A WAVELET–GALERKIN METHOD FOR THE SOLUTION OF PARTIAL DIFFERENTIAL EQUATION

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BY

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CERTIFICATE

This is to certify that the thesis entitled **"Wavelet-Galerkin Method for the Solution of Partial Differential Equation"** submitted by Ms. Anandita Dandapat, Roll No.409MA2062, for the award of the degree of Master of Science from National Institute of Technology, Rourkela, is absolutely best upon her own work under the guidance of Prof. (Dr.) S.Saha Ray. The results embodied in this thesis are new and neither this thesis nor any part of it has been submitted for any degree/diploma or any academic award anywhere before.

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DECLARATION

I declare that the topic 'A Wavelet-Galerkin method for the Solution of Partial Differential Equation' for my M.Sc. degree has not been submitted in any other institution or University for the award of any other degree or diploma.

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Abstract

Wavelet function generates significant interest from both theoretical and applied research given in the last ten years. In the present project work, the Daubechies family of wavelets will be considered due to their useful properties. Since the contribution of compactly supported wavelet by Daubechies and multi resolution analysis based on Fast Fourier Transform (FWT) algorithm by Beylkin, wavelet based solution of ordinary and partial differential equations gained momentum in attractive way. Advantages of Wavelet-Galerkin Method over finite difference or element method have led to tremendous application in science and engineering.

In the present project work the Daubechies families of wavelets have been applied to solve differential equations. Solution obtained may the Daubechies-6 coefficients has been compared with exact solution. The good agreement of mathematical results , with the exact solution proves the accuracy and efficiency of Wavelet-Galerkin Method.

CONTENTS

CHAPTER-1

Introduction

Wavelet Galerkin method is useful to solve partial differential equation. Wavelet analysis is a numerical concept which allows representing a function in terms of a set of basis functions, called wavelets, which are localized both in location and scale. Wavelets used in this method are mostly compact support introduce by Daubechies [1].

The wavelet based approximations of ordinary and partial differential equations [1-4] have been attracting the attention, since the contribution of orthonormal bases of compactly supported wavelet by Daubechies [5] and Multiresolution analysis based Fast Wavelet Transform Algorithm (F.W.T) by Beylkin [6] gained momentum to make wavelet approximations attractive. Among the wavelet approximations, the Wavelet-Galerkin technique [7-10] is the most frequently used scheme nowadays .Wavelet based numerical solutions of partial differential equations have been developed by several researchers [2, 3, 7, 10-14].

Daubechies constructed a family of orthonormal bases of compactly supported wavelets for the space of square-integrable function $L^2(R)$. The Wavelet-Galerkin scheme involves the evaluation of connection coefficients are integrals with integrands being products of wavelet bases and their derivatives.

Due to the derivatives of compactly supported wavelets, it is difficult and unstable to compute the connection coefficients by the numerical evaluation of integral. The connection coefficients and the associated computations algorithms have been developed in [8,12] for bounded and unbounded domains.

Wavelet $\Psi(x)$: An oscillatory function $\Psi(x) \in L^2(R)$ with zero mean is a wavelet if it has the desirable properties:

1.Smoothness: $\Psi(x)$ is n times differentiable and that their derivatives are continuous.

2.**Localization:** $\Psi(x)$ is well localized both in time and frequency domains, i.e., $\Psi(x)$ and its derivatives must decay very rapidly. For frequency localization $\hat{\Psi}(\omega)$ must decay sufficiently fast as $|\omega| \to \infty$ and that $\hat{\Psi}(\omega)$ becomes flat in the neighborhood of $\omega = 0$. The flatness is associated with number of vanishing moments of $\Psi(x)$, i.e.,

$$\int_{-\infty}^{\infty} x^k \Psi(x) dk = 0 \text{ or equivalently } \frac{d^k}{d\omega^k} \hat{\Psi}(\omega) = 0 \text{ for } k = 0,1,\ldots\ldots, n$$

in the sense that larger the number of vanishing moments more is the flatness when ω is small.

3.The **admissibility condition**

$$\int_{-\infty}^{\infty} \frac{\left|\hat{\Psi}(\omega)\right|^2}{|\omega|} d\omega < \infty$$

suggests that $\left|\hat{\Psi}(\omega)\right|^2$ decay at least as $|\omega|^{-1}$ or $|x|^{\varepsilon-1}$ for $\varepsilon > 0$.

Chapter-2

Wavelet Based Complete coordinate Function

The Daubechies [5, 15] defined the class of compactly supported wavelets. This means that they have non zero values within a finite interval and have a zero value everywhere else. Let $\Phi(x)$ be a solution of scaling relation

$$\Phi(x) = \sum_{k-0}^{L-1} a_k \Phi(2x-k) \tag{1}$$

The expression $\Phi(x)$ is called Scaling function. And wavelet function $\psi(x)$ is

$$\psi(x) = \sum_{k=-L}^{1} (-1)^k a_{1-k} \Phi(2x-k) \tag{2}$$

where L is positive even integral.

From the normalization $\int \Phi(x) = 1$ of the scaling function, the first condition can be written as follows,

$$\sum_{k=0}^{L-1} a_k = 2 \tag{3}$$

The translation of $\Phi(x)$ are required to be orthonormal

$$\int \Phi(x-k)\ \Phi(\mathrm{x-m}) = \delta_{k,m} \tag{4}$$

This formula (4) implies the second condition

$$\sum_{k=0}^{L-1} a_k a_{k-2m} = \delta_{0m} \tag{5}$$

Where δ is the Kronecker delta function.

Smooth wavelet function requires the moment of the wavelet to be zero

$$\int x^m \Psi(x) dx = 0 \tag{6}$$

This formula (6) implies the third condition

$$\sum_{k=0}^{L-1} (-1)^k k^m a_k = 0 \text{ for } m = 0, 1, ..., L/2 - 1 \tag{7}$$

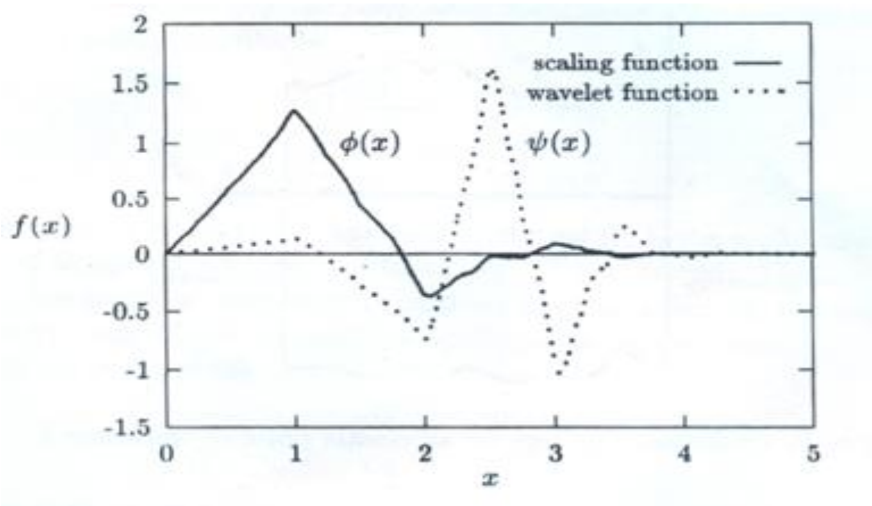

Figure1: Daubechies' scaling and wavelet function for $L = 6$.

For the coefficients satisfying with the above condition, the function, which consist of translation and dilations of the scaling function $\Phi(2^j x-k)$ or the wavelet function $\Psi(2^j x-k)$ form a complete and orthogonal basis. The relation between two functions is expressed as:

$$V_{j+1} = V_j \oplus W_j$$

$$\Rightarrow \int_{-\infty}^{\infty} \Phi(x)\Psi(x-m)dx = 0, \text{ for any integer } \quad m \qquad (8)$$

where $\oplus$ denotes the orthogonal direct sum. Also, V_j and W_j be the subspaces generated, respectively, as the L^2-closure of the linear spans of $\Phi_{jk}(x) = 2^{\frac{j}{2}}\Phi(2^j x-k)$ and $\Psi_{jk}(x) = 2^{\frac{j}{2}}\Psi(2^j x-k)$, $k \in Z$.

The condition (8) implies that

$$V_0 \subset V_1 \subset ..... \subset V_j \subset V_{j+1}$$

and $$V_{j+1} = V_0 \oplus W_0 \oplus W_1 \oplus ...... \oplus W_j \qquad (9)$$

Here, *j* is the dilation parameter as the scale. For a certain value of *j* and *L*, the support of the scaling function $\Phi(2^j x-k)$ is given as follows.

$$Supp(\Phi(2^j x-k)) = \left[\frac{k}{2^j}, \frac{L+k-1}{2^j}\right] \qquad (10)$$

As the scaling function yield a complete coordinate function basis, it can be used to expand a general function as follows

$$f(x) = \sum_k 2^j c_k \Phi(2^j x-k) \qquad (11)$$

For this expansion, we have the following convergence property,

$$\left\| f - \Sigma_k c_k \Phi(2^j x - k) \right\| \le C 2^{-jp} \left\| f^{(p)} \right\| \tag{12}$$

where

$$c_k = \int f(x)\Phi(2^j x - k)dx$$

and C and p are constants.

Here it is worth emphasizing that of a proper scale is very important. For example, to express a function having five periods in one interval, the scale j which at least has five translated components of the corresponding scaling function in the same interval must be selected. Besides this, there is another important point that scale j also affects the convergence in computational estimation. As we can see from the convergence property (12), the expanded function approaches the real value of, as $j \to \infty$.

CHAPTER 3

Multiresolution in $L^2(R)$

Multiresolution analysis is the method of most of the practically relevant discrete wavelet transform.

A Multiresolution analysis of the space $L^2(R)$ consist of a sequence of closed subspace V_j with the following properties:

1) $V_j \subset V_{j+1}, \forall j \in Z$

2) $f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1}$

3) $f(x) \in V_0 \Leftrightarrow f(x+1) \in V_0$

4) $\bigcup_{j\in Z} V_j$ is dense in $L^2(R), \bigcap_j V_{j\in Z} = \{0\}$

The existence of a scaling function $\phi(x)$ is required to generate a basis in each V_j by

$$V_j = \overline{span}\{\Phi_{ji}\}_{i\in Z}$$

With

$$\Phi_{ij} = 2^{j/2}\Phi(2^j x - i), j, i \in Z$$

In the classical case this basis orthonormal, so that

, of which the translates and dilates constitutes orthonormal bases of the spaces W_j. $\langle \phi_{ji}, \phi_{jk} \rangle_R = \delta_{ik}$,

With $\langle f, g \rangle = \int_{-\infty}^{\infty} f(x)\overline{g}(x)dx,$

being the usual inner product.

Let the W_j denote a subspace complementing the subspace V_j in V_{j+1} i.e. $V_{j+1} = V_j \oplus W_j$.

Each element of V_{j+1} can be uniquely written as the sum of an element in V_j, and an element in W_j which contains the details required to pass from an approximation at level j to an approximation at level $j+1$.

Based on the function $\phi(x)$ one can find $\psi(x)$, the so-called mother wavelet

$$W_j = \overline{span}\{\psi_{ji}\} i \in Z$$

Generated by the wavelets

$$\psi_{ji} = 2^{j/2} \psi(2^j x - i), j, i \in Z$$

Each function $f \in L^2(R)$, can now be expressed as

$$f(x) = \sum_{i \in Z} c_{j_0 i} \phi_{j_0 i}(x) + \sum_{j=j_0}^{\infty} \sum_{i \in Z} d_{ji} \psi_{ji}(x)$$

Where $c_{ji} = \langle f, \phi_{ji} \rangle_R \; d_{ji} = \langle f, \varphi_{ji} \rangle_R$

The scaling function $\phi(x)$ and its mother wavelet $\psi(x)$ have the following properties:

$$\int_{-\infty}^{\infty} \phi(x)dx = 1,$$

$$\int_{-\infty}^{\infty} \phi(x-j)\phi(x-i)dx = \delta_{i,j}$$

$$\int_{-\infty}^{\infty} x^k \psi(x)dx = 0, k = 0,1,2$$

and

$\int_{-\infty}^{\infty} \phi(x)\psi(x-k)dx = 0$, For any integer k

This condition implies that $V_{j+1} = V_j \oplus W_j$ on each fixed and scale j, the wavelets $\{\psi_{jk}(x)\}_{k\in Z}$ form an orthonormal basis W_j and the scaling functions $\{\phi_{jk}(x)\}_{k\in Z}$ form an orthonormal basis of V_j

The set of spaces of set V_j is called as Multiresolution analysis of $L^2(R)$.These spaces will be used to approximate the solutions of Partial Differential Equations using the Wavelet-Galerkin method.

CHAPTER 4

Connection coefficients

In order to solve the differential equation by using wavelet Galerkin method there we need connection coefficients,

$$\Omega_{l_1 l_2}^{d_1 d_2} = \int_{-\infty}^{\infty} \Phi_{l_1}^{d_1}(x)\Phi_{l_2}^{d_2}(x)dx$$

Taking the derivatives of the scaling function d times, we get

$$\varphi^d(x) = 2^d \sum_{k=0}^{N-1} a_k \varphi_k^d(2x-k)$$

After simplification and considering it for all $\Omega_{l_1 l_2}^{d_1 d_2}$, gives a system of linear equations with $\Omega^{d_1 d_2}$ as unknown vector:

$$T\Omega^{d_1 d_2} = \frac{1}{2^{d-1}}\Omega^{d_1 d_2}$$

where $d = d_1 + d_2$ and $T = \sum_i a_i a_{q-2l+i}$

The moments M_i^k of φ_i are defined as

$$M_i^k = \int_{-\infty}^{\infty} x^k \Phi_i(x)dx$$

with $M_0^0 = 1$

Latto et al derives a formula as

$$M_i^m = \frac{1}{2(2^m - 1)} \sum_{t=0}^{m} \binom{m}{t} i^{m-t} \sum_{l=0}^{t-1} \binom{t}{l} \sum_{i=0}^{L-1} a_i i^{t-l}$$

where a_i's are the Daubechies wavelet coefficients.

Tables regarding the value of Scaling function and Connection coefficients at j=0 and L=6 have been provided by Latto et al [8].

Table 1 scaling function $\phi(x)$

x	$\phi(x)$
0	0
0.5	0.60517847E+00
1	0.12863351E+01
1.5	0.44112248E+00
2	-0.38583696E+00
2.5	-0.14970591E-01
3	0.95267546E-01
3.5	-0.31541303E-01
4	0.42343456E-02
4.5	0.21094451E-02
5	0

Table 2 : Daubechies Wavelet filter coefficients, L=6

a_0	0.470467207784
a_1	1.14111691583
a_2	0.650365000526
a_3	-0.190934415568
a_4	-0.120832208310
a_5	0.0498174997316

Table 3: Connection coefficient at $L=6,\ \mathrm{j}=0$ $\Omega[n-k]=\int \phi''(x-k)\ \phi(x-n)dx$

$\Omega[-4]$	0.00535714285714
$\Omega[-3]$	0.11428571428571
$\Omega[-2]$	-0.87619047619052
$\Omega[-1]$	3.39047619047638
$\Omega[0]$	-5.26785714285743
$\Omega[1]$	3.39047619047638
$\Omega[2]$	-0.87619047619052
$\Omega[3]$	0.11428571428571
$\Omega[4]$	0.00535714285714

CHAPTER 5

Singularly perturbed second-order boundary value problem

Wavelet-Galerkin scheme for the singularly perturbed boundary value problem

$$\varepsilon u'' + \alpha u' + \beta u = f(x), \; 0 < x < 1, \tag{13}$$

subject to boundary condition

$$u(0) = 0, \quad u(1) = 0,$$

where (o< ε <<1), and α, β constant and $f(x)$ is a polynomial of degree any order in *x*, in [0, 1] otherwise, approximate by such a polynomial if necessary.

Let the solution $u_j(x)$ of the problem be approximated by its *j*th level wavelet series on the interval (0, 1), i.e.

$$u_j(x) = \sum_{k=2-L}^{2^j-1} c_k \phi_k(x), k \in Z \tag{14}$$

Where $\phi_{jk}(x) = 2^{j/2} \phi(2^j x - k), j > 0,$ and u_k, $\mathrm{k} = 2 - L, 3 - L, ..., 2^{\mathrm{j}} - 1$ are $2^{\mathrm{j}} + L - 2$ unknown coefficients to be determine. The integer j is used to control the smoothness of the solution. The larger integer j is used, the more accurate solution can be obtained.

The parameter L implies that the wavelet associated with the set of L Daubechies filter coefficients is used as the solution bases. Substituting the wavelet series approximation $u_j(x)$ in (14) for $u(x)$ in (13),

$$\varepsilon \sum_{k=2-L}^{2^j-1} c_k \frac{d^2}{dx^2}\phi_{jk}(x) + \alpha \sum_{k=2-L}^{2^j-1} c_k \frac{d}{dx}\phi_{jk}(x) + \beta \sum_{k=2-L}^{2^j-1} c_k \phi_{jk}(x) = f(x) \tag{15}$$

To determine the coefficient c_k, we take inner product of both sides of (15) with ϕ_{jl},

$$\varepsilon \sum_{k=2-L}^{2^j-1} c_k \int_0^1 \phi''_{jk}(x)\phi_{jll}(x)dx + \alpha \sum_{k=2-L}^{2^j-1} c_k \int_0^1 \phi'_{jk}(x)\phi_{jl}(x) + \beta \sum_{k=2-L}^{2^j-1} c_k \int_0^1 \phi_{jk}(x)\phi_{jl}(x)dx$$
$$= \int_0^1 f(x)\phi_{jl}(x)dx, \quad l = 2-L, 3-L, ..., 2^j - 1. \tag{16}$$

We assume that $f(x) = \sum_{l=0}^{m} a_i x^i$ is a polynomial of degree m in x. We write the equation in (16) as

$$\sum_{k=2-L}^{2^j-1} \varepsilon c_{kl}^j c_k + \alpha \sum_{k=2-L}^{2^j-1} b_{kl}^J c_K + \beta \sum_{k=2-L}^{2^{jJ}-1} a_{kl}^j c_K = d_{ml}^j, \quad l = 2-L, 3-L, ..., 2^j - 1 \tag{17}$$

Where

$$c_{kl}^j = \int_0^1 \phi''_{jk}(x)\,\phi_{jl}(x)dx, \quad b_{kl}^j = \int_0^1 \phi'_{jk}(x)\,\phi_{jl}(x)dx,$$

$$a_{kl}^j = \int_0^1 \phi_{jk}(x)\ \phi_{jl}(x)dx, \quad d_{ml}^j = \int_0^1 f(x)\ \phi_{jl}(x)dx$$

To find d_{ml}^{j} , we put the value of $f(x)$ yielding

$$d_{ml}{}^{j} = \sum_{i=0}^{m} a_i \int_0^1 x^i \phi_{jl}(x)dx$$

We know $\phi_{jl}(x) = 2^{j/2}\phi(2^j x - l)$

Put this in above equation then

$$d_{ml}{}^{j} = \sum_{i=0}^{m} a_i \int_0^1 x^i\, 2^{j/2}\phi(2^j x - l)dx$$

$$= \sum_{\mathrm{i}=0}^{m} a_i\, 2^{j/2} \int_0^1 x^i\, 2^{j/2}\phi(2^j x - l)dx$$

$$= \sum_{i=0}^{m} a_i \frac{2^{j/2}}{2^j 2^{ij}} \int_0^{2^J} y^i \phi(y - l)dy$$

Let $\quad M_k^n(x) = \int_o^x y^m \phi(y-k)dy$

So $\int_0^{2^j} y^i \phi(y-l)dy = M_l^i(2^j)$

Hence $d_{ml}^{j} = \sum_{i=0}^{m} a_i \dfrac{2^{j/2}}{2^j 2^{ij}} M_l^i(2^j)$

$$= \sum_{i=0}^{m} \frac{a_i}{2^{(i+\frac{1}{2})j}} M_l^i(2^j)$$

Equation (17) can be further put into the matrix-vector form as

$$A_1 U = D$$

where

$$A_1 = C + \alpha B + \beta A \ , \tag{18}$$

$C = \varepsilon\ [c_{kl}^{j}]_{\ 2-L \le k,l \le 2^j - 1}\ ,\quad B = \varepsilon\ [b_{kl}^{j}]_{\ 2-L \le k,l \le 2^j - 1,}$

$\mathrm{A} = \varepsilon\ [a_{kl}^{j}]_{\ 2-L \le \cdots k,l \le 2^j - 1,}\ ,\quad \mathrm{D} = [d_{kl}^{j}]_{2-L \le l \le 2^j \text{-}1,}$

and

$$U = [c_{2-L}, c_{3-L}, c_{2^j - 1}]^T$$

Now, we have a linear system of $2^j + L - 2$ equations of the $2^j + L - 2$ unknown coefficients. We can obtain the coefficient of the approximate solution by solving this linear system.

The solution U gives the coefficients in the Wavelet-Galerkin approximation $u_j(x)$ of $u(x)$

CHAPTER-6

Wavelet-Galerkin Solution of Shear Wave Equation:

Consider a plate of finite extent in the *z* & *y* direction & of thickness 1 in *x* direction. For horizontal polarized shear wave, the governing partial differential equation is

$$u_{xx} + u_{yy} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \tag{19}$$

where $u = u(x, y, t)$

We consider the solution of the wave equation as

.

$$u(x, y, t) = e^{i(\xi y - \omega t)} \tag{20}$$

substitute (20) in (19) we get

$$\frac{d^2 u(x)}{dx^2} + \beta^2 u(x) = 0 \tag{21}$$

where $\beta^2 = \frac{\omega^2}{c^2} - \gamma^2$

So exact solution is

$$u(x, y, t) = (A_1 \sin \beta x + A_2 \cos \beta x) e^{i(\xi y - \omega t)} \tag{22}$$

Wavelet Galerkin method solution

Here, we shall consider $L=6\,\&\,J=0$

Consider the solution of ordinary differential equation (21) is

$$u(x)=\sum_{k=L-1}^{2^j} c_k\, 2^{\frac{j}{2}}\Phi(2^j x-k), \qquad x\in[0,1]$$

$$=\sum_{k=-5}^{1} c_k\,\Phi(x-k)\ , \ x\in[0,1] \tag{23}$$

Where c_k are constants, the unknown co-efficient

Substitute (23) in (21) we get

$$\frac{d^2}{dx^2}\sum_{k=-5}^{1} c_k\Phi(x-k)+\beta^2\sum_{k=-5}^{1} c_k\Phi(x-k)=0$$

$$\sum_{k=-5}^{1} c_k\phi''(x-k)+\beta^2\sum_{k=-5}^{1} c_k\phi(x-k)=0$$

Without any loss of generality, let $\beta^2=1$ and taking inner product with $\phi(x-n)$,

we have

$$\sum_{k=-5}^{1} c_k\int_{\frac{1-L}{2^j}}^{\frac{L-1+2^j}{2^j}}\phi''(x-k)\phi(x-n)+\beta^2\sum_{k=-5}^{1} c_k\int_{\frac{1-L}{2^j}}^{\frac{L-1+2^j}{2^j}}\phi(x-k)\phi(x-n)=0$$

$$\Rightarrow \sum_{k=-5}^{1} c_k \Omega[n-k] + \sum_{k=-5}^{1} c_k \delta_{n,k} = 0 \quad (24)$$

$n = 1-L, 2-L, \cdots, 2^j$

i.e; $n = -5, -4, \cdots, 0, 1$

where $\Omega[n-k] = \int \phi''(x-k)\phi(x-n)dx$

$$\delta_{k,n} = \int \phi(x-k)\phi(x-n)dx$$

By using Dirichlet boundary conditions

$u(0) = 1, \quad u(1) = 0$

yielding this equation

$$u(0) = \sum_{k=-5}^{1} c_k \phi(-k) = 1 \quad (25)$$

and

$$u(1) = \sum_{k=-5}^{1} c_k \phi(1-k) = 0 \quad (26)$$

From left boundary conditions, we get equation (25) and from right boundary conditions, we get equation (26), which represents the relation of the coefficients c_k.

Now we eliminate first and last equations of (24) and in that places are including equation (25) and (26) respectively, we get the following matrix with $L = 6$.

$$TC = B$$

$$T = \begin{bmatrix} 0 & \phi(4) & \phi(3) & \phi(2) & \phi(1) & 0 & 0 \\ \Omega[1] & \Omega[0]+1 & \Omega[-1] & \Omega[-2] & \Omega[-3] & \Omega[-4] & \Omega[-5] \\ \Omega[2] & \Omega[1] & \Omega[0]+1 & \Omega[-1] & \Omega[-2] & \Omega[-3] & \Omega[-4] \\ \Omega[3] & \Omega[2] & \Omega[1] & \Omega[0]+1 & \Omega[-1] & \Omega[-2] & \Omega[-3] \\ \Omega[4] & \Omega[3] & \Omega[2] & \Omega[1] & \Omega[0]+1 & \Omega[-1] & \Omega[-2] \\ \Omega[5] & \Omega[4] & \Omega[3] & \Omega[2] & \Omega[1] & \Omega[0]+1 & \Omega[-1] \\ 0 & 0 & \phi(4) & \phi(3) & \phi(2) & \phi(1) & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} c_{-5} \\ c_{-4} \\ c_{-3} \\ c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By Gaussian elimination algorithm we get

$c_{-5} = -0.997181$

$c_{-3} = -0.877618$

$c_{-2} = 0.127868$

$c_{-1} = 1.08705$

$c_0 = 0.24756$

$c_1 = -0.505899$

The Exact solution by using Dirichlet boundary condition is

$$u(x) = \cos x - \cot 1 \sin x$$

Table-4 shows the comparison between Wavelet-Galerkin solution and Exact solution

Table 4 Comparison between wavelet Solution and Exact Solution

x	Wavelet solution	Exact solution	Absolute Error
0	1	1	0
0.125	0.921657	0.912145	0.00951209
0.25	0.829106	0.810056	.0190502
0.375	0.726413	0.69535	0.031086
0.5	0.609339	0.569747	0.0395922
0.625	0.477075	0.435276	0.041798
0.75	0.331501	0.294014	0.0374878
0.875	0.172689	0.148163	0.0245264
1	0	0	0

The value of above table & using MATLAB we obtain the following graph

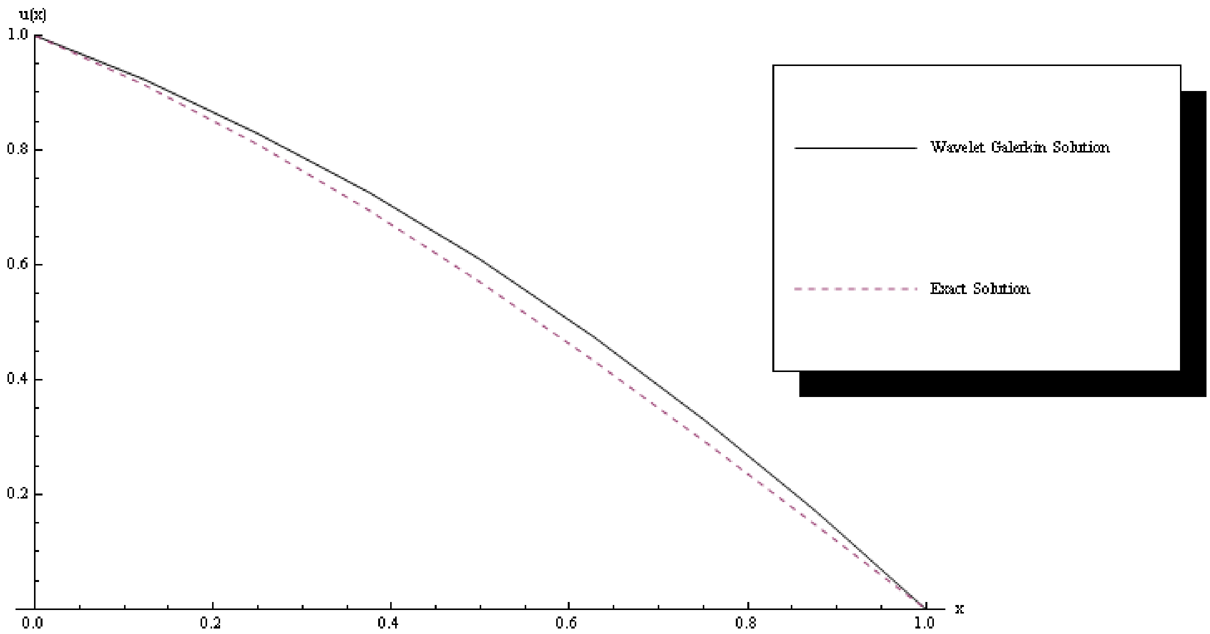

Figure 2: Comparison between wavelet Solution and Exact Solution

.To exhibit a comparison between Wavelet-Galerkin solution and Exact solution, figure2 has been diagrammed by MATLAB. A good agreement of result has been obtained as doted by figure 2.

CHAPTER-7

CONCLUSION

Wavelet-Galerkin method is the most frequenly used scheme now a days. In the present project work, the Daubechies family of the wavelet have been consider. Due to the fact that they posses several useful properties, such as orthogonality, compact support, exact representation of polynomials to a certain degree and ability to represent function at different levels resolution. Dabauchies' wavelets have gained great interest in the numerical solution of ordinary and partial differential equation.

An obtain advantages of this method of this method is that it uses Daubechies' coefficients and calculate the Scaling function, the connection coefficients as well as the rest of component only once.

This leads to a considerable saving of the computational time and improves numerical results through the reduction of round-off errors.

The Wavelet-Galerkin method has been shown to be a powerful numerical tool for fast and more accurate solution of differential equations, it can be observed from the result. Wavelet Galerkin method yields better result, which shows the effiency of the method.

Solution obtained using the Daubechies 6 coefficients wavelet has been compared with the exact solution. The good agreement of its numerical result with the exact solution proves its accuracy and efficiency .

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