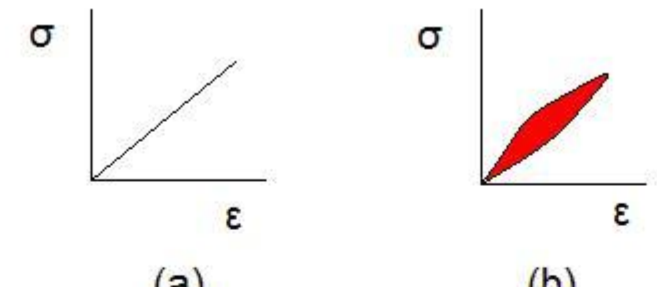
1.1 Introduction

A viscoelastic material stores energy and dissipates it in the thermal domain when subjected to dynamic loading and most interestingly the storage and loss of energy depend upon the frequency of excitation. To find a time domain model to represent the behaviour viscoelastic solids found interest of many researchers. Bagley and Torvik (1983, 1985) represented frequency dependent behaviour of viscoelastic solids by using four-model parameters and differential operators of fractional order. The time-domain model with ordinary integer differential operators was developed by Golla and Hughes (1985), who incorporated the hereditary integral form of the viscoelastic constitutive law in a finite element model. The finite-element equations are derived in the Laplace domain through the Ritz technique. McTavish and Hughes (1992, 1993) extended the Golla-Hughes model and formulated the GHM (Golla-Hughes-McTavish) model for linear viscoelastic structures. In this formulation, the material modulus is modelled as a collection of mini-oscillators. Lesieutre(1989) developed another time-domain model, using 'Augmenting Thermodynamic Fields (ATF)'approach to model frequency-dependent material-damping of linear viscoelastic structures in a finite element context. The procedure introduces a thermal coordinate to take into account the dissipation.Lesieutre and Mingori (1990) and later Lesieutre (1992) developed a one-dimensional formulation of the ATF model. In a recent paper, Roy et al. (2008) used the ATF approach to model a viscoelastic continuum of a rotor shaft, to obtain the equations of motion and studied the dynamic behaviour in terms of stability limit speed and unbalance response. The 'Anelastic Displacement Field' ADF, (Lesieutre and Bianchini (1995), Lesieutre et al. (1996)) approach was developed to extend the ATF method to three-dimensional states. In ADF approach, the displacement field was composed of an elastic component and an anelastic component, where the anelastic field is introduced to take in to account the dissipation. All the approaches, ATF, ADF and GHM thus employ additional co-ordinates to model damping more accurately. Whereas the 'dissipation coordinate' of GHM is internal to individual elements, it is continuous for ATF and ADF approaches from element to element. For this advantage ATF and ADF approaches are used in this work to represent the viscoelastic material behaviour.Viscoelastic parameters used in these approaches are determined initially from the storage modulus and loss factor reported by Lazan (1968). For this a Genetic Algorithm based approach is adopted to minimize the error between the hysteresis loops predicted by ATF, ADF approaches each and the same predicted by the complex modulus, for a viscoelastic solid bar under sinusoidally varying axial load. Subsequently the extracted ATF and ADF parameters are used to represent the constitutive equations and obtain the equations of motion for the continuum of a generally multilayered viscoelastic beam, discretized using finite beam elements. These equations of motion are studied for the dynamics of the viscoelastic composite beam.For an example the dynamics of a cantilevered composite beam made of two concentric layers of Steel and Aluminium is studied, where behaviours of both steel and aluminium are represented by viscoelastic models.

1.2 Brief Discussion on Damping

1.2.1 Elastic Behavior versus Viscoelastic Behavior

Stress-Strain Curves for a purely elastic material (a) and a viscoelastic material (b). The red area is a hysteresis loop and shows the amount of energy lost (as heat) in a loading and unloading cycle. It is equal to $\oint \sigma \, d\varepsilon$, where σ is stress and ε is strain. Unlike purely elastic substances, a viscoelastic substance has an elastic component and a viscous component. The viscosity of a viscoelastic substance gives the substance a strain rate dependent on time. Purely elastic materials do not dissipate energy (heat) when a load is applied, then removed. However, a viscoelastic substance loses energy when a load is applied, then removed. Hysteresis is observed in the stress-strain curve, with the area of the loop being equal to the energy lost during the loading cycle. Since viscosity is the resistance to thermally activated plastic deformation, a viscous material will lose energy through a loading cycle. Plastic deformation results in lost energy, which is uncharacteristic of a purely elastic material's reaction to a loading cycle Specifically, viscoelasticity is a molecular rearrangement. When a stress is applied to a viscoelastic material such as a polymer, parts of the long polymer chain change position. This movement or rearrangement is called Creep. Polymers remain a solid material even when these parts of their chains are rearranging in order to accompany the stress, and as this occurs, it creates a back stress in the material. When the back stress is the same magnitude as the applied stress, the material no longer creeps. When the original stress is taken away, the accumulated back stresses will cause the polymer to return to its original form. The material creeps, which gives the prefix visco-, and the material fully recovers, which gives the suffix –elasticity.



(a) (b)

1.2.2 Types of Viscoelasticity

Linear viscoelasticity is when the function is separable in both creep response and load. All linear viscoelastic models can be represented by a Volterra equation connecting stress and strain:

$$\epsilon(t) = \frac{\sigma(t)}{E_{\text{inst,creep}}} + \int_0^t K(t-t')\dot{\sigma}(t')dt'$$

or

$$\sigma(t) = E_{\text{inst,relax}}\epsilon(t) + \int_0^t F(t-t')\dot{\epsilon}(t')dt'$$

Where

- t is time
- $\sigma(t)$ is stress
- $\varepsilon(t)$ is strain
- $E_{\text{inst,creep}}$ and $E_{\text{inst,relax}}$ are instantaneous elastic moduli for creep and relaxation
- $K(t)$ is the creep function
- $F(t)$ is the relaxation function

Linear viscoelasticity is usually applicable only for small deformations.

Nonlinear viscoelasticity is when the function is not separable. It is usually happens when the deformations are large or if the material changes its properties under deformations.

An anelastic material is a special case of a viscoelastic material: an anelastic material will fully recover to its original state on the removal of load.

1.2.3 Linear Viscoelasticity

Viscoelastic materials, such as amorphous polymers, semicrystalline polymers, and biopolymers, can be modeled in order to determine their stress or strain interactions as well as their temporal dependencies. These models, which include the Maxwell model, the Kelvin-Voigt model, and the Standard Linear Solid Model, are used to predict a material's response under different loading conditions. Viscoelastic behavior has elastic and viscous components modeled as linear combinations of springs and dashpots, respectively. Each model differs in the arrangement of these elements, and all of these viscoelastic models can be equivalently modeled as electrical circuits. In an equivalent electrical circuit, stress is represented by voltage, and the derivative of strain (velocity) by current. The elastic modulus of a spring is analogous to a circuit's capacitance (it stores energy) and the viscosity of a dashpot to a circuit's resistance (it dissipates energy).

The elastic components, as previously mentioned, can be modeled as springs of elastic constant E, given the formula:

$$\sigma = E\varepsilon$$

Where σ is the stress, E is the elastic modulus of the material, and ε is the strain that occurs under the given stress, similar to Hooke's Law.

The viscous components can be modeled as dashpots such that the stress-strain rate relationship can be given as,

$$\sigma = \eta \frac{d\varepsilon}{dt}$$

Where σ is the stress, η is the viscosity of the material, and $d\varepsilon/dt$ is the time derivative of strain.

The relationship between stress and strain can be simplified for specific stress rates. For high stress states/short time periods, the time derivative components of the stress-strain relationship dominate. A dashpots resists changes in length, and in a high stress state it can be approximated as a rigid rod. Since a rigid rod cannot be stretched past its original length, no strain is added to the system

1.2.4 Effect of Temperature on Viscoelastic Behavior

Application of a stress favors some conformations over others, so the molecules of the polymer will gradually "flow" into the favored conformations over time. Because thermal motion is one factor contributing to the deformation of polymers, viscoelastic properties change with increasing or decreasing temperature. In most cases, the creep modulus, defined as the ratio of applied stress to the time-dependent strain, decreases with increasing temperature. Generally speaking, an increase in temperature correlates to a logarithmic decrease in the time required to impart equal strain under a constant stress. In other words, it takes less work to stretch a viscoelastic material an equal distance at a higher temperature than it does at a lower temperature.

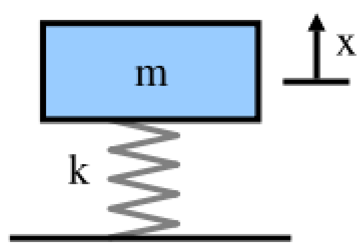
1.2.5 Measuring Viscoelasticity

Though there are many instruments that test the mechanical and viscoelastic response of materials, broadband viscoelastic spectroscopy (BVS) and resonant ultrasound spectroscopy (RUS) are more commonly used to test viscoelastic behavior because they can be used above and below ambient temperatures and are more specific to testing viscoelasticity. These two instruments employ a damping mechanism at various frequencies and time ranges with no appeal to time-temperature superposition. Using BVS and RUS to study the mechanical properties of materials is important to understanding how a material exhibiting viscoelasticity will perform.

1.2.6 Effect of Temperature on Viscoelastic Behavior

The secondary bonds of a polymer constantly break and reform due to thermal motion. Application of a stress favors some conformations over others, so the molecules of the polymer will gradually "flow" into the favored conformations over time. Because thermal motion is one factor contributing to the deformation of polymers, viscoelastic properties change with increasing or decreasing temperature. In most cases, the creep modulus, defined as the ratio of applied stress to the time-dependent strain, decreases with increasing temperature. Generally speaking, an increase in temperature correlates to a logarithmic decrease in the time required to impart equal strain under a constant stress. In other words, it takes less work to stretch a viscoelastic material an equal distance at a higher temperature than it does at a lower temperature.

1.2.7 Free vibration without damping



To start the investigation of the mass–spring–damper we will assume the damping is negligible and that there is no external force applied to the mass (i.e. free vibration).

The force applied to the mass by the spring is proportional to the amount the spring is stretched "x" (we will assume the spring is already compressed due to the weight of the mass). The proportionality constant, k, is the stiffness of the spring and has units of force/distance (e.g. lbf/in or N/m)

$$F_s = -kx.$$

The force generated by the mass is proportional to the acceleration of the mass as given by Newton's second law of motion.

$$\Sigma\ F = ma = m\ddot{x} = m\frac{d^2x}{dt^2}.$$

The sum of the forces on the mass then generates this ordinary differential equation:

$$m\ddot{x} + kx = 0.$$

If we assume that we start the system to vibrate by stretching the spring by the distance of *A* and letting go, the solution to the above equation that describes the motion of mass is:

$$x(t) = A\cos(2\pi f_n t).$$

This solution says that it will oscillate with simple harmonic motion that has an amplitude of *A* and a frequency of f_n. The number f_n is one of the most important quantities in vibration analysis and is called the **undamped natural frequency**. For the simple mass–spring system, f_n is defined as:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

Note: Angular frequency ω ($\omega = 2\pi f$) with the units of radians per second is often used in equations because it simplifies the equations, but is normally converted to "standard" frequency (units of Hz or equivalently cycles per second) when stating the frequency of a system.

If you know the mass and stiffness of the system you can determine the frequency at which the system will vibrate once it is set in motion by an initial disturbance using the above stated formula. Every vibrating system has one or more natural frequencies that it will vibrate at once it is disturbed. This simple relation can be used to understand in general what will happen to a more complex system once we add mass or stiffness. For example, the above formula explains why when a car or truck is fully loaded the suspension will feel "softer" than unloaded because the mass has increased and therefore reduced the natural frequency of the system.

1.2.8 Forced vibration with damping

In this section we will see the behavior of the spring mass damper model when we add a harmonic force in the form below. A force of this type could, for example, be generated by a rotating imbalance.

$$F = F_0 \cos(2\pi f t).$$

If we again sum the forces on the mass we get the following ordinary differential equation:

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos(2\pi f t).$$

The steady state solution of this problem can be written as:

$$x(t) = X\cos(2\pi ft - \phi).$$

The result states that the mass will oscillate at the same frequency, f, of the applied force, but with a phase shift φ.

The amplitude of the vibration "X" is defined by the following formula.

$$X = \frac{F_0}{k}\frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}.$$

Where "r" is defined as the ratio of the harmonic force frequency over the undamped natural frequency of the mass–spring–damper model.

$$r = \frac{f}{f_n}.$$

The phase shift, φ, is defined by the following formula.

$$\phi = \arctan\left(\frac{2\zeta r}{1-r^2}\right).$$

1.2.9 Application of Viscoelasticity

- In damping vibration, shock (Eg in vehicles).
- In reducing the vibration in the machines.
- In hydraulic servo Moto

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