DEVELOPMENT OF THE NON LINEAR MODEL FOR RC BEAMS

A thesis

Submitted by

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CERTIFICATE

This is to certify that this report entitled, "**Development of the Nonlinear Model for RC Beams**" submitted by **Anirban Sengupta (107CE002)** and **Bikash Kumar Pati (107CE021)** in partial fulfillment of the requirement for the award of Bachelor of Technology Degree in Civil Engineering at National Institute of Technology, Rourkela is an authentic work carried out by them under my supervision.

To the best of my knowledge, the matter embodied in this report has not been submitted to any other university/institute for the award of any degree or diploma

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Anirban Sengupta and Bikash Kumar Pati

ABSTRACT

KEYWORDS: *Moment-curvature relation, stress-strain relation, nonlinear analysis, reinforced concrete, rectangular beam, plastic hinge*

The relationship between the moment and curvature of reinforced sections is a very important parameter for nonlinear analysis of RC framed structure. A clear view of the strength, stiffness, ductility, and energy dissipation capacity structure can be obtained from this relationship. The moment-curvature relationship would enable us to observe the strength reduction beyond the peak point and degradation of the flexural rigidity. The present study focuses on the moment-curvature *(M-φ)* relation of rectangular beam section.

A user friendly software programme to generate moment-curvature relation of rectangular reinforced concrete beams developed using stress-strain curves of concrete and steel as per IS 456:2000. A thorough study is done to see the how different parameters (such as0 grade of concrete and steel, effective cover, amount of tension steel reinforcement *etc*.) affect the moment-curvature relation. Also a study is done on the variation of neutral axis in an under-reinforced rectangular doubly reinforced section with varying strain in the extreme compression fiber of concrete.

It is found from the study that increasing tension steel reinforcement substantially increases the moment capacity of the section but reduces the ductility. Increase in the grade of concrete marginally increases moment capacity but substantially increases the ductility. Both moment capacity and ductility of the beam decreases with the increase of effective cover. Fe 415 grade of steel makes the beam more ductile than that with Fe 500. It is observed that with the increase in the compressive strain in the extreme fiber the neutral axis depth initially increases followed by a steep decrease till the failure of the beam.

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NOTATIONS

A_{sc}	Area of compression reinforcement
A_{st}	Area of steel reinforcement
b	width of compression zone
C	Compression
D_{prime}	Effective depth of cross section
D_{prime}	Effective depth of cross section
Dc	Diameter of bar in compression
Dt	Diameter of bar in tension
E_s	Elastic modulus of steel =2 x10^5 MPa
E_c	modulus of elasticity of concrete=$5000\sqrt{fck}$
E	Modulus of elasticity
f_{yd}	design yield strength
f_y	specified yield strength
f_c	Design compressive stress corresponding to any strain
f_{ck}	28 day characteristic strength of concrete
f_{ct}	maximum tensile stress
f_{cr}	flexural tensile strength
I	Moment of Inertia
I_{eff}	Effective moment of inertia

M	Bending moment acting on a section at service load
M_{ur}	Ultimate moment of resistance
N_c	No. of bars in compression
N_t	No. of bars in tension
R	Radius of curvature
T	Tension
x_u	Depth of neutral axis
φ	Curvature (1/mm)
φ_{max}	Maximum curvature
φ_y	Section curvature at yielding
$\square_{eff}$	Effective curvature
ε_y	Design yield strain
ε_1	Extreme fiber strain at top
ε_2	Extreme fiber strain at bottom
ε_{cu}	maximum compressive strain
ε_c	Strain in concrete
ε_{cm}	Mean strain in concrete
ε_{sm}	Mean strain in steel
γ_s	Partial factor of safety for steel
γ_c	Partial factor of safety for concrete

ABBREVIATION

RC	Reinforcement Concrete
IS	Indian Standard
phi	Curvature (φ)
Epsilon	Strain in extreme compression fiber

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Reinforced concrete, one of the most widely used structural material today, despite its obvious non homogeneity remains amenable to the simplified methods of analysis and design. However linear behavior is limited to region of small response. At ultimate load conditions material and geometrical nonlinearity manifests. The simplified method of linear elastic analysis is no longer valid. It calls for nonlinear analysis of the structure.

There is a shift in design philosophy in recent times from linear static approaches to performance based approaches; non linear structural analysis is an important part of the performance based design approach. Moment-curvature relation is the important input parameter for non linear analysis.

The relationship between the moment and curvature of reinforced sections gives us a clear view of the strength, stiffness, ductility, and energy dissipation capacity for a structural section subjected to bending. The moment-curvature relationship also demonstrates the strength reduction beyond the peak point and degradation of the flexural rigidity.

In the present thesis development of moment-curvature *(M-φ)* relation for rectangular RC beam section and the effect of different parameters (such as grade of concrete and steel, effective cover, amount of steel reinforcement *etc*.) on the moment-curvature relation are studied Also the variation of neutral axis depth in an under reinforced rectangular doubly reinforced section is studied for different strain levels in the extreme compression fiber of concrete.

Development of moment-curvature relation is an iterative process. A computer programme should be developed to generate moment-curvature relation to make it attractive for design office environment. This is the primary motivation behind the present study

1.2 POST-YIELD STRUCTURAL BEHAVIOUR

As mentioned earlier analysis of structure is generally done by the conventional elastic theory even though the design is done by the limit state method by taking in to account the material non linearity. This is permitted by the code (Cl. 22.1 of IS: 456:2000) This method holds good in determinate as well as in indeterminate structures, even under factored load conditions till the moment-curvature relationship remains linear. For under reinforced sections this is valid until the reinforcing steel provided has not yielded.

However once yielding take place (at any section), the behavior of a structure enters an inelastic phase, and the simplified conventional linear elastic structural analysis is no longer valid. Inelastic analysis is thus called for to determine the bending moments for loading beyond the yielding stage. In simplified limit analysis the moment-curvature relation is an idealized bilinear elasto-plastic relation as shown in Fig 1.1

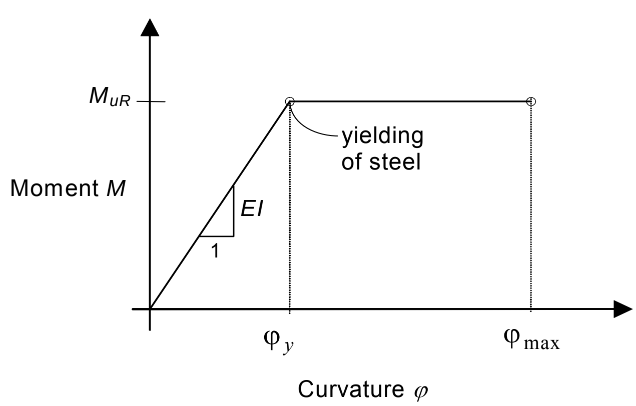


Fig 1.1 Idealized moment-curvature relation (Pillai and Menon, 2003)

With the yielding of the tension steel the ultimate moment of resistance (M_{ur}) is assumed to have reached the critical section. On increasing the strain the moment resisted by the section does not increase any further however there is a significant increase in curvature. Continued rotation take place in the zone of yielding as though a hinge is present but one that resists a constant moment M_{ur}. Formation of further plastic hinges take place, and finally the limitation in ductile behavior (i.e. the curvature φ reaching its ultimate value) at any one plastic hinge location results in the crushing of concrete. Therefore, moment-curvature relation for all flexural members is important modeling parameter for nonlinear structural analysis.

1.3 OBJECTIVES

Prior to defining the specific objectives of the present study, a detailed literature review was taken up. This is discussed in detail in Chapter 2. Based on the literature review the main objectives of the present study have been identified as follows.

a) To develop a computer program to determine the moment curvature relation of the reinforced concrete beam sections using concrete and steel stress-strain curves as per IS 456:2000

b) To assess the effect of the following parameters on the moment-curvature behavior of beam section: (i) Percentage of longitudinal tension steel reinforcement, (ii) Grade of concrete, (iii) Effective cover, (iv) Grade of steel and their variations

1.4 SCOPE OF WORK

a) The present study is limited to rectangular RC beam sections only. Analysis of column requires studies on axial force and bi-axial bending interaction and this issue is kept outside the scope of the present study

b) Presence of shear force may alter the moment-curvature relation of a beam section but for the present study uncoupled moment in one direction id=s considered.

c) Constitutive relation for concrete as given in IS 456:2000 does not consider the effect of confinement. Therefore effect of confinement on moment-curvature is not considered for the present study

1.5 METHODOLOGY

Development of the software programs requires clear understanding of programming language and the domain knowledge. The tool created was for generating moment-curvature relationship of reinforced concrete beams. The programming language used was 'GNU C++' and the platform used was 'Code blocks 10.0'. Both the language and platform used are open source and are available for free. The entire program has been backed by formal mathematical models which have been coded for ease of parametric study.

1.6 ORGANISATION OF THESIS

The introductory chapter has presented the background, objective, scope and methodology of the present study

Chapter 2 starts with a description of stress-strain curve for concrete and steel as per IS 456:2000. Later in the chapter the description of some of the previous works done on moment-curvature relation for RC sections are presented

Chapter 3 discusses the algorithm for generating moment-curvature relation and the assumptions considered

Chapter 4 presents moment-curvature behavior for typical beam section as obtained from the developed computer code. This chapter also discusses the results of parametric study carried out on moment-curvature relation

Finally chapter 5 presents a summary, significant conclusions from this study and future scope of research in the area

CHAPTER 2

LITERATURE REVIEW

2.1 GENERAL

In this section a study on the stress–strain curve of steel and concrete is presented as per IS 456:2000. The most relevant literature available on the study of the moment-curvature relation of reinforced concrete sections has also been reviewed and presented in this Chapter.

2.2 STRESS-STRAIN CHARACTERISTICS OF CONCRETE

Stress-strain curve of concrete is the basis of analysis and design of reinforced concrete sections. As specified by the IS 456:2000 the characteristic and design stress-strain curves for concrete in flexural compression is depicted in Fig 2.1 The maximum stress in the characteristic curve is restricted to $0.67f_{ck}$. The curve is idealized as parabolic in the initial ascending portion up to a strain of 0.002 (where the slope becomes zero), and a straight line thereafter, at a constant stress level of $0.67f_{ck}$, up to an ultimate strain of 0.0035.

For the purpose of limit state design a partial safety factor γ_c equivalent to 1.5 is applied. Thus the design curve is obtained simply by scaling down the ordinates of the characteristic curve-dividing by γ_c[Fig 2.1]. The design stress is taken accordingly as $0.447\,f_{ck}$. For any strain

$\varepsilon_c \leq 0.0035$ the design compressive stress f_c is given as:

$$f_c = \left\{0.447 f_{ck} \quad \left[2\left(\frac{\varepsilon_c}{0.002}\right) - \left(\frac{\varepsilon_c}{0.002}\right)^2\right]\right\} \qquad \text{for } \varepsilon_c < 0.002$$

$$= 0.447\, f_{ck} \quad \text{for } 0.002 \leq \varepsilon_c \leq 0.0035$$

As obtained from experiment the stress strain curve after reaching a peak stress at a strain of 0.002 descends, but for convenience in calculations it is idealized by a flat region thus keeping the stress uniform after a strain of 0.002. The model is silent about strength enhancement and ductility due to confinement. Our IS 456:2000 model is similar to British code [BS8110] model.

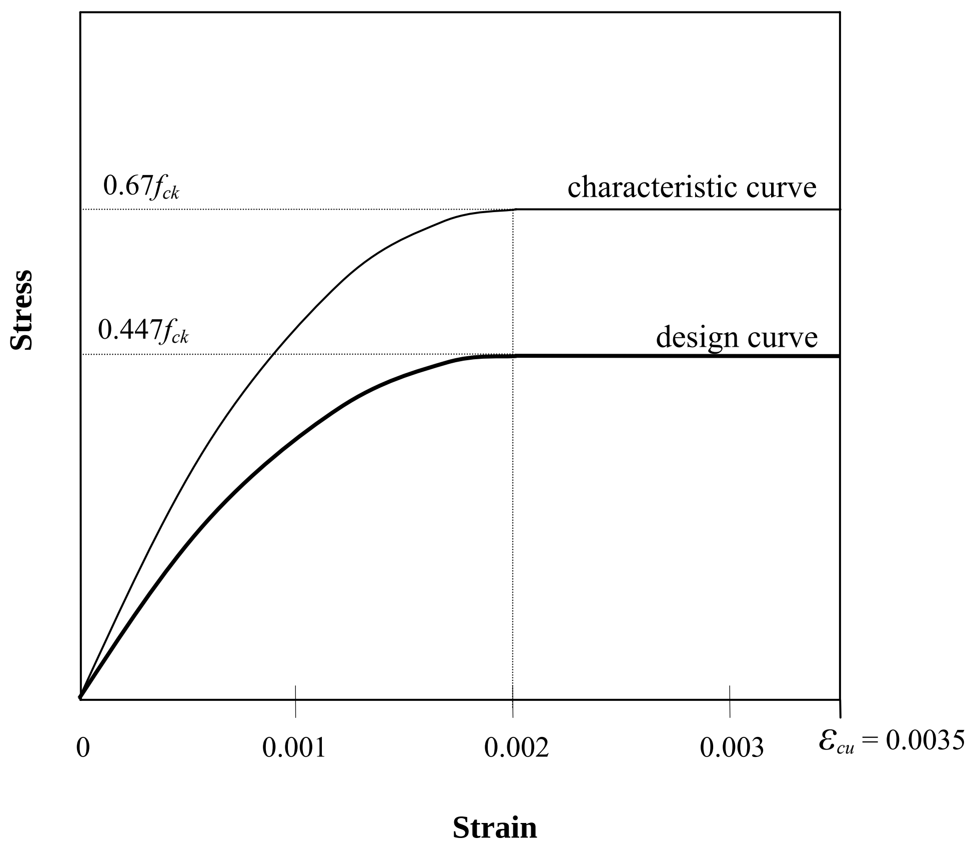


Fig 2.1 Characteristic and design stress-strain curves for concrete in flexural compression

2.3 STRESS-STRAIN CHARACTERISTIS OF STEEL

The characteristic and design stress-strain curves specified by IS 456:2000 for Fe 415 and Fe 500 grade of reinforcing steel (in tension and compression) is as shown in Fig 2.2. The partial safety factor $\gamma_s$is taken as 1.15 for ultimate limit state. The design yield strength f_{yd} is obtained by dividing the specified yield strength f_y by the partial safety factor, accordingly $f_{yd} = 0.87\ f_y$. However in the case of cold worked bars (Fe 415 and Fe 500), there is no specific yield point. The transition from linear elastic behavior to non-linear behavior is assumed to occur at a stress level equal to 0.8 f_y in the characteristic curve and 0.8f_{yd} in the design curve. The full design yield strength 0.87 f_y is assumed to correspond to a 'proof strain' of 0.002 i.e., the design yield strain ε_y is to be taken as $\frac{0.87 f_y}{E_s}$ +0.002 as shown in Fig 2.2. .The coordinates of the salient points of the design stress-strain curve for Fe 415 and Fe 500 are listed in Tab 1.The design stress corresponding to any strain can be obtained by interpolation.

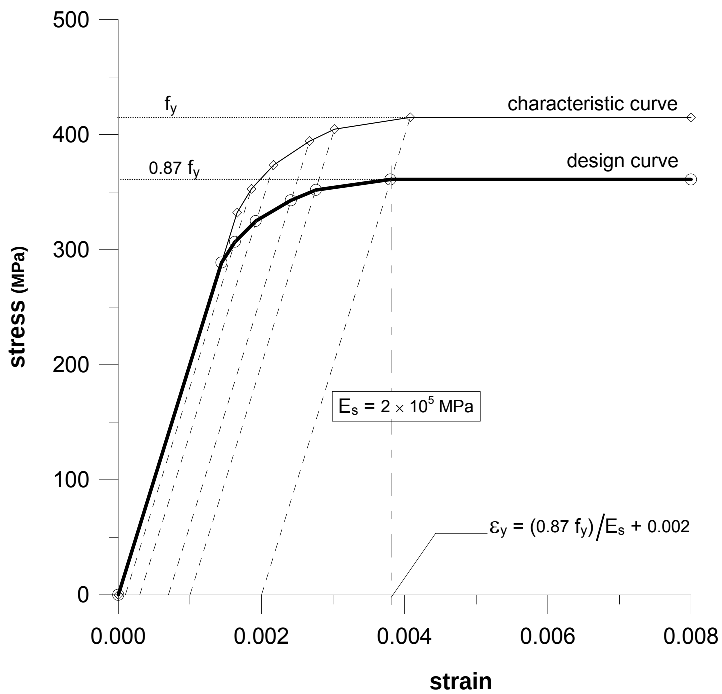


Fig 2.2 Characteristic and design stress-strain curves for Cold-worked steel

Table 1: Stress-strain variations for reinforcing steel

Fe 415		Fe 500	
Strain	Stress (MPa)	Strain	Stress (MPa)
0.000 00	0.0	0.000 00	0.0
0.001 44	288.7	0.001 74	347.8
0.001 63	306.7	0.001 95	369.6
0.001 92	324.8	0.002 26	391.3
0.002 41	342.8	0.002 77	413.0
0.002 76	351.8	0.003 12	423.9
≥0.003 80	360.9	≥0.004 17	434.8

2.4 PREVIOUS WORK ON THE NONLINEAR BEHAVIOUR OF RC SECTIONS

Menon (2006) reflects on the limitations of the simple elastic analysis to depict the true behavior of the structure under ultimate loads. Both geometrical and material non linearity manifests at the time of large deformations, also in performance based earthquake resistant design adequate ductility in the structure is an important aspect. Moment-curvature relation serves as an important tool for such non linear analysis of the structure giving us a clear view of the nature of variation of the ultimate moment of resistance (M_{ur}) of the section with the increasing curvature.

Chugh (2004) has done an elaborate work to develop tools for analysis and design of reinforced concrete beams and columns with main emphasis on design basis of moment-rotation curves .The study is done based on the classical strength of materials approach and also some empirical relations have been discussed. The entire study is dedicated to the study and formulation of the moment-rotation relationships for RC frame members, including effects of confinement, bond slip and axial force. Also analysis and design aids have been developed for rectangular beam and column sections. However the effect of shear has not been considered. In fact an important motive was to develop software tools by use of existing literature.

Ersoy and Ozcebe (1988) also developed a computer programme in a spread sheet environment to study the moment-curvature relation of reinforced concrete sections. Realistic material models were used for confined and unconfined concrete. Model for reinforcement included strain hardening of steel. The programme was used to study the influences of the confinement, strain hardening of steel, level of axial load, amount of tension and compression reinforcement on the behavior of reinforced concrete members subjected to bending and axial load.

2.5 SUMMARY

Stress-strain curve of concrete and steel as specified in the IS 456:2000 is described here. For the small displacement the moment-curvature relation of a structure remains linear and the simplified conventional elastic analysis remains valid. However, under large deformations both geometrical and material non-linearity manifests making it necessary to ensure that there is adequate ductility in the structure. These calls for proper design to ensure that plastic hinges develop at the right locations. Moment-curvature relation thus serves as an important tool to achieve it. Chugh (2004) developed analytical tools to generate moment-curvature curves of RC beams and columns using different concrete models considering confinement. Ersoy and Ozcebe (1988) developed a computer programme in a spread sheet environment for studying the moment-curvature of reinforced concrete section. However, none of these consider Indian code for their analysis. In the present study the stress-strain curve of concrete and steel as specified in the Indian code is used for the development of the moment-curvature relations.

CHAPTER 3

MODELLING

3.1 GENERAL

A computer programme has been developed to generate the moment-curvature relation for a rectangular beam section using the IS 456:2000 model of stress-strain curve for concrete and reinforcing steel. Dimension, reinforcing steel in the tension and compression side for the selected beams are the input parameters. The assumptions, procedure and logic have been discussed in this Chapter.

3.2 MOMENT-CURVATURE RELATIONS

Moment-curvature relation is the basic tool in the non-linear analysis of the structure. It plays an important role in predicting the behavior of the reinforced concrete members under flexure.

In nonlinear analysis it is used to model plastic hinge behavior.

3.2.1 CURVATURE

Curvature (φ) is defined as the reciprocal of radius of curvature (R) at any point along a curved line. When an initial straight beam segment is subjected to a uniform bending moment throughout its length, it is expected to bend in a segment of circle with a curvature (φ) that increases in some manner with increase in the applied moment (M). Curvature (φ) may be alternatively defined as the angle change in slope of the elastic curve per unit length$\Big(\varphi = 1/R = d\theta ds$. At any section, using the "plane sections remain plane" hypothesis under pure bending, the curvature can be computed as the ratio of the normal strain at any point across the depth to the distance measured from the neutral axis at that section (Fig 4.1)

If the bending produces in the extreme fiber strains of ε_1 and ε_2 at top and bottom at any section as shown in Fig 4.1 (compression at top and tension at bottom as assumed in this case), then for small deformations, it can be shown that

$$\varphi = \frac{\varepsilon_1 + \varepsilon_2}{D}$$

Where D is the depth of the beam in an initially straight beam section

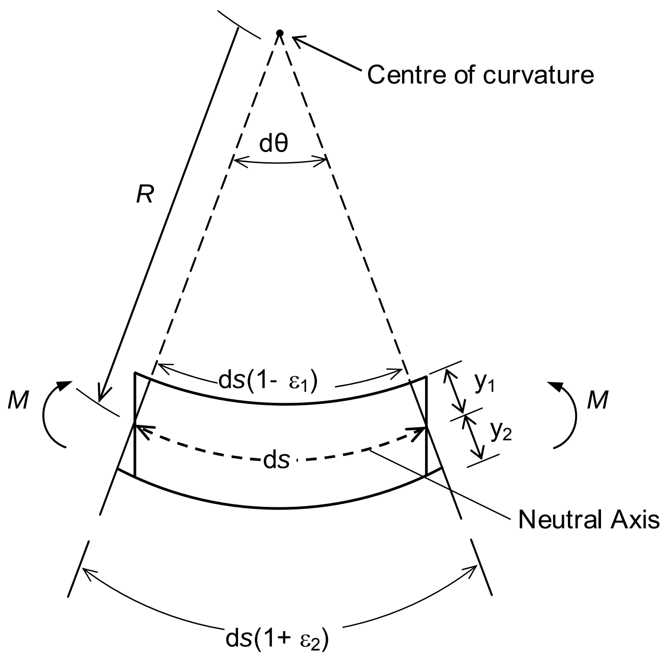


Fig 3.1 Curvature in an initially straight beam section

If the beam behavior is linear elastic, then there moment–curvature relationship is linear, and the curve is obtained as $\varphi = \frac{M}{EI}$, where EI is the flexural rigidity of the beam, obtained as the product of the modulus of the elasticity E and second moment of area of the section I.

When a RC flexural member under pure flexure is subjected to gradual increase in moment, its behavior transits through various stages, starting from initial uncracked state to ultimate limit state of collapse.

3.2.2 Un-cracked phase

Considering a simply supported beam subjected to gradually increasing load. The central segment of the beam is subjected to pure flexure. In the early stages of loading, the applied moment (at any section) is less than the cracking moment M_{cr} and the maximum tensile stress f_{ct} in the concrete is less than its flexural tensile strengthf_{cr}. The phase is uncracked phase, where in the entire section is effective in resisting the moment and is under stress. The uncracked phase reaches its limit when the applied moment M becomes equal to the cracking moment M_{cr} the behavior is linear elastic in this phase.

3.2.3 Cracked phase

The flexural tensile stress in concrete goes on increasing linearly with increase in applied moment until the applied moment exceeds the cracking moment M_{cr}. As the applied moment exceeds $M_{cr,}$ the maximum tensile stress in concrete exceeds the flexural tensile strength of concrete and the section begins to crack on the tension side. The cracks are initiated in the bottom (tensile) fibers of the beam, and with increasing loading, widen and propagate gradually towards the neutral axis. As the cracked portion of the concrete is now rendered ineffective in resisting the tensile stresses, the effective concrete section is reduced. The tension resisted by the concrete just prior to cracking is transferred to the reinforcing steel at the cracked section of the beam. For any further increase in applied moment, the tension component has to be contributed solely by the reinforcing steel. With the sudden increase in tension in the steel, there is associated increase in the tensile strain at the level of the steel resulting in upward shift of the neutral axis and an increase in curvature at the cracked section. However concrete in between the cracks can carry some tension, which contributes to the overall stiffness. This phenomenon is known as tension stiffening effect. This makes the problem much more complex because of the uncertainties associated with location, width and depth of the cracks. The behavior in this phase (called 'cracked phase') is very important, as usually the service load lies in this region

3.2.4 Limit state of collapse

The stresses in tension steel and concrete go on increasing as the moment increases further. Finally, collapse occurs when the strain in the extreme compression fiber reaches the ultimate strain of concrete. The behavior at the ultimate limit state depends on the percentage of steel provided, i.e. whether the section is 'under-reinforced' or 'over-reinforced'. In the case of under-reinforced section, failure is triggered by yielding of tension steel whereas in over reinforced section the steel does not yield at the limit state of failure. In both the cases, the failure eventually occurs due to crushing of concrete at the extreme compression fiber, when the ultimate strain in concrete reaches its limit.

3.2.5 Prediction of moment-curvature diagram

In practice, RC structural members are generally subjected to moments greater than the cracking moment M_{cr}. The member is cracked and the contribution of crack in the tension zone to the strength of the section is neglected. But the concrete in between the cracks has significant effect on the flexural rigidity. In a beam segment subjected to constant moment $M>M_{cr}$, theoretically the entire segment should be fully cracked on the tension side of the neutral axis. But in practice, the flexural cracks are dispersed somewhat randomly such that there are significant portions in between the cracks, which remain uncracked, as shown in the Fig. 4.2 (a)

The concrete in between the cracks resist some tension, and this is reflected by a reduction in tensile strain in the reinforcement [Fig.4.2 (b)], a lowering of the neutral axis [Fig.4.2 (a)], a fluctuation in the bond stress as well as a reduction in curvature [Fig 4.2 (c)].

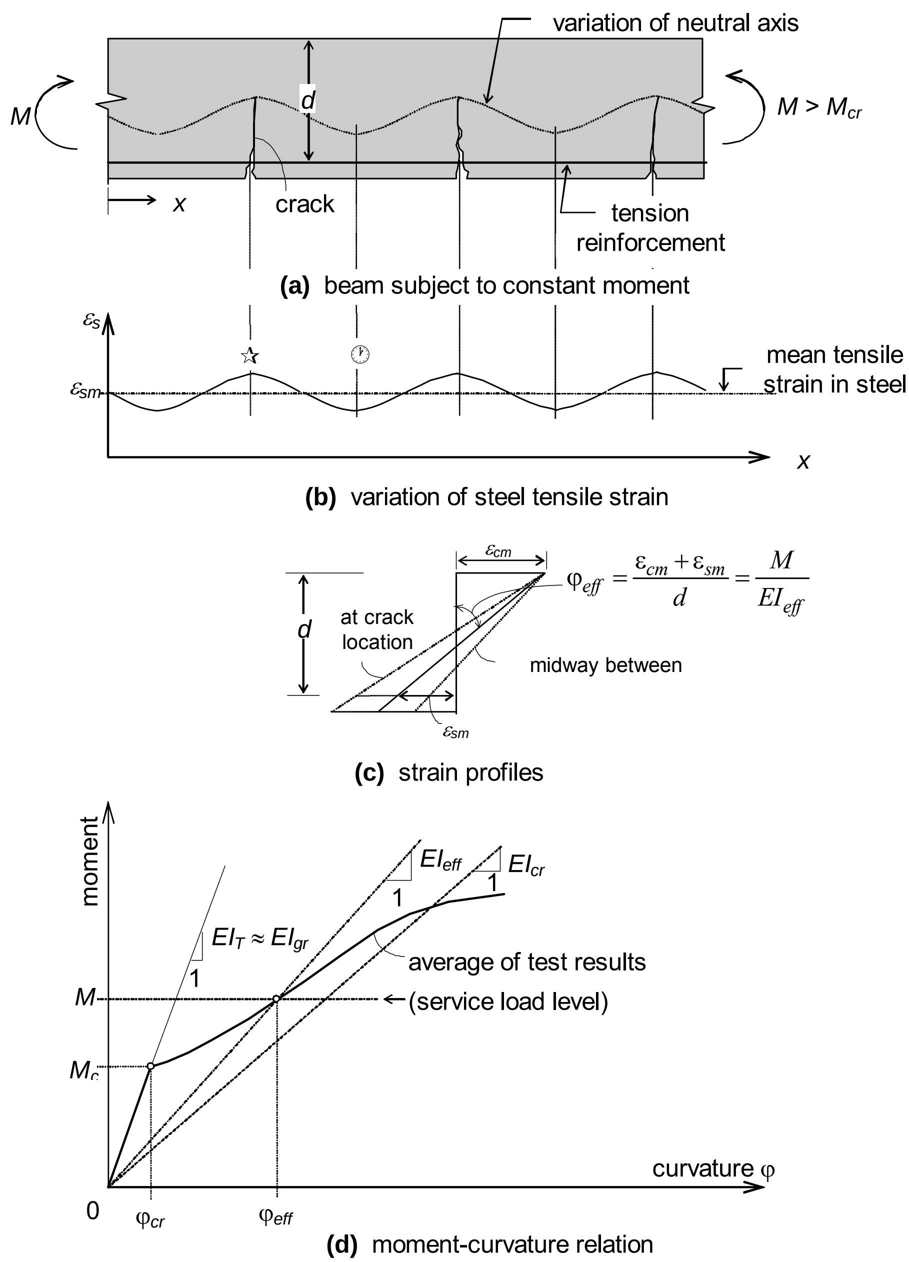


Fig 3.2 Effective flexural rigidity of a beam subjected to constant moment (Pillai and Menon, 2003)

An *effective curvature* φ_{eff} may be defined for the beam segment, as being representative of the mean curvature of the segment, under the action of a constant moment *M*. For this purpose, the strain profile to be considered may be reasonably based on the mean strain profile [Fig 4.2 (c)], rather than the strain profile at the crack location (which is obviously higher):

$$\varphi = \frac{\varepsilon_{cm} + \varepsilon_{sm}}{d} = \frac{M}{EI_{eff}}$$

where ε_{cm} and ε_{sm} are the mean strains in the extreme compression fiber in concrete and tension steel respectively, d is the effective depth, and EI_{eff} is the effective flexural rigidity of the section.

3.3 ASSUMPTIONS

1. The strain is linear across the height of the section, that is plane bending
2. The tensile strength of the concrete has been ignored
3. The maximum compressive strain ε_{cu} in concrete at the extreme compression fiber is taken as 0.0035
4. The modulus of elasticity of concrete E_c as adopted from IS 456:2000 is taken as $5000\sqrt{f_{ck}}$

3.4 Numerical Algorithm for Moment-Curvature of Beam Sections

1. Assign a very small starting strain value to the extreme compression fiber
2. A neutral axis depth is assumed from the extreme compression fiber
3. The strain and the corresponding stress at the centroid of the tension and compression reinforcements is calculated

4. The stress distribution in the concrete compressive region is calculated based on the IS 456:2000 model. The resultant compressive force is then calculated by the numerical integration of the stress over the entire compressive region
5. The tensile force from the stress in the tensile reinforcement and the area of the bar is calculated
6. If the net compressive force (Resultant compressive force + compressive force in compression reinforcement) is equal to the calculated tensile force, the neutral axis depth assumed is adopted
7. Otherwise a new neutral axis depth is assumed and steps 3 to 6 is repeated until the condition is satisfied
8. Assign the next higher value of strain to the extreme compression fiber and repeat steps 2 to 7
9. The procedure is then repeated to obtain the entire moment-curvature diagram

The computer programme for the moment-curvature relation is incorporated in Appendix A

CHAPTER 4

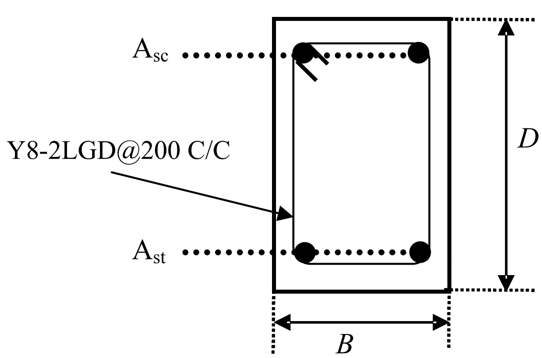
RESULTS AND DISCUSSIONS

4.1 GENERAL

In the present chapter the moment-curvature relations have been plotted using the developed computer program for a rectangular doubly reinforced section. A parametric study has been carried out by changing different input parameters. This chapter presents the results and discussions of the study.

4.2 Parametric Study of Moment-Curvature of Beam

A reinforced concrete section is taken from literature (Menon, 2006) for this study. The details of the section are shown in Fig. 4.1. The effects of various parameters (*e.g.*, the percentage of longitudinal tension steel reinforcement, grade of concrete, effective cover, and grade of steel) on the moment-curvature relation were studied and the corresponding results are presented as follows:



b	300 mm
D	450 mm
D_{prime}	40 mm
A_{st}	628 mm^2
A_{sc}	226 mm^2
f_y	415 MPa
f_{ck}	25 MPa

Fig 4.1 Properties of the assumed section

4.2.1 Influence of the amount of longitudinal tension steel reinforcement

The moment-curvature relation of a concrete section depends greatly on the percentage of longitudinal steel reinforcement. Fig. 4.2 presents the variation of moment-curvature relation as function of longitudinal tensile reinforcement. This figure shows that on increase in the percentage of tensile steel reinforcement the ascending portion of the moment-curvature curve gets steeper, *i.e.*, the moment carrying capacity of the section increases. However ultimate curvature decreases (therefore, ductility decreases) rapidly with the increase in the percentage of steel. For an increase in the tension steel from 0.9% to 2.1% there is a huge moment capacity increase of 57% and a decrease in curvature by 51%

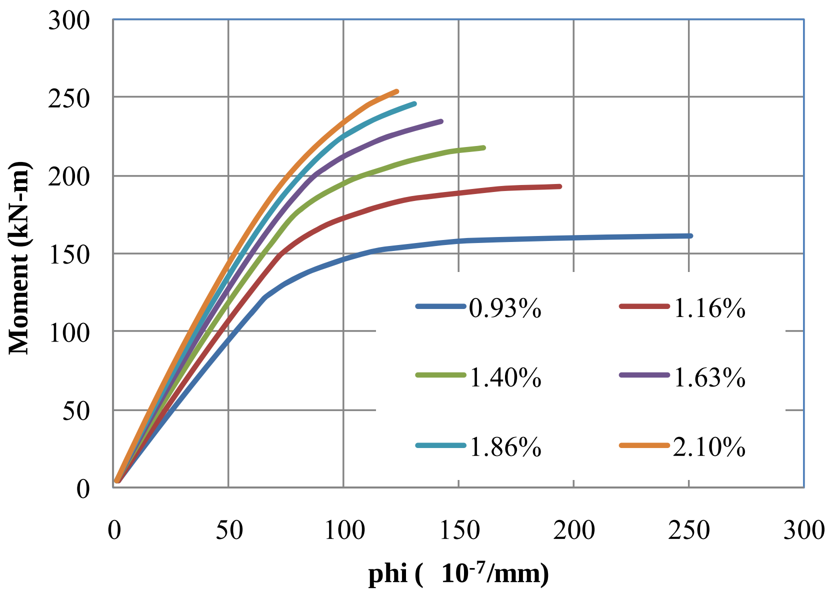


Fig 4.2 Influence of the percentage of longitudinal steel reinforcement

4.2.2 Influence of the grade of concrete

With increase in the grade of concrete the moment carrying capacity of the section increases marginally as expected. There is a comparatively larger increase in ductility with the increase in the grade of concrete as seen in the Fig. 4.3. For an increase in the grade of concrete from *M 25* to *M 50* there have been a 3% increase in the moment capacity of the section and 53% increase in the ductility.

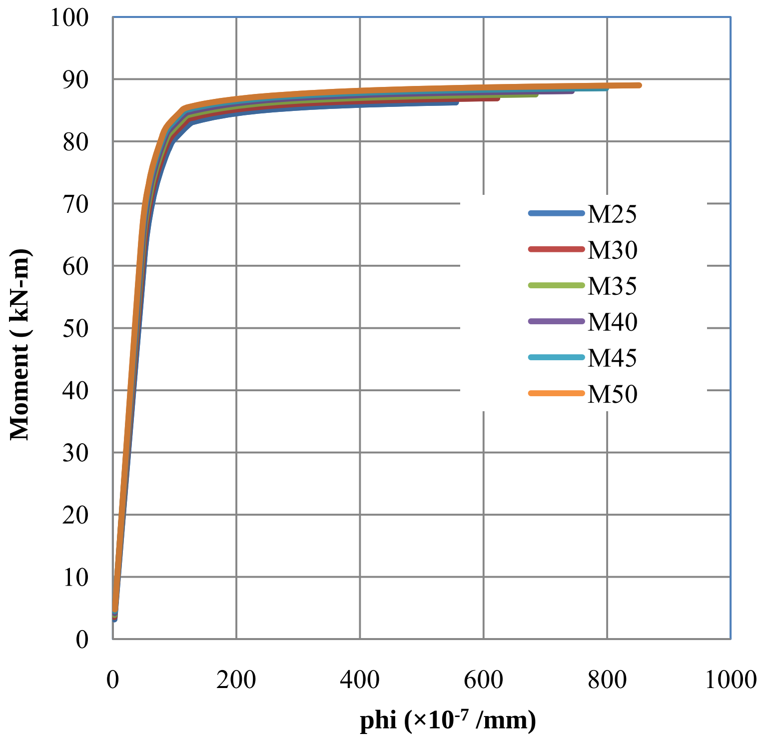


Fig 4.3 Influence of the grade of concrete

4.2.3 Influence of the effective cover

The moment-curvature of reinforced concrete beam with varying effective covers are studied and shown in Fig 4.4. Effective covers of 25, 30, 35, 40, and 45 have been considered while plotting the curves. It is observed that the moment capacity of the section and its curvature decreases with the increase in the effective cover. Moment capacity decreases by 6.5% and curvature decreases by 17% with a change in effective cover from 25 to 45

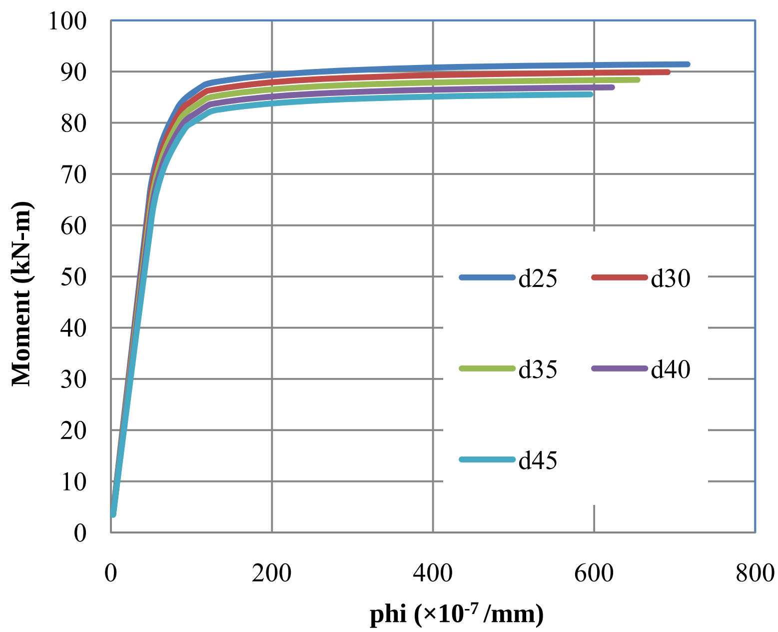


Fig 4.4 Influence of the effective cover

4.2.4 Influence of the grade of tension steel reinforcement

The effect of the grade of tension steel reinforcement is studied and shown in the Fig. 4.5

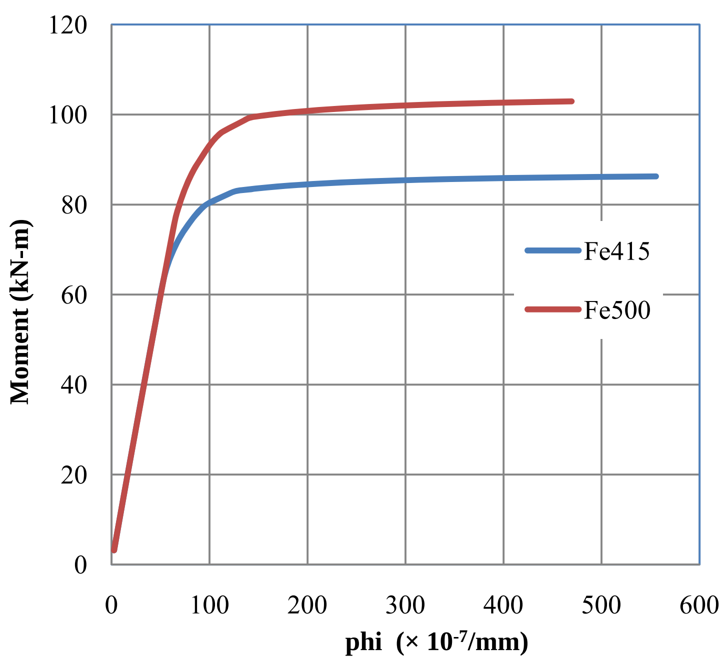


Fig 4.5 Influence of the tension steel reinforcement

As seen with the increase in the yield strength of the reinforcement the ductility reduces substantially by 15.5%.However there is an increase in moment capacity by 19 %.

4.3 Variation of the Neutral Axis with varying strain (in extreme compression fiber)

An under reinforced section (one in which the ultimate limit state is approached in steel before that in concrete) is considered in this study. The variation of the neutral axis depth with varying strain is plotted in the Fig 4.6.

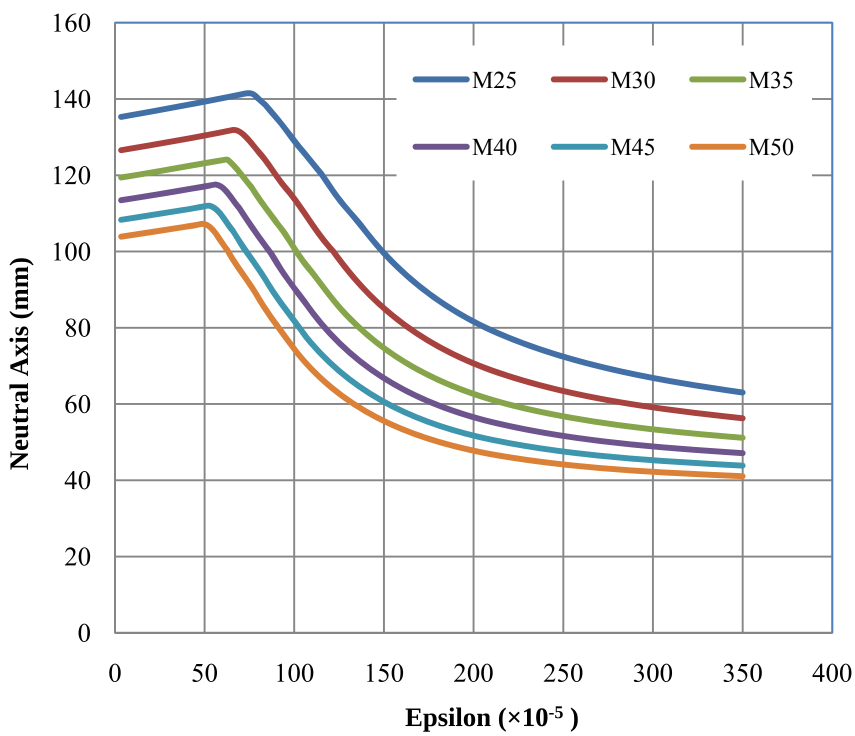


Fig 4.6 Variation of the neutral axis with strain in the extreme compression fiber

As seen in the figure there is a gradual increase in the depth of the neutral axis with increasing strain, followed by a steep decrease till the failure of the beam. A slight increase in the load moment causes the steel to elongate significantly, without any significant increase in stress. The marked increase in tensile strain causes the neutral axis to shift upwards, thus tending to reduce the area of concrete under compression. As the total tension T remains essentially constant at A_{st} f_y, the compressive stresses (and hence, the strains) have to increase in order to maintain equilibrium (C=T). This process is accompanied by wider and deeper tensile cracks increased beam curvatures and deflections, due to relatively rapid increase in tensile strain. The process continues until the maximum tensile strain in concrete reaches the ultimate compressive strain in concrete resulting in the crushing of concrete in the limited compression zone.

CHAPTER 5

SUMMARY AND CONCLUSION

5.1 SUMMARY

In the present study a computer programme for the generation of moment-curvature relation of rectangular RC sections. The stress-strain curves of concrete and steel as specified in IS 456:2000 has been used for this purpose A parametric study has been done on the various parameters affecting the moment-curvature relation like the percentage of longitudinal tension steel reinforcement, grade of steel and concrete, effective cover and their variations. Also a study is done on the change of the neutral axis depth with varying strain in the extreme compression fiber

5.2 CONCLUSIONS

Following are the important conclusions made from the present study:

1) Increasing the amount of tension steel reinforcement substantially increases the moment capacity of the section; however the ductility of the structure reduces drastically which is not at all desirable.
2) With the increase in the grade of concrete there is a marginal increase in the moment capacity of the section but a substantial increase in the ductility of the section
3) As the effective cover increases both moment-capacity and ductility of the beam decreases
4) Fe 415 steel when used as a tension reinforcement has more ductility than Fe 500, however the moment-capacity of Fe 500 is substantially more than Fe 415

5) It is observed that with the increase in the compressive strain in the extreme fiber the neutral axis depth initially increases followed by a steep decrease till the failure of the beam

5.3 SCOPE FOR FUTURE WORK

1) The study can be extended to RC column section that considers axial force coupled with biaxial bending moment
2) Circular cross section of columns is also popular in construction. This study can be further extended to circular RC sections
3) Steel stress-strain curves given in IS 456:2000 is consistent with the other international standards but the stress-strain curve of concrete as given in IS 456:2000 has many limitations. This study can be extended to include other improved models of concrete stress-strain curve
4) The effect of shear on the on the moment-curvature behavior of the beam has to be studied and the decrease in the flexural strength of the beam due to shear failure
5) A non linear analysis of the R.C structures using the moment-curvature relation generated through the program can be carried out

REFERENCES

1) **Chugh, R. (2004).** Studies on RC Beams, Columns and Joints for Earthquake-resistant Design, M. Tech. Thesis, IIT Madras

2) **Ersoy, U. and Ozcebe, G. (1998).** Moment–curvature relationship of confined concrete, Teknik Dergi, vol. 9, no. 4, pp. 1799-827

3) **IS: 456 –2000** Code of Practice for Plain and Reinforced Concrete. *Bureau of Indian Standards*. New Delhi. 2000.

4) **Menon, D. (2006)** "Modeling of RC Elements for Non-linear Analysis", Lecture Notes, IIT Madras

APPENDIX A

COMPUTER PROGRAMME FOR MOMENT CURVATURE RELATION OF RECTANGULAR RC SECTIONS

```
#include<iostream>
using namespace std;
#include<stdio.h>
#include<iomanip>
#include<math.h>
#include<conio.h>
#define max 50
#define pi 3.141592654

//global variables here
float b,D,Fck,Dprime;
float Xu,Fst,Fsc,Asc,Ast,Esc;//Esc is EpsilonDash
float Xu_max,Fy,Es;
float Epsilon,EpsilonS;
float Sigma_Value;
float Dc,Dt,Nc,Nt;
//Varibles for internal work
float Xu_Working_min=-1,Xu_Working_max=-1;
int grade=-1;
//initialize the values here

void initialize()
{
   //Change the values as per data;
   b=300;
   D=450;
   Fck=25;
   Dprime=40;
```

```
    Fy=415;
    Es=200000;
    Asc=638;
    Ast=638;
    //Fst=415;
    Dc=12;
    Dt=20;
    Nc=2;
    Nt=8;
    //temporary initializations:: internal use only
   Epsilon=0.0021;
    //Values for internal usage:::DO NOT CHANGE;
    Esc=-1;
    Xu=0;
    Fsc=-1;
}

// Calculation of sigma need to return a float floating point number
float Calculate_sigma()//check returntype
{
    float Sigma_Value;
    if(Esc<0.002)
    {
        Sigma_Value=0.447*Fck*(2*(Esc/0.002)-pow(Esc/0.002,2.0));
    }
    else if(0.002<=Esc && Esc<=0.0035)
    {
        Sigma_Value=0.447*Fck;
    }
    cout<<"\nSigma "<<Sigma_Value;
    return Sigma_Value;
}

//Calculation of integral of sigma
```

```
float Calculate_sigma_integral()//check returntype
{
   float Sigma_integral_Value;
   if(Epsilon<0.002)
   {
      Sigma_integral_Value=Fck*(223.5*pow(Epsilon,2)-37249.998*pow(Epsilon,3));
   }
   else if(0.002<=Epsilon && Epsilon<=(float)0.0035)
   {
      Sigma_integral_Value=Fck*(223.5*pow(0.002,2)-
37249.998*pow(0.002,3))+(0.447*Fck*(Epsilon-0.002));
   }


   cout<<"\nSigma integral"<<Sigma_integral_Value;
   return Sigma_integral_Value;
}

//End of sigma calcualtions reSume operatons as expexted.
//TODO change the formula to reflect changes
void Calculate_Xu_max()
{
   //Xu_max=0.0035/(0.0055+((0.87*Fy)/Es))*D;
   Xu_max=D;
   cout<<"\nXu max value\t"<<Xu_max;
}

void Calculate_Esc()//This is epsilon Dash (epsilon prime)
{
   if(Xu!=0)
   {
      Esc=Epsilon*(1-Dprime/Xu);
      cout<<"\nEsc=\t"<<Esc;
   }
   else
```

```
    cout<<"\nUnable to set Xu value,please check data";
}

void Calculate_EpsilonS()
{
    if(Xu!=0)
    {
        EpsilonS=Epsilon*((D-Dprime)/Xu-1);
        cout<<"\nEpsilonS=\t"<<EpsilonS;
    }
    else
    cout<<"\nUnable to set Xu value,please check data";
}

//volatile data handling
float Calculate_Fsc()
{
    float x[max], f[max];
            int i,j,n;
            //change this data if u want to change the lookup tables for fe415 and fe500
            n=7;
            float eg_Fe415_esc[7]={0.00000,0.00144,0.00163,0.00192,0.00241,0.00276,0.00380};
            float eg_Fe415_Fsc[7]={0.0,288.7,306.7,324.8,342.8,351.8,360.9};
    float eg_Fe500_esc[7]={0.00000,0.00174,0.00195,0.00226,0.00277,0.00312,0.00417};
    float eg_Fe500_Fsc[7]={0.0,347.8,369.6,391.3,413.0,423.9,434.9};
    //end of lookup tables
    //select grade enter 1 for fe415 and 2 for fe500
    if(grade==-1)
    {
        cout<<"\nWhich Grade do u need 1.Fe415 2.Fe500";
        grade=getch();
            }
            //cout<<"Enter the no of data: "; cin>>n;
    if(grade==49)
```

```
    {
        for(i=0; i<n; i++)
{
    x[i]=eg_Fe415_esc[i];
    cout<<"x["<<i<<"]:"<<x[i]<<setw(6); //cin>>x[i];
    f[i]=eg_Fe415_Fsc[i];
    cout<<"\tf["<<i<<"]:"<<f[i]; //cin>>f[i];
    cout<<endl;
}
    }
    else if(grade==50)
    {
        for(i=0; i<n; i++)
{
    x[i]=eg_Fe500_esc[i];
    cout<<"x["<<i<<"]:"<<x[i]; //cin>>x[i];
    f[i]=eg_Fe500_Fsc[i];
    cout<<"\tf["<<i<<"]:"<<f[i]; //cin>>f[i];
    cout<<endl;
}
    }

    float y,z;
    float fy=0,fz=0;
    y=Esc;
    cout<<"in"<<y<<"esc"<<Esc<<endl;
    for(i=0; i<n; i++)
    {
cout<<"a"<<x[i];
        if(y<x[i])
        break;
    }
    cout<<"hi"<<i<<endl;
    if(i!=n)
```

```
{cout<<"i am herte";
    fy=((f[i]-f[i-1])/(x[i]-x[i-1]))*(y-x[i-1])+f[i-1];//interpolation
Fsc=fy;
  }
  else
  {
    Fsc=f[n-1];
  }
  printf("\nFsc: %6f\n", Fsc);
  //calculation of FST(Es)
  z=EpsilonS;
  for(i=0; i<n; i++)
  {
    if(z<x[i])
    break;
  }
  //cout<<"hi"<<x[i]<<endl;
  if(i!=n)
  {
fz=((f[i]-f[i-1])/(x[i]-x[i-1]))*(z-x[i-1])+f[i-1];//interpolation
Fst=fz;
  }
  else
  {
    Fst=f[n-1];  //  CHECK THIS
  }
  printf("\nFst: %6f\n", Fst);
  printf("\nFsc: %6f\n", Fsc);
  return Fsc;
}
void Calculate_Asc()
{
  Asc= pi/4*Dc*Dc*Nc;
  //Asc=638;
```

```
}

void Calculate_Ast()
{
    Ast=pi/4*Dt*Dt*Nt;
  // Ast=638;
}

float Calculate_Cuc()
{
    //cout<<"\ncheck "<<b<<" "<<" "<<Xu<<" "<<Epsilon<<"gh"<<b*(Xu/Epsilon)<<endl;
    float Cuc=b*(Xu/Epsilon)*Calculate_sigma_integral();
    cout<<"\nCuc"<<Cuc;
    return Cuc;
}

float Calculate_Cus()
{
    cout<<"ASc"<<Asc;
    //float anir=Calculate_Fsc();
    //float maju=Calculate_sigma();
    float Cus=(Calculate_Fsc()-Calculate_sigma())*Asc;
    cout<<"\nCus"<<Cus;
    //cout<<"\nanir"<<anir;
    return Cus;
}

float Calculate_T()
{
        cout<<"\nAst"<<Ast;
    float T= (float)Fst*Ast;
     cout<<"\nt"<<T;
    return T;
```

```
}

void Estimate_Xu()
{
   if(Xu_Working_max==-1 && Xu_Working_min==-1)
   {
      Xu_Working_max=Xu_max;
      Xu_Working_min=Dprime;
   }
   Xu=(Xu_Working_max+Xu_Working_min)/2;
   cout<<"Xu Current Max"<<Xu_Working_max<<"Xu Current min"<<Xu_Working_min;
   cout<<"\n Xu Currently\t"<<Xu;
}

float Solve_Equation()
{
   float result=-1;int count=0;
   while(!(result>-1 && result<1))
   {
      Estimate_Xu();
      Calculate_Esc();
      Calculate_EpsilonS();
      Calculate_Fsc();

      result=Calculate_Cuc()+Calculate_Cus()-Calculate_T();//change here
      cout<<"\nError:\t"<<result<<endl;
      if(result>0)
      Xu_Working_max=Xu;
      else if(result<0)
      Xu_Working_min=Xu;
      //getch();
      count++;
      if(count==25)
      break;
```

```
    }
    //cout<<"Count"<<count;
    return Xu;
}
double Calculate_MuR()
{
    double Mur= Calculate_Cuc()*(D-Dprime-0.416*Xu)+Calculate_Cus()*(D-2*Dprime);
    cout<<"\nValue of Mur\t"<<Mur<<endl;
    return Mur;
}

double Calculate_Phi()
{
     double Phi=atan(Epsilon/Xu);
     cout<<"\nPhi\t"<<Phi;
     return Phi;
}
float epsilonArray[100],XuArray[100],MuRArray[100],PhiArray[100];
int main()
{
   double Mur,Phi;
   initialize();
        for(int p=1;p<=100;p++)
   {
        Epsilon=(0.0035/100)*p;
        cout<<"Epsilon"<<Epsilon<<endl;
        Calculate_Xu_max();
        Calculate_Asc();
        Calculate_Ast();
        Xu=Solve_Equation();
        cout<<"required Xu is::\t"<<Xu;
        Mur=Calculate_MuR ();
        Phi=Calculate_Phi();
        //making things back to initialization state
```

```
        epsilonArray[p-1]=Epsilon;
        XuArray[p-1]=Xu;
        MuRArray[p-1]=Mur;
        PhiArray[p-1]=Phi;
        Xu_Working_min=-1;
        Xu_Working_max=-1;
        Xu=0;
        //getch();
    }
    cout<<"Data Analysis section\n";
    for( int p=0;p<100;p++)
    {
cout<<epsilonArray[p]<<"\t"<<XuArray[p]<<"\t"<<MuRArray[p]<<"\t"<<PhiArray[p]<<endl;
    }
}
```