ANALYSIS OF BEAMS AND PLATES USING ELEMENT FREE GALERKIN METHOD

THESIS SUBMITTED IN PARTIAL FULLFILLMENT FOR THE DEGREE OF:

BACHELOR OF TECHNOLOGY
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CERTIFICATE

CERTIFICATE

This is to certify that the thesis entitled **"ANALYSIS OF BEAMS AND PLATES USING ELEMENT FREE GALERKIN METHOD"** submitted by **SLOKARTH DASH (107CE005)** and **ROSHAN KUMAR (107CE035),** in the partial fulfillment of the degree of **Bachelor of Technology** in **Civil Engineering**, National Institute of Technology, Rourkela, is an authentic work carried out by them under my supervision.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other university/institute for the award of any degree or diploma.

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SLOKARTH DASH

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ABSTRACT

The project has dealt with the study of beams and **thin plates** under **plane stress** using the mesh-less technique, **ELEMENT FREE GALERKIN METHOD**. This involves a detailed study of the Element Free Galerkin Method consisting of its formulation, mode of application, its advantages and disadvantages along with a brief study of the analysis of thin plates. A **MATLAB** code was written for the analysis of a **Timoshenko Beam Problem** using EFGM, so that the logic and modus operandi of the method may be fully understood. Several cases of plane stress were considered in the project such as cantilever beams subjected to point loads and uniformly distributed loads, and plates with varying geometries and boundary conditions using **MFree2D** simulation package. The major aims of this project were twofold. Firstly, **to check the accuracy of the Element Free Galerkin Method** by means of comparison with the exact theoretical values and secondly to **solve certain typical problems related to plates by the Element Free Galerkin Method.**

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1. INTRODUCTION

The use of the Finite Element Method of analysis is very common now-a-days. This technique has been used with great accuracy for determination of various parameters of importance in the system of consideration. But as the complexity of the problem statement increases, the accuracy of the FEM becomes an issue. Since this method requires the presence of a pre defined mesh for proper analysis to be carried, modeling of structures with complicated geometries requires a very fine mesh arrangement or sometimes more than one mesh thus increasing the time, load and cost of computation. Hence there arises a need to search for other computational methods which might bring out a greater accuracy with less time consumption and cost. Some of the major disadvantages of FEM are as follows:

- FEM mesh construction is very costly as most of the time used for analysis is consumed by mesh construction.

- Accuracy of the stress obtained is less. This is mainly because the displacement field is assumed to be piece-wise continuous. Thus errors are bound to occur at the interfaces.

- FEM analysis is not adaptive in nature which is one of the major drawbacks of this method. When a desired level of accuracy is not obtained at the chosen mesh; the entire mesh has to be refined from the start thus making all the previous works a waste.

As an alternative to FEM, a lot of research has been carried out for the development of mesh-free method of analysis. G.R.Liu(2002) defined the mesh-free method as "*a method used to establish system algebraic equations for the whole problem domain without the use of any predefined mesh for domain discretization.*"

Mesh free methods use distinct points known as nodes on the problem domain and the boundary to define the problem. These nodes do not form a mesh hence there is no need for any prior relationship to exist between nodes to create the interpolation function.

Some of the major mesh free methods are as follows:

- ELEMENT FREE GALERKIN METHOD
- MESHLESS LOCAL PETROV-GALERKIN METHOD
- POINT INTERPOLATION METHOD
- THE POINT ASSEMBY METHOD
- THE FINITE POINT METHOD
- FINITE IRREGULAR METHOD WITH ARBITARY IRREGULAR GRIDS
- SMOOTH PARTICLES HYDRODYNAMICS METHOD
- KERNEL PARTICLE METHOD

The Element Free Galerkin Method was developed by Belytschko in 1994, it is based on the Diffuse Element Method (Nayroles 1992).

The major features of the Element Free Galerkin Method are:

- Moving least square method is used to create shape functions.

- Galerkin Weak Form creates discretized equations.
- A background mesh is created to carry out integration to obtain the system matrices.

The mesh is solely used for the purpose of integration which is completely independent of the number of field nodes or its density. A detailed discussion of the EFG method is provided in the following chapter.

All the plates considered for the analysis are taken to be thin plates or Kirchhoff Plates of isotropic elastic materials.

2. LITERATURE REVIEW

The Element Free Galerkin Method was first developed by Belytschko in 1994 [1]. It is based on the Diffuse Element Method developed by Nayroles in1992. "He used the Moving Least Square Approximation" or the Local Regression or Loss which was invented by Lancaster and Salkauskas 1980[7][8] to determine the shape functions. Belytschko along with Krysl in 1999 developed the EFG method for thin plates or Kirchhoff plates [2]. Their study revealed the optimal quadrature order and the most favorable support size for EFG analysis. G.R. Liu in his book entitled "Mesh Free Methods: Moving beyond the Finite Element Method" [3] describes the EFG method in detail. Many numerical examples are used to show convergence studies and influence of factors such as number of nodes, quadrature points, support size domains etc. In addition to this, G.R. Liu along with Y.T. Gu created a source code for analysis of the EFG Method in FORTRAN, the code was published in their book entitled "An Introduction to Mesh Free Methods and their Programming"[4]. J. Dolbow and T. Belytschko[5] also created source codes for the EFG method in their paper "An introduction to programming the mesh-less Element Free Galerkin Method". The paper explained in detail the algorithm and flow chart for programming the EFGM for both 1D and 2D problem domains. A clarification of the algorithm and the logic behind it is expressed very lucidly in the paper. In addition to this the MATLAB source codes for the same are also given. In order to simplify the problem a uniform distribution of nodes is taken and a Gaussian quadrature order of 4 is assumed.

The content of the above mentioned books and journals have helped us in achieving a clear understanding of the Element Free Galerkin method as well as provide us with a clear idea of its coding logic, technique and procedure. The simulation package used for the analysis of some of the numerical examples has been developed by G.R.Liu at the Centre for advanced computation in engineering sciences.

3. THEORETICAL FORMULATION

3.1. ELASTIC ANALYSIS

Stress may be defined as the measure of the internal forces acting on the deformable structures. It may be quantitatively expressed as the average forces per unit area its standard unit being N/m^2.

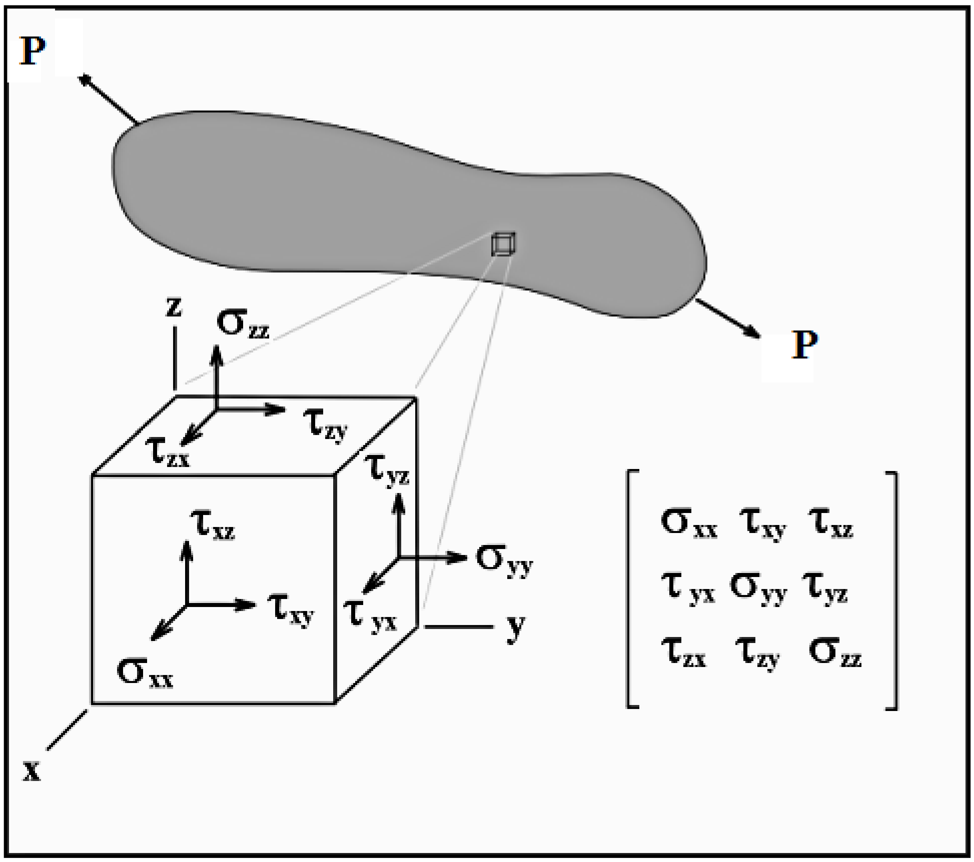


Figure 1 STRESS TENSOR

The above figure shows the general stress tensors for a three dimensional (3D) irregular solid subject to a force P on both sides. The stresses represented by σ show direct stress, where as τ represent shear forces. As is clearly visible each plane is subject to one direct stress and two shear forces. All three forces act at mutually perpendicular directions. The resultant stress tensor matrix is also shown in the figure, showing the stress acting along x, y and z axes of the Cartesian coordinate system.

When this general stress matrix is used for 2D analysis the forces along z axis may be neglected.

Now we define strain as the ratio of change in length to the original length. For a very small element dx strain can be represented as:

$$\varepsilon = \frac{\delta u}{\delta x}$$

Where, u may be defined as the displacement along x direction.

The general relationship between the strain and displacement is given by:

$$\boldsymbol{\varepsilon} = \mathbf{LU}$$

Where, L is the differential operator matrix and the U is the displacement matrix.

Again, from Hooke's law,

$$\sigma = C\varepsilon$$

Where, C is the constitutive matrix showing material properties.

3.2. PLATES

Plates are structures whose thickness is very small compared to its planar dimensions. This allows us to assume and analyze a plate as a 2D element neglecting its thickness. There are majorly three theories of plate analysis[3]. Namely:

- Kirchhoff or Classical Plate Theory(thin plates)

- Mindlin or thick Plate theory also known as First Order Shear Deformation Theory(thick plates).
- Third Order Shear Deformation Theory (laminates).

3.3. ELASTOSTATICS FOR 2-D SOLIDS

Let us consider a 2-D solid denoted by the Domain Ω and with the boundary condition Γ.

The stress condition when the solid is subject to an external force [3] ‘p’ may be denoted as:

$$K\sigma + p = 0$$

Where K = L^T.

And the boundary conditions may be defined as:

$u = \vec{u}$. at $\Gamma = \Gamma_e$ essential boundary condition.

$\sigma . n = \vec{t}$ at $\Gamma = \Gamma_n$ natural boundary condition.

Where, the boundary values are denoted on the Right hand Side.

3.4. ELEMENT FREE GALERKIN METHOD

3.4.1. M.L.S. APPROXIMATION AND SHAPE FUNCTIONS

The Moving Least Square [7][8] is widely used to generate the shape functions for various Mesh Free Methods. There are two salient Features of this method, firstly it creates a continuous and smooth approximation function in the field domain and secondly, the field function can be created with desired level of consistency.

Let us consider a displacement function u(x) on the domain Ω [3], the approximated value of u(x) can be represented as

$$u(x) = p_j(x)\, a_j(x) \qquad j=0,\, 1,2\ldots n.$$

In this case p(x represents) the polynomial matrix and a(x) the coefficient matrix.

The choice of the polynomial function is depends upon the basis and is decided by the Pascal's triangle. For example, a 1D bar having a linear basis=2, the polynomial matrix p(x) = [1, x]. Whereas for a 2D element having linear basis=3, the matrix will be p(x) = [1, x, y]. For cases in which deflection must be found a quadratic polynomial is used as continuity up to the second derivative of the shape function needs to be established for consistency. Therefore we take basis = 6 and $p(x) = [1, x, y, x^2, y^2, xy]$.

The coefficients are determined using the values at the field nodes represent inside the support domain. The support Domain may be defined as the small area neighboring a point inside which the field nodes exert influence over the point of consideration [2].

In addition to this the approximation is generally weighted to increase the accuracy of the interpolation function and to maintain continuity in the movement of nodes in and out of the support domain.

Now,

$$J=\sum w(x-x_i)\,[u^h(x-x_i)-u(x)_i]^2$$

Where $w(x-x_i)$ denotes the weight function and u_i is the value of the nodal displacement

Now, a(x) must be chosen in such a way that the weighted residual is, minimum.

Therefore,

$$\delta J/\delta a=0.$$

Thus the following linear equations are obtained.

$$A(x)a(x)=B(x)U_s$$

$$A(x) = \sum w(x)p(x_i)p^t(x_i)$$

And, $B(x) = w(x)\,p(x_i)$

Solving the above equations we obtain;

$$a(x) = A^{-1}(x)\,B(x)U_{s.}$$

And the shape function $\Phi(x) = \sum p(x)A^{-1}(x)\,B(x) = p^T A^{-1}B$.

Let us consider $A(x)\gamma(x) = p(x)$

Therefore $\Phi(x) = \gamma^T(x)B(x)$.

Some of the weight functions [4] chosen are:

$$\text{exponential:} \quad w(\bar{s}) = \begin{cases} e^{-(\bar{s}/\alpha)^2} & \text{for } \bar{s} \leq 1 \\ 0 & \text{for } \bar{s} > 1 \end{cases}$$

$$\text{cubic spline:} \quad w(\bar{s}) = \begin{cases} \frac{2}{3} - 4\bar{s}^2 + 4\bar{s}^3 & \text{for } \bar{s} \leq \frac{1}{2} \\ \frac{4}{3} - 4\bar{s} + 4\bar{s}^2 - \frac{4}{3}\bar{s}^3 & \text{for } \frac{1}{2} < \bar{s} \leq 1 \\ 0 & \text{for } \bar{s} > 1 \end{cases}$$

$$\text{quartic spline:} \quad w(\bar{s}) = \begin{cases} 1 - 6\bar{s}^2 + 8\bar{s}^3 - 3\bar{s}^4 & \text{for } \bar{s} \leq 1 \\ 0 & \text{for } \bar{s} > 1 \end{cases}$$

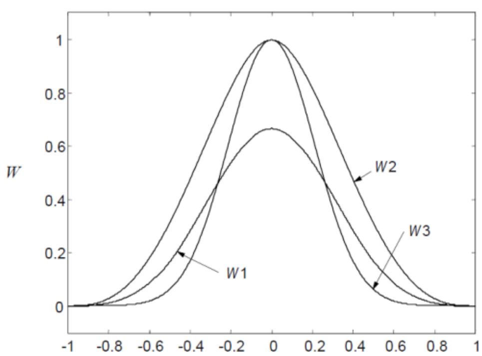


Figure 3 WEIGHT FUNCTIONS

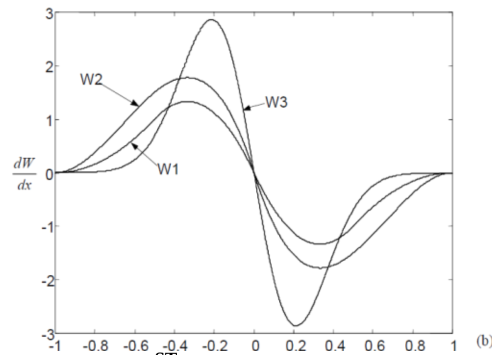


Figure 2 1ST ORDER DERIVATIVES

3.4.2. GALERKIN WEAK FORM

The Galerkin Weak form [1][2][3] in statics for a domain Ω is given by:

$$\int_\Omega \boldsymbol{\delta}(\boldsymbol{Lu})^T \mathbf{c}(\boldsymbol{Lu})\,\mathbf{d}\Omega - \int_\Omega \boldsymbol{\delta}\mathbf{u}^T\mathbf{b}\,\mathbf{d}\Omega - \int_{\Gamma_t} \boldsymbol{\delta}\mathbf{u}^T\,\mathbf{t}^*\mathbf{d}\Gamma = \mathbf{0}$$

3.4.3. EFGM FORMULATION

Using the result obtained from the MLS Approximation and the Galerkin Weak form. We can obtain the governing relation for the EFGM.

$$\mathbf{Lu}=\sum \mathbf{Bu}$$

3.4.4. LAGRANGE MULTIPLIERS

The shape functions obtained by the MLS Approximation do not satisfy the Kronecker Delta [3] property, therefore it is impossible for the shape function to satisfy the Essential Boundary Conditions. Hence we use Lagrange Multipliers to satisfy the boundary conditions. Now the nodal values of the Lagrange function are known, thus by means of Lagrange interpolation its values are determined and the final discretized equations for Ω are obtained.

3.4.5. DISCRETIZED EQUATIONS

The final discretized equations of the EFGM are as follows:

$$KU+G\lambda-F=0$$

$$G^TU-q=0$$

3.4.6. BACKGROUND MESH

The EFG method requires a background mesh [2] for the integration purpose. This mesh has no other use in the problem domain. Mesh just subdivides the domain into either squares triangles or any other chosen shape. There is no influence of this mesh in the formation of the shape function as is the case with FEM.

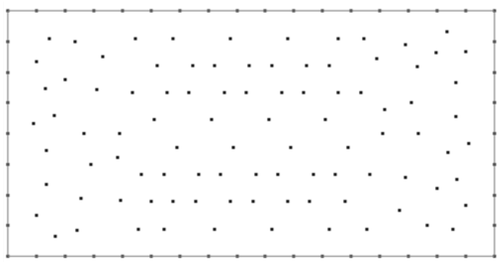


Figure 4 NODAL DISTRIBUTION

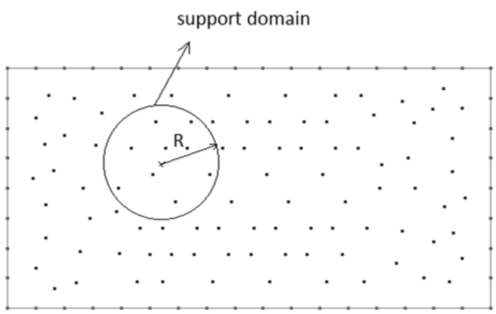


Figure 5 SUPPORT DOMAIN

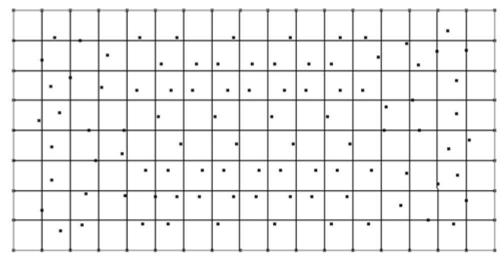


Figure 6 BACKGROUND MESH

4. ALGORITHM FOR EFGM IN 2-D

The following algorithm [4] is used for coding the EFGM:

STEP 1: DEFINED THE MATERIAL PROPERTIES AND PHYSICAL PARAMETERS

STEP 2: D FOR PLANE STRESS IS DEFINED

STEP 3: COORDINATES FOR BACKGROUND MESH WERE SETUP

STEP 4: INFLUENCE DOMAIN DETERMINED

STEP 5: QUADRATURE POINTS ARE SETUP

STEP 6: GUASS POINTS, WEIGHTS AND JACOBIAN FOR EACH CELL WERE DEFINED

STEP 7: LOOP OVER GUASS POINTS TO ESTABLISH NEIGHBOUR HOOD NODES, WEIGHTS AND SHAPE FUNCTIONS.

STEP 8: NODES ON NATURAL AND ESSENTIAL BOUNDARIES DEFINED.

STEP 9: GUASS POINTS FOR THE SAME ARE DEFINED.

STEP 10: INTEGRATION FOR f.

STEP 11: USING LAGRANGE MULTIPLIERS FOR G MATRIX AT ESSENTIAL BOUNDARY

STEP 12: ENFORCING ESSENTIAL BOUNDARY CONDITION.

STEP 13: OBTAIN NODAL PARAMETERS

STEP 14: DETERMINATION OF STRESS.

5. NUMERICAL EXAMPLES

5.1. CANTILEVER TYPE TIMOSHENKO BEAM WITH TRACTION

A Timoshenko beam of length L=100m and depth D = 36m is subjected to a traction force F = 1000KN at the free end. The beam is considered to be in plane stress and completely elastic in nature.

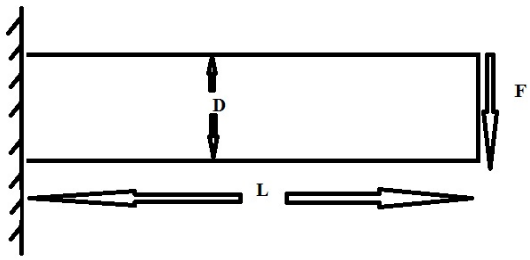


Figure 7 TIMOSHENKO BEAM PROBLEM

The exact solution of the Timoshenko beam is given by the following equations, given by Timoshenko and Goodier (1970) [6].

Stress:

$$\sigma_{xx} = -Fy(L-x)/I$$

$$\sigma_{yy} = o$$

$$\sigma_{xy} = -Fy(D^2/4-y^2)/I$$

The solution of the stresses obtained by the EFG program created in MATLAB [13][15] is shown in the following pages. The error of the EFGM method is shown in the form of percentage error and also in terms of the global energy error.

Table 1 TABLE SHOWS RESULTS OF EFGM FOR FIRST 30 QUADRATURE POINTS

NODE NUMBER	STRESS(XX) N/mm^2	STRESS(YY) N/mm^2	STRESS(XY) N/mm^2
1	391.0647	-4.1417	0.7989
2	378.1780	-2.9635	-11.3424
3	357.6344	-2.1964	-25.7797
4	340.2711	-1.8313	-36.0243
5	387.9502	-2.1911	-0.2940
6	374.0628	-1.8356	-11.6502
7	353.3418	-1.7325	-25.5655
8	336.2567	-1.6739	-35.6680
9	382.4395	-0.8520	-1.3763
10	367.6002	-1.1213	-11.9967
11	346.8370	-1.4731	-25.3927
12	330.1815	-1.6010	-35.3200
13	378.2035	-0.2872	-2.0883
14	362.6085	-0.8513	-12.3380
15	341.6609	-1.3989	-25.4582
16	325.1990	-1.6028	-35.2735
17	330.6604	-1.4330	-41.5621
18	311.9606	-0.7696	-51.5186
19	286.9381	-0.9345	-62.2734
20	267.9847	-0.8021	-69.8860
21	326.8750	-1.4009	-51.2087
22	308.6971	-0.8512	-62.1940
23	284.1575	-1.0187	-69.9980
24	265.3647	-0.9134	-40.7892
25	321.1214	-1.3983	-50.8137
26	303.6470	-0.9016	-61.9282
27	279.8461	-1.0703	-69.8748
28	261.4137	-1.0113	-40.7034
29	316.3157	-1.4279	-50.6846
30	299.2659	-0.9566	-61.8054

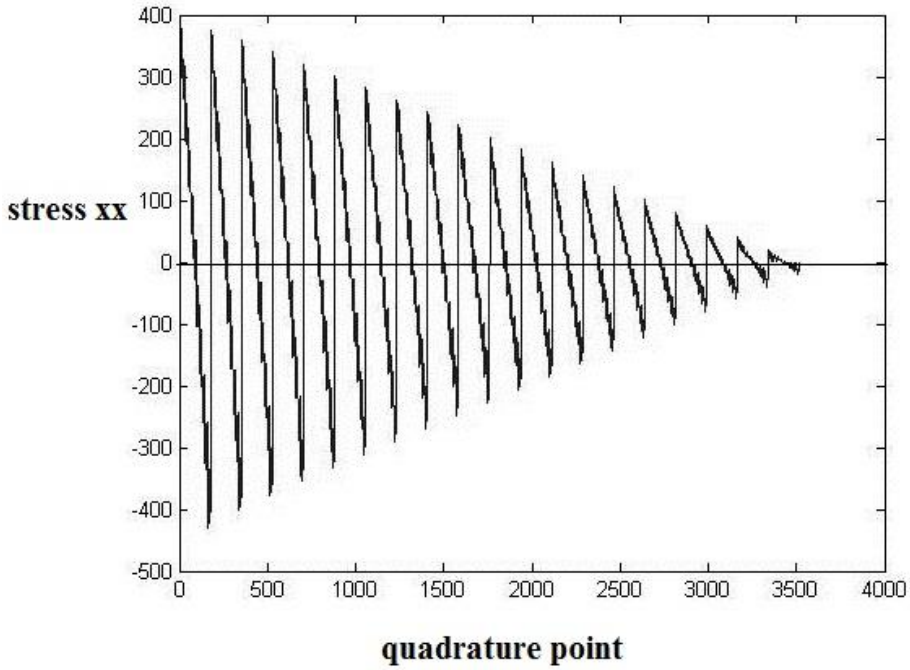


Figure 8 PLOT SHOWING VARIATION OF σ_{XX} WITH QUADRATURE POINTS

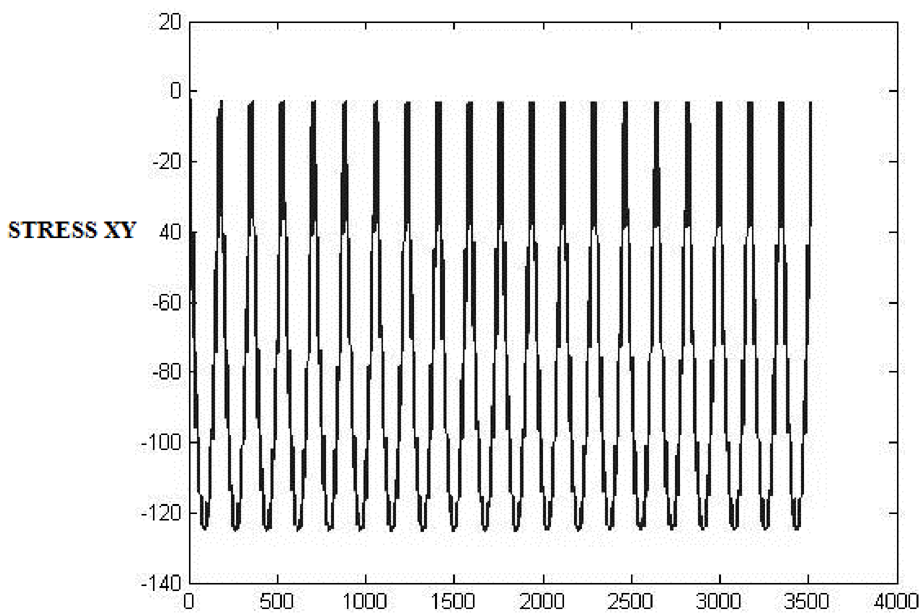


Figure 9 PLOT SHOWING VARIATION OF σ_{XY} WITH QUADRATURE POINTS

The above plots show the variations of direct stress σ_{xx} and shear stress σ_{xy} with the quadrature points. Quadrature points are arbitrary points taken in the plate domain for which the field parameters are computed for this case the total number of quadrature points taken was 3200. It must be noted that the values of the stresses obtained are related to the location of the point on the plate domain. Clearly if a point is located above the neutral axis the value of the stress will be tensile (positive) but if the point is located below the neutral axis the stress will be compressive (negative) even if both the points are equidistant from the fixed point.

The error in the EFG Method may be obtained in two methods by computing the percentage error or by evaluating the global strain energy error e_{norm}. The value of the e_{norm} helps us establish the rate of convergence of the method. The subsequent data show the influence of the total number of nodes on the value of error obtained through both the methods.

Table 2 CONVERGENCE TABLE FOR EFG METHOD

Number of nodes	σ_{xx} (N/mm^2) (EFG)	σ_{xx} (N/m^2) (Exact)	σ_{yy} (N/mm^2) (EFG)	σ_{yy} (N/mm^2) (EFG)	σ_{xy} (N/mm^2) (EFG)	σ_{xy} (N/mm^2) (EFG)
231	-1.6611	-1.5849	0.0339	0	-1.2234	-1.1492
1071	-0.6322	-0.6384	0.0094	0	-0.5782	-0.5766

Table 3 ERROR VALUES FOR DIFFERENT NUMBER OF NODES

Number of Nodes	Error e_{norm}	Percentage error in σ_{xx}	Percentage error in σ_{yy}	Percentage error in σ_{xy}
231	0.0074	4.8%	-------------	6.4%
1071	0.0034	0.97%	-------------	0.27%

It is quite clear that the EFG method is not very accurate for low number of nodes but as the number of nodes is increased, the accuracy of the method increases greatly. It should also be noted that the computation time does not differ by much with increase in number of nodes.

5.2. CANTILEVER BEAM WITH POINT LOAD AT FREE END:

A cantilever beam of length 2m and depth 0.5m is subject to a point load P= 1 KN. The beam was considered to be completely elastic with a Young's Modulus, E = $3x10^7$ N/m^2.

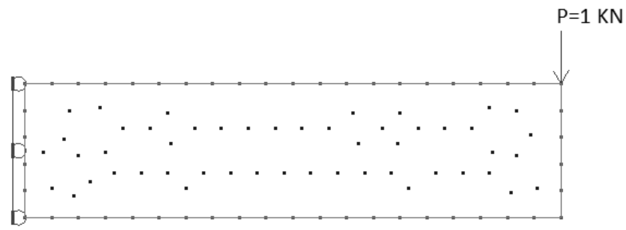


Figure 10 CANTILEVER BEAM WITH POINT LOAD

The exact solution for this example was computed manually and the EFG method analysis using the software package MFREE2D [11] developed by G.R. Liu and his co-workers.

We have considered two sections in the problem domain:

- Section 1-1 at x=1m.
- Section 2-2 at x=2m.

Table 4 COMPARISON OF STRESSES FOR NUMERICAL EXAMPLE 2

Sl. No.	σ_{xx} (N/mm^2) EFG Method	σ_{xx} (N/mm^2) Exact Analysis	Percentage Error
1.	480	485	1.041%
2.	240	251	4.583%

The analysis of the problem domain by the EFG Method is shown in the following sections by means of various graphs.

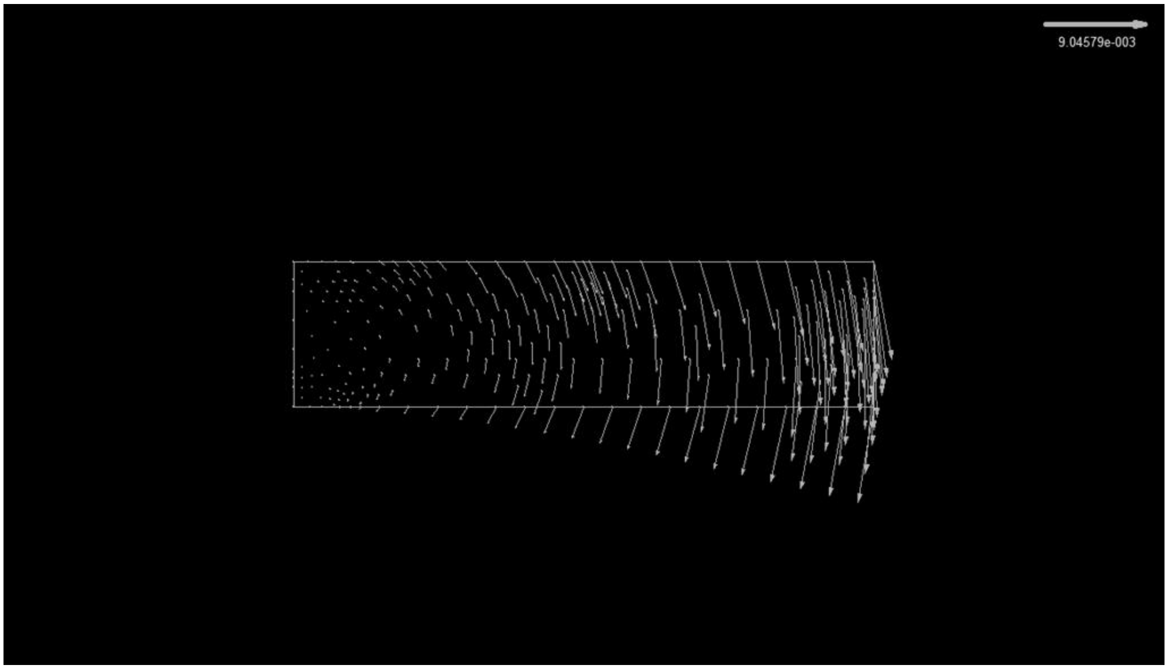


Figure 11 DISPLACEMENT OF BEAM

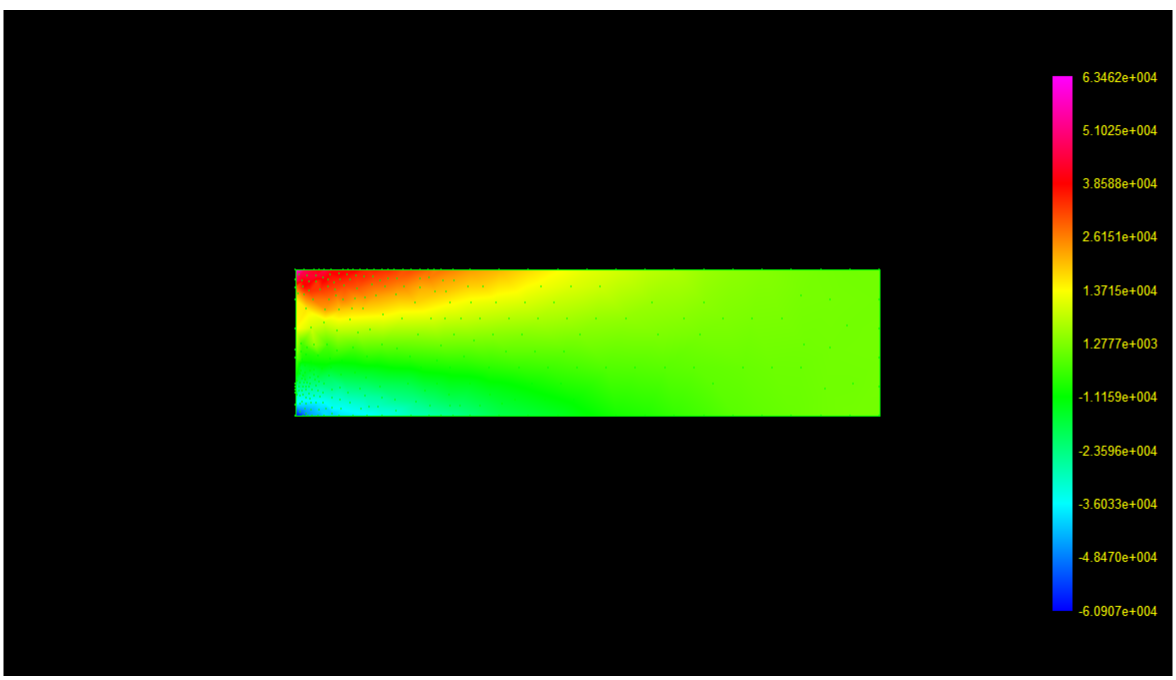


Figure 12 σ_{XX} FOR CANTILEVER BEAM

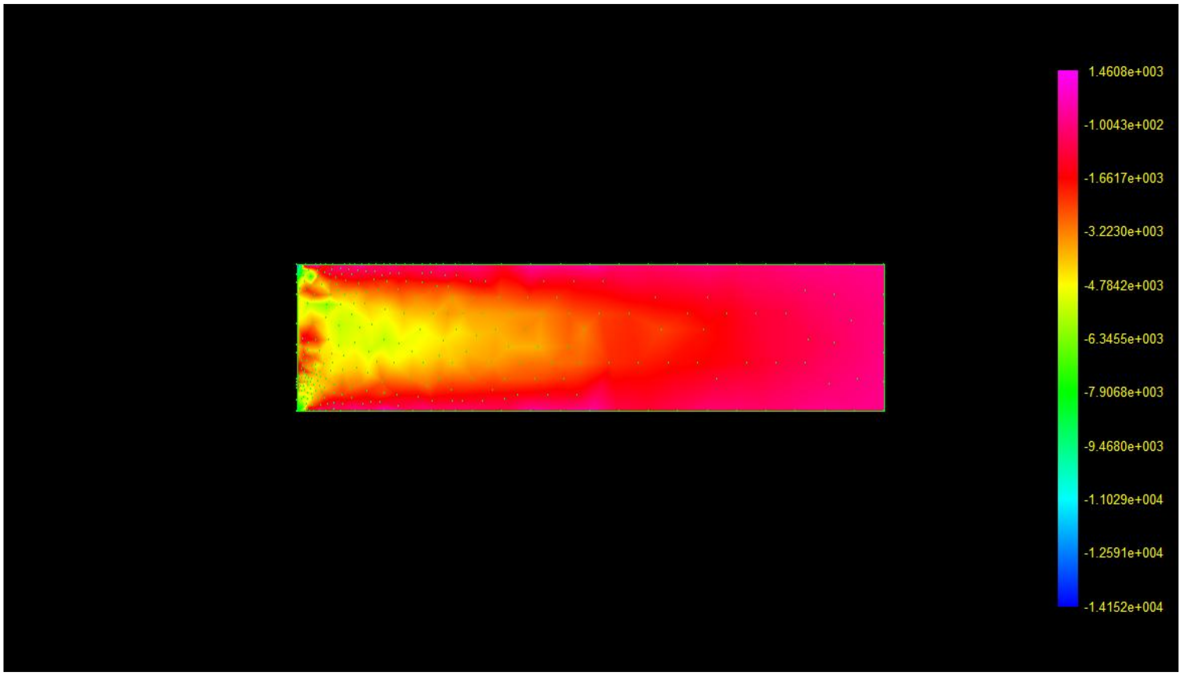


Figure 13 σ_{XY} FOR CANTILEVER BEAM

The above figures give the values of displacement, stress along x direction and the shear stresses. The first figure shows the displacement of each node owing to the external force. As expected the displacement of the free end is maximum, where as the displacement at the fixed end is almost zero. The second figure shows the distribution of σ_{xx} along the field. From this we infer that stress developed near the fixed end, both above and below the neutral axis is maximum. Also the magnitude of these stresses are equal, but they are opposite in sign. This is quite obvious as the top half of the beam is in tension where as the bottom half of the beam compression occurs. The third one shows the distribution of the shear stress along the field, the stress distribution shows higher values of shear stress near the edges of the beam as compared to the areas near the neutral axis.

The graphs for σ_{xx} and σ_{xy} at both the sections are shown in the following pages.

<u>**GRAPHS FOR SECTION 1-1**</u>

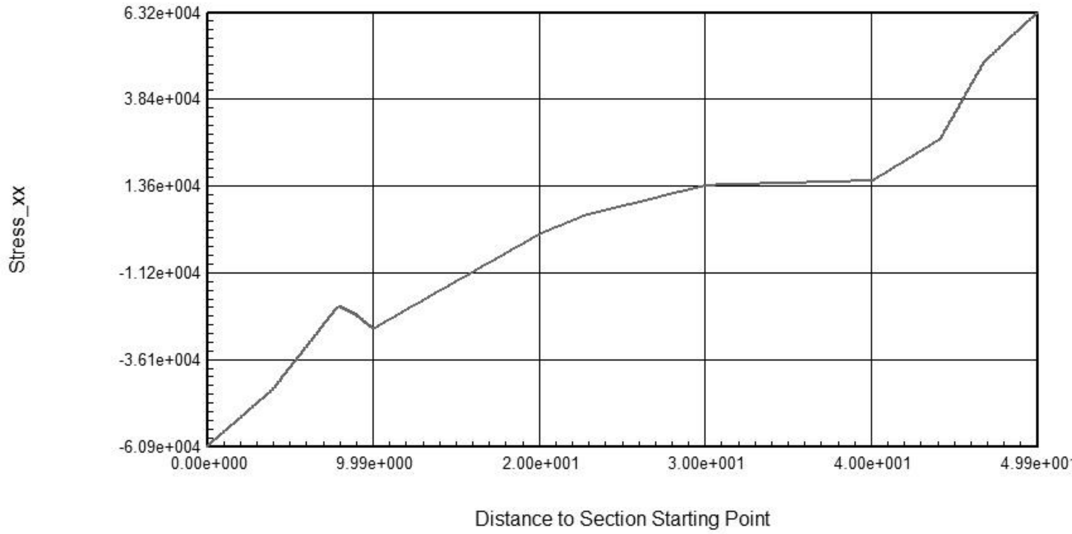


Figure 14 σ_{XX} FOR SECTION AT X=1

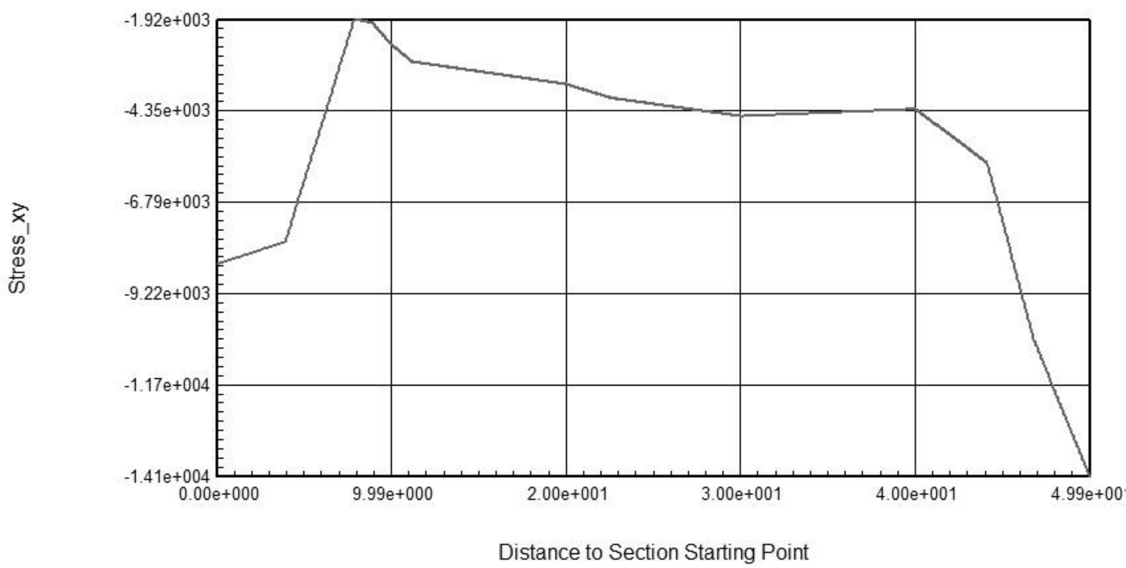


Figure 15 SHEAR STRESS AT SECTION 1-1

GRAPHS FOR SECTION 2-2

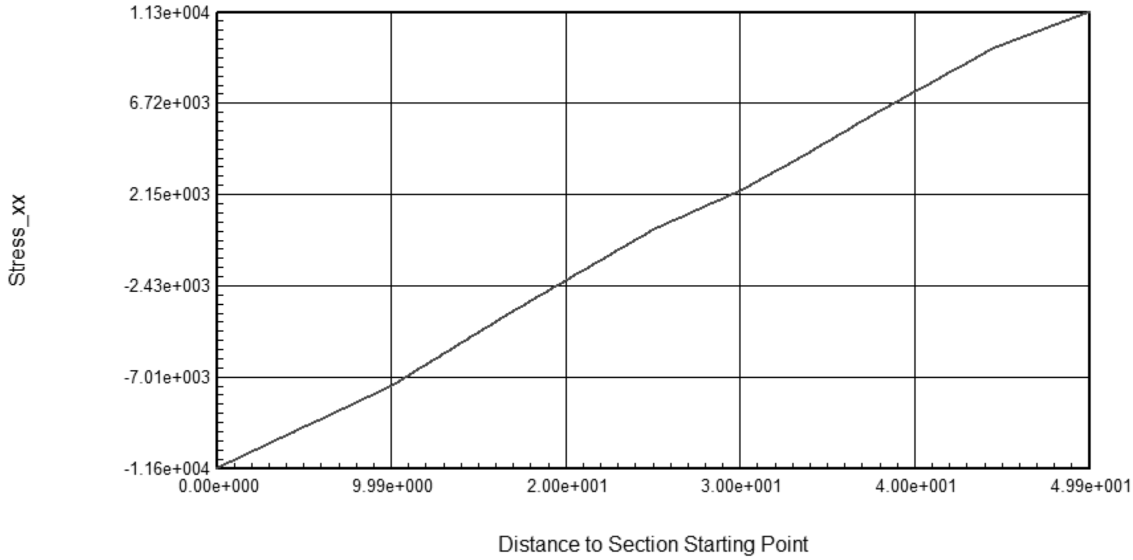


Figure 16 σ_{XX} FOR SECTION 2-2

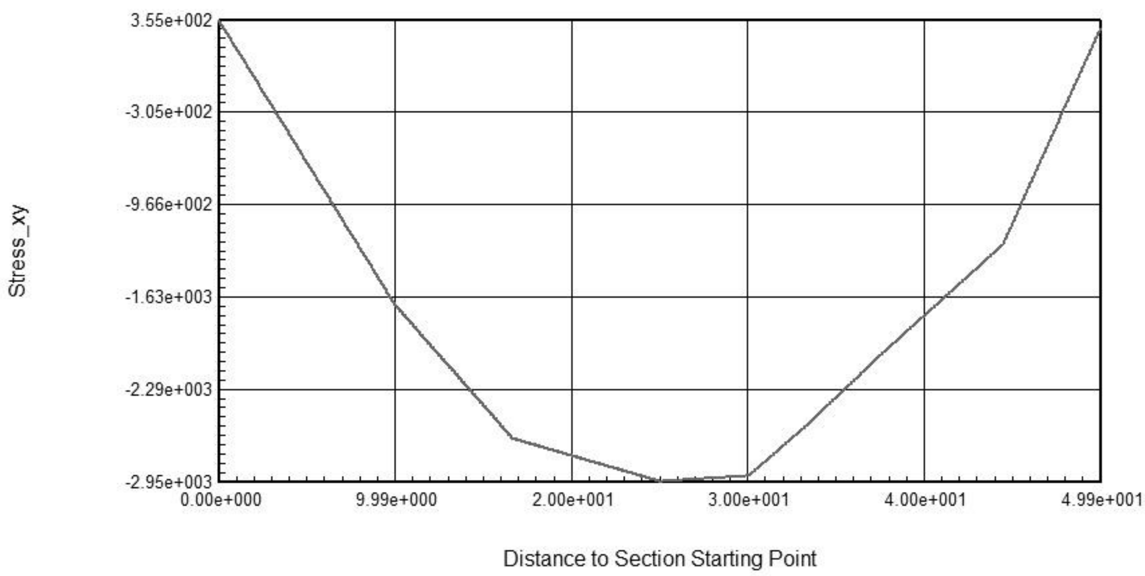


Figure 17 SHEAR STRESS AT SECTION 2-2

5.3. CANTILEVER BEAM WITH UNIFORMLY DISTRIBUTED LOAD

A cantilever beam of length = 2 m and depth = 0.5 m, subject to a traction force of 1KN. The beam is assumed to show elastic behavior and Young's Modulus = $3x10^7$ N/m^2.

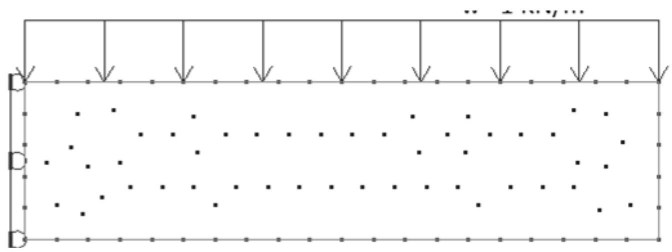


Figure 18 CANTILEVER BEAM WITH UNIFORM TRACTION

The exact analysis of the beam was done manually and the stresses from the EFG method were found out using MFree2D. A comparison between the exact and obtained stresses is shown. In the analysis of the domain two sections were considered.

- Section 1-1 at x=0m
- Section 2-2 at x=1m

Table 5 COMPARISON OF STRESSES OBTAINED BY EFG METHOD AND EXACT ANALYSIS

Sect. No.	σ_{xx} (N/mm^2) EFG Method	σ_{xx} (N/mm^2) Exact Analysis	Percentage Error
1-1	49850	48000	3.85%
2-2	11600	12000	3.33%

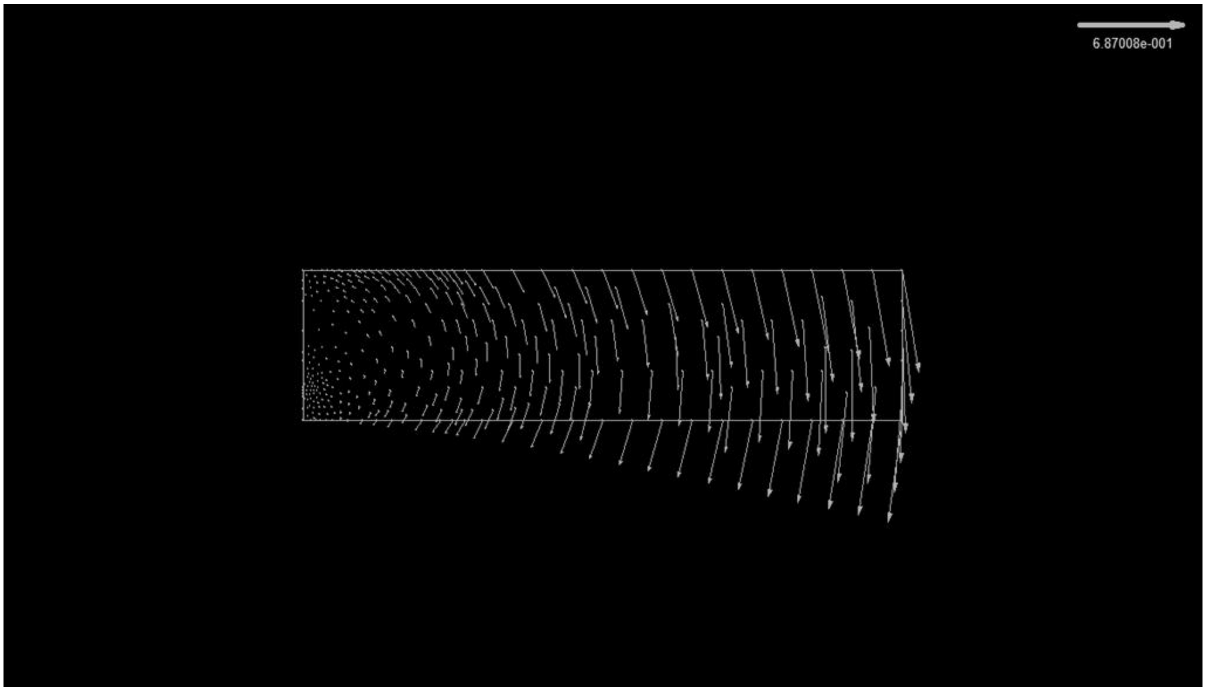


Figure 19 DISPLACEMENT FIELD VECTOR

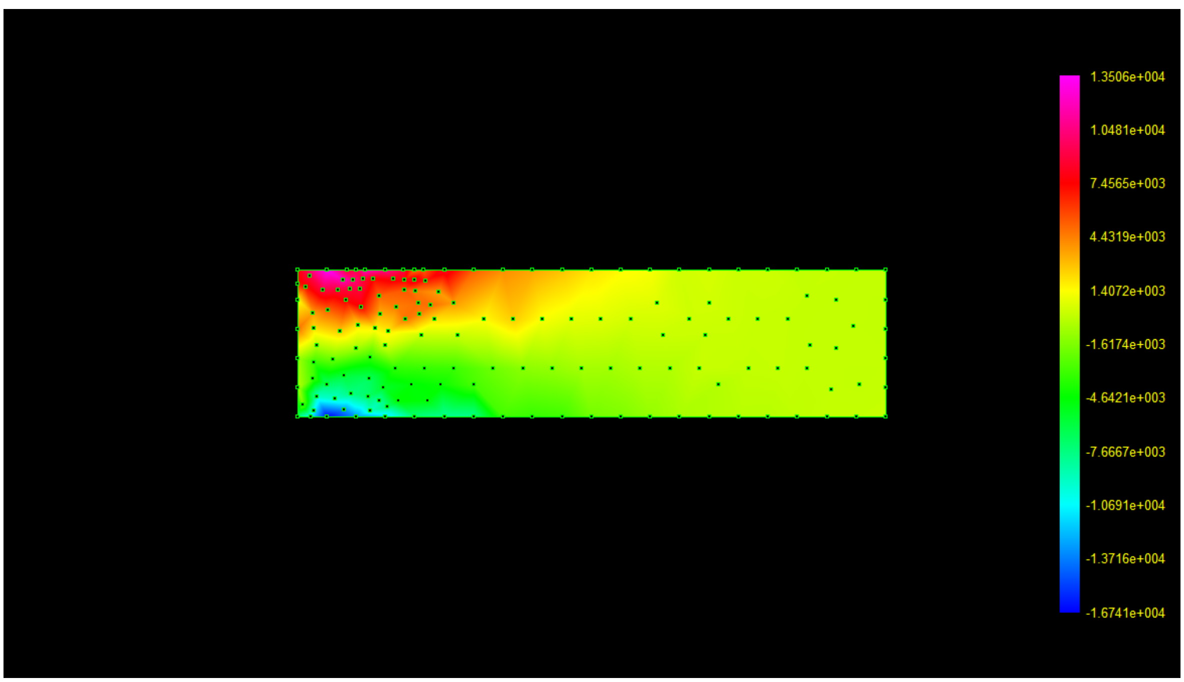


Figure 20 σ_{XX} FIELD VECTOR

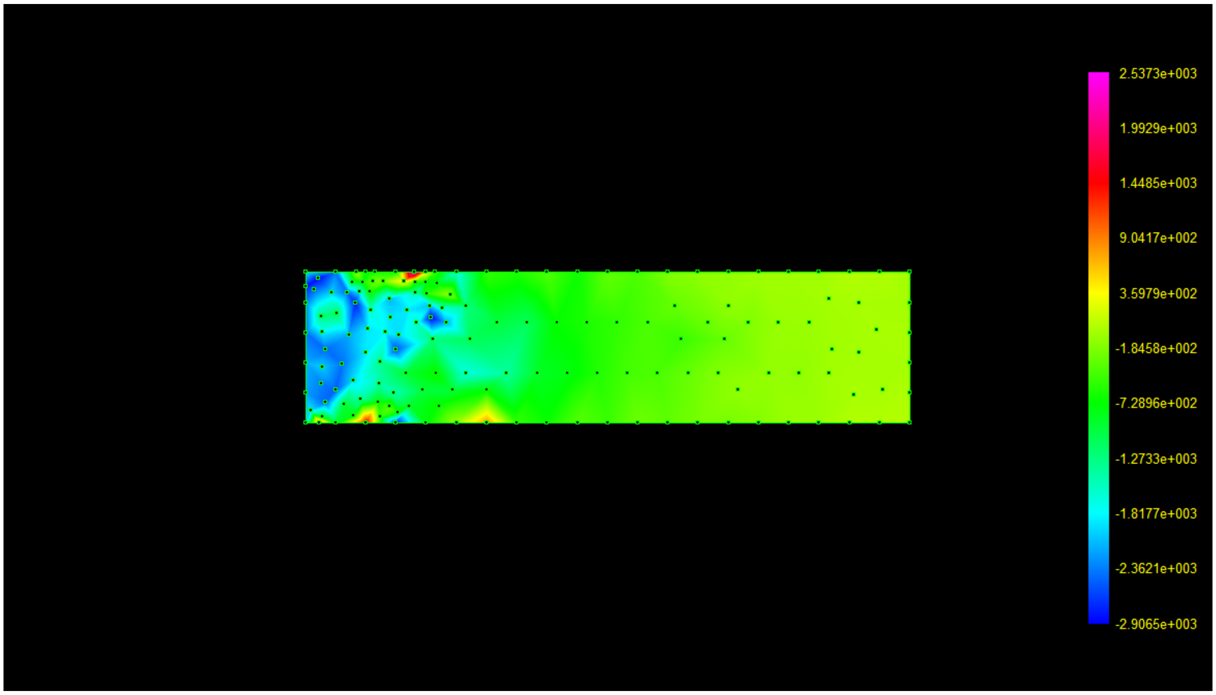


Figure 21 SHEAR STRESS FIELD VECTOR

It is found that the variation of the field parameters, are similar to that of the previous example. This is expected as both the test conditions are more or less the same, the only change being the presence of a distributed load instead of a point load. The shear stress field parameter increases with distance from the fixed end. This is because the effective load acting is dependent on the distance from the fixed end. The variation of σ_{xx} and σ_{xy} for both the above mentioned sections are shown in the subsequent pages.

GRAPHS FOR SECTION 1-1

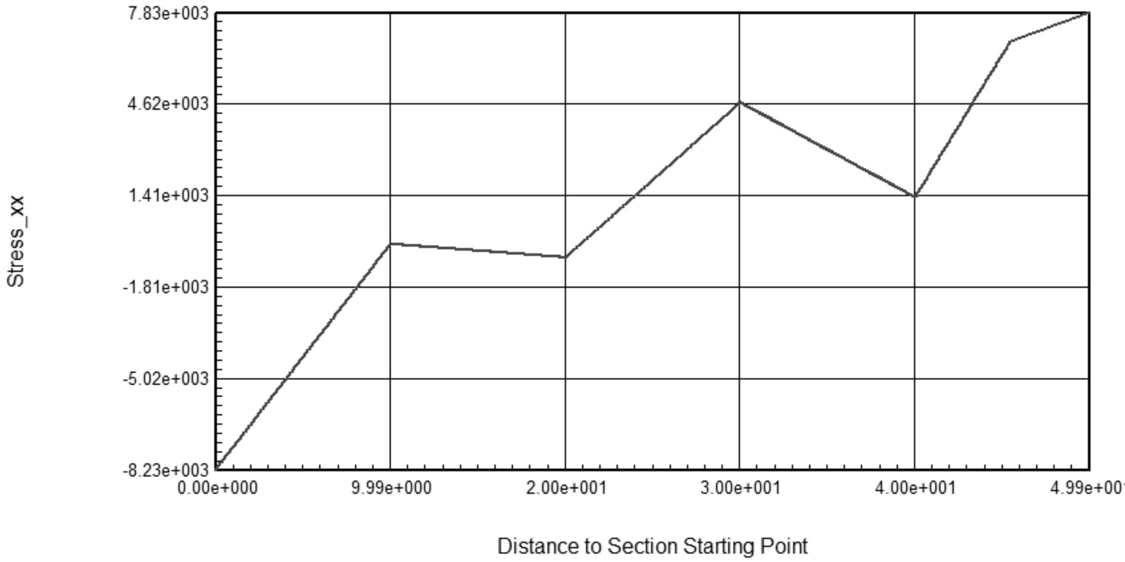


Figure 22 PLOT FOR STRESS AT SECTION 1-1

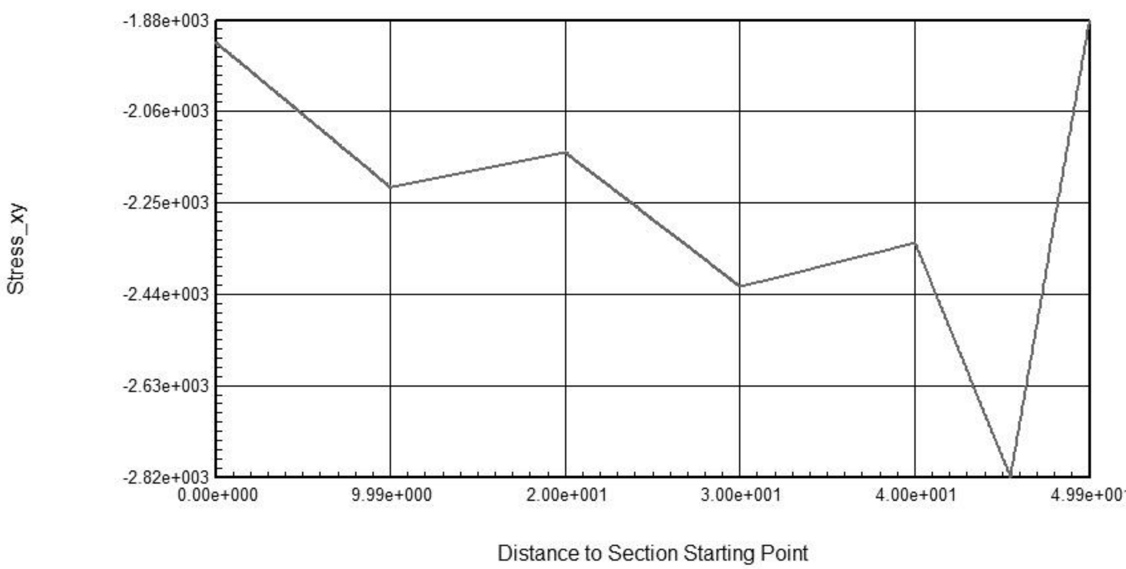


Figure 23 PLOT FOR SHEAR STRESS AT 1-1

GRAPHS FOR SECTION 2-2

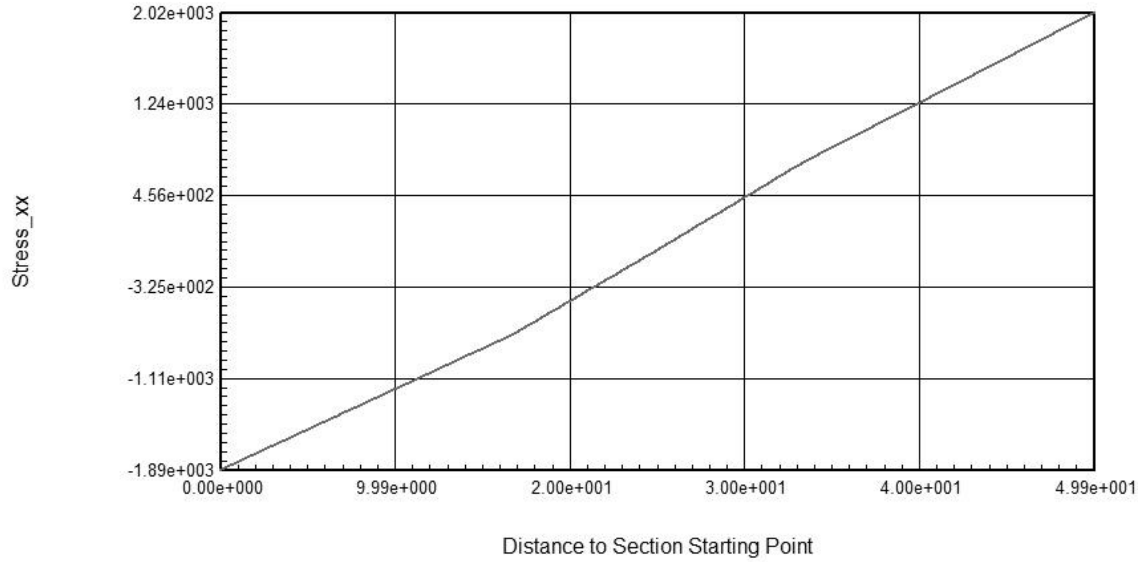


Figure 24 σ_{XX} AT SECTION 2-2

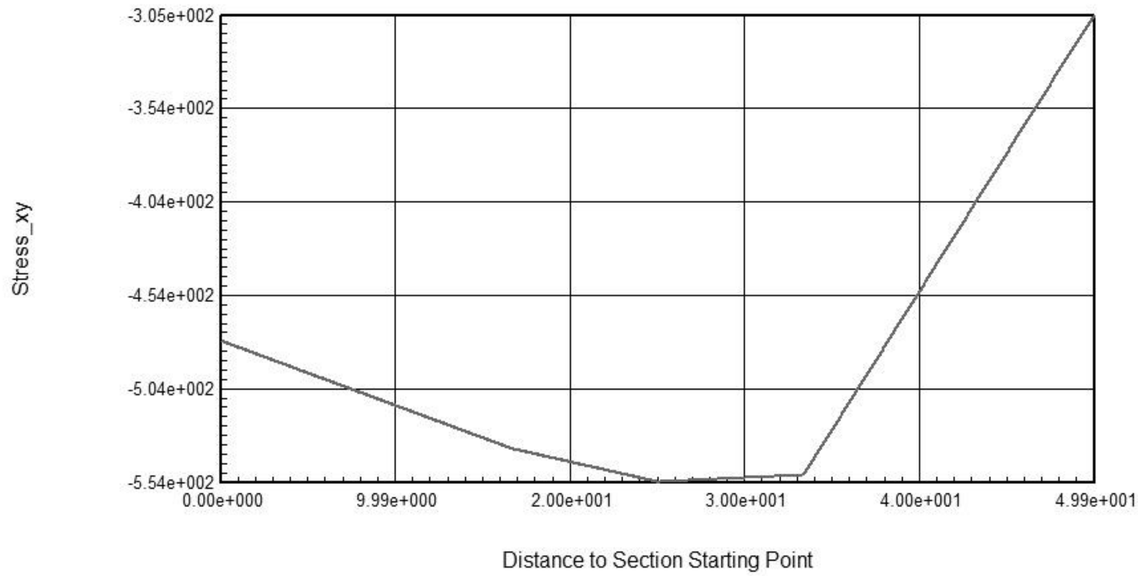


Figure 25 σ_{XY} AT SECTION 2-2

5.4. SQUARE PLATE WITH ONE EDGE FIXED AND SUBJECT TO UDL.

A thin Square plate of side 2m is subject to a distributed load of 1000N/m in x direction. It is fixed on the side opposite to that where the force is acting. All other sides are free. The material is assumed to be completely elastic in nature with a Young's Modulus = 3x 10^7 N/m^2.

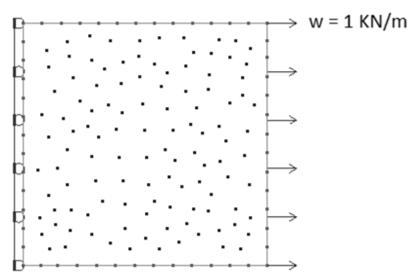


Figure 26 PROBLEM STATEMNT 4

The exact analysis for this problem has not been carried out. The problem only aims to highlight the usage of EFG method in calculation of various field parameters, namely; stress (σ_{xx} and σ_{xy}) and displacement. The plate problem is a very common one and has been chosen so that understanding and interpretation of the results are fairly easy. The plots of the field parameters are shown along with variation of these parameters along 2 section lines is shown.

Sections taken:

- Section 1-1 x=0(along fixed end)
- Section 2-2 (along the diagonal)

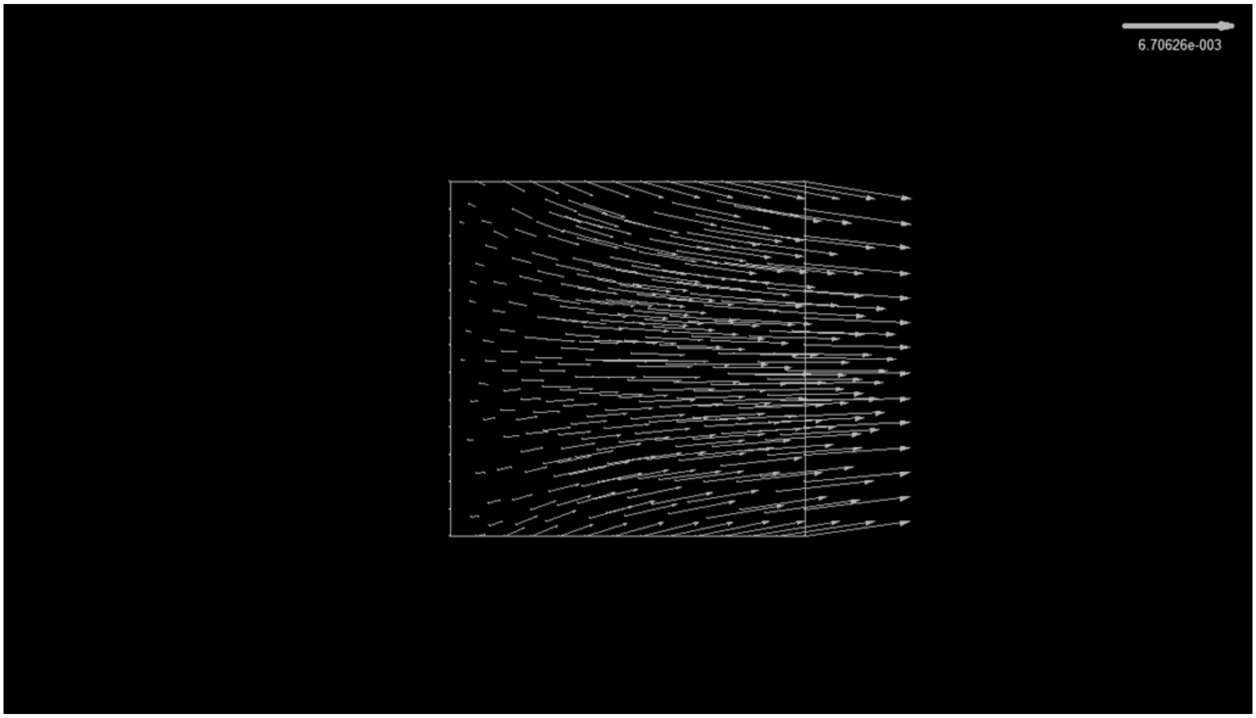


Figure 27 DISPLACEMENT VECTOR

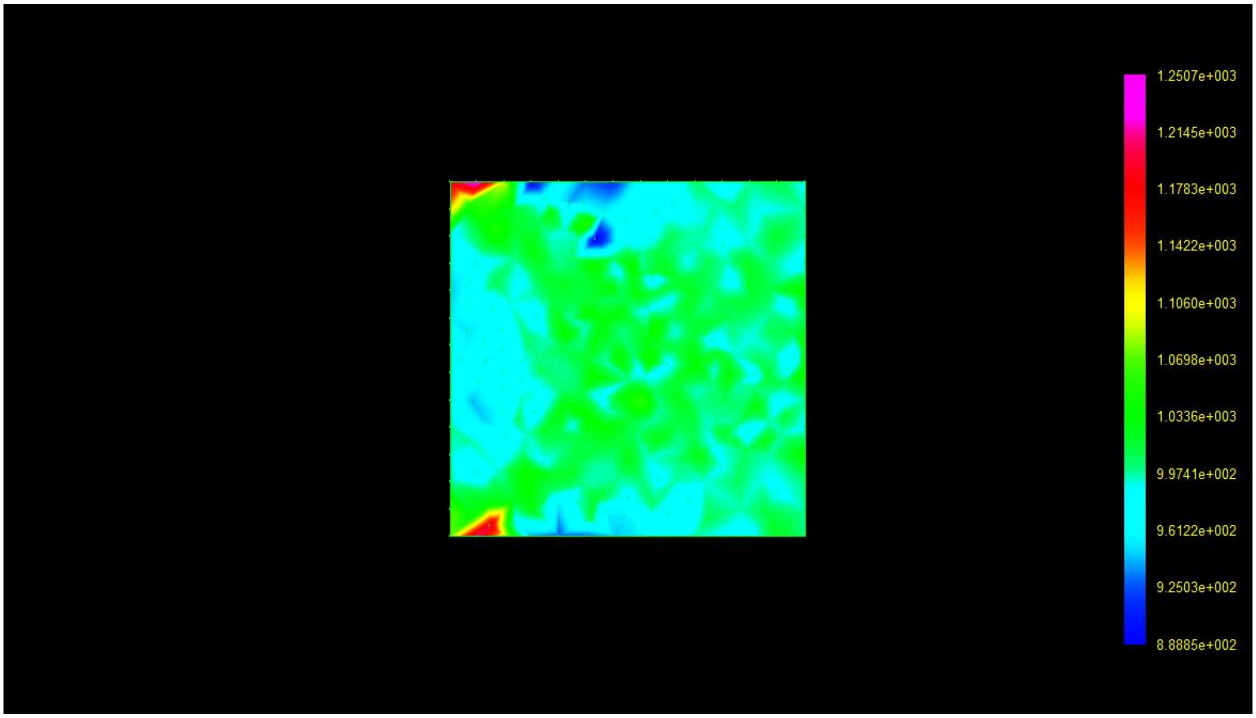


Figure 28 FIELD VECTOR σ_{XX}

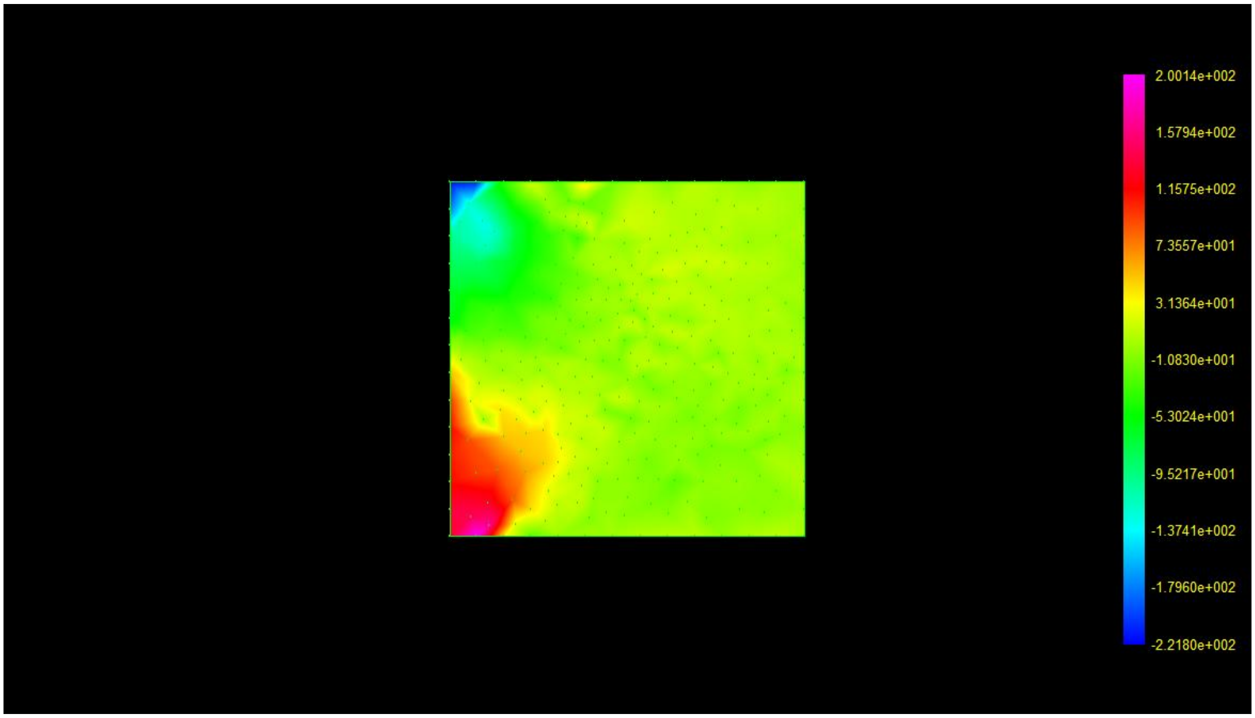


Figure 29 FIELD VECTOR σ_{XY}

The field vector displacement shows that the nodes closer to the fixed end are displaced less as compared to the nodes further away. Again the effect of the applied force on the plate domain is clearly visible by the displaced location of the nodes. The field vector σ_{xx} shows that the entire plate domain is subject to a more or less uniform stress (997.4 N/m2 to 1033.6N/m2). The stress is maximum at the corners of plate near the fixed end (1178.3 N/m2). The field vector σ_{xy} is similar to that of σ_{xx}, with the fact that the shear near the fixed end is maximum and for the rest of the plate is more or less uniform. The graphs showing the variation of these parameters at the pre-defined sections are shown on the following page.

GRAPH FOR SECTION 1-1

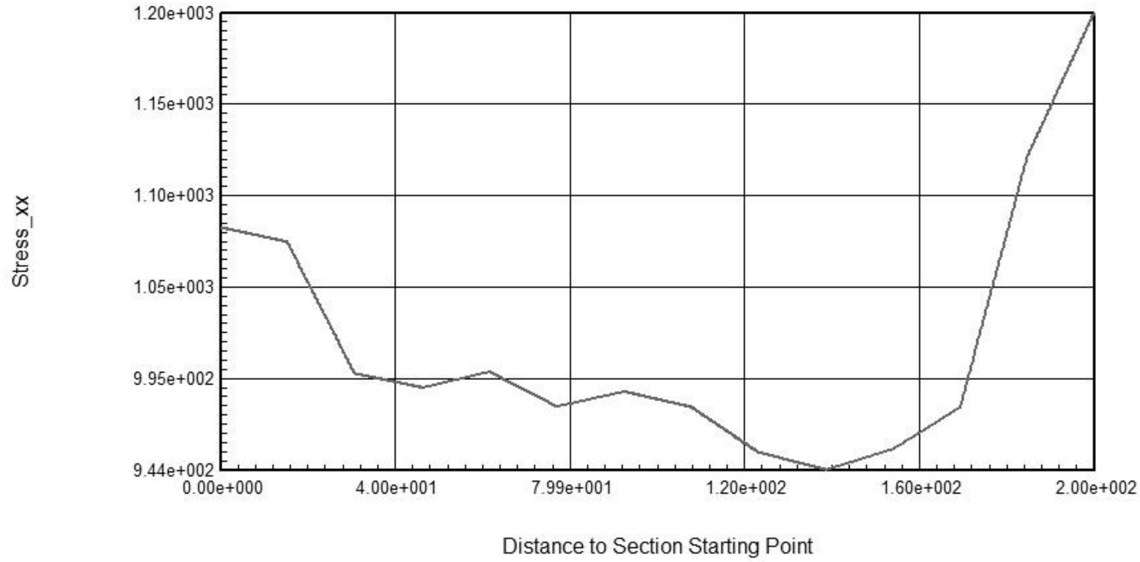


Figure 30 σ_{XX} AT SECTION 1-1

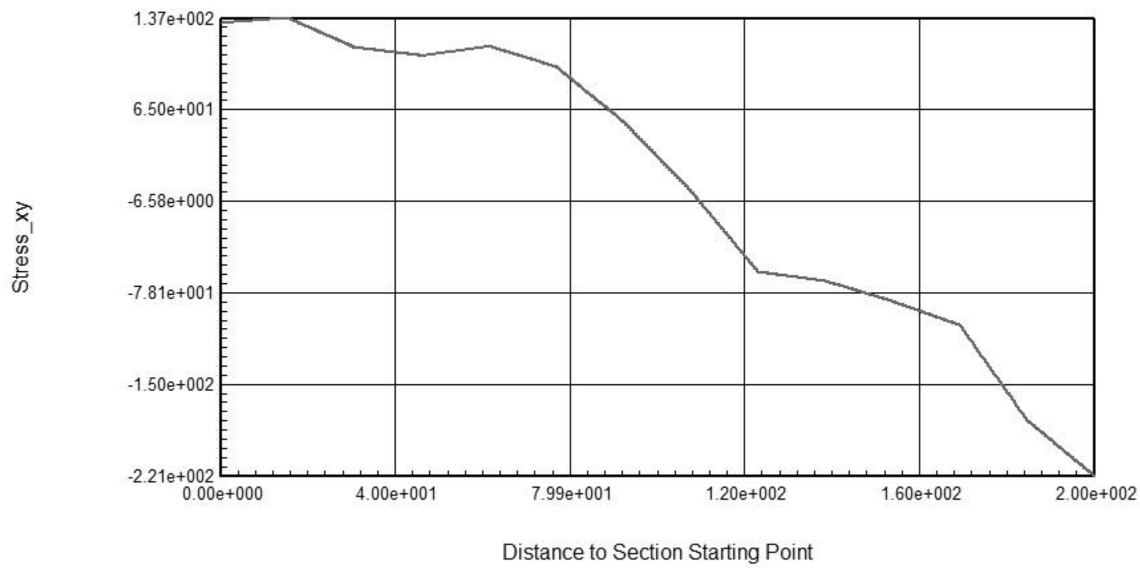


Figure 31 σ_{XY} AT SECTION 1-1

GRAPHS FOR SECTION 2-2

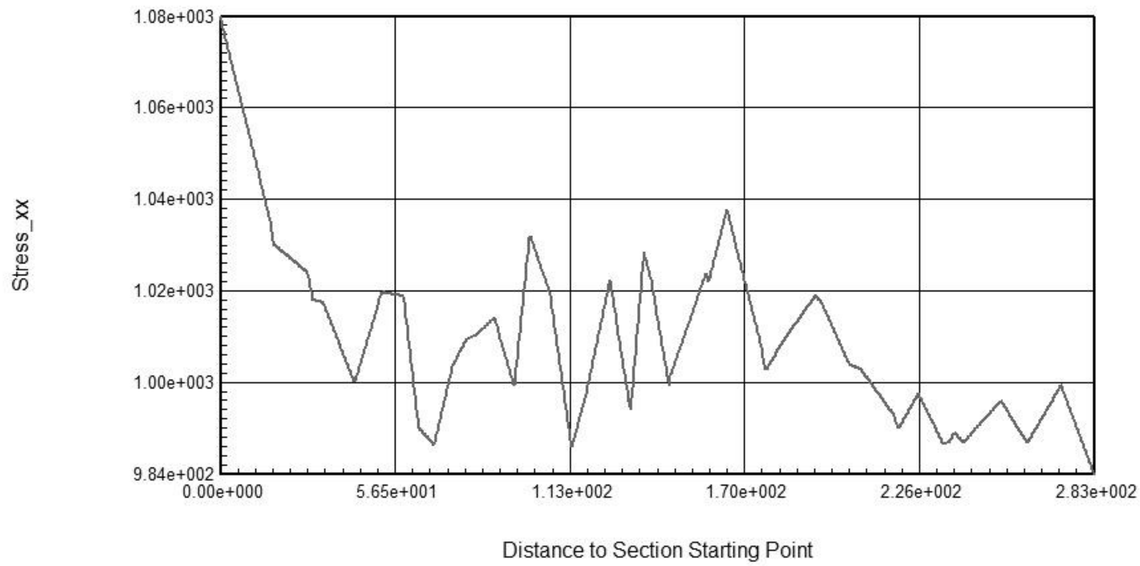


Figure 32 σ_{XX} FOR SECTION 2-2

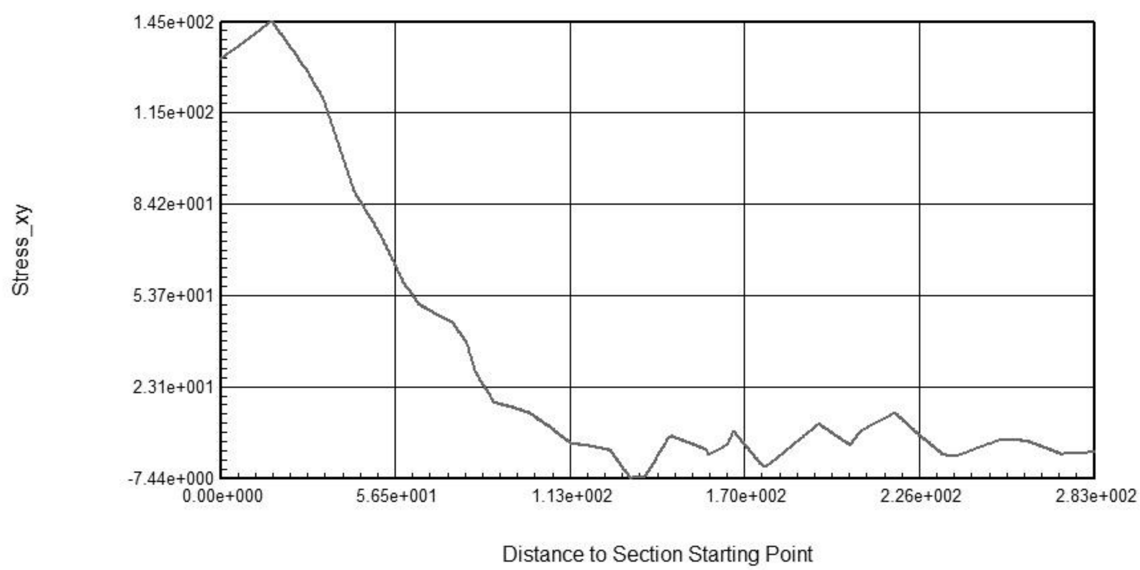


Figure 33 σ_{XY} AT SECTION 2-2

5.5. SQUARE PLATE WITH DEFORMED EDGE:

A thin square plate with a deformed edge is considered. The plate is fixed at two opposite ends and is subject to equal distributed force at the free edges. The plate is considered to be completely elastic with a Young's Modulus = 3×10^7.

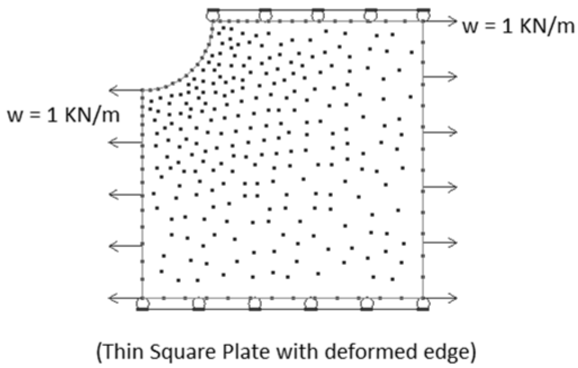


Figure 34 SQUARE PLATE WITH DEFORMED EDGE

Only EFG method analysis of this domain has been carried out. The direct stress along x, shear stress and the displacement field vectors are plotted. A discussion on the results of the analysis is given after the plots.

A section 1-1 is chosen such that it passes through the midpoint of the plate and through both the fixed edges.

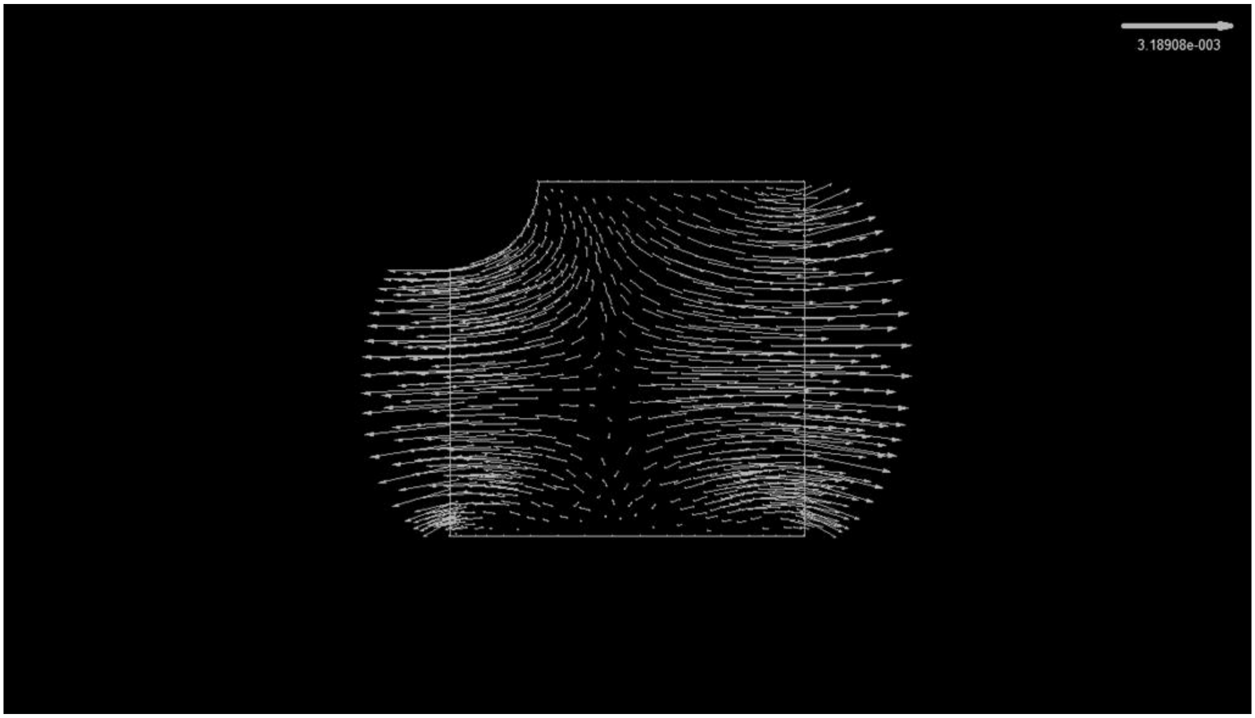


Figure 35 DISPLACEMENT FIELD VECTOR

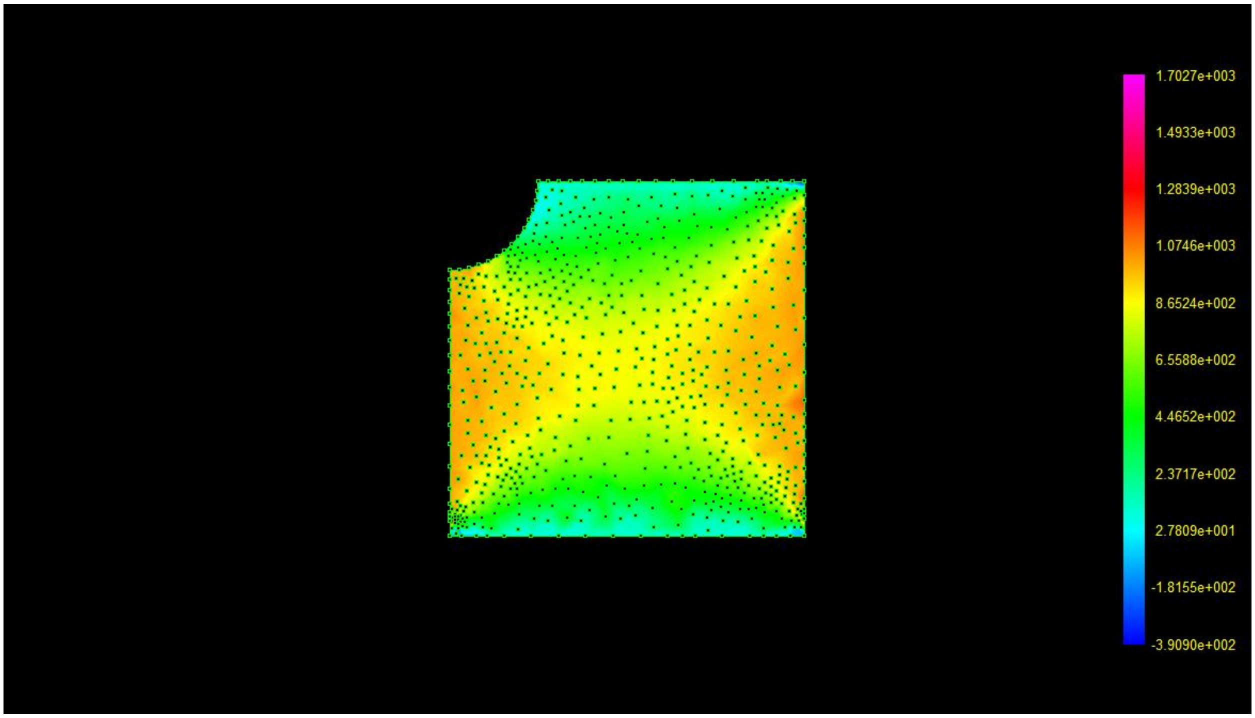


Figure 36 σ_{XX} FIELD VECTOR

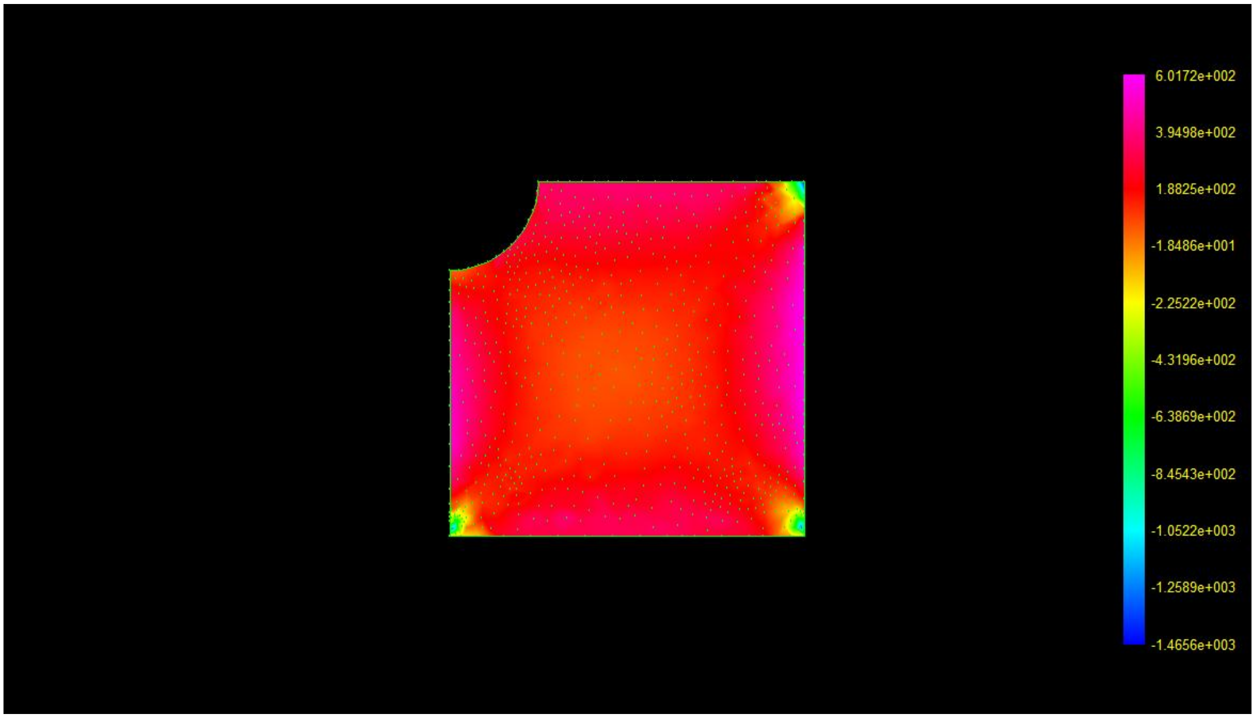


Figure 37 σ_{xy} FIELD VECTOR

The field vector shows the extent of the deformation due to the external forces. The study of the σ_{xx} field vector reveals that the stresses near the free edges are greatest where as that developed near the fixed edges is lower. The σ_{xy} field vector shows that the shear stress at the interior of the plate domain is highest. The edges face a relatively lower shear stress.

GRAPH AT SECTION 1-1

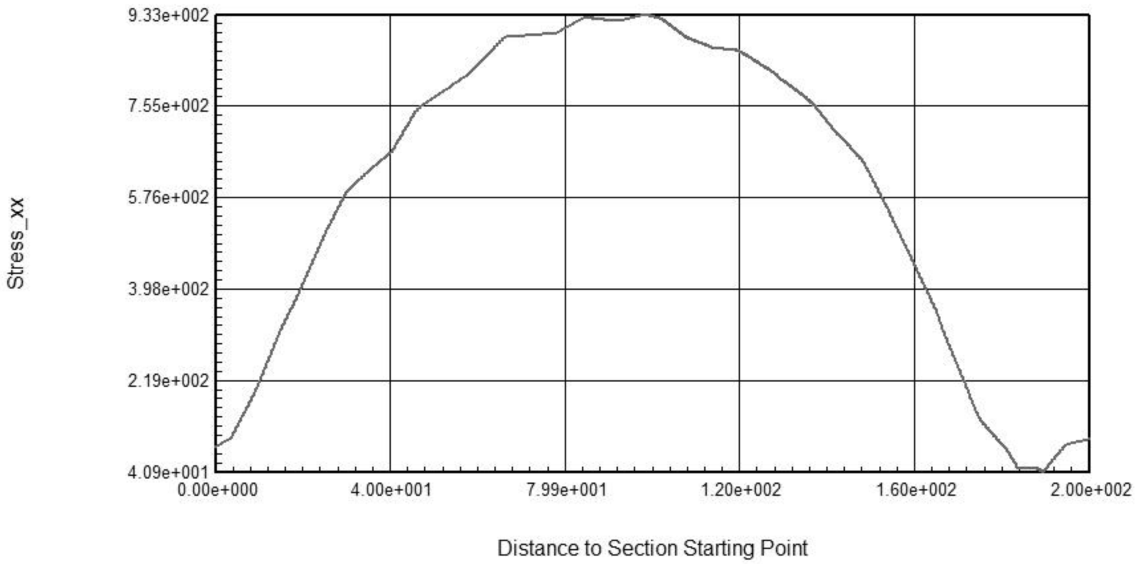


Figure 38 σ_{XX} AT SECTION 1-1

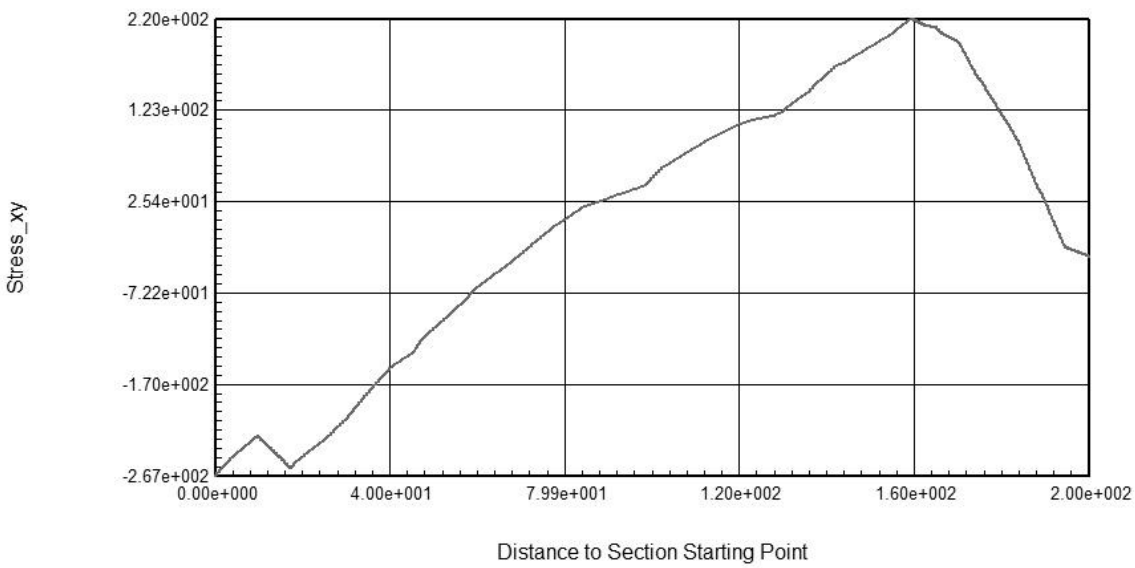
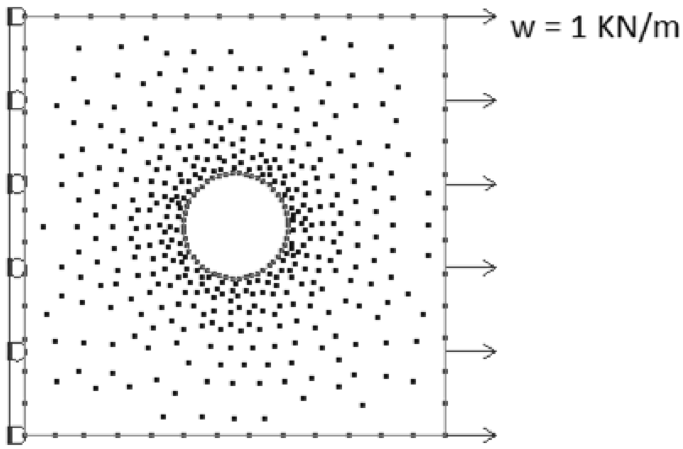


Figure 39 σXY FOR SECTION 1-1

5.6. SQUARE PLATE WITH HOLE

Consider a thin square plate with a hole at the centre of the plate. The plate is fixed at one edge and a uniformly distributed force is applied to the opposite edge.



(Thin Plate with hole at centre)

Figure 40 SQUARE PLATE WITH A HOLE

Here the nodes are distributed in such a way that there is greater concentration of nodes near the boundary of the hole this will help us determine the field vectors near the whole with greater accuracy. In other regions of the plate domain, stress distribution is relatively simpler and of lesser importance. In the Element Free Galerkin Method one can take the liberty of increasing density of the node near a specific point of interest. In this domain to sections have been taken;

- Section (1-1) at the fixed edge
- Section(2-2) passing through the centre of the hole parallel to section 1-1

The field vectors for the above domain are on the following pages.

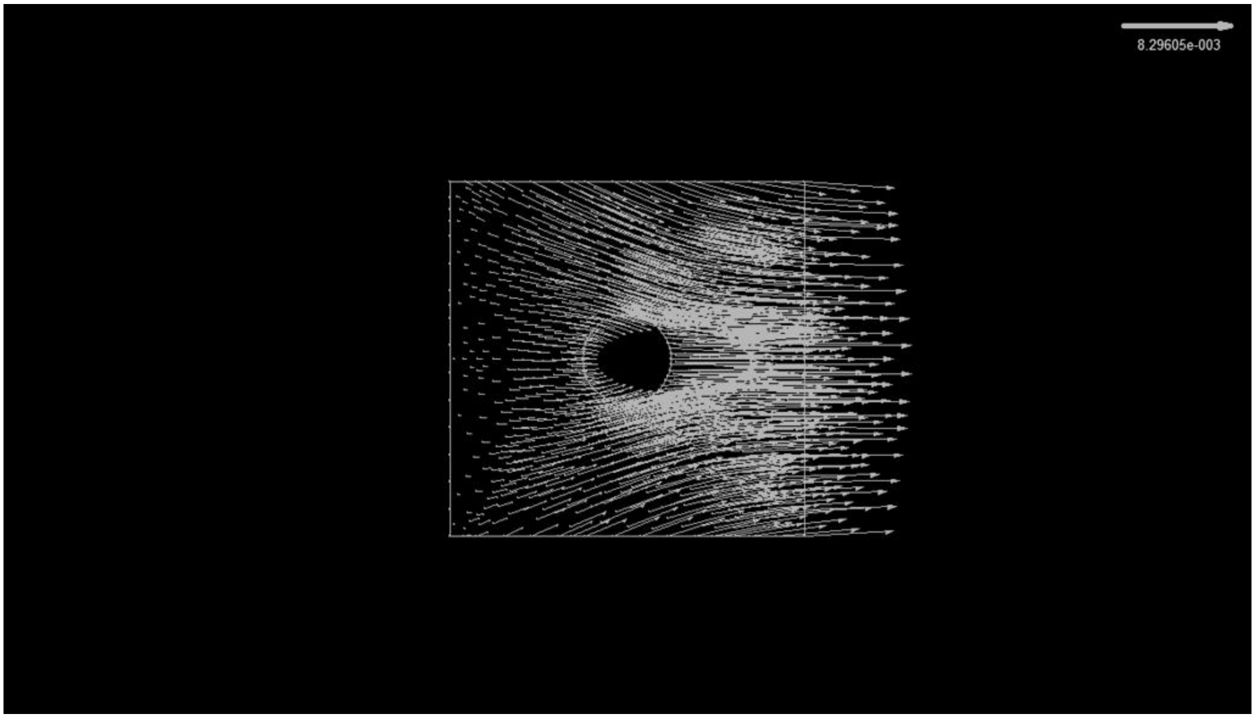


Figure 41 DISPLACEMENT FIELD VECTOR

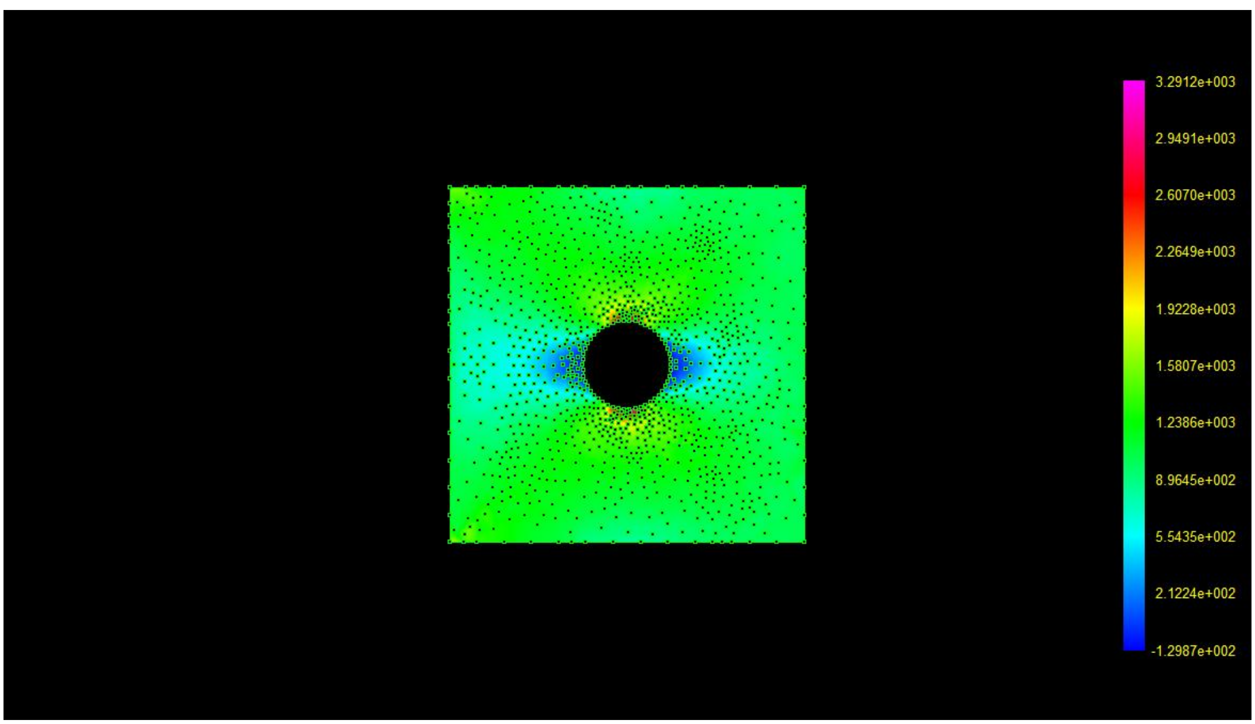


Figure 42 σ_{xx} FIELD VECTOR

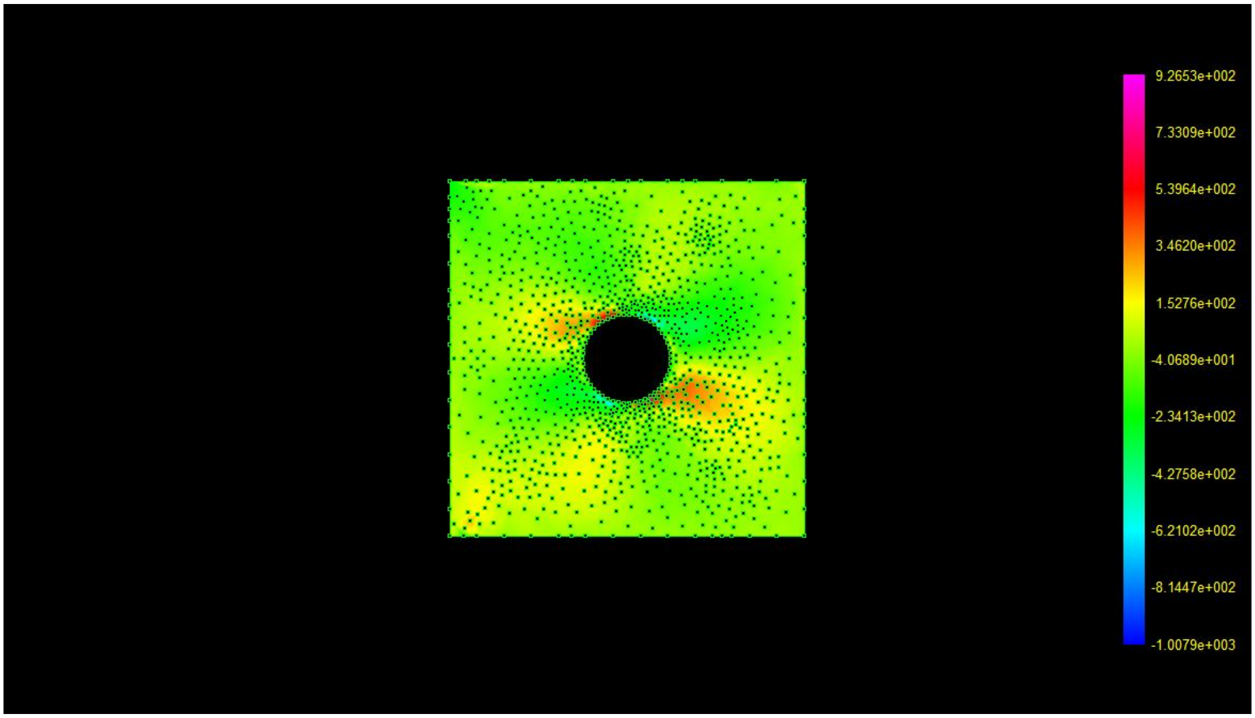


Figure 43 σ_{XY} FIELD VECTOR

The displacement vector shows that the hole will get deformed greatly owing to the applied force. The nodes at the fixed end will not suffer much deformation. The σ_{xx} field vector shows that in the region directly above and below the hole the stress will be maximum where as the region just left or right of hole will have minimum stress. The σ_{xy} field vector shows that the shear stress developed near the hole will be greater and the distribution pattern more complicated near the hole as compared to other areas of the plate domain. The field vectors for both direct stress and shear stress shows the stress distribution very clearly. Here the importance of the increased density of the node can be clearly noted as the variation of stress high compared with change in location of the point.

<u>**GRAPH AT SECTION 1-1**</u>

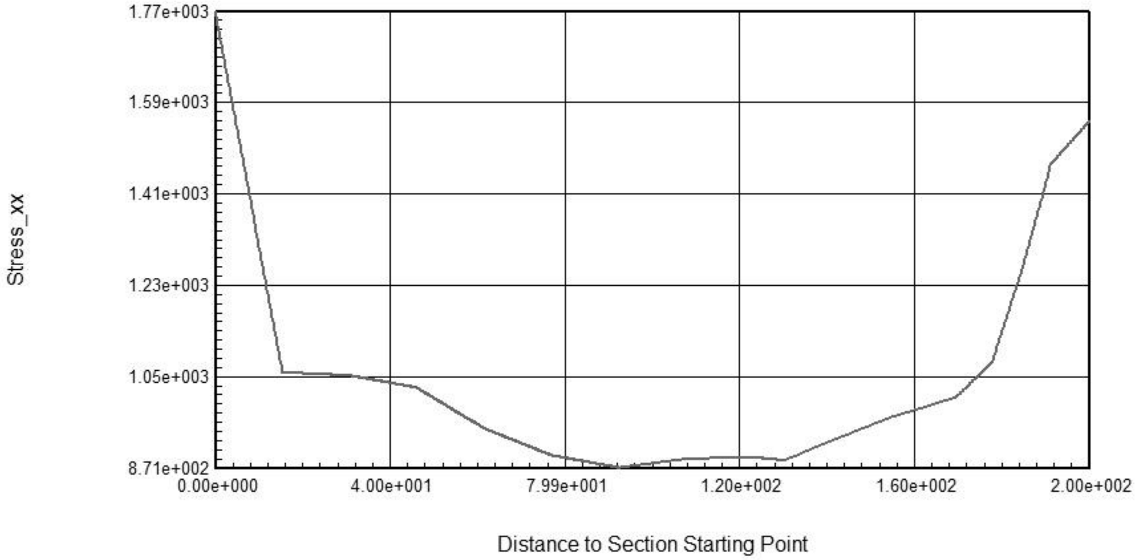


Figure 44 σ_{XX} AT SECTION 1-1

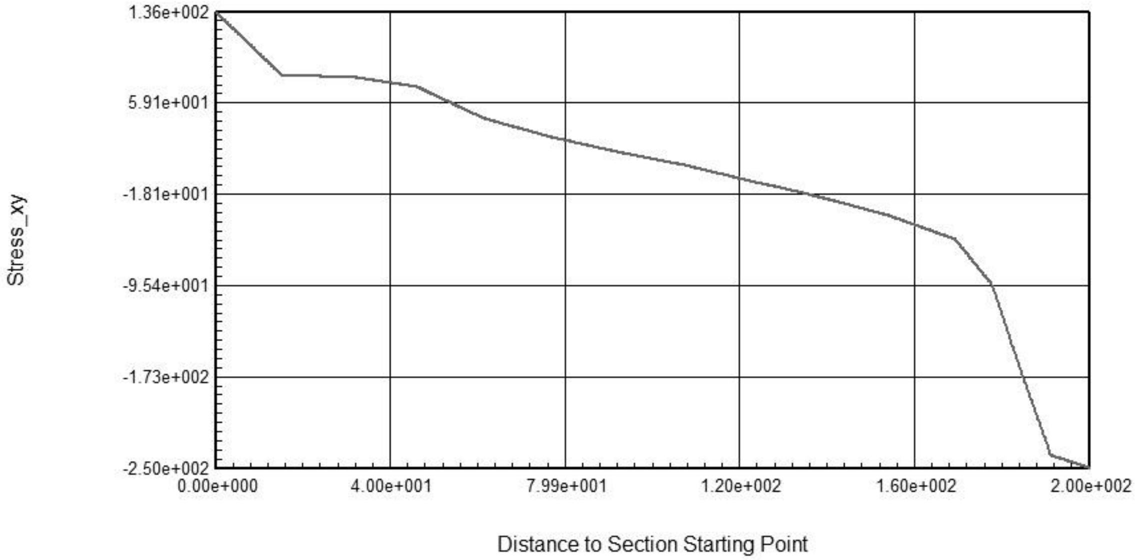


Figure 45 σ_{XY} AT SECTION 1-1

GRAPH AT SECTION 2-2

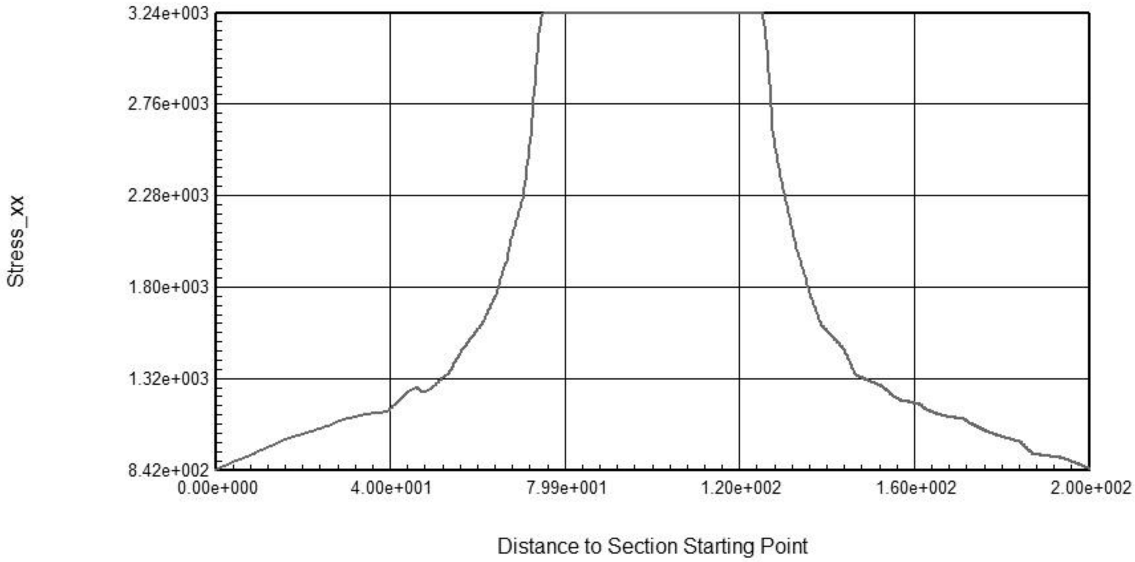


Figure 46 σ_{XX} AT SECTION 2-2

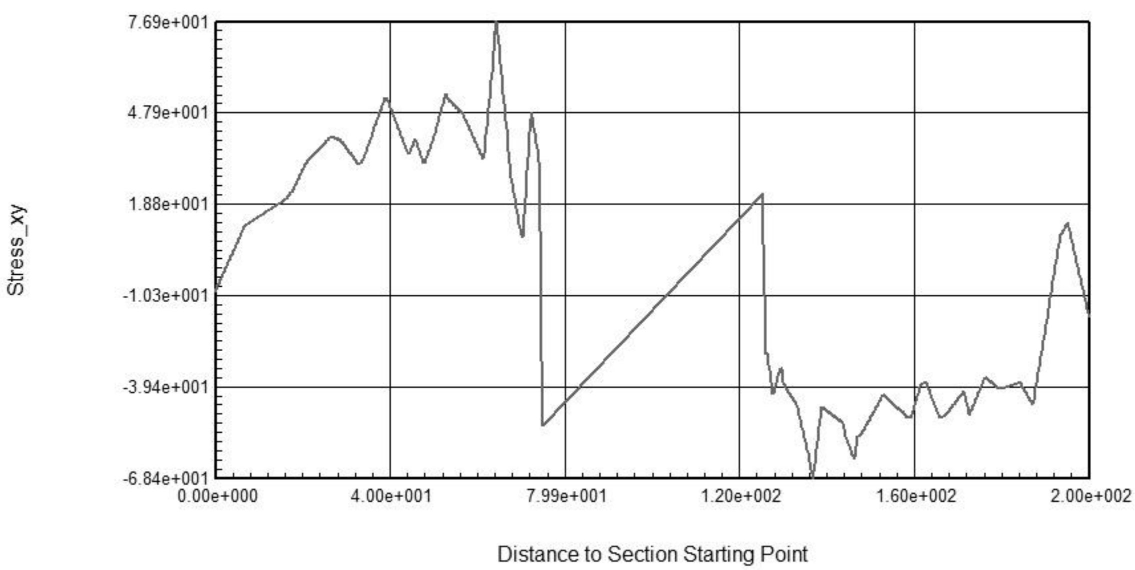
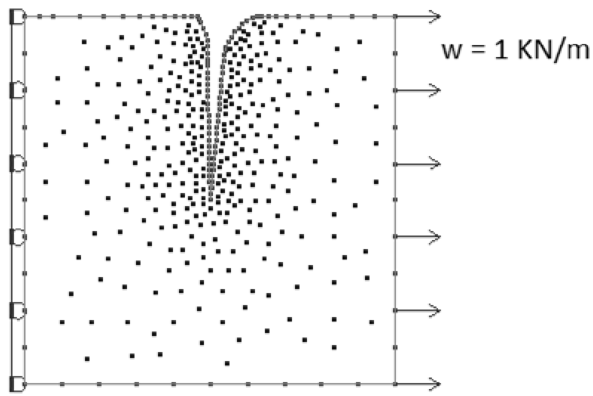


Figure 47 σ_{XY} AT SECTION 2-2

5.7. SQUARE PLATE WITH CRACK

Consider a thin square plate with one edge fixed and a crack at any one free end. The plate is subject to a uniformly distributed load at the free edge opposite of the fixed one.



(Thin Plate crack upto centre)

Figure 48 THIN PLATE CRACKED

This problem is a typical problem of crack analysis. In this case the crack has developed till the centre of the plate. Again the nodes are placed in such a way that their density is greater near the crack as compared to other areas of the plate. The plots for the stress distribution will help us analyze the nature of the cracks and help in predicting its propagation.

We have also taken a section (1-1) at the x= midpoint of the plate.

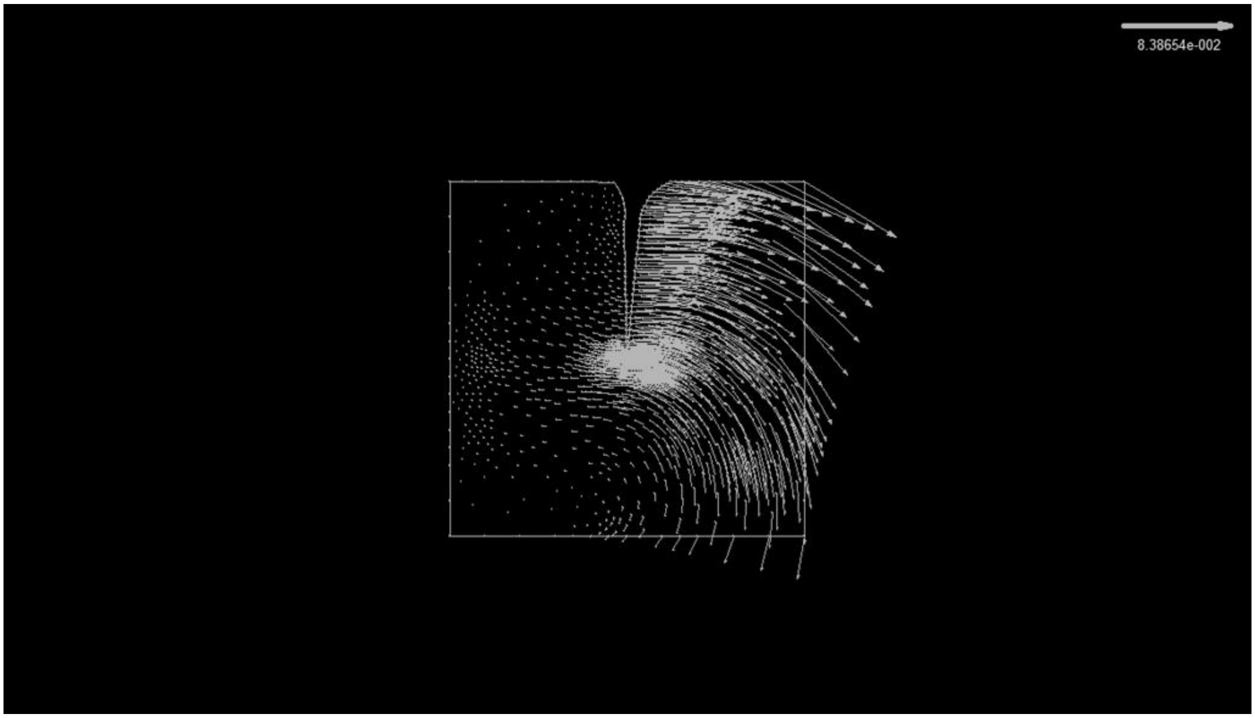


Figure 49 DISPLACEMENT FIELD VECTOR

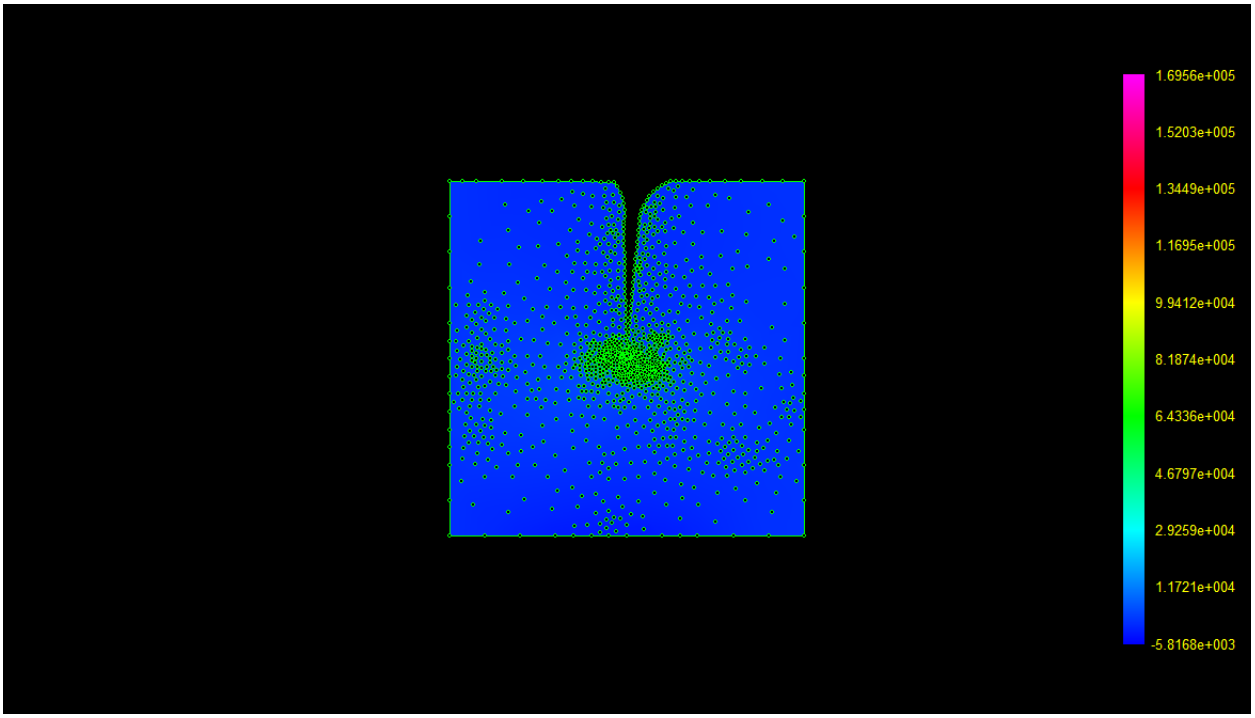


Figure 50 σ_{XX} FIELD VECTOR

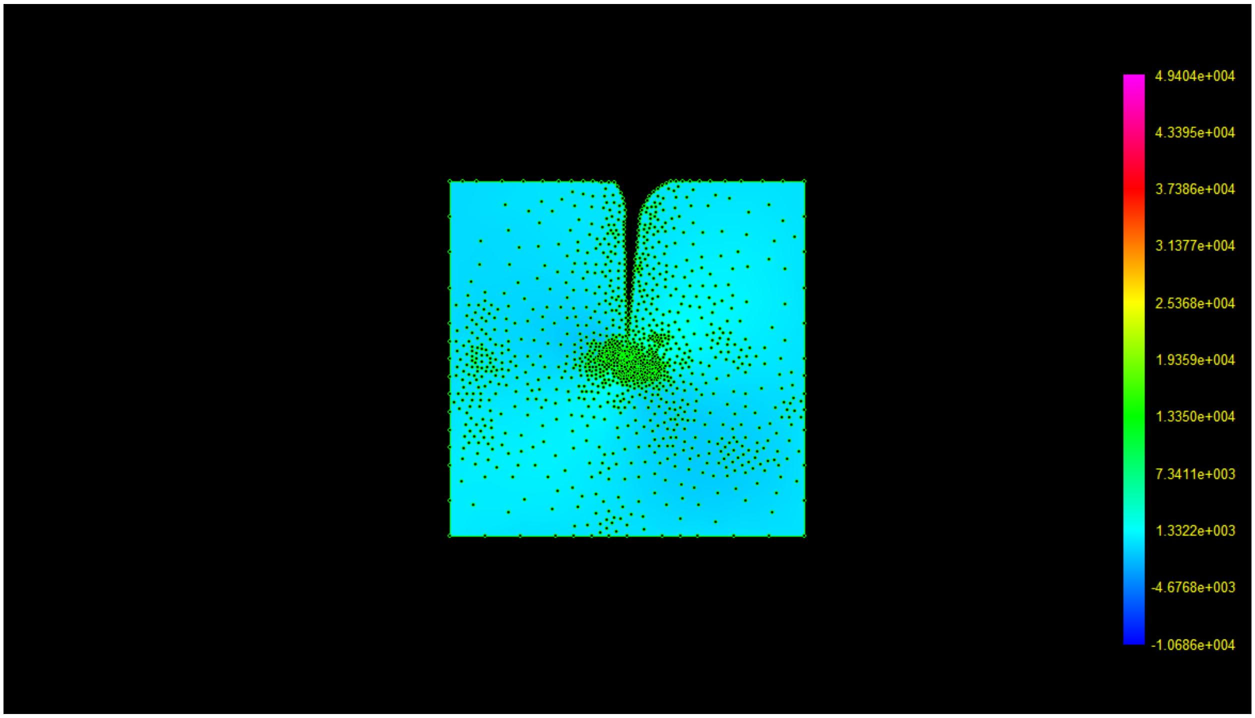


Figure 51 σ_{XY} FIELD VECTOR

The displacement vector shows that the force cause a greater displacement on the nodes to the right of the crack compared to the rest. Both the shear stress and the direct stress are greatest at the tip of the crack which is expected. The Stress distribution for the remaining domain is uniform in nature.

GRAPH AT SECTION1-1

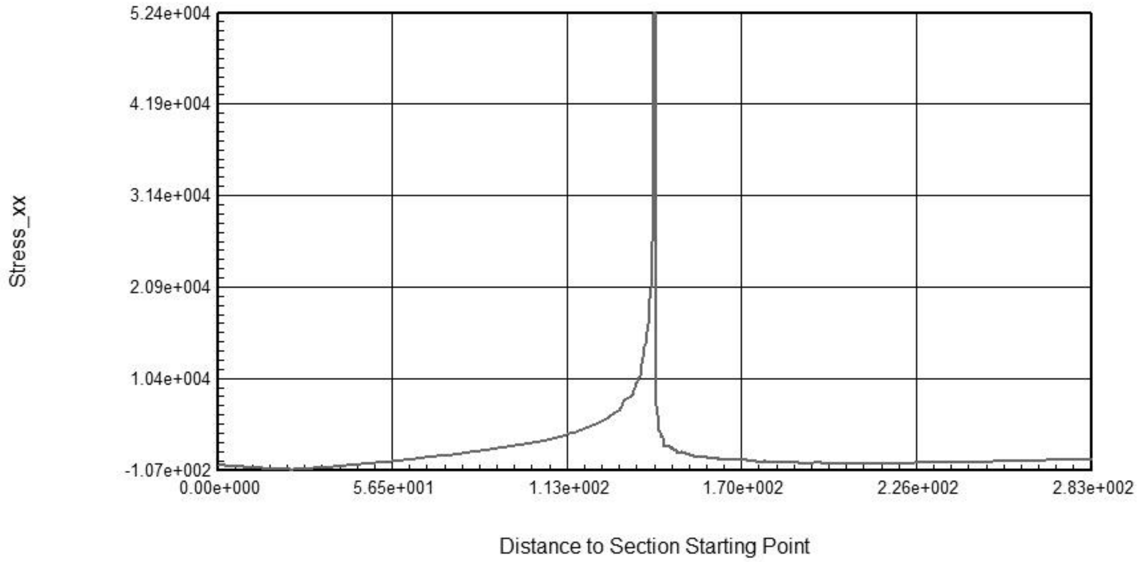


Figure 52 σ $_{XX}$ AT SECTION 1-1

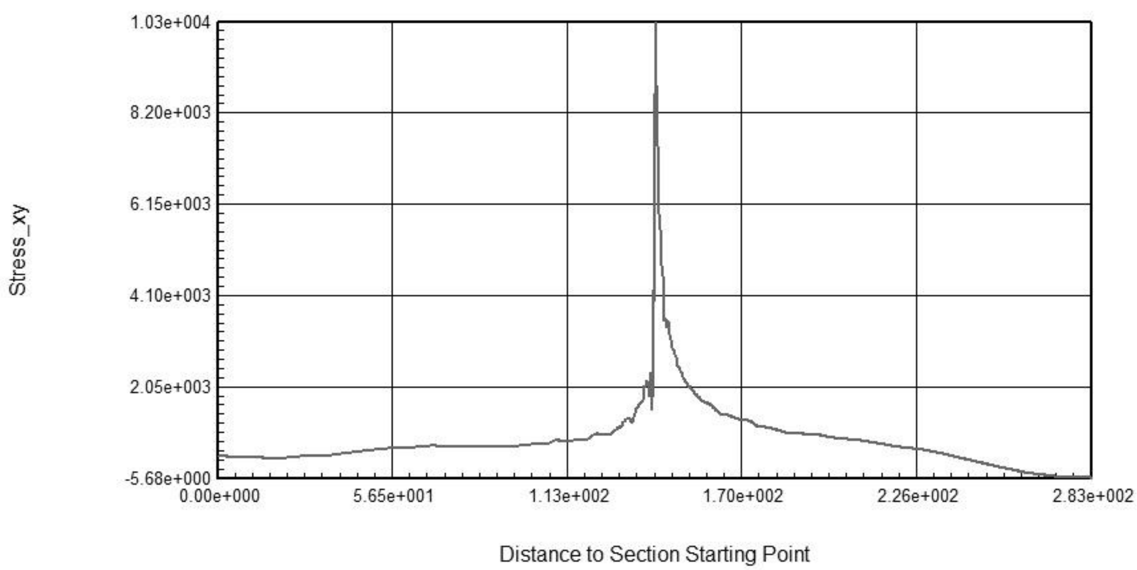


Figure 53 σ_{XY} AT SECTION 1-1

6. CONCLUSION

The Element Free Galerkin method is a mesh-free method used for analysis of various structures or problem domains. A detailed study of the method revealed that the method is not completely mesh less as it requires the use of a background mesh for integration purpose. But apart from this the mesh does not serve any other purpose and there is no need to predefine it. The density of the nodes can be varied as per the problem statement. Another key factor was that the shape function created by MLS approximation do not satisfy the essential boundary conditions, hence use of Lagrange multipliers was necessary to enforce the boundary conditions.

The first three numericals discussed were meant to establish the accuracy of the EFGM. The EFG analysis for the first method was coded using Matlab and the remaining numerical (including the last four) were solved using MFree2D software package. The following were the conclusions drawn.

- The accuracy of the EFGM is directly proportional to the number or nodes. With the increase in the number of nodes the accuracy of the EFGM automatically increases.
- Similarly, keeping the number of nodes constant, we can increase the quadrature points to decrease the error value. This can be achieved by refining the background mesh. Since the mesh is not predefined one can refine the mesh without having to remodel the entire domain. Thereby, not affecting the total computation time.

Thus with the proper choice of the number of nodes and the quadrature points, the EFGM proves to be a highly accurate analysis method for elasto-statics.

The last four cases discussed where analysis of the problems was done only by EFGM showed that the EFGM can be used as an effective tool for analysis for structures where there is large deformation or complicated geometry is involved. The importance of variation in node density was also highlighted, especially in the cases: Square plate with circular hole and Square plate with crack. The stress distribution obtained for each case can be very easily interpreted which may be used to predict failure or for optimization of the structure. The section graphs showed the variation of the stress along any particular section of the structure. The rapid increase in the stress developed at the section near the hole or the crack tip are bright examples of the potential of this method for analysis of structures.

From all the above findings it is clear that the Element Free Galerkin method can be used for analysis of 2D objects of various geometries, subject to various load conditions effectively and accurately.

11. REFERENCES

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