

# **INTERNAL MODEL CONTROL (IMC) AND IMC BASED PID CONTROLLER**

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF  
THE REQUIREMENTS FOR THE DEGREE OF

**Bachelor of Technology in**

**Electronics and Instrumentation Engineering  
Electronics and Communication Engineering**



By

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Under the Guidance of

**Prof. T K DAN**



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**CERTIFICATE**

This is to certify that the project report titled “INTERNAL MODEL CONTROL (IMC) AND IMC BASED PID CONTROLLER ” submitted by Garima Bansal (Roll No: 107EI011) and Abhipsa Panda (Roll No: 107EI022) and Sanyam Gupta(Roll No: 107EC018) in the partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Electronics and Instrumentation Engineering and Electronics and Communication Engineering during session 2007-2011 at National Institute of Technology, Rourkela (Deemed University) and is an authentic work carried out by them under my supervision and guidance.

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# ABSTRACT

**Internal Model Control (IMC)** is a commonly used technique that provides a transparent mode for the design and tuning of various types of control. In this report, we analyze various concepts of IMC design and IMC based PID controller has been designed for a plant transfer function to incorporate the advantages of PID controller in IMC.

The IMC-PID controller does good set-point tracking but poor disturbance response mainly for the process which have a small time-delay/time-constant ratio. But, for many process control applications, rejection of disturbance for the unstable processes is more important than set point tracking.

Thus, we assume an appropriate IMC filter to design an IMC-PID controller for better set-point tracking in unstable processes. The controller assumed works differently for different values of the filter tuning parameters to achieve the required response As the IMC approach is based on cancellation of pole zero, methods by which IMC is designed result in good set point responses. However, the IMC leads to a long settling time for the load disturbances in lag dominant processes which is not desirable in the control industry.

In our study we determined transfer function for the model of the actual process of a chemical reactor plant as we do not know the actual process which incorporates within it the effect of model uncertainties and disturbances entering into the process. As parameters of the physical system may vary with operating conditions and time and so it is essential to design a control system that shows robust performance in every situation. Then we tried to tune our IMC controller for different values of the filter tuning factor using SISO tool.

Since all the IMC-PID procedures include some kind of model factorization techniques to convert the IMC controller to the PID controller so approximation error usually occurs. This error becomes problematic for the processes which have time delay. For this we have taken some transfer functions with significant time delay or with non invertible parts i.e. containing RHP poles or the zeroes. Here we have used different techniques like factorization or elimination of RHP to get rid of these error containing stuffs. It is because if these errors are not removed then even if IMC filter gives best IMC performance but it is internally unstable leading to a false PID controller and poor performance.

Also, when a controller is designed based on an assumed model and implemented on the actual plant, its close loop performance may be arbitrarily poor depending on the extent of the mismatch between the model and the process. So we studied model uncertainty (model plant mismatch) more carefully and evaluated its impact on the expected performance of the control system. Apart from the objectives stability and performance in designing a control system we also concentrated on a third objective robustness which is the ability of a system to maintain its above properties in the presence of model uncertainty.

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# Chapter 1

## INTRODUCTION TO INTERNAL MODEL CONTROL (IMC)

## 1.1 IMC Background

In process control, model based control systems are mainly used to get the desired set points and reject small external disturbances. The internal model control (IMC) design is based on the fact that control system contains some representation of the process to be controlled then a perfect control can be achieved. So, if the control architecture has been developed based on the exact model of the process then perfect control is mathematically possible.

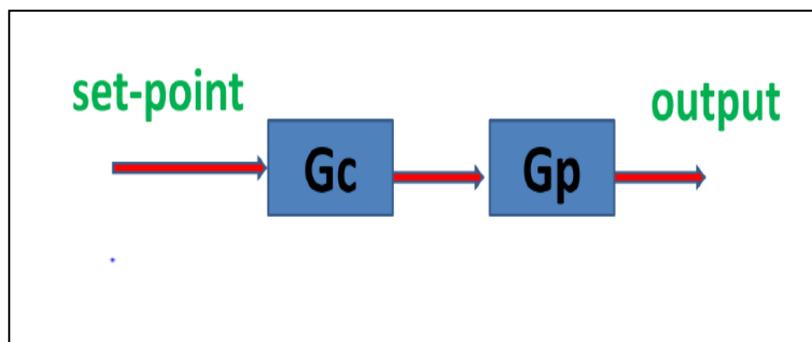


Fig 1. 1 Open loop control strategy

**Output =  $Q_c * G_p * \text{Set-point}$**

$Q_c$  = controller

$G_p$  = actual process

$G_p^*$  = process model

**$Q_c = \text{inverse of } G_p^*$**

If

**$G_p = G_p^*$**  (the model is the exactly same as the actual process)

Output is:

$$\begin{aligned}
 Y(s) &= Q_c * G_p * \text{Set-point} \\
 &= (1/ G_p^*) * G_p * \text{Set-point} \\
 &= \text{Setpoint}
 \end{aligned}$$

Hence, *for this condition the output will be equal to the set point*

Thus open loop configuration only, can give ideal performance if the process is exactly known before designing the controller and there is no need of feedback system.

IMC compensates for disturbances and model uncertainty while open loop control is not because the implementation of IMC leads to a feedback system. Also IMC must be detuned to assure stability if there is model uncertainty.

## 1.2 IMC basic structure

The exceptional characteristic of IMC structure is including the process model which is in parallel with the actual process or the plant. Here, '\*' has been used to represent signals associated with the model.

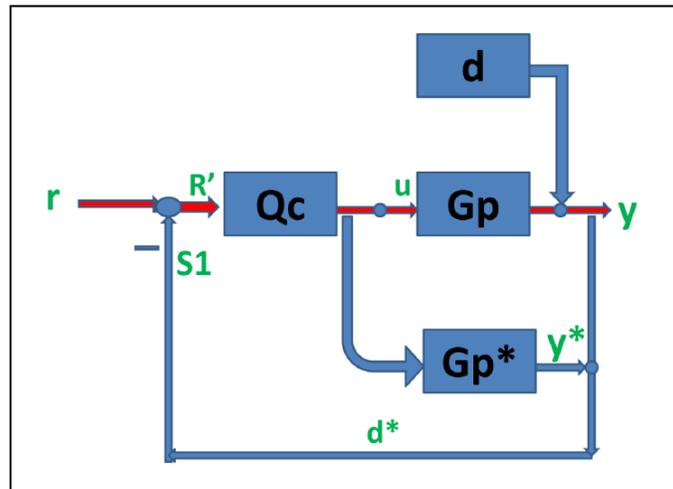


Fig 1. 2 IMC basic structure

## 1.3 IMC parameters

The various parameters used in the IMC basic structure shown above are as follows:

$Q_c$ = IMC

$G_p$ = actual process

$G_p^*$ = process model

$r$ = set point

$r'$ = modified set point

$u$ = manipulated variable (controller output)

$d$  = disturbance

$d^*$  = calculated new disturbance

$y$  = measured process output

$y^*$  = process model output

New calculated disturbance:

$$d^* = (G_p - G_{p^*})u + d$$

Modified set-point:

$$r' = r - d^* = r - (G_p - G_{p^*})u - d$$

## 1.4 IMC Strategy

Now we consider a different case

### **Perfect model without disturbance:**

A model is said to be perfect if

Process model is same as actual process

i.e.  $G_p = G_{p^*}$

no disturbance implies

$$d = 0$$

Thus relationship between the set point ( $r$ ) and the output ( $y$ ) is

$$y = G_p \cdot Q_c \cdot r$$

This relationship is same for as for open loop system. Thus if the controller  $Q_c$  and the process  $G_p$  are stable the closed loop system will be stable.

But in practical cases always the disturbances and the uncertainties do exist hence actual process or plant cannot be equal to the model of the process.

The error signal  $r'(s)$  comprises of the model mismatch and the disturbances which is send as modified set-point to the controller and is given by

$$r'(s) = r(s) - d^*(s)$$

And output of the controller is the manipulated variable  $u(s)$  which is send to both the process and its model.

$$u(s) = r'(s) * Q_c(s) = [r(s) - d^*(s)] Q_c(s)$$

$$= [r(s) - \{ [G_p(s) - G_p^*(s)].u(s) + d(s) \} ] \cdot Q_c(s)$$

$$u(s) = [ [r(s) - d(s)] * Q_c(s) ] / [ 1 + \{ G_p(s) - G_p^*(s) \} Q_c(s) ]$$

But

$$y(s) = G_p(s) * u(s) + d(s)$$

Hence, closed loop transfer function for IMC is

$$y(s) = \{ Q_c(s) \cdot G_p(s) \cdot r(s) + [1 - Q_c(s) \cdot G_p^*(s)] \cdot d(s) \} / \{ 1 + [G_p(s) - G_p^*(s)] Q_c(s) \}$$

Also to improve the robustness of the system the effect of model mismatch should be minimized. Since mismatch between the actual process and the model usually occurs at higher frequencies of the systems frequency response, a low pass filter  $f(s)$  is added to prevent the effects of mismatch.

**Thus the internal model controller is designed as inverse of the process model which is in series with the low pass filter i.e**

$$Q(s) = Q_c(s) * f(s)$$

The order of the filter is chosen to make it proper or at least semi proper (such that order of numerator is equal to the order of denominator) . The resulting closed loop then becomes

$$y(s) = \{ Q(s) \cdot G_p(s) \cdot r(s) + [1 - Q(s) \cdot G_p^*(s)] \cdot d(s) \} / \{ 1 + [G_p(s) - G_p^*(s)] Q(s) \}$$

# Chapter 2

## IMC DESIGN PROCEDURE

### 2.1 Introduction

The IMC design procedure is exactly the same as the open loop control design procedure. Unlike open loop control, the IMC structure compensates for disturbances and model uncertainties. The IMC filter tuning parameter “lem” is used to avoid the effect of model uncertainty. The normal IMC design procedure focuses on set point responses but with good set point responses good disturbance rejection is not assured, especially those occurring at the process inputs. A modification in the design procedure is proposed to enhance input disturbance rejection and to make the controller internally stable.

### 2.2 IMC design procedure

Consider a process model  $G_p^*(s)$  for an actual process or plant  $G_p(s)$ . The controller  $Q_c(s)$  is used to control the process in which the disturbances  $d(s)$  enter into the system. The various steps in the Internal Model Control (IMC) system design procedure are:

#### 2.2.1 FACTORIZATION

It includes factorizing the transfer function into invertible and non invertible parts. The factor containing right hand poles, zeros or time delays become the poles when the process model is inverted leading to internal stability. So this is non invertible part which has to be removed from the transfer function. Mathematically, it is given as

$$G_p^*(s) = G_{p+}^*(s) G_{p-}^*(s)$$

Where

$G_{p+}^*(s)$  is non-invertible part

$G_{p-}^*(s)$  is invertible part

There are two methods of factorization:

- (i) Simple
- (ii) All pass

Usually we use all pass factorization where the unstable RHP is compensated by a mirror image of it on the left hand side.

## 2.2.2 IDEAL IMC CONTROLLER

The ideal IMC is the inverse of the invertible part of the process model. It is given as

$$Qc^*(s) = \text{inv} [Gp.^*(s)]$$

## 2.2.3 ADDING FILTER

Now a filter is added to make the controller at least **semi-proper** because a transfer function is not stable if it is improper.

A transfer function is known as **proper** if the order of the denominator is greater than the order of the numerator and for exactly of the same order the transfer function is known as **semi-proper**.

So to make the controller proper mathematically it is given as

$$Q(s) = Qc^*(s) f(s) = \text{inv} [ Gp.^*(s)] f(s)$$

## 2.2.4 LOW PASS FILTER f(s)

We know to reduce the uncertainty at higher frequencies a filter is added and the resulting controller is given as:

$$Q(s) = Qc^*(s) .f(s) = \{\text{inv} [ Gp.^*(s)]\} f(s)$$

**Where**

$$f(s) = 1 / (lem * s + 1) ^ n$$

Where *lem* is the filter tuning parameter which varies the speed of the response of the closed loop system. When *lem* is smaller than the time constant of the first order process the response is faster.

The low pass filter is of three types:

a) For input as set point change, the filter used is

$$f(s) = 1 / (lem * s + 1) ^ n$$

here **n** is the order of the process.

b) For tracking ramp set point changes the filter used is

$$f(s) = (n. lem. s + 1) / (lem * s + 1) ^ n$$

c) For good rejection of step input load disturbances the filter used is

$$f = (\gamma \cdot s + 1) / (\lambda \cdot s + 1)^n$$

where  $\gamma$  is a constant.

### 2.3 IMC design for 1st order system

Now applying the above IMC design procedure for a first order system:

- **Given process model for 1st order system :  $G_p^*(s) = K_p^* / [T_p^*(s) + 1]$**

$$K_p = 1 \text{ and } T_p = 10$$

- $G_p^*(s) = G_{p+}^*(s) \cdot G_{p-}^*(s) = 1 \cdot (K_p^* / [T_p^*(s) + 1])$
- $Q_c^*(s) = \text{inv}[G_{p-}^*(s)] = [T_p^*(s) + 1] / K_p^*$
- $Q(s) = Q_c^*(s) \cdot f(s) = [T_p^*(s) + 1] / [K_p^* \cdot (\lambda \cdot s + 1)]$
- $f(s) = 1 / (\lambda \cdot s + 1)$
- $y(s) = Q(s) \cdot G_p(s) \cdot r(s) = G_{p+}^*(s) \cdot f(s) \cdot r(s)$
- **Output variable:**

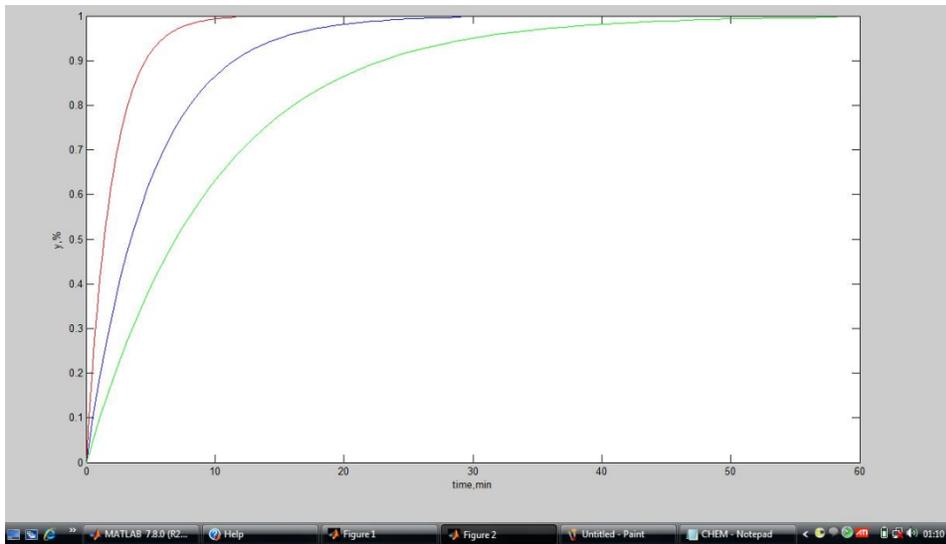
$$y(s) = r(s) / (\lambda \cdot s + 1)$$

- **Manipulated variable:**

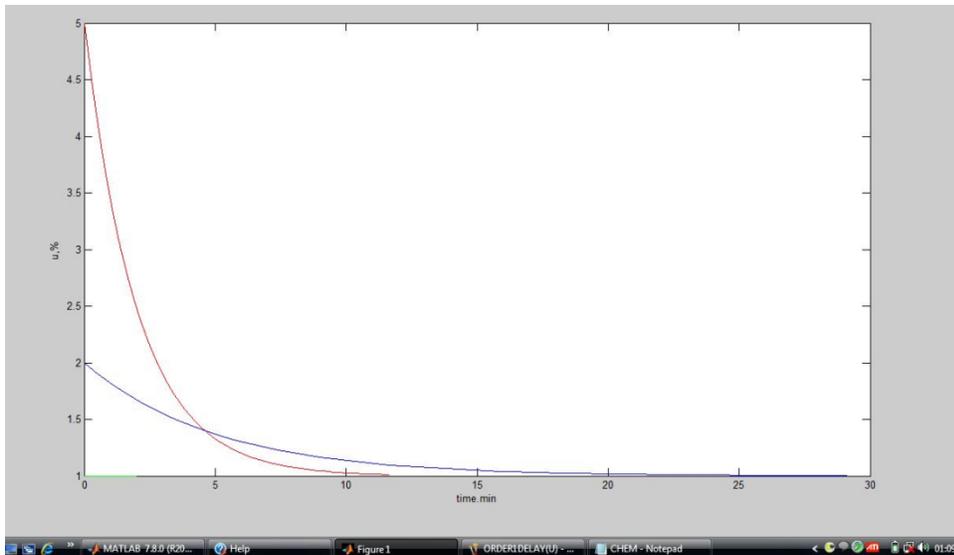
$$u(s) = Q(s) \cdot r(s) = [(T_p^*(s) + 1) \cdot r(s)] / [K_p \cdot (\lambda \cdot s + 1)]$$

### 2.3.1 Simulation plot for IMC 1st order system

#### a) Output variable response



#### b) Manipulated variable response



## 2.4 IMC design for 2nd order system

- **Given process model for 2nd order system:**  $G_p^*(s) = \frac{-9s + 1}{(15s + 1)(3s + 1)}$
- $G_p^*(s) = G_{p+}^*(s) \cdot G_{p-}^*(s) = \frac{(-9s + 1)}{(9s + 1)} * \frac{[9s + 1]}{(15s + 1)(3s + 1)}$
- $Q_c^*(s) = \text{inv}[G_{p-}^*(s)] = \frac{(15s + 1)(3s + 1)}{(9s + 1)}$
- $Q(s) = Q_c^*(s) * f(s) = \left[ \frac{(15s + 1)(3s + 1)}{(9s + 1)} \right] * \left[ \frac{1}{(\text{lem} \cdot s + 1)} \right]$
- $f(s) = \frac{1}{(\text{lem} \cdot s + 1)}$
- $y(s) = Q(s) \cdot G_p(s) \cdot r(s) = G_p^*(s) \cdot f(s) \cdot r(s)$
- **Output variable:**  

$$y(s) = \left\{ \frac{-9s + 1}{(15s + 1)(3s + 1)} \right\} * r(s)$$

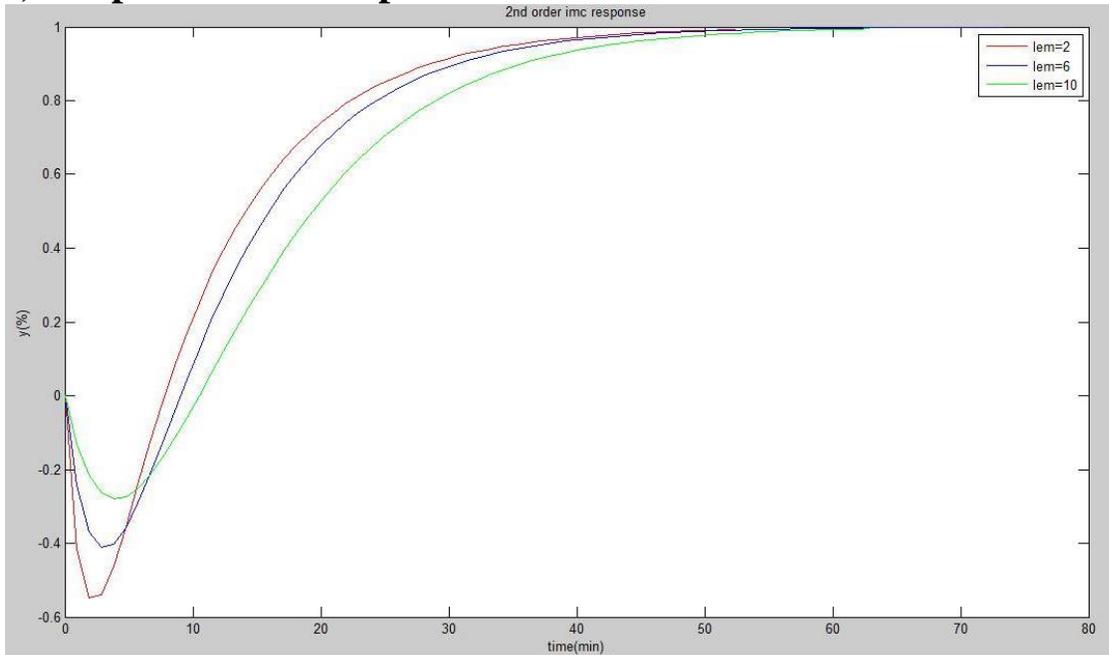
$$= \frac{-9s + 1}{9 \text{lem} s^2 + (9 + \text{lem}) s + 1}$$
- **Manipulated variable:**  

$$u(s) = Q(s) * r(s) = \left\{ \left[ \frac{(15s + 1)(3s + 1)}{(9s + 1)(\text{lem} \cdot s + 1)} \right] \right\} * r(s)$$

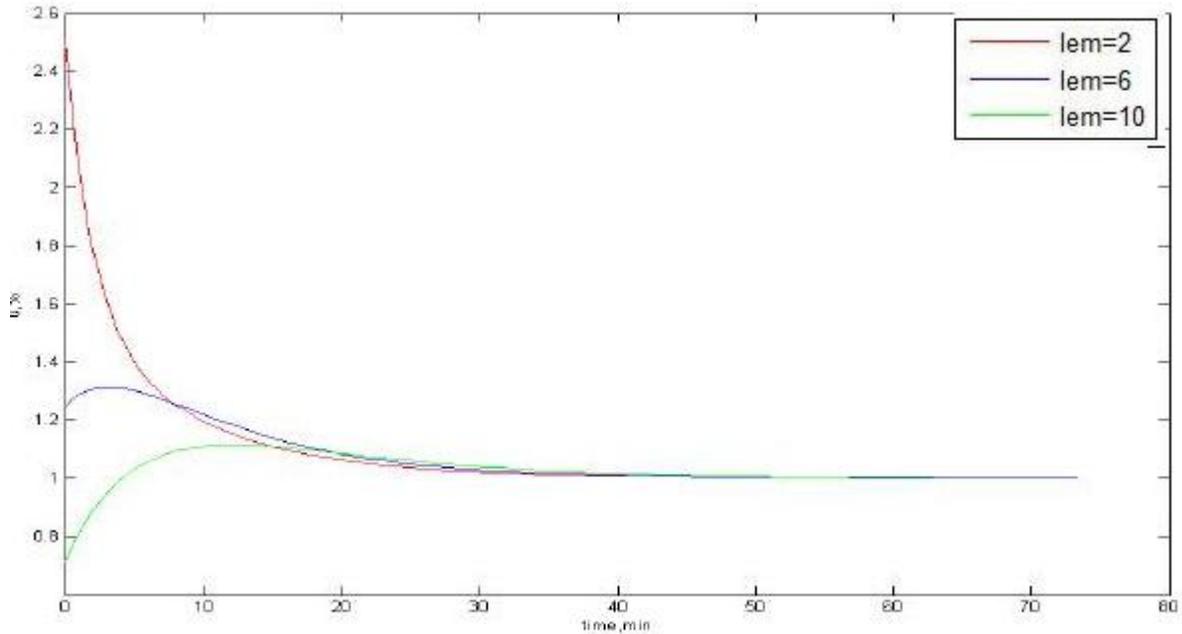
$$= \left[ \frac{(45 s^2 + 18 s + 1)}{(9 \text{lem} s^2 + (9 + \text{lem}) s + 1)} \right] * r(s)$$

## 2.4.1 Simulation plot for IMC 2st order system

### a) Output variable response



### b) Manipulated variable response



# Chapter 3

## IMC BASED PID

### 3.1 Introduction

The IMC structure is rearranged to get a standard feedback control system so that open loop unstable system can be handled. This is done to improve the input disturbance rejection. The IMC based PID structure uses the process model as in IMC design. In the IMC procedure the controller  $Q_c(s)$  is directly based on the invertible part of the process transfer function. The IMC results in only one tuning parameter which is filter tuning factor but the IMC based PID tuning parameters are the functions of this tuning factor. The selection of the filter parameter is directly related to the robustness. IMC based PID procedures uses an approximation for the dead time. And if the process has no time delays it gives the same performance as does the IMC.

### 3.2 IMC based PID structure

In ideal IMC structure the point of summation of the process and the model output is moved as shown in the figure to form a standard feedback controller which is known as IMC based PID controller.

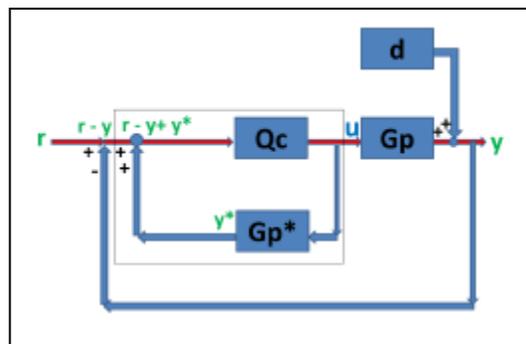


Fig 3.1: IMC based PID Design

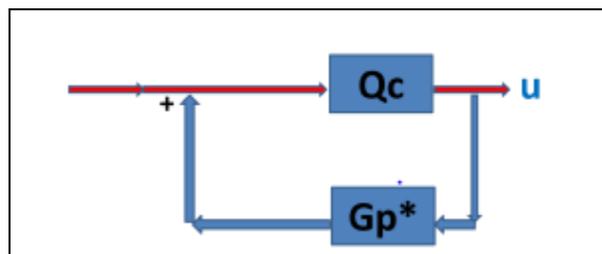


Fig 3.2: Inner loop of rearranged IMC structure

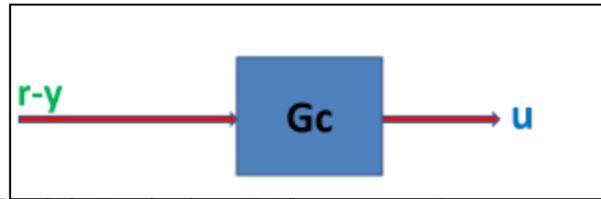


Fig 3.3: Equivalent IMC rearranged structure

### 3.3 IMC based PID design procedure

Consider a process model  $G_p^*(s)$  for an actual process or plant  $G_p(s)$ . The controller  $Q(s)$  is used to control the process in which the disturbances  $d(s)$  enter into the system. The IMC is designed as discussed in chapter two and then IMC based PID controller is designed

#### 3.3.1 Equivalent standard feedback controller

By rearranging the IMC we obtain equivalent standard feedback controller using :

$$G_c = Q_c / (1 - Q_c G_p^*)$$

Thus, output  $y(s)$  is the output of series of  $G_c(s)$  and  $G_p(s)$  and the unity feedback system.

The manipulated variable now is;

$$U(s) = [r(s) \cdot G_c(s)] / [1 + G_c(s) \cdot G_p(s)]$$

Output is;

$$Y(s) = [r(s) \cdot G_c(s) \cdot G_p(s)] / [1 + G_c(s) \cdot G_p(s)]$$

#### 3.3.2 Comparison with standard PID controller

Now we compare with PID Controller transfer function

For first order :

$$G_c(s) = [K_c \cdot (T_i \cdot s + 1)] / (T_i \cdot s)$$

And we find that  $K_c$  and  $T_i$  (PI tuning parameters)

$$K_c = T_p / (l_e m \cdot K_p)$$

$$T_i = T_p$$

Similarly for 2nd order we compare with the standard PID controller transfer function given by :

$$G_c(s) = K_c \cdot [(T_i \cdot T_d \cdot s^2 + T_i \cdot s + 1) / T_i \cdot s] \cdot [1 / T_f \cdot s + 1]$$

Where

T = Tau (constant)

T<sub>i</sub> = integral time constant

T<sub>d</sub> = derivative time constant

T<sub>f</sub> = filter tuning factor

K<sub>c</sub> = controller gain

Now we perform closed loop simulations for above procedure and adjust lem (lemda) considering a trade-off between performance and robustness (sensitivity to model error).

### 3.4 IMC based PID for 1st order system

Now we apply the above IMC based PID design procedure for a first order system with a given process model.

- **Given process model :  $G_p^*(s) = K_p^* / [T_p^*(s) + 1]$**
- $G_p^*(s) = G_{p+}^*(s) \cdot G_{p-}^*(s) = 1 \cdot K_p^* / [T_p^*(s) + 1]$
- $Q_c^*(s) = \text{inv}[G_{p-}^*(s)] = [T_p^*(s) + 1] / K_p^*$
- $Q_c(s) = Q_c^*(s) \cdot f(s) = [T_p^*(s) + 1] / [K_p^* \cdot (\text{lem}(s) + 1)]$
- $f(s) = 1 / (\text{lem} \cdot s + 1)$

- Equivalent feedback controller using transformation

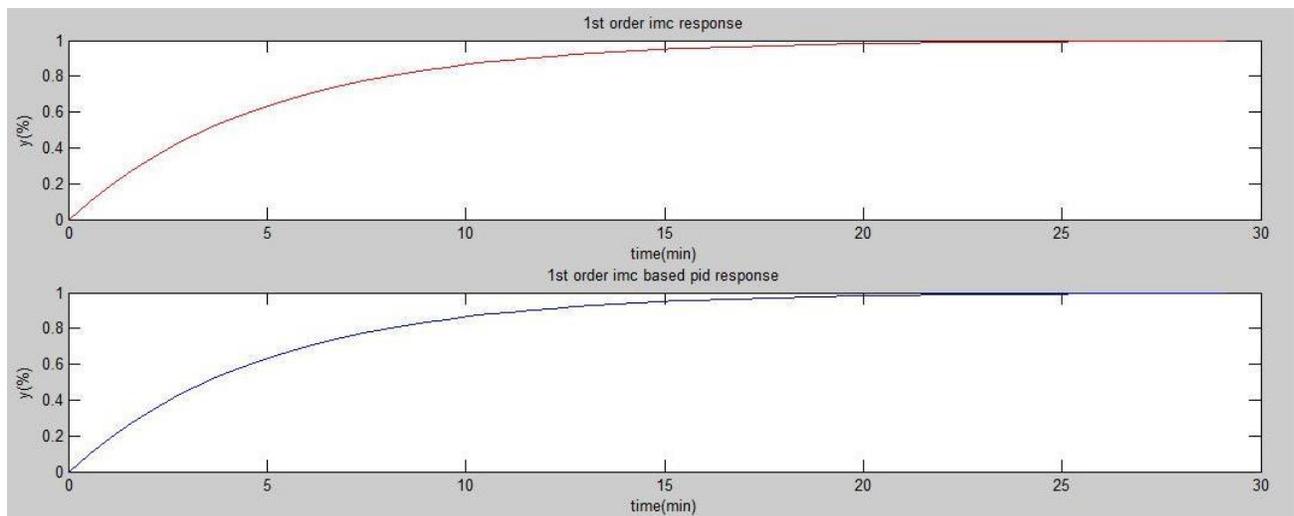
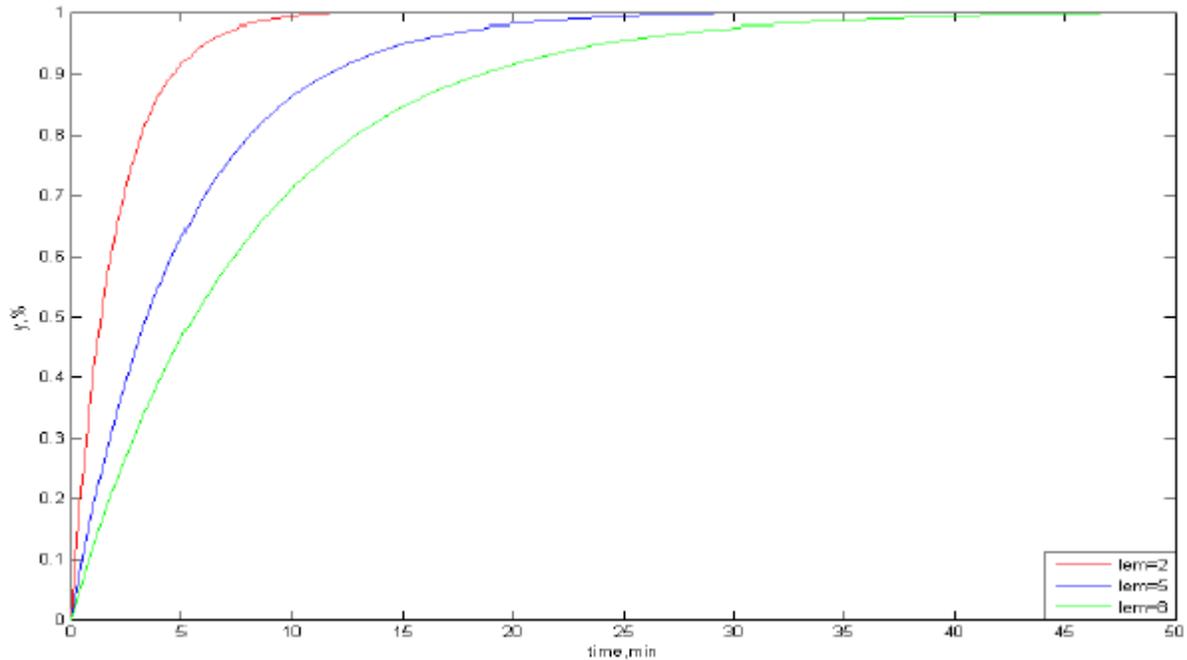
$$G_c(s) = Q_c(s) / (1 - Q_c(s) G_p^*(s)) = \frac{[T_p^*(s) + 1] / \{K_p^* \cdot (\text{lem}(s) + 1)\}}{[1 - K_p^* / (T_p^*(s) + 1)] \cdot \{T_p^*(s) + 1\} / \{K_p^* \cdot (\text{lem}(s) + 1)\}}$$

- $G_c(s) = \{T_p(s) + 1\} / K_p \cdot \text{lem} \cdot s$  (it is standard feedback controller for IMC)
- $G_c(s) = [K_c \cdot (T_i \cdot s + 1)] / (T_i \cdot s)$  (transfer function for PI controller)
- PI tuning parameters

$$K_c = T_p / (K_p \cdot \text{Lem})$$

$T_i = T_p$

### 3.4.1 Simulation for IMC based PID 1st order system and comparison with IMC



### Comparison of IMC and IMC based PID response

### 3.5 IMC based PID for 2nd order system

Now we apply the above IMC based PID design procedure for a second order system with a given process model.

- **Given process model :  $G_p^*(s) = K_p^*/[(T_{p1}^*(s)+1).(T_{p2}^*(s)+1)]$**
- $G_p^*(s) = G_{p+}^*(s) \cdot G_{p-}^*(s) = 1 \cdot K_p^*/[T_p^*(s)+1]$
- $Q_c^*(s) = \text{inv}[G_{p-}^*(s)] = [T_p^*(s)+1] / K_p^*$
- $Q_c(s) = Q_c^*(s) \cdot f(s) = [T_p^*(s)+1] / [K_p^* \cdot (\text{lem}(s) + 1)]$
- $f(s) = 1 / (\text{lem} \cdot s + 1)$
- Equivalent feedback controller using transformation

$$G_c(s) = Q_c(s) / (1 - Q_c(s) G_p^*(s))$$

$$= [(T_{p1} \cdot T_{p2} \cdot s^2) + (T_{p1} + T_{p2})s + 1] / [K_p \cdot \text{lem} \cdot s]$$

(it is the transfer function for the equivalent standard feedback controller )

- $G_c(s) = [K_c \cdot \{ (T_i \cdot T_d \cdot s^2 + T_i \cdot s + 1) \}] / [T_i \cdot s]$  (transfer function for ideal PID controller for second order)
- PID tuning parameters (on comparison)

$$K_c = (T_{p1} + T_{p2}) / (K_p \cdot \text{lem})$$

$$T_i = T_{p1} + T_{p2}$$

$$T_d = T_{p1}$$

# Chapter 4

## DESIGN A PLANT FUNCTION AND ANALYSIS IN SISO TOOL

### 4.1 Internal Model Control Design for a Chemical Reactor Plant

In process control industry, model-based control strategy is used to track set point and reject load disturbances. We have illustrated how to design an IMC controller for series chemical reactors, using the controller tuning available in SISO Design Tool available in Matlab.

#### 4.1.1 PLANT DESCRIPTION

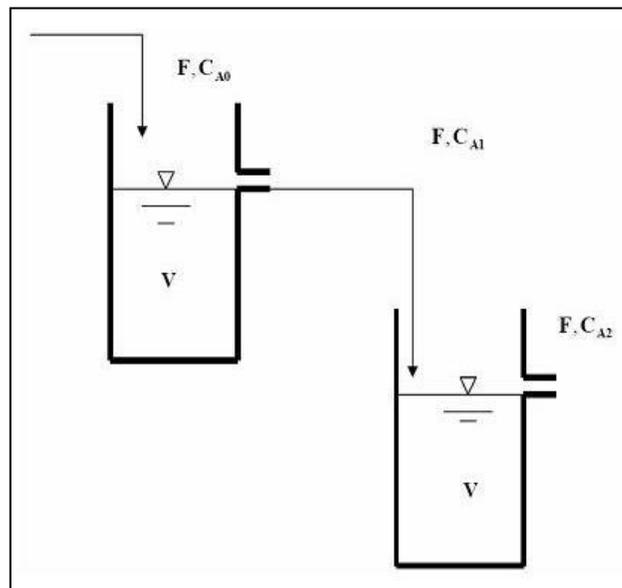


Fig 4.1: Chemical reactor system

The chemical reactor system shown in the above diagram comprises two well mixed tanks. Both the reactors are isothermal and the reactions are first order on component A:

$$r_A = -kC_A$$

Component balance is applied to both the tanks to generate the dynamic mathematical model for the system. The tank levels remain constant because the nozzle is at the same point for both tanks.

#### 4.1.2 EQUATIONS

We have the following differential equations to describe component balances:

$$V^* \{dC_{A1}/dt\} = F(C_{A0} - C_{A1}) - V_k C_{A1}$$

$$V^* \{dC_{A2}/dt\} = F(C_{A1} - C_{A2}) - V_k C_{A2}$$

At steady state, from

$$dC_{A1}/dt = 0$$

$$dC_{A2}/dt = 0$$

We have the following material balances:

$$F^*(C_{A0}^* - C_{A1}^*) - V_k C_{A1}^* = 0$$

$$F^*(C_{A1}^* - C_{A2}^*) - V_k C_{A2}^* = 0$$

where variables with \* denote steady state values.

By substituting the following design specifications and reactor parameters,

$$F^* = 0.083 \text{ mole/min}$$

$$C_{A0}^* = 0.920 \text{ mol/min}$$

$$V = 1.01 \text{ m}^3$$

$$k = 0.05 \text{ min}^{-1}$$

We obtain the steady state values of the concentrations in two reactors:

$$C_{A1}^* = K C_{A0}^* = 0.6190 \text{ mol/m}^3$$

$$C_{A2}^* = K^2 C_{A1}^* = 0.4142 \text{ mol/m}^3$$

where

$$K = F^*/\{F^*+Vk\} = 0.6688$$

### 4.1.3 CONTROL OBJECTIVE

The concentration of reactant coming out from the second tank  $C_{A2}$  is maintained by the molar flow rate of the reactant  $F$  that enters the first tank in the presence of feed concentration  $C_{A0}$  which acts as a disturbance.

In this control problem, the plant model is

$$G_p^* = C_{A2}/F(s)$$

and the disturbance function is

$$G_d = C_{A0}/C_{A2}$$

### 4.1.4 Linear Plant Models

The above chemical process is represented by the following diagram :

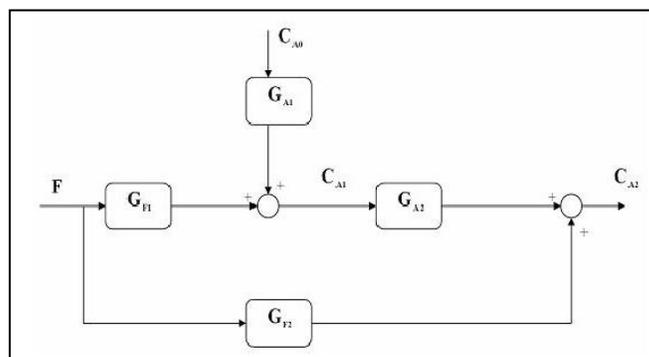


Fig 4.2: LTI Block Diagram

Considering the above block diagram, the plant and disturbance transfer functions are:

$$C_{A2}/F(s) = \{13.3259s+3.2239\}/\{(8.2677s+1)^2\}$$

$$C_{A0}/C_{A2} = G_{\{A1\}}G_{\{A2\}} = \{0.4480\}/\{(8.2677s+1)^2\}$$

## 4.2 Brief introduction to SISO tool

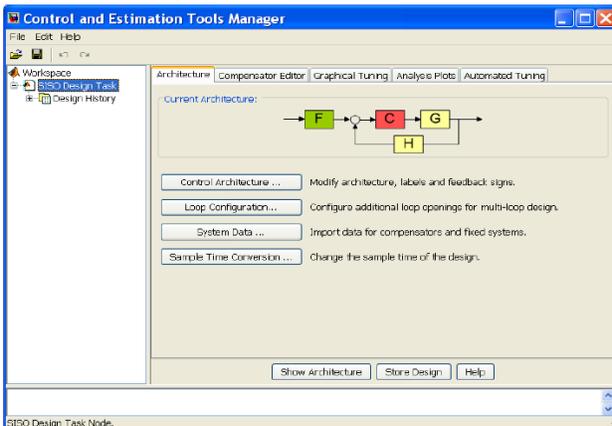
SISO TOOL is a Graphical User Interface (GUI) used to design single-input/single-output (SISO) compensators to analyze the system response.

When changes are introduced in the compensator, the LTI Viewer associated with our SISO Design Tool automatically updates the response plots chosen.

## 4.3 Using SISO TOOL for IMC implementation

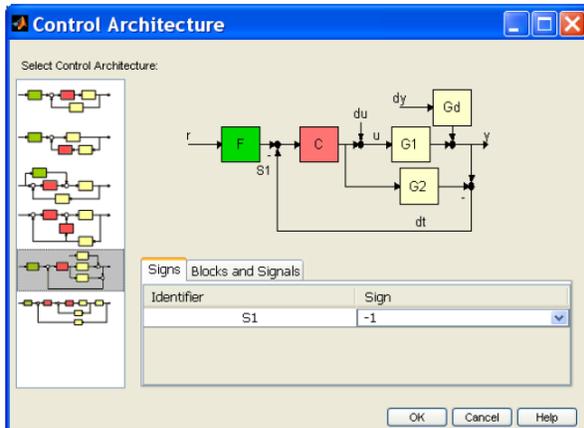
To design the compensator in an IMC structure in SISO Design Tool:

- (i) Open SISO Design Tool
- (ii) At the MATLAB® edit window, type SISOTOOL after which the Controls and Estimation Tools Manager opens.



### 4.3.1 Control architecture

- Click on the “Control Architecture” button
- Select Configuration available for IMC structure from Control Architecture dialog box.



### 4.3.2 Loading system data

First we create the following LTI models in MATLAB command prompt:

**G1** is the actual plant

**G2** is a plant model in the IMC structure.

**G1 = G2** implying **no model mismatch**.

**Gd** is the **disturbance function**.

Now the system data is loaded into the Controls and Estimation Tools Manager by clicking on the System Data button.

### 4.3.3 Automated tuning

For tuning the IMC compensator, Automated Tuning on the Controls and Estimation Tools Manager is selected with the Internal Model Control (IMC) Tuning as the design method.

Here controller of first order is taken and the time constant and gain of the process are varied and the system response to step input change and disturbance rejection are analyzed.

### 4.3.4 Analysis plots

For the process plant designed above for chemical reactor plant:

$$G1 = C_{A2} / F(s)$$

$$Gd = C_{A0} / C_{A2}$$

$$G1 = (13.3259*s + 3.2239) / (8.2677*s + 1)^2;$$

$$G2 = G1;$$

$$Gd = 0.4480 / (8.2677*s + 1)^2;$$

$$C = ((8.2677*s + 1)^2) / ((13.3259*s + 3.2239) * (5*s + 1));$$

To see the effect of model mismatch, the analysis of step response and disturbance rejection is done for different time constant and gain in the process.

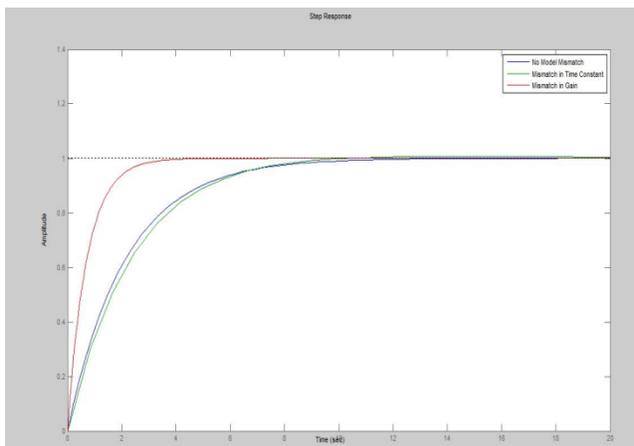


Figure 4.3

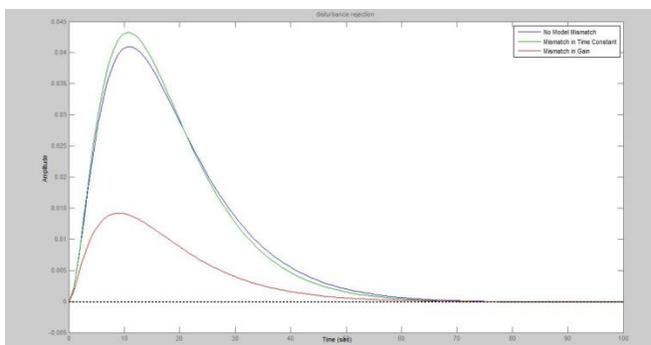


Figure 4.4

# Chapter 5

## **ROBUSTNESS: ROBUST STABILITY AND ROBUST PERFORMANCE**

## 5.1 Introduction

When a controller is designed based on an assumed model and implemented on the actual plant, its closed-loop performance may be arbitrarily poor depending on the extent of mismatch between the model and the plant. Hence, a study of model uncertainty and the quantification of its impact on the expected performance of the control system are necessary. The sources of model uncertainty can be varied:

Nonlinear effects when a linear model is used

High-order dynamics when model neglects such phenomena

Slow-varying parameters such as heat transfer coefficients(during fouling) and kinetic parameters (during catalyst decay)

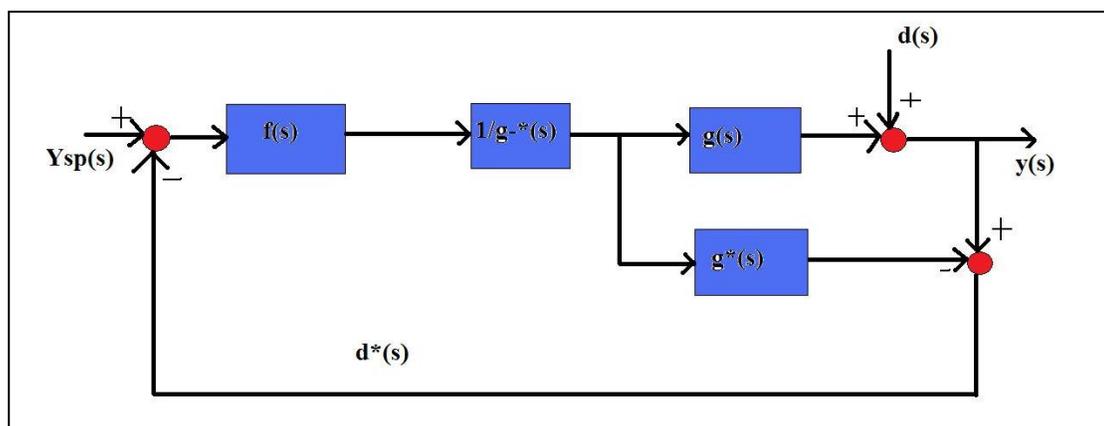
Unknown Phenomena(unmeasured disturbances)

To tackle these problems we introduce a new objective, robustness, in addition to stability and performance, to ensure that the closed loop control system requirements are fulfilled for all possible models that may represent the actual plant dynamics.

A closed loop system is robust with respect to a property (such as stability and performance), if it maintains that property in the presence of model uncertainty.

### 5.1.1 IMC Structure with Model Uncertainty

The dynamic behavior of a plant can never be described perfectly, but may be assumed to lie in some neighborhood of a nominal (reference) model. So we need to design a controller that ensures the performance and stability for all possible plant realizations in this neighborhood.



**Fig 5.1: IMC Structure with Model Uncertainty**

From the above figthe following relationships can be observed

$$d^*= d+(G_p - G_{p^*}) u$$

$$e=y- d^*=y- d-(G_p - G_{p^*}) u$$

Thus the presence of uncertainty leads to the feedback of the manipulated variable  $u$  which is the source of instability. This gives rise to a situation where the IMC filter  $f(s)$  needs to be designed for a particular input as well as for expected model uncertainty.

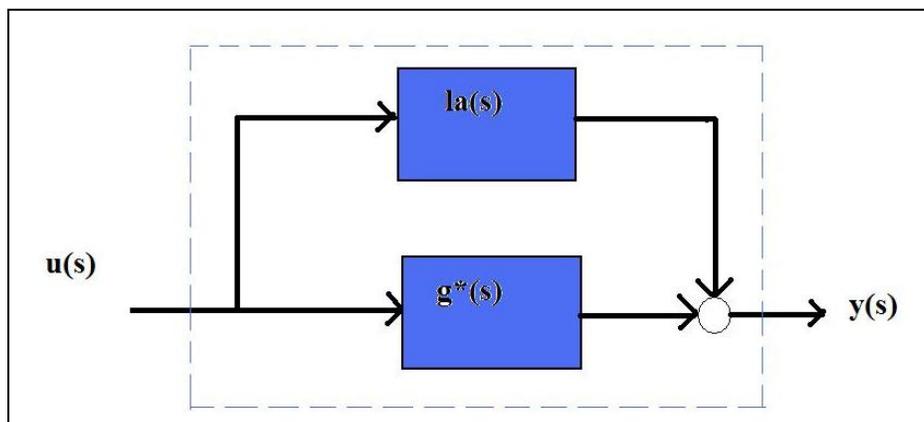
## 5.2 Description of Model Uncertainty

Quantification of the model uncertainty is required to incorporate this information in the control system design. The uncertainty description relies on the definition of bounds on the available nominal model information. Model uncertainty can be described as:

- Bounds on the parameters of a linear nominal model
- Bounds on the nonlinearities
- Bounds ion the frequency response of a nominal model

### 5.2.1 Additive Uncertainty

This form is appropriate for uncertainty associated with the process directly.



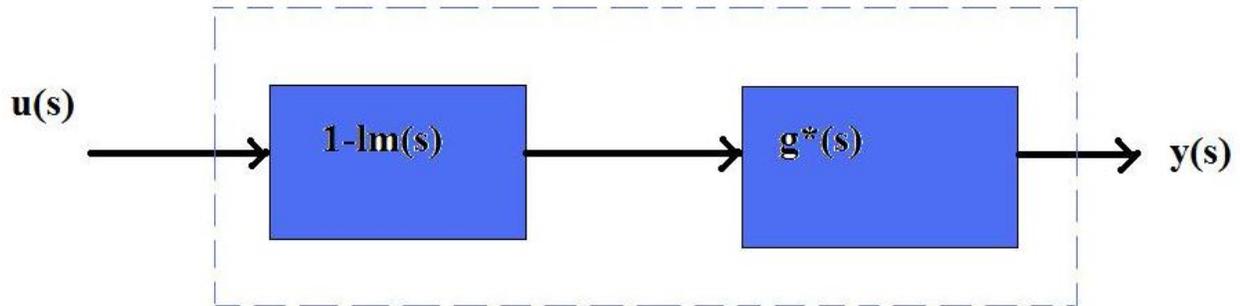
**Fig 5.2: Additive Uncertainty**

The actual process dynamics will be expressed in the form

$$g=g^*+l_a$$

where  $l_a$  is an unknown transfer function representing the uncertainty

## 5.2.2. Multiplicative Uncertainty



**Fig 5.3: Multiplicative Uncertainty**

This form is used to describe uncertainties associated with the sensors or dynamics that affect the output response.

Here the actual process dynamics will be expressed as

$$g=g^*(1+lm)$$

$lm$  is an unknown transfer function. Multiplicative Uncertainty is the form most often used due to its ability to capture a wide range of uncertain dynamics.

## 5.2.3 Estimation of Uncertainty Bounds

Sometimes the only information available about the above two uncertainty perturbations is some bound on their magnitudes in the frequency domain. The multiplicative uncertainty can be expressed in terms of the process model and a nominal process model as

$$lm=(g-g^*)/g^*$$

An upper bound on its magnitude can be proposed as

$$|lm| = |(g-g^*)/g^*| < lm(w)$$

$lm(w)$  is assumed to be a known function of frequency. Such a bound can be expressed graphically using either a Bode or Nyquist plot.

### 5.3 IMC Design under Model Uncertainty

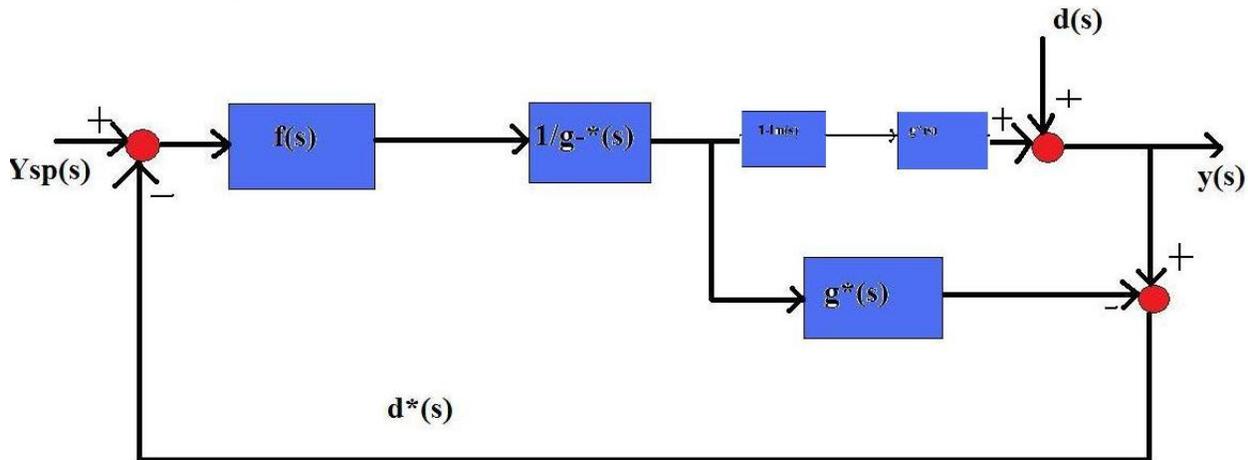


Fig 5.4: IMC Design under Model Uncertainty

The closed loop relationships for this structure can be expressed as

$$y = ((c^*g^*)/1 + (g-g^*))y_{sp} + ((1-cg^*)/(1-c(g-g^*)))d$$

$$y = ((c^*g^*)/1 + (g-g^*))(y_{sp}-d) + d$$

Substituting for controller as well as for uncertainty model, we obtain

$$y = \frac{g_+^* f(1+lm)}{1+g_+^* f lm} (y_{sp} - d) + d$$

Thus it can be seen that uncertainty directly affects the stability of the closed loop system as well as the closed loop performance to set=point changes and disturbances.

#### 5.3.1 Robust Stability

The following condition should be satisfied by the characteristic equation of the closed loop system for maintaining stability:

$$\det[1+g_+^* f lm] \neq 0$$

However since the uncertainty is not known, except for the bound on its magnitude, this condition is not practical. Hence a more practical but conservative condition is considered:

$$[1+g_+^* f lm] < 0$$

For all frequencies  $s=j\omega$ . The above equation is the Nyquist stability criterion rephrased for the case of model uncertainty. It can be rearranged to give

$$[g_+^* f lm] < 1$$

On further rearrangement and noting that  $|g_+^*|=1$  and  $|\text{Im}|<\text{Im}(w)$ , we get the condition on the IMC filter:

$$|f^*\text{Im}|<1 \rightarrow f|<1/\text{Im}(w)$$

If this conditions is satisfied , it guarantees the robust stability of closed loop performance.

This signifies that the frequency region where the uncertainty is large ( at high frequencies), the filter magnitude needs to be reduced leading to increase of the *lem* the tuning parameter and limiting the bandwidth of the closed-loop. Thus, the model uncertainty limits the speed of the response so that the stability can be maintained.

### 5.3.2 Robust Performance

First of all, the objective of the performance and the expectations for the nominal model is stated and then it is checked to maintain that for the actual plant. We define the sensitivity function for a closed loop performance as a parameter. For IMC:

$$\begin{aligned} Y(s) &= c(s)g^*(s)y_{sp}(s) + (1 - c(s)g^*(s))d(s) \\ &= \eta(s)y_{sp}(s) + \acute{e}(s)d(s) \end{aligned}$$

$\eta(s)$  is the complementary sensitivity function and  $\eta(s) + \acute{e}(s) = 1$  by definition. Good performance has two main criteria:

- (i) Supression of disturbance indicated by  $|\acute{e}| \rightarrow 0$
- (ii) Set point tracking indicated by  $|\eta| \rightarrow 1$

The above conditions follow for all frequencies.

Normally the complementary sensitivity amplitude approaches unity at low frequencies.

Thus, this is a key trade-off to robust performance.

### 5.4 Analysis of Im bound, Robust stability and performance

For the process:

$$g(s) = \frac{e^{-2.1s}}{(5s+1)(0.5s+1)(s+1)}$$

The estimated nominal model be:

$$g^*(s) = \frac{e^{-2s}}{(4s+1)}$$

## 5.4.1 Mathematical modeling

Now, the multiplicative error for the above mismatch or uncertainty is determined:

$$\overline{lm}(w) = \left| \frac{g(s) - g^*(s)}{g^*(s)} \right|$$

To find the IMC controller for the process  $g^*(s)$  is factorised:

$$\begin{aligned} g^*(s) &= g_-(s) \cdot g_+(s) \\ &= [1/(4s+1)] \cdot e^{-2s} \end{aligned}$$

So the IMC controller becomes;

$$Q(s) = (4s+1)/(l\epsilon m s + 1)$$

We also know that for robust performance:

$$|\eta| < 1$$

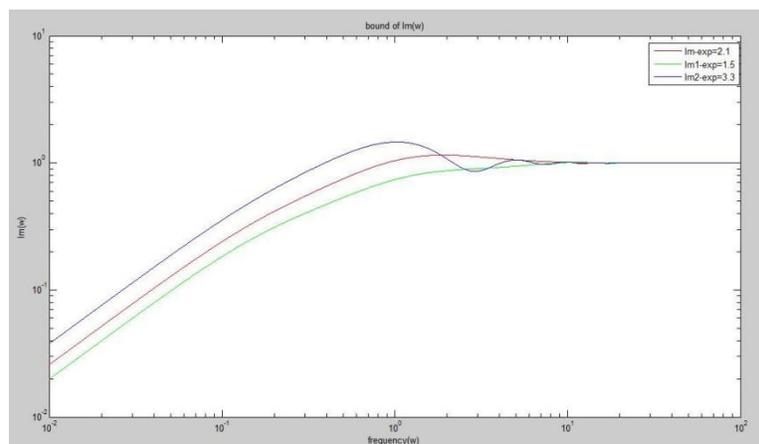
$$\eta = Qg^*$$

Robust stability is checked by:

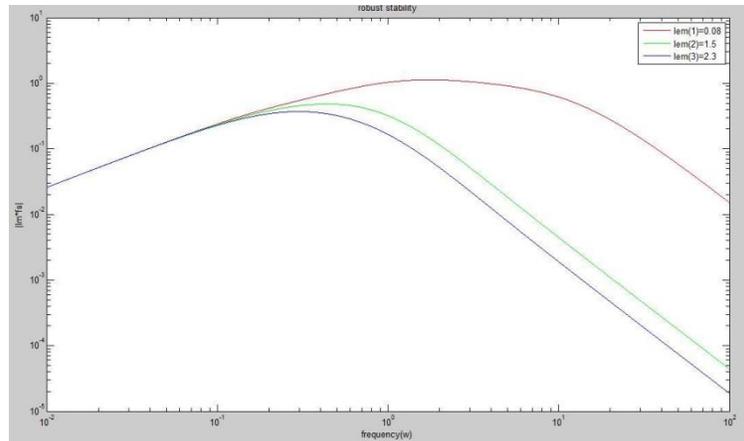
$$|f(s)| < 1/\overline{lm}(w)$$

## 5.4.2 Matlab simulation to analyze above conditions:

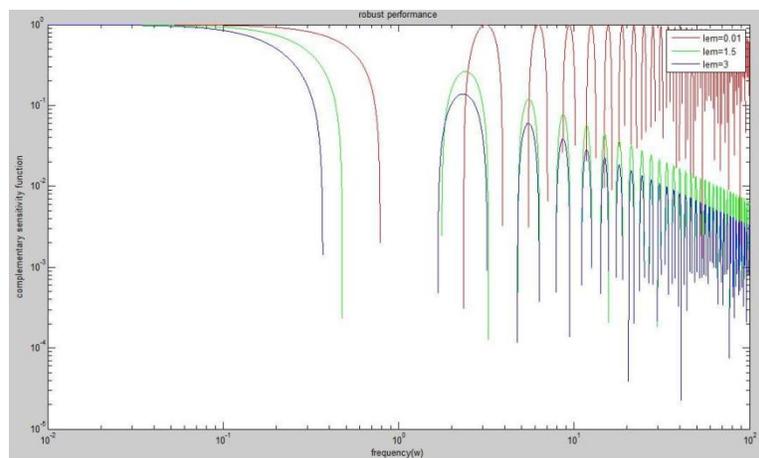
For various values of  $l\epsilon m$  bound, robust stability and performance is checked.



**Fig 5.5: limit of lm by bode plot**



**Fig 5.6: stability by magnitude of ( $f \cdot l_m$ )**



**Fig 5.7: performance by magnitude of  $\eta$**

From figure  $|l_m(w)| < 1$  only for frequency around 0.6 so for  $l_m = 3.3$  uncertainty can be handled up to this frequency only, so safe choice of  $l_m$  would be somewhere around below 3.

From figure  $|f \cdot l_m|$  is a little larger than 1 for  $l_m = 0.08$  so robust stability for the process can be achieved up to the value of  $l_m$  near 0.1 and all rest values greater than it.

From figure  $|\eta|$  is unity at low frequency for all  $l_m$  but it decreases for  $l_m = 1.5$  and 3 only, so performance is not good at  $l_m = 0.01$ . Thus, the greater the value of  $l_m$ , better is the performance.

# APPLICATIONS

- **At steady state, the controller response has no offset i.e. perfect control is obtained.**
- **The controller is used to get input tracking as well as disturbance rejection responses.**
- **Provides time delay compensation.**

# CONCLUSION

Our study of Internal Model Control (IMC) as well as its applications to design the compensator being used in IMC Model shows that the controller used can be successfully implemented for any industrial process as it is adequately robust towards uncertainty present in plant parameters. Also, for practical applications such an actual process in industries the IMC based PID controller algorithm is robust and simple to handle the uncertainty in model and hence the IMC-PID tuning method seems to be a useful trade-off between performance of the closed loop system and we achieve robustness to model inaccuracies with a single tuning parameter.

The IMC design procedure can be used to solve quite a few critical problems especially at the industrial level (using the concept of designing a model of the actual plant process). It also gives good solutions to processes having significant time delays which actually happens when working in a real time environment. For tuning the controller the filter tuning parameter  $\lambda$  (lambda) value is varied, and it also compromises the various effects of discrepancies that enter the system, and thus best performance is achieved. Hence, a good filter structure is one for which the optimum  $\lambda$  value gives the best PID performance.

We have also observed that IMC structure can be rearranged to design feedback controllers of the PID-type. When a process has no time delays we obtain same performance for both IMC based PID controller as well as the IMC .Existence of a RHP zero implies that a RHP zero must also be present for the specified closed loop response and IMC based PID procedure gives a satisfactory method to handle this. Also standard IMC filter has the advantage of good set point tracking. Although IMC design procedure is like the design procedure of open loop control system, the implementation of IMC is such that it results in being implemented as a feedback system. Thus, IMC has the added advantage of ability to compensate for model uncertainty and disturbances that open loop control does not have. But detuning of the IMC is also important if there is model uncertainty for assured stability and performance.

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