A STUDY ON PID CONTROLLER DESIGN
FOR SYSTEMS WITH TIME DELAY

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By

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CERTIFICATE

This is to certify that the thesis entitled, "**Study of PID controller design for systems with Time delay**" submitted by Shri **Anurag Mishra** in partial fulfilment of the requirements for the award of Bachelor of Technology degree in Electrical Engineering at the National Institute of Technology, Rourkela (Deemed University), is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any degree or diploma.

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ABSTRACT

PID controllers are being widely used in industry due to their well-grounded established theory, simplicity, maintenance requirements, and ease of retuning online. In the past four decades, there are numerous papers dealing with tuning of PID controller. Designing a PID controller to meet gain and phase margin specification is a well-known design technique. If the gain and phase margin are not specified carefully then the design may not be optimum in the sense that could be large phase margin (more robust) that could give better performance. This paper studies the relationship between ISE performance index, gain margin, phase margin and compares two tuning technique, based on these three parameters. These tuning techniques are particularly useful in the context of adaptive control and auto-tuning, where the control parameters have to be calculated on-line.

In the first part, basics of various controllers, their working and importance of PID controller in reference to a practical system (thermal control system) is discussed.

In the latter part of the work, exhaustive study has been done on two different PID controller tuning techniques. A compromise between robustness and tracking performance of the system in presence of time delay is tried to achieve. Results of simulation, graph, plots, indicate the validity of the study.

ACKNOWLEDGEMENTS

I have been very fortunate to start my thesis work under the supervision and guidance of Prof. Sandip Ghosh. He introduced me to the field of Control systems, educated me with the methods and principles of research, and guided me through the details of PID controllers. He is the whole Philosopher and Guide behind this thesis. Working with him, a person of values has been a rewarding experience.

I am highly indebted and express my deep sense of gratitude for his invaluable guidance, constant inspiration and motivation with enormous moral support during difficult phase to complete the work. I acknowledge his contributions and appreciate the efforts put by him for helping me complete the thesis.

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Chapter 1 **INTRODUCTION**

In the recent years, control system has assumed an increasingly important role in the development and advancement of modern civilization and technology. Practically every aspect of our day-to-day activities is affected by some type of control systems. Automatic control system are found in abundance in all sectors of industry, such as quality control of manufactured products, automatic assembly line, machine-tool control, space technology and weapon system, computer control, transportation systems, power systems, robotics and many others. It is essential in such industrial operations as controlling pressure, temperature, humidity, and flow in the process industries.

Recent application of modern control theory includes such non-engineering systems as biological, biomedical, control of inventory, economic and socioeconomic systems.

The basic ingredients of a control system can be described by:
- Objectives of control.
- Control system components.
- Results or output.
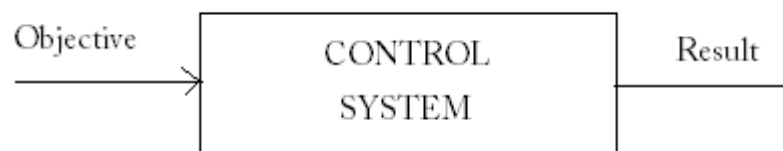
Fig1.1

 Automatic Controllers:-

An automatic controller is used to compare the actual value of plant result with reference command, determines the difference, and produces a control signal that will reduce this difference to a negligible value. The manner in which the automatic controller produces such a control signal is called the *control action*.

An industrial control system comprises of an automatic controller, an actuator, a plant, and a sensor (measuring element). The controller detects the actuating

error command, which is usually at a very low power level, and amplifies it to a very high level. The output of the automatic controller is fed to an actuator, such as a hydraulic motor, an electric motor or a pneumatic motor or valve (or any other sources of energy). The actuator is a power device that produces input to the plant according to the control signal so that the output signal will point to the reference input signal.

The sensor or the measuring element is a device that converts the output variable into another optimum variable, such as a displacement, pressure or voltage, that can be used to compare the output to the reference input command. This element is in a feedback path of the closed loop system. The set point controller must be converted to reference input with the same unit as the feedback signal from the sensor element.

Classification of Industrial controllers:-

Industrial controllers may be classified according to their control action as:
- Two-position or on-off controllers
- Proportional controllers
- Integral controllers
- Proportional-plus-integral controllers
- Proportional-plus-derivative controllers
- Proportional-plus-integral-plus-derivative controllers

Type of controller to use must be decided depending upon the nature of the plant and the operating condition, including such consideration as safety, cost, availability, reliability, accuracy, weight and size.

Two-position or on-off controllers:-

In a two-position control system, the actuating part has only two fixed positions, which are, in many simple cases, simply on and off. Due to its simplicity and inexpensiveness, it is being very widely used in both industrial and domestic control system.

Let the output signal from the controller be u(t) and the actuating error signal be e(t). Then mathematically,

$$u(t) = U_1, \text{ for } e(t) > 0$$

$$= U_2, \text{ for } e(t) < 0$$

Where,

U_1 and U_2 are constants and the minimum value of U_2 is usually either zero or - U1.

1.1 Proportional Control :-

A proportional control system is a type of linear feedback control system. Proportional control is how most drivers control the speed of a car. If the car is at target speed and the speed increases slightly, the power is reduced slightly, or in proportion to the error (the actual versus target speed), so that the car reduces speed gradually and reaches the target point with very little, if any, "overshoot", so the result is much smoother control than on-off control [5].

In the proportional control algorithm, the controller output is proportional to the error signal, which is the difference between the set point and the process variable. In other words, the output of a proportional controller is the multiplication product of the error signal and the proportional gain. This can be mathematically expressed as

$$P_{out} = K_p \, e(t)$$

Where

P_{out}: Output of the proportional controller

K_p: Proportional gain

$e(t)$: Instantaneous process error at time 't'. $e(t) = SP - PV$

SP: Set point

PV: Process variable

With increase in K_p :
- Response speed of the system increases.

- Overshoot of the closed-loop system increases.
- Steady-state error decreases.

But with high K_p value, closed-loop system becomes unstable.

1.2 Integral Control:-

In a proportional control of a plant whose transfer function doesn't possess an integrator 1/s, there is a steady-state error, or offset, in the response to a step input. Such an offset can be eliminated if integral controller is included in the system.

In the integral control of a plant, the control signal, the output signal from the controller, at any instant is the area under the actuating error signal curve up to that instant. But while removing the steady-state error, it may lead to oscillatory response of slowly decreasing amplitude or even increasing amplitude, both of which is usually undesirable [5].

1.3 Proportional-plus-integral controllers:-

In control engineering, a **PI Controller** (proportional-integral controller) is a feedback controller which drives the plant to be controlled by a weighted sum of the error (difference between the output and desired set-point) and the integral of that value. It is a special case of the PID controller in which the derivative (D) part of the error is not used.

The PI controller is mathematically denoted as:

$$G_c = K_p + \frac{Ki}{s}$$

or

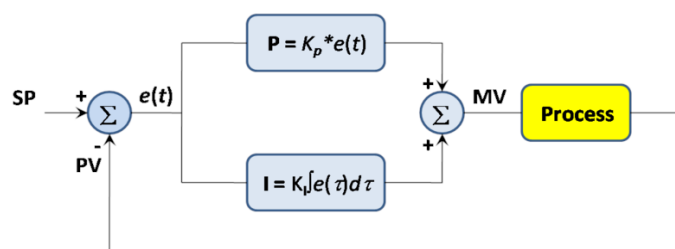$$G_c = K_p \left(1 + \frac{1}{sT_i}\right)$$

Fig.1.2 (courtesy-[5])

Integral control action added to the proportional controller converts the original system into high order. Hence the control system may become unstable for a large value of K_p since roots of the characteristic eqn. may have positive real part. In this control, proportional control action tends to stabilize the system, while the integral control action tends to eliminate or reduce steady-state error in response to various inputs. As the value of T_i is increased,

- Overshoot tends to be smaller

- Speed of the response tends to be slower.

1.4 Proportional-plus-derivative controllers:-

Proportional-Derivative or PD control combines proportional control and derivative control in parallel. Derivative action acts on the derivative or rate of change of the control error. This provides a fast response, as opposed to the integral action, but cannot accommodate constant errors (i.e. the derivative of a constant, nonzero error is 0). Derivatives have a phase of +90 degrees leading to an anticipatory or predictive response. However, derivative control will produce large control signals in response to high frequency control errors such as set point changes (step command) and measurement noise [5].

In order to use derivative control the transfer functions must be proper. This often requires a pole to be added to the controller.

$$G_{pd}(s) = K_p + K_d s \quad \text{or}$$

$$= K_p(1+T_d s)$$

With the increase of T_d

- Overshoot tends to be smaller

- Slower rise time but similar settling time.

1.5 Proportional-plus-integral-plus-derivative controllers:-

The PID controller was first placed on the market in 1939 and has remained the most widely used controller in process control until today. An investigation performed in 1989 in Japan indicated that more than 90% of the controllers used in process industries are PID controllers and advanced versions of the PID controller. PI controllers are fairly common, since derivative action is sensitive to measurement noise

"PID control" is the method of feedback control that uses the PID controller as the main tool. The basic structure of conventional feedback control systems is shown in Figure below, using a block diagram representation. In this figure, the process is the object to be controlled. The purpose of control is to make the process variable y follow the set-point value r. To achieve this purpose, the manipulated variable u is changed at the command of the controller. As an example of processes, consider a heating tank in which some liquid is heated to a desired temperature by burning fuel gas. The process variable y is the temperature of the liquid, and the manipulated variable u is the flow of the fuel gas. The "disturbance" is any factor, other than the manipulated variable, that influences the process variable. Figure below assumes that only one disturbance is added to the manipulated variable. In some applications, however, a major disturbance enters the process in a different way, or plural disturbances need to be considered. The error e is defined by $e = r - y$. The compensator $C(s)$ is the computational rule that determines the manipulated variable u based on its input data, which is the error e in the case of Figure. The last thing to notice about the Figure is that the process variable y is assumed to be measured by the detector, which is not shown explicitly here, with sufficient accuracy instantaneously that the input to the controller can be regarded as being exactly equal to y.
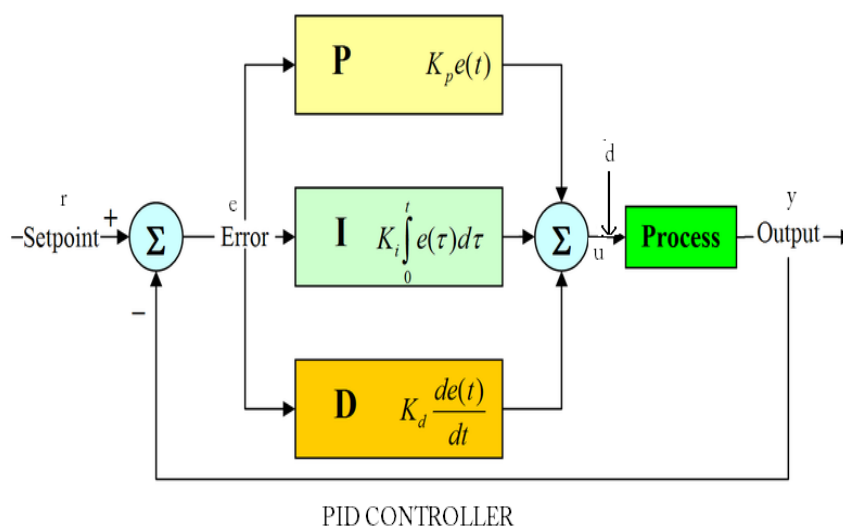

Fig. 1.3(courtesy-[5])

When used in this manner, the three element of PID produces outputs with the following nature:

- P element: proportional to the error at the instant t, this is the "present" error.
- I element: proportional to the integral of the error up to the instant t, which can be interpreted as the accumulation of the "past" error.

- D element: proportional to the derivative of the error at the instant *t*, which can be interpreted as the prediction of the "future" error.

Thus, the PID controller can be understood as a controller that takes the present, the past, and the future of the error into consideration. The transfer function $G_c(s)$ of the PID controller is :

$$G_c(s) = K_p \left(1 + \frac{1}{sT_i} + T_d s\right)$$

$$= K_p + \frac{K_i}{s} + K_d s$$

1.6 Application:-

In the early history of automatic process control the PID controller was implemented as a mechanical device. These mechanical controllers used a lever, spring and a mass and were often energized by compressed air. These pneumatic controllers were once the industry standard [5].

Electronic analog controllers can be made from a solid-state or tube amplifier, a capacitor and a resistance. Electronic analog PID control loops were often found within more complex electronic systems, for example, the head positioning of a disk drive, the power conditioning of a power supply, or even the movement-detection circuit of a modern seismometer. Nowadays, electronic controllers have largely been replaced by digital controllers implemented with microcontrollers or FPGAs.

Most modern PID controllers in industry are implemented in programmable logic controllers (PLCs) or as a panel-mounted digital controller. Software implementations have the advantages that they are relatively cheap and are flexible with respect to the implementation of the PID algorithm [5].
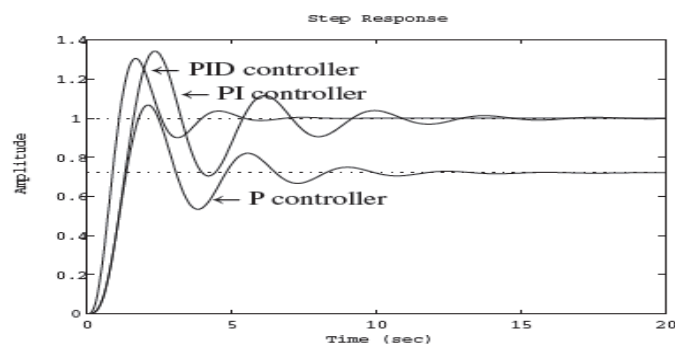

Fig.1.4 Close-loop step response.

Chapter 2 STUDY OF PROCESS CONTROL SIMULATOR (PCS 327)

2.1 Basic Introduction:-

A process control simulator contains electronic circuits for modifying set-point control signals by summing, integrating, differentiating and introducing step functions on continuous, single or periodic bases to determine off-line the behaviour of a process control system [6]. Provisions are made for several interacting control loops, for dead-time delays and for faster than actual response times to speed up simulation. The dead-time delays are provided by scanning a set of storage capacitors with a variable speed scanning motor through a scanning control cycle of operation. A three control loop controller section provides for setting the gain of the controller system and controlling integration and difference modes responsive to step disturbances introduced into an integration process after a selectable dead-time adjustment. The output signal is reproduced in the form of a process response waveform.

It is a special-purpose analogue simulator employing integrated circuit operational amplifiers laid out in such a manner to allow the principles of process control methods to be taught at both technician and technological levels. Numerous variable interconnections together with a rage of non-linear functions make the simulator sufficiently versatile to permit a detailed study of the dynamic responses of a wide – variety of linear and non-linear processes and the application of proportional, integral, derivative, two-step, three-step(with and without overlap) and many other modes of control to the improvement of their performance. At the same time provision is made for the process characteristics and controller configuration to be preset by the instructor and the full mimic diagrams concealed by simplified overlays; this helps to present the ideas of process control in the simplest possible terms to technicians whose acquaintance with process control methods will be of a more empirical nature.

Basically the motive here is to first, carry the physical investigation of the Process Control Simulator (PCS 327) and establish the ideal working condition of the device. For this purpose, first, all the individual Capacitances and Resistances were measured and compared to their specified values. Secondly, the Proportional, Integral,

Derivative units and the Non-linear units (Limits, Dead-band, Neutral zone) were individually studied and inferences were drawn. Then, after the ideal working apparatus is established, tuning of the controller is carried out and matched with the theoretical data from MATLAB to confirm its perfect working condition. All this is discussed in detail.
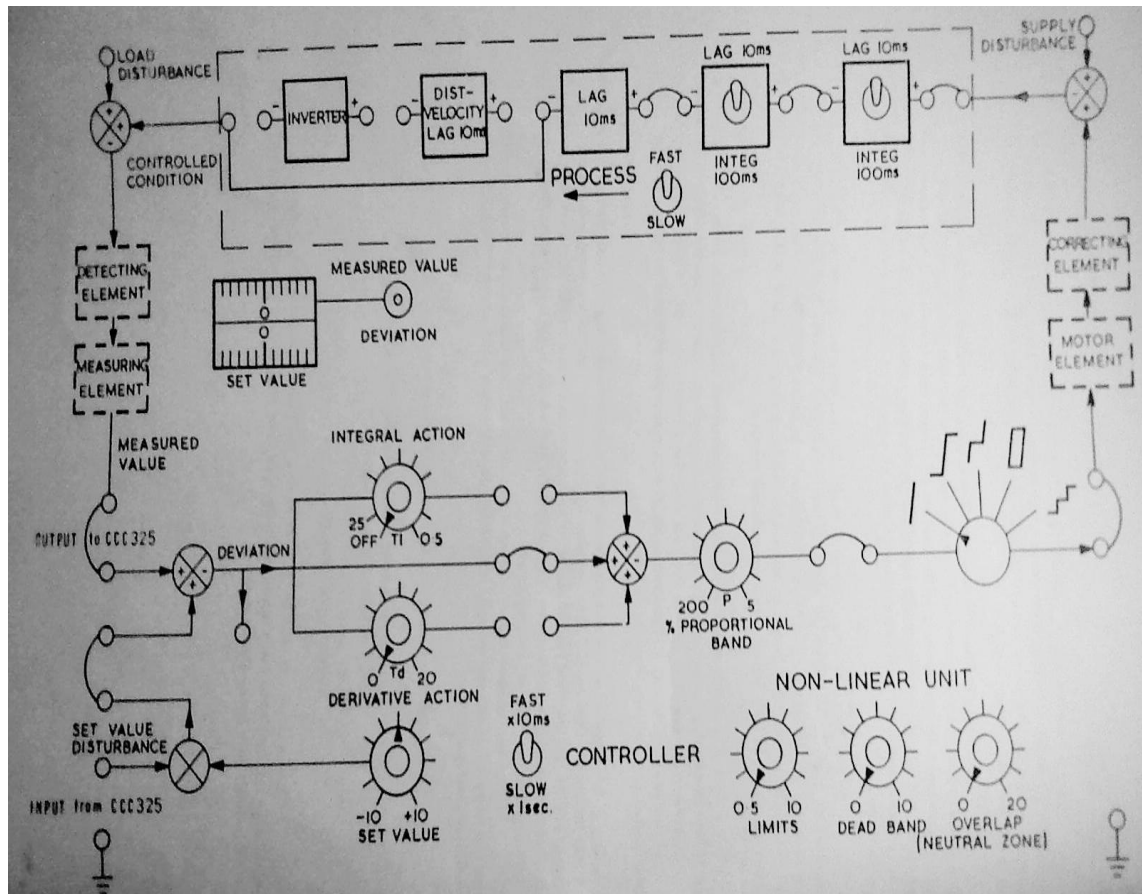
Fig.2.1 Diagram of Process Control Simulator PCS327(Courtesy: Feedback Instruments [6])

2.2 Experiments

2.2.1 Experiment-1

Aim:- Study of **'Proportional'** feature of PCS327.

Fig.2.2 Terminal diagram of Proportional part

Procedure:-

1. Using Multi-meter, the individual resistances across the terminals of the Proportional band were measured.

2. According to the change in the resistances, pattern was observed and inference was drawn.

Tabulation:-

SERIAL NO.	PROPORTIONAL %	RESISTANCE OF [1-C] (Ohms)	RESISTANCE OF [2-C] (Ohms)
1	0	6.16 M	5
2	30	45.6 K	0.99 K
3	50	38.9 K	1.059 K
4	100	25.57 K	1.102 K
5	200	3.3 K	1.126 K

Inference :-

As the resistances are varying properly (1-C is decreasing and 2-C is increasing), we can infer that the band is functioning properly.

2.2.2 Experiment-2

Aim:- Study of **'integral'** feature of PCS327.

Fig.2.3 Terminal diagram of Integral part

Procedure:-

1. Using Multi-meter, the individual resistances across the terminals of the Integral band were measured.

2. According to the change in the resistances, pattern was observed and inference was drawn.

Tabulation:-

SERIAL NO.	INTEGRAL	RESISTANCE OF [1-C] (Ohms)	RESISTANCE OF [2-C] (Ohms)
1	OFF	9	472.9
2	5	2.32 K	2.7 K
3	1.5	5.47 K	5.65 K
4	1	6.45 K	6.44 K
5	0.5	954	524.6

Inference :-

As the resistances are **NOT** varying properly (Both 1-C and 2-C is increasing first and then decreasing), we can infer that the band may have some problems and it is considered for further investigation.

2.2.3 Experiment-3

<u>**Aim:-**</u> Study of **'Derivative'** feature of PCS327.
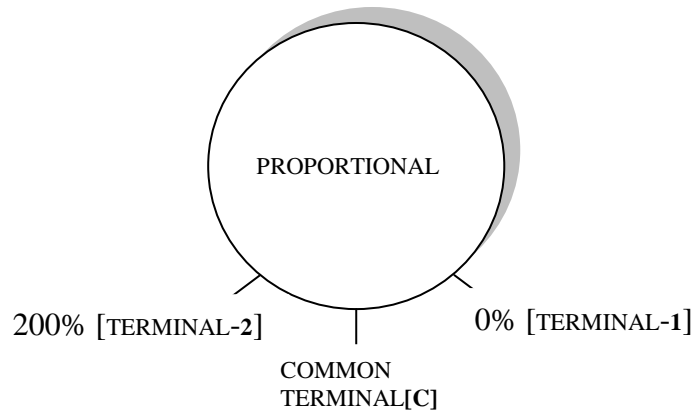

Fig.2.4 Terminal diagram of Derivative part

<u>**Procedure:-**</u>

1. Using Multi-meter, the individual resistances across the terminals of the Derivative band were measured.

2. According to the change in the resistances, pattern was observed and inference was drawn.

<u>**Tabulation:-**</u>

SERIAL NO.	DERIVATIVE	RESISTANCE OF [1-C] (Ohms)	RESISTANCE OF [2-C] (Ohms)
1	0	887	25.07 K
2	0.4	5.51 K	20.42 K
3	1	12.75 K	13.13 K
4	1.4	17.46 K	8.26 K
5	2	24.6 K	145.8

<u>**Inference :-**</u>

As the resistances are varying properly (1-C is increasing and 2-C is decreasing), we can infer that the band is functioning properly.

2.2.4 Experiment-4

Aim:- Study of **'Limits'(Non-linear Unit)** feature of PCS327.
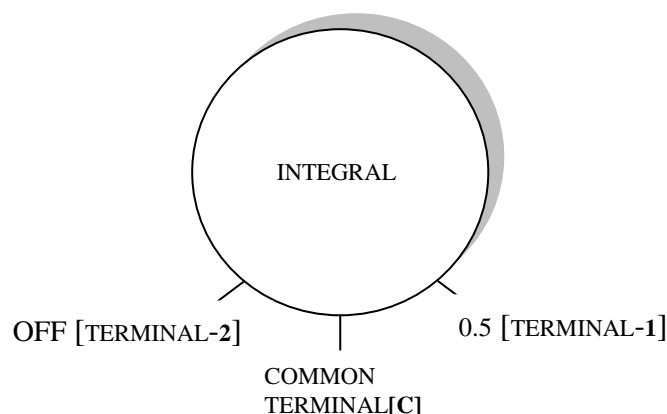

Fig.2.5 Terminal diagram of Limits part

Procedure:-

1. Using Multi-meter, the individual resistances across the terminals of the Limits band were measured.

2. According to the change in the resistances, pattern was observed and inference was drawn.

Tabulation:-

SERIAL NO.	LIMITS	RESISTANCE OF [1-C] (Ohms)	RESISTANCE OF [2-C] (Ohms)
1	0.5	6.2 K	946
2	2	5.62 K	3.0 K
3	4	4.69 K	4.07 K
4	8	2.82 K	5.79 K
5	10	1.66 K	6.64 K

Inference :-

As the resistances are varying properly (1-C is decreasing and 2-C is increasing), we can infer that the band is functioning properly.

2.2.5 Experiment-5

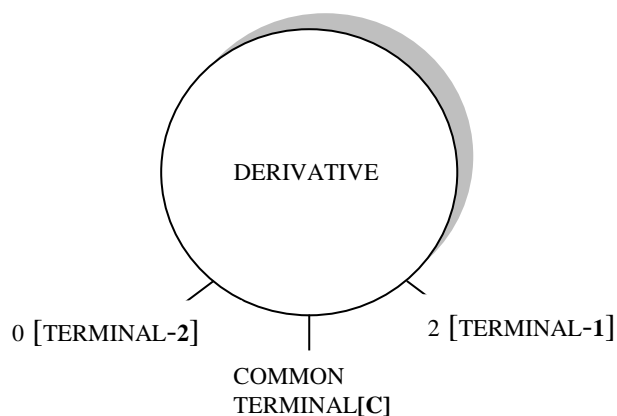**Aim:-** Study of **'Dead-Band'(Non-linear Unit)** feature of PCS327.

Fig.2.6 Terminal diagram of dead-band part

Procedure:-

1. Using Multi-meter, the individual resistances across the terminals of the Dead-band were measured.

2. According to the change in the resistances, pattern was observed and inference was drawn.

Tabulation:-

SERIAL NO.	DEAD-BAND	RESISTANCE OF [1-C] (Ohms)	RESISTANCE OF [2-C] (Ohms)
1	0	5.86 K	281.5
2	2	5.22 K	2.23 K
3	4	4.51 K	3.39 K
4	6	3.69 K	4.22 K
5	10	0.73 K	5.38 K

Inference :-

As the resistances are varying properly (1-C is decreasing and 2-C is increasing), we can infer that the band is functioning properly.

2.2.6 Experiment-6

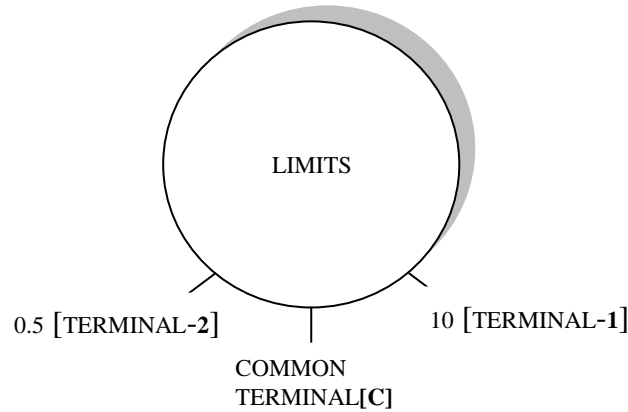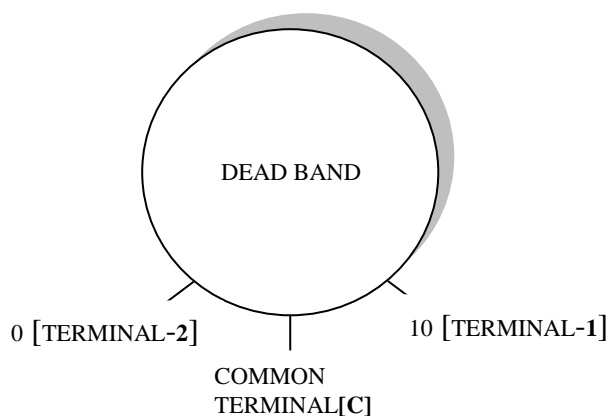**Aim:-** Study of **'Overlap'(Non-linear Unit)** feature of PCS327.
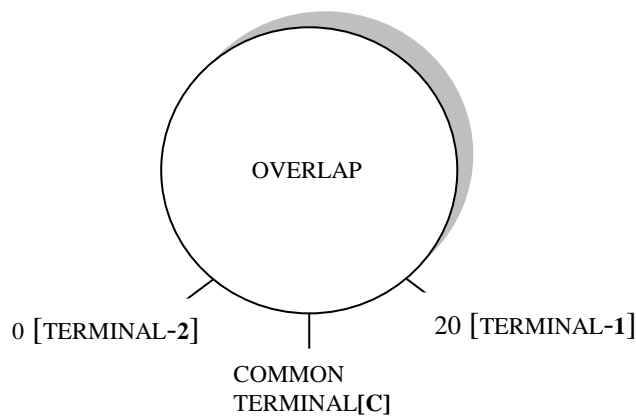
Fig.2.7 Terminal diagram of Overlap part

Procedure:-

1.Using Multi-meter, the individual resistances across the terminals of the Overlap band were measured.

2. According to the change in the resistances, pattern was observed and inference was drawn.

Tabulation:-

SERIAL NO.	OVERLAP	RESISTANCE OF [1-C] (Ohms)	RESISTANCE OF [2-C] (Ohms)
1	0	0.79 K	11.69 K
2	4	9.45 K	16.02 K
3	10	16.46 K	15.58 K
4	16	15.42 K	9.80 K
5	20	12.91 K	2.89 K

Inference :-

As the resistances are **NOT** varying properly (Both 1-C and 2-C is increasing first and then decreasing), we can infer that the band may have some problems and it is considered for further investigation.

2.2.7 Experiment-7

Aim:- Study of **Power Circuit** of PCS327.

Procedure:-

1.Using Multi-meter, all the individual resistances and Capacitances of the circuit were measured.

2. These experimental values were compared with the theoretical values and for those parts, which were either damaged or mismatching was found, inference was drawn that they need to be replaced.

Fig 2.8. Diagram of Power circuit of PCS327(Courtesy: Feedback Instruments)
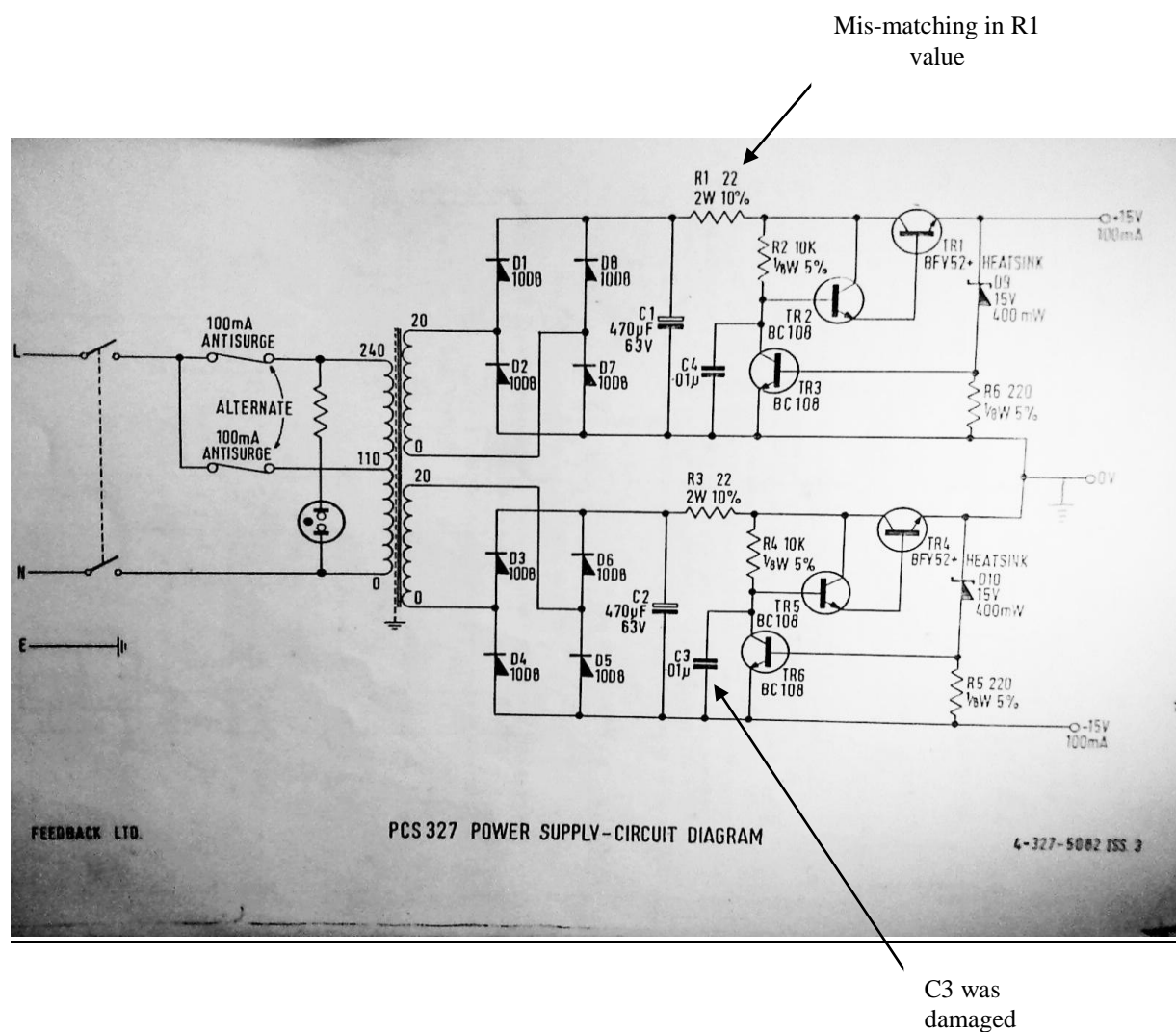
Observation:-

From the above experiment the marked parts (R1 and C3) did not satisfy the theoretical conditions and so it was concluded that they need to be replaced for perfect functioning of PCS327.

Chapter 3 **TUNING OF PID CONTROLLER**

3.1 Basic Introduction:-

"Tuning" is the engineering work to adjust the parameters of the controller so that the control system exhibits desired property. Currently, more than half of the controllers used in industry are PID controllers [5]. In the past, many of these controllers were analog; however, many of today's controllers use digital signals and computers. When a mathematical model of a system is available, the parameters of the controller can be explicitly determined. However, when a mathematical model is unavailable, the parameters must be determined experimentally. Controller tuning is the process of determining the controller parameters which produce the desired output. Controller tuning allows for optimization of a process and minimizes the error between the variable of the process and its set point [5].

Types of controller tuning methods include the trial and error method, and process reaction curve methods. The most common classical controller tuning methods are the Ziegler-Nichols and Cohen-Coon methods. These methods are often used when the mathematical model of the system is not available. The Ziegler-Nichols method can be used for both **closed** and **open loop systems**, while Cohen-Coon is typically used for open loop systems. A closed-loop control system is a system which uses feedback control. In an open-loop system, the output is not compared to the input [5].

The equation below shows the PID controller:-.

$$u(t) = K_p \ [\ e(t) + \frac{1}{T_i} \int_0^t e(t') \ dt' \ + T_d \ (\frac{de(t)}{dt}) \ ] + b$$

Where,

u is the control signal.

e is the difference between the current value and the set point.

K_c is the gain for a proportional controller.

T_i is the parameter that scales the integral controller.

T_d is the parameter that scales the derivative controller.

t is the time taken for error measurement.

b is the set point value of the signal, also known as bias or offset.

3.2 Ziegler-Nichols Rules for tuning PID Controller:-

It has been observed that step responses of many processes to which PID controllers are applied have monotonically increasing characteristics as shown in Figures a and b, so most traditional design methods for PID controllers have been developed implicitly assuming this property. However, there exist some processes that exhibit oscillatory responses to step inputs.

Two tuning methods were proposed by Ziegler and Nichols in 1942 and have been widely utilized either in the original form or in modified forms. One of them, referred to as Ziegler–Nichols' ultimate sensitivity method, is to determine the parameters as given in Table 1 using the data K_{cr} and T_{cr} obtained from the ultimate sensitivity test. The other, referred to as Ziegler–Nichols' step response method, is to assume the model FOPDT and to determine the parameters of the PID controller as given in Table 2 using the parameters *R* and *L* of FOPDT which are determined from the step response test.

Type of controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$0.833T_{cr}$	0
PID	$0.6K_{cr}$	$0.5T_{cr}$	$0.125T_{cr}$

Fig.3.1 Ziegler-Nichols ultimate sensitivity test [17].

Type of controller	K_p	T_i	T_d
P	1/RL	∞	0
PI	0.9/RL	L/0.3	0
PID	1.2/RL	2L	0.5L

Fig.3.2 Ziegler-Nichols step response method (RL≠0) [17].

Frequency-domain stability analysis tells that the above way of applying the Ziegler–Nichols' step response method to processes with self-regulation tends to set the parameters on the safe side, in the sense that the actual gain and phase margins become larger than the values expected in the case of integrating processes.

.These methods to determine PID parameter using empirical formula, as well as several other tuning methods developed on the same principle, are often referred to as "classical" tuning methods. Some of the other classical tuning methods are, Chien–Hrones–Reswick formula, Cohen–Coon formula, refined Ziegler–Nichols tuning, Wang–Juang–Chan formula.

Disadvantage :-

The classical tuning methods explained above have the following features:

• The process is assumed, implicitly (in the case of Ziegler–Nichols' ultimate sensitivity method) or explicitly (in the case of Ziegler–Nichols' step response method), to be modelled by the simple transfer function.

• The optimal values of the PID parameters are given by formulae of the process parameters that are determined directly and uniquely from experimental data.

The first feature is a weakness of these classical methods, in the sense that the applicable processes are limited, or in other words that the claimed "optimal" values are not necessarily, and are sometimes fairly far from, the true optimal in practical situations where the transfer function is nothing but an approximation of the real process characteristics. Specifically, the problem is serious when the pure delay L of the process is very short or very long, where "very short" and "very long" roughly means outside the range $0.05 \leq L/T \leq 1.0$ [17]. It can be interpreted as a weakness in the sense that there is no room to improve the results by making use of more detailed information about the process which is obtainable from theoretical study and accurate measurement.

Many attempts have been made to make up for these weaknesses of the classical methods. Many theoretical considerations have been used to develop sophisticated methods that use, as the basis of tuning, the shape of the frequency response of the return ratio, poles (and zeros) of the closed-loop transfer function, time-domain performance indices such as ISE, or frequency-domain performance indices.

3.3 Recent Trends :-

3.3.1 Optimum PID Controller Design:-

Optimum setting algorithms for a PID controller were proposed by Zhuang and Atherton for various criteria. Consider the general form of the optimum criterion

$$J_n(\theta) = \int_0^\infty [t^n \ e(\theta,t)]^2 \ dt$$

Where e(θ,t) is the error signal which enters the PID controller, with θ the PID controller parameters. For the system structure shown in Fig. 2, two setting strategies are proposed: one for the set-point input and the other for the disturbance signal *d(t)*. In particular, three values of *n* are discussed, i.e., for *n = 0, 1, 2*. These three cases correspond, respectively, to three different optimum criteria: the integral squared error (ISE) criterion, integral squared time weighted error (ISTE) criterion, and the integral squared time-squared weighted error (IST2E) criterion.

3.3.2 Automatic tuning of PID Controller:-

In 1984, Astrom and Hagglund proposed an automatic tuning method based on a simple relay feedback test, using the describing function analysis, the critical gain and the critical frequency of the system. This method is popularly known as Gain-phase margin (G-P) method.

This topic was further studied and discussed by Ho, Hang, and Cao in the year 1995.

All this studies on the approach of optimal design of PID controller based on Gain margin, phase margin have resulted in the following recommended range:

Gain Margin – 2 – 5;

Phase Margin – 30° – 70°

And two important factors that have been considered are:
- ROBUSTNESS: larger the phase margin greater is the robustness of the system.

- PERFORMANCE: lower the value of ISE better is the performance.

In this study on optimal design of PID Controller, we will be analysing the two technique, on how they perform to the given specification or range discussed above. The two methods are:

Method - 1 : Based on Gain and Phase margin i.e. numerical solution method.

Method – 2 : Based on Gain-Phase margin tester method.

Both the method is based on First Order Plus Delay Time (FOPDT) process. It is given by:

$$G_p(s) = (\frac{K}{1+sT})e^{-sL}$$

3.4 First order delay time process :-

A first-order system is one whose output y(t) is modelled by a first-order differential equation.[1] In the Laplace Domain, general first-order transfer functions are described by Equation:

$$G_p(s) = \frac{K}{1+sT}$$

Equation : First-Order Process

Dead time (θ) is the time delay between the process and the sensor. The transfer function for dead time is:

$$G_p(s) = e^{-sL}$$

Cause of dead time is:

- Transportation lag
- Sensor lag

Effect of Dead Time on the system is:

Time delay occurs in the control system when there is a delay between command response and the start of output response. The delay cause a decrease phase margin which implies a lower damping ratio and a more oscillatory response for the close-loop system. Further it decreases the gain margin thus moving the system to instability [5].

'**Pade**' approximation is used for approximation of dead time function in FOPDT transfer function.

FOPDT models are the combination of a first-order process model with dead time.

$$G_p(s) = (\frac{K}{1+sT})e^{-sL}$$

Where,

K : process gain

T : process time constant

L : dead time constant
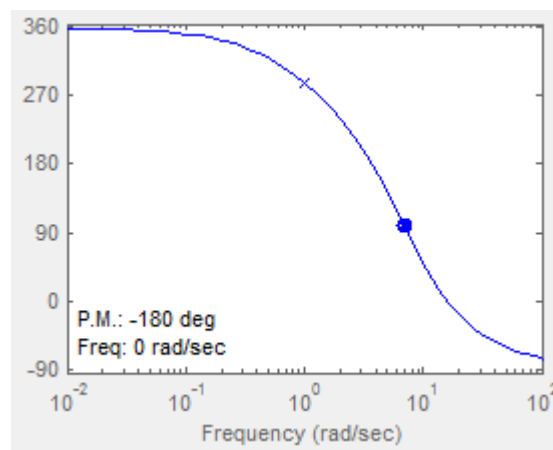
Frequency response of the dead time:

Fig 3.3

Example:

Thermal control system is a best example of FOPDT process. A heater takes time delay to attain the temperature desired or the set-point.

How PID works for this process?

Only very control of temperature can be achieved by causing heater power to be simply switched on and off according to an under or over temperature condition respectively. Ultimately, the heater power will be regulated to achieve a desired system temperature but refinement can be employed to enhance the control accuracy. Such refinement is available in the form of proportional (P), integral (I), and derivative (D) functions applied to the control loop. These functions, referred to as

control "terms" can be used in combination according to system requirements. The desired temperature is usually referred to as the set-point (SP).

Proportional (P) – A form of anticipatory action which slows the temperature rise when approaching set-point. Variations are more smoothly corrected but an offset will occur (between set and achieved temperatures) as conditions very.
Average heater power over a period of time is regulated and applied power is proportional to the error between sensor temperature and set-point (usually by time proportioning relay switching). The region over which power is thus varied is called the Proportional Band (PB) it is usually defined as a percentage of full scale.
.

Proportional + Integral + Derivative (PID) :-

Adding an integral term and derivative term to P control can provide automatic and continuous elimination of any offset. Integral action operates in the steady state condition by shifting the Proportional Band upscale or downscale until the system temperature and set-point coincide.

It is clear from the above example that to achieve optimum temperature control use of such PID technique is indispensible.

Thus next we will be studying two technique of optimal design of such PID controller applied to similar First order plus delay time (FOPDT) process as mentioned above and we will be analysing the performance of both the method. The controller so designed would compensate for the instability induced by the delay time and endows the system with robust safety margin in terms of gain and phase, as well greater performance by reducing ISE.

Chapter 4 DESIGN OF PID CONTROLLER

4.1 METHOD – I : Tuning based on Gain and Phase margin

4.1.1 Introduction:-

It is based on the approach to minimise a performance index i.e. ISE, IAE, ITAE, in order to get an optimum design of PID controller. The performance index considered here is ISE error, which is given by

$$ISE = \int_0^\infty e^2(t) \, dt$$

Where, e(t) is error signal at time t.

Gain and Phase margin are used as Measure of 'Robustness' and also the 'Performance' of closed-loop system. So such design is based on specifying the gain and phase margin wisely to give the best of performance and robustness [7].

The recommended range of Gain and Phase margin according to "Astrom and Hagglund, 1995" is given by:

$$G.M. - 2\text{-}5$$
$$P.M. - 30^o\text{-}70^o$$

4.1.2 Methodology:-

Various denotations used are as follows:

$G_p(s)$ = process transfer function.

$Gc(s)$ = controller transfer function.

A_m = gain margin.

φ_m = phase margin.

The PI controller transfer function is given as:

$$G_c(s) = K_c(1 + \frac{1}{sT_i}).$$ \hfill (1)

First order plus death-time process model given by:

$$G_p(s) = (\frac{K}{1+sT})e^{-sL}$$ \hfill (2)

From the basic definition of gain and phase margin following set of equations are obtained:

$$\varphi_m = \arg [G_c(jw_g)G_p(jw_g)] + \pi$$ \hfill (3)

$$A_m = 1/(|G_c(jw_p)G_p(jw_p)|)$$ \hfill (4)

Where w_g and w_p are given by

$$[G_c(jw_g)G_p(jw_g)] = 1;$$ \hfill (5)

$$\text{Arg}[G_c(jw_p)G_p(jw_p)] = -\pi;$$ \hfill (6)

From eqn.(1) and (2), forward loop transfer function is given by

$$G_c(s)G_p(s) = \frac{K_c K_p(1+sT_i)}{sT_i(1+sT)}e^{-sL}$$ \hfill (7)

Solving eqn (7) into (3) to (6) gives:

$$\pi/2 + \arctan(w_p T_i) - \arctan(w_p T) - w_p L = 0$$ \hfill (8)

$$A_m K_c K_p = w_p T_i \sqrt{\frac{(w_p^2 T^2 + 1)}{(w_p^2 T_i^2 + 1)}}$$ \hfill (9)

$$K_c K_p = w_g T_i \sqrt{\frac{(w_g^2 T^2 + 1)}{(w_g^2 T_i^2 + 1)}}$$ \hfill (10)

$$\varphi_m = \pi/2 + \arctan(w_g T_i) - \arctan(w_g T) - w_g L$$ \hfill (11)

For a given process K_p, T, L, and specification of A_m, φ_m, eqn. (8) – (11) can be solved for the PI control parameter K_c, T_i, and crossover frequency w_g, w_p, numerically but not analytically because of the presence of "arc tan" function. However approximate analytical solution can be obtained if we make the following approximation for the "arc tan" function.

$$\text{Arc tan } x \approx \pi/2 - \pi/4x \ (|x|>1) \tag{12}$$

The numerical solution of eqn. (8) – (11) shows that for

$$L/T > 0.3 \ , \ T_i \approx T$$

Thus eqn. (9) and (10) can be reduced to,

$$A_m K_c K_p = \ w_p T \tag{13}$$

$$K_c K_p = w_g T \tag{14}$$

Using the approximation in eqn. (12) for the "arc tan" function, eqn. (8) and (11) can be approximate as

$$\pi/2 - \pi/(4w_p T_i) + \pi/(4w_p T) - w_p L = 0 \tag{15}$$

$$\varphi_m = \pi/2 - \pi/(4w_g T_i) + \pi/(4w_g T) - w_g L \tag{16}$$

Solving eqn. (15) for w_p,

$$\pi/2 - w_p L = \ \pi/(4w_p T_i) - \pi/(4w_p T)$$

$$\pi - 2w_p L = (\pi T - \pi T_i) \ / \ (2w_p T_i T)$$

$$4w_p{}^2 T_i TL - 2\pi w_p T_i T + \ \pi T - \pi T_i$$

$$w_p = \ (2\pi T_i T \pm \sqrt{(4\pi^2 T_i{}^2 \ T^2 - 16\pi \ T^2 T_i L + 16\pi \ TT_i{}^2 L))} \ / \ (8 \ T_i TL)$$

$$= \ (2\pi T_i T \pm \sqrt{(4\pi T_i \ T \ [\pi T_i \ T - 4 \ TL + 4T_i L])))} \ / \ (8 \ T_i TL)$$

$$= \ \pi/4L \pm (\sqrt{(\pi T_i \ T \ [\pi T_i \ T - 4 \ TL + 4T_i L]))} / \ (4 \ T_i TL) \tag{17}$$

Putting the value of w_p in eqn. (13)

$$K_c = w_p T/(A_m K_p)$$

$$= [\pi/4L \pm (\sqrt{(\pi T_i\, T\, [\pi T_i\, T - 4\, TL + 4T_iL]))} / \,(4\, T_iTL)]* \, T/(A_mK_p)$$

$$= [\pi T_i\, T \pm \sqrt{(\pi T_i\, T\, [\pi T_i\, T - 4\, TL + 4T_iL])}] / (4A_mK_pT_iL) \qquad (18)$$

Solving eqn. (16) for w_g,

$$\pi/2 - w_gL - \varphi_m = \pi/(4w_gT_i)\, - \pi/(4w_gT)$$

$$\pi - 2w_gL - 2\varphi_m = (\pi T - \pi T_i) / (2w_gT_iT)$$

$$4w_g{}^2\, T_iTL + w_g(4T_iT\varphi_m - 2\,\pi T_i\, T) + \,\pi T - \pi T_i = 0$$

$$w_g = [2\,\pi T_i\, T - 4T_iT\varphi_m \pm \sqrt{(B^2 - 4AC)}] / \,(8\, T_iTL) \qquad (19)$$

where,

$$B^2 - 4AC = (16T_i{}^2T^2\varphi_m{}^2 + 4\pi^2T_i{}^2\, T^2 - 16\pi T_i{}^2\, T^2\varphi_m - \, 16\pi\, T^2T_iL\, + 16\pi\, TT_i{}^2L)$$

Putting the value of w_p in eqn. (14)

$$K_cK_p = w_gT$$

$$[\pi T_i\, T \pm \sqrt{(\pi T_i\, T\, [\pi T_i\, T - 4\, TL + 4T_iL])}] / (4A_mT_iL)$$

$$= [2\,\pi T_i\, T - 4T_iT\varphi_m \pm \sqrt{(B^2 - 4AC)}] / \,(8\, T_iL) \qquad (20)$$

Eqn. (20) has T_i in terms of known process parameters like T, L, π and specified gain and phase margin φ_m, A_m. So the value of T_i can be calculated by "numerical analysis method". The value of this T_i is then put into the eqn. (18) to get the corresponding value of K_c with given specification.

4.1.3 Computation:-

The procedure followed to calculate the value of K_c and K_i corresponding to the specified G.m. and P.m using the above method is given below. For simplicity we have taken a example of a case were,

$$A_m = 3;$$
$$\theta_m = 50;$$

4.1.4 Algorithm:-

Step 1: The first thing to do is to calculate the value of T_i by "numerical analysis method" for the given process parameters and a given specification of gain and phase margin. In Matlab file "**pid.m**" all the known values corresponding to the FOPDT system is entered. The value of T_i is varied within a range of 0.1 to 5.0 and the value of it corresponding to the minimum value on the R.H.S. is calculate. This value corresponds to the T_i value for the given specification. The code to do this job is as follows:

MATLAB CODE :-

```
function value = pid(tao)
Am=3;
L=0.5;
theta=50;
phi=(pi/180)*theta;
display(phi);
i1=1;
for Ti= 0.1:0.001:5.0
value1(i1)= (2*pi*Ti*tao*Am)- (4*Ti*tao*phi*Am)- (2*pi*Ti*tao)+
(16*Ti*tao*Am^2*(Ti*tao*phi^2+pi^2*Ti*tao/4-Ti*tao*pi*phi-pi*taon
*L+pi*Ti*L))^0.5 - (4*pi*Ti*tao*(pi*Ti*tao-4*tao+4*Ti))^0.5;
i1=i1+1;
end
[value,I]=min(value1);
display(value);
display(I);
Ti_value=0.9+(0.001*(I-1));
display(Ti_value);
```

Step 2: Function "pid.m" is run and result of 'tri_value' gives the value of minimum T_i value corresponding to the given specification. The following operation is performed on Matlab command window :

MATLAB Command Window:

$>>$ pid(1)

phi =

  0.8727

value =

  0.0015

I =

  688

Ti_value =

  0.7870

ans =

  0.0015

Step 3: The value of minimum T_i obtained in step – 2 is then put in M-file "pid4Kc.m" to get the corresponding K_c value. The code is given below:

MATLAB CODE:-

```
Am=5.0;
L=0.5;
Kp=1.0;
tao=1.0;
Ti=0.97;
display(Ti);
Kc= (pi*Ti*tao +(pi*Ti*tao*(pi*Ti*tao-
4*tao*L+4*Ti*L))^0.5)/(4*Ti*L*Am*Kp);
display(Kc);
```

Step 4: The M-file "pid4Kc.m" is run and the value of K_c is obtained in MAtlab command window.

OUTPUT:-

Ti =

  0.7870

Kc =

 1.0000

Step 5: The value obtained in step-4 gives the values of K_c(taken as K_p in the program) and T_i respectively for the specified parameters. This value is put in the function "exp2" written in M-file 'exp2.m'.To obtain the system transfer function i.e. $G_o(s)= Gc(s)*Gp(s)$.

MATLAB CODE:-

```
function tf = exp2(Kp,Ti)
syms s
Ki=Kp/Ti;
tf1=(s^4+13*s^3+60*s^2+48*s);
vpa(tf1,4);
tf=(s^3*Kp+s^2*(Ki-12*Kp)+s*(48*Kp-12*Ki)+48*Ki)/(tf1);
```

On running the above code on the Matlab command window we get the system forward transfer function.

MATLAB Command Window:

>> exp2(1.000,0.7870)

ans =

(s^3-8444/787*s^2+25776/787*s+48000/787)/(s^4+13*s^3+60*s^2+48*s)

 >> vpa(ans,4)
ans =

(s^3-10.73*s^2+32.75*s+60.99)/(s^4+13.*s^3+60.*s^2+48.*s)

Step 6: The system transfer function obtained in the step-5 is used to check for the actual G.M. and P.M. using the Matlab function "sisotool(sys)". The code is given below,

MATLAB Command Window:

>> num = [1.0 -10.73 32.75 60.99]

num =

 1.0000 -10.7300 32.7500 60.9900

>> den=[1 13 60 48 0]

den =

 1   13   60   48   0

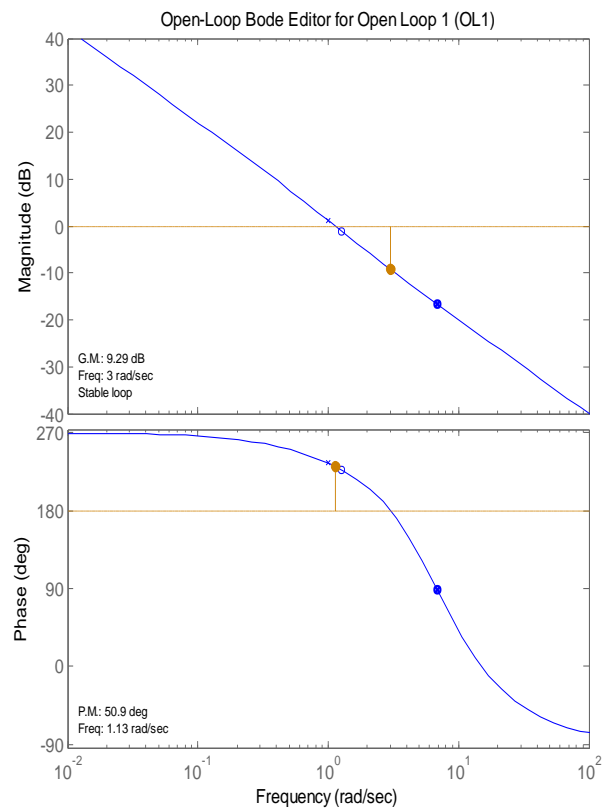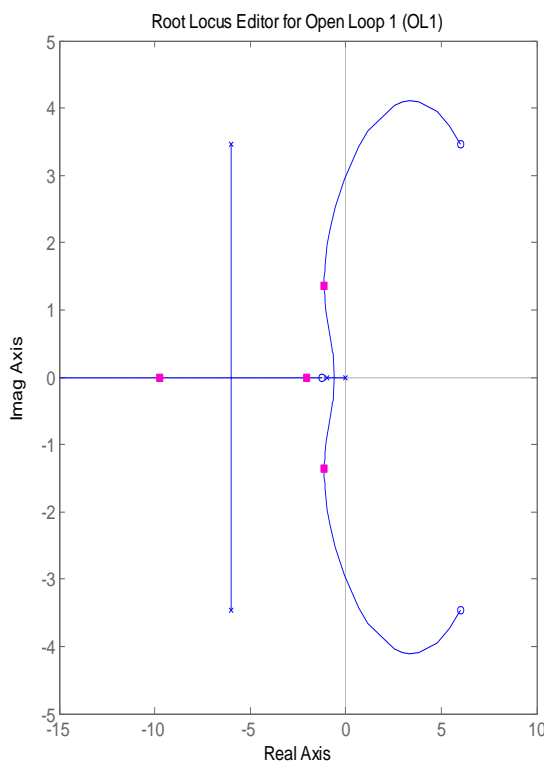>> sys=tf(num,den)

Transfer function:

s^3 - 10.73 s^2 + 32.75 s + 60.99

----

 s^4 + 13 s^3 + 60 s^2 + 48 s

>> sisotool(sys)

OUTPUT:-

Step 7: Actual values of G.M and P.M. are tabulated using the symbol *(asterisk) as a superscript. All the data obtained above are also tabulated.

Step 8: The value of 'ISE' error for the given specification can be obtained using SIMULINK. Using it a model is designed to display the values of :

- ISE Error
- IAE Error
- Output of the PROCESS.

The above values are calculated with *step input* and *step load disturbance* .
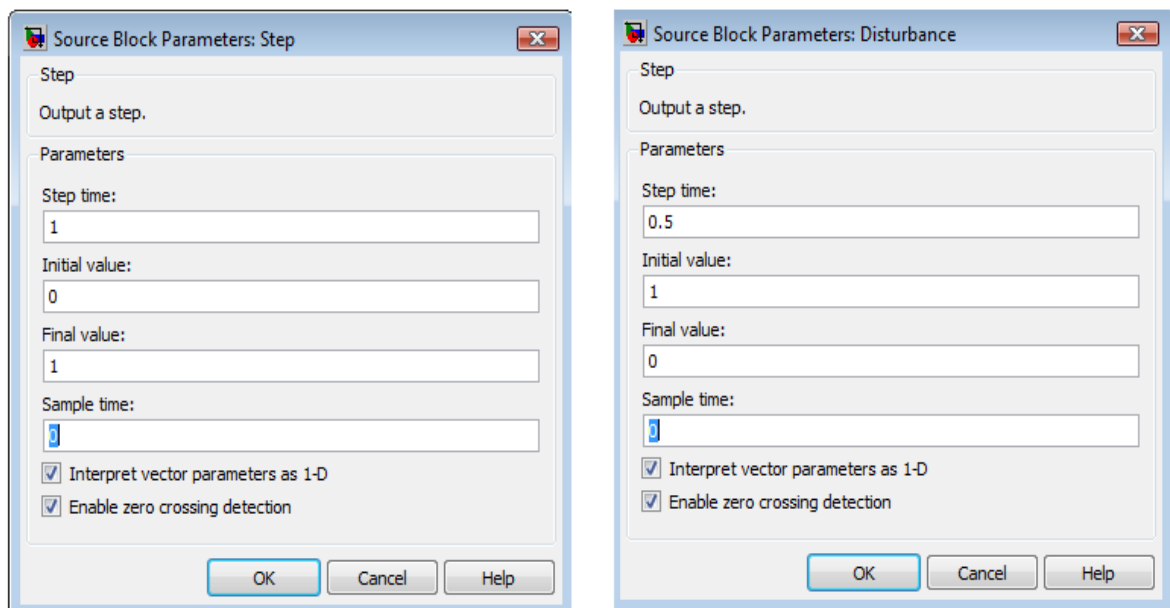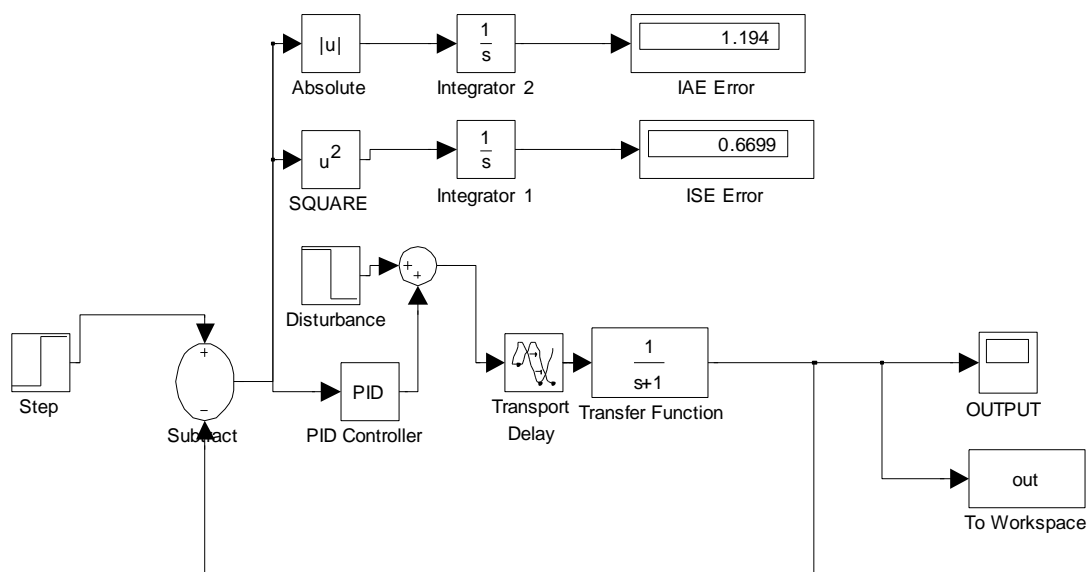

4.1.5 Simulink Model:-

Fig 4.1

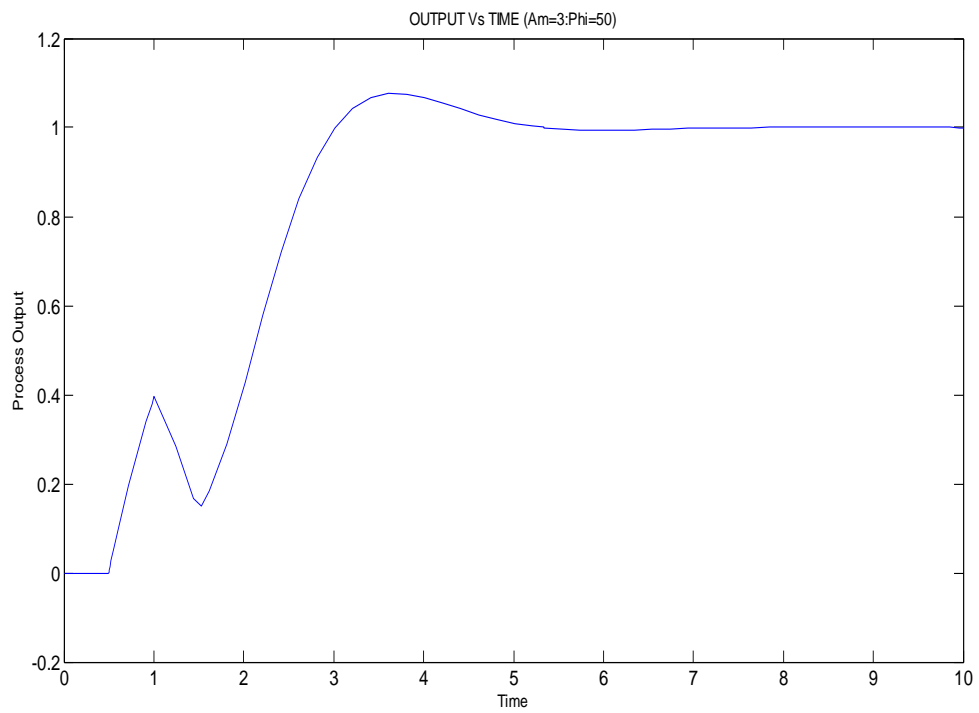Output of the process with PI controller is given below:

OUTPUT Vs TIME (Am=3:Phi=50)

Fig 4.2

4.1.6 Project Report:-

TABULATION – 4.1:-

α G.M. (gain margin) In db.	φ P.M. (phase margin) In degrees.	Kc (Proportional Constant)	Ki (integral constant)	Ti (Ti = Kp / Ki)	ISE Integral square error	φ^* (actual P.M) In degrees.	α^* (actual G.M) In db.
2.0 (6.02db)	30	1.3719	2.8252	0.5900	0.9446	27.5	5.17
	35	1.4221	2.1878	0.6500	0.7107	30.7	5.40
	40	1.4890	1.5458	0.7632	0.6263	36.1	5.72
	45	1.5710	1.5710	1.0000	0.6062	45.0	6.08
	50	1.6110	1.0739	1.5001	0.6031	50.6	6.24
	55	-	-	-	-	-	-
	60	-	-	-	-	-	-
	65	-	-	-	-	-	-
	70	-	-	-	-	-	-

TABULATION – 4.2 :-

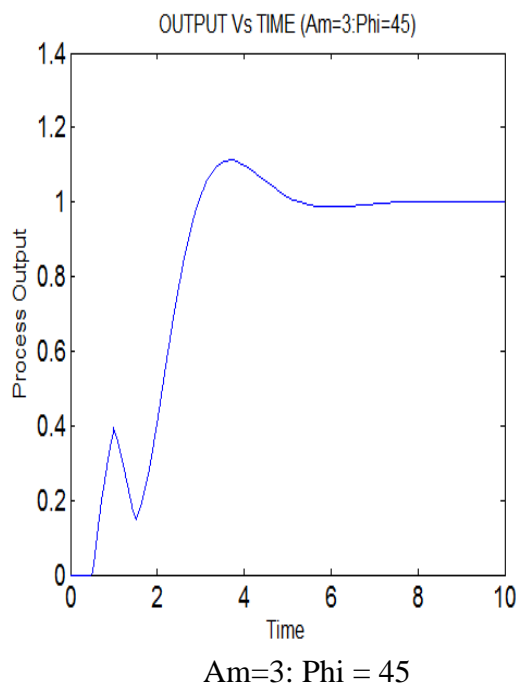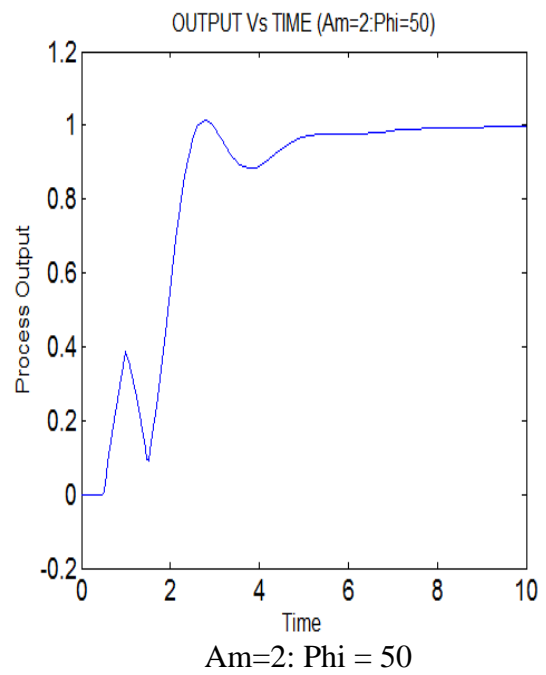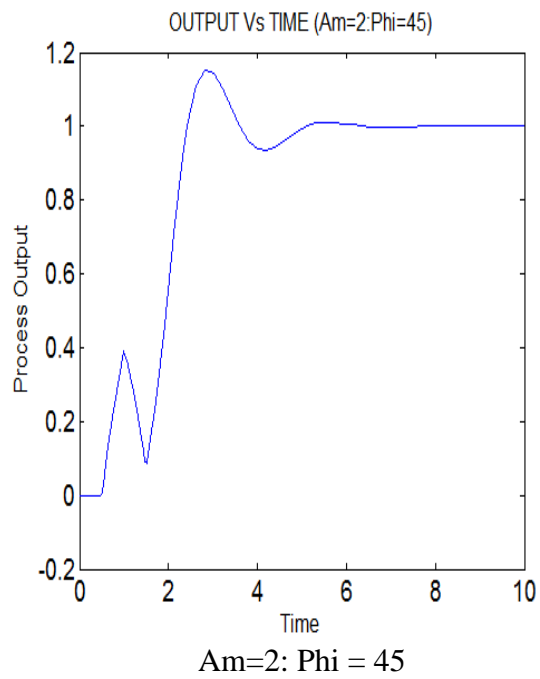| α G.M. (gain margin) In db.	φ P.M. (phase margin) In degrees.	Kc (Proportional Constant)	Ki (integral constant)	Ti (Ti = Kp / Ki)	ISE Integral square error	φ^* (actual P.M) In degrees.	α^* (actual G.M) In db.
3.0 (9.54db)	30	0.9146	1.5502	0.59	0.7492	40.4	8.69
	35	0.9379	1.4887	0.63	0.6930	42.6	8.85
	40	0.9574	1.4289	0.67	0.6838	44.9	8.99
	45	0.9778	1.3580	0.72	0.6801	47.5	9.13
	50	1.0080	1.2759	0.79	0.6699	51.0	9.30
	55	1.0240	1.1636	0.88	0.6790	55.1	9.46
	60	1.0470	1.0470	1.00	0.6742	60.0	9.61
	65	1.0732	0.9018	1.19	0.6837	66.6	9.76
	70	1.0994	0.7378	1.49	0.7193	74.9	9.90

TABULATION – 4.3:-

α G.M. (gain margin) In db.	φ P.M. (phase margin) In degrees.	Kc (Proportional Constant)	Ki (integral constant)	Ti (Ti = Kp / Ki)	ISE Integral square error	φ* (actual P.M) In degrees.	α* (actual G.M) In db.
4.0 (12.04db)	30	-	-	-	-	-	-
	35	0.7034	1.1165	0.63	0.7718	50.5	11.3
	40	0.7181	1.0568	0.67	0.7667	52.5	11.5
	45	0.7333	1.0184	0.72	0.7666	55.1	11.6
	50	0.7437	0.9735	0.76	0.7742	57.1	11.7
	55	0.7549	0.9319	0.81	0.7763	59.5	11.8
	60	0.7680	0.8727	0.88	0.7829	62.6	12.0
	65	0.7788	0.8197	0.95	0.7921	65.5	12.0
	70	0.7913	0.7536	1.05	0.8085	69.4	12.2

TABULATION – 4.4:-

α G.M. (gain margin) In db.	φ P.M. (phase margin) In degrees.	Kc (Proportional Constant)	Ki (integral constant)	Ti (Ti = Kp / Ki)	ISE Integral square error	φ* (actual P.M) In degrees.	α* (actual G.M) In db.
5.0 (13.97db)	30	-	-	-	-	-	-
	35	0.5627	0.8931	0.63	0.8734	55.7	13.3
	40	0.5744	0.8573	0.67	0.8752	57.4	13.8
	45	0.5844	0.8230	0.71	0.8791	59.8	13.5
	50	0.5950	0.7828	0.76	0.8863	62.2	13.7
	55	0.6039	0.7455	0.81	0.8955	64.4	13.8
	60	0.6116	0.7111	0.86	0.9062	66.6	13.9
	65	0.6183	0.6794	0.91	0.9183	68.6	13.9
	70	0.6252	0.6445	0.97	0.9341	70.9	14.0

4.1.7 Graphs and plots

Am=2: Phi = 45

Am=2: Phi = 50

Am=3: Phi = 45

Am=3: Phi = 60

Am=4: Phi = 50   Am=4: Phi = 60

Am=5: Phi = 50   Am=5: Phi = 60
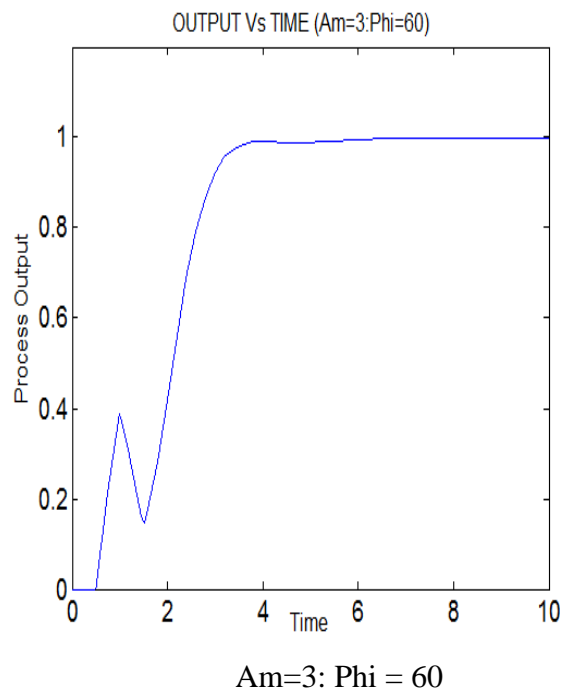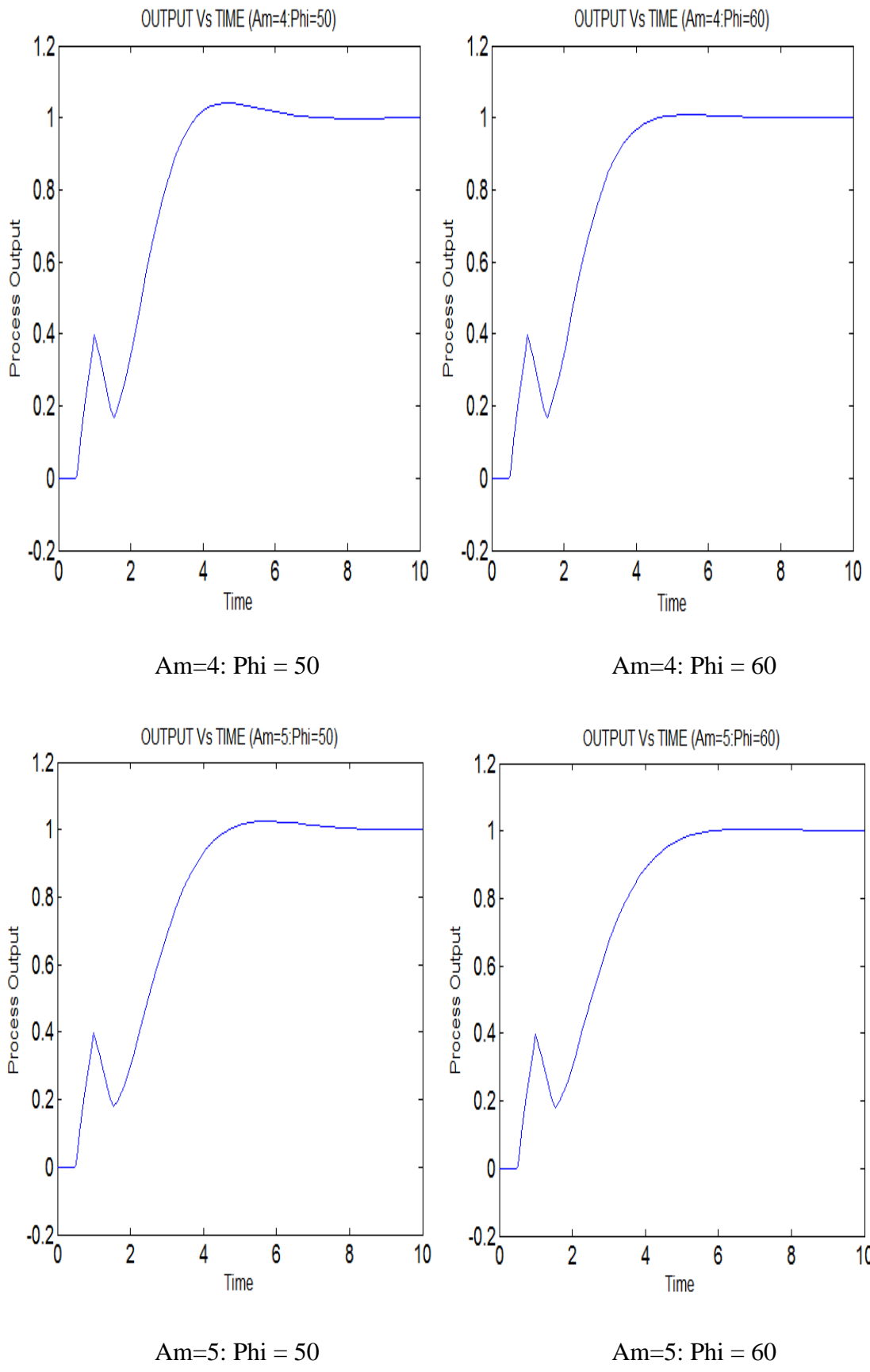
Fig 4.3

4.2 METHOD – II :- Gain-Phase margin tester method

4.2.1 Introduction :-

Gain margin and Phase margin plays an important role concerning the robustness of a system. This method is being applied to a non-minimum phase plant containing an uncertain delay time with specifications in terms of gain and phase. The gain-phase margin tester method is adopted to test the stability boundary in the parameter plane (2-D) for any given gain or phase margin specification. These margins serve as restriction to scheduling the controller. Such method guarantees both relative and absolute stability margin. Such parameter area in 2-D is used to achieve compromise between good tracking performance and system robustness with respect to external disturbance [14].

4.2.2 Advantage:-

- System performance resulting from such tuning can be realized completely.
- Especially when delay time is uncertain, this method works effectively well [14].
- It avoids extensive or unnecessary on-line tuning and results in easier implementation.
- It can be applied to both stable and unstable systems of higher order.

4.2.3 Methodology:-

For a first order plus delay time non-minimum system, its transfer function is shown as follows:-

$$G_p(s) = (\frac{K}{1+sT})e^{-sL}$$

(1)

Where, K = a constant.

T = 'tao'- time constant.

L = delay time constant.

Notations:

Gp(s) = process transfer function.

Gc(s) = controller transfer function.

D(s) = external disturbance transfer function.

An Error- actuated PID controller has the transfer function:

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \qquad . \qquad (2)$$

So, the forward open-loop transfer function is-

$$G_o(s) = Gc(s)*Gp(s) = N(s)/D(s). \qquad (3)$$

By putting $s = jw$ in equation (3), we get,

$$G_o(s) = |G_o(jw)| e^{j\theta} \qquad (4)$$

Now putting eqn. (4) in eqn. (3) we get ,

$$D(jw) - \frac{N(jw)}{|G_o(jw)| e^{j\theta}} \qquad (5)$$

Let,

$$A = 1/|G_o(jw)| \qquad (6)$$

$$\theta = \varphi + 180 \qquad (7)$$

When, $\theta = 0$; A gives the value of the gain margin.

and when, $A = 0$; θ gives the value of phase margin.

We can now define the gain-phase margin tester function as :

$$F(jw) = D(jw) + A\, e^{-j\theta} * N(jw) \qquad (8)$$

Eqn. (8) implies that the function F(jw) should always be equal to zero. This indicates that the G.M. and the P.M. can be calculated from the characteristic equation.

Now by adding a so-called gain-phase margin tester $A\exp(-j\theta)$ into the system as shown in the above fig.1., the characteristic equation is,

$$1 + A\, e^{-j\theta} * (Kp + \frac{Ki}{s} + K_d s) * (\frac{K}{1+sT} e^{-sL}) = 0 \qquad (9)$$

Now putting ,$A \exp(-j\theta) = A \cos\theta - jA \sin\theta$, $K = 1$, $T = 1$,eqn. (8) and (9) give rise to –

$$F(jw) = Xa(jw) + A(\cos(\theta+wL) - j \sin(\theta+wL))*[KpX_b(jw) + K_iX_c(jw) + K_dX_d(jw)] \quad (10)$$

Where,

$$X_a(jw) = -w^2+jw \; ; \tag{11}$$
$$X_b(jw) = jw \; ; \tag{12}$$
$$X_c(jw) = 1 \; ; \tag{13}$$
$$X_d(jw) = -w^2 \; ; \tag{14}$$

Taking eqn. (10) into consideration we obtain two more eqn. by separating all real and imaginary part of $F(jw)$ and by putting $\theta_1 = (\theta+wL)$, $K_d = 0$, we get,

$$
\begin{aligned}
F_r(jw) &= K_pB_1 + K_iC_1 + D_1 \\
&= Re(X_a) + A \cos\theta_1(K_pRe(X_b) + K_iRe(X_c) + K_dRe(X_d)) \\
&\quad + A \sin\theta_1(K_pIm(X_b) + K_iIm(X_c) + K_dIm(X_d)) = 0 \; ;
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
F_r(jw) &= K_pB_2 + K_iC_2 + D_2 \\
&= Im(X_a) + A \cos\theta_1(K_pIm(X_b) + K_iIm(X_c) + K_dIm(X_d)) \\
&\quad - A \sin\theta_1(K_pRe(X_b) + K_iRe(X_c) + K_dRe(X_d)) = 0 \; ;
\end{aligned}
\tag{16}
$$

Where,

$$
\begin{aligned}
B_1 &= A \cos\theta_1Re(X_b) + A \sin\theta_1Im(X_b) \tag{17} \\
&= wA \sin\theta_1
\end{aligned}
$$

$$
\begin{aligned}
C_1 &= A \cos\theta_1Re(X_c) + A \sin\theta_1Im(X_c) \tag{18} \\
&= A \cos\theta_1
\end{aligned}
$$

$$
\begin{aligned}
D_1 &= Re(X_a) + A \cos\theta_1K_dRe(X_d)+ A \sin\theta_1K_dIm(X_d) \tag{19} \\
&= -w^2 - w^2 K_d A \cos\theta_1
\end{aligned}
$$

`

$$
\begin{aligned}
B_2 &= A \cos\theta Im(X_b) - A \sin\theta_1Re(X_b) \tag{20} \\
&= w A \cos\theta_1
\end{aligned}
$$

$$C_2 = A \cos\theta_1Im(X_c) - A \sin\theta_1K_iRe(X_c) \tag{21}$$

$$= - A \sin\theta_1$$

$$D_2 = Im(X_a) + A \cos\theta_1 K_d Im(X_d) - A \sin\theta_1 K_d Re(X_d) \qquad (22)$$

$$= w$$

Solving eqn. (15) and (16) we get the value of K_p and K_i as,

$$K_p = (C_1 D_2 - C_2 D_1) / (B_1 C_2 - B_2 C_1) \qquad (23)$$

$$K_i = (D_1 B_2 - D_2 B_1) / (B_1 C_2 - B_2 C_1) \qquad (24)$$

Parameter Pane analysis:-

By varying one of the parameters, A, θ, and w, and fixing the others , it suffices to plot the constant gain margin boundary (A = constant , $\theta = 0$, w is varied over a range) and the constant phase margin boundary (A = 0 , θ = constant, w is varied over a range) in the parameter plane. The above locus representing the stability boundary of the system is plotted in the K_p-K_i plane. Region left of the stability boundary, facing the direction in which 'w' increases, is the "stable parameter area". The region characterizes all feasible control parameter sets which guarantees the controlled system "robust margin" i.e. G.M. and P.M. of the system [14].

4.2.4 Computation:-

The procedure followed to calculate the value of K_p and K_i corresponding to the specified G.m. and P.m using the K_p-K_i plot is given below. For simplicity we have taken a example of a case were,

$$A = 3;$$
$$\theta = 50;$$

4.2.5 Algorithm:-

Step 1: In the MATLAB, M-file ' **j1_final2.m**', all the known values corresponding to the given FOPDT system is entered. Code for the case considered is :

MATLAB CODE:-

```
    % PART ------------------> 1
% PM varying with a Constant GM Kp Vs Ki
```

```matlab
% for theta 50
A=1.0;
theta=50.0;
T=0.5;
Kd=0.0;
format short;
phi=(pi/180)*theta;
i1=1;
for w=0.7:0.01:10.8
B1=A*sin(phi+w*T)*w;
C1=A*cos(phi+w*T);
D1=-(w^2)-A*cos(phi+w*T)*Kd*w^2;
B2=A*cos(phi+w*T)*w;
C2=-A*sin(phi+w*T);
D2=w;
Ki(i1)=(D1*B2-D2*B1)/(B1*C2-B2*C1);
Kp(i1)=(C1*D2-C2*D1)/(B1*C2-B2*C1);
i1=i1+1;
end;
plot(Kp,Ki,'b')
xlabel('Kp'); ylabel('Ki');
hold on

% % % PART ---------------------------------------->2
% % % GM varying with constant PM Kp Vs Ki
% %A=3.0;
A=3.0;
theta=0.0;
T=0.5;
Kd=0.0;
format short;
phi=(pi/180)*theta;
i1=1;
for w=0.7:0.01:10.8
B1=A*sin(phi+w*T)*w;
C1=A*cos(phi+w*T);
D1=-(w^2)-A*cos(phi+w*T)*Kd*w^2;
B2=A*cos(phi+w*T)*w;
C2=-A*sin(phi+w*T);
D2=w;
Ki(i1)=(D1*B2-D2*B1)/(B1*C2-B2*C1);
Kp(i1)=(C1*D2-C2*D1)/(B1*C2-B2*C1);
i1=i1+1;
end;
plot(Kp,Ki,'b:')
xlabel('Kp'); ylabel('Ki');
hold on

hold off
% PART ---------------------------------------->DISPLAY PART
xlabel('Kp'); ylabel('Ki');
title(' PLOT of Kp Vs Ki w- 0 to 10.8')
```

Step 2: The M-file ' **j1_final2.m**' is run in the editor window. A plot is obtained between K_p Vs K_i for a specified range of 'w'. The plot is checked for the point of intersection.
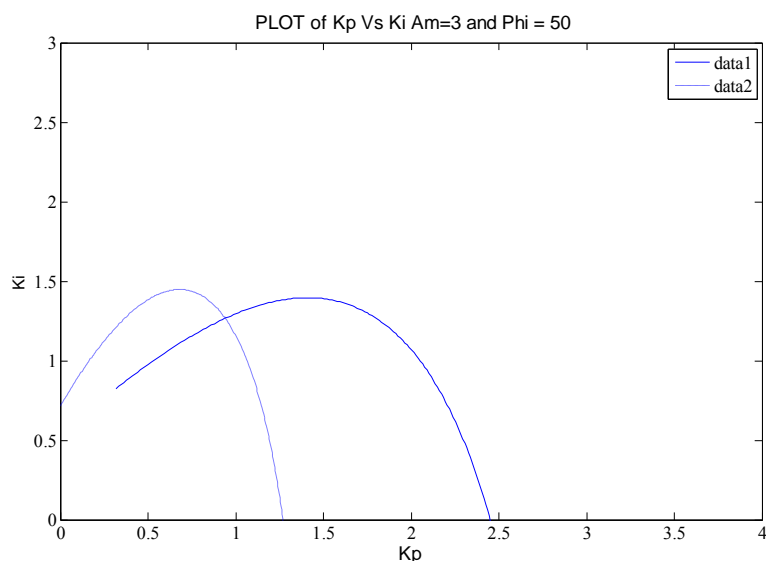
Fig 4.4

Step 3: The point of intersection of the locus in the plot of K_p Vs K_i is located using a command "ginput(1)" in the MATLAB command window. The window below shows the same:

MATLAB Command Window:

>> ginput(1)

ans =

0.9405   1.2666
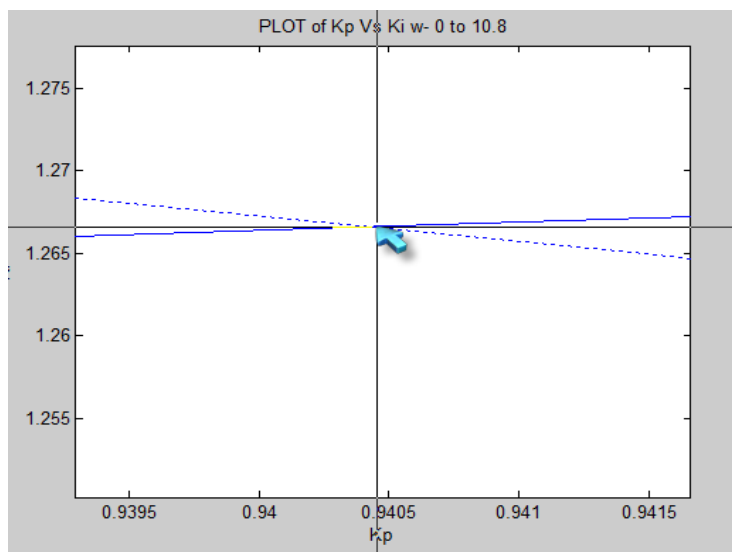
Step 4: The value obtained in step-3 gives the values of K_p and K_i respectively for the specified parameters. This value is put in the function "exp2" written in M-file 'exp2.m'.

To obtain the system transfer function i.e. $G_o(s) = G_c(s) * G_p(s)$.

MATLAB Code:-

```
function tf = exp2(Kp,Ki)
syms s
tf1=(s^4+13*s^3+60*s^2+48*s);
vpa(tf1,4);
tf=(s^3*Kp+s^2*(Ki-12*Kp)+s*(48*Kp-12*Ki)+48*Ki)/(tf1);
```

OUTPUT:-

>> exp2(0.9405,1.2666)

ans =

(1881/2000*s^3-50097/5000*s^2+37431/1250*s+37998/625)/(s^4+13*s^3+60*s^2+48*s)

>> vpa(ans,4)

ans =

(.9405*s^3-10.02*s^2+29.94*s+60.80)/(s^4+13.*s^3+60.*s^2+48.*s)

Step 5: The system transfer function obtained in the step-4 is used to check for the actual G.M. and P.M. using the Matlab function "sisotool(sys)". The code is given below,

MATLAB Code:

>> num=[0.9405 -10.02 29.94 60.80]

num =

  0.9405 -10.0200 29.9400 60.8000

>> den=[1 13 60 48 0]

den =

  1   13   60   48   0

>> sys=tf(num,den)

Transfer function:

0.9405 s^3 - 10.02 s^2 + 29.94 s + 60.8

----

    s^4 + 13 s^3 + 60 s^2 + 48 s

>> sisotool(sys)

OUTPUT:-
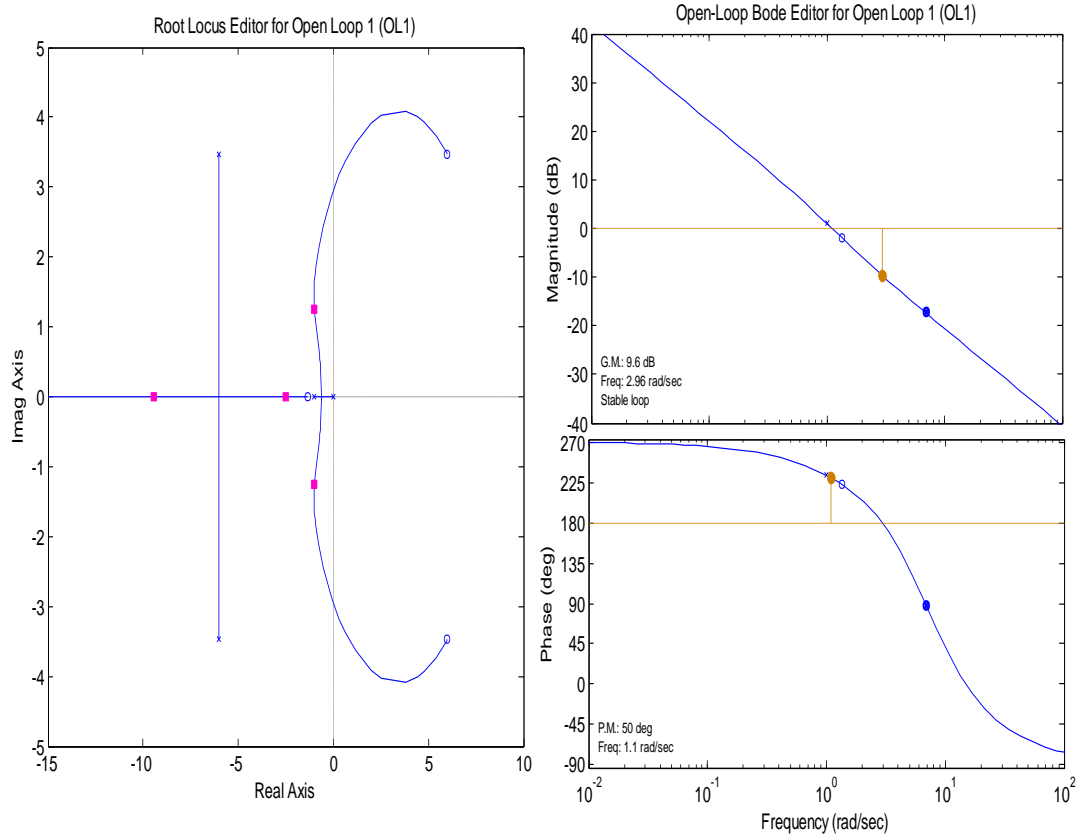
Step 6: Actual values of G.M and P.M. are tabulated using the symbol *(asterisk) as a superscript. All the data obtained above are also tabulated.

Step 7: The value of 'ISE' error for the given specification can be obtained using SIMULINK. Using it a model is designed to display the values of :

- ISE Error
- IAE Error
- Output of the PROCESS.

The above values are calculated with *step input* and *step load disturbance* .
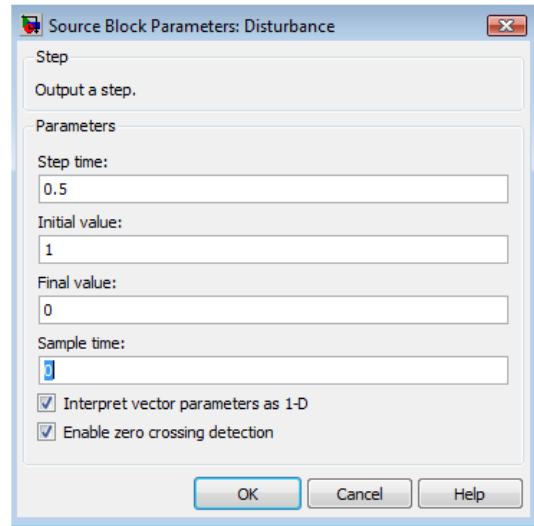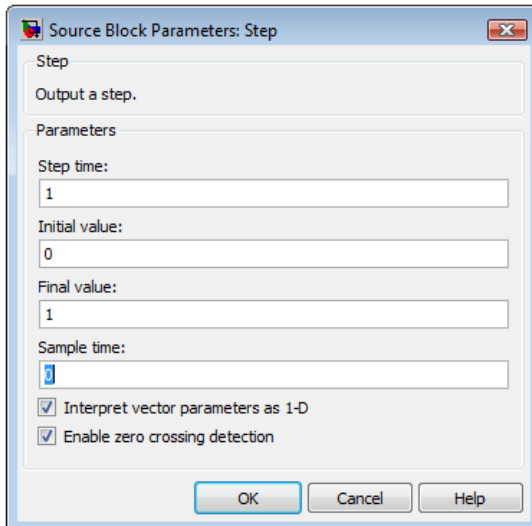
4.2.6 Simulink Model:-
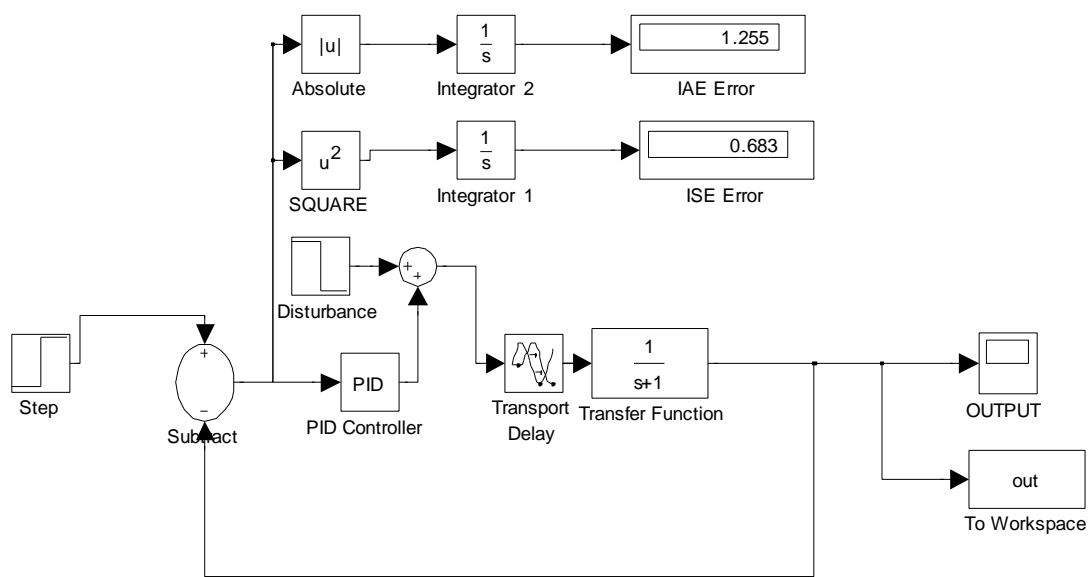
Fig 4.5

OUTPUT:-

4.2.7 Project Report:-

TABULATION – 4.5:-

| α G.M. (gain margin) In db.	φ P.M. (phase margin) In degrees.	Kc (Proportional Constant)	Ki (integral constant)	Ti (Ti = Kp / Ki)	ISE Integral square error	φ* (actual P.M) In degrees.	α* (actual G.M) In db.
	30	1.1811	2.1327	0.553	0.7483	29.6	6.05
	35	1.3597	1.9716	0.689	0.6604	35.0	6.08
	40	1.4790	1.7800	0.830	0.6329	40.0	6.08
	45	1.5720	1.5762	0.997	0.6072	44.9	6.07
2.0 (6.02db)	50	1.6512	1.3569	1.216	0.5906	49.8	6.05
	55	1.7070	1.1376	1.500	0.5868	54.8	6.08
	60	1.7602	0.9046	1.945	0.6199	59.8	6.09
	65	1.8085	0.6447	2.800	0.6738	65.0	6.10
	70	1.8540	0.3647	5.080	0.8241	70.0	6.11

TABULATION – 4.6-

α G.M. (gain margin) In db.	φ P.M. (phase margin) In degrees.	Kc (Proportional Constant)	Ki (integral constant)	Ti (Ti = Kp / Ki)	ISE Integral square error	φ* (actual P.M) In degrees.	α* (actual G.M) In db.
3.0 (9.54db)	30	-	-	-	-	-	-
	35	0.4875	1.3746	0.3546	0.5906	35.0	9.57
	40	0.7418	1.4391	0.5150	0.7721	40.0	9.58
	45	0.8600	1.3669	0.6290	0.7369	45.0	9.59
	50	0.9404	1.2666	0.7424	0.6830	50.0	9.62
	55	1.004	1.1574	0.8670	0.6874	55.1	9.58
	60	1.0472	1.0472	1.0000	0.6714	60.0	9.61
	65	1.0848	0.9398	1.1543	0.6745	65.0	9.61
	70	1.1157	0.8363	1.3340	0.6876	70.0	9.61

TABULATION – 4.7:-

α G.M. (gain margin) In db.	φ P.M. (phase margin) In degrees.	Kc (Proportional Constant)	Ki (integral constant)	Ti (Ti = Kp / Ki)	ISE Integral square error	φ* (actual P.M) In degrees.	α* (actual G.M) In db.
4.0 (12.04db)	30	-	-	-	-	-	-
	35	-	-	-	-	-	-
	40	-	-	-	-	-	-
	45	0.4816	1.0836	0.4440	0.9034	45.0	12.1
	50	0.6061	1.0563	0.5730	0.8184	50.0	12.1
	55	0.6789	0.9871	0.6870	0.8000	55.0	12.1
	60	0.7303	0.9075	0.8047	0.7907	60.0	12.1
	65	0.7678	0.8223	0.9337	0.7957	65.0	12.1
	70	0.7998	0.7458	1.0720	0.8074	70.0	12.1

TABULATION – 4.8:-

α G.M. (gain margin) In db.	φ P.M. (phase margin) In degrees.	Kc (Proportional Constant)	Ki (integral constant)	Ti (Ti = Kp / Ki)	ISE Integral square error	φ* (actual P.M) In degrees.	α* (actual G.M) In db.
5.0 (13.97db)	30	-	-	-	-	-	-
	35	-	-	-	-	-	-
	40	-	-	-	-	-	-
	45	-	-	-	-	-	-
	50	0.3579	0.8599	0.4160	1.0180	50.0	14.0
	55	0.4792	0.8486	0.5640	0.9334	55.0	14.0
	60	0.5421	0.7912	0.6850	0.59124	60.0	14.0
	65	-	-	-	-	-	-
	70	-	-	-	-	-	-

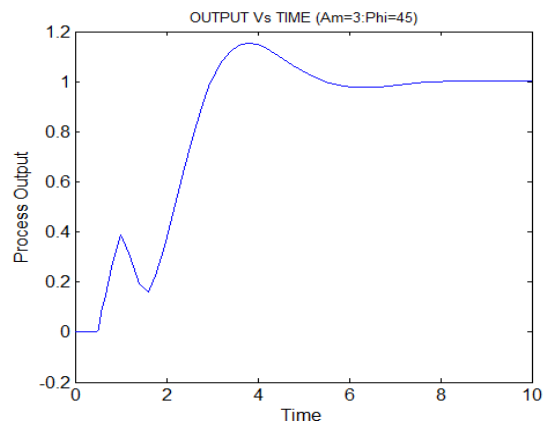**4.2.8 Graphs and Plots:-**

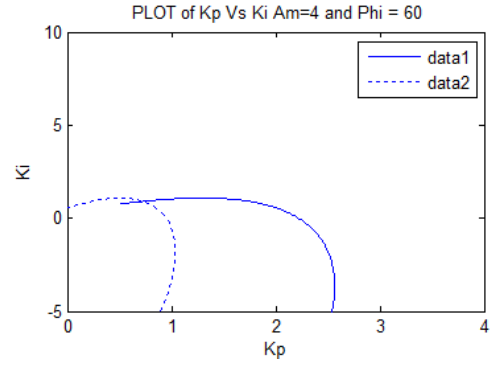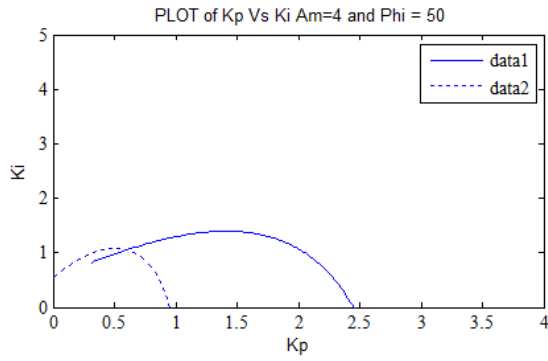NOTE: Here, Data 1 – Phase margin; Data 2 – Gain margin
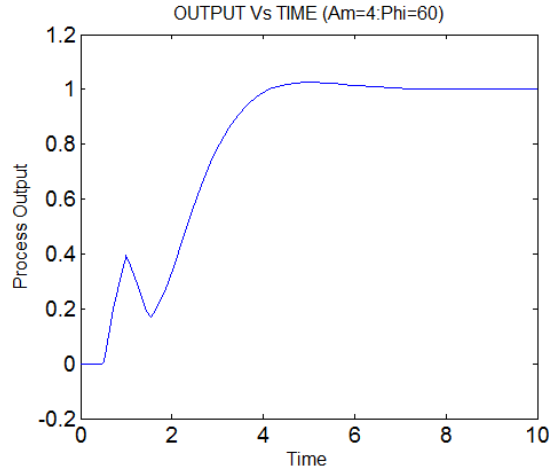PLOT:-
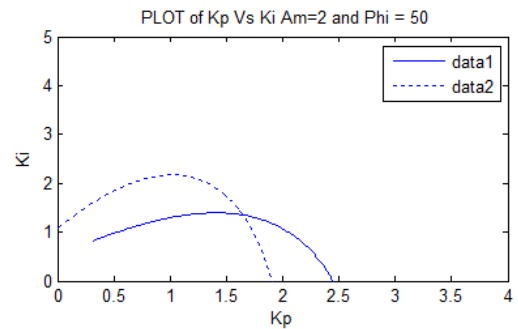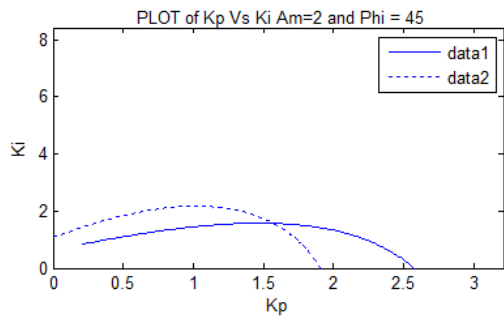
Corresponding Outputs:
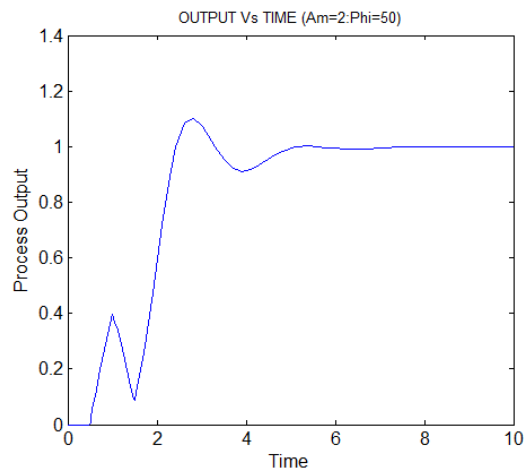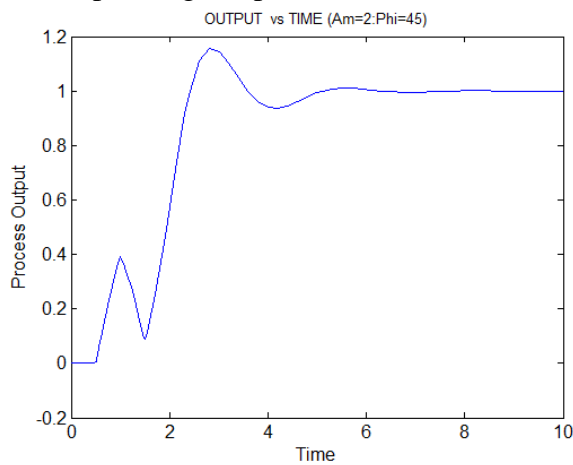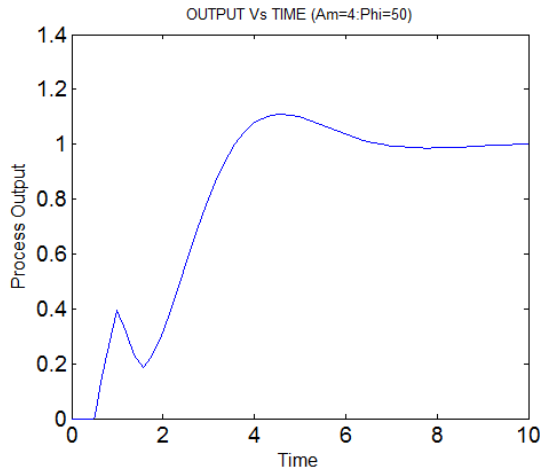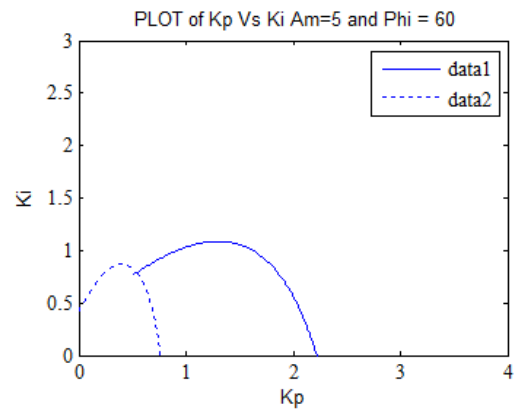

PLOT:-
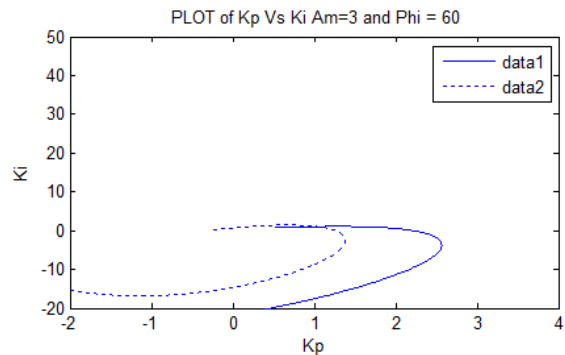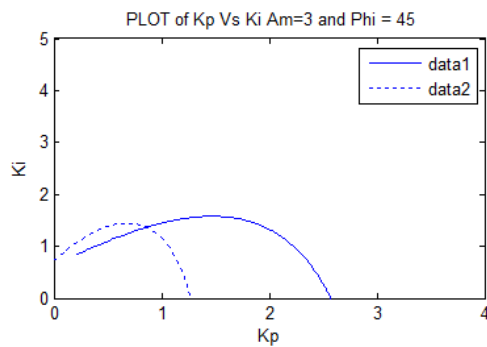
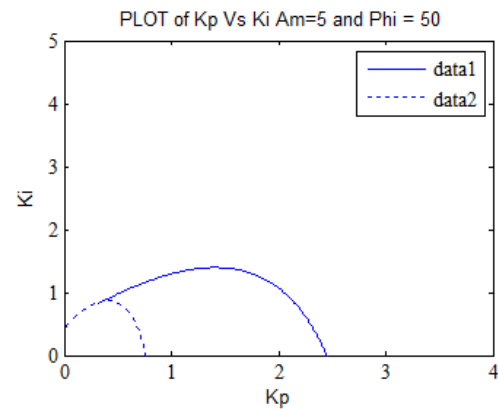Corresponding Output:
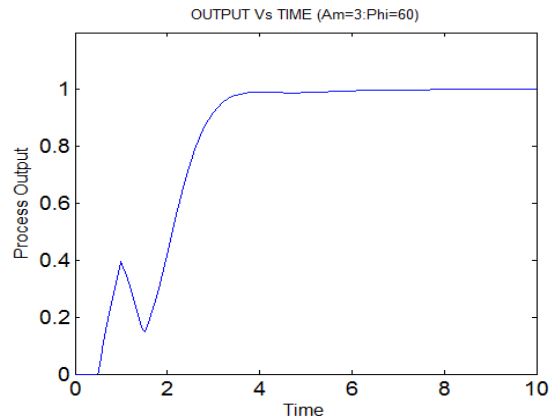
PLOT:-

Corresponding Output:-
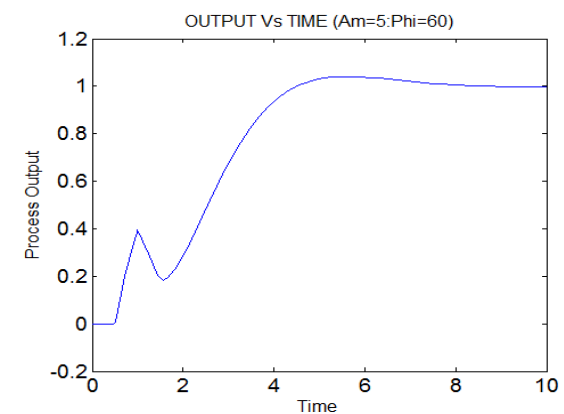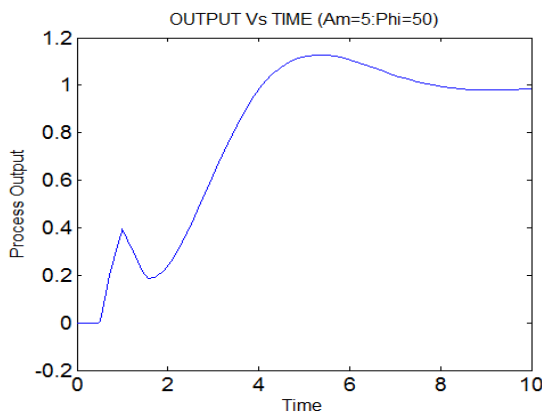

PLOT:-

Corresponding Output:-

Fig 4.7

RESULT OF THE ANALYSIS AND COMPARISION

Reliability:-

Method -2 is more reliable than Method – 1. The result obtained in the Method – 2 is almost near to accurate with minimal error. Method – 1 doesn't show desired result for low values of Phase margin.

Computational Simplicity:-

We can easily tune PID controller using Method – 1 given the T_i range is correctly chosen. Though there is some ambiguity in the results for some particular values of gain and phase margin. Citing one of such examples is the parameters value when gain margin is equal to 2 and phase margin is equal to 60. The tuning procedure may get cumbersome if the T_i is not correctly chosen.

Whereas method -1 though quite lengthy is easy to compile and the results obtained is almost satisfactory. The fact that data is being taken from a plot make it more valid.

Performance:-

The performance of both the method could be compared on the basis of ISE performance index value. The lower the values, better is the performance of the particular method.

From the tabulation it is quite clear that Method-2 gives lower values of ISE performance index values for most of the cases. This stands as a prove to the fact that Method -2 performs better than the Method-1.

Scope of Application:-

Method-2 has a wide range of scope of application. Two factors given below support the fact that this method is more flexible then the latter one.

- It's applicable to both lower order as well as higher orders of process transfer function.
- It gives more robust design parameter for a non-minimum phase system with uncertain delay time.

Method-1 has got some limitation. They are:
- They are applicable to process having L/T value within a particular range i.e. 0.1 - 1.0.
- They are specified for only FOPDT process system and doesn't hold good for higher order of process system

Validity of the Procedure:-

Both the method described above is based on well-grounded established theory. Method-1 is based on gain and phase margin specification as formulated by Astrom and Hagglund in 1995. But as the process is based on numerical analysis, it is not accurate. The fact that we are searching for the value of T_i that gives the minimum value on the L.H.S., we may not determine the exact value T_i desired.

Method-2 is based on the work of Shenton and Shafiei (1994) on graphical technique for calculating PID control parameters. The procedure is based on graphical approach, thus its validity can't be questioned. This very fact helps to determine the exact value of PID controller parameter i.e. value of K_p and K_i.

CONCLUSION AND FUTURE SCOPE

(Process Control Simulator)PCS327 may be used at high speed for oscilloscope observation or at a low speed for meter or chart recorder observation, speed selection being achieved by independent controls on the process and the Controller [6]. When the controller is set for low speed use it is suitable for the application of three-term control to the Feedback process trainer PT326.The equipment is fully compatible with all Feedback signal sources and all results may be observed on any laboratory oscilloscope with direct coupled horizontal and vertical amplifiers. Some additional features of the equipment are :-

- Simulated Distance-velocity lag(transport lag)
- Self-contained power supplies
- Integrated-circuit reliability
- Student-proof design

The fact that since last four decades there has been innumerable paper published on PID control design speak for itself the importance and demand of such controller in present day modern industry scenario. Despite of a huge number of theoretical and application papers on tuning techniques of PID controllers, this area still remains open for further research. There is a great scope of research into this field of control system. But what lacks is the comparative analysis between different tuning techniques. This study would thus surely come handy to such need of comparative analysis and also help in understanding the changing trends in the field of PID controller design. Few of the recent trends in the field of PID control design are optimal design through graphical approach and minimisation of error due to approximation in numerical analysis technique.

BIBLIOGRAPHY

[1] Shenton, A.T., & Shafiei, Z., Relative stability for control system with adjustable parameters, J. of guidance, Control and Dynamics 17(1994) 304-310.

[2] Xue Dingyu, Chen Yang Quan and Atherton P. Derek ,"Linear Feedback Control".

[3] Ogata Katsuhiko, Modern control Engineering, fourth edition, 2002.

[4] Kuo C. Benjamin, Automatic Control System, seventh edition, October 2000.

[5] Wikipedia.org

[6] Process Control Simultor PCS 327 Manual,Feedback Instruments.

[7] Ho, W. K., Hang C. C. & Cao L. S., Tuning of PID controllers based on gain and phase margin specifications. Automatica, 31(3)(1995),497-502.

[8] Astrom, K. J., & Hagglund, T. Automatic tuning of simple regulators with specifications on phase and amplitude margins. *Automatica*, *20*(1984), 645-651

[9] Ziegler, J. G., & Nichols, N. B. Optimum settings for automatic controllers. Transactions of the ASME, 64(1942), 759-768.

[10] Ho, W. K., Hang, C. C., Zhou, J. H., Self-tuning PID control of a plant with under-damped response with specifications on gain and phase margins. IEEE Transactions on Control Systems Technology, 5 (4) (1997), 446-452.

[11] Ho, W.K., Xu, W., PID tuning for unstable processes based on gain and phase-margin specifications, IEE Proc. – Control Theory and Appl. 145 (5) (1998) 392-396.

[12]] Ho, W. K., Lim, K. W., & Xu, W., Optimal gain and phase margin tuning for PID controllers. Automatica, 34(8) (1998), 1009-1014.

[13] Ho, W.K., Lim K.W. , Hang C.C. , Ni L.Y. , getting more phase margin and performance out of PID controllers. Automatica, 35 (1999), 1579-1585.

[14] Huang J. Ying, Wang Yuan-Jay, Robust PID Controller Design for Non-minimum phase time delay systems, ISA Transaction 40 (2001) 31-39.

[15] Kealy Tony and O'Dwyer Aidan, Analytical ISE Calculation and Optimum Control System Design, Proceedings of the Irish Signals and Systems Conference, University of Limerick, July 2003, pp. 418-423.

[16] Wen Tan, Jizhen Liu, Tongwen Chen, Horacio J.Marquez , Comparison of some well-known PID tuning formulas, Computers and Chemical Engineering 30 (2006) 1416–1423.

[17] Araki M., Control Systems, Robotics, and Automation – Vol. II - PID Control -, Kyoto University, Japan.