Axisymmetric Stress Analysis of Internally Pressurized Rotating Cylinder using Finite Element Method

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Of

BACHELOR OF TECHNOLOGY
IN
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CERTIFICATE

This is to certify that the thesis entitled, "Axisymmetric stress analysis of internally pressurized rotating cylinder using finite element method" submitted by Sri Bhagat Meghraj Vitthal in partial fulfillments for the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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Abstract

The present paper is devoted to stress analysis of a homogenous, orthotropic, internally pressurized rotating cylinder. Assuming the cylinder in plain strain condition and that the volume remains constant, finite element method is used to find out the stresses and displacement at each node of isoparametric elements (Bilinear and Quadratic). FEM results are then compared with the exact values, comparison is also done between different element numbers (for same element type) and between different element types (for same element number).

The reason for doing the comparison is to find out by how much the FEM results vary from the exact solution and to see how the FEM results converge to exact values by increasing the element number. Comparison also gives the brief idea about the two elements i.e which of the two elements gives better results for the same element number.

Contents

Introduction

The research on the determination of stresses and strains in a rotating thick hollow cylinder has never stopped because of their vast importance in the different fields of engineering (mechanical, electrical, civil, computer engineering, etc). Many standard and advanced textbooks like Ress, Timoshenko, Goodier and Uugural and Festes have pane strain and plane stress solutions in them for many years. Finite element is another important method of finding out the stresses and strains of any complex bodies for whom the exact solution has not been derived. Finite element method has made it possible to design any complex bodies.

In this paper an orthotropic rotating cylinder subjected to internal pressure is consider. Exact solution for stress and displacement is obtained by keeping the homogeneity constant in the derivation derived by Najed and Rahimi for an FGM rotating cylinder subjected to an internal pressure , finite element method is used to find the displacement and stresses at each nodes by using matlab. Here, two iso-parametric (Bilinear and Quadrilateral Quadratic elements) elements are used for solving the problem. Results obtained by FEM are then compared with the exact values and the comparison are discussed.

Literature Review

Stress analysis of cylinder vessels is important in the field of engineering. Cylindrical vessels have extensive use in power generating machines, chemical industries and oil refineries. Stress analysis of thick walled cylinder subjected to various types of axisymmetric loading has been carried out by many investigators for constant or varying material properties along the radius. The investigation done by [4]Nejad and Rahimi [2010] shows the stress variation along the radial direction of rotating FGM cylinder subjected to internal pressure.[5] H.R.Eipakchi [2008] solved the problem of thick walled conical shells with varying thickness subjected to varying pressure by second order shear deformation theory where in the calculated displacement and stresses are compared with the finite element method solution and the first order shear deformation theory. [3]George F. Hausenbauer [1966] derived the formula for finding out the stresses in thick walled conical shells. [9]S.A Tavares [1995] also derived the solution for finding out the displacement and stresses in a thick walled conical vessel subjected to various types of axisymmetric loading. Research in creep analysis of axisymmetric bodies has done by many of the investigators. [10]Taira and Wahl revealed that the creep occurs along the preferred orientations making initially isotropic material anisotropic. Notable contribution to anisotropy on creep behaviour of thick walled cylinder has been made by [8]Bhatnagar, Arya, pai, etc.

Chapter 1: Axisymmetric Solids (Structure of Revolution)

1.1 Axisymmetric Problems

The axisymmetric problem deals with the analysis of structures of revolution under axisymmetric loading. A structure of revolution (SOR) is obtained by a generating cross section which rotates 360 degrees about an axis of revolution, as shown in figure below. Such structures are said to be rotationally symmetric.

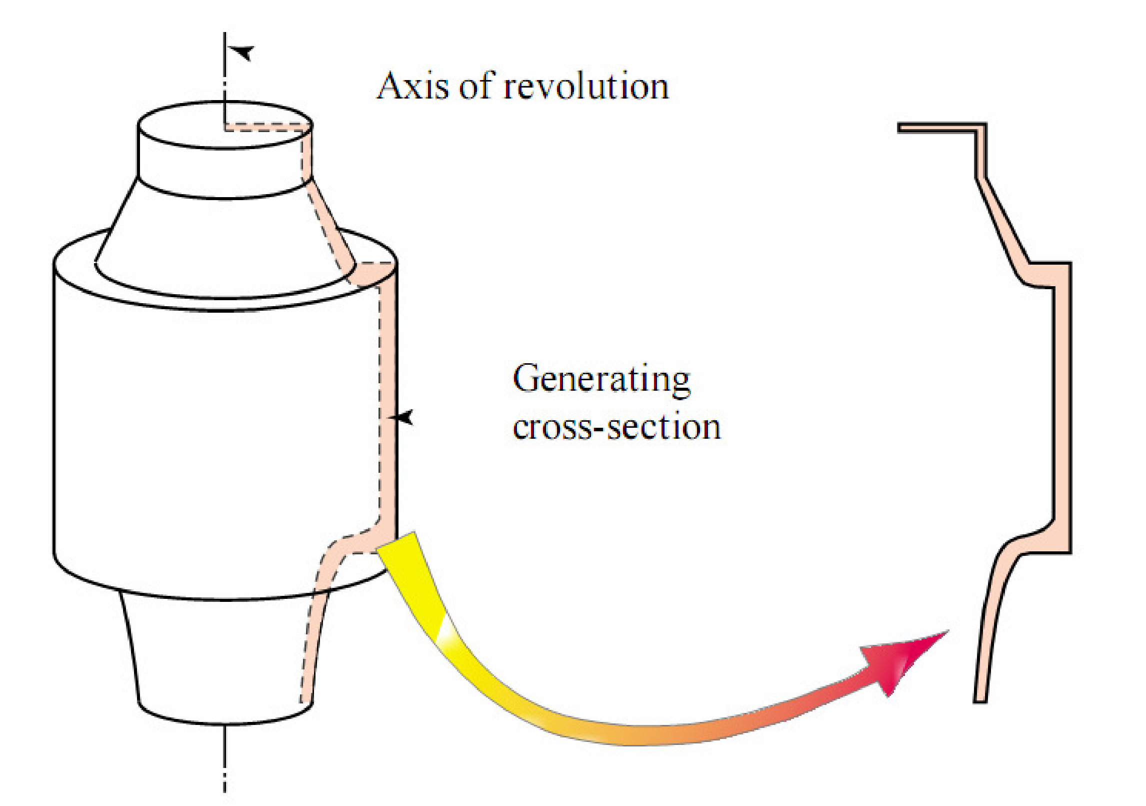

Figure 1: A SOR is generated by rotating a generating cross-section about an axis of revolution

The technical importance of Structure of revolution's is considerable because of the following practical considerations:

1. Fabrication : axisymmetric bodies are usually easier to manufacture compared to the bodies with more complex geometries. Eg pipes, piles, axles, wheels, bottles, cans, cups, nails, etc.
2. Strength : axisymmetric configuration are often more desireable in terms of strength to weight ratio because of the favorable distribution of the material.
3. Multipurpose : hollow axisymmetric can assume a dual purpose of both structure as well as shelter, as in a containers, vessels, tanks, rockets, etc.

Perhaps the most important application of Stucture of Revolution is storage and transport of liquid and gases. Examples of such structures are pressure vessels, pipes, containment vessels and rotating machinery (turbines, generators, shafts, etc.)

But a Structure of revolution (SOR) by itself does not necessarily define an axisymmetric problem. It is also necessary that the loading, as well as the support boundary conditions, be rotationally symmetric. This is shown in figure below for loads

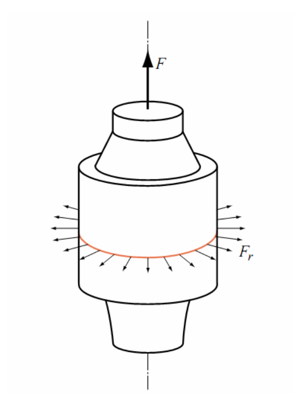

Figure 2 : Axisymmetric loading on a SOR: F = concentrated load, Fr = radial component of "ring" line load.

If these two structures are met

axisymmetric geometry and *axisymmetric loading*

The response of the structure is axisymmetric (also called radially symmetric). By this is meant that all quantities of interest in structural analysis: displacement, strains, and stress, are independent of the circumferential coordinate.

1.2 The Governing Equations

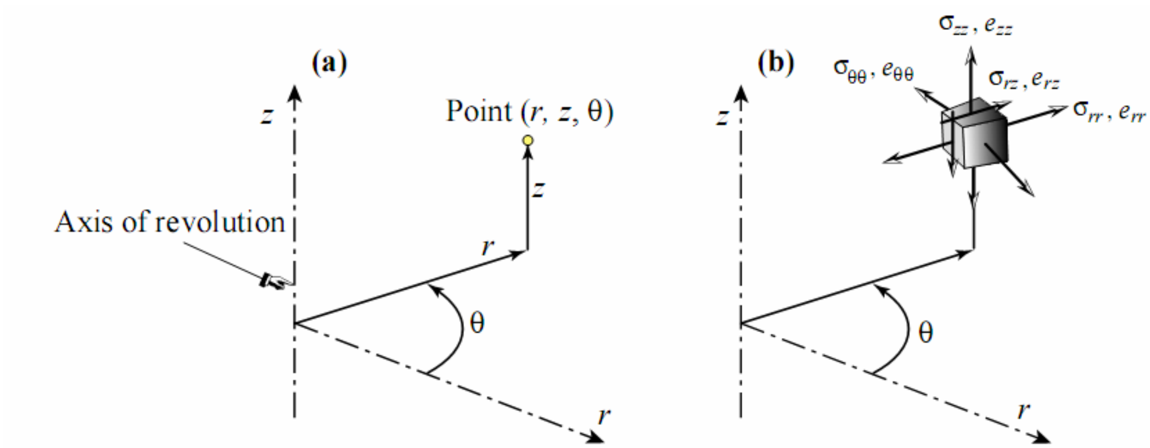

Figure 3.

1.2.1 Global Coordinate System

A global coordinate system is used to simplify the governing equations of the axisymmetric problem

r the radial coordinate: distance from the axis of revolution; always $r \succ 0$.

z the axial coordinate: directed along the axis of revolution.

θ the circumferential coordinate, also called the longitude.

The global coordinate system is sketched in figure 3.

1.2.2 Displacement, Strains, Stresses

The displacement field is a function of r and z only, defined by two components

$$\mathbf{u}(r,z) = \begin{bmatrix} u_r(r,z) \\ u_z(r,z) \end{bmatrix} \tag{1.1}$$

u_r is the radial displacement and uz is called the axial displacement. u_o,

circumferential displacement is zero on account of rotational symmetry.

The strain tensor in cylindrical coordinate is represented by the symmetric matrix:

$$[\,\mathbf{e}\,] = \begin{bmatrix} e_{rr} & e_{rz} & e_{r\theta} \\ e_{rz} & e_{zz} & e_{z\theta} \\ e_{z\theta} & e_{z\theta} & e_{\theta\theta} \end{bmatrix} \tag{1.2}$$

Due to assumed axisymmetric state , e and e vanish, leaving behind only four distinct components:

$$[\,\mathbf{e}\,] = \begin{bmatrix} e_{rr} & e_{rz} & 0 \\ e_{rz} & e_{zz} & 0 \\ 0 & 0 & e_{\theta\theta} \end{bmatrix} \tag{1.3}$$

Each of the vanishing components is a function of r and z only, the vanishing components are arranged as 4*1 strain vector:

$$\mathbf{e} = \begin{bmatrix} e_{rr} \\ e_{zz} \\ e_{\theta\theta} \\ \gamma_{rz} \end{bmatrix} \tag{1.4}$$

where $\gamma_{rz} = e_{rz} + e_{zr} = 2e_{rz}$

The stress tensor in cylindrical coordinates is a symmetric matrix

$$[\sigma] = \begin{bmatrix} \sigma_{rr} & \sigma_{rz} & \sigma_{r\theta} \\ \sigma_{rz} & \sigma_{zz} & \sigma_{z\theta} \\ \sigma_{r\theta} & \sigma_{z\theta} & \sigma_{\theta\theta} \end{bmatrix} \tag{1.5}$$

Due to axisymmetry, components $\sigma_{r\theta} and \sigma_{z\theta}$ vanishes. Thus,

$$[\sigma] = \begin{bmatrix} \sigma_{rr} & \sigma_{rz} & 0 \\ \sigma_{rz} & \sigma_{zz} & 0 \\ 0 & 0 & \sigma_{\theta\theta} \end{bmatrix} \tag{1.6}$$

Each nonvanishing components is a function of r and z. Stress vector becomes

$$\sigma = \begin{bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{\theta\theta} \\ \sigma_{rz} \end{bmatrix} \tag{1.7}$$

where $\sigma_{rz} = \sigma_{zr}$.

Chapter 2: Formulation of Exact Solution

The stress distribution in a FGM rotating thick-walled cylinder pressure vessel in plane stress and plane strain condition is given by Gholam Hosein Rahimi and Mohammad Zamani Nejad (2010) . Here in the problem the thick cylinder is homogenous in composition, so all its properties like modulus of elasticity, poisson's ratio, density and yield limit is constant throughout the material. Following is the derivation for radial displacement, radial stress and hoop stress for a rotating thick – walled cylinder pressure vessel by keeping the homogeneity constant in the derivation obtained by them.

A thick walled cylinder of inner radius a, and an outer radius b, subjected to internal pressure P which is axisymmetric, and rotating at a constant angular velocity w about it axis. Neglecting the body force component, the equilibrium equation is reduced to a single equation

$$\sigma'_{rr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = -\rho r \omega^2 \qquad (2.1)$$

Where σ_{rr} and $\sigma_{\theta\theta}$ are the radial and circumferential stress component, ρ is the density of the material and (') is differential with respect to r.

The radial strain ε_{rr} and circumferential strain $\varepsilon_{\theta\theta}$ are related to radial displacement u_r by

$$\varepsilon_{rr} = u_r^{'},$$

$$\varepsilon_{\theta\theta} = \frac{u_r}{r}$$

The stress strain relationship for isotropic material is

(2.2)

$$\sigma_{rr} = (C_{11}\varepsilon_{rr} = C_{12}\varepsilon_{\theta\theta})E,$$
$$\sigma_{\theta\theta} = (C_{11}\varepsilon_{\theta\theta} = C_{12}\varepsilon_{rr})E,$$

E is modulus of elasticity, C_{11} and C_{12} are related to Poisson's ratio ν

Plane strain condition:

$$C_{11} = \frac{1-\nu}{(1-\nu)(1-2\nu)}, C_{12} = \frac{\nu}{(1-\nu)(1-2\nu)} \tag{2.3}$$

Plane stress condition:

$$C_{11} = \frac{1}{1-\nu^2}, C_{12} = \frac{\nu}{1-\nu^2} \tag{2.4}$$

Using eq 1,3 the navier stroke in terms of radial displacement is

$$u_r'' + (\frac{1}{E}\frac{dE}{dr} + \frac{1}{r})u_r' + (\frac{n}{Er}\frac{dE}{dr} - \frac{1}{r^2})u_r = -\frac{\rho\omega^2}{C_{11}E}r, \tag{2.5}$$

Where

$$n = \frac{C_{11}}{C_{12}}$$

Plane strain condition :

$$n = {}^{\nu}\!/_{1-\nu}$$

Plane stress condition :

$$n = \nu$$

By substituting eq , the navier stroke becomes

$$r^2 u''_r + r u'_r - u_r = -\frac{\rho\omega^2}{EC_{11}} r^3 \tag{2.6}$$

Eq (12) is a non-homogenous Euler-Caushy equation whose solution is

$$u_r = A_1 r^{m_1} + A_2 r^{m_2} + A_3 r^{m_3}, \tag{2.7}$$

where

$$m_1 = -1, m_2 = 1, m_3 = 5$$

$$A_3 = -\frac{\rho a \omega^2}{24 C_{11} E}$$

The radial stress is obtained by substituting the eq

$$\sigma_{rr} = E[A_1(-C_{11} + C_{12}) r^{-2} + A_2(C_{11} + C_{12}) + A_3(5C_{11} + C_{12}) r^4] \tag{2.8}$$

Constants A_1 and A_2 are obtained by applying boundary conditions to the stresses given

by

$$\sigma_{rr|_{r=a}} = -P, \sigma_{rr|_{r=b}} = 0,$$

where P is the internal pressure, substituting the boundary conditions the obtained values

of A_1 and A_2 are

$$A_1 = \frac{P + A_3 E a^4 (5C_{11} + C_{12})(1 - k^4)}{E a^{-2} (-C_{11} + C_{12})(k^{-2} - 1)}$$

$$A_2 = \frac{P + A_3 E a^4 (5C_{11} + C_{12})(1 - k^6)}{E (C_{11} + C_{12})(k^2 - 1)}$$

where $k = b/a$

Thus, the radial stress, circumferential stress and radial displacement are

$$\sigma_{rr} = \frac{P + A(1-k^4)}{k^{-2}-1}(r/a)^{-2} + \frac{P + A(1-k^6)}{k^2-1} + A(r/a)^4 \qquad (2.9)$$

$$\sigma_{\theta\theta} = B_1 \frac{P + A(1-k^4)}{k^{-2}-1}(r/a)^{-2} + B_2 \frac{P + A(1-k^6)}{k^2-1} + B_3 A(r/a)^4 \qquad (2.10)$$

$$u_{rr} = \frac{[P + A(1-k^4)]a}{ED_1(k^{-2}-1)}(r/a)^{-1} + \frac{[P + A(1-k^6)]a}{ED_2(k^2-1)}(r/a) + \frac{Aa(r/a)^4}{ED_3} \qquad (2.11)$$

For plain strain condition

$$A = -\frac{\rho\omega^2 a^4}{24}[5 + \frac{\nu}{1-\nu}]$$

$$B_1 = \frac{-2\nu + 1}{2\nu - 1}$$

$$B_2 = 1$$

$$B_3 = \frac{4\nu + 1}{-4\nu + 5}$$

$$D_1 = \frac{2\nu - 1}{\nu(1+\nu)(1-2\nu)}$$

$$D_2 = \frac{1}{(1+\nu)(1-2\nu)}$$

$$D_3 = \frac{-4\nu + 5}{(1+\nu)(1-2\nu)}$$

For plane stress condition

$$A = -\frac{\rho\omega^2 a^4}{24}[5+\nu]$$

$$B_1 = \frac{-\nu+1}{\nu-1}$$

$$B_2 = 1$$

$$B_3 = \frac{5\nu+1}{\nu+5}$$

$$D_1 = \frac{-1+\nu}{1-\nu^2}$$

$$D_2 = \frac{1+\nu}{1-\nu^2}$$

$$D_3 = \frac{5+\nu}{1-\nu^2}$$

Chapter 3: ISO parametric elements

3.1 Bilinear Quadrilateral Element

Each bilinear element has four nodes with two in plane degrees of freedom at each node shown in the figure 4.

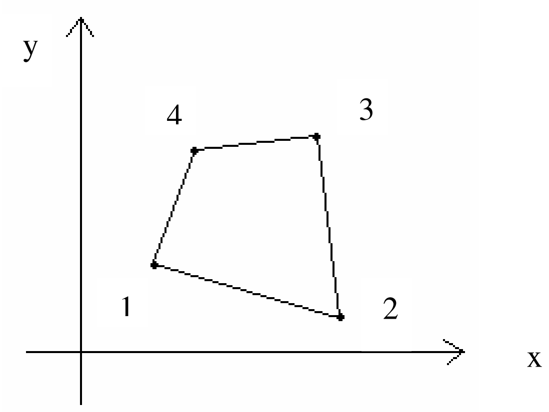

Figure 4. Bilinear Quadrilateral Element

The element is mapped to a rectangle through the use of the natural coordinates ξ and η as shown below

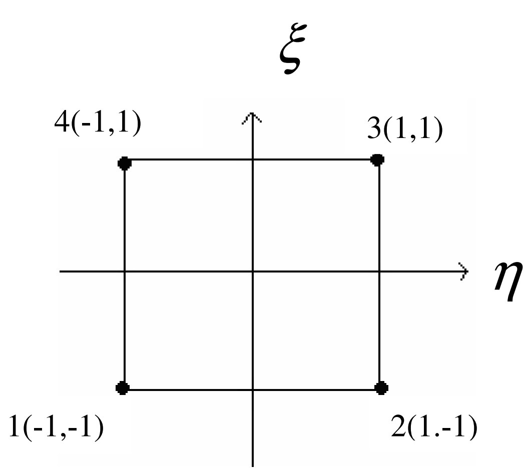

Figure 5. Bilinear Quadrilater Elment with natural coordinates

The shape functions are :

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$
$$N_2 = \frac{1}{4}(1+\xi)(1-\eta) \qquad (3.1)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$
$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

For this element the matrix [B] is given by:

$$[B] = \frac{1}{|J|}\begin{bmatrix} B_1 & B_2 & B_3 & B_4 \end{bmatrix} \qquad (3.2)$$

Where $[B_i]$ is given by:

$$[B_i] = \begin{bmatrix} a\frac{\partial N_i}{\partial \xi} - b\frac{\partial N_i}{\partial \eta} & 0 \\ 0 & c\frac{\partial N_i}{\partial \eta} - d\frac{\partial N_i}{\partial \xi} \\ c\frac{\partial N_i}{\partial \eta} - d\frac{\partial N_i}{\partial \xi} & a\frac{\partial N_i}{\partial \xi} - b\frac{\partial N_i}{\partial \eta} \end{bmatrix} \qquad (3.3)$$

Parameters a, b, c and d are given by:

$$a = \frac{1}{4}\left[y_1(\xi-1) + y_2(-1-\xi) + y_3(1+\xi) + y_4(1-\xi) \right]$$
$$b = \frac{1}{4}\left[y_1(\eta-1) + y_2(1-\eta) + y_3(1+\eta) + y_4(-1-\eta) \right]$$
$$c = \frac{1}{4}\left[x_1(\eta-1) + x_2(1-\eta) + x_3(1+\eta) + x_4(-1-\eta) \right] \qquad (3.4)$$
$$d = \frac{1}{4}\left[x_1(\xi-1) + x_2(-1-\xi) + x_3(1+\xi) + x_4(1-\xi) \right]$$

The determinant $\left|J\right|$ is given by:

$$[J]=\frac{1}{8}[x_1 \quad x_2 \quad x_3 \quad x_4]\begin{bmatrix} 0 & 1-\eta & \eta-\xi & \xi-1 \\ \eta-1 & 0 & 1+\xi & -\eta-\xi \\ \xi-\eta & -1-\xi & 0 & 1+\eta \\ 1-\xi & \eta+\xi & -1-\eta & 0 \end{bmatrix} \tag{3.5}$$

For plane stress case matrix [D] is

$$[D]=\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{3.6}$$

For the case of plane strain the matrix [D] is given by

$$[D]=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \tag{3.7}$$

The element stiffness matrix for the bilinear quadrilateral element is written in terms of double integration as follows

$$[k]=t\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\left|J\right|\partial\xi\partial\eta \tag{3.8}$$

3.2 Quadratic Quadrilateral Element

Each quadratic element has eight nodes with two in plane degrees of freedom at each node shown in the figure.

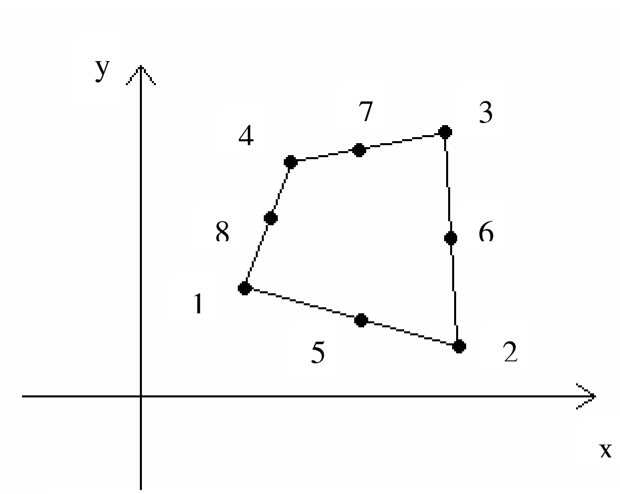

Figure 6: Quadratic Quadrilateral Element

The element is mapped to a rectangle through the use of the natural coordinates ξ and η as shown below

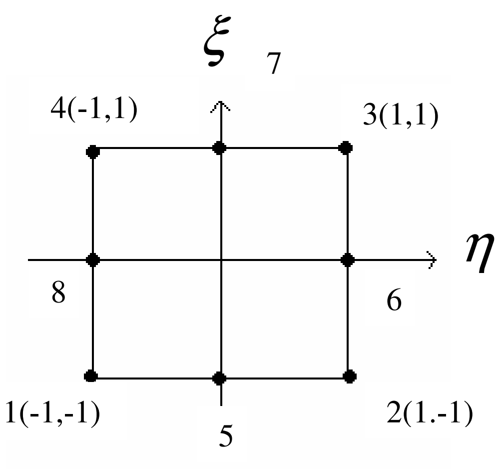

Figure 7: Quadratic Quadrilateral Element with natural coordinates

The shape functions are :

$$
\begin{aligned}
N_1 &= \frac{1}{4}(1-\xi)(1-\eta)(-\xi-\eta-1) \\
N_2 &= \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1) \\
N_3 &= \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1) \\
N_4 &= \frac{1}{4}(1-\xi)(1+\eta)(-\xi+\eta-1) \\
N_5 &= \frac{1}{4}(1-\eta)(1+\xi)(1-\xi) \\
N_6 &= \frac{1}{4}(1+\eta)(1+\xi)(1-\eta) \\
N_7 &= \frac{1}{4}(1+\eta)(1+\xi)(1-\xi) \\
N_8 &= \frac{1}{4}(1-\xi)(1+\eta)(1-\eta)
\end{aligned}
\tag{3.10}
$$

The Jacobian Matrix for this element is given by

$$
[J] = \begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} \end{bmatrix}
\tag{3.11}
$$

Where x and y are given by

$$
\begin{aligned}
x &= N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4 + N_5x_5 + N_6x_6 + N_7x_7 + N_8x_8 \\
y &= N_1y_1 + N_2y_2 + N_3y_3 + N_4y_4 + N_5y_5 + N_6y_6 + N_7y_7 + N_8y_8
\end{aligned}
\tag{3.12}
$$

The Matrix [B] is given as follows for this element

$$[B]=[D^{'}][N] \tag{3.13}$$

$$[D^{'}]=\frac{1}{|J|}\begin{bmatrix} \frac{\partial y}{\partial \eta}\frac{\partial ()}{\partial \xi}-\frac{\partial y}{\partial \xi}\frac{\partial ()}{\partial \eta} & 0 \\ 0 & \frac{\partial x}{\partial \xi}\frac{\partial ()}{\partial \eta}-\frac{\partial x}{\partial \eta}\frac{\partial ()}{\partial \xi} \\ \frac{\partial x}{\partial \xi}\frac{\partial ()}{\partial \eta}-\frac{\partial x}{\partial \eta}\frac{\partial ()}{\partial \xi} & \frac{\partial y}{\partial \eta}\frac{\partial ()}{\partial \xi}-\frac{\partial y}{\partial \xi}\frac{\partial ()}{\partial \eta} \end{bmatrix}$$

$$[N]=\begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 & N_5 & 0 & N_6 & 0 & N_7 & 0 & N_8 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 & N_5 & 0 & N_6 & 0 & N_7 & 0 & N_8 \end{bmatrix}$$

For the case of plane stress the matrix [D] is given by

$$[D]=\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{3.14}$$

For the case of plane strain the matrix [D] is given by

$$[D]=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \tag{3.15}$$

The element stiffness matrix for the quadratic quadrilateral element is written in terms of double integration as follows

$$\left[k\right]=t\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\left|J\right|\partial\xi\partial\eta \tag{3.16}$$

Chapter 4: Problem Description

A hollow cylinder tube of inner radius 'a' and outer radius 'b' is subjected to internal pressure 'P' and is rotating about the z-axis with angular velocity 'w'. The tube and its cross-section is shown in the fig below. The tube extends indefinitely along the z-axis and is in a plain strain condition along that direction. The material is isotropic with elastic modulus E and passion ratio 'v'. A "slice" of thickness'd' is extracted and discretized as shown in the fig below. The young's modulus, E = 200GPa, density = 7860kg/m3, poisson ratio = 0.33, inner and outer radius = 1.5m, 1.7m and angular velocity, w = 15rad/sec2.

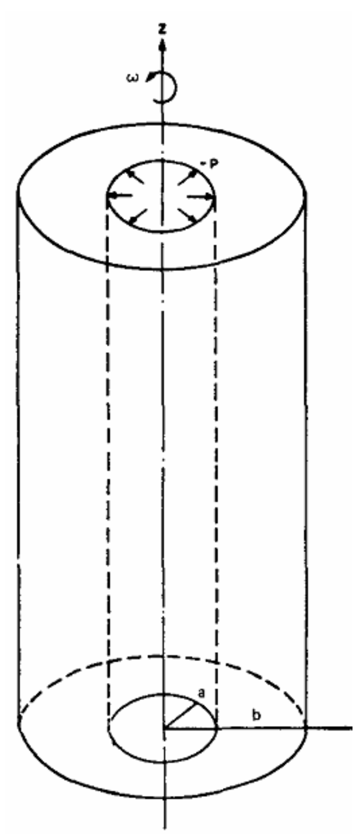

Figure 8: A rotating internally pressurized cylinder

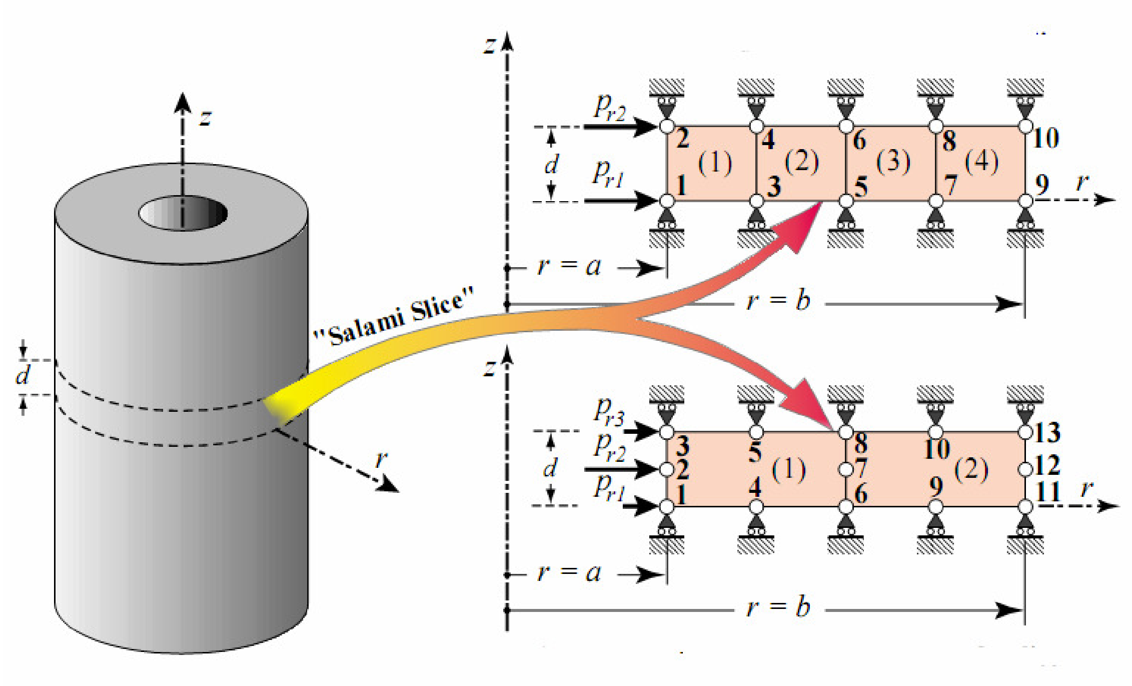

Figure 9: Element Discritisation

4.1 Element Discription

The length , breadth and thickness of the element is 0.2*0.034*0.0001, all the dimensions are in meters. The element is discritize into 2 and 4 as shown in the fig 9. For Quadratic Quadrilateral element 'Pr' is the total pressure force acting on the element which is equal to 0.0000034P which is distributed to the nodal points such that the pressure force at nodes 1 and 3 is one third of Pr and at node 2 is two third of Pr. For Bilinear Quadrilateral element the pressure force at node 1 and 2 is equal to half of Pr. Along with pressure force centrifugal force will also be acting along r : $b_r = \rho\omega^2 r, b_z = 0$

Problem is solved first by using 2 elements both bilinear and quadratic and then with 4 element again with both bilinear and quadratic. The results obtained are then compared with the exact results.

The matlab program was used to find out the results (displacement and stresses)

4.2 Matlab programming

4.2.1 For Bilinear Quadrilateral Element (4-element)

```
%%
E=200000000000;
v=0.33;
d=0.0001;
%%
k=zeros(20);
k1=zeros(8);
k2=zeros(8);
k3=zeros(8);
k4=zeros(8);
%%
a=BilinearElementArea(x1,y1,x2,y2,x3,y3,x4,y4);
%%
k1=BilinearElementStiffness(E,v,d,x1,y1,x2,y2,x3,y3,x4,y4,p);
%%
k2=BilinearElementStiffness(E,v,d,x1,y1,x2,y2,x3,y3,x4,y4,p);
%%
k3= BilinearElementStiffness(E,v,d,x1,y1,x2,y2,x3,y3,x4,y4,p);
%%
k4= BilinearElementStiffness(E,v,d,x1,y1,x2,y2,x3,y3,x4,y4,p);
%%
k= BilinearAssemble(k,k1,1,6,8,3,4,7,5,2);
%%
```

```
k= BilinearAssemble(k,k2,1,6,8,3,4,7,5,2);
%%
k= BilinearAssemble(k,k3,1,6,8,3,4,7,5,2);
%%
k= BilinearAssemble(k,k4,1,6,8,3,4,7,5,2);
%%
```

s1= BilinearElementStresses(E,v, $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$,p,u);

%%

s2= BilinearElementStresses(E,v, $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$,p,u);

%%

s3= BilinearElementStresses(E,v, $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$,p,u);

%%

s4= BilinearElementStresses(E,v, $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$,p,u);

end;

4.2.1.1 Matlab functions used

(Obtained from the book : Matlab Guide to finite elements – P I Kattan)

BilinearElementArea($x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$)- This gives the area of the bilinear quadrilateral element

BilinearElementStiffness(E,v,d, $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$,p)-This gives the element stiffness matrix for a bilinear

BilinearAssemble(K,k,i,j,m,n)- This gives the element stiffness matrix k of the bilinear quadrilateral element

BilinearElementStresses(E,v, $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$,p,u)- This function gives the element stress vector for a bilinear quadrilateral element

4.2.2 For Quadratic Quadrilateral Element (4-element)

```
%%
E=200000000000;
v=0.33;
d=0.0001;
%%
k=zeros(46);
k1=zeros(16);
k2=zeros(16);
k3=zeros(16);
k4=zeros(16);
%%
a1= QuadraticElementArea(x1,y1,x2,y2,x3,y3,x4,y4);
%%
a2= QuadraticElementArea(x1,y1,x2,y2,x3,y3,x4,y4);
%%
a3= QuadraticElementArea(x1,y1,x2,y2,x3,y3,x4,y4);
%%
a4= QuadraticElementArea(x1,y1,x2,y2,x3,y3,x4,y4);
%%
k1= QuadraticElementStiffness(E,v,d,x1,y1,x2,y2,x3,y3,x4,y4,p);
%%
k2= QuadraticElementStiffness(E,v,d,x1,y1,x2,y2,x3,y3,x4,y4,p);
%%
k3= QuadraticElementStiffness(E,v,d,x1,y1,x2,y2,x3,y3,x4,y4,p);
```

```
%%
k4= QuadraticElementStiffness(E,v,d, x1,y1,x2,y2,x3,y3,x4,y4,p);
%%
K=QuadraticAssemble(K,k1,1,6,8,3,4,7,5,2);
%%
K=QuadraticAssemble(K,k2,6,11,13,8,9,12,10,7);
%%
K=QuadraticAssemble(K,k3,11,16,18,13,14,17,15,12);
%%
K=QuadraticAssemble(K,k4,16,21,23,18,19,22,20,17);
%%
s1=QuadraticElementSresses(E,v, x1,y1,x2,y2,x3,y3,x4,y4,p,u);
%%
s2=QuadraticElementSresses(E,NU,x1,y1,x2,y2,x3,y3,x4,y4,p,u);
%%
s3=QuadraticElementSresses(E,NU,x1,y1,x2,y2,x3,y3,x4,y4,p,u);
%%
s4=QuadraticElementSresses(E,NU,x1,y1,x2,y2,x3,y3,x4,y4,p,u);
end;
```

4.2.2.1 Matlab functions used

(Obtained from the book : Matlab Guide to finite elements – P I Kattan)

QuadraticElementArea($x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$) –This gives the area of the quadratic quadrilateral element.

QuadraticElementStiffness(E,v,d, $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$,p) –This gives the element stiffness matrix for a quadratic quadrilateral element .

QuadraticAssemble(K,k,i,j,m,p,q,r,s,t)- This gives the element stiffness matrix k of the quadratic quadrilateral element

`s1=QuadraticElementSresses(E,v,` $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$ `,p,u)` –This function gives the element stress vector for a bilinear quadrilateral element.

Chapter 5: The Global Stiffness Matrix

```
2-element Bilinear Quadilateral Element

K =

  1.0e+007 *

    0.9915    0.3731   -0.6100    0.0028    0.1142   -0.0028   -0.4958   -0.3731
    0.3731    2.2856   -0.0028   -2.1578    0.0028    1.0150   -0.3731   -1.1428
   -0.6100   -0.0028    0.9915   -0.3731   -0.4958    0.3731    0.1142    0.0028
    0.0028   -2.1578   -0.3731    2.2856    0.3731   -1.1428   -0.0028    1.0150
    0.1142    0.0028   -0.4958    0.3731    1.9830         0   -1.2199         0
   -0.0028    1.0150    0.3731   -1.1428         0    4.5712         0   -4.3156
   -0.4958   -0.3731    0.1142   -0.0028   -1.2199         0    1.9830         0
   -0.3731   -1.1428    0.0028    1.0150         0   -4.3156         0    4.5712
         0         0         0         0    0.1142    0.0028   -0.4958    0.3731
         0         0         0         0   -0.0028    1.0150    0.3731   -1.1428
         0         0         0         0   -0.4958   -0.3731    0.1142   -0.0028
         0         0         0         0   -0.3731   -1.1428    0.0028    1.0150
```

Columns 9 to 12

```
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    0.1142   -0.0028   -0.4958   -0.3731
    0.0028    1.0150   -0.3731   -1.1428
   -0.4958    0.3731    0.1142    0.0028
    0.3731   -1.1428   -0.0028    1.0150
    0.9915   -0.3731   -0.6100   -0.0028
   -0.3731    2.2856    0.0028   -2.1578
   -0.6100    0.0028    0.9915    0.3731
   -0.0028   -2.1578    0.3731    2.2856
```

4-element Bilinear Quadrilateral Element

```
K =

  1.0e+007 *

  Columns 1 through 8

    0.8773    0.3731   -0.1142    0.0028   -0.3245   -0.0028   -0.4387   -0.3731
    0.3731    1.2706   -0.0028   -1.0150    0.0028    0.3797   -0.3731   -0.6353
   -0.1142   -0.0028    0.8773   -0.3731   -0.4387    0.3731   -0.3245    0.0028
    0.0028   -1.0150   -0.3731    1.2706    0.3731   -0.6353   -0.0028    0.3797
   -0.3245    0.0028   -0.4387    0.3731    1.7546         0   -0.2284         0
   -0.0028    0.3797    0.3731   -0.6353         0    2.5413         0   -2.0300
   -0.4387   -0.3731   -0.3245   -0.0028   -0.2284         0    1.7546         0
   -0.3731   -0.6353    0.0028    0.3797         0   -2.0300         0    2.5413
         0         0         0         0   -0.3245    0.0028   -0.4387    0.3731
         0         0         0         0   -0.0028    0.3797    0.3731   -0.6353
         0         0         0         0   -0.4387   -0.3731   -0.3245   -0.0028
         0         0         0         0   -0.3731   -0.6353    0.0028    0.3797
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
```

Columns 9 to 15

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.3245	-0.0028	-0.4387	-0.3731	0	0	0
0.0028	0.3797	-0.3731	-0.6353	0	0	0
-0.4387	0.3731	-0.3245	0.0028	0	0	0
0.3731	-0.6353	-0.0028	0.3797	0	0	0
1.7546	0	-0.2284	0	-0.3245	-0.0028	-0.4387
0	2.5413	0	-2.0300	0.0028	0.3797	-0.3731
-0.2284	0	1.7546	0	-0.4387	0.3731	-0.3245
0	-2.0300	0	2.5413	0.3731	-0.6353	-0.0028
-0.3245	0.0028	-0.4387	0.3731	1.7546	0	-0.2284
-0.0028	0.3797	0.3731	-0.6353	0	2.5413	0
-0.4387	-0.3731	-0.3245	-0.0028	-0.2284	0	1.7546
-0.3731	-0.6353	0.0028	0.3797	0	-2.0300	0
0	0	0	0	-0.3245	0.0028	-0.4387
0	0	0	0	-0.0028	0.3797	0.3731
0	0	0	0	-0.4387	-0.3731	-0.3245
0	0	0	0	-0.3731	-0.6353	0.0028

Columns 16 through 20

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
-0.3731	0	0	0	0
-0.6353	0	0	0	0
0.0028	0	0	0	0
0.3797	0	0	0	0
0	-0.3245	-0.0028	-0.4387	-0.3731
-2.0300	0.0028	0.3797	-0.3731	-0.6353
0	-0.4387	0.3731	-0.3245	0.0028
2.5413	0.3731	-0.6353	-0.0028	0.3797
0.3731	0.8773	-0.3731	-0.1142	-0.0028
-0.6353	-0.3731	1.2706	0.0028	-1.0150
-0.0028	-0.1142	0.0028	0.8773	0.3731
0.3797	-0.0028	-1.0150	0.3731	1.2706

2-element Quadratric Quadrilateral Element

k =

1.0e+008 *

Columns 1 through 8

0.1840	0.1044	-0.1840	-0.0727	0.0864	0.0059	-0.0779	-0.0256
0.1044	0.5044	-0.0256	-0.7508	-0.0059	0.2684	-0.0727	0.0331
-0.1840	-0.0256	0.4372	0	-0.1840	0.0256	0	0.0983
-0.0727	-0.7508	0	1.5192	0.0727	-0.7508	0.0983	0
0.0864	-0.0059	-0.1840	0.0727	0.1840	-0.1044	-0.0604	0.0246
0.0059	0.2684	0.0256	-0.7508	-0.1044	0.5044	0.0246	-0.0681
-0.0779	-0.0727	0	0.0983	-0.0604	0.0246	0.2990	0
-0.0256	0.0331	0.0983	0	0.0246	-0.0681	0	0.4983
-0.0604	-0.0246	0	-0.0983	-0.0779	0.0727	-0.0224	0
-0.0246	-0.0681	-0.0983	0	0.0256	0.0331	0	-0.4282
0.0728	0.0059	-0.1024	0.0246	0.0814	-0.0430	-0.0779	0.0256
-0.0059	0.1681	0.0246	-0.3780	-0.0430	0.2231	0.0727	0.0331
-0.1024	-0.0246	0.1356	0	-0.1024	0.0246	0	-0.0983
-0.0246	-0.3780	0	0.7385	0.0246	-0.3780	-0.0983	0
0.0814	0.0430	-0.1024	-0.0246	0.0728	-0.0059	-0.0604	-0.0246
0.0430	0.2231	-0.0246	-0.3780	0.0059	0.1681	-0.0246	-0.0681
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Columns 9 to 15

-0.0604	-0.0246	0.0728	-0.0059	-0.1024	-0.0246	0.0814
-0.0246	-0.0681	0.0059	0.1681	-0.0246	-0.3780	0.0430
0	-0.0983	-0.1024	0.0246	0.1356	0	-0.1024
-0.0983	0	0.0246	-0.3780	0	0.7385	-0.0246
-0.0779	0.0256	0.0814	-0.0430	-0.1024	0.0246	0.0728
0.0727	0.0331	-0.0430	0.2231	0.0246	-0.3780	-0.0059
-0.0224	0	-0.0779	0.0727	0	-0.0983	-0.0604
0	-0.4282	0.0256	0.0331	-0.0983	0	-0.0246
0.2990	0	-0.0604	0.0246	0	0.0983	-0.0779
0	0.4983	0.0246	-0.0681	0.0983	0	-0.0256
-0.0604	0.0246	0.3681	0	-0.3681	0	0.1728
0.0246	-0.0681	0	1.0088	0	-1.5017	0
0	0.0983	-0.3681	0	0.8744	0	-0.3681
0.0983	0	0	-1.5017	0	3.0384	0
-0.0779	-0.0256	0.1728	0	-0.3681	0	0.3681
-0.0727	0.0331	0	0.5367	0	-1.5017	0
0	0	-0.0779	-0.0727	0	0.0983	-0.0604
0	0	-0.0256	0.0331	0.0983	0	0.0246
0	0	-0.0604	-0.0246	0	-0.0983	-0.0779
0	0	-0.0246	-0.0681	-0.0983	0	0.0256
0	0	0.0728	0.0059	-0.1024	0.0246	0.0814
0	0	-0.0059	0.1681	0.0246	-0.3780	-0.0430
0	0	-0.1024	-0.0246	0.1356	0	-0.1024
0	0	-0.0246	-0.3780	0	0.7385	0.0246
0	0	0.0814	0.0430	-0.1024	-0.0246	0.0728
0	0	0.0430	0.2231	-0.0246	-0.3780	0.0059

Columns 16 to 22

0.0430	0	0	0	0	0	0
0.2231	0	0	0	0	0	0
-0.0246	0	0	0	0	0	0
-0.3780	0	0	0	0	0	0
0.0059	0	0	0	0	0	0
0.1681	0	0	0	0	0	0
-0.0246	0	0	0	0	0	0
-0.0681	0	0	0	0	0	0
-0.0727	0	0	0	0	0	0
0.0331	0	0	0	0	0	0
0	-0.0779	-0.0256	-0.0604	-0.0246	0.0728	-0.0059
0.5367	-0.0727	0.0331	-0.0246	-0.0681	0.0059	0.1681
0	0	0.0983	0	-0.0983	-0.1024	0.0246
-1.5017	0.0983	0	-0.0983	0	0.0246	-0.3780
0	-0.0604	0.0246	-0.0779	0.0256	0.0814	-0.0430
1.0088	0.0246	-0.0681	0.0727	0.0331	-0.0430	0.2231
0.0246	0.2990	0	-0.0224	0	-0.0779	0.0727
-0.0681	0	0.4983	0	-0.4282	0.0256	0.0331
0.0727	-0.0224	0	0.2990	0	-0.0604	0.0246
0.0331	0	-0.4282	0	0.4983	0.0246	-0.0681
-0.0430	-0.0779	0.0256	-0.0604	0.0246	0.1840	-0.1044
0.2231	0.0727	0.0331	0.0246	-0.0681	-0.1044	0.5044
0.0246	0	-0.0983	0	0.0983	-0.1840	0.0256
-0.3780	-0.0983	0	0.0983	0	0.0727	-0.7508
-0.0059	-0.0604	-0.0246	-0.0779	-0.0256	0.0864	0.0059
0.1681	-0.0246	-0.0681	-0.0727	0.0331	-0.0059	0.2684

Columns 23 to 26

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
-0.1024	-0.0246	0.0814	0.0430
-0.0246	-0.3780	0.0430	0.2231
0.1356	0	-0.1024	-0.0246
0	0.7385	-0.0246	-0.3780
-0.1024	0.0246	0.0728	0.0059
0.0246	-0.3780	-0.0059	0.1681
0	-0.0983	-0.0604	-0.0246
-0.0983	0	-0.0246	-0.0681
0	0.0983	-0.0779	-0.0727
0.0983	0	-0.0256	0.0331
-0.1840	0.0727	0.0864	-0.0059
0.0256	-0.7508	0.0059	0.2684
0.4372	0	-0.1840	-0.0256
0	1.5192	-0.0727	-0.7508
-0.1840	-0.0727	0.1840	0.1044
-0.0256	-0.7508	0.1044	0.5044

```
4-element Quadratric Quadrilateral element

K =

  1.0e+008 *

  Columns 1 through 8

    0.1521    0.0705   -0.0881   -0.0328    0.0632   -0.0001   -0.1283   -0.0335
    0.0705    0.2202   -0.0335   -0.2900    0.0001    0.1123   -0.0328   -0.0234
   -0.0881   -0.0335    0.2780         0   -0.0881    0.0335         0    0.0663
   -0.0328   -0.2900         0    0.6140    0.0328   -0.2900    0.0663         0
    0.0632    0.0001   -0.0881    0.0328    0.1521   -0.0705   -0.0752    0.0166
   -0.0001    0.1123    0.0335   -0.2900   -0.0705    0.2202    0.0166   -0.0447
   -0.1283   -0.0328         0    0.0663   -0.0752    0.0166    0.3303         0
   -0.0335   -0.0234    0.0663         0    0.0166   -0.0447         0    0.2669
   -0.0752   -0.0166         0   -0.0663   -0.1283    0.0328    0.0767         0
   -0.0166   -0.0447   -0.0663         0    0.0335   -0.0234         0   -0.1306
    0.0684   -0.0001   -0.0593    0.0166    0.0673   -0.0290   -0.1283    0.0335
    0.0001    0.0783    0.0166   -0.1501   -0.0290    0.0974    0.0328   -0.0234
   -0.0593   -0.0166    0.0169         0   -0.0593    0.0166         0   -0.0663
   -0.0166   -0.1501         0    0.2661    0.0166   -0.1501   -0.0663         0
    0.0673    0.0290   -0.0593   -0.0166    0.0684    0.0001   -0.0752   -0.0166
    0.0290    0.0974   -0.0166   -0.1501   -0.0001    0.0783   -0.0166   -0.0447
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
```

Columns 9 to 15

-0.0752	-0.0166	0.0684	0.0001	-0.0593	-0.0166	0.0673
-0.0166	-0.0447	-0.0001	0.0783	-0.0166	-0.1501	0.0290
0	-0.0663	-0.0593	0.0166	0.0169	0	-0.0593
-0.0663	0	0.0166	-0.1501	0	0.2661	-0.0166
-0.1283	0.0335	0.0673	-0.0290	-0.0593	0.0166	0.0684
0.0328	-0.0234	-0.0290	0.0974	0.0166	-0.1501	0.0001
0.0767	0	-0.1283	0.0328	0	-0.0663	-0.0752
0	-0.1306	0.0335	-0.0234	-0.0663	0	-0.0166
0.3303	0	-0.0752	0.0166	0	0.0663	-0.1283
0	0.2669	0.0166	-0.0447	0.0663	0	-0.0335
-0.0752	0.0166	0.3041	0	-0.1762	0	0.1265
0.0166	-0.0447	0	0.4405	0	-0.5800	0
0	0.0663	-0.1762	0	0.5559	0	-0.1762
0.0663	0	0	-0.5800	0	1.2281	0
-0.1283	-0.0335	0.1265	0	-0.1762	0	0.3041
-0.0328	-0.0234	0	0.2247	0	-0.5800	0
0	0	-0.1283	-0.0328	0	0.0663	-0.0752
0	0	-0.0335	-0.0234	0.0663	0	0.0166
0	0	-0.0752	-0.0166	0	-0.0663	-0.1283
0	0	-0.0166	-0.0447	-0.0663	0	0.0335
0	0	0.0684	-0.0001	-0.0593	0.0166	0.0673
0	0	0.0001	0.0783	0.0166	-0.1501	-0.0290
0	0	-0.0593	-0.0166	0.0169	0	-0.0593
0	0	-0.0166	-0.1501	0	0.2661	0.0166
0	0	0.0673	0.0290	-0.0593	-0.0166	0.0684
0	0	0.0290	0.0974	-0.0166	-0.1501	-0.0001
0	0	0	0	0	0	0
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Columns 16 through 23

0.0290	0	0	0	0	0	0	0
0.0974	0	0	0	0	0	0	0
-0.0166	0	0	0	0	0	0	0
-0.1501	0	0	0	0	0	0	0
-0.0001	0	0	0	0	0	0	0
0.0783	0	0	0	0	0	0	0
-0.0166	0	0	0	0	0	0	0
-0.0447	0	0	0	0	0	0	0
-0.0328	0	0	0	0	0	0	0
-0.0234	0	0	0	0	0	0	0
0	-0.1283	-0.0335	-0.0752	-0.0166	0.0684	0.0001	-0.0593
0.2247	-0.0328	-0.0234	-0.0166	-0.0447	-0.0001	0.0783	-0.0166
0	0	0.0663	0	-0.0663	-0.0593	0.0166	0.0169
-0.5800	0.0663	0	-0.0663	0	0.0166	-0.1501	0
0	-0.0752	0.0166	-0.1283	0.0335	0.0673	-0.0290	-0.0593
0.4405	0.0166	-0.0447	0.0328	-0.0234	-0.0290	0.0974	0.0166
0.0166	0.3303	0	0.0767	0	-0.1283	0.0328	0
-0.0447	0	0.2669	0	-0.1306	0.0335	-0.0234	-0.0663
0.0328	0.0767	0	0.3303	0	-0.0752	0.0166	0
-0.0234	0	-0.1306	0	0.2669	0.0166	-0.0447	0.0663
-0.0290	-0.1283	0.0335	-0.0752	0.0166	0.3041	0	-0.1762
0.0974	0.0328	-0.0234	0.0166	-0.0447	0	0.4405	0
0.0166	0	-0.0663	0	0.0663	-0.1762	0	0.5559
-0.1501	-0.0663	0	0.0663	0	0	-0.5800	0
0.0001	-0.0752	-0.0166	-0.1283	-0.0335	0.1265	0	-0.1762
0.0783	-0.0166	-0.0447	-0.0328	-0.0234	0	0.2247	0
0	0	0	0	0	-0.1283	-0.0328	0
0	0	0	0	0	-0.0335	-0.0234	0.0663
0	0	0	0	0	-0.0752	-0.0166	0
0	0	0	0	0	-0.0166	-0.0447	-0.0663
0	0	0	0	0	0.0684	-0.0001	-0.0593
0	0	0	0	0	0.0001	0.0783	0.0166
0	0	0	0	0	-0.0593	-0.0166	0.0169
0	0	0	0	0	-0.0166	-0.1501	0
0	0	0	0	0	0.0673	0.0290	-0.0593
0	0	0	0	0	0.0290	0.0974	-0.0166
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Column 24 to 30

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0	0	0	0	0	0	0
0	0	0	0	0	0	0
-0.0166	0.0673	0.0290	0	0	0	0
-0.1501	0.0290	0.0974	0	0	0	0
0	-0.0593	-0.0166	0	0	0	0
0.2661	-0.0166	-0.1501	0	0	0	0
0.0166	0.0684	-0.0001	0	0	0	0
-0.1501	0.0001	0.0783	0	0	0	0
-0.0663	-0.0752	-0.0166	0	0	0	0
0	-0.0166	-0.0447	0	0	0	0
0.0663	-0.1283	-0.0328	0	0	0	0
0	-0.0335	-0.0234	0	0	0	0
0	0.1265	0	-0.1283	-0.0335	-0.0752	-0.0166
-0.5800	0	0.2247	-0.0328	-0.0234	-0.0166	-0.0447
0	-0.1762	0	0	0.0663	0	-0.0663
1.2281	0	-0.5800	0.0663	0	-0.0663	0
0	0.3041	0	-0.0752	0.0166	-0.1283	0.0335
-0.5800	0	0.4405	0.0166	-0.0447	0.0328	-0.0234
0.0663	-0.0752	0.0166	0.3303	0	0.0767	0
0	0.0166	-0.0447	0	0.2669	0	-0.1306
-0.0663	-0.1283	0.0328	0.0767	0	0.3303	0
0	0.0335	-0.0234	0	-0.1306	0	0.2669
0.0166	0.0673	-0.0290	-0.1283	0.0335	-0.0752	0.0166
-0.1501	-0.0290	0.0974	0.0328	-0.0234	0.0166	-0.0447
0	-0.0593	0.0166	0	-0.0663	0	0.0663
0.2661	0.0166	-0.1501	-0.0663	0	0.0663	0
-0.0166	0.0684	0.0001	-0.0752	-0.0166	-0.1283	-0.0335
-0.1501	-0.0001	0.0783	-0.0166	-0.0447	-0.0328	-0.0234
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Column 31 to 38

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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0.0684	0.0001	-0.0593	-0.0166	0.0673	0.0290	0	0
-0.0001	0.0783	-0.0166	-0.1501	0.0290	0.0974	0	0
-0.0593	0.0166	0.0169	0	-0.0593	-0.0166	0	0
0.0166	-0.1501	0	0.2661	-0.0166	-0.1501	0	0
0.0673	-0.0290	-0.0593	0.0166	0.0684	-0.0001	0	0
-0.0290	0.0974	0.0166	-0.1501	0.0001	0.0783	0	0
-0.1283	0.0328	0	-0.0663	-0.0752	-0.0166	0	0
0.0335	-0.0234	-0.0663	0	-0.0166	-0.0447	0	0
-0.0752	0.0166	0	0.0663	-0.1283	-0.0328	0	0
0.0166	-0.0447	0.0663	0	-0.0335	-0.0234	0	0
0.3041	0	-0.1762	0	0.1265	0	-0.1283	-0.0335
0	0.4405	0	-0.5800	0	0.2247	-0.0328	-0.0234
-0.1762	0	0.5559	0	-0.1762	0	0	0.0663
0	-0.5800	0	1.2281	0	-0.5800	0.0663	0
0.1265	0	-0.1762	0	0.3041	0	-0.0752	0.0166
0	0.2247	0	-0.5800	0	0.4405	0.0166	-0.0447
-0.1283	-0.0328	0	0.0663	-0.0752	0.0166	0.3303	0
-0.0335	-0.0234	0.0663	0	0.0166	-0.0447	0	0.2669
-0.0752	-0.0166	0	-0.0663	-0.1283	0.0328	0.0767	0
-0.0166	-0.0447	-0.0663	0	0.0335	-0.0234	0	-0.1306
0.0684	-0.0001	-0.0593	0.0166	0.0673	-0.0290	-0.1283	0.0335
0.0001	0.0783	0.0166	-0.1501	-0.0290	0.0974	0.0328	-0.0234
-0.0593	-0.0166	0.0169	0	-0.0593	0.0166	0	-0.0663
-0.0166	-0.1501	0	0.2661	0.0166	-0.1501	-0.0663	0
0.0673	0.0290	-0.0593	-0.0166	0.0684	0.0001	-0.0752	-0.0166
0.0290	0.0974	-0.0166	-0.1501	-0.0001	0.0783	-0.0166	-0.0447

Column 39 to 46

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0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
-0.0752	-0.0166	0.0684	0.0001	-0.0593	-0.0166	0.0673	0.0290
-0.0166	-0.0447	-0.0001	0.0783	-0.0166	-0.1501	0.0290	0.0974
0	-0.0663	-0.0593	0.0166	0.0169	0	-0.0593	-0.0166
-0.0663	0	0.0166	-0.1501	0	0.2661	-0.0166	-0.1501
-0.1283	0.0335	0.0673	-0.0290	-0.0593	0.0166	0.0684	-0.0001
0.0328	-0.0234	-0.0290	0.0974	0.0166	-0.1501	0.0001	0.0783
0.0767	0	-0.1283	0.0328	0	-0.0663	-0.0752	-0.0166
0	-0.1306	0.0335	-0.0234	-0.0663	0	-0.0166	-0.0447
0.3303	0	-0.0752	0.0166	0	0.0663	-0.1283	-0.0328
0	0.2669	0.0166	-0.0447	0.0663	0	-0.0335	-0.0234
-0.0752	0.0166	0.1521	-0.0705	-0.0881	0.0328	0.0632	0.0001
0.0166	-0.0447	-0.0705	0.2202	0.0335	-0.2900	-0.0001	0.1123
0	0.0663	-0.0881	0.0335	0.2780	0	-0.0881	-0.0335
0.0663	0	0.0328	-0.2900	0	0.6140	-0.0328	-0.2900
-0.1283	-0.0335	0.0632	-0.0001	-0.0881	-0.0328	0.1521	0.0705
-0.0328	-0.0234	0.0001	0.1123	-0.0335	-0.2900	0.0705	0.2202

Chapter 6: Results

Exact solution

Node	ur (mm)	Radial stress (Mpa)	Circumferential stress (Mpa)
1.5	0.8489	-30	270.09
1.525	0.8475	-25.53	267.36
1.55	0.8466	-21.28	264.08
1.575	0.8455	-17.2	261.06
1.6	0.8448	-13.43	258.28
1.625	0.8443	-9.8	255.75
1.65	0.844	-6.34	253.02
1.675	0.8438	-3.06	251.38
1.7	0.838	0	249.52

Table 1

2-element (Bilinear Quadrilateral Element)

Node	ur (mm)	Radial stress (Mpa)	Circumferential stress (Mpa)
1	0.848	-24.83	267.58
2	0.848	-24.83	267.58
3	0.8444	-12.8	259.85
4	0.8444	-12.8	259.85
5	0.8375	-6.52	249.51
6	0.8375	-6.52	249.51

Table 2

2-element (Quadratic Quadrilateral Element)

Node	ur (mm)	Radial stress (Mpa)	Circumferential stress (Mpa)
1	0.8486	-28.157	269.75
2	0.8486	-28.157	269.75
3	0.8486	-28.157	269.75
4	0.8462	-20.125	263.15
5	0.8462	-20.125	263.15
6	0.8448	-12.579	258.08
7	0.8448	-12.579	258.08
8	0.8448	-12.579	258.08
9	0.844	-6.251	252.01
10	0.844	-6.251	252.01
11	0.838	0	249.52
12	0.838	0	249.52
13	0.838	0	249.52

Table 3

4-element(Bilinear Quadratic Element)

Node	ur (mm)	Radial stress (Mpa)	Circumferential stress (Mpa)
1	0.8485	-26.515	269.57
2	0.8485	-26.515	269.57
3	0.8461	-22.578	262.15
4	0.8461	-22.578	262.15
5	0.8448	-14.178	258.35
6	0.8448	-14.178	258.35
7	0.844	-6.247	254.51
8	0.844	-6.247	254.51
9	0.838	0	249.58
10	0.838	0	249.58

Table 4

4-element (Quadrilateral Quadratic Element)

Node	ur (mm)	Radial stress (Mpa)	Circumferential stress (Mpa)
1	0.8488	-29.573	270.85
2	0.8488	-29.573	270.85
3	0.8488	-29.573	270.85
4	0.8474	-25.187	267.3
5	0.8474	-25.187	267.3
6	0.84625	-21.138	264
7	0.84625	-21.138	264
8	0.84625	-21.138	264
9	0.8548	-17.253	261
10	0.8548	-17.253	261
11	0.84477	-13.334	258.2
12	0.84477	-13.334	258.2
13	0.84477	-13.334	258.2
14	0.84428	-9.832	255.68
15	0.84428	-9.832	255.68
16	0.844	-6.345	252.88
17	0.844	-6.345	252.88
18	0.844	-6.345	252.88
19	0.8438	-3.061	251.3
20	0.8438	-3.061	251.3
21	0.8438	0	249.51
22	0.8438	0	249.51
23	0.8438	0	249.51

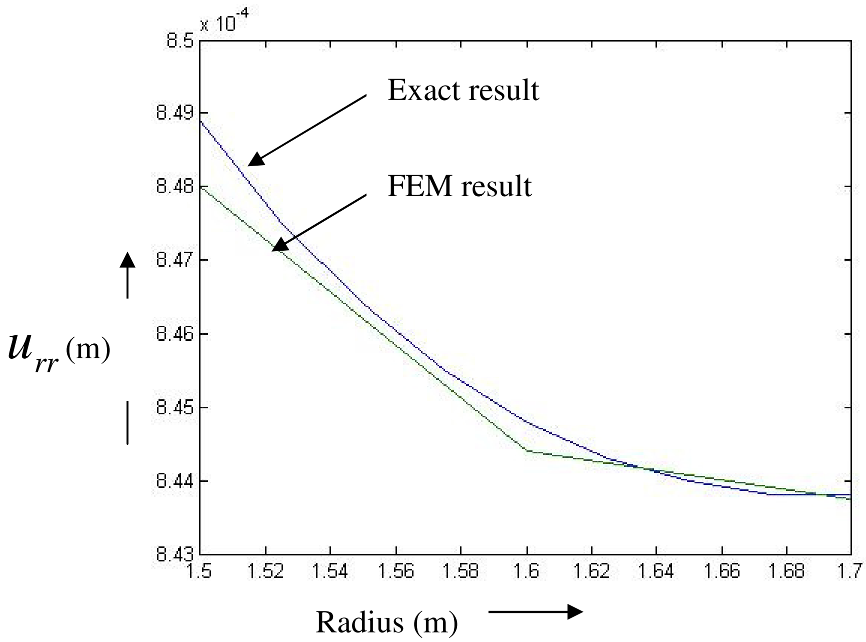
Table 5

Note : $\sigma_{zz} and \sigma_{rz}$ is zero for all the elements discussed above.

Chapter 7: Graphs

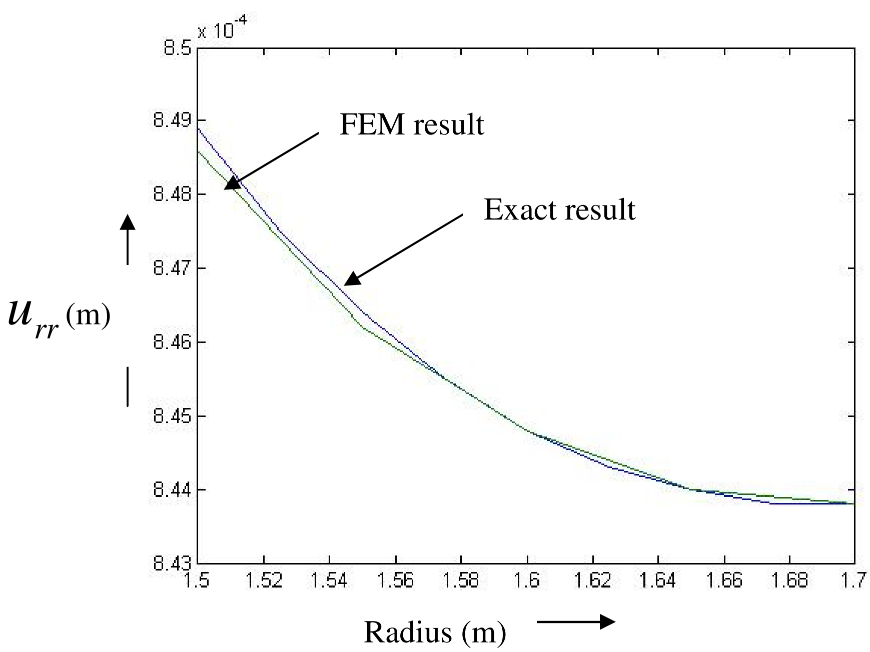
Radial displacement (Exact values Vs FEM values)

1.1 For 2-element bilinear quadrilateral element



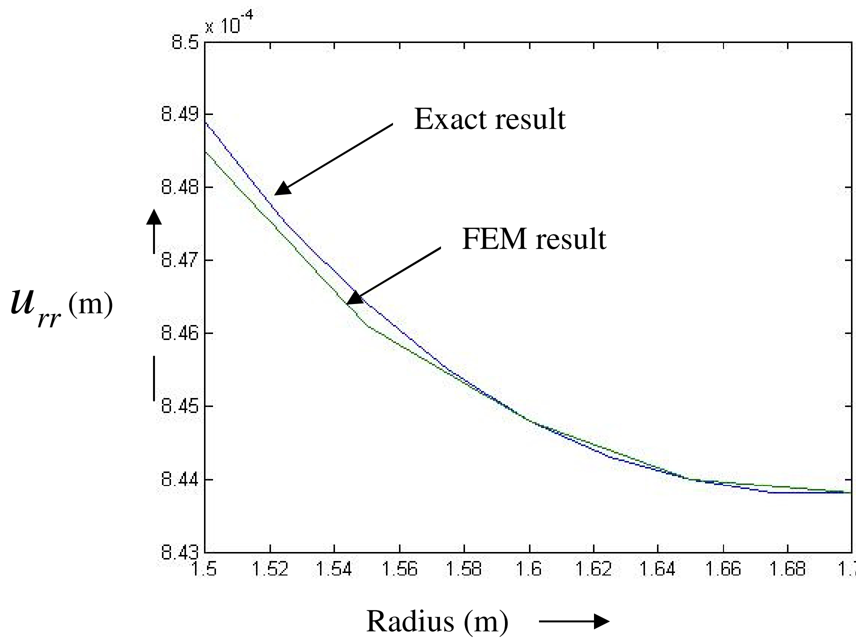
Graph 1

1.2 For 2-element Quadratic quadrilateral element



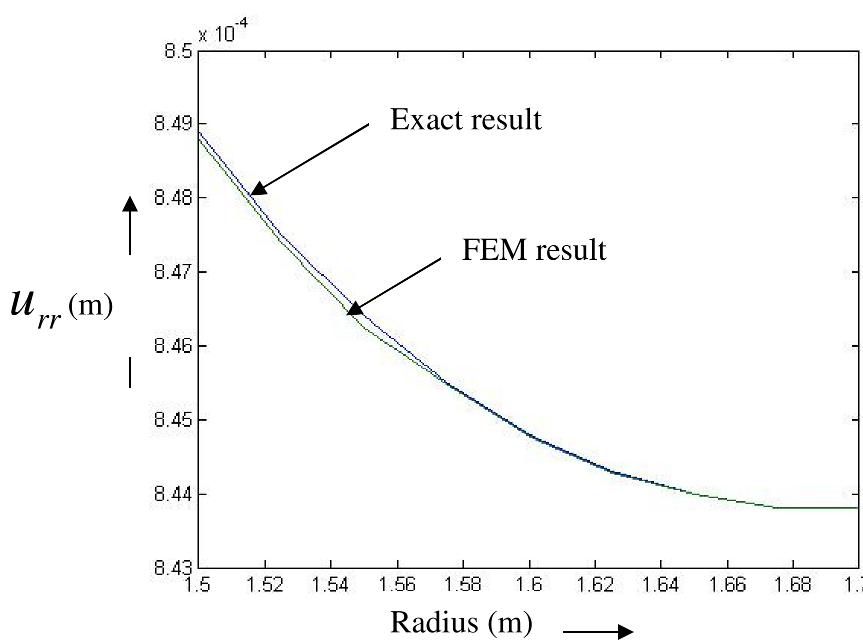
Graph 2

1.3 For 4-element Bilinear Quadrilateral element



Graph 3

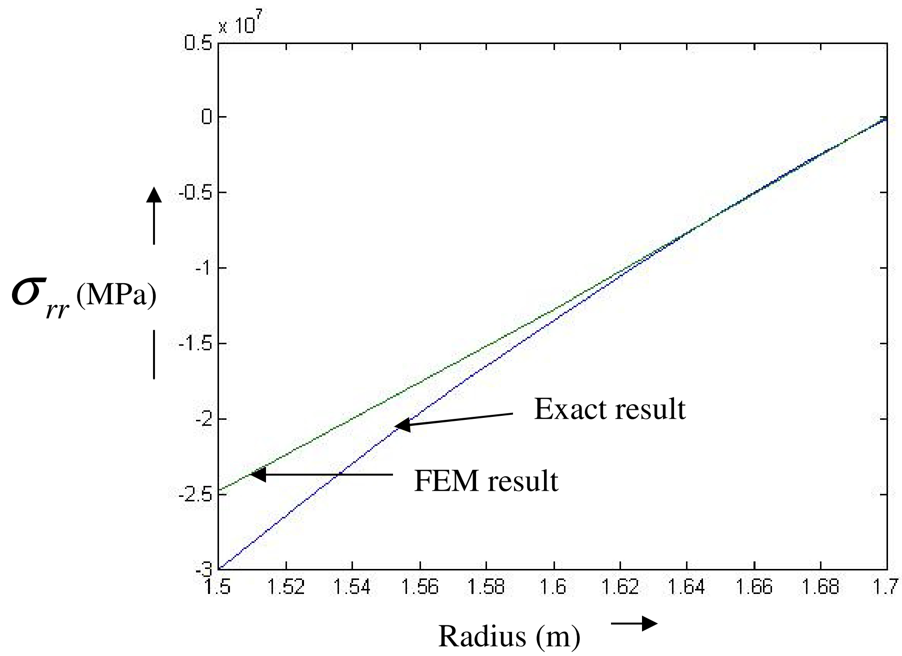
1.4 For 4 –element Quadratic Quadrilateral Element



Graph 4

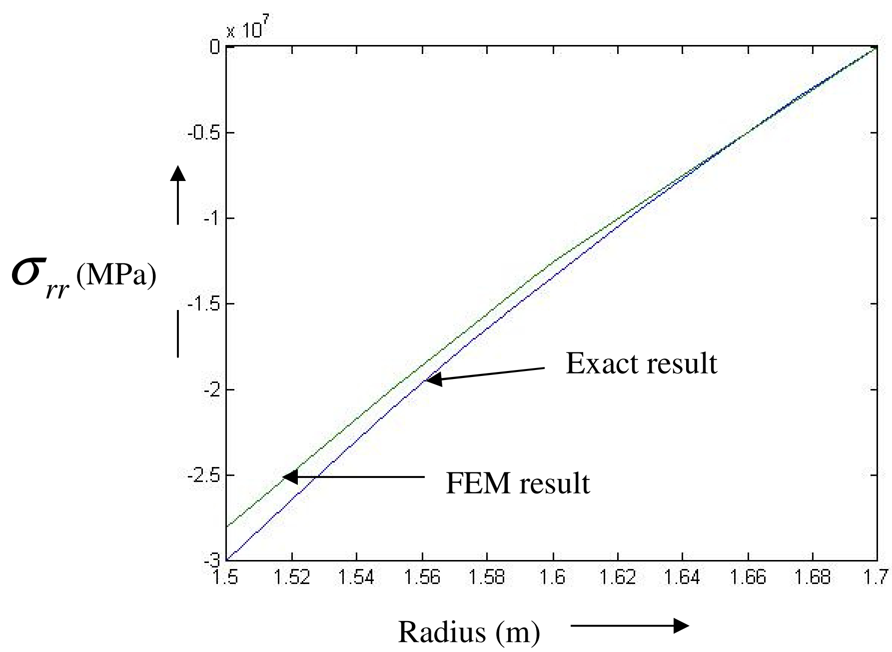
(2) **Radial Stress** (σ_{rr})

2.1 For 2-element bilinear quadrilateral element



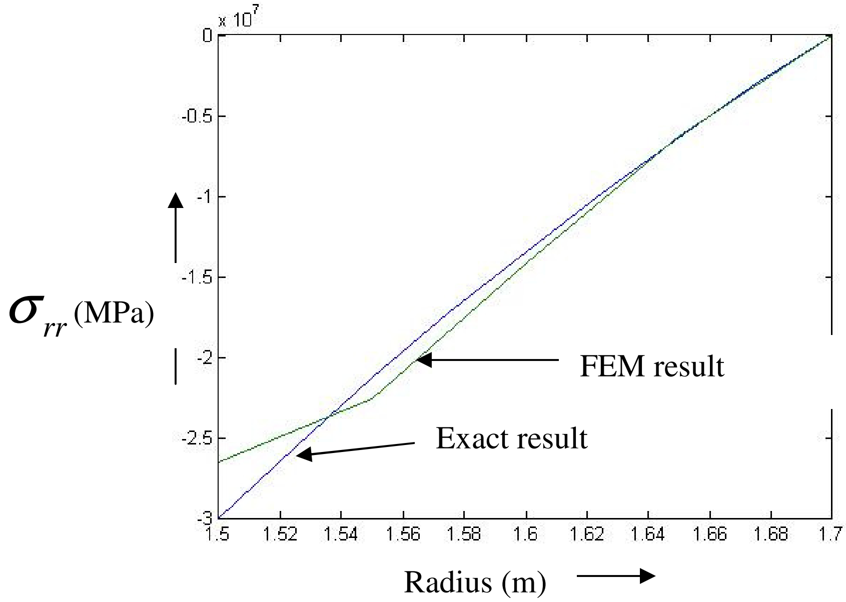
Graph 5

2.2 For 2-element Quadratic quadrilateral element



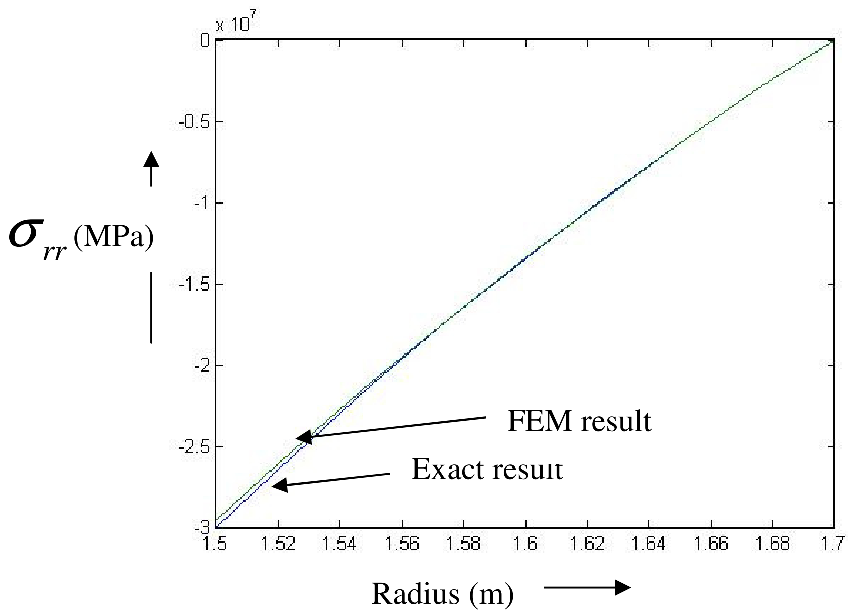
Graph 6

2.3 For 4-element Bilinear quadrilateral element



Graph 7

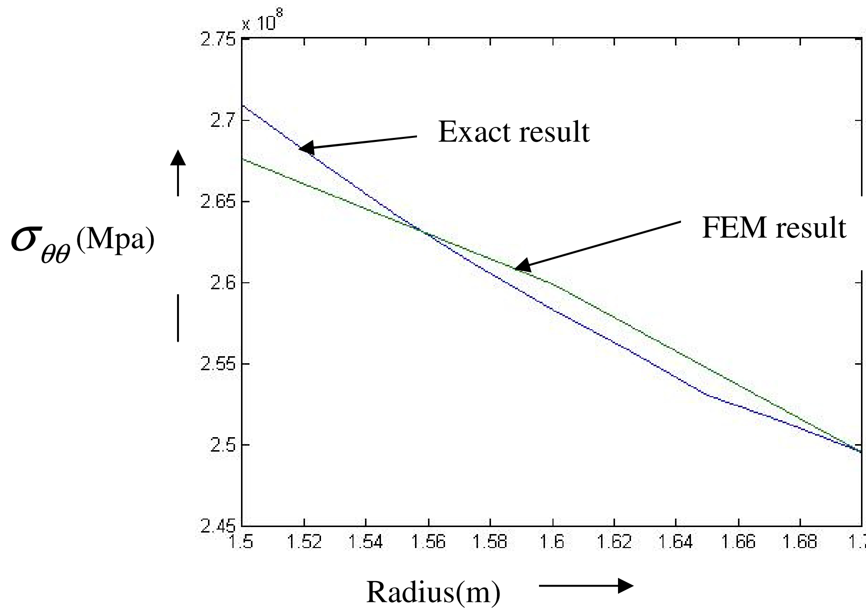
2.4 For 4-element Quadratic quadrilateral element



Graph 8

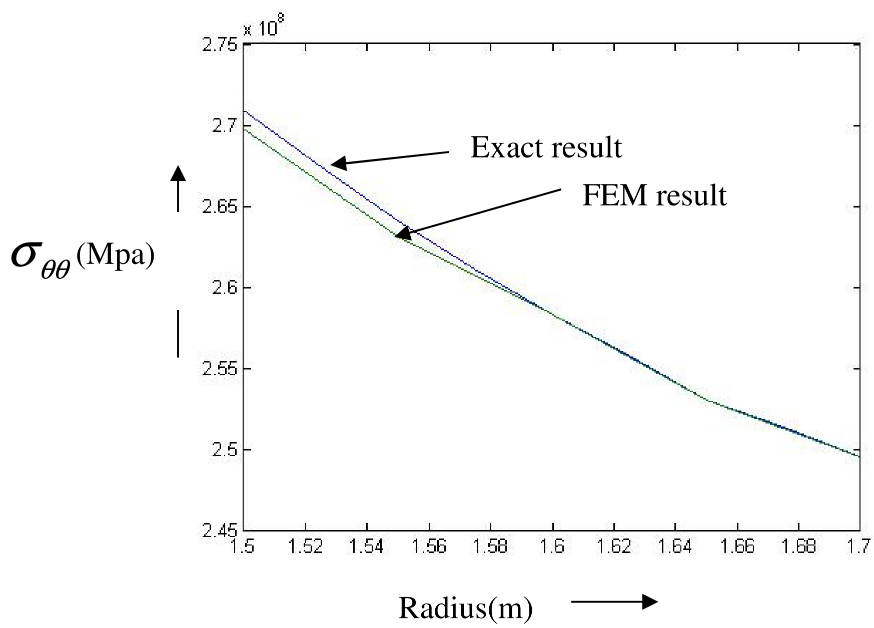
(1) **Circumferential Stress** ($\sigma_{\theta\theta}$)

For 2-element bilinear quadrilateral element



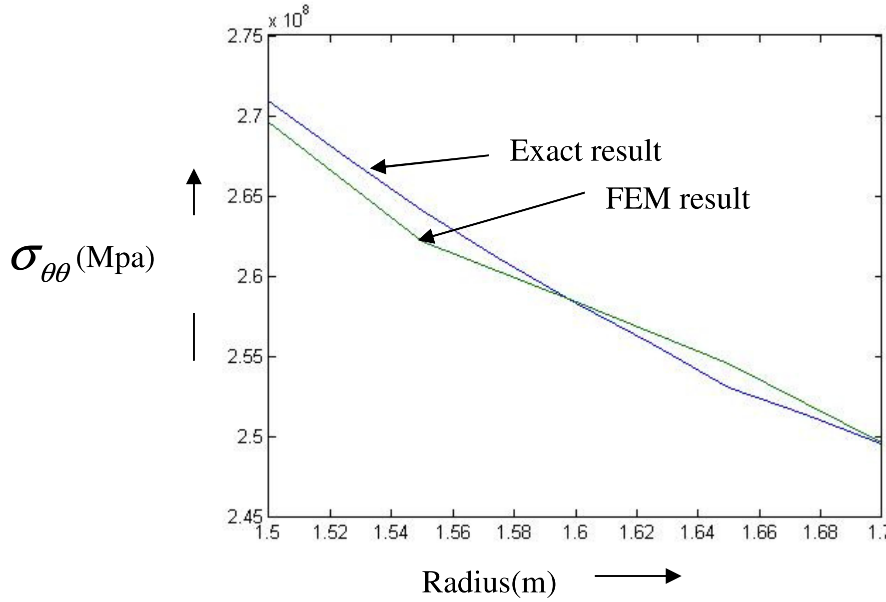
Graph 9

3.2 For 2-element Quadratic quadrilateral element



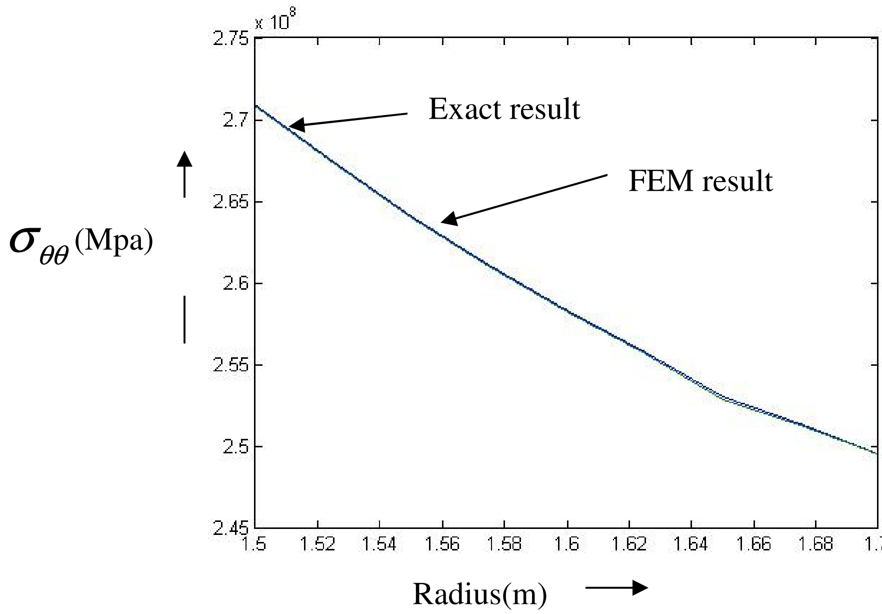
Graph 10

3.3 For 4-element Bilinear quadrilateral element



Graph 11

3.4 For 4-element Quadratic quadrilateral element



Graph 12

Chapter 8: Discussion

Computed and Exact values are given in table 1, 2, 3, 4, 5. Radial displacement, radial stress and hoop stress are graphically compared over a<r<b with the exact solutions.

Radial displacement, u_r :

The radial displacement for 2-element (Bilinear) shows more variation compared to others. Out of four types 4-element (Quadratic) show better results. It is seen that for either element type the values converges to exact values by increasing the element numbers.

Radial stress, σ_{rr} :

The radial stress also follows the same trend followed by radial displacement. For lesser number of elements large variation is observed but the variation vanishes on increasing the element number as seen in the graphs of 2 and 4 elements. Quadratic element has upper hand when compared with bilinear and its better to use quadratic elements for better results.

Circumferential stress, $\sigma_{\theta\theta}$:

Circumferential stresses are very important; these stresses are larger in values than any other stress induced in the cylinder so their study is important. Bilinear elements do not show good results compared to quadratic element as seen in the graphs. The results are more close to the exact values as the number of elements increases also seen in the graphs.

Chapter 9: Conclusion

Finite element techniques were applied to the above problems and calculate the deflections and stresses at each of the nodes. The results obtained in the method differ slightly from the results obtained from the exact solution. The errors may be due to some problems in the computational techniques. The FEM results obtained were within the acceptable ranges. When compared between two isoparametric elements (Bilinear and Quadratic) for the same element number; bilinear showed more variation from the exact values than the quadratic as seen in the graph and for the same element type with different element numbers results were as expected i.e. results converge to exact values as the number of element increases.

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