OPTIMAL VIBRATION CONTROL OF FIBER REINFORCED POLYMER COMPOSITE SHELL PANEL

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By

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CERTIFICATE

This is to certify that the thesis entitled, "**OPTIMAL VIBRATION CONTROL OF FIBER REINFORCED POLYMER COMPOSITES SHELL PANEL"** submitted by VIJAY KUMAR and SURAT PRAKASH DUNDUNG in partial fulfillment of the requirements for the award of Bachelor of Technology in **Mechanical Engineering** at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any Degree or Diploma.

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ABSTRACT

The concept of optimal vibration control using LQR (Linear Quadratic Regulator) is a new area of research of the shell structure. Many research have been done previously for the optimal vibration control. In this thesis it is mainly focused on the optimal vibration control of the FRP composites of shell structures using sensors and actuators. The vibration occurs when impulse loads is applied for certain period of time and the types of vibration depend on the material properties. So using LQR technique the vibration is controlled of the shell structures of FRP composites.

CONTENTS

LIST OF SYMBOLS

ξ ,η = curvilinear coordinates

ζ = linear coordinate

Φ_a^e = degree of freedom at piezoelectric actuator

Φ_s^e = degree of freedom at piezoelectric sensor

Ψ = modal matrix.

ξ_{di} = damping ratio.

$\alpha_1, \alpha_2, \gamma$ = coefficients associated with total kinetic energy, strain energy and input energy respectively

$\overline{\prod}$ =weighted energy

$[\varepsilon]$ = strain component

$\{\sigma\}$ = stress vector

$\{\varepsilon\}$ = strain vector

[e] = piezoelectric coupling constant

$[\epsilon]$ = dielectric constant matrix.

$[\Phi_a]$ = control input

[Q], [R]. = weighting matrices

J_N =quadratic loss function

Q_0, Q_1 Q_2 = semi definite symmetric matrices

i = discrete time index

v_i, w_i = discrete time Gaussian

J = Minimized quadratic cost function

K_i = Kalman gain

P_i = Riccati difference equation which runs forward in time

L_i = feedback gain matrix

S_i = Riccati difference equation which runs backward in time

N_k = shape function at node k

{D} = electric displacement vector

{E} = electric field vector

$\{E_a^e\}$ = electric field potential

$[B_a^e]$, $[B_s^e]$ = electric field gradient matrices of the actuators and sensors respectively

$[M_{uu}^e]$ = structural mass

$[K_{uu}^e]$ = structural stiffness

$[K_{ss}^e]$ = Dielectric conductivity

$[M_{uu}]$ = global mass matrix,

$[K_{uu}]$ = global elastic stiffness matrix,

$[K_{ua}]$, $[K_{us}]$ = global piezoelectric coupling matrices of actuator and sensor patches

$[K_{aa}]$, $[K_{ss}]$ = global dielectric stiffness matrices of actuator and sensor patches respectively.

$[u_d]$ = disturbance input vector,

$[K_{\Phi w}]$ = sensor coupling matrix

J = cost function

[Q] = positive definite weighting matrices on the output

[R] = positive definite weighting matrices on the control input

$[G_c]$ = optimal gain

$[\hat{R}]$ = dielectric coupling matrix of the actuator

J_N = quadratic loss function

LIST OF ABBREVIATIONS

LQR= linear quadratic regulator

LQG=linear quadratic Gaussian

LQE=linear quadratic estimator

GA=genetic algorithm

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1. INTRODUCTION

Composite materials are made up of two or more materials having different chemical and physical properties which are separate and distinct in a macroscopic level within the finished structure. Composite materials are extensively used in the manufacture of aerospace structures and effort is being put for the development of smart and intelligent structures. A lot of research is going on this technical development especially in the field of health monitoring, vibration and control of flexible structures using sensors and actuators placement in the host structure. Composite materials being light weight structure has low internal damping and higher flexibility and are susceptible to large vibration having large decay time. These structure require suitable integration of active control for better performance under operation. Piezoelectric materials which has flexible structure can act as sensors and actuators and it can provide self-monitoring and self-controlling capabilities to these structures. In many practical conditions, these structure experiences mechanical loading which are needed to be controlled for mechanical responses. Sensors, actuators and a controller are required for this type of vibration control. The design process of this kind of system has three main phases of structural design, optimal placement of sensors and actuators and design of controllers. A complete tool for the electro-mechanical analysis of such structure is necessary to solve such problem and control scheme is required for control.

Fiber Reinforced Polymer (FRP) composites are thin laminates which uses epoxy adhesives to bond externally the structural members. It is a combination of two or more material in macroscopic scale. It consists of two phase i.e 1.fiber 2.matrix. physical and chemical properties of individual material is never lost. The combination is done so as to get properties that are better and consecutively more reliable. FRP increase the members load carrying capacity. These are made up of high strength fibers embedded in a resin matrix. The resin protects the fibers, maintain their alignment and distributes the loads evenly among them. FRP has non-corrosive properties, speed and ease of installation, lower cost, and aesthetics.

Controllers are required to find the optimal gain to minimize the performance index. Open-loop and closed-loop are used for the placement of actuators and sensors. Design of controller avoids the work of finding out the control gain arbitrarily to solve the objectives and overcome the problems of saturation. Linear Quadratic Control (LQR) has been used to find the optimal gain by reducing the performance index in vibration control using the weighting matrices [Q] and [R]. These weighting matrices affects the output performance and the input cost, thus, the selection of weighting matrices is of much importance. Linear Quadratic Gaussian (LQG) is another method for determination of optimal gain

A number of works have been done for the active vibration control of smart structures and also for mechanical loading of these structures. Till now LQR technique has been found effective for the active vibration control with weighting matrices, which gives optimal control gain by minimizing the performance index. In this present work LQG control scheme has been proposed for controlling the dynamic oscillation due to mechanical loading gradient and to control all types of vibration of different types of loading of functionally graded composite materials.

2. LITERRATURE REVIEW

2.1 GENERAL

Composite materials offers superior properties of high strength and high stiffness to the metallic materials which has resulted in the large use of composite materials is the aircraft and aerospace industry. In order to develop such materials, Bailey and Hubbard [1] used the angular velocity at the tip of cantilever beam with constant gain and constant amplitude and experimentally achieved the control gain. Bhattacharya et al. [2] used LQR method for vibration suppression of spherical shells made up of laminated composites by trial and error method of selection of weighting matrices [Q] and [R]. Ang et al [3] proposed a method of selecting weighting matrices which is total energy method. Narayan and Balmurugun [4] presented a finite element modeling with distributed actuators and sensors and used LQR method to control the displacement by trial and error selection of matrices [Q] and [R]. Christensen and Santos [5] proposed an active control system to control blade and rotor vibration in a couple rotor blade system using tip mass actuators and sensors. Roy and Chakraborty [6] developed a GA based LQR control scheme to control the vibration of smart FRP structures with piezoelectric surface at surface minimizing the maximum displacement. Abdullah et al [7] used GA to simultaneously place actuators and sensors in the multi storey bulding and using the output feedback as the control law in terms of weighted energy of the system and concluded that the decision variable is hugely dependent on the selection of weighting matrices [Q] an [R]. Robandi et al [8] proposed a use of GA for optimal feedback control in multi machine power system. Yang et al [9] presented a simultaneous optimization by placing the sensors and actuators and the size of the sensor and actuator and feedback control gain for the vibration suppression of simply supported beam by minimizing the total mechanical energy of the system.

They did not consider the input energy and so the actuator voltage is was shown. Wang [10] presented the optimization of sensors and actuators pairs for torsional vibration control of a laminated composite cantilever plate using output feedback control. Reddy and Cheng [11] presented three dimensional solutions for smart functionally graded plates. Zhang and Kirpitchenko(2002) performed vibration suppression analysis of cantilevered beam with piezoelectric actuators and sensors to an exciting force. They considered two set of piezoelectric patches with three locations of patches and performed experiments which showed that the damping increased by 8-10 times of combined beam-piezoelectric patches in comparison to that of mechanical system. Bhattacharya et al. (2002) used LQR strategy for vibration suppression of spherical shells made of laminated composites by trial and error selection of [Q] and [R] matrices.

2.2 CONTROL THEORY

Control theory is a branch of engineering which deals with the behavior of dynamic system. The output of the system is called reference. When one or more output variables need to follow a certain reference overtime, controller is required which manipulates the inputs of a system to obtain the desired effect on the output of the system. There are two types of controllers open-loop controllers and closed loop controller. In open loop controller there is no direct connection between the output of the system and the conditions given to the system, which means that the unexpected errors cannot be compensated by the system. In a closed loop control system, a sensors monitors the output of the system and feeds the data into the system continuously as required to keep the minimum control error. A feedback allows the controller to compensate for disturbance of the system. An ideal feedback of the system cancel out all the errors, and it mitigates the effects of all the forces that may or may not arise during the operation and produces the response of the system which perfectly matches with the wishes of the user. In reality this cannot be achieved by the system because of error measurement of the sensors, delays in the controllers and imperfection in the control input. To avoid the problems of open loop controller feedback is used, process inputs have the effect on the process output which is measured by the sensors and processed by the controllers, the result is used as input to the process and closing the loop.

Closed loop transfer function have certain advantage over open loop transfer function

Disturbance rejection

Certain performance with modal uncertainties

Unstable process can be stabilized

Reduced sensitivity to parameter variations

improved reference tracking performance

Modern Control Theory in contrast to the time domain analysis, it uses the time domain state space representation, a mathematical model as a set of input, output and state variables related by first order differential equations. The input, output and state variables are represented in the matrix form and in the state space representation it provides a convenient way to analyze system with multiple input and outputs.

Optimal Control is a control technique in which the control signal optimizes a certain cost index and its application is widely used in industries

2.3 LQR (Linear Quadratic Regulator)

Linear Quadratic Regulator s concerned with the theory of control theory which operates under dynamic system at minimum cost. The system dynamic is expressed by a set of liner differential equations and cost is described by a quadratic functional called the LQ problem. The LQR is an important part for the solution obtained by the LQG problem. In LQR by Lyman's term the setting of a controller governing the controller or machine are found out using an algorithm which minimizes the cost function with the help of weighting matrices supplied by the humans. Cost is defined as the deviations in the measurements from the desired values. The algorithm finds the controller settings which minimizes the undesired deviations. The LQR algorithm finds the solution of the tedious work done by the control system in optimizing the controller. The engineer needs to specify the weighting matrices and compare the results. LQR is just an automated way of finding out the state-feedback-controller. Difficulty in finding the right weighting matrices reduces the use of LQR control technique.

2.4 LQG(Linear Quadratic Gaussian)

LQG (Linear Quadratic Gaussian) control deals with uncertain linear system which is distributed by additive white Gaussian noise and quadratic costs, LQG controller is a combination of LQE (Linear Quadratic Estimator) and LQR (Linear Quadratic Regulator). LQG control can be applied to linear time variant system and linear time invariant system. Linear time varying system is used for the designing of linear feedback controller for non-linear system. The solution of LQG problem is the most appropriate result in the control and system field. The LQG system is dynamic system and both have the same state dimension. if the dimensions is large then the LQG controller can be problematic and so to overcome this problem a number of states is fixed.

It is more difficult to solve as the problem is not separable and the solution is no more unique. Numerical algorithm solve the associated optimal projection equations which constitute n conditions for a locally optimal reduced-order LQG controller. A caution,. LQG optimality does not ensure good robustness properties. The robust stability of the closed loop system must be separately checked after the LQG controller has been designed.. The associated more difficult control problem leads to a similar optimal controller of which only the controller parameters are different

2.5 OBJECTIVES

1. To find the optimal gain using the Linear Quadratic Regulator (LQR) control and Linear Quadratic Gaussian (LQG) control
2. To find the responses of the vibration using unit impulse and unit step functions
3. To find the optimal feedback using the gain and response and finally controlling the vibration.

3. FORMULATION

3.1 FINITE ELEMNET FORMULATION

Here the Reissner - Mindlin assumption has been considered to describe the kinematics using the first order shear deformation theory. The basic assumptions are

i. The straight line normal to the mid surface may not remain straight during deformation
ii. The straight line corresponding to the stress component which is orthogonal to the mid surface is disregarded.

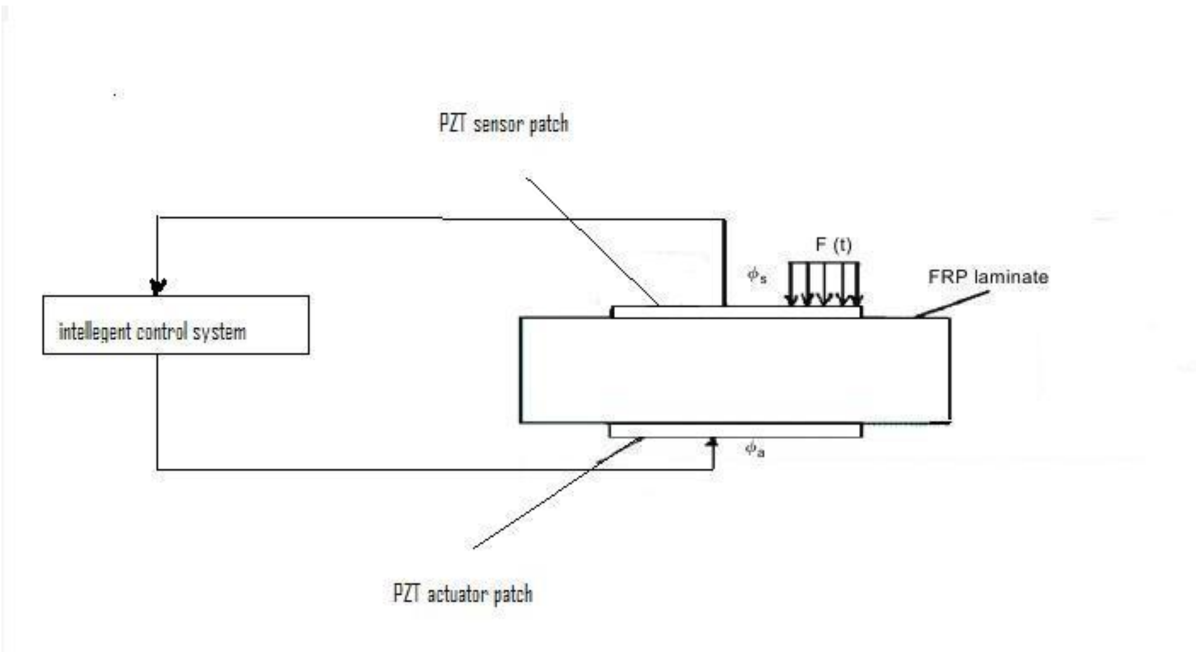
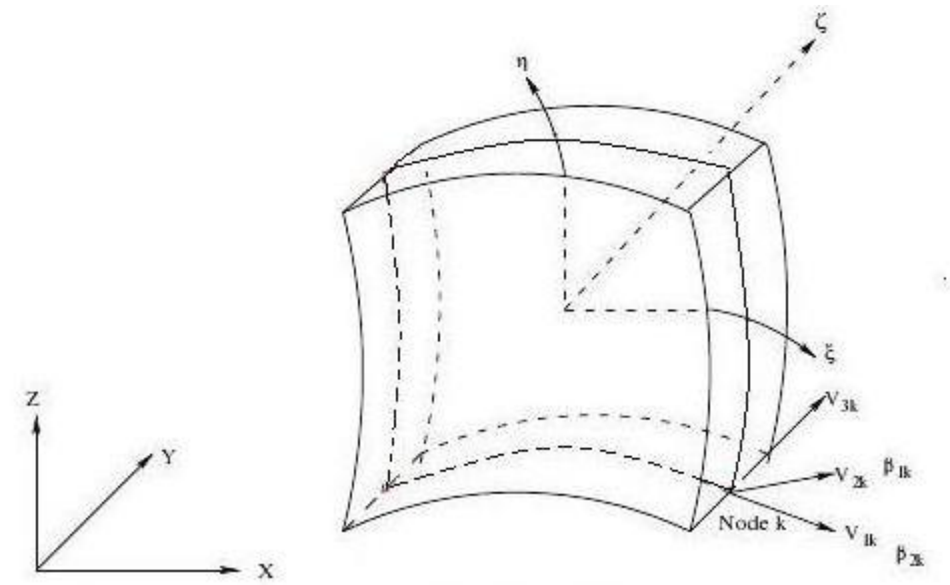


Figure 1: Front view of a smart PZT patches bonded laminated plate with feedback control



Global coordinate system Nodal and curvilinear coordinates

Figure 2 : Various coordinates system of the shell element

The piezoelectric patches are bonded to the surface of the structures and the bonding layers are thin. The global coordinate system (X-Y-Z) represents the displacement components of the mid-point of the normal, the nodal coordinates, global stiffness matrix and applied force vectors. v_1, v_2, v_3 are three mutually perpendicular vectors at each nodal point. Vector v_{1k} is perpendicular to v_{3k} and parallel to x-z plane and is assumed to be parallel to x-axis. v_{2k} is obtained from the cross product of v_{1k} and v_{3k}. V_{1k} , V_{2k} , V_{3k} are the unit vectors in the direction of v_{1k}, v_{2k}, v_{3k} . $\xi - \eta - \zeta$ is a natural coordinate system, where ξ and η are curvilinear coordinates and ζ is the linear coordinate with ζ= -1 and ζ= 1 in the top and bottom surfaces.

3.2 ELEMENT GEOMETRY AND DISPLACEMENT FIELD

The coordinate of a point within a element in isoparametric formulation is obtained as

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum_{k=1}^{8} N_k\left(\xi,\eta\right)\begin{Bmatrix} x_k \\ y_k \\ z_k \end{Bmatrix}_{mid} + \sum_{k=1}^{8} N_k\left(\xi,\eta\right)\frac{h_k}{2}\zeta V_{3k} \tag{1}$$

Where

$$\begin{Bmatrix} x_k \\ y_k \\ z_k \end{Bmatrix}_{mid} = \frac{1}{2}\left(\begin{Bmatrix} x_k \\ y_k \\ z_k \end{Bmatrix}_{top} + \begin{Bmatrix} x_k \\ y_k \\ z_k \end{Bmatrix}_{bottom}\right) \tag{2}$$

h_k is the shell at the k node. The displacement field has five degree of freedom, three displacement of its mid-point $\left(u_k \; v_k \; w_k\right)^T$ and two rotations $\left(\beta_{1k} \; \beta_{2k}\right)$. The displacement of a point calculated from the two rotations by

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{k=1}^{8} N_k \begin{Bmatrix} u_k \\ v_k \\ w_k \end{Bmatrix}_{mid} + \sum_{k=1}^{8} N_k \zeta \frac{h_k}{2}\begin{bmatrix} V_{1k}^{x} & -V_{2k}^{x} \\ V_{1k}^{y} & -V_{2k}^{y} \\ V_{1k}^{z} & -V_{2k}^{z} \end{bmatrix}\begin{Bmatrix} \beta_{1k} \\ \beta_{2k} \end{Bmatrix} \tag{3}$$

Where u_k, v_k, w_k are the displacement of node k along the mid surface of the global coordinate system, N_k is the shape function at node k.

3.3 STRAIN DISPLACEMENT REALTION

Five strain components and one tensor strain is neglected, in the local coordinate system is following:-

$$[\varepsilon]=\begin{bmatrix}\varepsilon_{x'}\\ \varepsilon_{y'}\\ \gamma_{x'y'}\\ \gamma_{x'y'}\\ \gamma_{y'z'}\end{bmatrix}=\begin{bmatrix}\frac{\partial u'}{\partial x'}\\ \frac{\partial v'}{\partial y'}\\ \frac{\partial u'}{\partial y'}+\frac{\partial v'}{\partial x'}\\ \frac{\partial u'}{\partial z'}+\frac{\partial w'}{\partial x'}\\ \frac{\partial v'}{\partial z'}+\frac{\partial w'}{\partial y'}\end{bmatrix} \tag{4}$$

The strain-displacement matrix can be formed using the displacement derivatives in the global coordinate system. The relation between strain components $\{\varepsilon\}$ in global coordinate system and the nodal variable can be expressed as

$$\{\varepsilon\}=\sum_{k=1}^{8}\left[\left(B_u\right)_k^e\right]\left\{d_k^e\right\}=\left[B_u^e\right]\left\{d^e\right\} \tag{5}$$

3.4 DIRECT AND CONVERSE PIEZOELECTRIC RELATION

The linear piezoelectric constitutive equations which is coupled with elastic and electric field is expressed as

$$\{D\}=[e]\{\varepsilon\}+[\in][E] \tag{6}$$

$$\{\sigma\}=[C]\{\varepsilon\}-[e]^T\{E\} \tag{7}$$

Where $\{D\}$= electric displacement vector, {σ} = stress vector, {ε} = strain vector and {E} = electric field vector. [e] = piezoelectric coupling constant and [Є] = dielectric constant matrix.

3.5 ELECTRIC POTENTIAL IN PIEZOELECTRIC PATCH

The element is assumed to have one degree of freedom at the top of the piezoelectric actuator and sensor patches, Φ_a^e and Φ_s^e respectively. The electric potential over an element is assumed to be constant all over and it varies linearly through the thickness of the piezoelectric patch. As the electric field is dominant in the thickness direction the electric field can be accurately approximated with a non-zero component only. The electric field potential is expressed as with this approximation

$$\left\{-E_a^e\right\} = \left[B_a^e\right]\left\{\phi_a^e\right\} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{h_a} \end{bmatrix}\left\{\phi_a^e\right\} \tag{8}$$

$$\left\{-E_s^e\right\} = \left[B_s^e\right]\left\{\phi_s^e\right\} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{h_a} \end{bmatrix}\left\{\phi_s^e\right\} \tag{9}$$

Subscript a and s represents the actuators and sensor patch, $[B_a^e]$ and $[B_s^e]$ are the electric field gradient matrices of the actuators and sensors respectively.

3.6 DYNAMIC FINITE ELEMENT EQUATION

The coupled finite element equation derived for one element model after the application of variational principle and finite element discretization becomes

$$\begin{pmatrix} [M_{uu}^e] & [0] & [0] \\ [0] & [0] & [0] \\ [0] & [0] & [0] \end{pmatrix} \begin{Bmatrix} \{\ddot{d}\} \\ \{\ddot{\phi}_a\} \\ \{\ddot{\phi}_s\} \end{Bmatrix} + \begin{pmatrix} [K_{uu}^e] & [K_{ua}^e] & [K_{us}^e] \\ [K_{au}^e] & [K_{aa}^e] & [0] \\ [K_{su}^e] & [0] & [K_{ss}^e] \end{pmatrix} \begin{Bmatrix} \{d\} \\ \{\phi_a\} \\ \{\phi_s\} \end{Bmatrix} = \begin{Bmatrix} \{F^e\} \\ \{G^e\} \\ \{0\} \end{Bmatrix} \tag{10}$$

Structural mass :

$$\left[M_{uu}^e\right] = \int_V \rho \left[N^T\right]\left[N\right] dV$$

Structural stiffness :

$$\left[K_{uu}^e\right] = \frac{2}{T}\int_{-1}^{1}\int_{-1}^{1}\sum_{k=1}^{N}\frac{t_k - t_{k-1}}{2}\int_{-1}^{1}\left[B_u\right]^T\left[C\right]\left[B_u\right]|J|\,d\xi d\eta d\zeta \tag{11}$$

Dielectric conductivity

$$\left[K_{ss}^e\right] = -\frac{2}{T}\int_{-1}^{1}\int_{-1}^{1}\sum_{k=1}^{N}\frac{t_k - t_{k-1}}{2}\int_{-1}^{1}\left[B_\phi\right]^T\left[\in\right]\left[B_\phi\right]|J|\,d\xi d\eta d\zeta \tag{12}$$

PIezoelectric coupling matrix:-

$$\left[K_{us}^e\right] = \frac{2}{T}\int_{-1}^{1}\int_{-1}^{1}\sum_{k=1}^{N}\frac{t_k - t_{k-1}}{2}\int_{-1}^{1}\left[B_u\right]^T\left[e\right]^T\left[B_\phi\right]|J|\,d\xi d\eta d\zeta$$

The overall dynamic finite element equation is

$$\left[M_{uu}\right]\left\{d''\right\} + \left[\left[K_{uu}\right] - \left[K_{ua}\right]\left[K_{aa}\right]^{-1}\left[K_{au}\right] - \left[K_{us}\right]\left[K_{ss}\right]^{-1}\left[K_{su}\right]\right]\left\{d\right\} = \left\{F\right\} - \left[K_{ua}\right]\left\{\phi_a\right\} \tag{13}$$

Where $[M_{uu}]$ = global mass matrix, $[K_{uu}]$ = global elastic stiffness matrix, $[K_{ua}]$ and $[K_{us}]$ = global piezoelectric coupling matrices of actuator and sensor patches respectively. $[K_{aa}]$ and $[K_{ss}]$ = global dielectric stiffness matrices of actuator and sensor patches respectively.

3.7 STATE-SPACE REPERSENTATION

The modes of vibration which are low have lower energy and are most excitable ones. These are more significant to the global response of the system. A modal matrix Ψ s used as transformation matrix between the generalized coordinates $d(t)$ and the modal coordinate matrix $\eta(t)$.

The displacement vector can be approximated as

$$\{d(t)\} \approx [\psi]\{\eta(t)\} \tag{14}$$

Where $[\Psi]$ = $[\ \Psi_1\ \Psi_2\ \Psi_3 \ldots\ldots..\Psi_r]$ is the truncated modal matrix.

The decoupled dynamic equation when modal damping is considered is

$$\{\eta_i(\ddot{t})\} + 2\xi_{di}\omega_i\{\eta_i(t)\} + \omega_i^2\{\eta_i(t)\} = [\psi]^T\{F\} - [\psi]^T[K_{ua}]\{\phi_a\} \tag{15}$$

Where ξ_{di} is the damping ratio.

In the state space form it can be written as

$$\{\dot{x}\} = [A]\{X\} + [B]\{\phi_s\} + [\hat{B}]\{u_d\} \tag{16}$$

$$[A] = \begin{bmatrix} [0] & [1] \\ -\omega_i^2 & 2\xi_{di}\omega_i \end{bmatrix}$$

Is the system matrix

$$[B] = \begin{bmatrix} [0] \\ -[\psi]^T[K_{ua}] \end{bmatrix}$$

Is the disturbance matrix, $[u_d]$ = disturbance input vector, $[\Phi_a]$ = control input

$$\left[\dot{X}\right] = \begin{Bmatrix} \dot{\eta} \\ \ddot{\eta} \end{Bmatrix} \text{ and } \left[X\right] = \begin{Bmatrix} \eta \\ \eta \end{Bmatrix}$$

The sensor output equation is

$$\{y\} = [C_0][X]$$

Where $[C_o]$ depends on the modal matrix $[\Psi]$ and the sensor coupling matrix $[K_{\Phi w}]$

3.8 LQR OPTIMAL FEEDBACK

Linear quadratic regulator (**LQR**) is used to find the control gains. Here, the feedback control is used to minimize the cost function which is proportional to the system responses. The cost function is

$$J = \frac{1}{2}\int_{t_0}^{t_1}\left(\{y\}^T[Q]\{y\} + \{\phi_a\}^T[R]\{\phi_a\}\right)dt \tag{17}$$

Where, *[Q]* = positive definite weighting matrices on the output

[R] = positive definite weighting matrices on the control input

The Ricatti equation in steady matrix is

$$[K] + [K][A] - [K]\{B\}[R]^{-1}[B]^T[K] + [C]^T[Q][C] = 0 \tag{18}$$

The optimal gain after solving the Ricatti equation

$$[G_c] = [R^{-1}][B]^T[K] \tag{19}$$

The output voltage can be calculated using the output feedback by

$$\phi_a = -[G_c]\{y\} \tag{20}$$

3.9 DETERMINATION OF WEIGHTING MATRIX

In LQR optimization process weighting matrices [Q] and [R] are important components as it influences the system performance. [Q] and [R] are assumed to be semi-positive definite matrix and positive definite matrix respectively by Lewis. As per Ang et al. [Q] and [R] can be determined by

$$[Q]=\begin{bmatrix}\alpha_2[\psi]^T[K][\psi] & 0\\ 0 & \alpha_1[\psi]^T[M][\psi]\end{bmatrix} \quad \text{and} \quad [R]=\gamma[R] \tag{21}$$

The weighted energy of the system is

$$\prod=\frac{1}{2}\alpha_1\left\{\dot{X}\right\}^T[M]\left\{\dot{X}\right\}+\frac{1}{2}\alpha_2\{X\}^T[K]\{X\}+\frac{1}{2}\gamma\{\phi_a\}^T\left[\hat{R}\right]\{\phi_a\} \tag{22}$$

Where $[\hat{R}]$ = dielectric coupling matrix of the actuator , α_1, α_2 and γ = coefficients associated with total kinetic energy, strain energy and input energy respectively.

3.10 LQG OPTIMAL FEEDBACK

LQG is also used to find the control gains like the LQR control. It is used to minimize the quadratic loss function which is dependent on the system responses

The quadratic loss function is

$$J_N=E\left[x(N)^T Q_0 x(N)+\sum_{t=t_0}^{N=1}\left\{x(t)^T Q_1 x(t)+u(t)^T Q_2 u(t)\right\}\right] \tag{23}$$

Where $N>t_0$

Q_0,Q_1 and Q_2= semi definite symmetric matrices

Discrete time linear equations

$$x_{i+1}=A_i x_i+B_i u_i+v_i \tag{24}$$

$$y_i=C_i x_i+w_i \tag{25}$$

Where, i = discrete time index

v_i, w_i = discrete time Gaussian

Minimized quadratic cost function

$$J=E\left(x_N^{'}Fx_N+\sum_{i=0}^{N-1}x_i^{'}Q_i x_i+u_i^{'}R_i u_i\right) \tag{26}$$

$F\geq 0, Q_i\geq 0, R_i>0$

Discrete time LQG controller

$$\hat{x}_{i+1} = \mathrm{A}_i x_i + \mathrm{B}_i u_i + \hat{K}_i (y_i - C_i x_i)\,,\, \hat{x}_0 = \mathrm{E}(x_0) \tag{27}$$

$$u_i = -L_i \hat{x}_i$$

The Kalman gain

$$K_i = A_i P_i C_i^{'} \left(C_i P_i C_i^{'} + W_i \right)^{-1} \tag{28}$$

Where P_i is determined Riccati difference equation which runs forward in time,

$$P_{i+1} = A_i \left(P_i - P_i C_i^{'} \left(C_i P_i C_i^{'} + W_1 \right)^{-1} C_i P_i \right) A_i^{'} + V_i , P_0 = E\left(x_0 x_0^{'} \right) \tag{29}$$

The feedback gain matrix

$$L_i = \left(B_i^{'} S_{i+1} B_i + R_i \right)^{-1} B_i^{'} S_{i+1} A_i \tag{30}$$

Where S_i is determined Riccati difference equation which runs backward in time,

$$S_i = A_i^{'} \left(S_{i+1} - S_{i+1} B_i \left(B_i^{'} S_{i+1} B_i + R_i \right)^{-1} B_i^{'} S_{i+1} \right) A_i + Q_i , S_N = F \tag{31}$$

4.RESULTS AND DISCUSSION

4.1 PROBLEM DEFINATION

The smart spherical shell structures is made up of two piezoelectric layers both at top and bottom of the shell structures and four FRP composite layers in between the two piezoelectric layers. In the figure shown, the structure is in x and y direction with ten elements on each axes, thus making it to total of hundred elements. In the shell structure sensors and actuators (pink color) are attached on the bottom and top surface respectively. Load is applied at the centre of the shell which is determined by the actuators and sensors gives the response and feedback which is to be controlled.

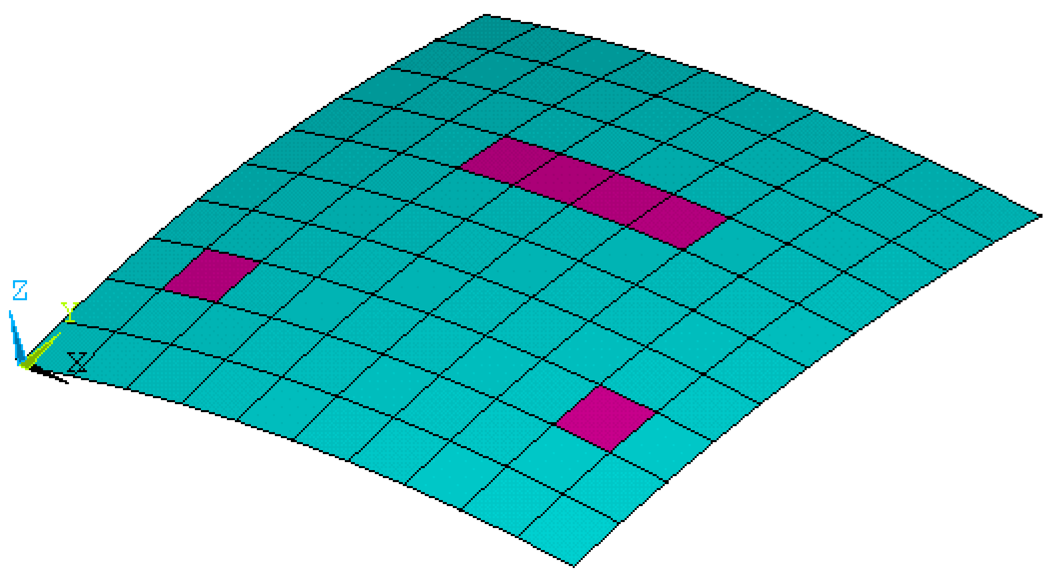


Figure:3 Smart spherical panel with piezoelectric patches

4.2 MATERIAL PROPERTIES

t= thickness in meters

θ =orientation in degrees

μ_{12} , μ_{21} = Poisson's ratio

E_1 , E_2 = Young's Modulus of Elasticity

g_{12} , g_{23} , g_{13} = shear modulus

ε =permittivity

ρ = density in kg/m^3

k = piezoelectric constant

Table 1: material properties

t	θ	μ_{12}	μ_{21}	E_1	E_2	g_{12}	g_{23}	g_{13}	ε	ρ	k
5	0	0.28	0.28	63×10^9	63×10^9	24.6×10^9	24.6×10^9	24.6×10^9	15.55×10^{-9}	7600	-2.5×10^{-4}
2.5	0	0.25	0.01	172.5×10^9	6.9×10^9	3.45×10^9	1.38×10^9	3.45×10^9	0	1600	0
2.5	90	0.25	0.01	172.5×10^9	6.9×10^9	3.45×10^9	1.38×10^9	3.45×10^9	0	1600	0

2.5	90	0.25	0.01	172.5×10^9	6.9×10^9	3.45×10^9	1.38×10^9	3.45×10^9	0	1600	0
2.5	0	0.25.	0.01	172.5×10^9	6.9×10^9	3.45×10^9	1.38×10^9	3.45×10^9	0	1600	0
5	0	0.28	0.28	63×10^9	63×10^9	24.6×10^9	24.6×10^9	24.6×10^9	3.45×10^9	7600	-2.5×10^{-4}

Table 2: piezoelectric coupling properties

e_{31}	e_{32}	e_{33}	e_{34}	e_{35}
10.62	10.62	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
10.62	10.62	0	0	0

In table 1 and in table 2 the first and the last row is for piezoelectric materials and the rest are for FRP composites materials. In the shell structure there are 1705 nodes, out of which 100 elements are used and 6 modes are considered. The modal damping value is 1%. An impulse load of 10N is applied at the centre of the shell for a duration of 3.3×10^{-6} s. All the edges of the shell are simply supported. 6 sensors and 6 actuators are used.

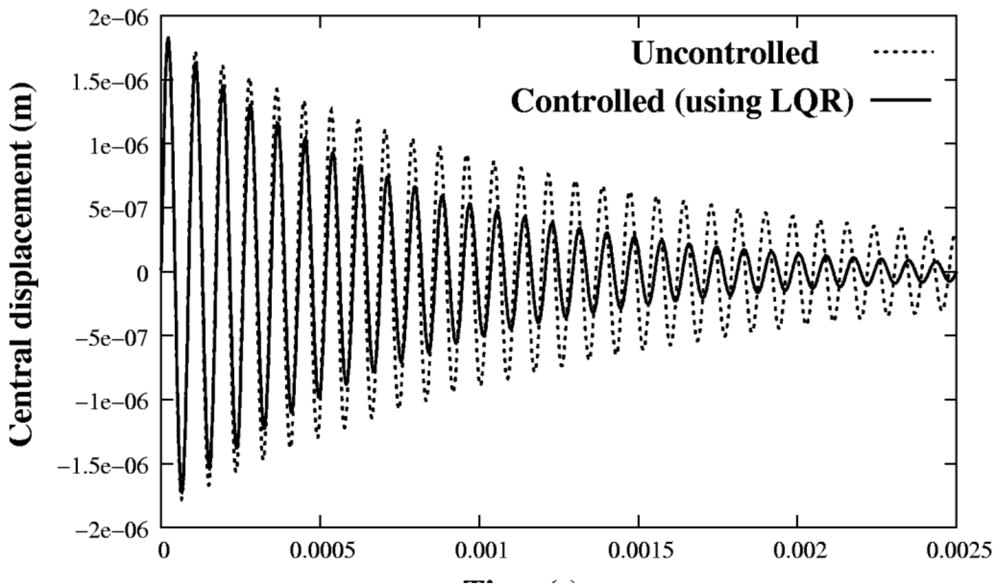


Figure: 4 Central displacements of smart FRP composite shell under impulse load

When a impulse load of 10N is applied for a duration of 3.3×10^{-6} s then the actuator determines the voltage and the sensors finds the uncontrolled damping vibration which is needed to be controlled. So the sensors sends the signal and the feedback. LQR is then used to control the vibration. In the figure the dotted lines shows the uncontrolled damping vibration and the solid lines shows the controlled vibration which is controlled by using LQR technique. Thus the uncontrolled vibration is controlled.

4. CONCLUSION AND SCOPE OF FUTURE WORK

The vibration problem of the composite shell structure is analyzed and concluded that with the use of sensors and actuators the vibration in the shell structures of the composites can be found out, which is uncontrolled and is needed to be controlled. An LQR technique is used to control this damping vibration and it can be used to control the vibrations of such types of structures.

In the present work LQR technique is used to control the damping vibration which can occurs anytime. In future another technique can be used to control this type of vibration like Linear Quadratic Gaussian (LQG). LQG can be used to control the vibration and can overcome the problems faced by LQR.

REFERENCE

1) T Bailey, J. E Hubbard, Distributed piezoelectric polymer active vibration control of cantilever beam, Journal of Guidance Control and Dynamics. 85(1985) 605-611

2) P. Bhattacharya, H. Suhail, P. K. Sinha, Finite Element Analysis and Distributed Control of Laminated Composite Shells Using LQR/IMSC Approach, Aerospace science and technology 6 (2002) 273-281

3) K. K. Ang, S.Y. Wang, S. T. Quck, Weighted energy linear quadratic regulator vibration control of linear piezoelectric composite plates, Journal of Smart Elements and Structures 11 (2002) 98-106

4) S. Narayanan, V. Balmurugan, Finite element modeling of piezolaminated smart structures for active vibration control with distributed sensors and actuators, Journal of Sound and Vibration 262 (2003) 529-562

5) R. H. Christensen, I. F. Santos, Design of active controlled rotor-blade system based on time-variant modal analysis, Journal of Sound and Vibration 280 (2005) 863-882.

6) T. Roy, D. Chakraborty, GA-LQR based optimal vibration control of smart FRP structures with bonded PZT ptches. J. Reinforced Plastics Composites (2008). Doi:10.1177/0731684408089506

7) M.M Abdullah, A. Richardson, J. Hanif, Placement of sensors and actuators on civil structures using genetic algorithm, Earthquake Engineering and Structural Dynamics 30 (8) (2001) 1167-1154.

8) I. Robandi, K. Nishimori, R. Nishimura, N. Ishihara, Optimal feedback control design usng genetic algorithm in multimachine power system, Electrical Power and Energy System 23 (2001) 263-271

9) Y. Yang, Z. JIN, C.K. Soh, Integrated optimal design of vibration control system for smart beams using genetic algorithm, Journal of Sound and Vibration 282 (2005) 1293-1307.

10) S. Y. Wang, K. Tai, S.T Quck, Topology optimization of piezoelectric sensors and actuators for torsional vibration control of composite plates. Journal of Smart Materials and Structures 15(2) (2006) 253-269

11) J. N. Reddy, Exact solution of moderately thick shell laminated shells, ASCE Journal of Engineering Mechanics 110 (5) (1984) 794-809.

12) S. Ahamad, B.M Irons, O.C. Zienkiewecz, Analysis of thick and thin shell structures by curved elements, International Journal of Numerical Methods in Engineering 2(1970) 419-451

13) Zhang, N, Kirpitchenko, L : Modelling dynamics of a continous structure with a piezoelectric sensors and actuators for passive structural control. Journal of Sound and Vibration. 249(2), 251-261 (2002)

14) Kwakemernaak, Huibert and Sivan, Raphel (1972). Linear Optimal Control System. First edition. Wiley-Interscience. ISBN 0-471-511102

15) Sontag, Edurado (1998), Mathematical Control Theory: Deterministic Finite Dimensional System. Second Edition. Springer ISBN 0-387-984895

16) en.wikipedia.org/wiki/Linear-Quadratic_Regulator

17) Athans M (1971). The role and use of the stochastic Linear-Quadratic Gaussian problem in control system design. IEEE Transaction on Automation Control AC-16 (6) 529-552 doi:10.1109/TAC.197111099818

18) Van Willigenburg L.G., De Koning W.L. (2000). Numerical algorithms and issues concerning the discrete-time optimal projection equations. European Journal of Control **6** (1): 93–100.

19) Zigic D., Watson L.T., Collins E.G., Haddad W.M., Ying S. (1996). Methods for solving the optimal projection equations for the H2 reduced order model problem. International Journal of Control **56** (1): 173–191.

20) Collins Jr. E.G, Haddad W.M., Ying S. (1996). A algorithm for reduced-order dynamic compensation using the Hyland-Bernstein optimal projection equations. Journal of Guidance Control & Dynamics **19**: 407–417.

21) en.wikipedia.org/wiki/Linear-quadratic-Gaussian_control#cite_note-Athans-0

22) Maxwell, J.C. (1867). On governors Proceedings of the Royal Society of London **16**: 270–283 doi: 10.1098/rspl.1867.0055 Retrieved 2008-04-14.

23) Routh, E.J.; Fuller, A.T. (1975). Stability of motion. Taylor & Francis.

24) Routh , E.J. (1877). A Treatise on the Stability of a Given State of Motion, Particularly Steady Motion: Particularly Steady Motion. Macmillan and co

25) Hurwitz, A. (1964). "On The Conditions under which an equation has only roots with negative real parts". Selected Papers on Mathematical Trends in Control Theory.

26) Kang , Y.K., Park, H.C., Kim, J., Choi, S.B, : Interaction of active and passive vibration control of laminated composite beams with piezoceramics sensors and actuators. Mater Des. 23 277-286 (2002)

27) Hiramoto, K., Doki, H., Obinata, G,: Optimal sensor and actuators placement for active vibration control using explicit solution of algebraic Riccati equation. Journal of Sound Vibration 229(5), 1057-1075 (2000). Doi: 10.1006/jsvi.1999.2530.