

ECONOMIC DESIGN OF X-BAR CONTROL CHART USING SIMULATED ANNEALING

A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF TECHNOLOGY

in

MECHANICAL ENGINEERING

by

Mr. DARUN PRASATH.L

(ROLL NO. 107ME003)



Department of Mechanical Engineering

National Institute of Technology

Rourkela-769008

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C E R T I F I C A T E

This is to certify that the work in this thesis entitled “**ECONOMIC DESIGN OF X-BAR CONTROL CHART USING SIMULATED ANEALING**” by **DARUN PRASATH.L** has been carried out under my supervision in partial fulfillment of the requirements for the degree of **Bachelor of Technology in Mechanical Engineering** during session 2010- 2011 in the Department of Mechanical Engineering, National Institute of Technology, Rourkela. To the best of my knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.

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A C K N O W L E D G E M E N T

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ABSTRACT

Control charts are widely used in industry for monitoring and controlling manufacturing processes. They should be designed economically in order to achieve minimum quality control costs. The major function of control chart is to detect the occurrence of assignable causes so that the necessary corrective action can be taken before a large quantity of nonconforming product is manufactured. The X-bar control chart dominates the use of any other control chart technique if quality is measured on a continuous scale. In the present project, the economic design of the X-bar control chart using Simulated Annealing has been developed to determine the values of the sample size, sampling interval, width of control limits such that the expected total cost per hour is minimized. Simulated annealing is a solution method in the field of combinatorial optimization based on an analogy with the physical process of annealing. Solving a combinatorial optimization problem amounts to finding the best or optimal solution among a finite or countable infinite number of alternative solutions. A program has been developed using Matlab software to optimize the cost. The result was compared with the literature and found to be superior to the initial cost obtained.

INTRODUCTION

Control chart:

Control chart is a tool used to monitor processes and to assure that they remain in Control or stable.

Elements of a Control chart:

A control chart consists of:

1. A central line,
2. An upper control limit,
3. A lower control limit and
4. Process values plotted on the chart.

Designing a control chart:

All the process values are plotted on the chart. If the process values fall within the upper and control limits and the process is referred to as In Control. If the process values plotted fall outside the control limits, the process is referred to as Out of Control.

Economic Design of Control Charts:

Traditionally, control charts have been designed with respect to statistical criteria only. This usually involves selecting the sample size and control limits. The frequency of sampling is

rarely treated analytically. But the practitioners are advised to consider another factor such as a sampling frequency. Thus the selection of three parameters: (1) sample size (2) a sampling frequency or interval between samples and (3) the control limits, is usually called the design of the control charts.

The design of control chart has economic consequences in which the costs of sampling and testing are associated with investigating out of control signals. Correcting the assignable causes and costs of allowing non-conforming units to reach the consumer are all affected by the choice of the control chart from an economic viewpoint.

Process Characteristics:

To formulate an economic model for the design of a control chart, it is necessary to make certain assumptions about the behavior of the process.

- When no assignable causes are present, the process is characterized by a single in-control state corresponding to the mean or fraction non-conforming corresponding to measurable or an attribute quality characteristic respectively.
- The process may have, in general, one or more out-of-control states. Each out-of-control state is usually associated with a particular type of assignable cause.
- The nature of transitions between the in-control and out-of-control states, having assignable causes during an interval of time, follow a Poisson process.
- The process transitions between states and nature of the failure mechanism imply states are instantaneous, among that the process needs interruption for correction.

Cost Parameters:

Three categories of costs are customarily considered in the economic design of control charts are;

1. The cost of sampling and testing.
2. The costs associated with investigating an out-of-control signal and with the repair or correction of any assignable causes found.
3. Costs associated with the production of non-conforming items.

Usually, the cost of sampling and testing is assumed to consist of both fixed and variable components say a and b , respectively, such that the total cost of sampling and testing is

$$a+bn \tag{1}$$

The cost of investigating and possibly correcting the process following an out-of-control signal has been treated in several ways. Some authors have suggested that the costs of investigating false alarms will vary the costs of correcting assignable causes. Consequently, these two situations must be represented in the model by different cost coefficients. Furthermore, the cost of repairing or correcting the process could depend on the type of assignable cause present in the process. If, in the model there are 's' out-of-control states then there will be 's+1' cost coefficients required to model the search and adjustment procedures associated with out-of-control signals.

Usually, it is observed that small shifts are difficult to find but easy to correct, whereas large shifts are easy to find but difficult to correct.

Economic models are generally formulated using a total cost function, which expresses the relationship between the control chart design parameters and the three types of costs discussed above. The production, monitoring, and adjustment process may be thought of as a series of independent cycles over a specific period of time. Each cycle begins with production process in the in-control state and continues until the process monitoring via the control chart results in an out-of-control signal. The process is set to return in the in-control state when certain adjustments are performed to correct the process, thus resulting a cycle to begin. Using, the process failure, search, and repair pattern, we can define few expected quantities as follows:

Let

$E(T)$: expected length of a cycle.

$E(C)$: expected total cost incurred during a cycle.

$E(A)$: expected cost per unit time.

Thus

$$E(A) = E(C)/E(T) \tag{2}$$

Optimization techniques are then applied to Eq (2) to determine the optimal control charts design economically. The Eq (2) can also be replaced by the expected number of units produced during the cycle, resulting in the expected cost on per item rather than per unit time basis. The sequence of production-monitoring-adjustment with accumulation of costs over the cycle can be represented by a particular type of stochastic process called a renewal reward process. Stochastic processes of this type have the property that their average time cost is given

by the ratio of the expected reward per cycle to the expected cycle length. Much work is done on the process-failure, inspection, and repairing mechanism.

TYPES OF CONTROL CHARTS

Variables Charts

The classical type of control chart is constructed by collecting data periodically and plotting it versus time. If more than one data value is collected at the same time, statistics such as the mean, range, median, or standard deviation are plotted. Control limits are added to the plot to signal unusually large deviations from the centreline, and run rules are employed to detect other unusual patterns.

X-Bar & Range Charts

X-bar & Range Charts are a set of control charts for variables data (data that is both quantitative and continuous in measurement, such as a measured dimension or time). The X-bar chart monitors the process location over time, based on the average of a series of observations, called a subgroup. The Range chart monitors the variation between observations in the subgroup over time.

Attributes Charts

For attribute data, such as arise from PASS/FAIL testing, the charts used most often plot either rates or proportions. When the sample sizes vary, the control limits depend on the size of the samples.

Attribute Charts are a set of control charts specifically designed for Attributes data. Attribute charts monitor the process location and variation over time in a single chart.

The family of Attribute Charts include the:

- **np-Chart:** for monitoring the number of times a condition occurs, relative to a constant sample size, when each sample can either have this condition, or not have this condition
- **p-Chart:** for monitoring the percent of samples having the condition, relative to either a fixed or varying sample size, when each sample can either have this condition, or not have this condition
- **c-Chart:** for monitoring the number of times a condition occurs, relative to a constant sample size, when each sample can have more than one instance of the condition.
- **u-Chart:** for monitoring the percent of samples having the condition, relative to either a fixed or varying sample size, when each sample can have more than one instance of the condition.

Time-Weighted Charts

When data is collected one sample at a time and plotted on an individual's chart, the control limits are usually quite wide, causing the chart to have poor power in detecting out-of-control situations. This can be remedied by plotting a weighted average or cumulative sum of the data, not just the most recent observation. The average run length of such charts is usually much less than that of a simple X chart.

CUSUM Charts

A CUSUM Chart is a control chart for variables data which plots the cumulative sum of the deviations from a target. A V-mask is used as control limits. Because each plotted point on the CUSUM Chart uses information from all prior samples, it detects much smaller process shifts than a normal control chart would. CUSUM Charts are especially effective with a subgroup size of one. Run tests should not be used since each plotted point is dependent on prior points as they contain common data values.

Moving Average (MA) Chart

To return to the piston ring example, suppose we are mostly interested in detecting small trends across successive sample means. For example, we may be particularly concerned about machine wear, leading to a slow but constant deterioration of quality (i.e., deviation from specification). The CUSUM chart described above is one way to monitor such trends, and to detect small permanent shifts in the process average. Another way is to use some weighting

scheme that summarizes the means of several successive samples moving such a weighted mean across the samples will produce a moving average chart.

Exponentially-weighted Moving Average (EWMA) Chart

The idea of moving averages of successive (adjacent) samples can be generalized. In principle, in order to detect a trend we need to weight successive samples to form a moving average; however, instead of a simple arithmetic moving average, we could compute a geometric moving average.

Introduction of Economic Models of X-Bar Control Charts

The most popular and widely used control chart is X-Bar chart when dealing with a variable quality characteristic. It is usually a standard practice to control σ the mean of quality characteristic. The construction and application of \bar{x} chart is easily illustrated.

Most of the standard types of control charts, such as the X-Bar chart, p-bar chart and the cumulative-sum chart have been extensively investigated. The economic model can be appropriately chosen as single assignable cause models. If assignable causes are present which result in out-of-control signal, then the immediate action is to detect and eliminate the cause.

Single Assignable-Cause Models

Duncan (1956) first presented economic model for the optimum economic design of the X-Bar control chart. The model determines the control chart parameters while incorporating formal optimization methodology. Duncan (1956) also utilized a design criterion that maximized the expected net income per unit of time from the process. He assumed, that the process is characterized by an in-control state u_0 and that a single assignable cause of magnitude δ which make product's performance characteristic out-of-control resulting in a shift in the mean u_0 to either $u_0 - \delta\sigma$ or to $u_0 + \delta\sigma$. The process is monitored by \bar{x} chart with

Centerline= u_0 and

Upper and Lower Control Limits = $(u_0 \pm L (\sigma/\sqrt{n}))$.

To search for the assignable cause occurring at random, samples at a fixed interval (h per unit time) are taken. The process is allowed to continue to operate during the search. The parameters u_0, δ and σ are assumed known, while $n, L,$ and h are to be determined. The values n, L and h are called design parameters.

Literature Review

Statistical control charts are widely used in manufacturing industry. However, for processes exhibiting certain trend pattern, which stems mainly from deteriorating elements such as power consumption or tool-wear, traditional control charts have to be interpreted differently. Cai et al (2002) discuss about the appropriate timing problem of making adjustment to such trended processes through economical design of its control chart. This extends the traditional study on economic design of control chart, which mainly focuses on the setting of control limits.

Economic design of \bar{x} control charts is to control normal process means and ensure that an economic design control chart actually lowers the cost, compared with a Shewhart control chart. Many authors have studied the control charts from economic viewpoint from then on. An economic design does not consider the statistical properties, such as type I or type II error and average time to signal (ATS). To improve these issues, an economic statistical design of control charts has been developed by Yu et al (2010) under the consideration of one assignable cause. However, there are multiple assignable causes in the real practice such as machine problem, material deviation, human errors, etc. In order to have a real application, this research will extend the original research from single to multiple assignable causes to establish an economic-statistical model of \bar{x} control chart.

Most of the studies in economic design of control charts focus on a fixed-sampling interval (FSI), however, it has been discovered that variable-sampling-interval (VSI) control charts are substantially quicker in detecting shifts in the process than FSI control charts due to a higher frequency in the sampling rate when a sample statistic shows some indication of a process change. In this paper, an economic design for a VSI X-Bar control chart is proposed by Yu and Chen (2005) for a continuous-flow production process.

Control charts are widely implemented in firms to establish and maintain statistical control of a process which leads to the improved quality and productivity. Design of control charts requires that the engineer selects a sample size, a sampling frequency and the control limits for the chart. In this paper, a possible combination of design parameters is considered by Asadzadeh and Khoshalhan (2008) as a decision making unit which is identified by three attributes: hourly expected cost, detection power of the chart and in-control average run length. Optimal design of control charts can be formulated as multiple objective decision making (MODM).

Simulated Annealing (SA) is a popular global minimization method. Two weaknesses are associated with standard SA: firstly, the search process is memory-less and therefore cannot avoid revisiting regions that are less likely to contain global minimum; and secondly the randomness in generating a new trial does not utilize the information gained during the search and therefore, the search cannot be guided to more promising regions. In this paper, the Learning-Enhanced Simulated Annealing (LESA) method to overcome these two difficulties has

been presented by Sun et al (2008). It adds a Knowledge Base (KB) trial generator, which is combined with the usual SA trial generator to form the new trial for a given temperature. LESA does not require any domain knowledge and, instead, initializes its knowledge base during a burn-in phase using random samples of the search space, and, following that, update the knowledge base at each iteration.

In many cases, the combination of goals and resources exponentially increases the search space, and thus the generation of consistently good scheduling is particularly difficult, because we have a very large combinatorial search space and precedence constraints between operations. Exact methods such as the branch and bound method and dynamic programming take considerable computing time to obtain the optimum solution. In order to overcome this difficulty, it is more sensible to obtain a good solution near the optimal one. Stochastic search techniques such as evolutionary algorithms can be used to find a good solution. In this paper a new method for solving job-shop scheduling problem using hybrid Genetic Algorithm (GA) with Simulated Annealing (SA) has been presented by Tamilarasi and Kumar (2010). This method introduces a reasonable combination of local search and global search for solving JSSP.

Simulated annealing is a probabilistic method proposed for finding the global minimum of a cost function that may possess several local minima. It works by emulating the physical process whereby a solid is slowly cooled so that when eventually its structure is frozen, this happens at a minimum energy configuration. Extensions of simulated annealing to the case of functions defined on continuous sets have also been introduced in the literature. In this review,

Bertsimas and Tsitsiklis (1993) goal is to describe the method, its convergence and its behavior in applications

Objectives

The main objective of this thesis is to optimize the cost function in designing an X-Bar control chart. Simulated annealing process gets terminated when the minimum temperature is obtained. Based on the simulated annealing process, a program has to be generated for determining the cost function.

A Unified Approach of the Economic Design of Control Charts

To differentiate between the assignable causes and the inevitable random causes in a process, Shewhart invented control chart. The main philosophy attributed to these charts is to study the process-failure mechanism. If random causes are at work, leave the process alone and if assignable causes are present, detect and eliminate them.

Three basic types of control charts are the \bar{x} chart, used to control a continuous process; the p-chart, used to control a Bernoulli process; the u chart, used to control the number of defects per unit. To use any of these charts, three design parameters must be specified by the sample size n , the sampling interval h and the control limits L .

Simulated Annealing

The simulated annealing method resembles the cooling process of molten metals through annealing. At high temperature, the atoms in the molten metal can move freely with respect to each other, but as the temperature is reduced, the movement of the atoms gets restricted. The atoms start to get ordered and finally form crystals having minimum possible energy. However, the formation of the crystal mostly depends on the cooling rate. If the temperature is reduced at a very fast rate, the crystalline state may not be achieved at all, instead the system may end up in a polycrystalline state, which may have higher energy state than the crystalline state. Therefore, in order to achieve the absolute minimum energy state, the temperature needs to be reduced at a slow rate. The process of slow cooling is known as annealing in metallurgical parlance.

Simulated annealing improves this strategy through the introduction of two tricks. The first is the so-called "Metropolis algorithm", in which some trades that do not lower the mileage are accepted when they serve to allow the solver to explore more of the possible space of solutions. Such bad trades are allowed using the criterion that

$$e^{\Delta D/T} > R(0,1)$$

where, ΔD the change of distance is implied by the trade (negative for a "good" trade; positive for a "bad" trade), T is a synthetic temperature, and $R(0,1)$ is a random number in the interval $(0,1)$. D is called a cost function, and corresponds to the free energy in the case of annealing a metal (in which case the temperature parameter would actually be the kT , where k is Boltzmann's Constant and T is the physical temperature, in the Kelvin absolute temperature scale). If T is large, many bad trades are accepted, and a large part of solution space is accessed.

Objects to be traded are generally chosen randomly, though more sophisticated techniques can be used.

The second trick is, again by analogy with annealing of a metal, to lower the temperature. After making many trades and observing that the cost function declines only slowly, one lowers the temperature, and thus limits the size of allowed bad trades. After lowering the temperature several times to a low value, one may then quench the process by accepting only good trades in order to find the local minimum of the cost function. There are various annealing schedules for lowering the temperature, but the results are generally not very sensitive to the details.

There is another faster strategy called threshold acceptance. In this strategy, all good trades are accepted, as are any bad trades that raise the cost function by less than a fixed threshold. The threshold is then periodically lowered, just as the temperature is lowered in annealing. This eliminates exponentiation and random number generation in the Boltzmann criterion. As a result, this approach can be faster in computer simulations.

The simulated annealing procedure simulates this process of slow cooling of molten metal to achieve the minimum function value in a minimization problem. The cooling rate phenomenon is simulated by controlling a temperature like parameter introduced with the concept of the Boltzmann probability distribution. According to the Boltzmann probability distribution, a system in thermal equilibrium at a temperature T has its energy distributed probabilistically according to $P(E) = \exp(-E/kT)$, where k is the Boltzmann constant. This expression suggests that a system at a high temperature has almost uniform probability of being at any energy state, but at a low temperature it has a small probability of being at a high energy

state. Therefore, by controlling the temperature T and assuming that the search process follows the Boltzmann probability distribution, the convergence of an algorithm can be controlled. Metropolis suggested one way to implement the Boltzmann probability distribution in simulated thermodynamic systems. The same can also be used in the function minimization context. Let us say, at any instant the current point is x power (t) and the function value at that point is $E(t) = f_n(x \text{ power } (t))$. Using the metropolis algorithm, we can say that the probability of the next point being at x power $(t+1)$ depends on the difference in the function values at these two points or on $dE = E(t+1) - E(t)$ and is calculated using the Boltzmann probability distribution:

$$P(E(t+1)) = \min [1, \exp(-dE/kT)]$$

If $dE < 0$, the probability is one and the point x power $(t+1)$ is always accepted. In the function minimization context, this makes sense because if the function value at x power $(t+1)$ is better than that at x power (t) , the point x power $(t+1)$ must be accepted. The interesting situation happens when $dE > 0$, which implies that the function value at x , power $(t+1)$ is worse than that at x power (t) . According to many traditional algorithm, the point x power $(t+1)$ must not be chosen in this situation. But according to the metropolis algorithm, there is some finite probability of selecting the point x power $(t+1)$ even though it is the same in all situations. This probability depends on relative magnitude of dE and T values. If the parameter T is large, this probability is more or less high for points with largely disparate function values. Thus, any point is almost acceptable for a large value of T . On the other hand, if the parameter T is small, the probability of accepting an arbitrary point is small. Thus, for small values of T , the points with only small deviation in function value are accepted. Later, we shall illustrate this feature of simulated annealing algorithm better through an exercise problem.

Simulated annealing is a point-by-point method. The algorithm begins with an initial point and a high temperature T . A second point is created at random in the vicinity of the initial point and the difference in the function values (dE) at these two points is calculated. If the second point has a smaller function value, the point is accepted; otherwise the point is accepted with a probability $\exp(-dE/kT)$. This completes one iteration of the simulated annealing procedure. In the next generation, another point is created at random in the neighborhood of the current point and the Metropolis algorithm used to accept or reject the point. In order to simulate the thermal equilibrium at every temperature, a number of points (n) is usually tested at a particular temperature, before reducing the temperature. The algorithm is terminated when a sufficiently small temperature is obtained or a small enough change in the function values is found

Algorithm

Step 1 Choose an initial point $x^{(0)}$, a termination criterion ϵ .

Set T a sufficiently high value, number of iteration to be performed at a particular temperature n , and set $t = 0$.

Step 2 Calculate a neighboring point $x^{(t+1)} = N(x^{(t)})$.

Usually, a random point in the neighborhood is created.

Step 3 If $dE = E(x^{(t+1)}) - E(x^{(t)}) < 0$, set $t = t+1$;

Else create a random number (r) in the range $(0, 1)$. If $r \leq \exp(-dE/kT)$ set $t = t+1$;

Else go to step 2.

Step 4 If $|x^{(t+1)} - x^{(t)}| < \epsilon$ and T is small, Terminate;

Else if $(t \bmod n) = 0$ then lower T according to a cooling schedule. Go to step 2;

Else go to step 2.

The initial temperature (T) and the number of iterations (n) performed at a particular temperature, are two important parameters which govern the successful working of the simulated annealing procedure. If a large initial T is chosen, it takes a number of iterations for convergence. On the other hand, if a small initial T is chosen, the search is not adequate to thoroughly investigate the search space before converging to the true optimum. A large value of n is recommended in order to achieve quasi-equilibrium state at each temperature, but the computation time is more. Unfortunately, there are no unique values of the initial temperature and n that work for every problem. However, an estimate of the initial temperature can be obtained by calculating the average of the function values at a number of random points in the search space. A suitable value of n can be chosen (usually between 20 to 100) depending on the available computing resource and the solution time. Nevertheless, the choice of the initial temperature and subsequent cooling schedule still remain an art and usually require some trial-and-error efforts.

An Economic Design of the Control Chart

The design of an \bar{x} -control chart requires the determination of various design parameters. These include the size of the sample (n) drawn in each interval, the sampling interval (h) and the upper and lower control limits coefficient(k).

So we can conclude that cost is a function of n, h, k . i.e. $f(n, h, k)$.

Here cost is directly proportional to sample size, no of false alarm and inversely proportional to sampling interval, width of control limits. Inspection is necessary to determine the control state of the process in order that the penalties associated with the probabilities of Type I and Type II errors can be minimized. The following costs are important in determining the decision variables in the economic design of \bar{x} -control charts: sampling cost, search cost and the cost of operating both in control and out of control. It is assumed that output quality is measurable on a continuous scale and is normally distributed. When the process is in control, the initial mean is μ_0 ; however, due to the occurrence of an assignable cause, the initial mean may be shifted from μ_0 to $\mu_0 + \delta\sigma$ or $\mu_0 - \delta\sigma$ (out-of-control state), where δ is the shift parameter and σ is the standard deviation. The control limit of the \bar{x} - control charts shown in fig.1 is set at $\mu_0 \pm k$ times the standard deviation of the sample means, where k is known as the control limit coefficient, such that

$$UCL = \mu_0 + k\sigma/\sqrt{n}$$

$$LCL = \mu_0 - k\sigma/\sqrt{n}$$

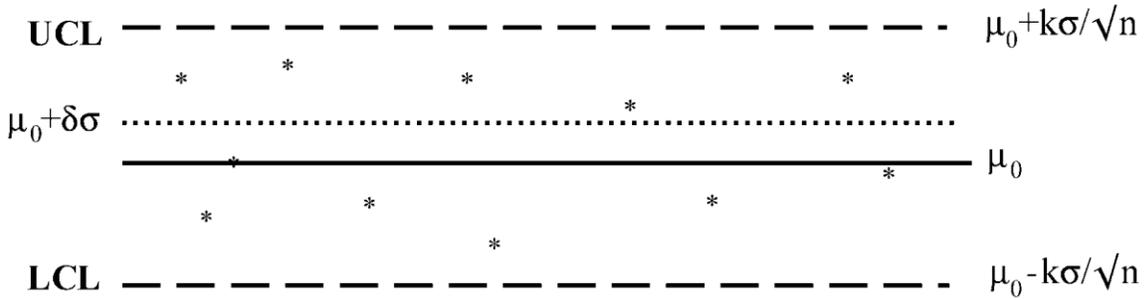


Fig.1 Control limit of the \bar{x} - control charts

Here a fixed length of sampling and a constant failure rate over each interval were assumed. A fixed sample of size n is taken from output every h hour. Whenever the sample mean falls outside the specification limits of the product, the result signals that the process has shifted to an out-of-control state as shown in fig.2

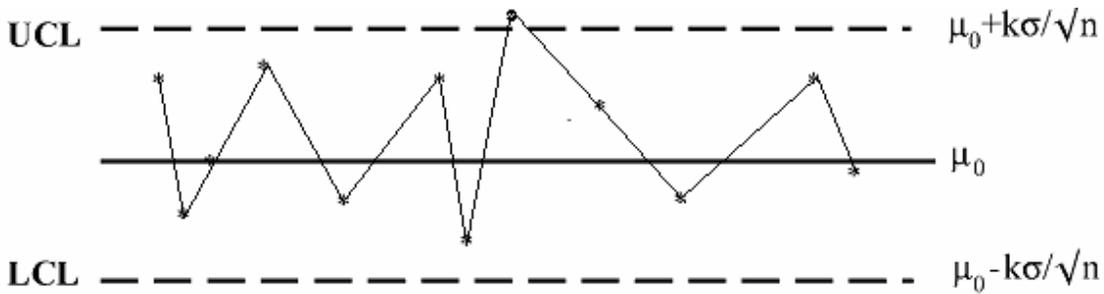


Fig.2 The assignable cause

Hence, appropriate actions such as identifying the assignable cause and restorative work may be undertaken to bring the process back to an in- control state. Otherwise, the out-of-control state will continue until the end of the production run.

The assignable cause is assumed to according to a Poisson's process with an intensity of λ occurrences per hour. That is the process begins in the in- control state, the time interval that the process remains in control is an exponential random variable with mean $1/\lambda h$.

Therefore the occurrence of the assignable cause between two consecutive interval (let j th & $(j+1)$ th is:

$$\tau = \frac{\int_{jh}^{(j+1)h} \exp(-\lambda t) (t-jh) dt}{\int_{jh}^{(j+1)h} \exp(-\lambda t) \lambda dt} = \frac{1-(1+\lambda h) \exp(-\lambda h)}{\lambda (1-\exp(-\lambda h))}$$

There are two situations which may result in wrong decisions. The first situation is caused by a Type I error, meaning that the process is in control, but an out-of-control signal is reported. Here the symbol α is used to represent the probability of a Type I error. The diagram is shown in Fig.3:

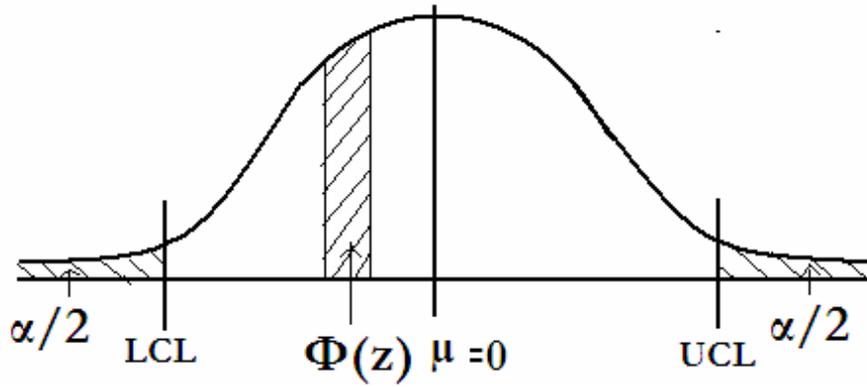


Fig.3 Type I error

The probability of false alarm is:

$$\alpha = 2 \int_k^{\infty} \Phi(z) dz$$

The second situation occurs when the process is shifted to an out-of-control state and the control chart fails to report the out-of control condition, this is defined as a Type II error. Here

the symbol β is used to represent the probability of a Type II error. So β is the probability of not detecting the shift. Whereas $(1 - \beta)$ is the probability of detecting the error, so it is called power of the test. The diagram is shown in Fig.4:

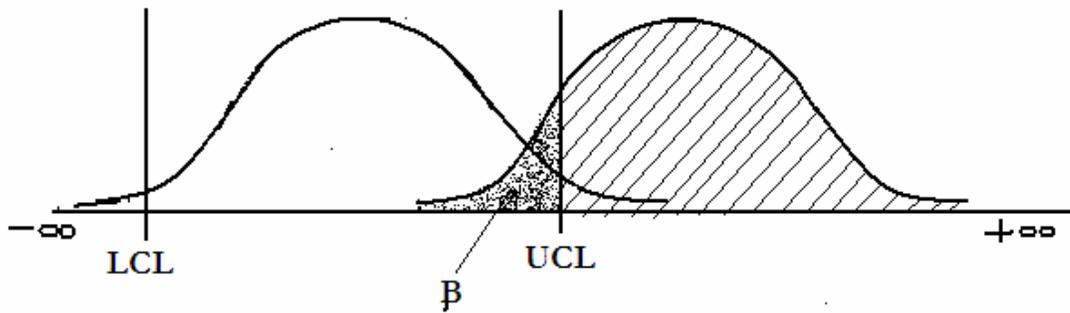


Fig.4 Type II error

The probability that it will be detected on any subsequent sample is:

$$1 - \beta = \int_{(-\infty)}^{(-k - \delta\sqrt{n})} \Phi(z) dz + \int_{(k - \delta\sqrt{n})}^{(\infty)} \Phi(z) dz$$

A production cycle is defined as the interval of time from the start of production where the process is assumed to start in in-control state following an adjustment to the detection and elimination of the assignable cause. The cycle shown in Fig.5 consists of four periods:

- (i) in-control period
- (ii) out-of-control period
- (iii) time to take a sample and interpret the result
- (iv) time to find the assignable cause

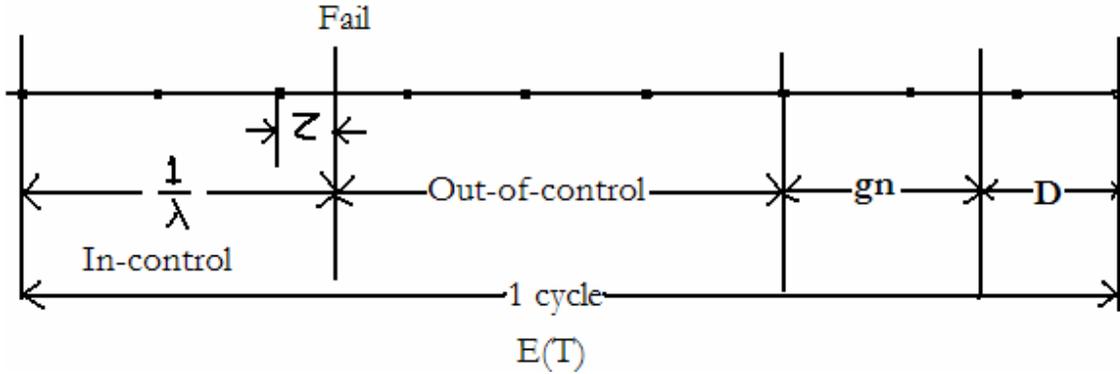


Fig.5 Production cycle

The expected length of in-control period is $1/\lambda$. Noting the no of samples required to produce an out-of-control signal is a geometric random variable with a mean $1/(1-\beta)$ which is called Average Run Length(**ARL**). Average Run Length is the average no of samples required for the sample to fall outside control limits. We conclude that the expected length of out-of-control is $1/(1-\beta)-\tau$. The time required to take a sample and interpret the result is a constant **g** proportional to the sample size, so that **gn** is the length of the cycle. The time required to find the assignable cause following an action signal is a constant **D**. Therefore the expected length of a cycle is:

$$E(T) = 1/\lambda + (h/(1-\beta) - \tau) + gn + D$$

Let the net income per hour of operation in the in-control state is V_0 and the net income per hour of operation in the out-of-control state is V_1 . The cost of taking a sample of size n is assumed to be of the form $(a_1 + a_2 * n)$.

Where a_1 & a_2 represents the fixed and variable components of sampling cost. The expected no of samples taken within a cycle is the expected cycle length divided by the interval

between samples i.e. $E(T)/h$. The cost of finding an assignable cause is a_3 and the cost of investigating a false alarm is a_3^1 . The expected no of false alarms generated during a cycle is α times the expected number of samples taken before the shift, so:

$$\alpha \sum_{j=0}^{\infty} \int_{j^{\text{th}}}^{(j+1)^{\text{th}}} j \exp(-\lambda t) dt = \alpha \exp(-\lambda h)/(1 - \exp(-\lambda h))$$

Therefore, the expected net income per cycle is:

$$V_0(1/\lambda) + V_1(h/(1 - \beta) - \tau + gn + D) - a_3 - a_3^1 \alpha \exp(-\lambda h)/(1 - \exp(-\lambda h)) - (a_1 + a_2 * n)E(T)/h$$

The expected net income per hour can be found by dividing $E(C)$ by $E(T)$.

So the result becomes:

$$E(A) = E(C)/E(T)$$

$$E(A) =$$

$$\frac{V_0(1/\lambda) + V_1(h/(1 - \beta) - \tau + gn + D) - a_3 - a_3^1 \alpha \exp(-\lambda h)/(1 - \exp(-\lambda h)) - (a_1 + a_2 * n)E(T)/h}{1/\lambda + (h/(1 - \beta) - \tau) + gn + D}$$

Let $a_4 = V_0 - V_1$ where a_4 is the hourly penalty cost associated with production in the out-of-control state.

So $E(A)$ can also be written as:

$$E(A) =$$

$$V_0 - \frac{(a_1 + a_2 * n)}{h} - \frac{a_4(h/(1-\beta) - \tau + gn + D) + a_3 + a_1^3 \alpha \exp(-\lambda/h) / (1 - \exp(-\lambda h))}{1/\lambda + (h/(1-\beta) - \tau) + gn + D}$$

$$\text{Or } E(A) = V_0 - E(L)$$

$$E(L) =$$

$$\frac{(a_1 + a_2 * n)}{h} - \frac{a_4(h/(1-\beta) - \tau + gn + D) + a_3 + a_1^3 \alpha \exp(-\lambda/h) / (1 - \exp(-\lambda h))}{1/\lambda + (h/(1-\beta) - \tau) + gn + D}$$

Here $E(L)$ represents the expected loss per hour by the process. $E(L)$ is a function of the control parameters n, k, h . So it is clear that by minimizing $E(L)$ we can maximize $E(A)$.

So to optimize $E(A)$ we take the first partial derivative of $E(L)$ with respect to n, k, h . An iterative procedure is applied to solve for the optimal n, k . $E(L)$ can also be minimized by using an unconstrained optimization or search technique coupled with a digital computer program for repeated evaluations of the cost function.

In this project, the values for optimization solved by Montgomery have been taken, where data given are:

1. $a_1 = 1$

2. $a_2 = 0.01$
3. $a_3 = 25$
4. $a^1_3 = 50$
5. $a_4 = 100$
6. $\lambda = 0.05$
7. $g = 0.0167$
8. $D = 1.0$

Here only unknowns are τ , α & β which can be calculated by the formulae given below.

$$\tau = \frac{\int_{jh}^{(j+1)h} \exp(-\lambda t) (t-jh) dt}{\int_{jh}^{(j+1)h} \exp(-\lambda t) \lambda dt} = \frac{1-(1+\lambda h) \exp(-\lambda h)}{\lambda (1-\exp(-\lambda h))}$$

Using above formula we can calculate τ .

The probability of false alarm is:

$$\alpha = 2 \int_k^{\infty} \Phi(z) dz$$

The probability that it will be detected on any subsequent sample is:

$$1-\beta = \int_{(-\infty)}^{(-k-\delta\sqrt{n})} \Phi(z) dz + \int_{(k-\delta\sqrt{n})}^{(\infty)} \Phi(z) dz$$

So, β can be calculated as,

$$\beta = 1 - \int_{(-\infty)}^{(-k-\delta\sqrt{n})} \Phi(z) dz + \int_{(k-\delta\sqrt{n})}^{(\infty)} \Phi(z) dz$$

So to minimize $\mathbf{E(L)}$ which is loss function per hour Simulated Annealing program have been developed to get the optimal solution for $\mathbf{n, h, k}$ which is a iterative method itself.

RESULT & DISCUSSION

In this project work, for each value of sample size n varying from 1 to 15, the cost function has been minimized using the program developed in Matlab based on Simulated Annealing algorithm and the results of optimum values for width of control limits k , sampling interval h and corresponding minimum values of cost function have been shown in Table 1.

Table 1: Optimum solution

n	k	h	Cost
1	2.40	0.40	14.187
2	2.64	0.47	11.420
3	2.82	0.52	10.404
4	2.97	0.56	9.956
5	3.11	0.59	9.755
6	3.25	0.61	9.676
7	3.38	0.63	9.662
8	3.51	0.65	9.682
9	3.63	0.67	9.720
10	3.76	0.69	9.768
11	3.87	0.71	9.820
12	3.99	0.72	9.872
13	4.11	0.74	9.924
14	4.22	0.76	9.975
15	4.33	0.77	10.042

As shown in this table, as sample size n increased the optimum cost values first decrease and then increase after $n = 7$.

The minimum cost was found to be 9.662 for the values of Sample size (n) = 7, Width of the control limits (k) = 3.38, Sampling interval (h) = 0.63

CONCLUSION

In this thesis, using Simulated Annealing method, the minimum cost was found to be 9.662. $n=7$, $k=3.38$, $h=0.63$

The result of Montgomery's economic design was found to be 10.38. Comparing our result with Montgomery's result, the cost is less. So, this report suggests that simulated annealing is an efficient method for the optimization of cost for X-Bar control chart.

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