# POWER SYSTEM STABILITY STUDIES *USING MATLAB*

*A Project Report Submitted in partial fulfillment of the requirements*

*for the degree of*

*Bachelor of Technology in Electrical Engineering*

*By*

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Under the guidance of

**PROF. P.C. PANDA**

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# NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

## CERTIFICATE

This is to certify that the project entitled "**POWER SYSTEM STABILITY USING MATLAB**" submitted by **Ms. Pranamita Basu ( Roll No. 10502064 ) and Ms. Aiswarya Harichandan (Roll No. 10502019** in partial fulfillment of the requirements for the award of **Bachelor of Technology Degree** in **Electrical Engineering** at **NIT Rourkela** is an authentic work carried out by them under my supervision and guidance.

Date:

Rourkela

(Prof. P. C. Panda)
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# ACKNOWLEDGEMENT

I would like to thank NIT Rourkela for giving me the opportunity to use their resources and work in such a challenging environment. .

First and foremost I take this opportunity to express my deepest sense of gratitude to my guide Prof. P. C. Panda for his able guidance during my project work. This project would not have been possible without his help and the valuable time that he has given me amidst his busy schedule.

I would also like to extend my gratitude to my friends and senior students of  this department who have always encouraged and supported me in doing my work.

Last but not the least I would like to thank all the staff members of Department of Electrical Engineering who have been very cooperative with us.

**Pranamita Basu**

**Aiswarya Harichandan**

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# ABSTRACT

The stability of an interconnected power system is its ability to return to normal or stable operation after having been subjected to some form of disturbance. With interconnected systems continually growing in size and extending over vast geographical regions, it is becoming increasingly more difficult to maintain synchronism between various parts of the power system.

- In our project we have studied the various types of stability- steady state stability, transient state stability and the swing equation and its solution using numerical methods using *MATLAB* and *Simulink* .
- We have presented the solution of swing equation for transient stability analysis using three different methods – Point-by-Point method, Modified Euler method and Runge-Kutta method.
- Modern power systems have many interconnected generating stations, each with several generators and many loads. So our study is not limited to one-machine system but we have also studied multi-machine stability.
- We study the small-signal performance of a machine connected to a large system through transmission lines. We gradually increase the model detail by accounting for the effects of the dynamics of the field circuit. We have analysed the small-signal performance using eigen value analysis.
- Further a more detailed transient stability analysis is done whereby the classical model is slightly improved upon by taking into account the effect of damping towards transient stability response. Characteristics of the various components of a power system during normal operating conditions and during disturbances have been examined, and effects on the overall system performance are analyzed.

# Chapter 1

## INTRODUCTION

# INTRODUCTION

Successful operation of a power system depends largely on the engineer's ability to provide reliable and uninterrupted service to the loads. The reliability of the power supply implies much more than merely being available. Ideally, the loads must be fed at constant voltage and frequency at all times. The first requirement of reliable service is to keep the synchronous generators running in parallel and with adequate capacity to meet the load demand. Synchronous machines do not easily fall out of step under normal conditions. If a machine tends to speed up or slow down, synchronizing forces tend to keep it in step. Conditions do arise, however, such as a fault on the network, failure in a piece of equipment, sudden application of a major load such as a steel mill, or loss of a line or generating unit., in which operation is such that the synchronizing forces for one or more machines may not be adequate, and small impacts in the system may cause these machines to lose synchronism.

A second requirement of reliable electrical service is to maintain the integrity of the power network. The high-voltage transmisssion system connects the generating stations and the load centers. Interruptions in this network may hinder the flow of power to the load. This usually requires a study of large geographical areas since almost all power systems are interconnected with neighboring systems.

Random changes in load are taking place at all times, with subsequent adjustments of generation. We may look at any of these as a change from one equilibrium state to another. Synchronism frequently may be lost in that transition period, or growing oscillations may occur over a transmission line, eventually leading to its tripping. These problems must be studied by the power system engineer and fall under the heading ***"power system stability"***.

# Chapter 2

## STUDY OF SWING EQUATION

## 2.1 STABILITY

The tendency of a power system to develop restoring forces equal to or greater than the disturbing forces to maintain the state of equilibrium is known as "***STABILITY***".

The problem of interest is one where a power system operating under a steady load condition is perturbed, causing the readjustment of the voltage angles of the synchronous machines. If such an occurrence creates an unbalance between the system generation and load, it results in the establishment of a new steady-state operating condition, with the subsequent adjustment of the voltage angles. The perturbation could be a major disturbance such as the loss of a generator, a fault or the loss of a line, or a combination of such events. It could also be a small load or random load changes occurring under normal operating conditions. Adjustment to the new operating condition is called the transient period. The system behavior during this time is called the dynamic system performance, which is of concern in defining system stability. The main criterion for stability is that the synchronous machines maintain synchronism at the end of the transient period.

So we can say that if the oscillatory response of a power system during the transient period following a disturbance is damped and the system settles in a finite time to a new steady operating condition, we say the system is stable. If the system is not stable, it is considered unstable. This primitive definition of stability requires that the system oscillations be damped. This condition is sometimes called asymptotic stability and means that the system contains inherent forces that tend to reduce oscillations. This is a desirable feature in many systems and is considered necessary for power systems. The definition also excludes continuous oscillation from the family of stable systems, although oscillators are stable in a mathematical sense. The reason is practical since a continually oscillating system would be undesirable for both the supplier and the user of electric power. Hence the definition describes a practical specification for an acceptable operating condition. The stability problem is concerned with the behavior of the synchronous machines after a disturbance. For convenience of analysis, stability problems are generally divided into two major categories-steady state stability and transient state stability and transient state stability.

## 2.2 SWING EQUATION

Under normal operating conditions, the relative position of the rotor axis and the resultant magnetic field axis is fixed. The angle between the two is known as the power angle or torque angle. During any disturbance, rotor will decelerate or accelerate with respect to the synchronously rotating air gap mmf, a relative motion begins. The equation describing the relative motion is known as the swing equation.

### Synchronous machine operation:

- Consider a synchronous generator with electromagnetic torque Te running at synchronous speed $\omega_{sm}$.
- During the normal operation, the mechanical torque Tm = Te.
- A disturbance occur will result in accelerating/decelerating torque Ta=Tm-Te (Ta>0 if accelerating, Ta<0 if decelerating).
- By the law of rotation –

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e$$

where J is the combined moment of inertia of prime mover and generator

- $\theta_m$ is the angular displacement of rotor w.r.t. stationery reference frame on the stator
- $\theta_m = \omega_{sm}t+\delta_m$, $\omega_{sm}$ is the constant angular velocity
- Taking the derivative of θm, we obtain –

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

- Taking the second derivative of θm –

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

- Substituting into law of rotation-

$$J\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e$$

- Multiplying $\omega_m$ to obtain power equation

$$J\omega_m\frac{d^2\delta_m}{dt^2} = M\frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e = P_m - P_e$$

Where $P_m$ and $P_e$ are mechanical power and electromagnetic power.

- Swing equation in terms of inertial constant M

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_e$$

- Relations between electrical power angle δ and mechanical power angle $\delta_m$ and electrical speed and mechanical speed

$$\delta = \frac{p}{2}\delta_m, \qquad \omega = \frac{p}{2}\omega_m \quad \text{where } p \text{ is pole number}$$

- Swing equation in terms of electrical power angle δ

$$\frac{2}{p}M\frac{d^2\delta}{dt^2} = P_m - P_e$$

- Converting the swing equation into per unit system

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)}, \quad \text{where } M = \frac{2H}{\omega_s}$$

where H is the inertia constant

# Chapter 3

## STEADY STATE STABILITY

# STEADY STATE STABILITY

The ability of power system to remain its synchronism and returns to its original state when subjected to small disturbances. Such stability is not affected by any control efforts such as voltage regulators or governor.

## 3.1 Analysis of steady-state stability by swing equation

- starting from swing equation

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} = P_m - P_{\max}\sin\delta \qquad P_S = \frac{dP}{d\delta}\Big|_{\delta_0} = P_{\max}\cos\delta_0$$

- introduce a small disturbance $\Delta\delta$
- derivation is from $\delta=\delta_0+\Delta\delta$
- simplify the nonlinear function of power angle $\delta$
- Analysis of steady-state stability by swing equation
- swing equation in terms of $\Delta\delta$

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_m\cos\delta_0\Delta\delta = 0$$

- PS= Pmax cosδ0: the slope of the power-angle curve at δ0, PS is positive when $0^0 < \delta < 90^o$
- the second order differential equation

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_S\Delta\delta = 0$$

- Characteristic equation:

$$s^2 = -\frac{\pi f_0}{H}P_S$$

rule 1: if PS is negative, one root is in RHP and system is unstable

rule 2: if PS is positive, two roots in the jω axis and motion is oscillatory and undamped, system is marginally stable

The oscillatory frequency of the undamped system

## 3.2 Damping torque:

- phenomena: when there is a difference angular velocity between rotor and air gap field, an induction torque will be set up on rotor tending to minimize the difference of velocities
- introduce a damping power by damping torque

$$P_d = D\frac{d\delta}{dt}$$

- introduce the damping power into swing equation
- Characteristic equation:

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_S\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + 2\varsigma\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0$$

- Analysis of characteristic equation

$$s^2 + 2\varsigma\omega_n s + \omega_n^2 = 0$$

- for damping coefficient $\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{HP_S}} < 1$
- roots of characteristic equation

$$\mathrm{s}_1, \mathrm{s}_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

- damped frequency of oscillation

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

- positive damping (1>ζ>0): s1,s2 have negative real part if PS is positive, this implies the response is bounded and system is stable

- Solution of the swing equation

$$\frac{d^2\Delta\delta}{dt^2}+2\varsigma\omega_n\frac{d\Delta\delta}{dt}+\omega_n^2\Delta\delta=0$$

- roots of swing equation

$$\Delta\delta=\frac{\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_d t+\theta),\ \delta=\delta_0+\frac{\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_d t+\theta)$$

- rotor angular frequency

$$\Delta\omega=-\frac{\omega_n\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_d t),\ \omega=\omega_0+\frac{\omega_n\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_d t)$$

- response time constant

$$\tau=\frac{1}{\zeta\omega_n}=\frac{2H}{\pi f_0 D}$$

- settling time: $t_S \cong 4\tau$
- relations between settling time and inertia constant H: increase H will result in longer $t_s$, decrease $\omega_n$ and $\zeta$

**3.3 Illustration:** A 60 Hz synchronous generator having inertia constant H =9.94 MJ/MVA and a transient reactance $X_d$'=0.3 p.u. is connected to an infinite bus through a purely reactive circuit as shown in figure 3.1. Reactances are marked on the diagram on a common system base. The generator is delivering real power of 0.6 p.u., 0.8 power factor lagging to the infinite bus at a voltage of V=1 per unit.

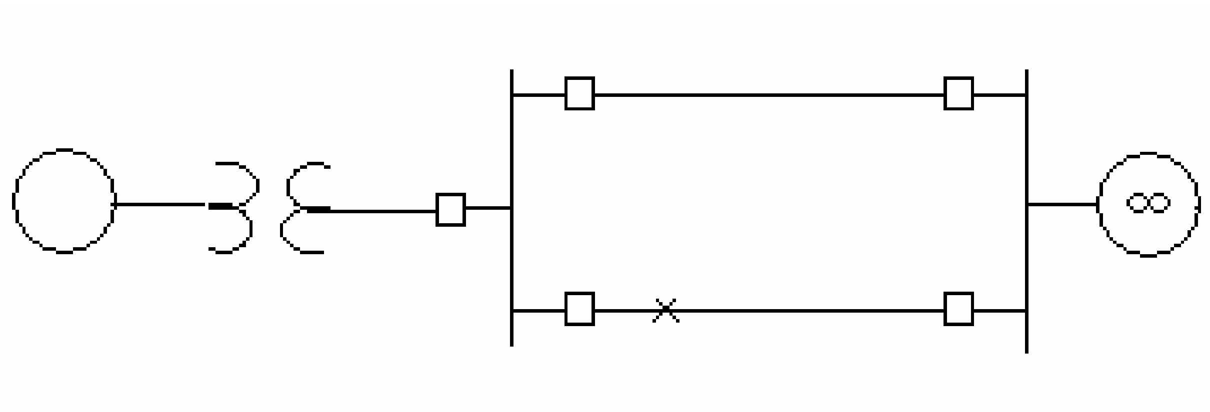

Fig. 3.1 Diagram for steady state stability problem

## 3.4 MATLAB CODE FOR STEADY STATE STABILITY DESIGN

```
global  Pm f H E V X1 X2 X3

Pm = 0.80;  E = 1.17;  V = 1.0;

X1 = 0.65; X2 = 1.80; X3 = 0.8;

H = 5.0; f = 60; tf = 1; Dt = 0.01;

%  Fault is cleared in 0.3 sec.

tc = 0.3;

swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)

%  Fault is cleared in 0.4 sec. and 0.5 sec.

tc = .5;

swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)

tc = .4;

swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)

disp('Parts (a) & (b) are repeated using swingrk4')

disp('Press Enter to continue')

pause

tc = 0.3;
```

```
swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf)

tc = .5;

swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf)

tc = .4;

swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf)
```

## 3.5 Wave form for steady state response

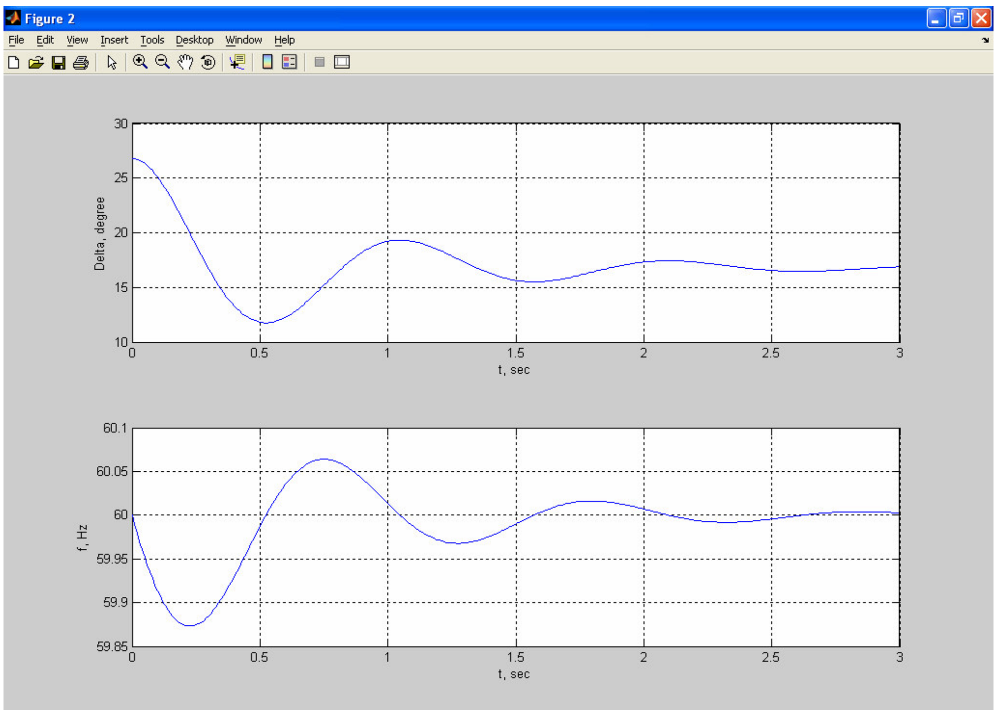


Fig. 3.2 Rotor angle and frequency vs time

# Chapter 4

## TRANSIENT STABILITY STUDIES

The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbance. This may be sudden application of load, loss of generation, loss of large load, or a fault on the system. In most disturbances, oscillations are of such magnitude that linearization is not permissible and the nonlinear swing equation must be solved.

## 4.1 NUMERICAL SOLUTION OF SWING EQUATION

The transient stability analysis requires the solution of a system of coupled non-linear differential equations. In general, no analytical solution of these equations exists. However, techniques are available to obtain approximate solution of such differential equations by numerical methods and one must therefore resort to numerical computation techniques commonly known as digital simulation. Some of the commonly used numerical techniques for the solution of the swing equation are:

- Point by point method
- Euler modified method
- Runge-Kutta method

In our analysis, we have used Euler modified method and Point-by Point Method.

The swing equation can be transformed into state variable form as

$$\frac{d\delta}{dt} = \Delta\omega$$
$$\frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H} P_a$$

We now apply modified Euler's method to the above equations as below

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega^p_{i+1}} = \Delta\omega^p_{i+1}, \quad \text{where} \quad \Delta\omega^p_{i+1} = \Delta\omega_i + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} \Delta t$$
$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta^p_{i+1}} = \frac{\pi f_0}{H} P_a\Big|_{\delta^p_{i+1}} \quad \text{where} \quad \delta^p_{i+1} = \delta_i + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} \Delta t$$

Then the average value of the two derivatives is used to find the corrected values.

$$\delta_{i+1}^{c} = \delta_i + \left( \frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i^p}}{2} \right)\Delta t, \quad \Delta\omega_{i+1}^{c} = \Delta\omega_i + \left( \frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_i^p}}{2} \right)\Delta t$$

This is illustrated in the following design.

**4.2 Illustration**: A 60 Hz synchronous generator having inertia constant, H = 5MJ/MVA and a direct axis transient reactance $X_d$' = 0.3p.u. is connected to an infinite bus through a purely reactive circuit as shown in Fig. 1. Reactances are shown on the diagram in a common system base. The generator is delivering reactive power $P_e$=0.8p.u. and Q = 0.074p.u. to the infinite bus at a voltage of 1p.u. A three phase fault occurs at the middle of one line and is cleared by isolating the faulted circuit simultaneously at both ends as shown in Fig.4.1. The fault is cleared in 0.3 second. The numerical solution is obtained for 1.0 second using the modified Euler method with a step size of Δt= 0.01second in Matlab7.0. The swing curve is used to determine the system stability and the critical clearing time is determined. The simulation was repeated and the swing plots obtained using SIMULINK.

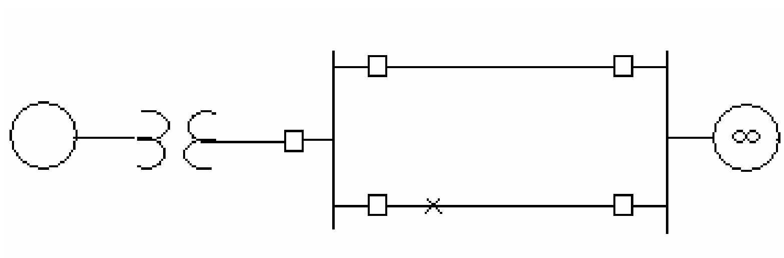

Fig.4.1 Diagram for transient state stability design

### 4.3 MATLAB CODE FOR TRANSIENT STATE STABILITY DESIGN

```
global  Pm f H E V X1 X2 X3

Pm = 0.80;  E = 1.17;  V = 1.0;

X1 = 0.65; X2 = 1.80; X3 = 0.8;
```

```
H = 5.0; f = 60; tf = 1; Dt = 0.01;
%  Fault is cleared in 0.3 sec.
tc = 0.3;
swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)
%  Fault is cleared in 0.4 sec. and 0.5 sec.
tc = .5;
swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)
tc = .4;
swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)
disp('Parts (a) & (b) are repeated using swingrk4')
disp('Press Enter to continue')
pause
tc = 0.3;
swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf)
tc = .5;
swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf)
tc = .4;
swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf)
% This program solves the swing equation of a one-machine system
% when subjected to a three-phase fault with subsequent clearance
% of the fault. Modified Euler method
function swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)
%global  Pm f H E V X1 X2 X3
```

```
clear t

if exist('Pm')~=1

Pm = input('Generator output power in p.u. Pm = '); else, end

if exist('E')~=1

E = input('Generator e.m.f. in p.u. E = '); else, end

if exist('V')~=1

V = input('Infinite bus-bar voltage in p.u. V = '); else, end

if exist('X1')~=1

X1 = input('Reactance before Fault in p.u. X1 = '); else, end

if exist('X2')~=1

X2 = input('Reactance during Fault X2 = '); else, end

if exist('X3')~=1

X3 = input('Reactance after Fault X3 = '); else, end

if exist('H')~=1

H  = input('Generator Inertia constant in sec. H = '); else, end

if exist('f')~=1

f  = input('System frequency in Hz f = '); else, end

if exist('Dt')~=1

Dt = input('Time interval  Dt = '); else, end

if exist('tc')~=1

tc = input('Clearing time of fault in sec tc = '); else, end

if exist('tf')~=1

tf = input('Final time for swing equation in sec tf = '); else, end
```

```
Pe1max = E*V/X1; Pe2max=E*V/X2; Pe3max=E*V/X3;
clear t x1 x2  delta
d0 =asin(Pm/Pe1max);
t(1) = 0;
x1(1)= d0;
x2(1)=0;
np=tf /Dt;
Pemax=Pe2max;
ck=pi*f/H;
for k = 1:np
   if t(k) >= tc
   Pemax=Pe3max;
   else, end
t(k+1)=t(k)+Dt;
Dx1b=x2(k);
Dx2b=ck*(Pm-Pemax*sin(x1(k)));
x1(k+1)=x1(k)+Dx1b*Dt;
x2(k+1)=x2(k)+Dx2b*Dt;
Dx1e=x2(k+1);
Dx2e=ck*(Pm-Pemax*sin(x1(k+1)));
Dx1=(Dx1b+Dx1e)/2;
Dx2=(Dx2b+Dx2e)/2;
x1(k+1)=x1(k)+Dx1*Dt;
```

```
x2(k+1)=x2(k)+Dx2*Dt;

end

delta=180*x1/pi;

clc

fprintf('\nFault is cleared at %4.3f Sec. \n', tc)

head=['                              '

     '    time     delta      Dw   '

     '     s       degrees    rad/s'

     '                             '];

disp(head)

disp([t', delta' x2'])

h=figure; figure(h)

plot(t, delta), grid

title(['One-machine system swing curve. Fault cleared at ', num2str(tc),'s'])

xlabel('t, sec'), ylabel('Delta, degree')

cctime(Pm, E, V, X1, X2, X3, H, f)    % Obtains the critical clearing time

% This program solves the swing equation of a one-machine system

% when subjected to a three-phase fault with subsequent clearance

% of the fault.

function swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)

%global  Pm f H E V X1 X2 X3

if exist('Pm') ~= 1

Pm = input('Generator output power in p.u. Pm = '); else, end
```

```
if exist('E') ~= 1

E = input('Generator e.m.f. in p.u. E = '); else, end

if exist('V') ~= 1

V = input('Infinite bus-bar voltage in p.u. V = '); else, end

if exist('X1') ~= 1

X1 = input('Reactance before Fault in p.u. X1 = '); else, end

if exist('X2') ~= 1

X2 = input('Reactance during Fault X2 = '); else, end

if exist('X3') ~= 1

X3 = input('Reactance after Fault X3 = '); else, end

if exist('H') ~= 1

H  = input('Generator Inertia constant in sec. H = '); else, end

if exist('f') ~= 1

f  = input('System frequency in Hz f = '); else, end

if exist('tc') ~= 1

tc = input('Clearing time of fault in sec tc = '); else, end

if exist('tf') ~= 1

tf = input('Final time for swing equation in sec tf = '); else, end

Pe1max = E*V/X1; Pe2max=E*V/X2; Pe3max=E*V/X3;

clear t  x  delta

d0 =asin(Pm/Pe1max);

t0 = 0;

x0 = [d0;  0];
```

```
tol=0.001;
tspan = [t0; tc];
[t1,xf]=ode45('pfpower', tspan, x0); % During fault solution
x0c =xf(length(xf), :);
tspan = [tc, tf];
[t2,xc] =ode45('afpower', tspan, x0c); % After fault solution
t =[t1; t2]; x = [xf; xc];
delta = 180/pi*x(:,1);
clc
fprintf('\nFault is cleared at %4.3f Sec. \n', tc)
head=['                              '
    '    time     delta      Dw   '
    '      s       degrees    rad/s'
    '                              '];
disp(head)
disp([t, delta, x(:, 2)])
h=figure; figure(h)
plot(t, delta), grid
title(['One-machine system swing curve. Fault cleared at ', num2str(tc),'s'])
xlabel('t, sec'), ylabel('Delta, degree')
cctime(Pm, E, V, X1, X2, X3, H, f)    % Obtains the critical clearing time
% This function  Simulates the swing equation of a one-machine system
% and returns the critical clearing time for stability.
```

```
function  cctime(Pm, E, V, X1, X2, X3, H, f)

Pe1max = E*V/X1; Pe2max=E*V/X2; Pe3max=E*V/X3;

d0 =asin(Pm/Pe1max);

dmax = pi-asin(Pm/Pe3max);

cosdc = (Pm*(dmax-d0)+Pe3max*cos(dmax)-Pe2max*cos(d0))/(Pe3max-Pe2max);

  if abs(cosdc) > 1

  fprintf('No critical clearing angle could be found.\n')

  fprintf('System can remain stable during this disturbance.\n\n')

  return

  else, end

dc = acos(cosdc);

  if dc > dmax

  fprintf('No critical clearing angle could be found.\n')

  fprintf('System can remain stable during this disturbance.\n\n')

  return

  else, end

tf = 0.4;

x0 = [d0; 0];

tspan = [0, tf];

options = odeset('RelTol', 0.00001);

[t1,xf] =ode23('pfpower', tspan, x0, options);

kk=find(xf(:,1) <= dc); k=max(kk);

tt=t1(k);
```

```
while tf <= tt & tf <= 3.6
tf=tf+.4;
   fprintf('\nSearching with a final time of %3.2f Sec. \n', tf)
   tol=0.00001+tf*2.5e-5;
   tspan = [0, tf];
   options = odeset('RelTol', tol);
   [t1,xf] =ode23('pfpower', tspan, x0, options);
   kk=find(xf(:,1) <= dc); k=max(kk);
      tt= t1(k);
end
tmargin = t1(k);
if tf >= 3.6
  fprintf('\nA clearing time could not be found up to 4 sec. \n\n')
  return
  else, end
fprintf('\nCritical clearing time =  %4.2f seconds \n', tmargin)
fprintf('Critical clearing angle = %6.2f degrees \n\n', dc*180/pi
```

## 4.4 SIMULINK DESIGN

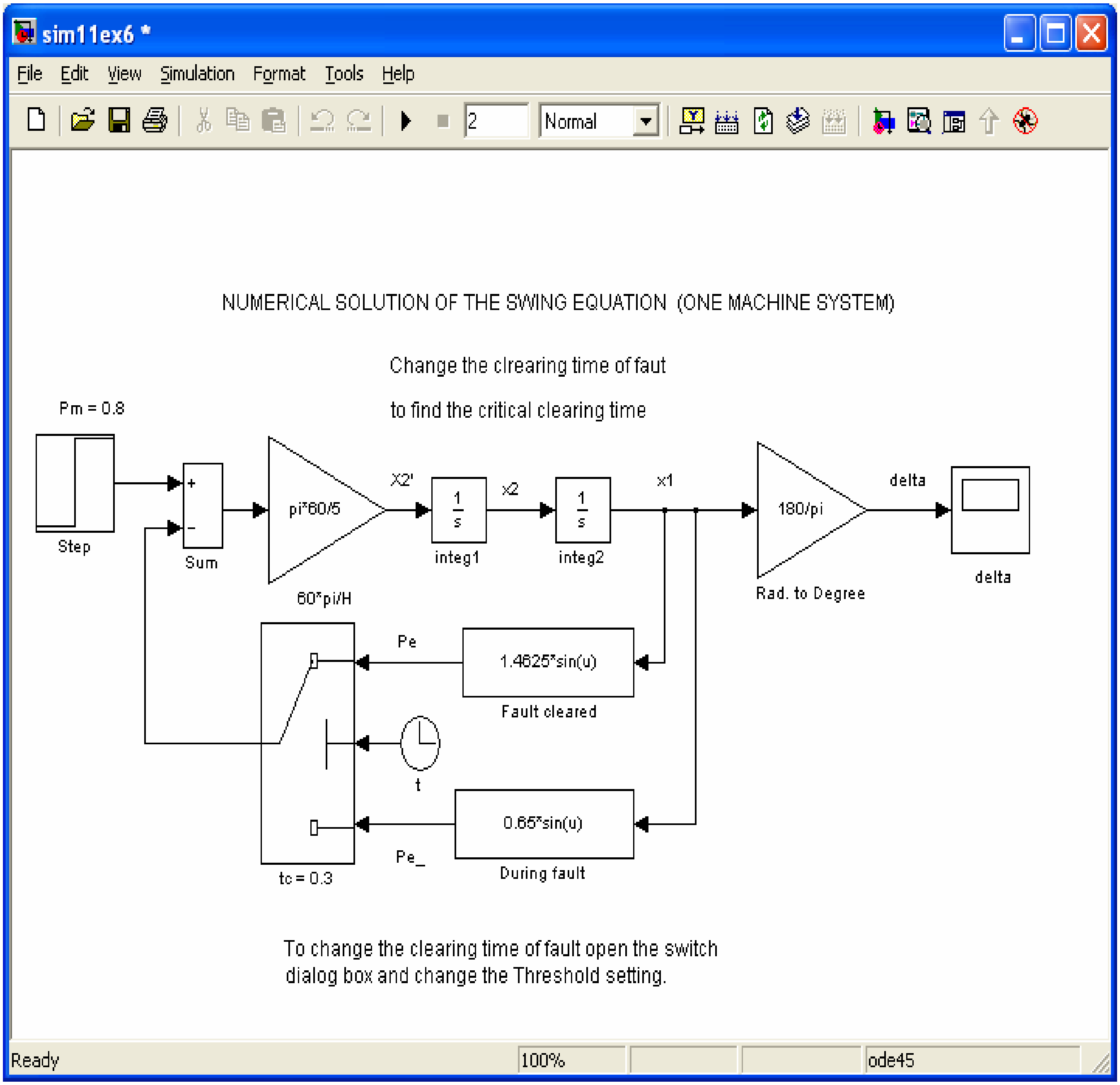


Fig. 4.2 Simulink Design for Transient Stability Design

## 4.5 OUTPUT WAVEFORMS

### USING MODIFIED EULER METHOD

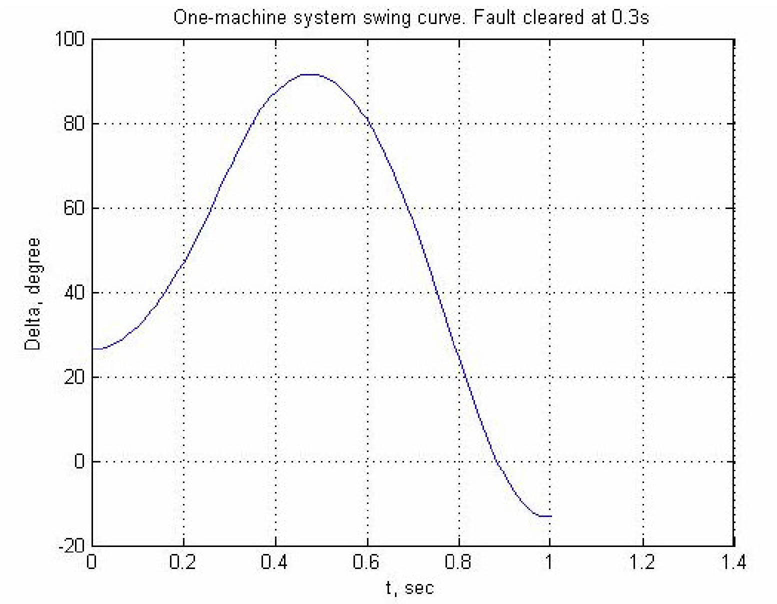


Fig. 4.3 Swing Curve using Modified Euler Method for fault cleared at 0.3s

### USING RUNGE-KUTTA METHOD

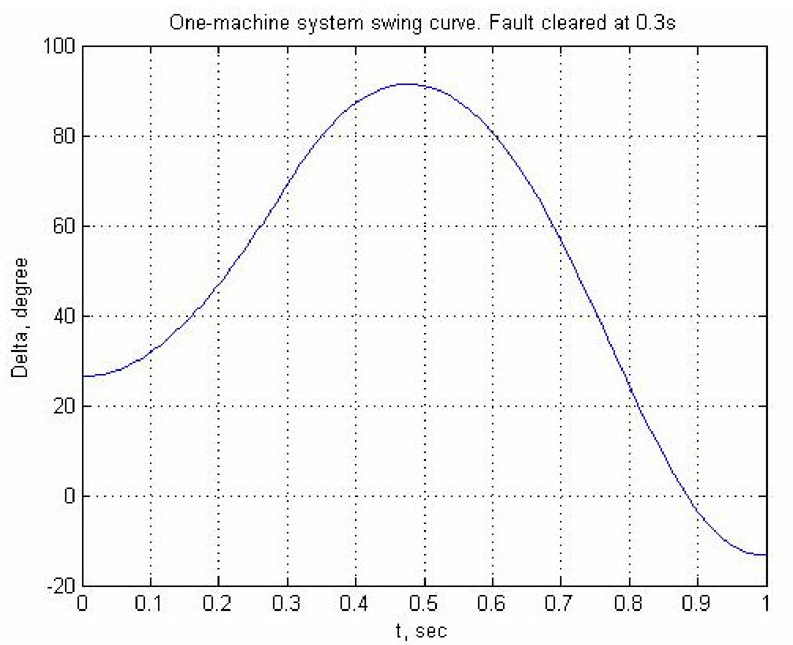


Fig. 4.4 Swing Curve using Runge-Kutta Mehod for fault cleared at 0.3s

## USING MODIFIED EULER METHOD

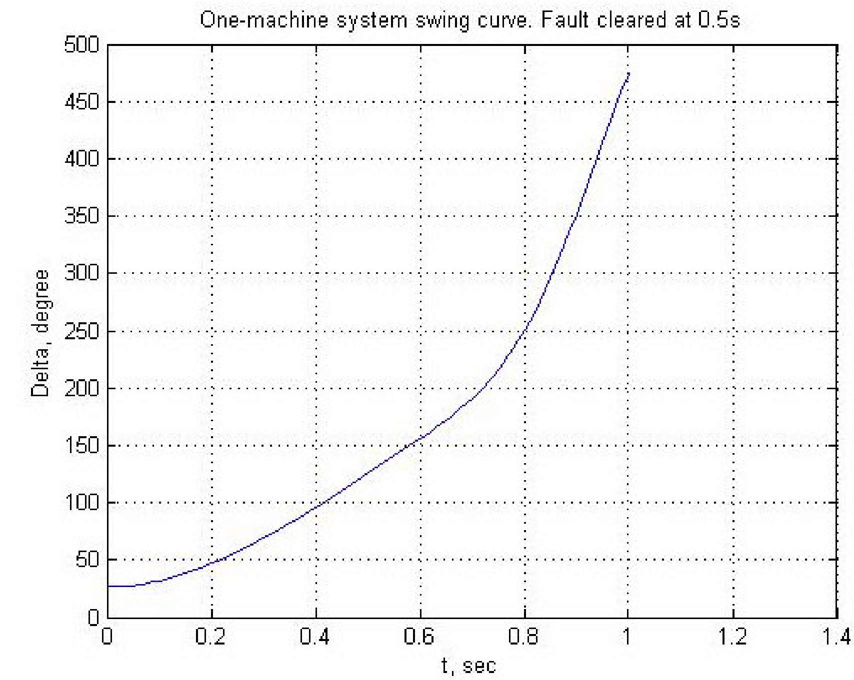


Fig. 4.5 Swing Curve using Modified Euler Method for fault cleared at 0.5s

## USING RUNGE-KUTTA METHOD

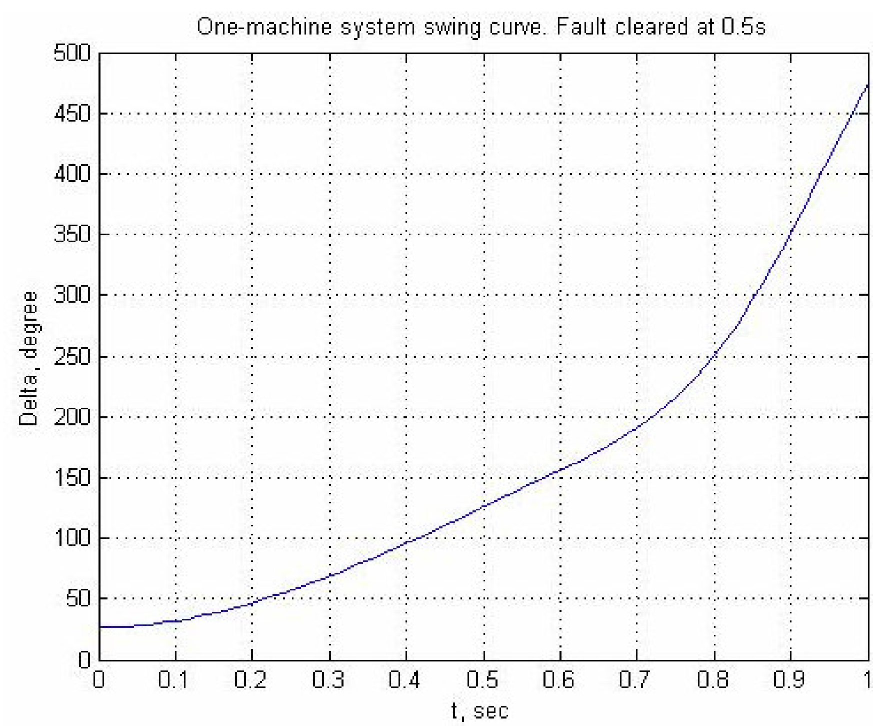


Fig. 4.6 Swing Curve using Modified Euler Method for fault cleared at 0.5s

## USING MODIFIED EULER METHOD

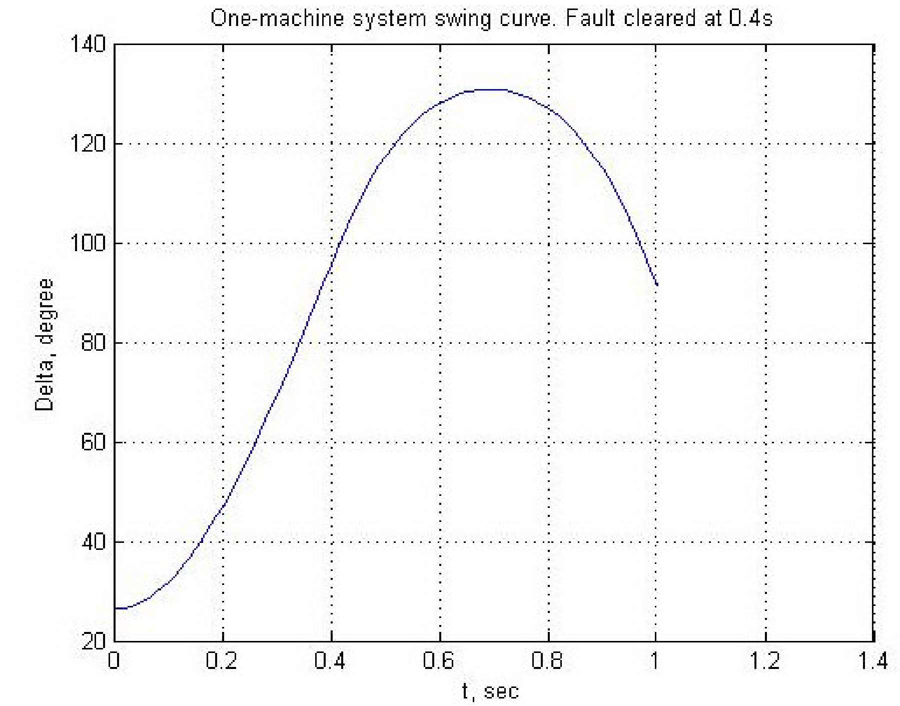


Fig. 4.7 Swing Curve using Modified Euler Method for fault cleared at 0.4s

## USING RUNGE-KUTTA METHOD

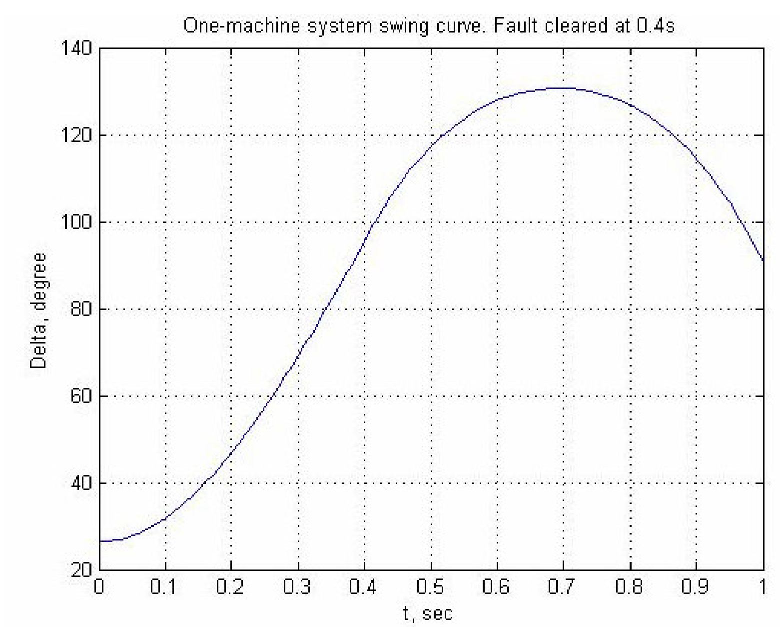


Fig. 4.8 Swing Curve using Runge-Kutta Method for fault cleared at 0.4s

The swing curve shows that the power angle returns after a maximum swing indicating that with system damping, the oscillation will subside and a new operating angle is attained. Hence the system is found to be stable for this fault clearing time. The critical clearing time is determined by the program to be

Critical clearing time = 0.41 seconds

Critical clearing angle = 98.83 degrees

The above program is run for a clearing time of $t_c$ =0.4 second and $t_c$=0.5 second with the results shown in figure. The swing curve for $t_c$ =0.4 second corresponds to the critical clearing time. The swing curve for $t_c$ = 0.5 second shows that the power angle $\delta$ is increasing with out limit. Hence the system is unstable for this clearing time.

## 4.6 Point-by-Point Method

It is always required to know the critical clearing time corresponding to critical clearing angle so as to design the operating times of the relay and circuit breaker so that time taken by them should be less than the critical clearing time for stable operation of the system. So the point-by-point method is used for the solution of critical clearing time associated with critical clearing angle and also for the solution of multi machine system. The step-by-step or point-by-point method is the conventional, approximate but proven method. This involves the calculation of the rotor angle as time is incremented. The accuracy of the solution depends upon the time increment used in the analysis.

The following parameters are evaluated for each interval (n)

The accelerating power $Pa(n-1)=Ps - Pe(n-1)$

From the swing equation $\alpha(n-1)=Pa(n-1)/M$

$\Delta\omega n-1/2= \alpha n-1\Delta t$

$\omega n-1/2 = \omega n-3/2+\alpha n-1\Delta t$

$\Delta\delta n = \omega n\text{-}1/2\ \Delta t = (\ \omega n\text{-}3/2 + \alpha n\text{-}1\Delta t)\ \Delta t$

$= \Delta\delta n\text{-}1 + \alpha n\text{-}1\Delta t2$

$= \Delta\delta n\text{-}1 + Pa(n\text{-}1)\ \Delta t2/M$

$\delta n == \delta n\text{-}1 + \Delta\delta n$

The above calculations have been programmed using MATLAB-7.0 for a 20 MVA,

50 Hz generator delivering 18 MW over a double circuit line to an infinite bus. The generator has kinetic energy of 2.52MJ/MVA at rated speed. The generator has transient reactance of Xd'=0.32 p.u. Each transmission circuit has zero resistance and a reactance of 0.2 p.u. on a 20 MVA base .Magnitude of E' is 1.1p.u. and infinite base voltage of 1.0∟00. A three phase circuit occurs at the mid point of one of the transmission line. The fault is cleared by simultaneous opening of breakers and both ends of line at 2.5 cycles and 6.25 cycles after the occurrence of the fault.

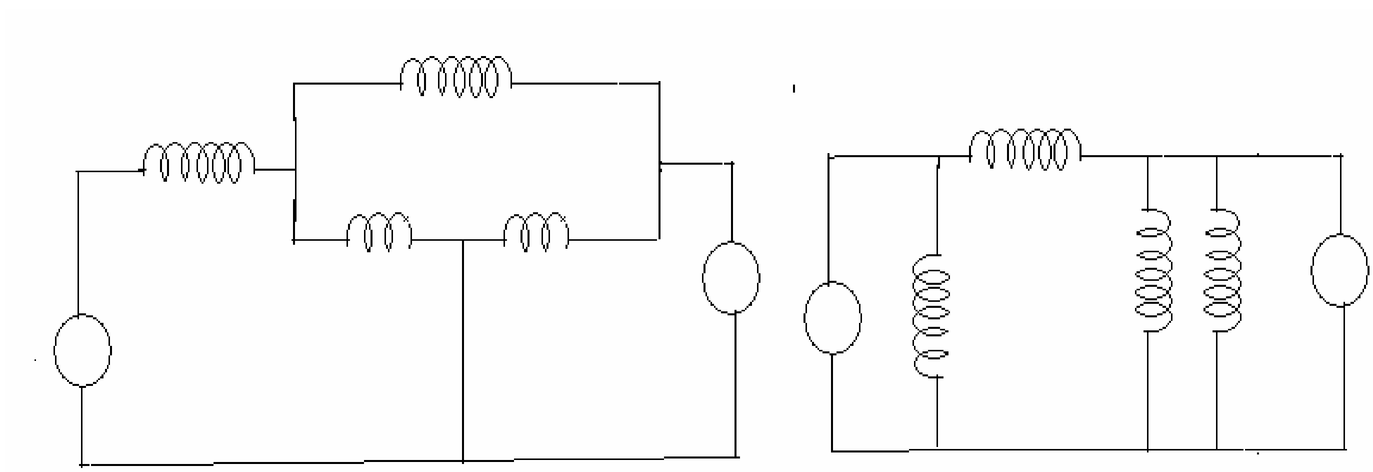

Fig 4.9. Diagram for study of Point by point Method

To find the critical clearing time swing curves can be obtained, similarly, for progressively greater clearing time till the torque angle δ increases without bound.

### 4.7 MATLAB CODE FOR POINT-BY-POINT METHOD DESIGN

```
t=0
tf=0
tfinal=0.5
tc=0.125
tstep=0.05
M=2.52/(180*50)
i=2
delta=21.64*pi/180
ddelta=0
time(1)=0
ang(1)=21.64
Pm=0.9
Pmaxbf=2.44
Pmaxdf=0.88
Pmaxaf=2.00
while t<tfinal,
   if (t==tf),
       Paminus=0.9-Pmaxbf*sin(delta)
       Paplus=0.9-Pmaxdf*sin(delta)
       Paav=(Paminus+Paplus)/2
       Pa=Paav
   end
```

```
    if (t==tc),
        Paminus=0.9-Pmaxdf*sin(delta)
        Paplus=0.9-Pmaxaf*sin(delta)
        Paav=(Paminus+Paplus)/2
        Pa=Paav
    end
    if(t>tf & t<tc),
        Pa=Pm-Pmaxdf*sin(delta)
    end
    if(t>tc),
        Pa=Pm-Pmaxaf*sin(delta)
    end
    ddelta=ddelta+(tstep*tstep*Pa/M)
    delta=(delta*180/pi+ddelta)*pi/180
    deltadeg=delta*180/pi
    t=t+tstep
    pause
    time(i)=t
    ang(i)=deltadeg
    i=i+1
end
axis([0 0.6 0 160])
plot(time,ang,'ko-')
```

## 4.8 OUTPUTS

Critical clearing angle = 118.62 degrees

Critical clearing time = 0.38 seconds

### USING POINT-BY-POINT METHOD

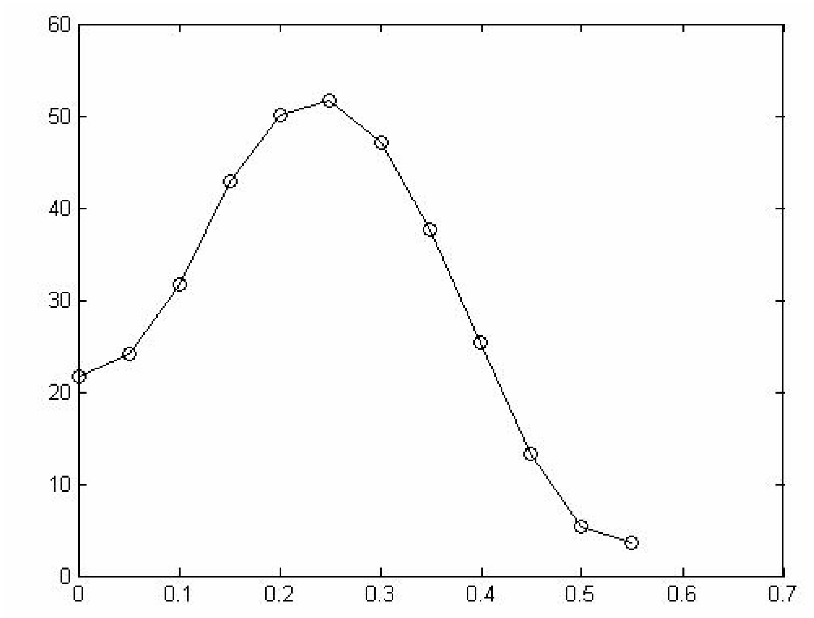


Fig. 4.10 Swing Curve: Fault cleared in 0.125s

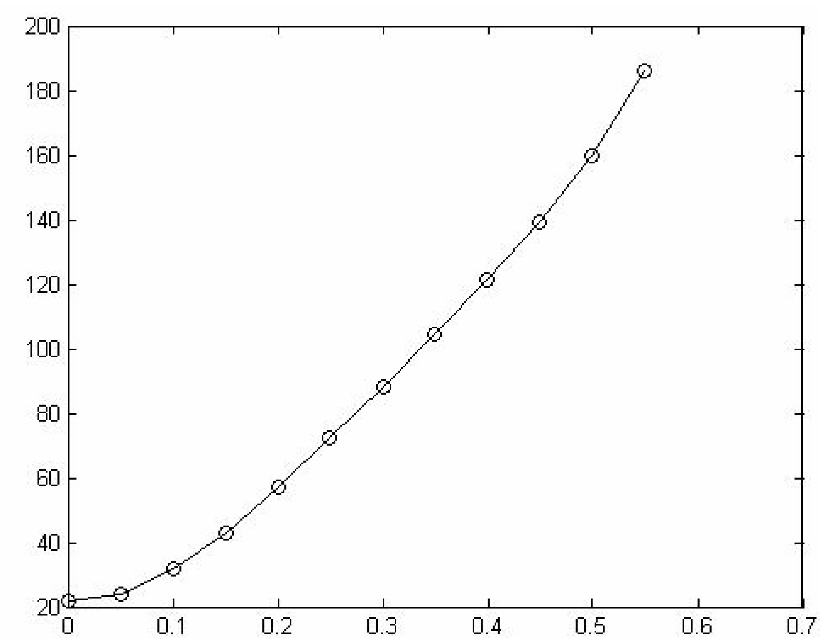


Fig. 4.11 Swing Curve: Fault cleared in 0.5s

# Chapter 5

## MULTIMACHINE SYSTEM ANALYSIS

## 5.1 MULTIMACHINE SYSTEMS

- Multi-machine system can be written similar to one-machine system by the following assumptions:
- Each synchronous machine is represented by a constant voltage E behind Xd (neglect saliency and flux change)
- Input power remain constant
- using prefault bus voltages, all loads are in equivalent admittances to ground
- damping and asynchronous effects are ignored
- δmech = δ
- machines belong to the same station swing together and are said to be coherent, coherent machines can equivalent to one machine
- Solution to multi-machine system:
- solve initial power flow and determine initial bus voltage magnitude and phase angle

$$I_i = \frac{S_i^*}{V_i^*} = \frac{P_i - jQ_i}{V_i^*}, \quad E_i' = V_i + jX_d' I_i$$

- calculating load equivalent admittance

$$y_{i0} = \frac{P_i - jQ_i}{|V_i|^2}$$

- nodal equations of the system

$$\begin{bmatrix} 0 \\ I_m \end{bmatrix} = \begin{bmatrix} Y_{nn} & Y_{nm} \\ Y_{nm}^t & Y_{mm} \end{bmatrix} \begin{bmatrix} V_n \\ E_m' \end{bmatrix}$$

- electrical and mechanical power output of machine at steady state prior to disturbances
- Classical transient stability study is based on the application of the three-phase fault

$$P_{ei} = P_{mi} = \mathrm{Re}\{E_i^* I_i\} = \sum_{j=1}^{m} |E_i'||E_j'||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

- Swing equation of multi-machine system

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{m} |E_i^{'}||E_j^{'}||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) = P_{mi} - P_{ei}$$

- Yij are the elements of the faulted reduced bus admittance matrix
- state variable model of swing equation

$$\frac{d\delta_i}{dt} = \Delta\omega_i, \quad i = 1,\mathrm{K}\ , n$$

$$\frac{d\Delta\omega_i}{dt} = \frac{\pi f_0}{H_i}(P_{mi} - P_{ei})$$

### 5.2 Illustration:

The power system network of an electrical company is shown in Fig-5.1.

The load data, voltage magnitude, generation schedule and the reactive power limits for the regulated buses are tabulated below in table-1, table-2, table-3 respectively.

Bus 1, whose voltage is specified as V1=1.04∟00,is taken as slack bus.

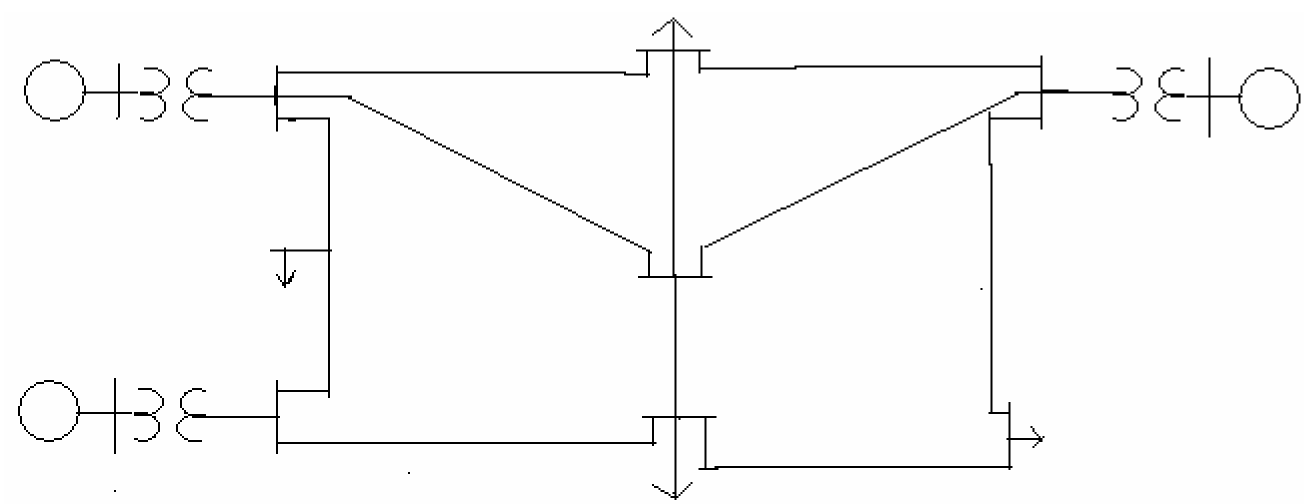

Fig 5.1 Diagram for multimachine stability

### 5.3 MATLAB CODE FOR MULTIMACHINE STABILITY DESIGN

```
basemva =100;   accuracy=0.0001;  maxiter=10;

busdata=[1   1   1.06    0  0     0     0      0  0      0  0;
```

```
        2   2   1.04    0   0     0     150   0   0     140 0;
        3   2   1.03    0   0     0     100   0   0     90  0;
        4   0   1       0   100   70    0     0   0     0   0;
        5   0   1       0   90    30    0     0   0     0   0;
        6   0   1       0   160   110   0     0   0     0   0];
      %Sixth column is Transformer Tap position
linedata=[1   4   0.035     0.225    0.0065     1;
         1   5   0.025     0.105    0.0045   1;
         1   6   0.040     0.215    0.0055   1;
         2   4   0.0       0.035    0.0   1;
         3   5   0.0       0.042    0.0   1;
         4   6   0.026     0.125     0.0035   1;
         5   6   0.026     0.175     0.0300   1];
      gendata=[1    0   0.2    20;
              2   0   0.15   4;
              3   0   0.25   5];
 %  This program obtains th Bus Admittance Matrix for power flow solution
j=sqrt(-1); i = sqrt(-1);
nl = linedata(:,1); nr = linedata(:,2); R = linedata(:,3);
X = linedata(:,4); Bc = j*linedata(:,5); a = linedata(:, 6);
nbr=length(linedata(:,1)); nbus = max(max(nl), max(nr));
Z = R + j*X; y= ones(nbr,1)./Z;        %branch admittance
for n = 1:nbr
```

```
if a(n) <= 0  a(n) = 1;

else

end

Ybus=zeros(nbus,nbus);     % initialize Ybus to zero

               % formation of the off diagonal elements

for k=1:nbr;

       Ybus(nl(k),nr(k))=Ybus(nl(k),nr(k))-y(k)/a(k);

       Ybus(nr(k),nl(k))=Ybus(nl(k),nr(k));

    end

end

              % formation of the diagonal elements

for  n=1:nbus

     for k=1:nbr

         if nl(k)==n

         Ybus(n,n) = Ybus(n,n)+y(k)/(a(k)^2) + Bc(k);

         elseif nr(k)==n

         Ybus(n,n) = Ybus(n,n)+y(k) +Bc(k);

         else, end

     end

end

clear pgg

%   Power flow solution by Newton-Raphson method

ns=0; ng=0; Vm=0; delta=0; yload=0; deltad=0;
```

```
nbus = length(busdata(:,1));
for k=1:nbus
n=busdata(k,1);
kb(n)=busdata(k,2); Vm(n)=busdata(k,3); delta(n)=busdata(k, 4);
Pd(n)=busdata(k,5); Qd(n)=busdata(k,6); Pg(n)=busdata(k,7); Qg(n) = busdata(k,8);
Qmin(n)=busdata(k, 9); Qmax(n)=busdata(k, 10);
Qsh(n)=busdata(k, 11);
   if Vm(n) <= 0  Vm(n) = 1.0; V(n) = 1 + j*0;
   else delta(n) = pi/180*delta(n);
       V(n) = Vm(n)*(cos(delta(n)) + j*sin(delta(n)));
       P(n)=(Pg(n)-Pd(n))/basemva;
       Q(n)=(Qg(n)-Qd(n)+ Qsh(n))/basemva;
       S(n) = P(n) + j*Q(n);
   end
end
for k=1:nbus
if kb(k) == 1, ns = ns+1; else, end
if kb(k) == 2 ng = ng+1; else, end
ngs(k) = ng;
nss(k) = ns;
end
Ym=abs(Ybus); t = angle(Ybus);
m=2*nbus-ng-2*ns;
```

```
maxerror = 1; converge=1;

iter = 0;

% Start of iterations

clear A  DC   J  DX

while maxerror >= accuracy & iter <= maxiter % Test for max. power mismatch

for i=1:m

for k=1:m

   A(i,k)=0;      %Initializing Jacobian matrix

end, end

iter = iter+1;

for n=1:nbus

nn=n-nss(n);

lm=nbus+n-ngs(n)-nss(n)-ns;

J11=0; J22=0; J33=0; J44=0;

   for i=1:nbr

     if nl(i) == n | nr(i) == n

        if nl(i) == n,  l = nr(i); end

        if nr(i) == n,  l = nl(i); end

        J11=J11+ Vm(n)*Vm(l)*Ym(n,l)*sin(t(n,l)- delta(n) + delta(l));

        J33=J33+ Vm(n)*Vm(l)*Ym(n,l)*cos(t(n,l)- delta(n) + delta(l));

        if kb(n)~=1

        J22=J22+ Vm(l)*Ym(n,l)*cos(t(n,l)- delta(n) + delta(l));

        J44=J44+ Vm(l)*Ym(n,l)*sin(t(n,l)- delta(n) + delta(l));
```

```
    else, end
    if kb(n) ~= 1  & kb(l) ~=1
    lk = nbus+l-ngs(l)-nss(l)-ns;
    ll = l -nss(l);
   % off diagonalelements of J1
    A(nn, ll) =-Vm(n)*Vm(l)*Ym(n,l)*sin(t(n,l)- delta(n) + delta(l));
        if kb(l) == 0  % off diagonal elements of J2
        A(nn, lk) =Vm(n)*Ym(n,l)*cos(t(n,l)- delta(n) + delta(l));end
        if kb(n) == 0  % off diagonal elements of J3
        A(lm, ll) =-Vm(n)*Vm(l)*Ym(n,l)*cos(t(n,l)- delta(n)+delta(l)); end
        if kb(n) == 0 & kb(l) == 0  % off diagonal elements of  J4
        A(lm, lk) =-Vm(n)*Ym(n,l)*sin(t(n,l)- delta(n) + delta(l));end
    else end
  else , end
end
Pk = Vm(n)^2*Ym(n,n)*cos(t(n,n))+J33;
Qk = -Vm(n)^2*Ym(n,n)*sin(t(n,n))-J11;
if kb(n) == 1 P(n)=Pk; Q(n) = Qk; end   % Swing bus P
  if kb(n) == 2  Q(n)=Qk;
     if Qmax(n) ~= 0
       Qgc = Q(n)*basemva + Qd(n) - Qsh(n);
       if iter <= 7                  % Between the 2th & 6th iterations
         if iter > 2               % the Mvar of generator buses are
```

```
            if Qgc  < Qmin(n),       % tested. If not within limits Vm(n)
            Vm(n) = Vm(n) + 0.01;    % is changed in steps of 0.01 pu to
            elseif Qgc  > Qmax(n),   % bring the generator Mvar within
            Vm(n) = Vm(n) - 0.01;end % the specified limits.
           else, end
         else,end
       else,end
    end
   if kb(n) ~= 1
    A(nn,nn) = J11;  %diagonal elements of J1
    DC(nn) = P(n)-Pk;
   end
   if kb(n) == 0
    A(nn,lm) = 2*Vm(n)*Ym(n,n)*cos(t(n,n))+J22;  %diagonal elements of J2
    A(lm,nn)= J33;        %diagonal elements of J3
    A(lm,lm) =-2*Vm(n)*Ym(n,n)*sin(t(n,n))-J44;  %diagonal of elements of J4
    DC(lm) = Q(n)-Qk;
   end
end
DX=A\DC';
for n=1:nbus
  nn=n-nss(n);
  lm=nbus+n-ngs(n)-nss(n)-ns;
```

```
    if kb(n) ~= 1
    delta(n) = delta(n)+DX(nn); end
    if kb(n) == 0
    Vm(n)=Vm(n)+DX(lm); end
 end
  maxerror=max(abs(DC));
     if iter == maxiter & maxerror > accuracy
   fprintf('\nWARNING: Iterative solution did not converged after ')
   fprintf('%g', iter), fprintf(' iterations.\n\n')
   fprintf('Press Enter to terminate the iterations and print the results \n')
   converge = 0; pause, else, end
end
if converge ~= 1
   tech= ('                       ITERATIVE SOLUTION DID NOT CONVERGE'); else,
   tech=('                    Power Flow Solution by Newton-Raphson Method');
end
V = Vm.*cos(delta)+j*Vm.*sin(delta);
deltad=180/pi*delta;
i=sqrt(-1);
k=0;
for n = 1:nbus
     if kb(n) == 1
     k=k+1;
```

```
    S(n)= P(n)+j*Q(n);
    Pg(n) = P(n)*basemva + Pd(n);
    Qg(n) = Q(n)*basemva + Qd(n) - Qsh(n);
    Pgg(k)=Pg(n);
    Qgg(k)=Qg(n);
    elseif  kb(n) ==2
    k=k+1;
    S(n)=P(n)+j*Q(n);
    Qg(n) = Q(n)*basemva + Qd(n) - Qsh(n);
    Pgg(k)=Pg(n);
    Qgg(k)=Qg(n);
  end
yload(n) = (Pd(n)- j*Qd(n)+j*Qsh(n))/(basemva*Vm(n)^2);
end
busdata(:,3)=Vm'; busdata(:,4)=deltad';
Pgt = sum(Pg);  Qgt = sum(Qg); Pdt = sum(Pd); Qdt = sum(Qd); Qsht = sum(Qsh)
%   'busout'  Prints the power flow solution on the screen
disp(tech)
fprintf('                      Maximum Power Mismatch = %g \n', maxerror)
fprintf('                              No. of Iterations = %g \n\n', iter)
head =['    Bus  Voltage  Angle    ------Load------    ---Generation---   Injected'
       '    No.  Mag.     Degree     MW       Mvar       MW       Mvar       Mvar '
       '                                                              '];
```

```
disp(head)

for n=1:nbus

     fprintf(' %5g', n), fprintf(' %7.3f', Vm(n)),

     fprintf(' %8.3f', deltad(n)), fprintf(' %9.3f', Pd(n)),

     fprintf(' %9.3f', Qd(n)),  fprintf(' %9.3f', Pg(n)),

     fprintf(' %9.3f ', Qg(n)), fprintf(' %8.3f\n', Qsh(n))

end

    fprintf('      \n'), fprintf('    Total              ')

    fprintf(' %9.3f', Pdt), fprintf(' %9.3f', Qdt),

    fprintf(' %9.3f', Pgt), fprintf(' %9.3f', Qgt), fprintf(' %9.3f\n\n', Qsht)

   %global Pm f H E  Y th ngg

f=60;

%zdd=gendata(:,2)+j*gendata(:,3);

ngr=gendata(:,1);

%H=gendata(:,4);

ngg=length(gendata(:,1));

%%

for k=1:ngg

zdd(ngr(k))=gendata(k, 2)+j*gendata(k,3);

%H(ngr(k))=gendata(k, 4);

H(k)=gendata(k,4);   % new

end

%%
```

```
for k=1:ngg

I=conj(S(ngr(k)))/conj(V(ngr(k)));

%Ep(ngr(k)) = V(ngr(k))+zdd(ngr(k))*I;

%Pm(ngr(k))=real(S(ngr(k)));

Ep(k) = V(ngr(k))+zdd(ngr(k))*I;  % new

Pm(k)=real(S(ngr(k)));            % new

end

E=abs(Ep); d0=angle(Ep);

for k=1:ngg

nl(nbr+k) = nbus+k;

nr(nbr+k) = gendata(k, 1);

%R(nbr+k)  = gendata(k, 2);

%X(nbr+k)  = gendata(k, 3);

R(nbr+k)  = real(zdd(ngr(k)));

X(nbr+k)  = imag(zdd(ngr(k)));

Bc(nbr+k)  = 0;

a(nbr+k) = 1.0;

yload(nbus+k)=0;

end

nbr1=nbr; nbus1=nbus;

nbrt=nbr+ngg;

nbust=nbus+ngg;

linedata=[nl, nr, R, X, -j*Bc, a];
```

```
[Ybus, Ybf]=ybusbf(linedata, yload, nbus1,nbust);
fprintf('\nPrefault reduced bus admittance matrix \n')
Ybf
Y=abs(Ybf); th=angle(Ybf);
Pm=zeros(1, ngg);
disp(['      G(i)    E''(i)     d0(i)      Pm(i)'])
for ii = 1:ngg
for jj = 1:ngg
Pm(ii) = Pm(ii) + E(ii)*E(jj)*Y(ii, jj)*cos(th(ii, jj)-d0(ii)+d0(jj));
end,
fprintf('       %g', ngr(ii)), fprintf('   %8.4f',E(ii)), fprintf('   %8.4f', 180/pi*d0(ii))
fprintf('  %8.4f \n',Pm(ii))
end
respfl='y';
while respfl =='y' | respfl=='Y'
nf=input('Enter faulted bus No. -> ');
fprintf('\nFaulted reduced bus admittance matrix\n')
Ydf=ybusdf(Ybus, nbus1, nbust, nf)
%Fault cleared
[Ybus,Yaf]=ybusaf(linedata, yload, nbus1,nbust, nbrt);
fprintf('\nPostfault reduced bus admittance matrix\n')
Yaf
resptc='y';
```

```
while resptc =='y' | resptc=='Y'

tc=input('Enter clearing time of fault in sec. tc = ');

tf=input('Enter final simulation time in sec.  tf = ');

clear t  x  del

t0 = 0;

w0=zeros(1, length(d0));

x0 = [d0,  w0];

tol=0.0001;

Y=abs(Ydf); th=angle(Ydf);

%[t1, xf] =ode23('dfpek', t0, tc, x0, tol);  % Solution during fault (use with MATLAB 4)

tspan=[t0, tc];                                    %use with MATAB 5

[t1, xf] =ode23('dfpek', tspan, x0);  % Solution during fault (use with MATLAB 5)

x0c =xf(length(xf), :);

Y=abs(Yaf); th=angle(Yaf);

%[t2,xc] =ode23('afpek', tc, tf, x0c, tol); % Postfault solution (use with MATLAB 4)

tspan = [tc, tf];                          % use with MATLAB 5

[t2,xc] =ode23('afpek', tspan, x0c);        % Postfault solution (use with MATLAB 5)

t =[t1; t2]; x = [xf; xc];

fprintf('\nFault is cleared at %4.3f Sec. \n', tc)

for k=1:nbus

   if kb(k)==1

   ms=k; else, end

end
```

```
fprintf('\nPhase angle difference of each machine \n')
fprintf('with respect to the slack in degree.\n')
fprintf('   t - sec')
kk=0;
for k=1:ngg
   if k~=ms
   kk=kk+1;
   del(:,kk)=180/pi*(x(:,k)-x(:,ms));
   fprintf('    d(%g,',ngr(k)), fprintf('%g)', ngr(ms))
   else, end
end
fprintf(' \n')
disp([t, del])
h=figure; figure(h)
plot(t, del)
title(['Phase angle difference (fault cleared at ', num2str(tc),'s)'])
xlabel('t, sec'), ylabel('Delta, degree'), grid
  resp=0;
  while strcmp(resp, 'n')~=1 & strcmp(resp, 'N')~=1 & strcmp(resp, 'y')~=1 & strcmp(resp, 'Y')~=1
  resp=input('Another clearing time of fault? Enter ''y'' or ''n'' within quotes -> ');
  if strcmp(resp, 'n')~=1 & strcmp(resp, 'N')~=1 & strcmp(resp, 'y')~=1 & strcmp(resp, 'Y')~=1
  fprintf('\n Incorrect reply, try again \n\n'), end
  end
```

```
resptc=resp;
end
   resp2=0;
   while strcmp(resp2, 'n')~=1 & strcmp(resp2, 'N')~=1 & strcmp(resp2, 'y')~=1 & strcmp(resp2,
'Y')~=1
   resp2=input('Another fault location: Enter "y" or "n" within quotes -> ');
   if strcmp(resp2, 'n')~=1 & strcmp(resp2, 'N')~=1 & strcmp(resp2, 'y')~=1 & strcmp(resp2,
'Y')~=1
   fprintf('\n Incorrect reply, try again \n\n'), end
   respf1=resp2;
   end
   if respf1=='n' | respf1=='N', return, else, end
```

## 5.4 OUTPUT WAVEFORMS

### MULTIMACHINE STABILITY

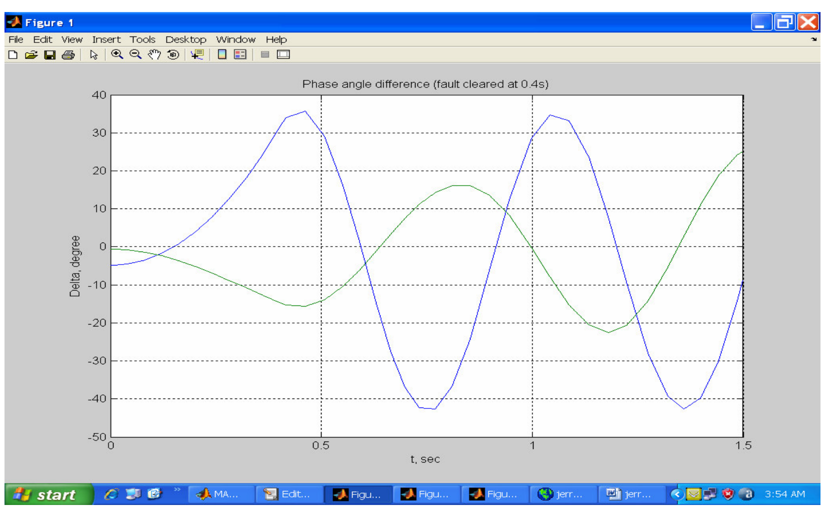


Fig. 5.2 Multimachine Stability for Fault cleared at 0.4 sec

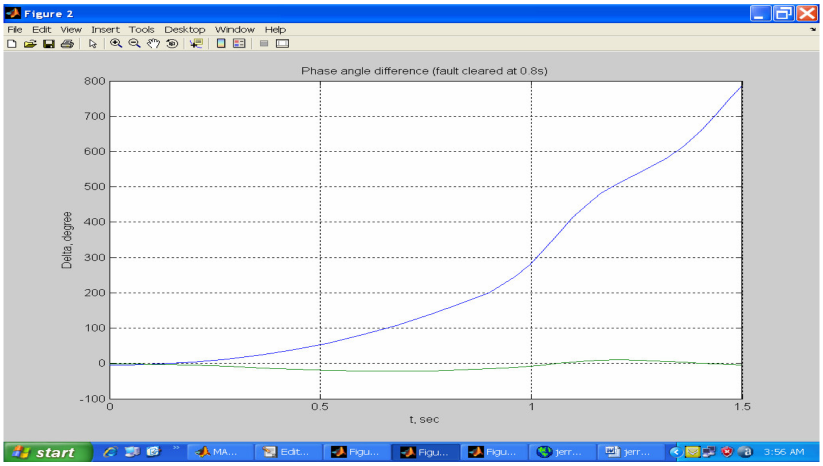

Fig. 5.3 Multimachine Stability for Fault cleared at 0.8 sec

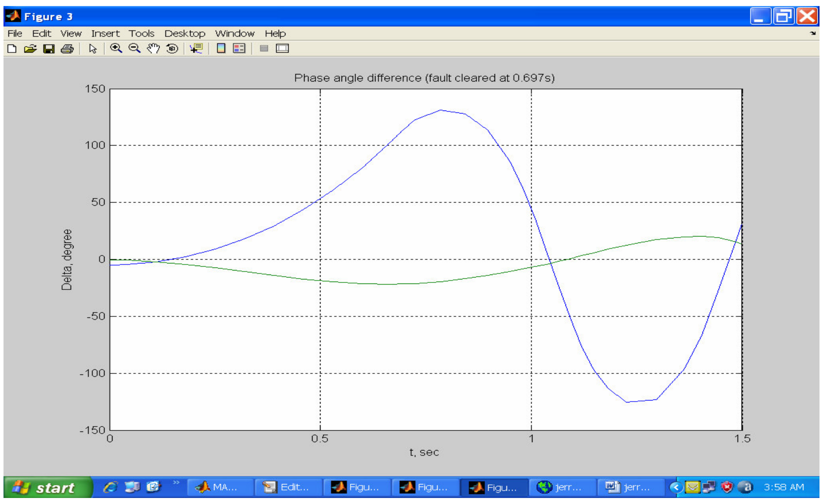

Fig. 5.4 Multimachune Stability for Fault cleared at 0.697 sec

Figure shows that the phase angle differences, after reaching a maximum of $\delta_{21}$=123.9$^0$ and $\delta_{31}$=62.95$^0$ will decrease, and the machines swing together. Hence, the system is found to be stable when fault is cleared in 0.4 second.

The swing curves shown in figure show that machine 2 phase angle increases without limit. Thus, the system is unstable when fault is cleared in 0.5 second. The simulation is repeated for a clearing time of 0.45 second which is found to be critically stable

# Chapter 6

## SMALL SIGNAL STABILITY INCLUDING EFFECT OF ROTOR CIRCUIT DYNAMICS

## 6.1 SMALL-SIGNAL STABILITY

Small signal stability is the ability of the power system to maintain synchronism when subjected to small disturbances. Here we study the small-signal performance of a machine connected to a large system through transmission lines. A general system configuration is shown below in Fig. 6.1. For the sake of analysis Fig.6.1(a) can be reduced to Fig.6.1(b) by using Thevenin's equivalent of the transmission network external to the machine and adjacent transmission.

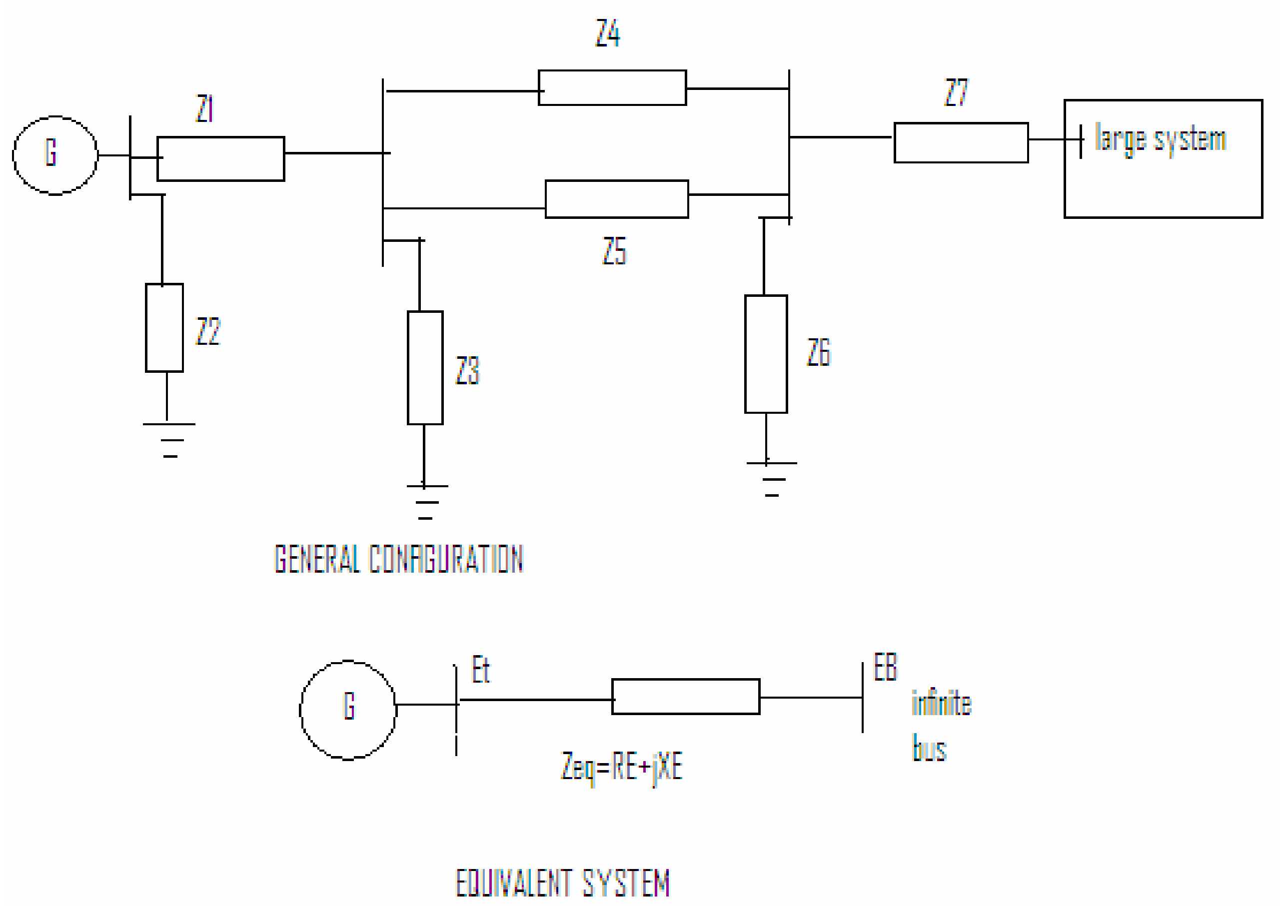


Fig. 6.1 Small-signal Studies (a) General Configuration (b) Equivalent System

We had already discussed the classical model of the generator in the first half of our project work done before. So here we will gradually increase the model detail by accounting for the effects of the dynamics of the field circuit and excitation systems. We will develop the expressions for the elements of the state matrix as explicit functions of system parameters. While this method is not suited for a detailed study of large systems, it is useful in gaining a physical insight into the effects of field circuit dynamics and in establishing the basis for methods of enhancing stability through excitation control.

## 6.2 EFFECT OF SYNCHRONOUS MACHINE FIELD CIRCUIT DYNAMICS

We consider the system performance including the effect of the field flux variations. The field voltage will be assumed constant (manual excitation control). We will develop the state-space model of the system by first reducing the synchronous machine equations to an appropriate form and then combining them with the network equations. We will express time in seconds, angles in electrical radians and all other variables in per unit.

### 6.2.1 Synchronous Machine Equations

The rotor angle $\delta$ is the angle ( in electrical radian) by which the q-axis leads the reference $E_B$. With amortisseurs neglected, the equivalent circuits relating the machine flux linkages and current are as shown min Fig. 6.2.

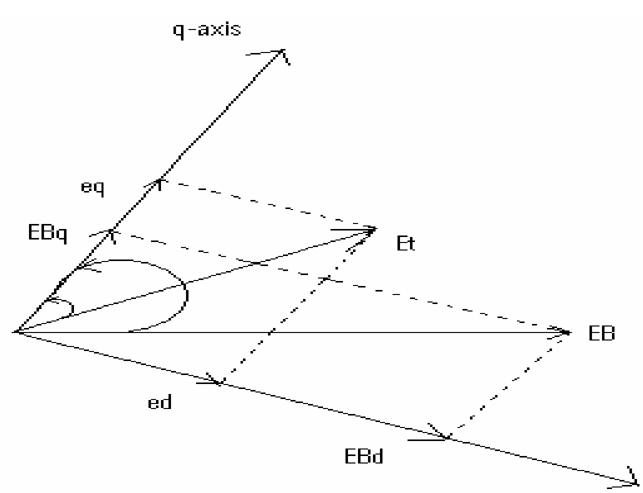


Fig. 6.2 Representation of the rotor angle and $E_B$

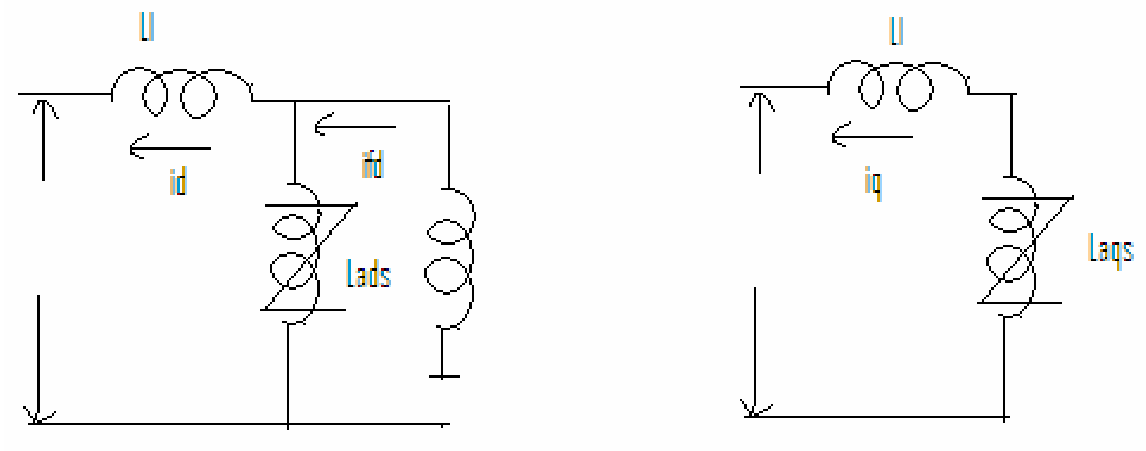


Fig. 6.3 The equivalent circuit relating the machine flux linkages and currents

The stator and rotor flux linkages are given by

$\psi_d = -L_l i_d + L_{ads}(-i_d+i_{fd})$

$= -L_l i_d + \psi_{ad}$ (1)

$\psi_q = -L_l i_q + L_{aqs}(-i_q)$

$= -L_l i_q + \psi_{aq}$ (2)

$\psi_{fd} = -L_{ads}(-i_d+i_{fd}) + L_{fd} i_{fd}$

$= \psi_{ad} + L_{fd} i_{fd}$ (3)

In the above equations, $\boldsymbol{\psi_{ad}}$ and $\boldsymbol{\psi_{aq}}$ are the air-gap(mutual) flux linkages and $\boldsymbol{L_{ads}}$ and $\boldsymbol{L_{aqs}}$ are the saturated values of mutual inductances.

From eqn, the field current may be expressed as

$$i_{fd} = (\psi_{fd}-\psi_{ad})/L_{fd} \quad (4)$$

The d-axis mutual flux linkage can be written in terms of $\boldsymbol{\psi_{fd}}$ and $\mathbf{i_d}$ as follows

$$\psi_{ad} = -L_{ads} i_d + L_{ads} i_{fd}$$

$$= -L_{ads} i_d + L_{ads}(\psi_{fd}-\psi_{ad})/L_{fd}$$

$$= L_{ads}'(-i_d + \psi_{fd}/L_{fd}) \quad (5)$$

where

$$L_{ads}'=1/(1/L_{ads}+1/L_{fd}) \qquad (6)$$

Since there are no rotor circuits considered in the q-axis, the mutual flux linkage is given by

$$\Psi_{aq} = -L_{aqs}i_q \qquad (7)$$

The air-gap torque is

$$T_e = \psi_d i_q - \psi_q i_d$$

$$= \psi_{ad} i_q - \psi_{aq} i_d \qquad (8)$$

With pψ terms and speed variations neglected the stator voltage equations become

$$e_d = -R_a i_d - \psi_q$$

$$= - R_a i_d + (L_l i_q - \Psi_{aq}) \qquad (9)$$

$$e_q = -R_a i_q - \psi_d$$

$$= - R_a i_q + (L_l i_d - \Psi_{ad}) \qquad (10)$$

### 6.2.2 Network Equations

Since there is only one machine, the machine as well as network equations can be expressed in terms of one reference frame, ie. the d-q reference frame of the machine. Referring to Fig. , the machine terminal and infinite bus voltages in terms of the d and q components are

$$E_t = e_d + je_q \qquad (11)$$

$$E_B = E_{BD} + jE_{bq} \qquad (12)$$

The network constraint equation for the system of Fig. 2 is

$$E_t = E_B + (R_E + jX_E)I_t \qquad (13)$$

$$e_d + je_q = (E_{BD} + jE_{bq}) + (R_E + jX_E)(i_d + ji_q) \qquad (14)$$

Resolving into d and q components gives

$$e_d = R_E i_d - X_E i_q + E_{Bd} \qquad (15)$$

$$e_q = R_E i_q - X_E i_d + E_{Bq} \tag{16}$$

where

$$E_{Bd} = E_B \sin\delta \tag{17}$$

$$E_{Bq} = E_B \cos\delta \tag{18}$$

Using equations and to eliminate $e_d$, $e_q$ in equations and and using the expressions for $\psi_{ad}$ and $\psi_{aq}$ given by equations and, we obtain the following expressions for id and iq in terms of state variables $\psi_{fd}$ and $\delta$.

$$i_d = (Xtq[\psi_{fd}(L_{ads}/(L_{ads}+L_{fd}) - E_B\cos\delta] - R_T E_B \sin\delta)/D \tag{19}$$

$$i_q = (R_T[\psi_{fd}(L_{ads}/(L_{ads}+L_{fd}) - E_B\cos\delta] - X_{Td} E_B \sin\delta)/D \tag{20}$$

where

$$R_T = R_a + R_E \tag{21}$$

$$X_{Tq} = X_E + (L_{aqs} + L_l) = X_E + X_{qs} \tag{22}$$

$$X_{Td} = X_E + (L_{ads}' + L_l) = X_E + X_{ds}' \tag{23}$$

$$D = R_T^2 + X_{Tq} X_{Td} \tag{24}$$

The reactances $X_{qs}$ and $X_{ds}'$ are saturated values. In per unit they are equal to the corresponding inductances.

### 6.2.3 Linearized System Equations

Expressing equations and in terms of perturbed values, we may write

$$\Delta i_d = m_1 \Delta\delta + m_2 \Delta\psi_{fd} \tag{25}$$

$$\Delta i_q = n_1 \Delta\delta + n_2 \Delta\psi_{fd} \tag{26}$$

where

$$m_1 = E_B(X_{Tq}\sin\delta_0 - R_T\cos\delta_0)/D$$

$$n_1 = E_B(R_T\sin\delta_0 - X_{Td}\cos\delta_0)/D$$

$$m_2 = (X_{Tq}/D)^* L_{ads}/(L_{ads}+L_{fd}) \qquad (27)$$

$$n_2 = (R_T/D)^* L_{ads}/(L_{ads}+L_{fd})$$

By linearizing equations (5) and (7) and substituting in them the above expressions for $\Delta i_d$ and $\Delta i_{q,}$ we get

$$\Delta\psi_{ad} = L_{ads}'(-\Delta i_d + \Delta\psi_{fd}/L_{fd})$$

$$= (1/ L_{fd} - m_2) L_{ads}' \Delta\psi_{fd} - m_2 L_{ads}' \Delta\delta \qquad (28)$$

$$\Delta \psi_{aq} = L_{ads}'(-\Delta i_d + \Delta\psi_{fd}/L_{fd})$$

$$= - n_2 L_{aqs}\Delta\psi_{fd} - n_1 L_{aqs}\Delta\delta \qquad (29)$$

Linearizing equation (4) and substituting for $\Delta\psi_{ad}$ from equation (28) gives

$$\Delta i_{fd} = (\Delta\psi_{fd}- \Delta\psi_{ad})/L_{fd}$$

$$= (1- L_{ads}'/ L_{fd}+m L_{ads}') \Delta\psi_{fd}/ L_{fd} \qquad (30)$$

The linearized form of equation (8) is

$$\Delta T_e = \psi_{ad0}\Delta i_q + i_{q0}\Delta\psi_{ad} - \psi_{aq0}\Delta i_d + i_{d0}\Delta\psi_{aq} \qquad (31)$$

Substituting for $\Delta i_d$, $\Delta i_q$, $\Delta\psi_{ad}$ and $\Delta\psi_{aq}$ from equations (25) to (29), we obtain

$$\Delta T_e = K_1\Delta\delta +K2\ \Delta\psi_{fd} \qquad (32)$$

where $K_1 = n_1(\psi_{ad0}+ L_{aqs}i_{d0}) - m_1(\psi_{aq0}+ L_{ads}'i_{q0})$

$$K_2 = n_2(\psi_{ad0}+ L_{aqs}i_{d0}) - m_2(\psi_{aq0}+ L_{ads}'i_{q0}) + L_{ads}'/ L_{fd}\ i_{q0} \qquad (33)$$

By using the expressions for $\Delta i_{fd}$ and $\Delta T_e$ given by equations (30) and (32), we obtain the system equations in the desired final form:

where

$$a_{11} = -K_D/(2H)$$

$$a_{12} = -K_1/(2H)$$

$$a_{13} = -K_2/(2H)$$

$a_{21} = \omega_0 = 2\Pi f_0$

$a_{32} = -(\omega_0 R_{fd}/ L_{fd}) m_1 L_{ads}'$ (35)

$a_{33} = -\omega_0 R_{fd}/ L_{fd}[1- L_{ads}'/ L_{fd}+m_2 L_{ads}']$

$b_{11}=1/(2H)$

$b_{32} = \omega_0 R_{fd}/ L_{adu}$

and $\Delta T_m$ and $\Delta E_{fd}$ depend on prime-mover and excitation controls. With constant mechanical input torque, $\Delta T_m=0$; with constant exciter output voltage, $\Delta E_{fd} =0$.

The mutual inductances $L_{ads}$ and $L_{aqs}$ in the above equations are saturated values.

### 6.2.4 Representation of saturation in small-signal studies

Since we are expressing small-signal performance in terms of perturbed values of flux linkages and currents, a distinction has to be made between total saturation and incremental saturation.

Total saturation is associated with total values of flux linkages and currents.

Incremental saturation is associated with perturbed values of flux linkages and currents. Therefore, the incremental slope of the saturation curve is used in computing the incremental saturation as shown in figure.

Denoting the incremental saturation factor $K_{sd(incr)}$, we have

$$L_{ads(incr)} = K_{sd(incr)} L_{adu} \qquad (36)$$

Based on the definitions of $A_{sat}$, $B_{sat}$ and $\psi_{T1}$ it is shown that

$$K_{sd(incr)} = 1/(1+ A_{sat}B_{sat}e^{B_{sat}(\psi_{at0}-\psi_{T1})}) \qquad (37)$$

A similar treatment applies to q-axis saturation.

For computing the initial values of system variables (denoted by subscript 0) total saturation is used.

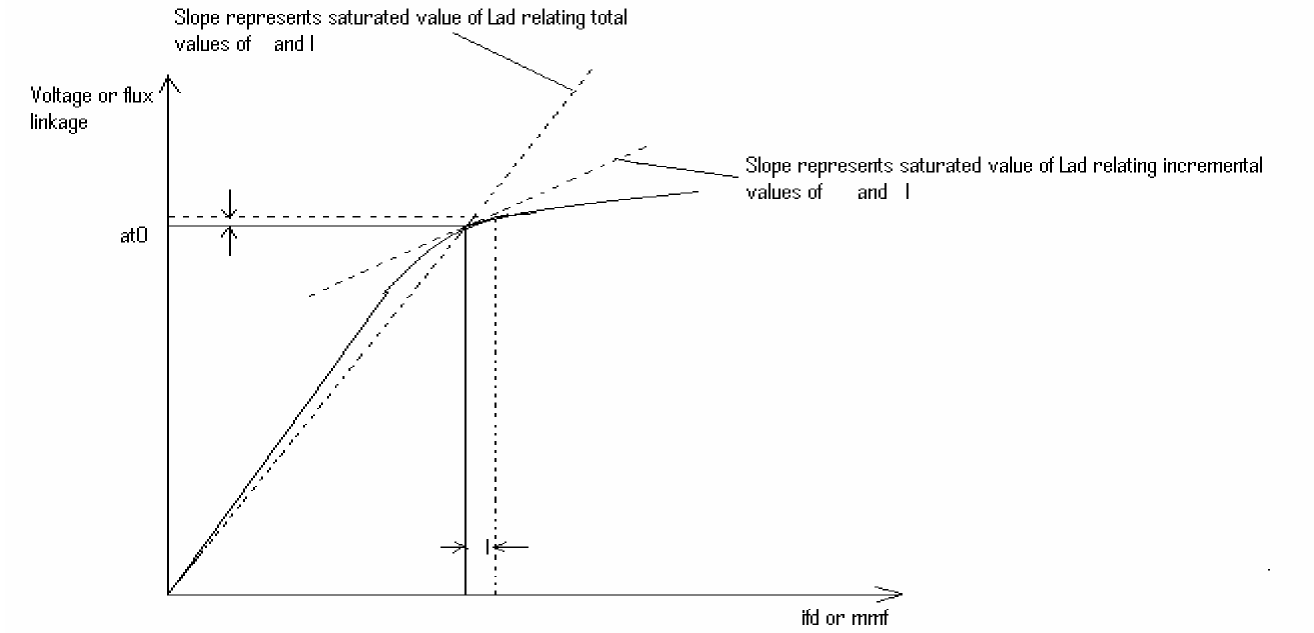


Fig. 6.4 Distinction between incremental and total saturation

### 6.2.5 Summary of procedure for formulating the state matrix

(a) The following steady-state operating conditions, machine parameters and network parameters are given:

$P_t \quad Q_t \quad E_t \quad R_E \quad X_E$

$L_d \quad L_q \quad L_l \quad R_a \quad L_{fd} \quad R_{fd} \quad A_{sat} \quad B_{sat} \quad \psi_{T1}$

Alternatively $E_B$ may be specified instead of $Q_t$ or $E_t$.

(b) The first step is to compute the initial steady-state values of system variables:

$I_t$, power factor angle $\Phi$

Total saturation factors $K_{sd}$ and $K_{sq}$

$X_{ds} = L_{ds} = K_{sd}L_{adu} + L_l$

$X_{qs} = L_{qs} = K_{sq}L_{aqu} + L_l$

$\delta_i = \tan^{-1}((I_tX_{qs}\cos\Phi - I_tR_a\sin\Phi)/( E_t + I_tR_a\cos\Phi - I_tX_{qs}\sin\Phi))$

$e_{d0} = E_t\sin\delta_i$

$e_{q0} = E_t\cos\delta_i$

$i_{d0} = I_t\sin(\delta_i+\Phi)$

$i_{q0} = I_t\cos(\delta_i+\Phi)$

$E_{Bd0} = e_{d0} – R_E i_{d0}+X_E i_{q0}$

$E_{Bq0} = e_{q0} – R_E i_{q0}+X_E i_{d0}$

$\delta_0 = \tan^{-1}(E_{Bd0}/ E_{Bq0})$

$E_B = (E_{Bd0}^2 + E_{Bq0}^2)^{1/2}$

$i_{fd0} = (e_{q0} + R_a i_{q0} + L_{ds} i_{do})/L_{ads}$

$E_{fd0} = L_{adu} i_{fd0}$

$\Psi_{ad0} = L_{ads}(-i_{d0}+i_{fd0})$

$\Psi_{aq0} = -L_{aqs} i_{q0}$

(c) The next step is to compute incremental saturation factors and the corresponding saturated values of $L_{ads}$, $L_{aqs}$, $L'_{ads}$ and then

$R_T$, $X_{Tq}$, $X_{Td}$, D

$m_1$,$m_2$, $n_1$,$n_2$

$K_1$,$K_2$

(d) Finally, compute the elements of matrix A from equation (35)

### 6.3 BLOCK DIAGRAM REPRESENTATION

Figure 6.5 shows the block diagram representation of the small-signal performance of the system. In this representation, the dynamic characteristics of the system are expressed in terms of the so-called K constants. The basis for the block diagram and the expressions for the associated constants are developed below.

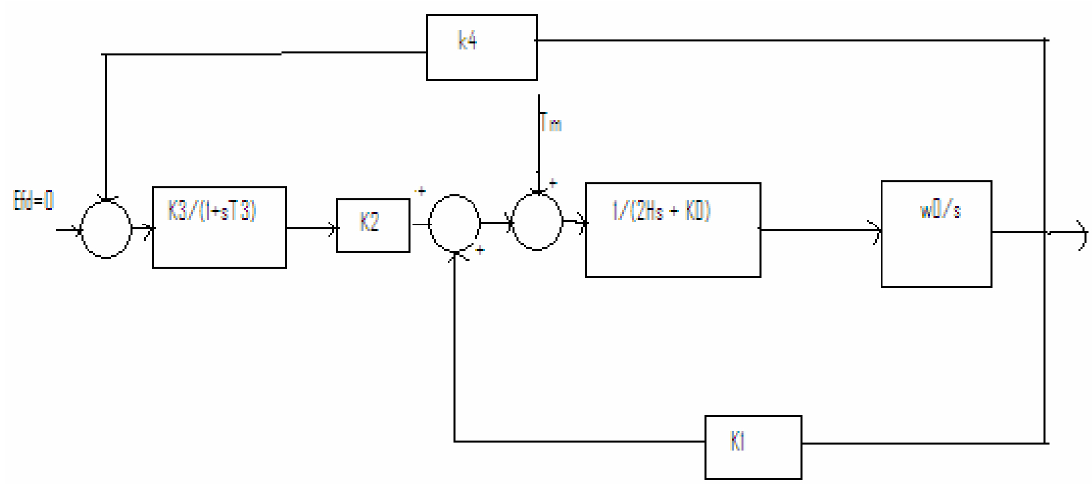


Fig. 6.5 Block Diagram Representation of Small-Signal Performance

From equation 32 we may express the change in air-gap torque as a function of $\Delta\delta$ and $\Delta\psi_{fd}$ as follows:

$$\Delta T_e = K_1\Delta\delta + K_2\,\Delta\psi_{fd}$$

Where

$K_1 = \Delta T_e / \Delta\delta$ with constant $\psi_{fd}$

$K_2 = \Delta T_e / \Delta\psi_{fd}$ with constant rotor angle $\delta$

The expressions for $K_1$ and $K_2$ are given by Equations 33 and 34.

The component of torque given by $K_1\Delta\delta$ is in phase with $\Delta\delta$ and hence represents a synchronizing torque component.

The component of torque resulting from variations in field flux linkage is given by $K_2\Delta\psi_{fd}$.

The variation of $\psi_{fd}$ is determined by the field circuit dynamic equation:

$$p\Delta\psi_{fd} = a_{32}\Delta\delta + a_{33}\Delta\psi_{fd} + b_{32}\Delta E_{fd}$$

By grouping terms involving $\Delta\psi_{fd}$ and rearranging, we get

$$\Delta\psi_{fd} = K_3\,(\Delta E_{fd} - K_4\,\Delta\delta)/(1 + pT_3) \qquad (38)$$

where

$$K_3 = -b_{32}/a_{33}$$

$$K_4 = -a_{32}/b_{32} \qquad (39)$$

$$T_3 = -1/a_{33} = K_3\, T'_{d0} L_{adu}/L_{ffd}$$

Equations (38) with s replacing p, accounts for the field circuit block in figure

***6.3.1* Expressions for the K constants in the expanded form*:***

We have expressed K constants in terms of the elements of matrix A. In the literature, they are usually expressed explicitly in terms of the various system parameters, as summarized below.

The constant K1 was expressed in Equation (33) as

$$K_1 = n_1 (\psi_{ad0} + L_{aqs} i_{d0}) - m_1 (\psi_{aq0} + L'_{ads} L_{aqs} i_{q0})$$

From equation (10), the first term in parentheses in the above expression for $K_1$ may be written as:

$$\psi_{ad0} + L_{aqs}\, i_{d0} = e_{q0} + R_a i_{q0} + X_{qs}\, i_{d0} = E_{q0} \qquad (40)$$

where $E_{q0}$ is the pre disturbance value of the voltage behind $R_a + jX_q$ . The second term in parentheses in the expression for $K_1$ may be written as

$$\psi_{aq0} + L'_{aqs}\, i_{q0} = -L_{aqs}\, i_{q0} + L'_{ads}\, i_{q0}$$

$$(41)$$

$$= -(X_q - X'_d)\, i_{q0}$$

Substituting for n1, m1 from Equation (27) and for the terms given by Equations (40) and (41) in the expression for $K_1$, yields

$$K_1 = E_B E_{q0} (R_T \sin\delta_0 + X_{Td} \cos\delta_0)/D + E_B i_{q0} (X_q - X'_d)( X_{Tq} \sin\delta_0 - RT \cos\delta_0)/D \quad (42)$$

Similarly, the expanded form of the expression for the constant $K_2$ is

$$K_2 = L_{ads} [R_T E_{q0}/D + ((X_{Tq} (X_q - X'_d)/D) + 1)\, i_{q0}]/(L_{ads} + L_{fd}) \qquad (43)$$

From Equations (6),(27) and (35) we may write

$a_{33} = (-\omega_0 R_{fd}/L_{fd})\ [1- (L_{ads}/(L_{ads} + L_{fd}))+ (X_{Tq}\ L_{ads}\ L_{ads}\ L_{fd}/D(L_{ads} + L_{fd})(L_{ads} + L_{fd}))]$

$= -\omega_0\ R_{fd}/(L_{ads} + L_{fd})[1 + (X_{Tq}\ L_{ads}{}^2/D(L_{ads} + L_{fd}))]$ (44)

$= -\omega_0\ R_{fd}/(L_{ads} + L_{fd})[1 + X_{Tq}(X_d - X'_d)/D]$

Substitution of the above in the expression for $K_3$ and $T_3$ given by Equation (39) yields

$K_3 = (L_{ads} + L_{fd})/\ (L_{adu}(1 + X_{Tq}(X_d - X'_d)/D))$ (45)

$T_3 = (L_{ads} + L_{fd})/\ (\omega_0 R_{fd}\ (1 + X_{Tq}(X_d - X'_d)/D))$ (46)

$= T'_{d0s}/(1 + X_{Tq}(X_d - X'_d)/D)$

Where T'd0s is the saturated value of T'd0. Similarly, from Equations (6),(27) and (35) we may write

$a_{32} = (-\omega_0\ R_{fd}\ E_B/D\ L_{fd})(\ X_{Tq}\ \sin\delta_0 - R_T\ \cos\delta_0)\ L_{ads}L_{fd}/(L_{ads} + L_{fd})$

Substitution of the above in the expression for $K_4$ given by Equation (39) yields

$K_4 = L_{adu}\ L_{ads}\ E_B\ (X_{Tq}\ \sin\delta_0 - R_T\ \cos\delta_0)\ /(D\ (L_{ads} + L_{fd}))$ (47)

If the effect of saturation is neglected, this simplifies to

$K_4 = E_B\ (X_d - X'_d)\ (X_{Tq}\ \sin\delta_0 - R_T\ \cos\delta_0)/D$ (48)

If the elements of matrix A are available, the K constants may be computed directly from them. The expanded forms are derived here to illustrate the form of expressions used in the literature. An advantage of these expanded forms is that the dependence of the K constants on the various system parameters is more readily apparent. A disadvantage, however, is that some inconsistencies appear in representing saturation effects.

In the literature, $E'_q = (L_{ad}/\ L_{ffd})\ \psi_{fd}$ is often used as a state variable instead of $\psi_{fd}$. The effect of this is to remove the $L_{ad}/(\ L_{ad} + L_{fd})$ term from the expressions for $K_2$ and $K_3$. The product $K_2K_3$ would, however remain the same.

### 6.3.2 Effect of field flux linkage variation on system stability

We see from the block diagram of figure 23452342 that, with constant field voltage ($\Delta E_{fd}$ =0), the field flux variations are caused only by feedback of $\Delta\delta$ through the coefficient $K_4$. This represents the demagnetizing effect of the armature reaction.

The change in air-gap torque due to field flux variations caused by rotor angle changes is given by

$$(\Delta T_e / \Delta\delta) \text{ (due to } \Delta\psi_{fd}) = -K_2K_3K_4/(1+sT_3) \qquad (49)$$

The constants K2, K3 and K4 are usually positive. The condition of $\Delta\psi_{fd}$ to synchronizing and damping torque components depends on the oscillating frequency as discussed below.

(a) In the steady state and at very low oscillating frequencies ($s = j\omega \rightarrow 0$):

$\Delta T_e$ due to $\Delta\psi_{fd} = -K_2K_3K_4\ \Delta\delta$

The field flux variation due to $\Delta\delta$ feed back (i.e. , due to armature reaction) introduces a negative synchronizing torque component. The system becomes monotonically unstable when this exceeds $K_1\Delta\delta$. The steady state stability limit is reached when

$$K_2K_3K_4 = K_1$$

(b) At oscillating frequencies much higher than $1/T_3$

$$\Delta T_e \approx -K_2K_3K_4\Delta\delta \,/\, j\omega T_3$$

$$= K_2K_3K_4\ j\Delta\delta \,/\, \omega T_3$$

Thus, the component of air-gap torque due to $\Delta\psi_{fd}$ is $90^o$ ahead of $\Delta\delta$ or in phase with $\Delta\omega$ . Hence, $\Delta\psi_{fd}$ results in a positive damping torque component.

(b) At typical machine oscillating frequencies of about 1 Hz (2∏ rad/s), $\Delta\psi_{fd}$ results in a positive damping torque component and a negative synchronizing torque component. The net effect is to reduce slightly the synchronizing torque component and increase the damping torque component.

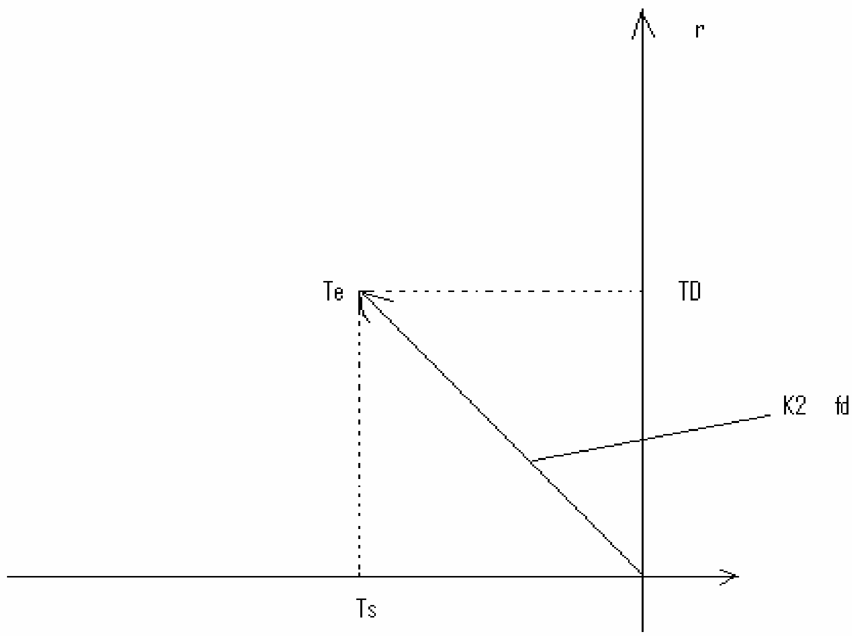


Fig. 6.6 Positive damping torque and negative synchronizing torque due to $K_2\Delta\psi_{fd}$

### 6.3.3 Special situations with $K_4$ negative

The coefficient $K_4$ is normally positive. As long as it is positive, the effect of field flux variation due to armature reaction ($\Delta\psi_{fd}$ with constant $E_{fd}$) is to introduce a positive torque component. However, there can be situations where $K_4$ is negative. From the expression given by Equation 58978698, $K_4$ is negative when $(X_E+X_Q)\sin\delta_0 - (R_a+R_E)\cos\delta_0$ is negative. This is the situation when a hydraulic generator without damper windings is operated at light load and is connected by a line of relatively high resistance to reactance ratio to a large system

Also $K_4$ can be negative when a machine is connected to a large local load, supplied partly by the generator and partly by the remote large system. Under such conditions, the torques produced by induced currents in the field due to armature reaction have components out of phase with $\Delta\omega$, and produce negative damping.

### 6.4 Illustration

The analysis of small-signal stability of the system of the figure 6.7 including the effects of the generator field circuit dynamics. The parameters of each of the four generators of the plant in pu on its rating are as follows:

$X_d = 1.81$ $X_q = 1.76$ $X_d' = 0.3$ $X_l=0.16$ $R_a=0.003$ $T_{d0}'=8.00$

$H=3.5$ $K_d=0$

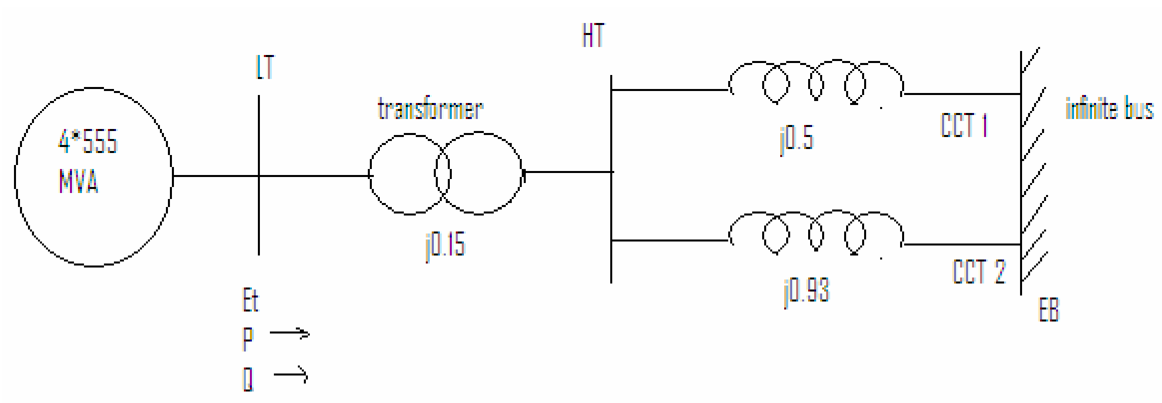


Fig. 6.7 A thermal generating station consisting of four 555MVA, 24 kV, 60Hz units

The above parameters are unsaturated values. The effect of saturation is to be represented by assuming that d and q axes have similar saturation characteristics with $A_{sat}=0.031$ $B_{sat} = 6.93$

$\Psi_{T1} = 0.8$

The effects of the amortisseurs may be neglected. The exciting system is on manual control ( constant Efd) and transmission circuit 2 is out of service.

If the plant output in pu on 2220MVA, 24KV base is P=0.9 Q=0.3(over excited), Et=1.0

Compute the following:

(i) The elements of the state matrix A representing the small signal performance of the system.

(ii) The constants K1 to K4 and T3 associated with the block diagram representation of figure.

(iii) Eigenvalues of A and the corresponding eigen vectors and participation matrix; frequency and damping ratio of the oscillatory mode.

(iv) Steady state synchronizing torque coefficient, damping and synchronizing torque coefficients at the rotor oscillating frequency.

The four units of the plant may be represented by a single generator whose parameters on 2220MVA base are the same as those of each unit on its rating. The circuit model of the system in pu on 2220MVA base is shown in fig.6.8

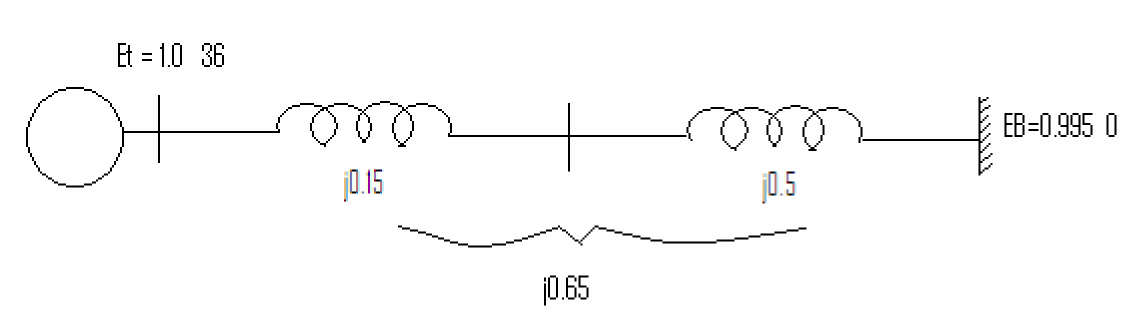


Fig. 6.8 The Equivalent circuit model of the system

## 6.5 MATLAB CODE

```
%-------------------------- INPUTS -------------------------%
Xd=1.81;      Xq=1.76;     Xl=0.16;
Ldh=0.3;      Xdh=0.3;     Ra=0.003;
f0=60;        Td0h=8.0;    H=3.5;
Kd=0;         Pt=0.9;      RE=0;
XE=0.65i;     Qt=0.3;      Et=1;
Asat=0.031;   Bsat=6.93;   SIt1=0.8;
```

```
%------ COMPUTATION OF INITIAL STEADY STATE VALUES OF THE SYSTEM ------%

fprintf('\nCOMPUTATION OF INITIAL STEADY STATE VALUES OF THE SYSTEM\n');

Ladu=Xd-Xl;

Laqu=Xq-Xl;

Ll=Xl;

Lfd=Ladu*(Ldh-Ll)/(Ladu-Ldh+Ll);

Rfd=(Ladu+Lfd)/(377*Td0h);

It=sqrt(power(Pt,2)+power(Qt,2));

Phi=acosd(Pt/It*Et);

Ith=It*(cosd(Phi)-sind(Phi)*i);

Ea=Et+(Ra+Xl*i)*Ith;

SIat=abs(Ea);

SIi=Asat*exp(Bsat*(SIat-SIt1));

Ksd=SIat/(SIat+SIi)

Ksq=Ksd

Xadu=Ladu;

Xad=Ksd*Xadu;

Xd=Xad+Xl;

Xaqu=Laqu;

Xaq=Ksq*Xaqu;

Xq=Xaq+Xl;

DELi=atand((Xq*It*cosd(Phi)-Ra*It*sind(Phi))/(Et+Ra*It*cosd(Phi)+Xq*It*sind(Phi)))

ed0=Et*sind(DELi)
```

```
eq0=Et*cosd(DELi)

id0=It*sind(DELi+Phi)

iq0=It*cosd(DELi+Phi)

EBd0=ed0-RE*id0+abs(XE)*iq0;

EBq0=eq0-RE*iq0-abs(XE)*id0;

DEL0=atand(EBd0/EBq0)

EB=sqrt(power(EBd0,2)+power(EBq0,2));

Lds=Ksd*Ladu+Ll;

Lads=Ksd*Ladu;

Laqs=Ksq*Laqu;

ifd0=(eq0+Ra*iq0+Lds*id0)/Lads;

Efd0=Ladu*ifd0

Ksdincr=1/(1+(Asat*Bsat*exp(Bsat*(SIat-SIt1))))

Ksqincr=Ksdincr

%---------------- COMPUTATION OF THE VALUES AFTER PERTURBATION ---------%

fprintf('\nCOMPUTATION OF THE VALUES AFTER PERTURBATION\n');

Laqsi=Ksqincr*Laqu

Ladsi=Ksdincr*Ladu

XTq=abs(XE)+(Laqsi+Ll)

Ladsh=1/((1/Ladsi)+(1/Lfd))

XTd=abs(XE)+Ladsh+Ll

RT=Ra+RE

D=power(RT,2)+(XTq*XTd)
```

```
m1=EB*(XTq*sind(DEL0)-RT*cos(DEL0))/D

n1=EB*((RT*sind(DEL0))+(XTd*cosd(DEL0)))/D

m2=(XTq*Ladsi)/(D*(Ladsi+Lfd))

n2=(RT*Ladsi)/(D*(Ladsi+Lfd))

SIad0=Lads*(ifd0-id0);

SIaq0=-Laqs*iq0;

K1=n1*(SIad0+(Laqsi*id0))-m1*(SIaq0+(Ladsh*iq0));

K2=n2*(SIad0+(Laqsi*id0))-m2*(SIaq0+(Ladsh*iq0))+(Ladsh*iq0/Lfd);

a11=-Kd/2*H;

a12=-K1/(2*H);

a13=-K2/(2*H);

a21=2*pi*f0;

a32=-(2*pi*f0*Rfd*m1*Ladsh)/Lfd;

a33=-2*pi*f0*Rfd*(1-(Ladsh/Lfd)+(m2*Ladsh))/Lfd;

b11=1/(2*pi);

b32=2*pi*f0*Rfd/Ladu;

K3=-b32/a33;

K4=-a32/b32;

T3=-1/a33;

A=[a11,a12,a13;a21,0,0;0,a32,a33];

[V,D]=eig(A);

Shi=inv(V);

for r=1:3
```

```
    for c=1:3
        P(r,c)=V(r,c)*Shi(c,r);
    end
end
Ks=K1-K2*K3*K4;
fprintf('\n Steady-state torque coefficient Ks = %4.3f \n',Ks);
s=imag(D(1,1))*i;
Ksrf=K1-abs(((K2*K3*K4)/(1-power(s,2)*power(T3,2))));
Kdrf=abs((K2*K3*K4*T3*2*pi*f0)/(1-power(s,2)*power(T3,2)));
Wn=sqrt((Ksrf*2*pi*f0)/(2*H));
Ep=(1/2)*(Kdrf/sqrt(Ksrf*2*H*2*pi*f0));
fprintf('\n State matrix A \n');
A
fprintf('\n Constants associated with the block diagram \n');
fprintf('\n K1 = %4.3f \n',K1);
fprintf('\n K2 = %4.3f \n',K2);
fprintf('\n K3 = %4.3f \n',K3);
fprintf('\n K4 = %4.3f \n',K4);
fprintf('\n T3 = %4.3f \n',T3);
fprintf('\n Eigen values L1= %4.3f , L2=%4.3f , L3=%4.3f \n',D(1,1),D(2,2),D(3,3));
fprintf('\n Eigen vectors matrix \n');
V
fprintf('\n Participation matrix \n');
```

P

```
fprintf('\n Steady-state synchronizing torque coefficient Ks = %4.3f \n',Ks);

fprintf('\n Synchronizing torque coefficient at rotor oscillating frequency Ksrf = %4.3f \n',Ksrf);

fprintf('\n Damping coefficient at rotor oscillating frequency Kdrf = %4.3f \n',Kdrf);

fprintf('\n Undamped natural frequency of the oscillatory mode Wn = %4.3f \n',Wn);

fprintf('\n Damping ratio of the oscillatory mode Ep = %4.3f \n',Ep)
```

## 6.6 RESULTS

COMPUTATION OF INITIAL STEADY STATE VALUES OF THE SYSTEM

Ksd =0.8491

Ksq = 0.8491

DELi = 43.1255

ed0 = 0.6836

eq0 = 0.7299

id0 = 0.8342

iq0 = 0.4518

DEL0 = 79.1317

Efd0 = 2.3947

Ksdincr = 0.4337

Ksqincr = 0.4337

COMPUTATION OF THE VALUES AFTER PERTURBATION

Laqsi = 0.6940

Ladsi = 0.7156

XTq = 1.5040

Ladsh = 0.1260

XTd = 0.9360

RT = 0.0030

D = 1.4078

m1 = 1.0458

n1 = 0.1268

m2 = 0.8802

n2 = 0.0018

Steady-state torque coefficient Ks = 0.368

State matrix A

A =

| | | |
|---|---|---|
| 0 | -0.1094 | -0.1236 |
| 376.9911 | 0 | 0 |
| 0 | -0.1942 | -0.4229 |

Constants associated with the block diagram

K1 = 0.765

K2 = 0.865

K3 = 0.323

K4 = 1.422

T3 = 2.365

Eigen values L1= -0.110 , L2=-0.110 , L3=-0.204

Eigen vectors matrix

V =

| | | |
|---|---|---|
| -0.0003 + 0.0170i | -0.0003 - 0.0170i | 0.0004 |
| 0.9994 | 0.9994 | -0.7485 |
| -0.0015 + 0.0302i | -0.0015 - 0.0302i | 0.6631 |

Participation matrix

P =

| | | |
|---|---|---|
| 0.5005 - 0.0085i | 0.5005 + 0.0085i | -0.0011 - 0.0000i |
| 0.5005 - 0.0085i | 0.5005 + 0.0085i | -0.0011 - 0.0000i |
| -0.0011 + 0.0171i | -0.0011 - 0.0171i | 1.0022 + 0.0000i |

Steady-state synchronizing torque coefficient Ks = 0.368

Synchronizing torque coefficient at rotor oscillating frequency Ksrf = 0.764

Damping coefficient at rotor oscillating frequency Kdrf = 1.531

Undamped natural frequency of the oscillatory mode Wn = 6.413

Damping ratio of the oscillatory mode Ep = 0.017

# Chapter 7

## TRANSIENT STABILITY ANALYSIS INCLUDING DAMPING

## 7.1 AN ELEMENTARY VIEW OF TRANSIENT STABILITY

Consider the system shown in figure (7.1) consisting of a generator delivering power to a large system represented by an infinite bus through transmission circuits. An infinite bus represents a voltage source of constant voltage magnitude and constant frequency.

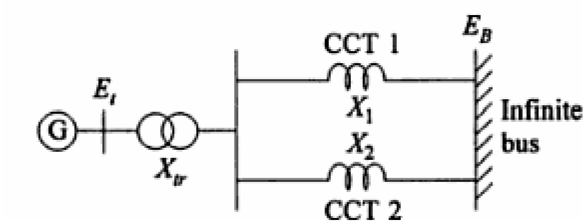


Fig. 7.1 Single-machine infinite bus system

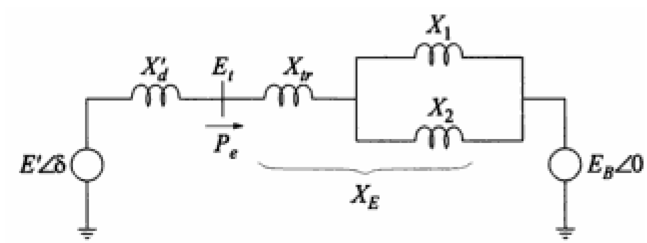


Fig 7.2(a) Equivalent circuit

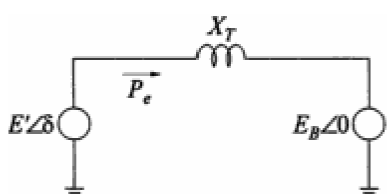


Fig 7.2(b) Reduced equivalent circuit

Fig 7.2 System representation with generator represented by classical model

We will present fundamental concepts and principles of transient stability by analyzing the system response to large disturbances, using very simple models. All resistances are neglected. The generator is represented by the classical model (fig 7.1) and the speed governor effects are neglected. The corresponding system representation is shown in Figure (7.2 a). The voltage behind the transient reactance ($X_d$') is denoted by E'. The rotor angle δ represents the angle by which E' leads $E_B$. When the system is perturbed, the magnitude of E' remains constant at its pre disturbance value and $\delta$ changes as the generator rotor speed deviates from synchronous speed $\omega_0$.

The system model can be reduced to the form shown in Figure (7.2 b). It can be analyzed by using simple analytical methods and is helpful in acquiring a basic understanding of the transient stability phenomenon.

The generator's electrical output is

$$P_e = \frac{E'E_B}{X_T}\sin\delta = P_{max}\sin\delta \quad P_{max} = \frac{E'E_B}{X_T}$$

Since we have neglected the stator resistance, $P_e$ represents the air-gap power as well as the terminal power. The power angle relationship with both transmission circuits in service (I/S) is shown graphically in Figure (7.3) as curve 1. With a mechanical power input of $P_m$, the steady-state electrical power output $P_e$ is equal to $P_m$, and the operating condition is represented by point *a* on the curve. The corresponding rotor angle is $\delta_a$.

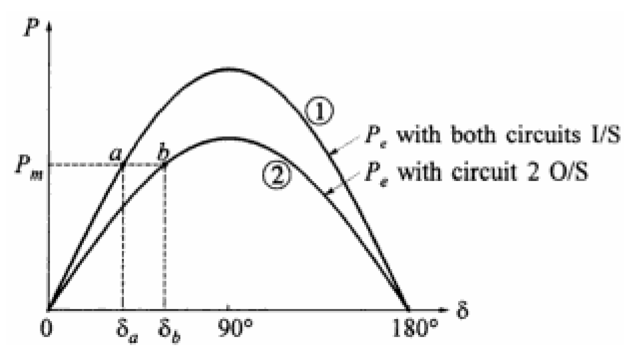


Fig.7.3 Power-angle relationship

If one of the circuits is out of service (O/S), the effective reactance $X_T$ is higher. The power-angle relationship with circuit 2 out of service is shown in Figure (7.3) as curve 2.The maximum power is now lower. With a mechanical power input of $P_m$, the rotor angle is now $\delta_b$ corresponding to the operating point *b* on curve 2; with a higher reactance, the rotor angle is higher in order to transmit the same steady-state power.

During a disturbance, the oscillation of $\delta$ is superimposed on the synchronous speed $\omega_0$, but the speed deviation ($\Delta\omega_r = d\delta/dt$) is very much smaller than $\omega_0$. Therefore, the generator speed is practically equal to $\omega_0$ and

the per unit (pu) air-gap torque may be considered to be equal to the pu air-gap power. We will therefore use torque and power interchangeably when referring to the swing equation.

The *equation of motion* or the *swing equation* may be written as

Where

$P_m$ = mechanical power input, in pu

$P_{max}$ = maximum electrical power output, in pu

$H$ = inertia constant, in elec.rad

$t$ = time, in s

## 7.2 RESPONSE TO A STEP CHANGE IN $P_m$

Let us now examine the transient behavior of the system, with both circuits in service, by considering a sudden increase in the mechanical power input from an initial value of $P_{m0}$ to $P_{m1}$ as shown in Figure (7.4). Because of the inertia of the rotor, the rotor angle can not change instantly from the initial value of $\delta_0$ to $\delta_1$ corresponding to the new equilibrium point b at which $P_e = P_{m1}$. The mechanical power is now in excess of the electrical power. The resulting accelerating torque causes the rotor to accelerate from the initial operating point *a* toward the new equilibrium point *b*, tracing the $P_e$-$\delta$ curve at a rate determined by the swing equation. The difference between $P_{m1}$ and $P_e$ at any instant represents the accelerating power.

When point *b* is reached, the accelerating power is zero, but the rotor speed is higher than the synchronous speed $\omega_0$ (which corresponds to the frequency of the infinite bus voltage). Hence, the rotor angle continues to increase. For values of $\delta$ higher than $\delta_1$, $P_e$ is greater than $P_{m1}$ and the rotor decelerates. At some peak value $\delta_m$, the rotor speed recovers to the synchronous value $\omega_0$, but $P_e$ is higher than $P_{m1}$. The rotor continues to decelerate with the speed dropping below $\omega_0$; the operating point retraces the $P_e$-$\delta$ curve from *c* to *b* and then to *a*. The rotor angle oscillates indefinitely about the new equilibrium angle $\delta_1$ with constant amplitude as shown by the time plot of $\delta$ in Figure (7.4 b).

In our representation of the power system in the above analysis, we have neglected all resistances and the classical model is used to represent the generator. In effect, this neglects all sources of damping. Therefore, the rotor oscillates continue unabated following the perturbation. There are many sources of positive damping including field flux variations and rotor amortisseur circuits. In a system which is small-signal stable, the oscillations damp out.

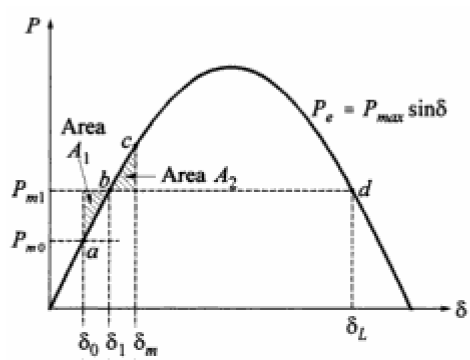


Fig 7.4(a) Power angle variations

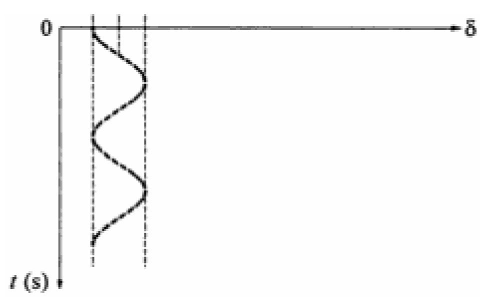


Fig 7.4(b) Rotor angle time response

Figure 7.4 Response to a step change in mechanical power output

## 7.3 RESPONSE TO A SHORT-CIRCUIT FAULT

Let us consider the response of the system to a three-phase fault at location F on transmission circuit 2, as shown in Figure 5(a). The corresponding equivalent circuit, assuming a classical generator model, is shown in Figure (5 b). The fault is cleared by opening circuit breakers at both the ends of the faulted circuit, the fault clearing time depending upon the relaying time and breaker time.

If the fault location F is at the sending end (HT bus) of the faulted circuit, no power is transmitted to the infinite bus. The short-circuit current from the generator flows through pure reactances to the fault. Hence, only reactive power flows and the active power $P_e$ and the corresponding electrical torque $T_e$ at the air-gap are zero during the fault. If we had included generator stator and transformer resistances in our model $P_e$ would have a small value, representing the corresponding resistive losses.

If the fault location F is at some distance away from the sending end as shown in Figures 5(a) and (b) some active power is transmitted to the infinite bus while the fault is still on.

Figures 5(c) and (d) show $P_e$-$\delta$ plots for the network conditions :

(i) pre fault (both circuits in service)

(ii) with a three phase fault on circuit 2 at a location some distance from the sending end

(iii) post fault (circuit 2 out of service)

## 7.4 FACTORS INFLUENCING TRANSIENT STABILITY

We conclude that transient stability of the generator is dependent on the following:

(a) How heavily the generator is loaded

(b) The generator output during fault. This depends on the fault location and type

(c) The fault-clearing time

(d) The post fault transmission system reactance

(e) The generator reactance. A lower reactance increases peak power and reduces initial rotor angle.

(f) The generator inertia. The higher the inertia, the slower the rate of change in angle. This reduces the kinetic energy gained during fault; i.e., area A1 is reduced.

(g) The generator internal voltage magnitude (E'). This depends on the field excitation.

(h) The infinite bus voltage magnitude $E_B$

As a means of introducing basic concepts, we have considered a system having a simple configuration and represented by a simple model. This has enabled the analysis of stability by using a graphical approach. Although rotor angle plots as a function of time are shown in Figures 4 and 5, we have not actually computed them, and hence the time scales have not been defined for these plots. Practical power systems have complex network structures. Accurate analysis of their transient stability requires detailed models for generating units and other equipment. At present, the most practical available method of transient stability analysis is time-domain simulation in which non-linear differential equations are solved by using step-by-step numerical integration techniques.

## 7.5 NUMERICAL INTEGRATION METHODS

The differential equations to be solved in power system stability analysis are nonlinear ordinary differential equations with known initial values:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x},t)$$

where **x** is the state vector of *n* dependent variables and *t* is the independent variable (time). Our objective is to solve **x** as a function of *t*, with the initial values of **x** and *t* equal to $\mathbf{x_0}$ and $t_0$ respectively.

In this section we provide a general description of numerical integration methods applicable to the solution of equations of the above form. In describing these methods, without loss of generality, we'll treat above equation as if it were a first order differential equation.

### 7.5.1 RUNGE-KUTTA (R-K) METHODS

The R-K methods approximate the Taylor series solution; however, unlike the formal Taylor series solution, the R-K methods do not require explicit evaluation of derivatives higher than the first. The effects of higher derivatives are included by several evaluations of the first derivative. Depending on the number of terms effectively retained in the Taylor series, we have R-K methods of different orders.

#### 7.5.1.1 Second-order R-K method

Referring to the above differential equation, the second order R-K formula for the value of $x$ at $t = t_0 + \Delta t$ is

$$x_1 = x_0 + \Delta x = x_0 + (k_1 + k_2)/2$$

where

$$k_1 = f(x_0, t_0)\,\Delta t$$

$$k_2 = f(x_0 + k_1, t_0 + \Delta t)\,\Delta t$$

This method is equivalent to considering first and second derivative terms in the Taylor series; error is on the order of $\Delta t$.

A general formula giving the value of x for (n + 1)$^{st}$ step is

$$x_{n+1} = x_n + (k_1 + k_2)/2$$

where

$$k_1 = f(x_n, t_n)\,\Delta t$$

$$k_2 = f(x_n + k_1, t_n + \Delta t)\,\Delta t$$

### 7.5.1.2 Fourth-order R-K method

The general formula giving the value of x for the $(n + 1)^{st}$ step is

$$x_{n+1} = x_n + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

where

$$k_1 = f(x_n, t_n)\,\Delta t$$

$$k_2 = f(x_n + k_1/2, t_n + \Delta t/2)\,\Delta t$$

$$k_3 = f(x_n + k_2/2, t_n + \Delta t/2)\,\Delta t$$

$$k_4 = f(x_n + k_3, t_n + \Delta t)\,\Delta t$$

The physical interpretation of the above solution is as follows:

$k_1$ = (slope at the beginning of time step) $\Delta t$

$k_2$ = (first approximation to slope at midstep) $\Delta t$

$k_3$ = (second approximation to slope at midstep) $\Delta t$

$k_4$ = ( slope at the end of step) $\Delta t$

$\Delta x = (k_1 + 2k_2 + 2k_3 + k_4)/6$

Thus $\Delta x$ is the incremental value of $x$ given by the weighted average of estimates based on slopes at the beginning, midpoint, and end of the time step.

This method is equivalent to considering up to fourth derivative terms in the Taylor series expansion; it has an error on the order of $\Delta t^5$

## 7.6 Illustration:

We examine the transient stability of a thermal generating station consisting of four 555 MVA, 24 KV, 60 Hz units supplying power to an infinite bus through two transmission circuits as shown in Figure 7.5.

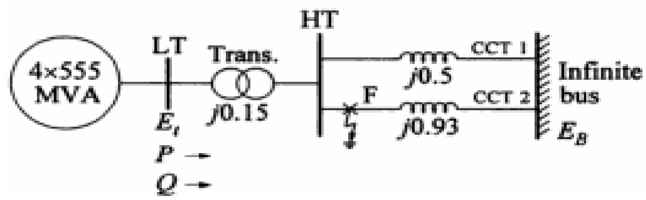


Fig. 7.5 Equivalent circuit of the thermal generating station

The network reactances shown in the figure are in per unit on 2220 MVA, 24 kV base (referred to the LT side of the step-up transformer). Resistances are assumed to be negligible.

The initial system –operating condition, with quantities expressed in per unit on 2220 MVA and 24 kV base, is as follows:

$P = 0.9$ $Q = 0.436$ (overexcited) $Et = 1.0\angle 28.34^\circ$ $E_B = 0.90081\angle 0$ The generators are modelled as a single equivalent generator represented by the classical model with the following parameters expressed in per unit on 2220 MVA, 24 kV base:

$X_d' = 0.3$ $H = 3.5$ MW.s/MVA $K_D = 0$

Circuit 2 experiences a solid three-phase fault at point F, and the fault is cleared by isolating the faulted circuit.

Determine the critical fault clearing time and the critical clearing angle by computing the time response of the rotor angle, using numerical integration.

## 7.7 SOLUTION

With the generator represented by the classical model, the reduced equivalent circuit representing the three system conditions: (i) prefault (ii) during fault (iii) post fault. Also shown in the figure are the corresponding expressions for the electrical power output as a function of δ.

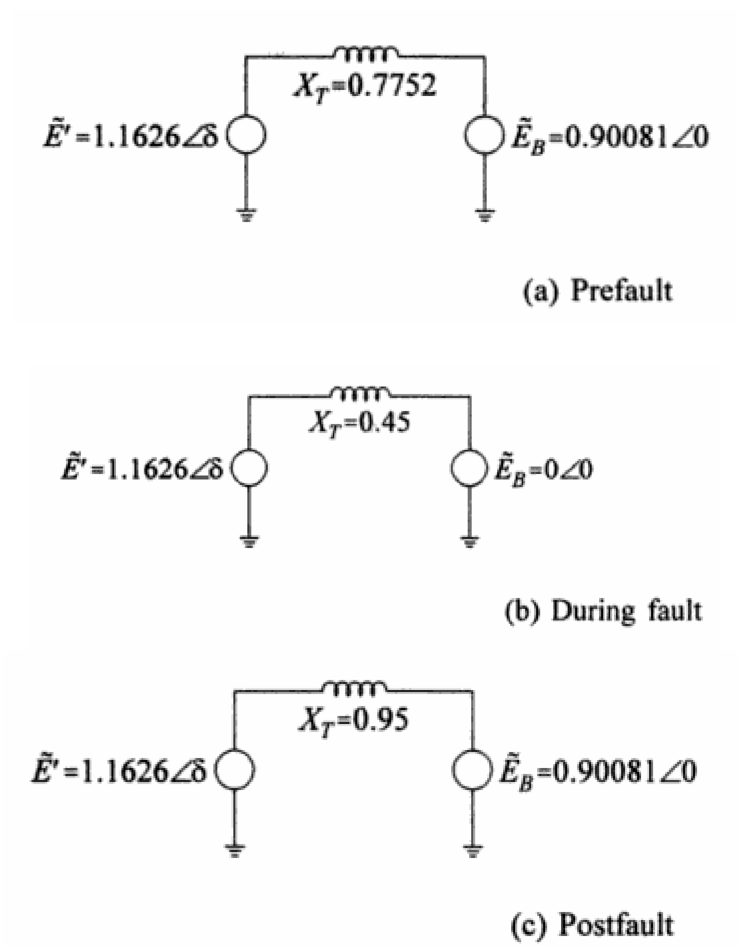


Fig. 7.6 Equivalent Circuit for prefault, during fault and post fault conditions

The equations of motion can be written as

$$p(\Delta\omega_r) = \frac{1}{2H}(P_m - P_{max}\sin\delta)$$

$$= \frac{1}{7.0}(0.9 - P_{max}\sin\delta)$$

$$p(\delta) = \omega_0 \Delta\omega_r$$

## 7.8 MATLAB CODE

```
global  Pm f H E V X1 X2 X3

Pm = 0.9;  E = 1.1626;  V = 0.90081;

X1 = 0.775; X2 = 0.45; X3 = 0.95;
```

```
H = 3.5; f = 60; tf = 10.0; Dt = 0.05;

disp('Parts (a) & (b) are repeated using swingrk4')

disp('Press Enter to continue')

pause

tc = 0.07;

swingrk2(Pm, E, V, X1, X2, X3, H, f, tc, tf)

tc = 0.086;

swingrk2(Pm, E, V, X1, X2, X3, H, f, tc, tf)

tc = 0.087;

swingrk2(Pm, E, V, X1, X2, X3, H, f, tc, tf)

This program solves the swing equation of a one-machine system

% when subjected to a three-phase fault with subsequent clearance

% of the fault.

function swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)

%global  Pm f H E V X1 X2 X3

if exist('Pm') ~= 1

Pm = input('Generator output power in p.u. Pm = '); else, end

if exist('E') ~= 1

E = input('Generator e.m.f. in p.u. E = '); else, end

if exist('V') ~= 1

V = input('Infinite bus-bar voltage in p.u. V = '); else, end

if exist('X1') ~= 1

X1 = input('Reactance before Fault in p.u. X1 = '); else, end
```

```
if exist('X2') ~= 1

X2 = input('Reactance during Fault X2 = '); else, end

if exist('X3') ~= 1

X3 = input('Reactance after Fault X3 = '); else, end

if exist('H') ~= 1

H  = input('Generator Inertia constant in sec. H = '); else, end

if exist('f') ~= 1

f  = input('System frequency in Hz f = '); else, end

if exist('tc') ~= 1

tc = input('Clearing time of fault in sec tc = '); else, end

if exist('tf') ~= 1

tf = input('Final time for swing equation in sec tf = '); else, end

Pe1max = E*V/X1; Pe2max=E*V/X2; Pe3max=E*V/X3;

clear t  x  delta

d0 =asin(Pm/Pe1max);

t0 = 0;

x0 = [d0;  0];

%tol=0.001;

%[t1,xf] =ode23('pfpower', t0, tc, x0, tol);  % During fault solution (use with MATLAB 4)

tspan = [t0, tc];                              % use wint MATLAB 5

[t1,xf] =ode23('pfpower', tspan, x0);  % During fault solution (use with MATLAB 5)

x0c =xf(length(xf), :);

%[t2,xc] =ode23('afpower', tc, tf, x0c, tol); % After fault solution (use with MATLAB 4)
```

```
tspan = [tc, tf];
[t2,xc] =ode23('afpower', tspan, x0c); % After fault solution (use with MATLAB 5)
t =[t1; t2]; x = [xf; xc];
delta = 180/pi*x(:,1);
clc
fprintf('\nFault is cleared at %4.3f Sec. \n', tc)
fprintf('\n %4.3f   %4.3f  %4.3f', x0,d0, Pe1max)
head=['                              '
      '    time     delta      Dw    '
      '     s       degrees    rad/s'
      '                              '];
disp(head)
disp([t, delta, x(:, 2)])
h=figure; figure(h)
plot(t, delta), grid
title(['One-machine system swing curve. Fault cleared at ', num2str(tc),'s'])
xlabel('t, sec'), ylabel('Delta, degree')
cctime(Pm, E, V, X1, X2, X3, H, f)    % Obtains the critical clearing time
% This function  Simulates the swing equation of a one-machine system
% and returns the critical clearing time for stability.
function  cctime(Pm, E, V, X1, X2, X3, H, f)
Pe1max = E*V/X1; Pe2max=E*V/X2; Pe3max=E*V/X3;
d0 =asin(Pm/Pe1max);
```

```
dmax = pi-asin(Pm/Pe3max);
cosdc = (Pm*(dmax-d0)+Pe3max*cos(dmax)-Pe2max*cos(d0))/(Pe3max-Pe2max);
  if abs(cosdc) > 1
  fprintf('No critical clearing angle could be found.\n')
  fprintf('System can remain stable during this disturbance.\n\n')
  return
  else, end
dc = acos(cosdc);
  if dc > dmax
  fprintf('No critical clearing angle could be found.\n')
  fprintf('System can remain stable during this disturbance.\n\n')
  return
  else, end
tf = 0.4;
x0 = [d0; 0];
%[t1,xf] = ode23('pfpower', 0, tf, x0, 0.00001);   % use with MATLAB 4
tspan = [0, tf];                                  % use with MATLAB 5
options = odeset('RelTol', 0.00001);              % use with MATLAB 5
[t1,xf] = ode23('pfpower', tspan, x0, options);    % use with MATLAB 5
kk=find(xf(:,1) <= dc); k=max(kk);
tt=t1(k);
while tf <= tt & tf <= 3.6
tf=tf+.4;
```

```
  fprintf ('\nSearching with a final time of %3.2f Sec. \n', tf)
  tol = 0.00001+tf*2.5e-5;
  %[t1,xf] =ode23('pfpower', 0, tf, x0, tol);    % use with MATLAB 4
  tspan = [0, tf];                              % use with MATLAB 5
  options = odeset('RelTol', tol);              % use with MATLAB 5
  [t1,xf] = ode23('pfpower', tspan, x0, options);  % use with MATLAB 5
  kk = find(xf(:,1) <= dc); k=max(kk);
    tt= t1(k);
end
%end
tmargin = t1(k);
if tf >= 3.6
  fprintf('\nA clearing time could not be found up to 4 sec. \n\n')
  return
  else, end
fprintf('\nCritical clearing time =  %4.2f seconds \n', tmargin)
fprintf('Critical clearing angle = %6.2f degrees \n\n', dc*180/pi)
% State variable representation of the swing equation of
% the one-machine system during fault.
function xdot = pfpower(t,x)
global Pm E V X1 X2 X3 H f
xdot = [x(2); pi*f/H*(Pm-E*V/X3*sin(x(1)))];
%State variable representation of the swing equation of
```

% the one-machine system after fault clearance.

```
function xdot = afpower(t,x)

global Pm f H E  V X1 X2 X3

xdot = [x(2); pi*f/H*(Pm-E*V/X3*sin(x(1))-0.02)];
```

## 7.9 RESULTS

Critical clearing time = 0.22 seconds

Critical clearing angle = 52.23 degrees

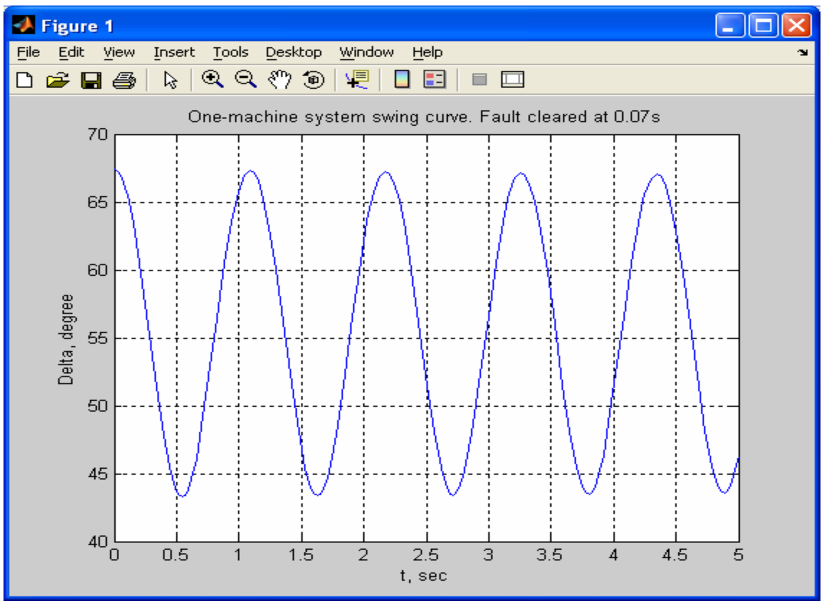

Fig. 7.7 One-machine system swing curve Fault cleared at 0.07 second

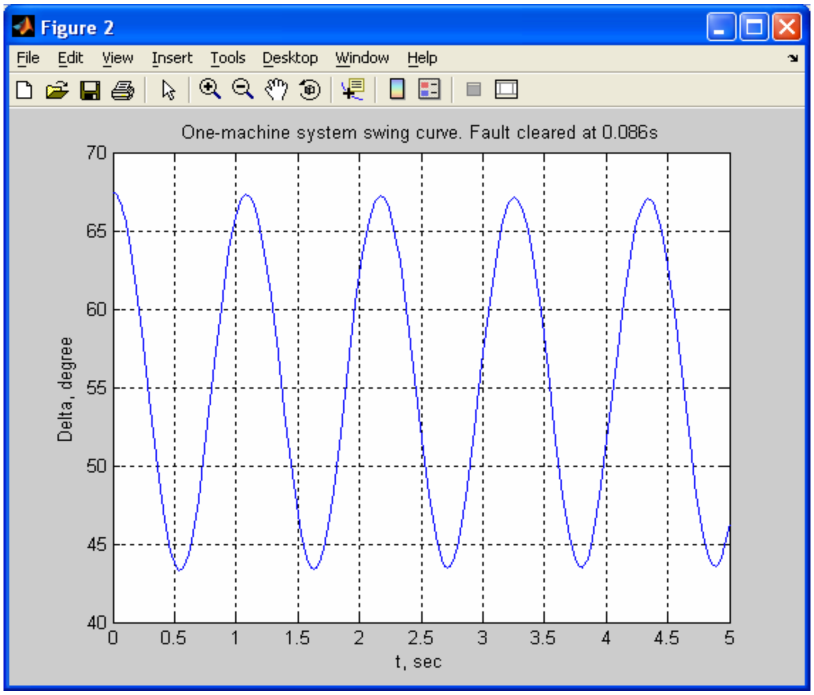


Fig. 7.8 One-machine system swing curve Fault cleared at 0.07 second

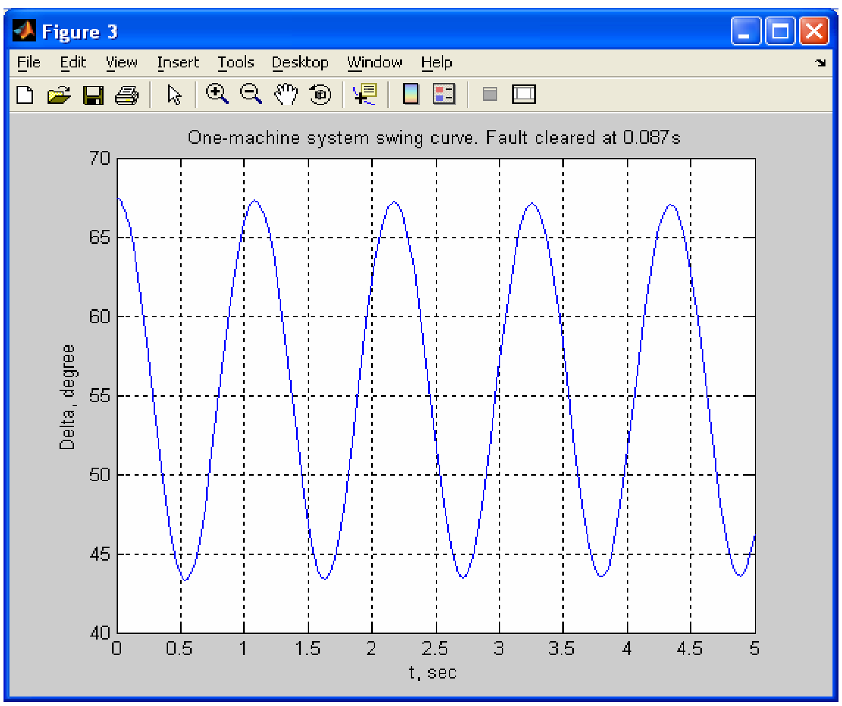


Fig. 7.9 One-machine system swing curve Fault cleared at 0.07 second

## 7.10 SIMULATION OF POWER SYSTEM DYNAMIC RESPONSE

### 7.10.1 Structure of the Power System Model

Analysis of transient stability of power systems involves the computation of their nonlinear dynamic response to large disturbances, usually a transmission network fault, followed by the isolation of the faulted element by protective relaying.

For transient stability analysis, non-linear system equations are solved. In addition, large discontinuities due to faults and network switching and small discontinuities due to limits on system variables appear in the system model. Bus voltages, line flows and performance of protection systems are of interest in addition to the basic information related to the stability of the system.

The overall system representation includes models for the following individual components:

- Synchronous generators and the associated excitation systems and prime movers
- Interconnecting transmission network including static loads
- Induction and synchronous motor loads
- Other devices such as HVDC converters and SVCs

The model used for each component should be appropriate for transient stability analysis and the system equations must be organized in a form suitable for applying numerical methods.

As we will see in what follows, the complete system model consists of a large set of ordinary differential equations and large sparse algebraic equations. The transient stability analysis is thus a differential algebraic initial-value problem.

### 7.10.2 SYNCHRONOUS MACHINE REPRESENTATION (INCLUDING DAMPING)

To illustrate the implementation of the generator model for transient stability analysis, we assume that the generator is represented by a model with one $d$-axis and two $q$-axis amortisseurs as shown in figure. However the equations presented here can be readily modified to account for a model with an arbitrary number of amortisseurs.

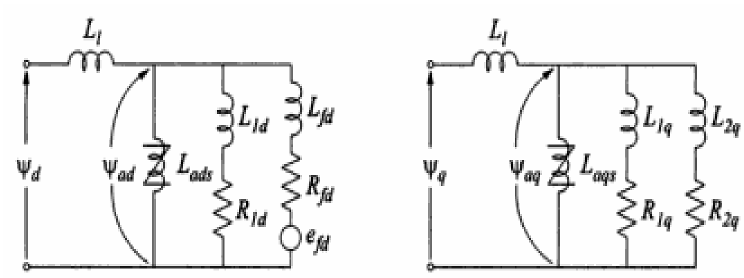


Fig. 7.10 Synchronous machine equivalent circuits

The following is a summary of the synchronous machine equations as a set of first order differential equations, with time *t* in seconds, rotor angle δ in electrical radians, and all other quantities in per unit.

### 7.10.2.1 Equations of motion

$$p\Delta\omega_r = \frac{1}{2H}(T_m - T_e - K_D\Delta\omega_r)$$

$$p\delta = \omega_0\Delta\omega_r$$

Where

$\omega_0 = 2\Pi f_0$ electrical radian/sec

$\Delta\omega_r$ = pu rotor speed deviation

*P* = derivative operator *d/dt*

### 7.11 Illustration:

We analyze the transient stability of the system considered in the previous example and including a more detailed model which would take into consideration the effects of damping at various stages which is consolidated and included as a parameter with constant $K_D$ in the equation of motion. The new system is simulated in Matlab and the resulting transient response is displayed.

## 7.12 MATLAB CODE

```
global  Pm f H E V X1 X2 X3

Pm = 0.9;  E = 1.1626;  V = 0.90081;

X1 = 0.775; X2 = inf; X3 = 0.95;

H = 3.5; f = 60; tf = 10.0; Dt = 0.05;

disp('Parts (a) & (b) are repeated using swingrk4')

disp('Press Enter to continue')

pause

tc = 0.07;

swingrk2(Pm, E, V, X1, X2, X3, H, f, tc, tf)

% The function swingrk2 is same as used in the previous example.

% State variable representation of the swing equation of

% the one-machine system during fault.

function xdot = pfpower(t,x)

global Pm E V X1 X2 X3 H f

xdot = [x(2); pi*f/H*(Pm-E*V/X3*sin(x(1))-0.02*x(2))];

%State variable representation of the swing equation of

% the one-machine system after fault clearance.

function xdot = afpower(t,x)

global Pm f H E  V X1 X2 X3

xdot = [x(2); pi*f/H*(Pm-E*V/X3*sin(x(1))-0.02*x(2))];
```

## 7.13 RESULTS

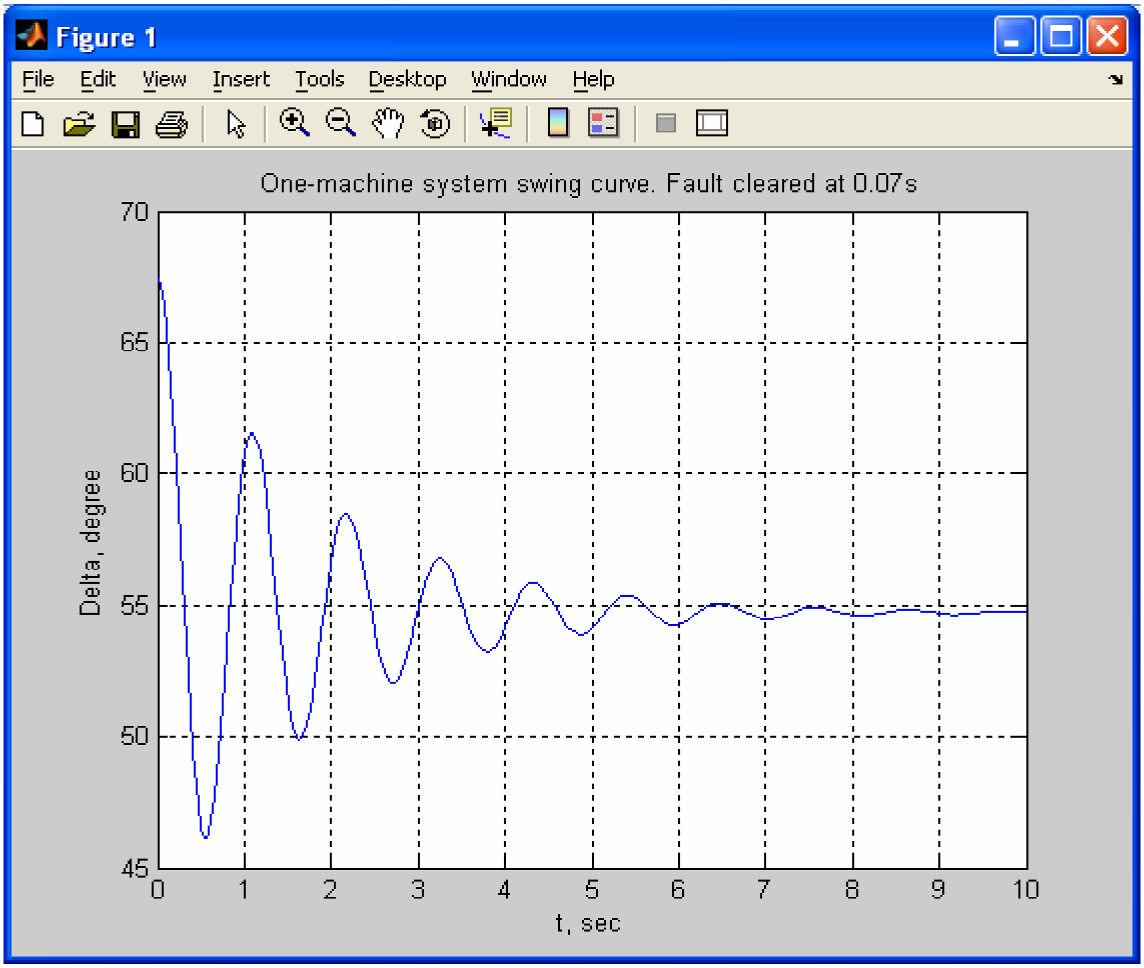


Fig. 7.11 One-machine system swing curve Fault cleared at 0.07 second

The waveform clearly shows that the effect of damping on the dynamic response of the system. The damping factor used is $K_D$=0.02. The oscillations are clearly damped almost completely within a few cycles which give a beeter idea about stability.

# Chapter 8

## CONCLUSION AND REFERENCES

## 8.1 CONCLUSION

Thus we see that a two-machine system can be equivalently reduced to a one machine system connected to infinite bus bar. In case of a large multi-machine system, to limit the computer memory and time requirements, the system is divided into a study subsystem and an external subsystem. The study subsystem is modeled in details whereas approximate modeling is carried out for the rest of the subsystem. The qualitative conclusions regarding system stability drawn from a two-machine or an equivalent one-machine infinite bus system can be easily extended to a multi-machine system.

It can be seen that transient stability is greatly affected by the type and location of a fault so that a power system analyst must at the very outset of a stability study decide on these two factors. For the case of one-machine system connected to infinite bus it can be seen that an increase in the inertia constant M of the machine reduces the angle through which it swings in a given time interval offering a method of improving stability. But this can not be employed in practice because of economic reasons and for the reason of slowing down of the response of the speed-governor loop apart from an excessive rotor weight.

For a given clearing angle, as the maximum power limit of the various power angles is raised, it adds to the transient stability limit of the system. The maximum steady power of a system can be increased by raising the voltage profile of a system and by reducing the transfer reactance.

Thus we see that by considering the effect of rotor circuit dynamics we study the model in greater details. We have developed the expressions for the elements of the state matrix as explicit functions of system parameters. In addition to the state-space representation, we also use the block diagram representation to analyse the system stability characteristics.

While this approach is not suited for a detailed study of large systems, it is useful in gaining a physical insight into the effects of field circuit dynamics and in establishing the basis for methods of enhancing stability through excitation control.

We have explored a more detailed model for transient stability analysis taking into account the effect of damping which is clearly visible from the dynamic response of the system. We have included a damping factor in the original swing equation which accounts for the damping taking place at various points within the system.

Our aim should be to improvise methods to increase transient stability. A stage has been reached in technology whereby the methods of improving stability have been pushed to their limits. With the trend to reduce machine inertias there is a constant need to determine availability, feasibility and applicability of new methods for maintaining and improving stability.

## 8.2 **REFERENCES**

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