OUT-OF-PLANE VIBRATION OF CURVED BEAMS

A THESIS SUBMITTED IN PARTIAL REQUIREMENTS FOR THE DEGREE OF
Bachelor of Technology
In
Civil Engineering

By
Subhrajeet Das(10601001)
Abhishek Majumdar(10601019)

Under the guidance of
Dr. M. R. Barik

Department of Civil Engineering
National Institute of Technology, Rourkela
2010

National Institute of Technology
Rourkela

CERTIFICATE

This is to certify that the thesis entitled, "**OUT-OF-PLANE VIBRATION OF CURVED BEAMS**" submitted by Subhrajeet Das (10601001) and Abhishek Majumdar (10601019) in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Civil Engineering at the National Institute Of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

Date:

Prof. M. R. Barik
Dept. Of Civil Engineering
National Institute of Technology, Rourkela

ACKNOWLEDGEMENT

We welcome this opportunity to express our heartfelt gratitude and regards to our project guide Dr. M. R. Barik, Department of Civil Engineering, National Institute of Technology, Rourkela, for his unconditional guidance. He always bestowed parental care upon us and evinced keen interest in solving our problems. An erudite teacher, a magnificent person and a strict disciplinarian, we consider ourselves fortunate to have worked under his supervision.

We are highly grateful to Prof. M. Panda, Head of the department, Civil Engineering, NIT Rourkela, for providing necessary facilities during the course of the work.

We admit thanks to Dr. N. Roy, Project Coordinator, Department of Civil Engineering, NIT Rourkela for giving us such an opportunity to carry on such mind stimulating and innovative project.

We extend our utmost gratefulness towards Abhishek Gupta, 8th sem Civil Engineering, NIT Rourkela, for his immense help in carrying on the project.

We greatly appreciate & convey our heartfelt thanks to my friends' flow of ideas, dear ones & all those who helped me in completion of this work.

Date: Subhrajeet Das (10601001)
Abhishek Majumdar (10601019)

CONTENTS:

List of Tables:

List of figures:

ABSTRACT:

A very thin circular curved beam is analyzed for its natural frequency in this project. Only out of plane vibrations are considered in this project. The stiffness matrix and mass matrix are derived from the strain energy and kinetic energy. This is done with the help of natural shape functions. The derivations are done in local coordinate system or Global Cartesian coordinate system. The out of plane deformations considered are the rigid body displacement of the centre of curvature in the axial direction, the rigid body rotation about the centre of curvature in the radial direction, and the rigid body rotation about the centre of curvature in the circumferential direction at the mid cross section. For analysis FEM is used. Study of change of slenderness ratio on different modes of frequencies is done in this project. For tedious calculation Matlab 7.0 is used.

1.INTRODUCTION:

Curved beams have found many applications in civil, mechanical and aerospace engineering. Exact and efficient nonlinear analysis of structures, built up from beam components, using robust Numerical methods, e.g. finite element methods, should be based on proper nonlinear beam theories.

For computationally analyzing curved beams or arches, many prefer using straight beam elements based on straight beam theories. This is a simple and good approximation for slender curved beams or flexible curved beams although more elements will be used to get a satisfactory accuracy. Others prefer using curved beam/arch elements to analyze curved beams or arches based on slender beam theories to reduce the number of elements used

.

However, for thick and moderately thick curved beams, an increase in the accuracy of the finite element solution by increasing the number of straight beam elements or curved beam elements based on the slender beam theories has its limit, especially when long-term dynamic responses as well as strains and stresses in three-dimensional level are needed for design purposes. In this case, more refined curved beam theories should be used.

1.1 Mechanical vibration

This is the continuing and repetitive motion (often periodic) of a solid or liquid body. Vibration occurs in a variety of natural phenomena such as the oceanic tidal motion, in stationary and rotating machinery, in varied nature structures like ships and buildings, in vehicles, etc. It is observed that there exists a strong coupling between the mechanical vibration notions and the propagation of vibration and acoustic signals through both the air and the ground to create a possible source of annoyance, discomfort and physical damage to structures and people.[10]

1.2 Complex systems

For study of vibration, many simplistic assumptions are taken into consideration. These may include the input and response being periodic; the input being of discrete nature, which it is temporal in nature having no reference to spatial distribution; and a single resonant frequency and a single set of parameters are required to define the stiffness, the mass, and the damping. The real world is much more complex. Many sources of vibration are not always periodic. These may include impulsive forces and shock or impact loading, where a force is suddenly applied on the body or on the system for a very short time; random excitations, where the signal varies in time in such a way that its amplitude at any given point is expressed only in terms of a probability.

1.3 Sources of vibration

There are many sources of vibration, both mechanical and structure. The most common form of mechanical vibration problem is induced by machinery of variety, often (but not always) of the rotating type. Other sources of vibration are: ground-borne propagation due to construction; vibration due to movement of heavy vehicles on any type of pavement as well as vibrations generated due from the railway systems common in many big and developed areas; and vibrations induced by natural phenomena and events, like earthquakes and wind forces.

1.4 Effect of vibrations

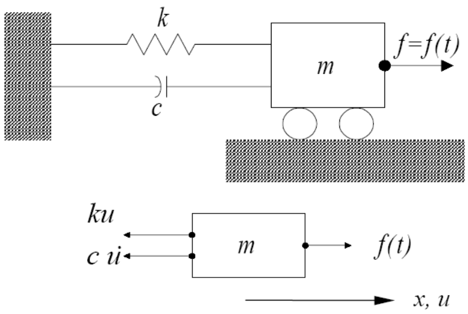
The most serious effect of vibration, especially in the case of machinery, is that sufficiently high alternating stresses can produce fatigue failure in machine and structural parts. Less serious effects include increased wear of parts, general malfunctioning of apparatus, and the propagation of vibration through foundations and buildings to locations where the vibration of its acoustic realization is intolerable either for human comfort or for the successful operation of sensitive measuring equipment.

1.5 Vibration in Mass-spring-damper system:

A mass spring damper system is used to understand some basic principles of vibration without undergoing any difficulty of complex system. Such a system contains a spring with spring constant k that restore the mass to a neutral position, a mass M and a damping element which opposes the motion of the spring with a force proportional to the velocity of the system, the damping constant c being the constant of proportionality. The damping force is dissipative in nature, and without the presence the resistive force of this mass-spring system will continue to be in a periodic motion.

Single DOF System

Fig.-1



$$\begin{cases} m \text{ - mass} \\ k \text{ - stiffness} \\ c \text{ - damping} \\ f(t) \text{ - force} \end{cases}$$

From Newton's law of motion ($ma = F$), we have

$$m\ddot{u} = f(t) - ku - c\dot{u},$$

i.e.

$$m\ddot{u} + c\dot{u} + ku = f(t), \qquad (1)$$

where u is the displacement, $\dot{u} = du/dt$ and $\ddot{u} = d^2u/dt^2$.

Free Vibration: $f(t) = 0$ and no damping ($c = 0$)

Eq. (1) becomes

$$m\ddot{u} + ku = 0 \qquad (2)$$

(meaning: inertia force + stiffness force = 0)

Assume:

$$u(t) = U \sin(\omega t),$$

where ω is the frequency of oscillation, U the amplitude.

Eq. (2) yields

$$-U\omega^2 m \sin(\omega t) + kU \sin(\omega t) = 0$$

i.e.,

$$\left[-\omega^2 m + k\right] U = 0 \, .$$

For nontrivial solutions, we must have

$$\left[-\omega^2 m + k\right] = 0 \, ,$$

which yields

$$\omega = \sqrt{\frac{k}{m}} \, . \qquad (3)$$

1.6 Modes of vibration

Mode of an oscillating system is a pattern of motion in which whole system move in sinusoidal with the same frequency. Any physical object has a set of normal modes that depend on its materials, structure and boundary conditions. The mode of vibration is characterized by a mode shape and modal frequency, and is numbered according to the number of half waves formed in the vibration. In a system with two or more dimensions, each dimension is given a mode number.

2.LITERATURE REVIEW:

W. P. Howson and A. K. Jemah [1] deduced a very effective method of determining the exact out of plane natural frequencies of curved beams. Stiffnesses are derived from the governing differential equations. These stiffnesses are used to yield a transcendental eigen value problem. Values obtained from this method are considered to be exact analytical solution to vibration of curved beam problem.

Bo Yeon Kim, Chang-Boo Kim [2] considered a thin finite circular beam element for the out-of-plane vibration analysis of curved beams. Its stiffness matrix and mass matrix were derived, respectively, from the strain energy and the kinetic energy. The effects of transverse shear deformation, transverse rotary inertia, and torsional rotary inertia, were presented.

C. G. Culver and D. J. Oestel [3] had developed the method of determining natural frequency in multispan curved beam. The method is illustrated in a two span beam. In their work they have used Rayleigh-Ritz method together with the Lagrange multiplier concept. Both methods led to very accurate results. The beam element considered in this case was of double symmetry, due to which the nature of the response was an uncoupled response for in-plane bending, and a coupling of the out of plane normal bending and the rotational responses.

R. Emre Erkmen and Mark A. Bradford [4] developed the 3D elastic total Lagrangian formulation for the numerical analysis of steel concrete composite beams which are curved in-plan. On the basis of geometric nonlinearities the strain expressions and the partial interaction at the interface in the tangential direction as well as in the radial direction were derived. The beam with large initial curvatures may behave as slender beams at the elastic range geometric nonlinearity. They had also shown that if the initial curvature of the beam is increased the behavior becomes significantly softer.

Jong-keun Choi and Jang-keun Lim [5] used the curved beam elements as their consistent form of strain fields simplifies their formulation. Hence on the basis of the assumed strain field and Timoschenko beam theory they developed two-noded and three noded curved beam element. These two elements include the axial, in plane and out of plane shear, bending and torsional deformations. Whereas the two-noded beam has the constant strain fields and the three-noded beam has the linear strain fields. The displacement functions were considered in the local curvilinear coordinate system, which were again used in the derivation of stiffness matrix by applying the total potential energy theorem. Then these local stiffness matrices were transformed into a global Cartesian coordinate system in order to obtain the global stiffness matrix.

By using the curved beam elements, John-Shyong Wu and Lieh-Kwang Chiang [6] obtained the dynamic responses of a circular curved Timoshenko beam under the application of a moving load. They considered the effect of shear deformation and that of rotary inertia resulted due to bending and torsional vibrations. Then the stiffness matrix and mass matrix were obtained from the force displacement relation and the kinetic energy equations respectively. As the element matrices of the curved beam element were based on local co-ordinate system, hence the coefficients were independent on d curved beam elements having a constant radius of curvature and the transformation from local stiffness matrix to global stiffness matrix was not needed.

Dipak Sengupta and Suman Dasgupta[7] used Lagrange Polynomials in natural co-ordinates for beam geometry interpolation and its vertical displacements. But the angles of transverse rotation and twist were interpolated by another set of three degree polynomial. After assuming the elastic deformations to be proportional to the reactive forces, the effect of shear deformations were considered in the stiffness matrix, whereas the translational and rotary inertias were considered in the formulation of mass matrix. But the flexural rotary inertia and torsion rotary inertia were neglected in dynamic loading cases. Four-point Gaussian scheme was used in numerical integration. Considering static loading with and without elastic foundation, displacements, bending moments and torque was calculated.

J. R. Hutchison [8] has done some of the work on shear coefficients for Timoshenko beam. His work shows that in a Timoshenko beam shear deformations and rotary inertia have effects on the vibration in slender beams. The formulation for shear coefficient is done in his work. The values of shear coefficient from his work have been used in the problems done in this paper.

Jong Shyong Wu and Lieh Kwang Chiang [9] tried to determine the dynamic responses of a circular curved Timoshenko beam due to a moving load using the curved beam elements. In addition to the typical circular curved beams, a curved beam composed of one curved beam segment and two straight beam segments subjected to a moving load was also studied. Influence on the dynamic responses of the curved beams of the slenderness ratio, moving-load speed, shears deformation and rotary inertias were investigated.

In this report, a thin beam element is considered for which cross section remains same throughout the length. Matrix calculation is done by stabilization of strain and kinetic energy. Analysis and determination of shape functions is similar to that given in [2]. The matrix calculation is done with a simplistic approach. The effect of change of local coordinate system is not considered for matrix assembly as the elements are compatible at connecting nodes.

3.Analysis of Thin circular beam element:

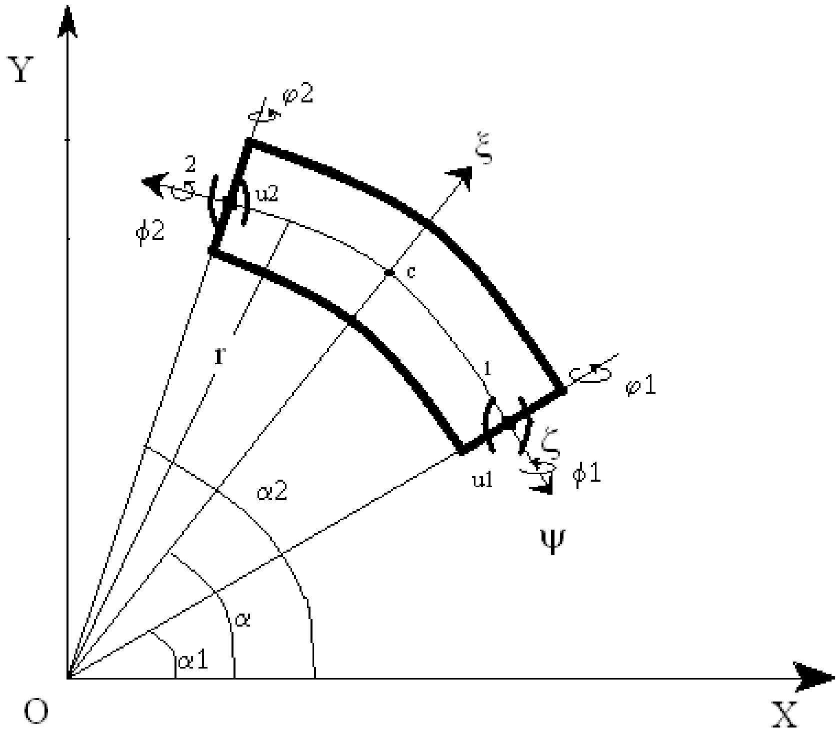


Fig.2 *Thin circular beam showing out of plane Deformations in both Cartesian and local coordinate system*

3.1 Out-of-plane deformations

Along with the Cartesian system, the local coordinate system of the beam is also shown in polar form. ‘O’ is the center of curvature of the circular thin beam element and C is the center of cross section. The radius of the centroidal line passing through the center of cross section is ‘*r*’. Half of the subtended angle of the element at center is $\theta = (\alpha 2 - \alpha 1)/2$. The nodes of the element, *1* and *2* are on the centroidal line.

ξ is the local centroidal axis, ψ is the local circumferential axis and ζ is the local vertical axis of cross section. When only out of plane deformations are considered, ‘*u’* is the rigid body displacement of the center of curvature along ζ axis, ‘φ’ is the radial component of rotation and ‘ϕ ’ is the circumferential component of the rotation at C.

The bending curvature κ_ξ, twist τ_ψ, and shear strain γ_ζ at C, are expressed as

$$\kappa_\xi = (\varphi' - \phi)/ r \qquad (1a)$$
$$\tau_\psi = (\phi' + \varphi)/ r \qquad (1b)$$
$$\gamma_\zeta = -\varphi + u'/ r \qquad (1c)$$

Where ( )' is a partial differential with respect to circumferential coordinate α.

The internal bending moment M_ξ, torsion moment M_ϕ, and shear force N_ζ at the point C can be expressed as

$$M_\xi = EI_\xi \kappa_\xi \qquad (2a)$$
$$M_\phi = GJ_\psi \tau_\psi \qquad (2b)$$
$$N_\zeta = K_\zeta GA \gamma_\zeta \qquad (2c)$$

Where A is area, I_ξ is he area moment of inertia about ξ –axis, J_ψ is the torsional moment of inertia, K_ζ is the shear coefficient of the cross section. E is Young's modulus of the material. G is the shear modulus, which is expressed as $G=E/2(1+\nu)$ with the Poisson ratio ν.

3.2 Shape functions:

Out-of-plane forces and moments are applied at nodes 1 and 2 of the circular beam element in equilibrium. The internal bending moment, torsion moment, and shear force on the cross section at C can be expressed in terms of the internal bending moment $M_{\xi 0}$, torsion moment $M_{\phi 0}$ and shear force $N_{\zeta 0}$ on the mid cross section at $\theta=0$ as

$$M_\xi = M_{\xi 0} c\theta + (M_{\phi 0} - r N_{\zeta 0}) s\theta \qquad (3a)$$
$$M_\phi = -M_{\xi 0} s\theta + (M_{\phi 0} - r N_{\zeta 0}) c\theta + r N_{\zeta 0} \qquad (3b)$$
$$N_\zeta = N_{\zeta 0} \qquad (3c)$$

Where $s\theta=\sin\theta$, $c\theta=\cos\theta$, $\theta=\alpha-(\alpha 2+\alpha 1)/2$.

By substituting Eq. (3) into Eq. (2), we get

κ_ξ=($B_5 c\theta$+ $B_6 s\theta$)/ r^2 (4a)
τ_ψ =β*(B_4−$B_5 s\theta$+ $B_6 c\theta$)/ r^2 (4b)
γ_ζ =a* B_4/r (4c)

where

B_4= r^3 $N_{\zeta 0}$ / EI_ξ (5a)
B_5= r^2 $M_{\xi 0}$ /EI_ξ (5b)
B_6= r^2 ($M_{\phi 0}$ −r $N_{\zeta 0}$)/ EI_ξ (5c)

a = EI_ξ/ K_ζ GA r^2 (6a)
β= EI_ξ/ G J_ψ (6b)

If a =0, then the effect of transverse shear deformation is neglected, i.e γ_ζ =0 .

By substituting Eq. (4) into Eq. (1), we get

($B_5 c\theta$ + $B_6 s\theta$)/ r^2= (φ’−ϕ)/ r (7a)
β*(B_4−$B_5 s\theta$ + $B_6 c\theta$)/ r^2= (ϕ’+φ)/ r (7b)
a* B_4/r=−φ + u’/ r (7c)

B_1, B_2/r and B_3/r are the constants of integration of the differential equations. They are the rigid body displacement of the center of curvature in the axial direction, the rigid body rotation about the center of curvature in the radial direction, and the rigid body rotation about the center of curvature in the circumferential direction at the mid cross section, respectively.

Adding Eqn(7a)’ and (7b) we get

(φ’’+ φ)/ r= (- $B_5 s\theta$+ $B_6 c\theta$)/ r^2+ β(B_4−$B_5 s\theta$+ $B_6 c\theta$)/ r^2

Solving this differential eqn with boundary condition $\varphi(0) = B_2/r$ and $\varphi'(0)= (B_5+ B_3)/a$ we get

$$\varphi = \{B_2 c\theta+ B_3 s\theta+ B_4 f_2 (1-c\theta) + B_5 (f_4 s\theta+f_3\theta c\theta) +B_6 f_3\theta s\theta\}/ r \qquad (8a)$$

Subtracting eqn (7a) from (7b) ' we get

$$(\phi''+\phi)/ r= \beta(-B_5 c\theta- B_6 s\theta)/ r^2-(B_5 c\theta+ B_6 s\theta)/ r^2$$

solving this differential eqn with boundary condition $\phi (0)= B_3/r$ and $\phi'(0)= \beta(B_4 +B_6 c\theta)/ r - B_2/r$ we get

$$\phi = \{-B_2 s\theta+B_3 c\theta+B_4 f_2 s\theta-B_5 f_3\theta s\theta-B_6(f_4 s\theta-f_3\theta c\theta)\}/ r \qquad (8b)$$

Putting eqn (8b) in (7c) we get

$$(u)'=a*B_4+\{B_2 c\theta+ B_3 s\theta + B_4 f_2 (1-c\theta) + B_5 (f_4 s\theta+f_3\theta c\theta)+B_6 f_3\theta s\theta\}$$

solving this differential eqn with boundary condition $u(0)=B_1$ we get

$$u = B_1+B_2 s\theta-B_3 c\theta+B_4 (f_1\theta+f_2\theta-f_2 s\theta) +B_5 (-f_2+f_2 c\theta+f_3\theta s\theta) + B_6 (f_2 s\theta+f_4 s\theta- f_3\theta c\theta) \qquad (8c)$$

Where

$$f_1 =a \qquad (9a)$$
$$f_2 =\beta \qquad (9b)$$
$$f_3=(1+\beta)/ 2 \qquad (9c)$$
$$f_4 =(1-\beta)/ 2 \qquad (9d)$$

The static deformations represented by Eq. (8) are used as the shape functions for the out-of-plane motion of thin circular beam element. They are the rigid body modes associated with B_1, B_2/r, and B_3/r and the flexible modes associated with B_4, B_5, and B_6.

The displacements and rotations at nodes, 1 and 2, can be expressed as

$$\{v\}=[a]\{B1\} \tag{10}$$

where

$$\{v\}=\begin{bmatrix} \varphi_1 \\ \phi_1 \\ u_1 \\ \varphi_2 \\ \phi_2 \\ u_2 \end{bmatrix} \tag{11a}$$

$$\{B1\} = \{B_1\ B_2\ B_3\ B_4\ B_5\ B_6\}^T \tag{11b}$$

$$[a]=\begin{bmatrix} 0 & c\theta/r & -s\theta/r & f_2\,(1-c\theta)/r & -(f_4 s\theta+f_3\theta c\theta)/r & f_3\theta s\theta/r \\ 0 & s\theta/r & c\theta/r & -f_2 s\theta/\,r & -f_3\theta s\theta/\,r & (f_4 s\theta+f_3\theta c\theta)/\,r \\ 1 & -s\theta & -c\theta & -(f_1\theta+f_2\theta-f_2 s\theta) & -f_2+f_2 c\theta+f_3\theta s\theta & -(f_2 s\theta+f_4 s\theta)+f_3\theta c\theta \\ 0 & c\theta/r & s\theta/r & f_2\,(1-c\theta)/r & (f_4 s\theta+f_3\theta c\theta)/r & f_3\theta s\theta/r \\ 0 & -s\theta/r & c\theta/r & f_2 s\theta/\,r & -f_3\theta s\theta/\,r & -(f_4 s\theta-f_3\theta c\theta)/\,r \\ 1 & s\theta & -c\theta & f_1\theta+f_2\theta-f_2 s\theta & -f_2+f_2 c\theta+f_3\theta s\theta & (f_2 s\theta+f_4 s\theta)-f_3\theta c\theta \end{bmatrix}$$

The coefficient vector of shape functions, $\{B1\}$ then can be expressed in terms of the nodal displacement vector with respect to the local polar coordinate system, $\{v\}$, as

$$\{B1\}=\{a\}^{-1}\{v\} \tag{12}$$

3.3 Stiffness matrix:

Eq.4 can be written in the matrix form as

$$[strain]=[R]\{B1\}$$

Or, $[strain]=[R]\ \{a\}^{-1}\{\ v\}$

$$=[B]\{v\}$$

Where, $[strain]=\{\kappa_{\xi}\ \tau_{\psi}\ \gamma_{\zeta}\}$

$[B]$ =strain-displacement matrix.= $[R]\ \{a\}^{-1}$ (13)

$[R]=1\ /\ r^{2}\ *$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & c\theta & s\theta \\ 0 & 0 & 0 & \beta & -\beta s\theta & \beta c\theta \\ 0 & 0 & 0 & r*\alpha & 0 & 0 \end{bmatrix}$$

From eq.(2)

$$[Stress] = [D][strain]$$

Where, $[D]$ = strain stress matrix

=

$$\begin{bmatrix} E*I_{\xi} & 0 & 0 \\ 0 & G*J_{\psi} & 0 \\ 0 & 0 & K_{\zeta}G*A \end{bmatrix}$$

Therefore, ***stiffness matrix*** is given by

$$[K] = \int_{\alpha 1}^{\alpha 2} [B]^{T}[D][B]*r*d\theta \qquad (14)$$

3.4 Mass Matrix

The kinetic energy of the thin circular beam element is expressed as

$$T=1/2 *_{\alpha1}\int^{\alpha2} \rho I_{\xi}(\varphi')^2+\rho I_{\psi}(\phi')^2+\rho A(u')^2 * r^*d\theta \qquad (15)$$

Taking ρ*r common and arranging the terms inside bracket in form of matrix we get

$$T= 1/2 *_{\alpha1}\int^{\alpha2} \rho.r^* \begin{Bmatrix} \varphi' \\ \phi' \\ u' \end{Bmatrix} \begin{bmatrix} I_{\xi} & 0 & 0 \\ 0 & I_{\psi} & 0 \\ 0 & 0 & A \end{bmatrix} \begin{Bmatrix} \varphi' & \phi' & u' \end{Bmatrix} d\theta$$

$$=1/2*_{\alpha1}\int^{\alpha2}\{v'\}^T([N]\{a\}^{-1})^T [N]\{a\}^{-1}\{v'\}\rho.r^* d\theta$$

(16)

Where, [N] is derived from eqn (8) as $[\varphi \ \ \phi \ \ u]^T = [N]\ \{B1\}$

$$[N] = \begin{bmatrix} 0 & c\theta/r & s\theta/r & f_2 (1-c\theta)/r & (f_4 s\theta+f_3\theta c\theta)/r & f_3\theta s\theta/r \\ 0 & -s\theta/r & c\theta/r & f_2 s\theta/ r & -f_3\theta s\theta/ r & -(f_4 s\theta-f_3\theta c\theta)/ r \\ 1 & s\theta & -c\theta & (f_1\theta+f_2\theta-f_2 s\theta) & -f_2+f_2 c\theta+f_3\theta s\theta & (f_2 s\theta+f_4 s\theta)-f_3\theta c\theta \end{bmatrix}$$

(18)

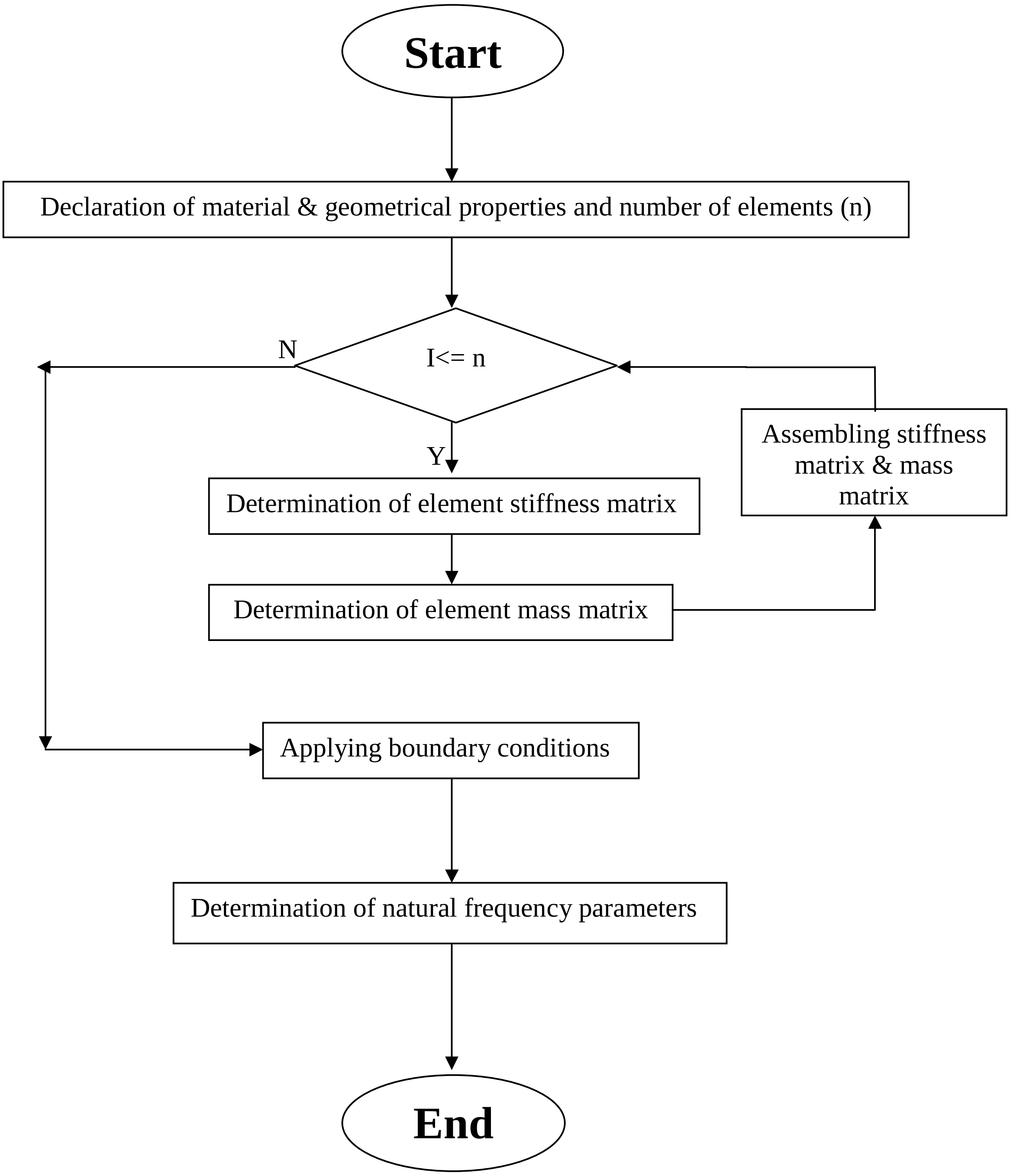
So, ***mass matrix*** is

$$[M] = \int_{\alpha 1}^{\alpha 2} \rho.r * ([N] \{a\}^{-1})^{T} \begin{bmatrix} I_{\xi} & 0 & 0 \\ 0 & I_{\psi} & 0 \\ 0 & 0 & A \end{bmatrix} [N] \{a\}^{-1} *d\theta \qquad (19)$$

Since all the element property matrices for the curved beam element are derived based on the local polar coordinate system (rather than the local Cartesian one), their coefficients are invariant for any curved beam element with constant radius of curvature and subtended angle and one does not need to transform the property matrices of each curved beam element from the local coordinate system to the global one to achieve the overall property matrices for the entire curved beam structure before they are assembled. The elements are compatible at nodes.

4. Flow-Chart

The flow-chart of the program used in determination of stiffness matrix, mass matrix and natural frequency parameter:



5. RESULTS AND DISCUSSIONS

Table 1, 2 and 3 shows the natural frequencies of circular beam clamped at both ends. Ref.-1 shows the values obtained by Howson [1] and Ref.-2 are the values obtained by Yeon Kim [2]. It is very much evident from the graph and tables that there is a good agreement between all results.

E = 200 GPa, $\nu = 0.3$, $\rho = 7830$ kg/m^3, r = 1 m, $I_\xi = 1.5\times10^{-5}$ m^4, $I_\psi = 3.0\times10^{-5}$ m^4, $J_\psi = 3.0\times10^{-5}$ m^4, $K_\zeta = 0.89$

Frequency parameter, $\lambda = \sqrt{(A*r^2/ I_\xi)}$

And slenderness ratio, $s_\xi = \omega\sqrt{(\rho* A*r^4/E\, I_\xi)}$

Table 1: Comparison of frequency parameter of out of plane of a clamped circular arc subtending angle 60^o

s_ξ	**Mode Number**	α= 60^o		
		Ref.-1	**Ref.-2**	**PRESENT**
20	1	16.885	16.885	16.8853
	2	39.700	39.706	39.7048
	3	40.934	40.940	40.9399
	4	70.581	70.612	70.6110
100	1	19.454	19.454	19.4538
	2	54.148	54.148	54.1482
	3	105.86	105.87	105.8652
	4	173.16	173.18	173.1774

Fig 3: Plot of three natural frequency results for **α**= 60^o and s_ξ=20

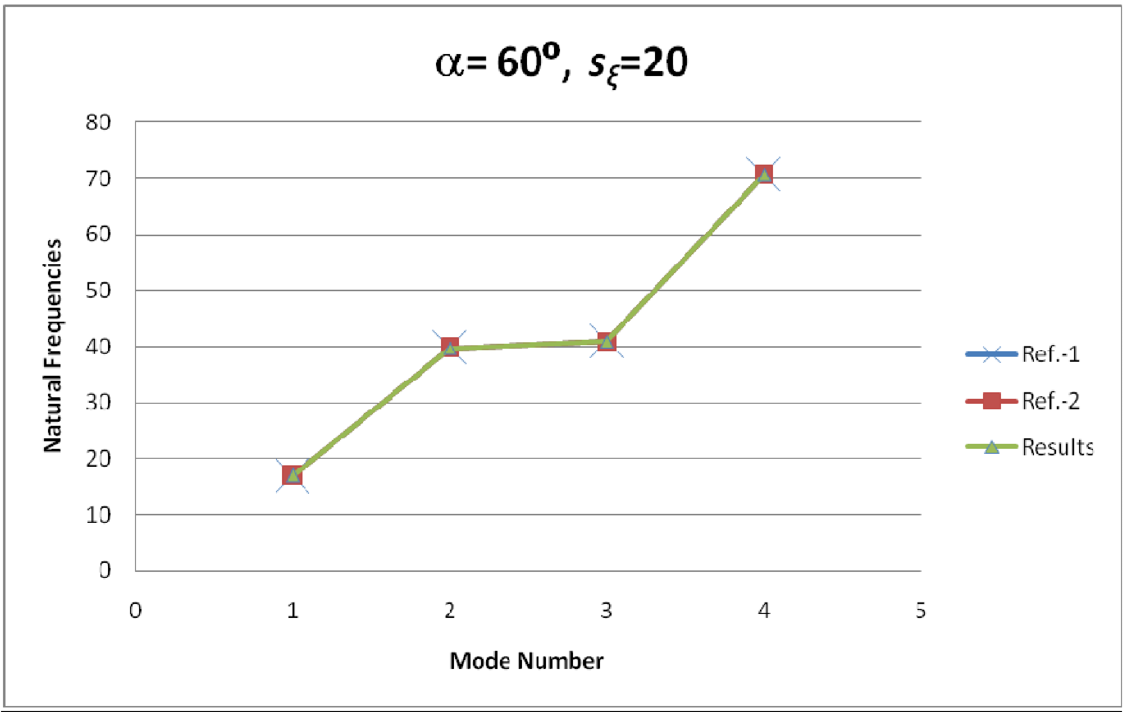


Fig 4: Plot of three natural frequency results for **α**= 60^o and s_ξ=100

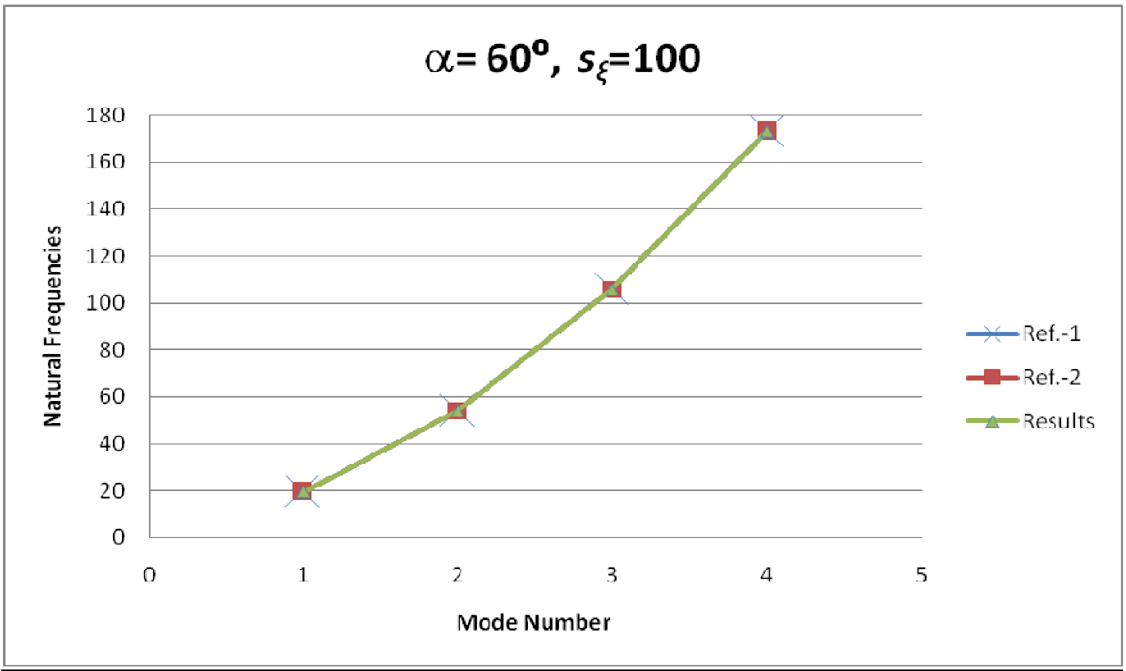


Table 2: Comparison of frequency parameter of out of plane of a clamped circular arc subtending angle 120^o

s_ξ	Mode Number	α =120^o		
		Ref.-1	**Ref.-2**	**PRESENT**
20	1	4.3094	4.3094	4.3094
	2	11.796	11.796	11.7961
	3	22.510	22.511	22.5111
	4	23.303	23.304	23.3035
100	1	4.4731	4.4731	4.4731
	2	12.892	12.892	12.8916
	3	26.081	26.081	26.0806
	4	43.684	43.684	43.6843

Fig 5: Plot of three natural frequency results for **α=** 120^o and s_ξ=20

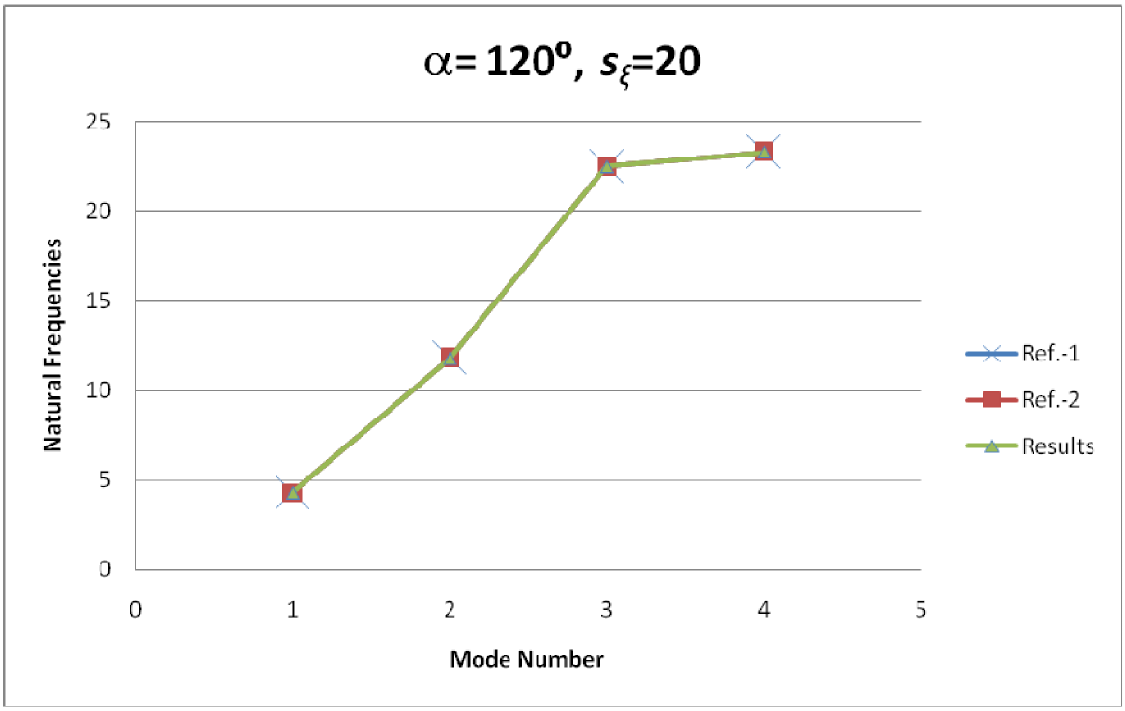


Fig 6: Plot of three natural frequency results for α= 120^{o} and s_{ξ}=100

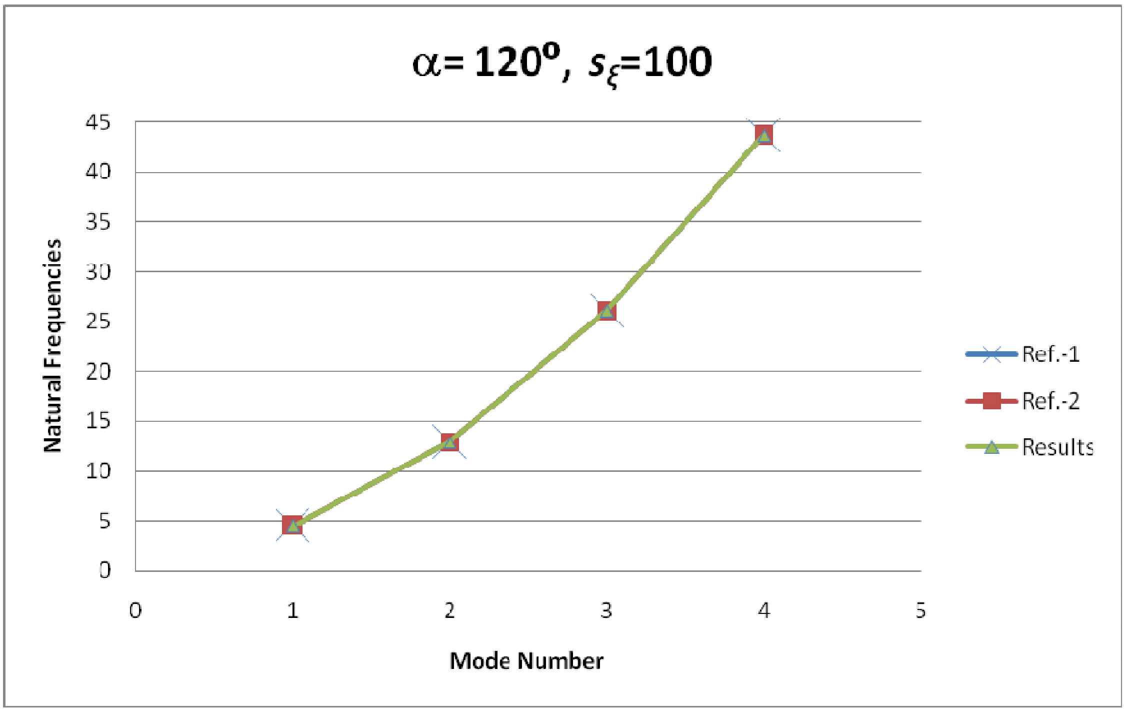


Table 3: Comparison of frequency parameter of out of plane of a clamped circular arc subtending angle 180^{o}

s_{ξ}	Mode Number	α=180 ^{o}		
		Ref.-1	**Ref.-2**	**PRESENT**
20	1	1.7908	1.7908	1.7908
	2	5.0324	5.0324	5.0324
	3	10.232	10.232	10.2323
	4	16.917	16.918	16.9177
100	1	1.8182	1.8182	1.8182
	2	5.2415	5.2415	5.2415
	3	10.989	10.989	10.9889
	4	18.813	18.813	18.8134

Fig 7: Plot of three natural frequency results for α= 180° and s_ξ=20

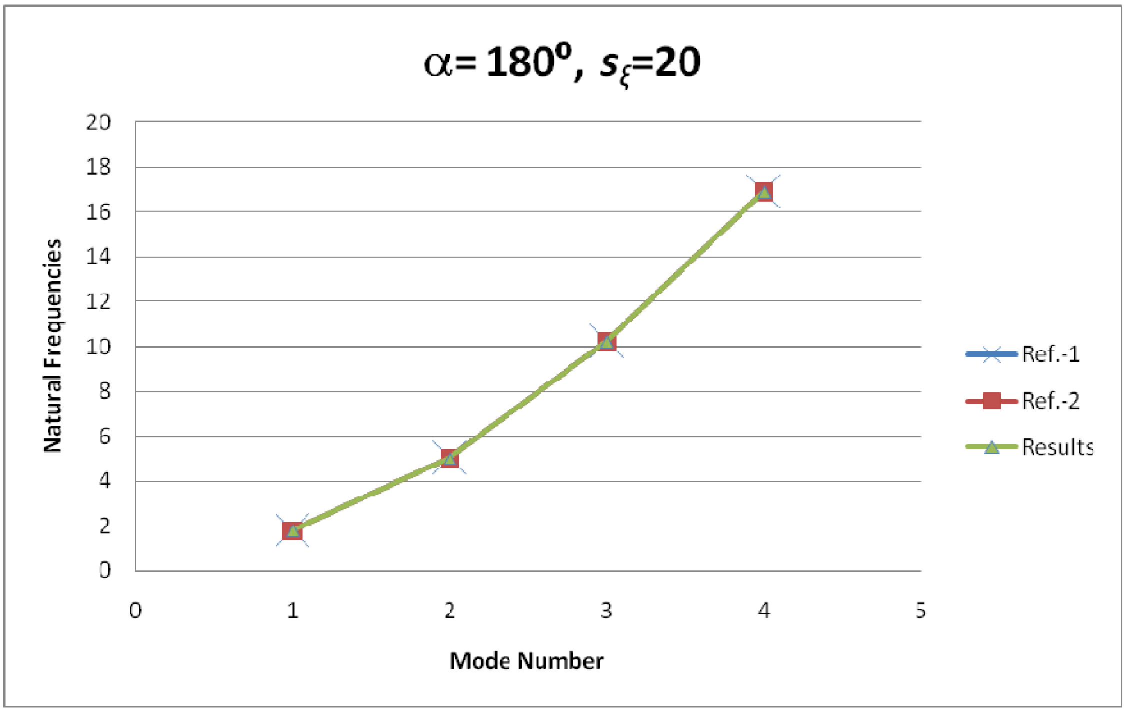
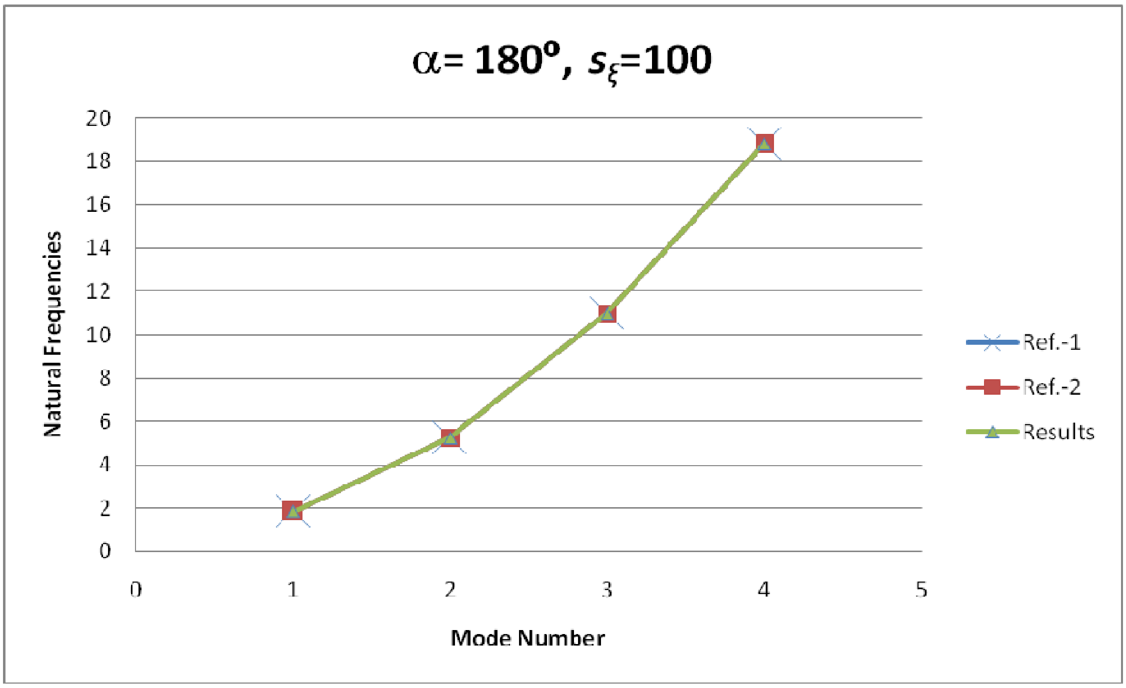


Fig 8: Plot of three natural frequency results for α= 180° and s_ξ=100



New results:

Keeping other parameter same such as material properties and shear coefficients, and by changing the slenderness ratio and angle subtended by the circular arc at centre of curvature following results were obtained.

Table 4: Frequency parameter of a clamped circular arc subtending angle 45°

α	Mode Number	s_ξ			
		20	**50**	**80**	**100**
45°	1	27.8613	33.7647	34.7424	34.9813
	2	51.4874	88.9404	94.5267	95.9843
	3	63.4709	128.9513	180.6708	185.3154
	4	100.1443	164.1954	206.3712	257.9775

Fig 9: Plot between natural frquency parameter and mode number of frequency for **α**= 45° and different s_ξ values.

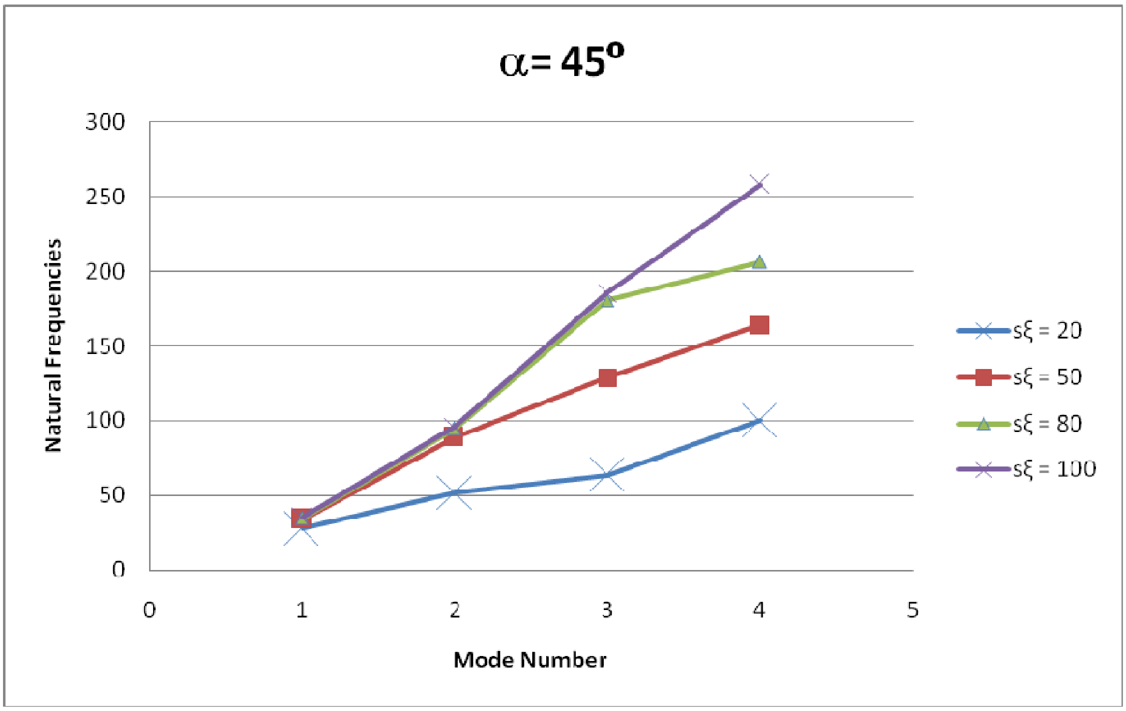


Table 5: Frequency parameter of a clamped circular arc subtending angle 60°

α	Mode Number	s_{ξ}			
		20	**50**	**80**	**100**
60 °	1	16.8853	19.0624	19.3784	19.4538
	2	39.7048	51.7326	53.6675	54.1482
	3	40.9399	98.1597	104.2694	105.8652
	4	70.6110	99.4945	159.2184	173.1774

Fig 10: Plot between natural frquency parameter and mode number of frequency for α= 60° and different s_{ξ} values.

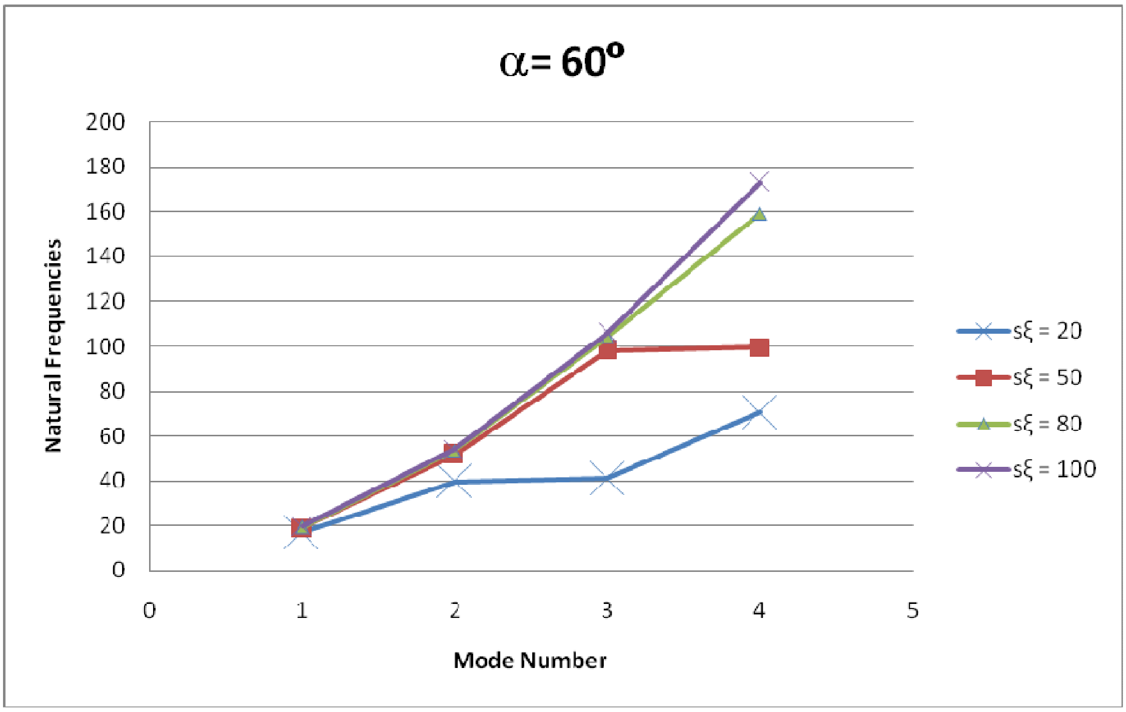


Table 6: Frequency parameter of a clamped circular arc subtending angle 90^o

α	**Mode Number**	s_ξ			
		20	**50**	**80**	**100**
90^o	1	7.7846	8.2495	8.3097	8.3238
	2	20.4151	23.1607	23.5623	23.6582
	3	28.4540	45.3880	46.7420	47.0740
	4	37.3564	71.3495	77.2028	78.0383

Fig 11: Plot between natural frquency parameter and mode number of frequency for **α**= 90^o and different s_ξ values.

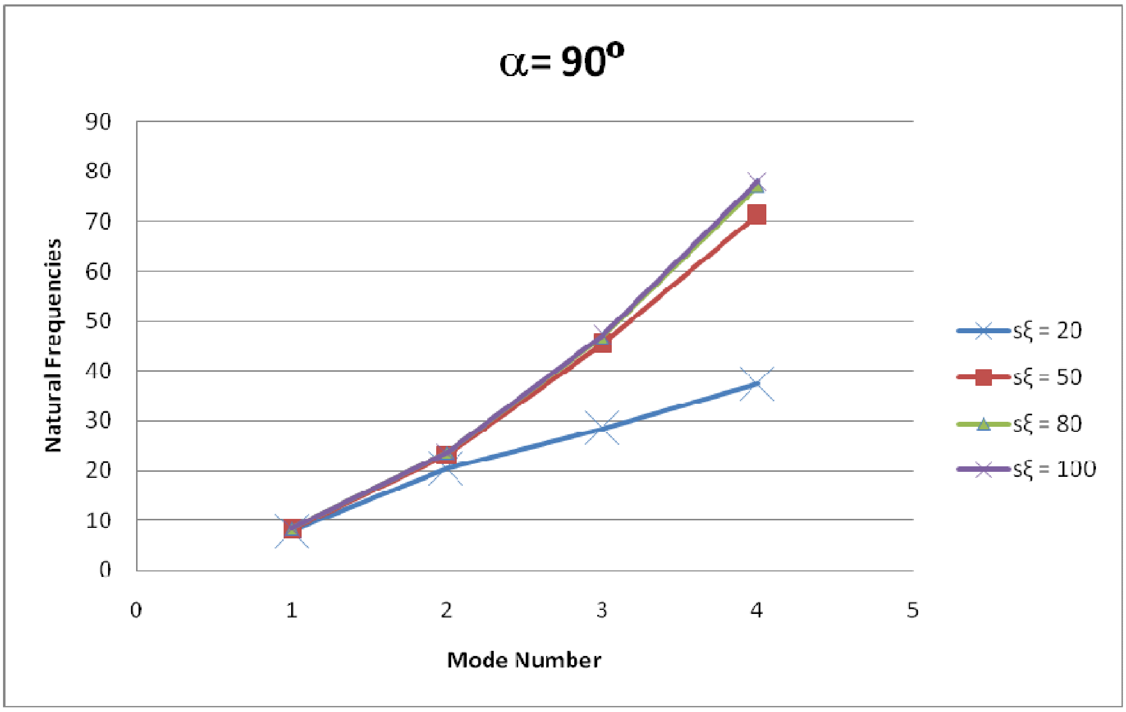


Table 7: Frequency parameter of a clamped circular arc subtending angle 120°

α	Mode Number	s_ξ			
		20	**50**	**80**	**100**
120°	1	4.3094	4.4515	4.4690	4.4731
	2	11.7961	12.7365	12.8621	12.8916
	3	22.5111	25.5343	25.9752	26.0806
	4	23.3035	42.3015	43.4133	43.6843

Fig 12: Plot between natural frquency parameter and mode number of frequency for **α**= 120° and different s_ξ values.

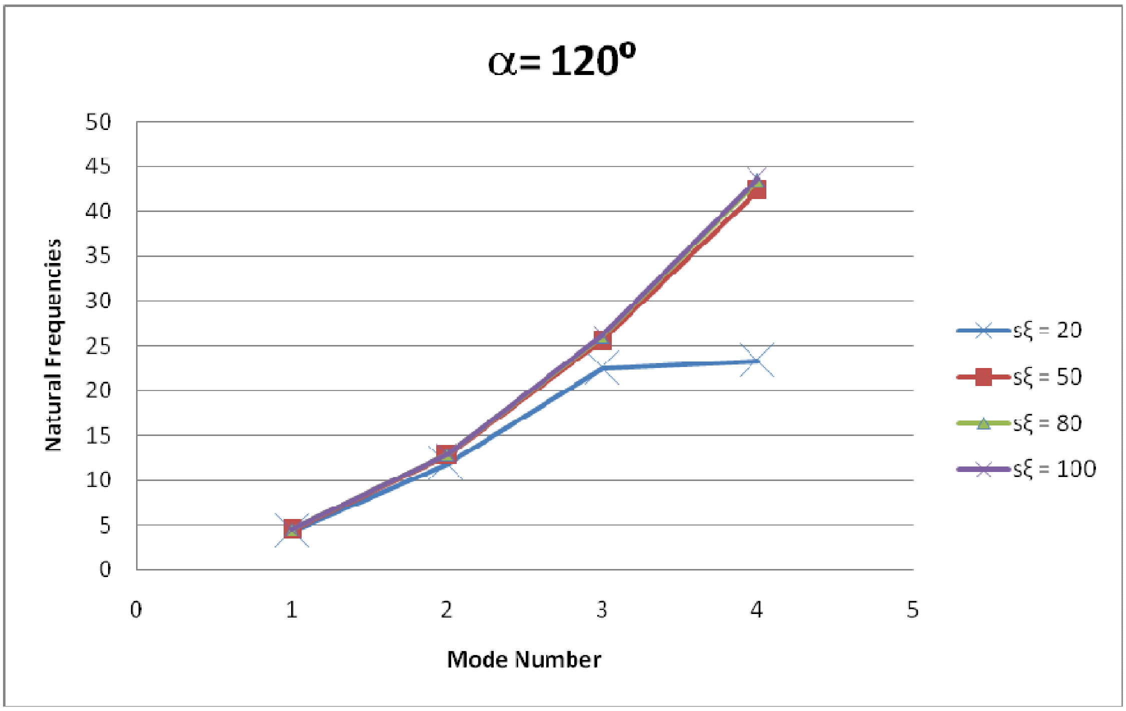


Table 8: Frequency parameter of a clamped circular arc subtending angle 180°

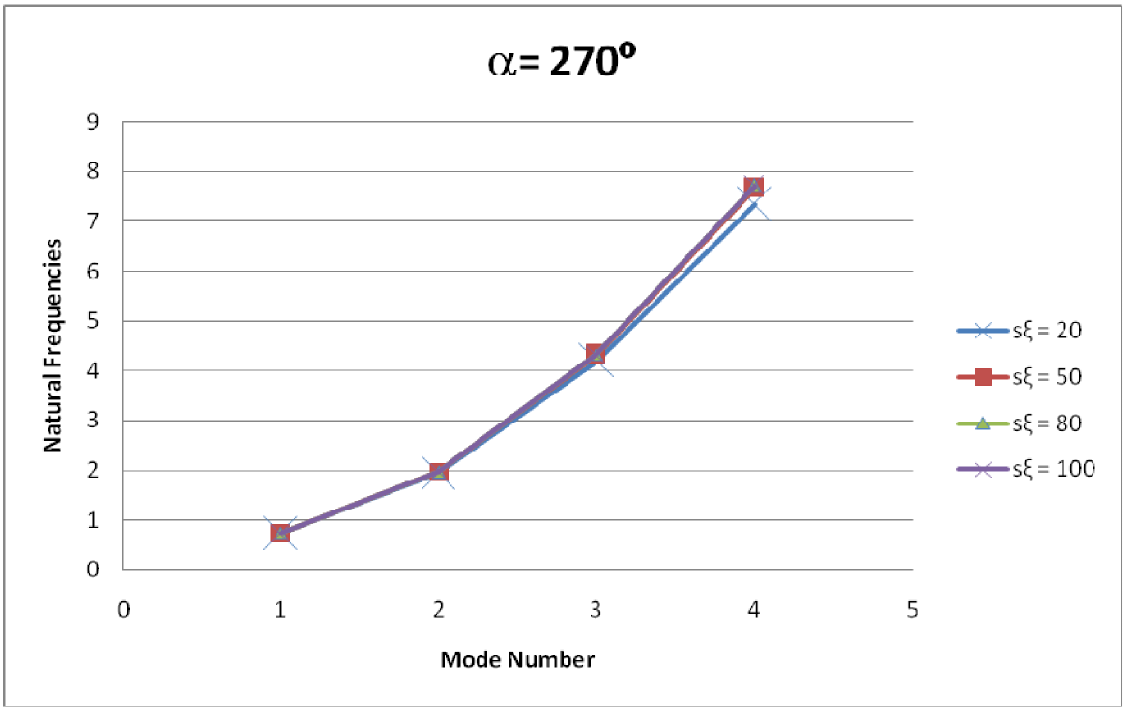
α	Mode Number	s_ξ			
		20	**50**	**80**	**100**
180°	1	1.7908	1.8147	1.8175	1.8182
	2	5.0324	5.2138	5.2363	5.2415
	3	10.2323	10.8842	10.9690	10.9889
	4	16.9177	18.5367	18.7604	18.8134

Fig 13: Plot between natural frquency parameter and mode number of frequency for **α**= 180° and different s_ξ values.

Table 9: Frequency parameter of a clamped circular arc subtending angle 270°

α	**Mode Number**	s_ξ			
		20	**50**	**80**	**100**
270°	1	0.7311	0.7349	0.7353	0.7354
	2	1.9356	1.9646	1.9680	1.9688
	3	4.1990	4.3189	4.3336	4.3370
	4	7.3449	7.6729	7.7143	7.7239

Fig 14: Plot between natural frquency parameter and mode number of frequency for **α**= 270° and different s_ξ values.



5. CONCLUSION

The analysis of vibration of a thin walled circular curved beam is presented in this report. The results obtained are in good agreement with published results. The results are accurate and precise. This analysis holds good for linear static problems. The natural frequencies converge to the theoretical results. From the graph (Fig 5 to 14), it is very much evident that for smaller angle subtended by beam, frequencies vary greatly with variation in slenderness ratio for higher mode number. As the angle subtended increases, the frequency curve converges gradually. And for greater angle subtended (keeping other parameters constant) the natural frequency obtained is less.

6. SCOPE FOR FUTURE WORKS:

- Beams of different geometrical shapes (like elliptical etc.) can be formulated and analyzed so that it will be useful in enhancing the aestheticism.
- Beams of various material (like thin wall sandwiched beam) can be analyzed that will impart more strength and stability to structures.
- Improvements can be done by the application of combination of differential materials in multi-span curved beams.
- Non-linear analysis of curved beams can be done which will help in vibrational analysis of thick curved beams.

7. References:

[1] W. P. Howson and A. K. Jemah, Exact out-ofplane natural frequencies of curved Timoshenko beams, *J. Eng. Mech* 125 (1) (1999) 19-25.

[2] Bo Yeon Kim, Chang-Boo Kim*, Seung Gwan Song, Hyeon Gyu Beom and Chongdu Cho, A finite thin circular beam element for out-of-plane vibration analysis of curved beams, Journal of Mechanical Science and Technology 23 (2009) 1396~1405.

[3] C. G. Culver, D. J. Oestel;**,** Natural Frequencies Of Multispan Curved Beams, J.S ound Vib.(1 969) 10(3),3 80-389.

[4] R. Emre Erkmen, Mark A. Bradford, Nonlinear elastic analysis of composite beams curved in-plan, Engineering Structures 31 (2009) 1613_1624.

[5] J. K. Choi and J. K. Lim, General curved beam elements based on the assumed strain fields, *Comp and Struct.* 55 (3) (1995) 379-386.

[6] J. S. Wu and L. K. Chiang, Out-of-plane responses of a circular curved Timoshenko beam due to a moving load, *Int. J. Solids Structures* 40 (2003) 7425-7448.

[7] Dipak Sengupta, Suman Dasgupta, Static And Dynamic Applications Of A Five Noded Horizontally Curved Beam Element With Shear Deformation, International Journal For Numerical Methods In Engineering, Vol. 40, 1801*Đ*1819 (1997).

[8] J. R. Hutchinson, Shear Coefficients for Timoshenko Beam Theory, DOI:10.1115/1.1349417.

[9] Jong-Shyong Wu, Lieh-Kwang Chiang, Out-of-plane responses of a circular curved Timoshenko beam due to a moving load, International Journal of Solids and Structures 40 (2003) 7425–7448.

[10] Daniel J. Inman, Engineering Vibration(3rd edition).

[11] David H. Hutton, Fundamentals Of Finite Element Analysis.