# DEVELOPMENT OF ALTERNATIVE METHODS FOR ROBOT KINEMATICS

A THESIS REPORT SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

**Master of Technology (Research)**

**In**

**Mechanical Engineering**

By


**Santosini Sahu**

**DEPARTMENT OF MECHANICAL ENGINEERING**

**NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA**

**ORISSA, INDIA, PIN-769008**


***DECEMBER 2008***

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**NATIONAL INSTITUTE OF TECHNOLOGY**
**ROURKELA-769008, ORISSA**

# CERTIFICATE

This is to certify that the work in the thesis entitled **“**Development of Alternative Methods For Robot Kinematics**”** submitted by **Ms Santosini Sahu** in partial fulfillment of the award of **Master of Technology (Research) Degree** during the session **2006 -2008** in the Department of Mechanical Engineering, National Institute of Technology Rourkela is an authentic work carried out by her under my supervision and guidance.

To the best of my knowledge, the work reported in this thesis is original and has not been submitted to any other Institution or University for the award of any degree or diploma.

She bears a good moral character to the best of my knowledge and belief.

(Prof. B.B. Biswal)
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Date:

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# ABSTRACT

The problem of finding mathematical tools to represent rigid body motions in space has long been on the agenda of physicists and mathematicians and is considered to be a well-researched and well-understood problem. Robotics, computer vision, graphics, and other engineering disciplines require concise and efficient means of representing and applying generalized coordinate transformations in three dimensions. Robotics requires systematic ways to represent the relative position or orientation of a manipulator rigid links and objects. However, with the advent of high-speed computers and their application to the generation of animated graphical images and control of robot manipulators, new interest arose in identifying compact and computationally efficient representations of spatial transformations.

The traditional methods for representing forward kinematics of manipulators have been the homogeneous matrix in line with the D-H algorithm. In robotics, this matrix is used to describe one coordinate system with respect to another one. However for online operation and manipulation of the robotic manipulator in a flexible manner the computational time plays an important role. Although this method is used extensively in kinematic analysis but it is relatively neglected in practical robotic systems due to some complications in dealing with the problem of orientation representation. On the other hand, such matrices are highly redundant to represent six independent degrees of freedom. This redundancy can introduce numerical problems in calculations, wastes storage, and often increases the computational cost of algorithms. Keeping these drawbacks in mind, alternative methods are being sought by various researchers for representing the same and reducing the computational time to make the system fast responsive in a flexible environment. Researchers in robot kinematics tried alternative methods in order to represent rigid body transformations based on concepts introduced by mathematicians and physicists such as Euler angle or Epsilon algebra. In the present work alternative representations, using quaternion algebra and lie algebra are proposed, tried and compared.

In chapter 1, a brief introduction on robots, their development, classification is presented. The general configurations of robots are illustrated and their advantages and disadvantages are discussed along with some of the application. They have found applications in many areas of geometric analysis and modeling. The problems of forward and inverse kinematics are discussed and solution to forward kinematics problem through traditional homogeneous matrix method is introduced. In order to achieve the objectives of the research work and arrive at the desired result a systematic study of the basic theories of representing the transformation has been done.

In chapter 2, the study and analysis of some of the important literatures in the area of the robot manipulator kinematics was done. The study prompted to carry out further research work in this area with an objective to understand and analyze systematically the geometrical significance and develop new algorithms which will be efficient and easily understood by robotics community. This study comes out with broad objectives of finding alternative representations of robot manipulator's forward kinematics with the help of higher mathematical theories such as lie algebra and quaternion algebra and to test the capabilities of the developed representations for higher DOF manipulators.

In chapter 3, homogeneous transformation matrix, quaternion algebra and lie algebra are extensively discussed along with their history, development and application. The mathematical models are developed and equations are presented for kinematic representation of the robot arm. An example problem of 6-dof revolute robot is taken and solved by using aforementioned methods.

Chapter 4 presents the results of the proposed methods as described in Chapter 3 followed by a vivid discussion on the same. The comparisons are made in terms of computational cost and efficiency for all the methods. Finally conclusion and future scope of the work have been presented in chapter 5.

# Table of Contents

**Chapter 2: Literature survey**



**Chapter 3: Description of Methods**

# List of Figures

# List of Tables

# Chapter-I

## INTRODUCTION

# CHAPTER 1

# Introduction

## 1.1 Introduction

Robotics is a relatively young field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their applications requires knowledge of electrical engineering, mechanical engineering, systems and industrial engineering, computer science, economics, and mathematics. New disciplines of engineering, such as manufacturing engineering, applications engineering, and knowledge engineering have emerged to deal with the complexity of the field of robotics. The science of robotics has grown tremendously over the past twenty years, fueled by rapid advances in computer and sensor technology as well as theoretical advances in control and computer vision. At the present time, the vast majority of robot applications deal with industrial robot arms operating in structured factory environments so that a first introduction to the subject of robotics must include a rigorous treatment of the topics in this text.

The industrial robot manipulator can be considered as an open chain mechanism consisting of rigid links and joints. The problem of finding mathematical tools to represent rigid body motions in space has long been on the agenda of physicists and mathematicians and is considered to be a well-researched and well-understood problem. Robotics, computer vision, graphics, and other engineering disciplines require concise and efficient means of representing and applying generalized coordinate transformations in three dimensions. Robotics requires systematic ways to represent the relative position or orientation of a manipulator rigid links and objects. A number of different representations have been developed. However, with the advent of high-speed computers and their application to the generation of animated graphical images and control of robot manipulators, new interest arose in identifying compact and computationally efficient representations of spatial transformations.

### 1.2 Robots: Evolution and Present Status

The term ‘robot’ was first introduced by the Czech play wright Karel Capek in his 1920 play Rossum's Universal Robots, the word ‘robota’ being the Czech word for work. Since then the term has been applied to a great variety of mechanical devices, such as teleoperators, underwater vehicles, autonomous land rovers, etc. Virtually anything that operates with some degree of autonomy, usually under computer control, has at some point been called a robot. Such devices, though far from the robots of science fiction, are nevertheless extremely complex electro-mechanical systems whose analytical description requires advanced methods, and which present many challenging and interesting research problems. An official definition of such a robot comes from the Robot Institute of America (RIA): A robot is a reprogrammable multifunctional manipulator designed to move material, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks. The key element in the above definition is the re-programmability of robots. It is the computer brain that gives the robot its utility and adaptability. The so-called robotics revolution is, in fact, part of the larger computer revolution. Even this restricted version of a robot has several features that make it attractive in an industrial environment. Among the advantages often cited in favor of the introduction of robots are decreased labor costs, increased precision and productivity, increased flexibility compared with specialized machines, and more humane working conditions as dull, repetitive, or hazardous jobs are performed by robots. The first successful applications of robot manipulators generally involved some sort of material transfer, such as injection molding or stamping where the robot merely attended a press to unload and either transfer or stack the finished part. These first robots were capable of being programmed to execute a sequence of movements, such as moving to a location A, closing a gripper, moving to a location B, etc., but had no external sensor capability. More complex applications, such as welding, grinding, deburring, and assembly require not only more complex motion but also some form of external sensing such as vision, tactile, or force-sensing, due to the increased interaction of the robot with its environment.

It should be pointed out that the important applications of robots are by no means limited to those industrial jobs where the robot is directly replacing a human worker. There are many other applications of robotics in areas where the use of humans is impractical or undesirable. Among these are undersea and planetary exploration, satellite retrieval and repair, the defusing of explosive devices, and work in radioactive environments. Finally artificial limbs are themselves robotic devices requiring methods of analysis and design similar to those of industrial manipulators.

## 1.3 Classification of Robots

The robots are typically classified according to various criteria such as their

a) Degree of freedom and kinematic characteristics
b) Kinematic structure
c) Drive technology
d) Workspace geometry, and
e) Motion characteristics.

### 1.3.1 Classification by degree of freedom and kinematic characteristics

Degrees of freedom are specific, defined mode in which a mechanical device or a system can move. The number of degree of freedom is equal to the total number of independent displacement or aspect of motion. A manipulator possesses 6 degrees of freedom in order to manipulate an object freely in three dimensional space. From this point of view robots are classified as follows:

**a) General purpose robot**

A robot is called general purpose robot if it possesses six degree of freedom. Fanuc S-900W robot is an example of general purpose type of robot.

**b) Redundant robot**

The major problem with definitions of redundancy is that it is a term used for quite disparate, but related, situations. This note looks at a number of widely used definitions with a view to identifying the key features and proposing some workable definitions. Starting at the highest level, redundancy concerning robotic manipulators can be categorized as sensor redundancy and mechanical redundancy. Sensor redundancy occurs when there are more sensors than theoretically necessary, usually when high reliability is required. Although sensory redundancy is important, it is not considered in this work. Mechanical redundancy can be further divided into kinematic and actuation redundancy. The term redundancy used here means kinematic redundancy. Redundancy is described as ‘When a manipulator can reach a specified position with more than one configuration of the linkages, the manipulator is said to be redundant. ’ According to this, redundancy means more than one solution to the inverse kinematic transform. From a general point of view, any robotic system in which the way of achieving a given task is not unique may be called redundant.

**c) Flexible robot**

The assumption that robot arms are rigid bodies is not valid when considering faster, lighter and more precise robots handling relatively heavy payloads accurately. The flexible motions influence both the dynamics and kinematics of such robots to such an extent that their exclusion from the analysis may lead to substantial errors in motion control. This is the case of space robots with very long light arms that usually need some settling time to damp elastic deformations and of manipulators like the next generation of industrial robots foreseen by developers which will have lower ratio of arm weight to payload weight and better energy efficiency. It would also be very useful for tele-operated arms working in areas such as medical surgery or nuclear installations, where safety factors such as very light arms should prevent the robots from causing any undesired damage to the working environment due to human error.

The ever increasing utilization of robotic manipulators for various applications in recent years has been motivated by the requirements and demands of industrial automation. Among these, attention is focused more towards flexible manipulators, due to various advantages they offer compared to their rigid counterparts. Flexural dynamics have constituted the main research challenge in modeling and control of such systems; research activities have accordingly concentrated on the development of methodologies to cope with this.

**d) Deficient robot**

A robot is called deficient robot if it posses less than six degrees of freedom. Adept-one is an example of deficient robot.

### 1.3.2 Classification by kinematic structure

According to kinematic structure, the robots are classified as

**a) Serial robot**

A robot is said to be a serial robot if its kinematic structure takes the form of an open-loop chain. Adept-one is an example of this type robot.

**b) Parallel robot**

A robot is said to be parallel robot if its kinematic structure takes the form of closed-loop chain. Generally this type of robot has the advantages of the higher stiffness, payload capacity and lower inertia than the serial robot at the price of smaller workspace and more complex mechanism. Pac Drive Robot D2 manufactured by ELAU Company is an example of this type robot.

**c) Hybrid robot**

A robot said to be hybrid robot if its kinematic structure takes the form of both open and closed loop chain. Fanuc S-9000W is an example of this type robot. Many industrial robots employ this type structure because it allows the third motor to be mounted on the waist so that it reduces inertia of the manipulator.

### 1.3.3 Classification by drive technology

Manipulators are classified by their drive technology such as;

**a) Electric drive**

It is the most popular drive so that most manipulators use either electric dc motor or stepper motor because they are clean and easy to control.

**b) Hydraulic drive**

When high speed and high load carrying capacity are needed then hydraulic drive is used. But disadvantages of this type are the leaking of liquids.

**c) Pneumatic drive**

When high speed and high load carrying capacity are needed then pneumatic drive is used. Though it is clean and fast, but it is difficult to control because air is compressible fluid.

### 1.3.4 Classification by workspace

The workspace of a manipulator is defined as the aggregate of all possible position of a point attached to the free end of the manipulator. Two different types of workspace is used. A reachable workspace is the volume of space within which every point can be reached by the end-effector in at least one orientation. According to the workspace the robots are classified as:

**a) Cartesian robot**

The kinematic structure of this type robot arm is made of three mutually perpendicular prismatic joints. The wrist centre position of a Cartesian robot can be conveniently described by the three Cartesian co-ordinates associated with the three prismatic joints. The shape of the workspace of the Cartesian robot is rectangular box.

**b) Cylindrical robot**

A robot is called a cylindrical robot if either the first or second joint is replaced by a revolute robot. The wrist centre position of a cylindrical robot can be described by the set of cylindrical co-ordinates associated with the three joint variables. The workspace of the cylindrical robot is confined by two concentric cylinder of finite length.

**c) Spherical robot**

A robot is said to be a spherical robot if the first two joints are made up of two intersecting revolute joints and the third is prismatic joints. The wrist centre position of a spherical robot can be described by the set of spherical co-ordinates associated with three joint variables. The workspace of spherical robot is confined by two concentric spheres.

**d) Articulated robot**

A robot arm articulated if all three joints are revolute. The workspace of articulated robot is very complex, typically crescent-shaped cross-section. Example of this type robot is PUMA robot.

**e) SCARA robot**

SCARA (Selective Compliance Assembly Robot Arm) robot is a special type of robot. It consists of three revolute joint followed by a prismatic joint. In addition all three joint axes are parallel to each other and point along the direction of gravity. This type of robot is used for assembling parts on the plane. Adept One belongs to this category of robot.

**f) Revolute robot**

The robot in which all the joints are revolute joint, it is known as revolute robot. The revolute joint is that which permits the relative rotation about a unique pair axes and has a single degree of freedom. Revolute robot has six degrees of freedom. Three are in X, Y and Z axes. The other three are pitch, yaw and roll. Pitch is when the wrist moves up and down. When the hand moves left and right it is known as yaw. When the forearm entirely rotates, it is known as roll. PUMA series, Fanuc robot (model S-215) and Staubli robot (model RX-130) robots are example of this type robot. The revolute robots are classified as 3R, 4R, 5R, 6R etc robots. The 6R robot is classified as type A1, A2, B1, B2, C and D. The structures of the types of robots are shown in the figure 1.1. Examples of different types of robot,

1. Type A1 robot: Nordson robot, ASEA IRS6 robot.
2. Type A2 robot: ABB robot.
3. Type B1 robot: Cincinnati Milacron T3 robot, Polar 6000 robot.

4. Type B2 robot: EPSON robot, MITSUBISHI Electric robot.

5. Type C robot: MA23 robot, Unimation Puma250.

6. Type D robot: Denso robot.

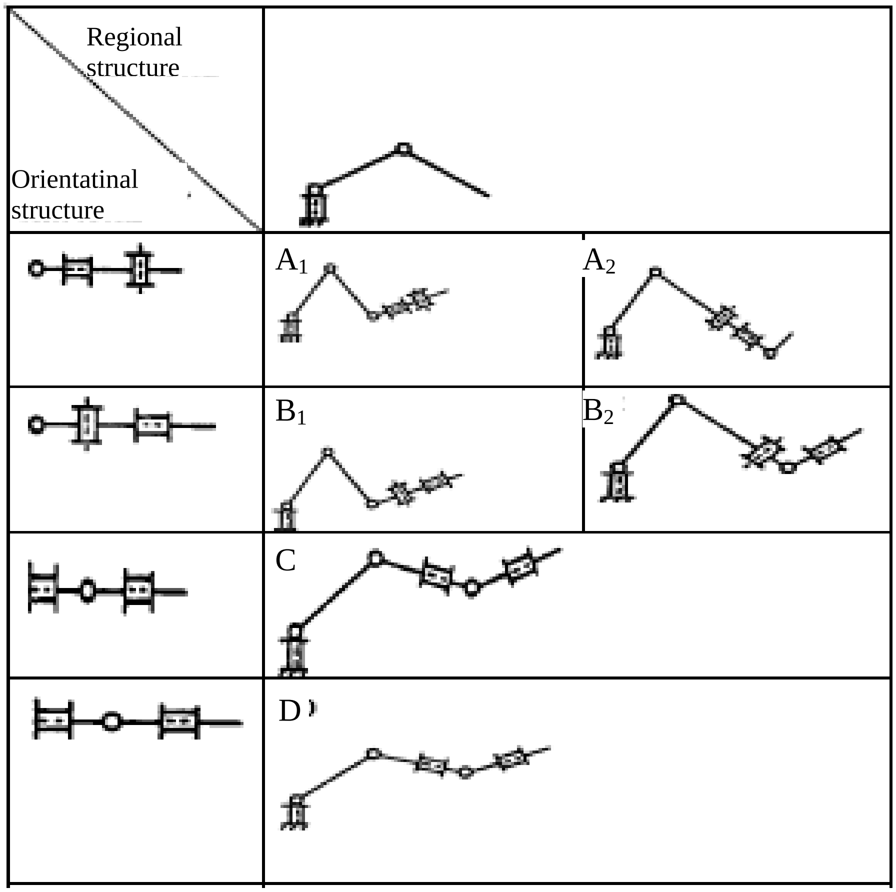



Figure 1.1: Different types of 6R robot structure

### 1.3.5 Classification by motion characteristics

Robot manipulators can also be classified according to their nature of motion such as;

**a) Planar manipulator**

A manipulator is said to be a planar manipulator if its mechanism is planar mechanism. The planar mechanism is that all the moving links in the mechanism perform planar motion that is all parallel to one another. Planar mechanism that utilizes only lower pair joints called planar linkage. Revolute and prismatic joints are only permissible lower pairs for planar linkages. In planar linkage, the axes of all revolute joints must be normal to plane of motion, while the direction of translation of a prismatic joint must be parallel to the plane of motion. Planar manipulators are useful for manipulating an object on a plane.

**b) Spherical manipulator**

A manipulator is called spherical manipulator if it is made up of a spherical mechanism. In spherical mechanism all the moving links perform spherical motion about a common stationary point and the motion of all particles can be described by the radial projection on the surface of unit sphere. A revolute joint is only possible lower pair for construction for all spherical linkage. A spherical manipulator is used as a pointing device.

**c) Spatial manipulator**

A rigid body is said to be perform spatial motion if its motion cannot be characterized as a planar or spherical motion. A manipulator is said to be a spatial manipulator if at least one of the moving links in the mechanism posses several co-ordinates system, a leading superscript is used to indicate the co-ordinate system to which vector is referred.

In view of the growing trend of application of robots in the present day's industry and the prospect of robotics in the future industries, the present study is focused on industrial robots only.

## 1.4 General Configuration of Industrial Robots

### 1.4.1 Cartesian robot

In this type of robot all the joints are prismatic joints as shown in figure 1.2. The work envelope of a rectangular robot is a cube or rectangle, so that any work performed by robot must only involve motions inside the space. The robot configuration has three linear axes of motion: the first one represents left and right motion along X-direction. The second one describes forward and backward motion along Y-direction. The third one depicts up-and-down motion along Z direction.

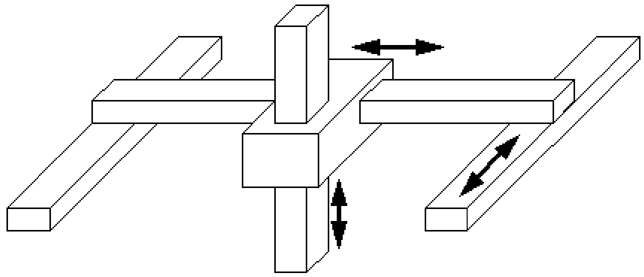


Figure 1.2: Cartesian robot

The main advantages of Cartesian robot are they can obtain large work envelope and their linear movement allows for simpler controls. They have high degree of mechanical rigidity, accuracy, and repeatability due to their structure and they can carry heavy loads because the weight-lifting capacity does not vary at different locations within the work envelope. The disadvantages of these robots are they make maintenance more difficult and access to the volume region by overhead crane or other material-handling equipment may be impaired by the robot-supporting structure. Further their movement is limited to one direction at a time.

### 1.4.2 Cylindrical robot

The cylindrical co-ordinate robot in figure 1.3 is a variation of the Cartesian robot. This robot consists of a base and a column, but the column is able to rotate i.e. it has two linear motions and one rotary motion. The first coordinate describe the angle theta of base rotation about the up-down axis. The second coordinate correspond to a radical or y in out motion at whatever angle the robot is positioned. It also carries an extending arm that can move up and down on the column to provide more freedom of movement. The cylindrical co-ordinate robot is designed for handling machine tools and assembly. Rotational ability gives the advantage of moving rapidly to the point in z -plane of rotation. It results in a larger work envelope than a rectangular robot manipulator and Suited for pick-and-place operations.

The advantage of this configuration is their vertical structure conserves floor space and their deep horizontal reach is useful for far-reaching operations. They are capable of carrying large payloads. The main demerit is their overall mechanical rigidity is lower than that of the rectilinear robots because their rotary axis must overcome inertia. The repeatability and accuracy are also lower in the direction of rotary motion. This configuration requires a more sophisticated control system than the rectangular robots.

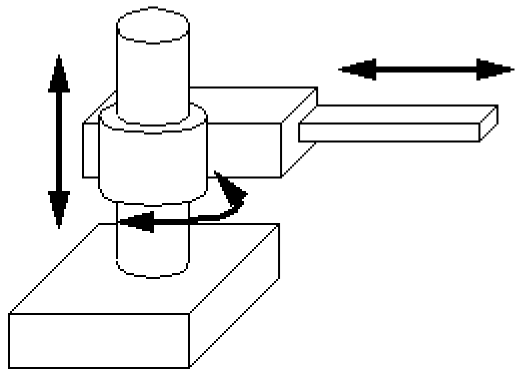


Figure 1.3: Cylindrical robot

### 1.4.3 Polar or spherical robot

In the spherical co-ordinate configuration, shown in figure 1.4, the robot has one linear motion and two rotary or angular motions. The linear motion corresponds to a radial in or out translation; the first angular motion corresponds to base rotation about a vertical axis. The second angular motion corresponds to an elbow rotation. The work volume is like a section of sphere. A spherical-coordinated robot provides a larger work envelope than the rectilinear or cylindrical robot. The two rotation s along with the in or out motion enable the robot to reach any specified point in the space bounded by an outer and inner hemisphere. Design gives weight lifting capabilities. Advantages and disadvantages same as cylindrical-coordinated design.

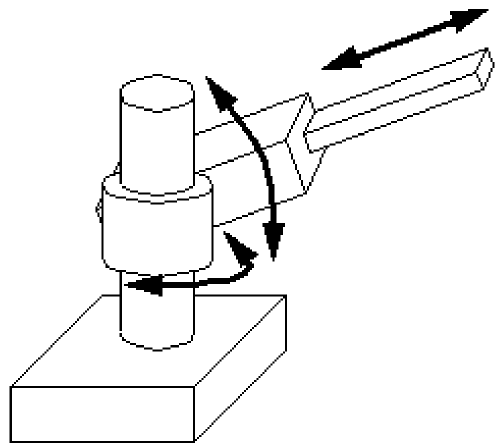


Figure 1.4: Spherical robot

The robot in which all the joints are revolute joint, it is known as revolute robot. The revolute robot is also called anthropomorphic or jointed arm configuration and uses three rotations. The anthropomorphic design corresponds to the design of human arm having waist, shoulder and elbow joints. The link of the arm mounted on the base joint can rotate around the base about the z-axis and the two links, shoulder and elbow. The shoulder can rotate about a horizontal axis and the elbow motion may either be a rotation about a horizontal axis or may be at any location in space depending on the rotational motion of the base and the shoulder. The anthropomorphic robot can move in space bounded between a spherical outer surface having scallops due to the constraints of the joints.

### 1.4.4 Revolute robot

This robot resembles the human arm.  In figure 1.5, all the joints are revolute. It has six degrees of freedom. Three are in X, Y and Z axes. The other three are pitch, yaw and roll. Pitch is when the wrist moves up and down. Yaw is when the hand moves left and right. Roll is when the forearm entirely rotates. PUMA series robots are example of this type robot. The revolute robot can move in a space bounded between a spherical outer surface and inner surface having scallops due to the constraint joints.

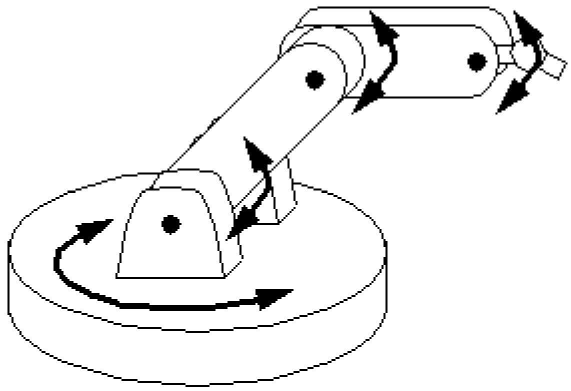


Figure 1.5: Revolute robot

**1.4.5 SCARA robot**

SCARA stands for Selective Compliance Assembly Robot Arm, figure 1.6. As may be known from the name, the robot has compliance only in specific directions (X and Y directions) and has high rigidity in other direction (Z direction), and thus, has been designed mainly for automation of assembling works.

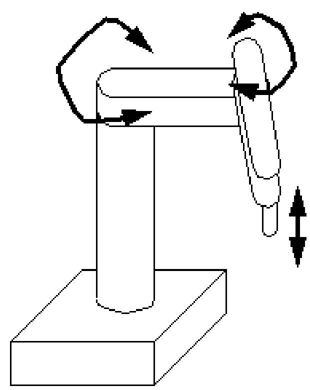


Figure 1.6: SCARA robot

At present, it is used in various production sites as a robot that is very effective not only in assembling works but also in component carrying works (pick & place works) because of its outstanding speed. Although SCARA robot is examined by comparing with Cartesian coordinate robot in many cases owing to its operating range, it can be said that SCARA robot is suitable for works requiring speeds on three axes or four axes motions because of its excellent cost-performance ratio. The features of this robot include its small installation area that provides higher degree of freedom in design of a system, and in addition, provides an advantage that, in case the system is disused in the future, the robot can be installed easily on other system.

The above mentioned configurations and their corresponding workspaces are given in figure 1.7 in a tabular form.

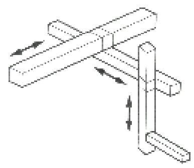
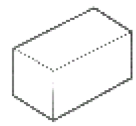
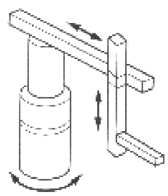
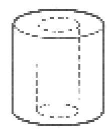
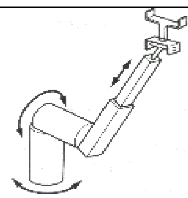
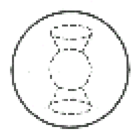
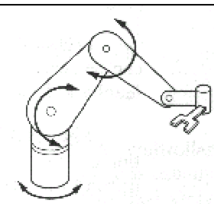
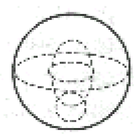
| Types of robot | Structure | Joint type | Shape of the workspace |
|---|---|---|---|
| Cartesian robot |  | P-P-P |  |
| Cylindrical robot |  | R-P-P |  |
| Spherical robot |  | R-R-P |  |
| Revolute robot |  | R-R-R |  |

Figure 1.7: Shape of the workspace of different robots

## 1.5 Robot Kinematics

Before going into details of kinematics, some basic terms used in the robotics are explained in this introductory part of the thesis. Kinematic Chain consists of nearly rigid links which are connected with joints or kinematics pair allowing relative motion of the neighboring links.

Links and Joints: An industrial robot can be thought as a chain of N+l bodies (links) interconnected by N rotary and/or prismatic joints. Each joint can be controlled by a motor. The torque delivered by the motors, which constitute the inputs to the system, are to be designed so that the end body follows the desired trajectory. The trajectories are usually programmed depending on the environment and according to optimality criteria on task performances. Links are the solid structural members of a robot, and joints are the movable couplings between them. Closed Loop Chain consists of every link in the kinematic chain connected to any other link by at least two distinct paths.

Open loop chain is the manipulator in which every link in the kinematic chain is connected to any other link by one and only one distinct path. A serial chain manipulator consists of serial chain of rigid links, connected by generally revolute joints, forming a "shoulder", an "elbow", and a "wrist". Their main advantage is their large workspace with respect to their own volume and occupied floor space.

A parallel manipulator consists of a fixed "base" platform, connected to an end-effector platform by means of a number of "legs". These legs often consist of an actuated prismatic joint, connected to the platforms through passive spherical and/or universal joints. Hence, the links feel only traction or compression, not bending, which increases their position accuracy and allows a lighter construction. The actuators for the prismatic joints can be placed in the motionless base platform, so that their mass does not have to be moved, which makes the construction lighter. Parallel manipulators have high structural stiffness, since the end effector is supported in several places at the same time. All these features result in manipulators with a high bandwidth motion capability. The major drawback is their limited workspace, because the legs may collide and, in addition, each leg has five passive joints that each has its own mechanical limits. Another drawback is that they lose stiffness in singular positions completely.

Flexible mechanism: A mechanical system such as a serial manipulator in which there is more independent joints than are necessary to define the desired output i. e. end effector position and orientation it can be labeled a redundantly actuated system or kinematically redundant system.

Degree of Freedom (dof): Each joint on the robot introduces a degree of freedom. Each dof can be a slider, rotary, or other type of actuator. Robots typically have five or six degrees of freedom. Out of these three of the degrees of freedom allow positioning in 3D space, while the other two or three are used for orientation of the end effecter. Six degrees of freedom are enough to allow the robot to reach all positions and orientations in 3D space.

Orientation Axes: Roll, pitch and yaw are the common orientation axes used. Basically, if the tool is held at a fixed position, the orientation determines which direction it can be pointed in. Looking at the figure 1.8, it can be understood that the tool can be positioned at any orientation in space. This can be understood as rolling in a plane would turn down an object upside down. The pitch changes for takeoff and landing and when flying in a crosswind the plane will yaw.

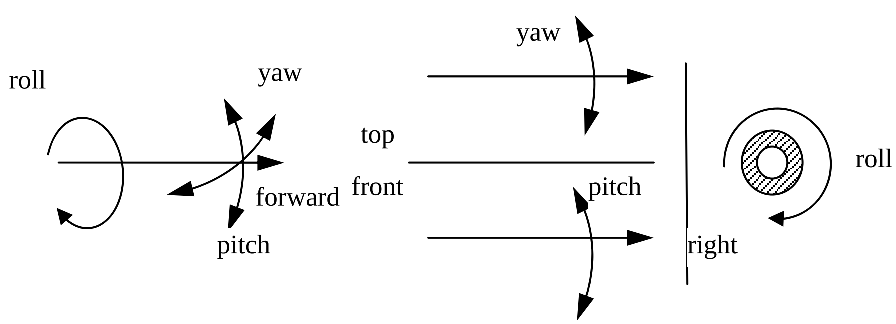



Figure 1.8: Roll, pitch and yaw of robot

Position Axes: The tool, regardless of orientation, can be moved to a number of positions in space. Various robot geometries are suited to different work geometries. Tool Centre Point (TCP): The tool centre point is located either on the robot, or the tool. TCP is shown for a robot arm in figure 1.9. Typically the TCP is used when referring to the robots position, as well as the focal point of the tool (e.g. the TCP could be at the tip of a welding torch). The TCP can be specified in cartesian, cylindrical, spherical, etc. coordinates depending on the robot. As tools are changed the robot is often reprogrammed for the TCP.

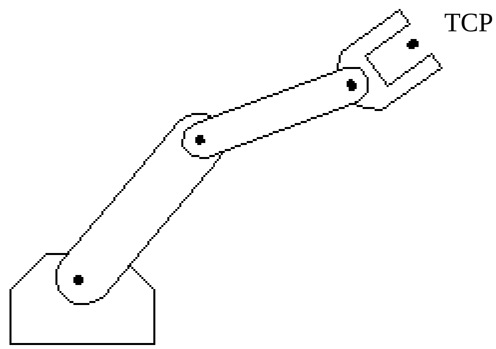



Figure 1.9: Tool centre point (TCP)

Work envelope/workspace: The robot tends to have a fixed and limited geometry. The work envelope as understood from figure 1.10 is the boundary of positions in space that the robot can reach.

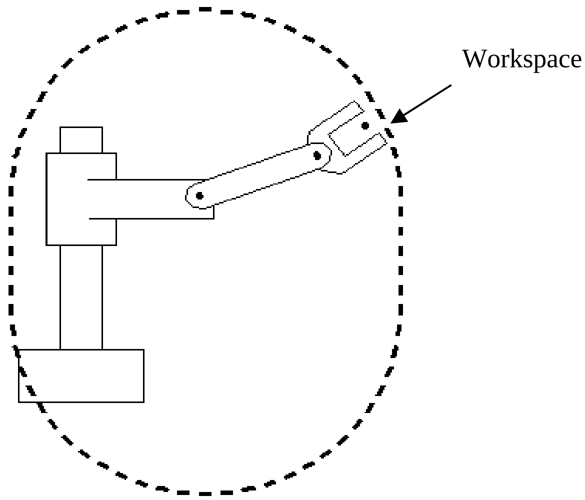



Figure 1.10: Work envelop

For a cartesian robot (like an overhead crane) the workspace might be a square, for more sophisticated robots the workspace might be a shape that looks like a `clump of intersecting bubbles'. There are two different types workspace used, one is reachable workspace and other is dexterous workspace. A reachable workspace is the volume of space within which every point can be reached by the end effector in at least one orientation. The dexterous workspace is the volume of the space within which every point can be reached by the end effector in all possible orientation. The dexterous workspace is the subset of reachable workspace.

Although it is not a necessary condition, but many serial robots are designed with their first three moving links longer than the remaining link. Thus the first three links are used primarily for manipulating the position and the remaining links for controlling the orientation of the end effector. For this reason the subassembly associated with the first three link called arm and last is called wrist. Except redundant manipulator, the arm usually possesses three degree of freedom while the wrist may have 1 to 3 degree of freedom. Further wrist assembly is often designed with its joints axes intersecting at a common point called wrist centre. The arm assembly can assume various kinematic structures and therefore generate different work-envelopes called workspace. The workspace supplied by robot manufacturing usually shows regional workspace. There are different types of workspace for different types of robot. The shape of workspace of Cartesian robot is rectangular box type due to its wrist centre can be described by three Cartesian coordinate associated with the three prismatic joints. The shape of the workspace of cylindrical robot is confined by two concentric cylinder of finite length. It is due to the wrist centre position of the cylinder robot can be described by a set of cylindrical coordinate system associated with the three joint variable. The workspace of spherical robot is confined by two concentric spheres. It is due to the wrist centre position of the spherical robot is described by a set of spherical coordinates associated with three joint variable. The workspace of revolute robot is very complex, typically crescent-shaped cross-section. The SCARA robot is a special type of robot. It consists of two revolute joints followed by prismatic joints. In addition, all three joints axes are parallel to each other and usually a point along the direction of gravity. Thus the first two actuators do not have to work against the gravitational force of the links and the payload. The workspace is formed by the kinematic synthesis of robot manipulator.

Types of coordinates: The robot can move, therefore it is necessary to define positions. Different coordinate frames are shown in figure 1.11.

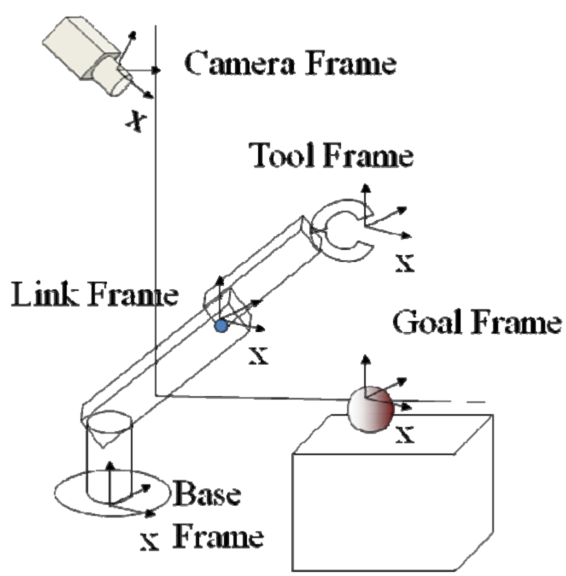



Figure 1.11: Coordinate frames in robotic system

The coordinates are a combination of both the position of the origin and orientation of the axes. They are of following types.

World coordinate: This is the position of the tool measured relative to the base and the orientation of the tool is assumed to be the same as the base.

Tool coordinate: Here the tool orientation is considered and the coordinates are measured against a frame attached to the tool.

Joint coordinate: The position of each joint are used to describe the position of the robot.

A robot manipulator is composed of a set of links connected together by various joints. The joints can either be very simple, such as a revolute joint or a prismatic joint, or else they can be more complex, such as a ball and socket joint. The forward kinematics problem is concerned with the relationship between the individual joints of the robot manipulator and the position and orientation of the tool or end-effector. The forward kinematics problem is to determine the position and orientation of the end-effector, given the values for the joint variables of the robot. The joint variables are the angles between the links in the case of revolute or rotational joints, and the link extension in the case of prismatic or sliding joints. The inverse kinematic problem deals with finding the joint variables like joint angles in terms of the end-effector position and orientation which is given. This is the problem of inverse kinematics, and it is, in general, more difficult than the forward kinematics problem.

The first problem encountered is to describe both the position of the tool and the locations with respect to a common coordinate system. Vectors and matrix algebra are utilized to develop systematic and generalized approach to describe and represent the location of the links of a robot arm with respect to a fixed reference frame. Since the links of the robot arm may rotate and or translate with respect to a reference coordinate frame, a body-attached coordinate frame will be established along the joint axis for each link. A 3x3 rotation matrix is used to describe the rotational operation nof the body attached frame with respect to the reference frame. The homogeneous coordinates are then used to represent position vectors in the three- dimensional space. And the rotation matrices will be expanded to 4x4 homogeneous transformation matrices to include the translational operations of the body-attached frame. This matrix representation of a rigid mechanical link to describe the spatial geometry of a robot arm was first used by Denavit-Hartenberg.

The second problem of robot arm kinematics is the inverse kinematics solution which is just reverse of the forward kinematic equation. In order to control the position and orientation of the end effector of a robot to reach its object the inverse kinematics solution is more important. In other words, given the position and orientation of the end effector and its joints and link parameters the corresponding joint angles are obtained so that end effector can be positioned as desired. The above discussion can be summarized as there are two types kinematics, i.e. forward kinematics and reverse kinematics. If the position and orientation of the end effector are derived from the given joint angle and link parameters, the scheme is called forward kinematics. If the joint angle and the different configuration of the manipulator are derived from the position and orientation of end effector, the scheme is called reverse kinematic.

**1.6 Method of Representation**

Homogeneous matrix method is the classical method to describe the relationship between two adjacent rigid mechanical links. To use homogeneous matrix method for displacement analysis of a spatial linkage we need to attach a coordinate frame to each link. These coordinate systems are established in a systematic manner following Denavit-Hartenberg’s algorithm. The concept of homogeneous-coordinate representation of points in a three –dimensional euclidean space is useful in developing matrix transformations that include rotation, translation, scaling and perspective transformation. In general the representation of an N-component position vector by an (N+1) component vector is called homogeneous coordinate representation. There is no unique homogeneous coordinate representation for apposition vector in three-dimensional space. The fourth component of the homogeneous coordinate can be viewed as a scale factor. If this coordinate is unity than the transformed homogeneous coordinates of a position vector are same as the physical coordinate of the vector. In robotics application the csale factor is always taken as unity. The homogeneous transformation matrix is a 4X4 matrix which maps apposition vector expressed in homogeneous coordinates from from one coordinate to another coordinate system. The other methods of representation like epsilon algebra, lie algebra, quaternion algebra, and Euler angle methods are also applied in robot kinematics to describe motion and orientation.

### 1.7 Objective of the Research

The study and analysis of some of the important literatures in the area of the robot manipulator kinematics suggest that there is a need to refine the kinematic solution of higher DOF manipulators to a higher degree than the traditional methods to obtain greater accuracy of motion. It is also desirable to compute the kinematics in lesser time to make the online control and monitoring of the manipulator more efficient. Therefore, the present work is envisaged with the following broad objectives.

- To find out alternative representations of robot manipulator's forward kinematics with the help of higher mathematical theories such as lie algebra and quaternion algebra.
- To test the capabilities of the developed representations for higher DOF manipulators.
- To carry out a comprehensive study of the developed methods with the existing one in terms of representation convenience and computational convenience.

## 1.8 Methodology

In order to achieve the aforementioned objectives of the research work and arrive at the desired result a systematic study of the basic theories of representing the transformation has to be done. The steps that have been planed for the present work as follows:

- Exploring the existing method of representation and identifying the bottler-necks so far as higher DOF robot manipulators are concerned.
- Studying the specific area of mathematical theories having greater capabilities for handling transformation of rigid body frames in space and utilizing these theories for developing forward kinematics of higher DOF robot manipulators.
- Implementing the developed kinematics representation as higher DOF industrial robotic manipulators
- Testing the correctness and capability of the developed methods in terms of computational convenience and faster communication.
- Making a comparative study of the methods for the benefit of robot users.

## 1.9 Organization of Thesis

The thesis describing the present research work is divided into five chapters. The subject of the topic its contextual relevance and the related matters including the objectives of the work and the methodology to be adopted are presented in Chapter 1. The reviews on several diverse streams of literature on different issues of the topic such as epsilon algebra, quaternion theory and their algebra, lie group and lie algebra along with their applications are presented in Chapter 2. In Chapter 3, selected methods are explained and applied on the six DOF revolute robots. In Chapter 4, the pros and cons of different methods have been discussed and their comparative study is made in connection with the kinematic synthesis of robot arm. Finally, Chapter 5 presents the conclusion and future scope of the research work.

## 1.10 Summary

In this chapter a brief introduction on robots, their development, classification is presented. The general configurations of robots are illustrated and their advantages and disadvantages are discussed along with some of the application. A brief introduction on robot kinematics is given with definitions of serial chain, open and closed loop manipulator, parallel and flexible robots. The problems of forward and inverse kinematics were discussed and solution to forward kinematics problem through traditional homogeneous matrix method was introduced.

# Chapter-II

## LITERATURE SURVEY

# CHAPTER 2

## Literature Survey

### 2.1 Overview

Robotics requires systematic ways to represent the relative position or orientation of a manipulator rigid links and objects. So a number of different representations have been developed. However, with the advent of high-speed computers and their application to the generation of animated graphical images and control of robot manipulators, new interest arose in identifying compact and computationally efficient representations of spatial transformations. A lot of literature survey has been done regarding this area, some of which are discussed as follows.

### 2.2 Manipulator Kinematics and Homogeneous Transforms

Yutaka and Gary [1] proposed a new “heterogeneous” two-dimensional (2-D) transformation group to solve motion analysis/planning problems in robotics. In the new method they used a 3X1 matrix to represent a transformation which is as capable as the homogeneous theory. This requires less memory space and less computation time as opposed to a 3X3 matrix in the homogeneous formulation and it does not have the rotational matrix inconsistency problem. This heterogeneous formulation has been successfully implemented in the MML software system for the autonomous mobile robot Yamabico-11. Bhaumik and Ray [2] developed kinematics for a new mobile robot using traditional method.

Low and Dubey [3] studied two different approaches to the inverse- kinematics problem for a six-degree-of-freedom robot manipulator having three revolute joint axes intersecting at the wrist. In the first method three rotational generalized coordinates are used to describe the orientation of the body. The second method uses equivalent Euler parameters with one constraint equation. These two approaches have been incorporated into two different computer algorithms, and the results from each are compared on the basis of computational complexity, time simulation, and singularity. It was found that Euler parameters were less efficient than three rotational angles for solving the inverse-kinematics problem of the robot considered, and that the physical singularities caused by the robot mechanism could not be eliminated by using either of the two approaches. Tiwari et.al [4], used forward kinematics principle in a machine loading problem in Flexible Manufacturing System.

Michael W. Walker [5] presented the position of a manipulator expressed as either in joint coordinates or in Cartesian coordinates. Manipulator tasks are more easily specified by Cartesian coordinates. A new algebra has been defined for the use in solving the forward and inverse kinematics problem of manipulators. The properties of the algebra are investigated and functions of an epsilon numbers are defined. The Ada language was used for illustration because of the ease in implementing the algebra and it is being used to solve the forward and inverse kinematics problems. However, the program actually used epsilon numbers and used the overloading feature of the Ada language to implement the epsilon algebra. By simply changing the order of the algebra, the resulting program can compute a time derivative of the end-effector's position when used-to solve the forward kinematics problem and any time derivative of joint positions when used to solve the inverse kinematics problem.

Nicholas and Dimitros [6] presented three methods for the formulation of the kinematic equations of robots with rigid links. The first and most common method in the robotics community is based on homogeneous matrix transformation, the second one is based on Lie algebra, and the third one on screw theory expressed via dual quaternion algebra. These three methods are compared in this paper for their use in the kinematic analysis of robot arms. Three analytic algorithms are presented for the solution of the direct kinematic problem corresponding to each method. Finally, a comparative study on the computation and storage requirements for the three methods is worked out. However the application has not been done in higher DOF manipulators and it is applied to five DOF robots only.

Kinsey and Whitcomb [7] report a novel stable adaptive identifier on the group of rigid body rotations, and its application to a sensor calibration problem arising in underwater vehicle navigation. The problem addressed is the identification of a rigid-body rotation map from input-output data. General least-square and adaptive identification techniques are commonly employed to identify general linear maps from input-output data, but do not guarantee that the resulting identified map is a rigid body rotation. At present, a least square singular value decomposition approach is the standard method for identification constrained to the group of rigid body rotations. This paper reports the first exact adaptive identifier on the group of rigid body rotations, together with a proof of stability. The performance of this adaptive identifier is evaluated on actual experimental data and found to compare favorably with results obtained via previously reported least-squares techniques. The methodology reported herein is of broader interest because of its applicability to general problems in the identification, dynamics, and control on the group of rigid body motions.

Tiwari et.al.[8] formulated the operation allocation problem in FMS considering the homogeneous matrices of the mechanism within the system. Dai [9] contributed towards the displacement and transformation of a rigid body, and on their mathematical formulation and its progress. He studied some contemporary developments of the finite screw displacement and the finite twist representation in the late 20th century.

Eric Lee and Merlet [10] solved the geometric design problem of serial-link robot manipulators with three revolute (R) joints using an interval analysis method. In this problem, five spatial positions and orientations are defined and the dimensions of the geometric parameters of the 3-R manipulator are computed so that the manipulator will be able to place its end-effector at these pre-specified locations. Denavit and Hartenberg parameters and 4X4 homogeneous matrices are used to formulate the problem and obtain the design equations and an interval method is used to search for design solutions within a predetermined domain. This is an important new result for a very difficult problem related to the exact synthesis of spatial manipulators that has not been solved before. It is useful as it can give insight on both the number and the nature of design solutions for the synthesis of the 3R.

## 2.2. Kinematic Representation and Quaternion Algebra

Funda, Taylor and Paul [11] implemented three-dimensional modeling of rotations and translations in robot kinematics that are most commonly performed using homogeneous transforms. In their work an alternate approach, employing quaternion/vector pairs as spatial operators, is compared with homogeneous transforms in terms of computational efficiency and storage economy. The conclusion drawn is that quaternion/vector pairs are efficient, more compact, and more elegant than their matrix counterparts. A robust algorithm for converting rotational matrices into equivalent unit quaternion is described, and an efficient quaternion-based inverse kinematics solution for the Puma 560 robot arm is presented.

Funda and Paul [12] proposed a computational analysis and comparison of line-oriented representations of rotational and translational spatial displacements of rigid bodies. Four mathematical formalisms for effecting a general spatial screw displacement are discussed and analyzed in terms of computational efficiency in performing common operations needed in kinematic analysis of multilinked spatial mechanisms. The corresponding algorithms are analyzed in terms of both sequential and parallel execution. They concluded that the dual-unit quaternion representation offers the most compact and most efficient screw transformation formalism but that line-oriented methods are not well suited for efficient kinematic computations in real-time control applications. However, the mathematical redundancy inherent in Plucker coordinate space representation makes them computationally less attractive than the corresponding point-oriented formalisms.

Perrier et al. [13] suggested a mobile manipulator, composed of a manipulator mounted on a vehicle, is a very useful system to achieve tasks in dangerous environments. The use of a wheeled vehicle generally introduces non-holonomic constraints, and the combination of a vehicle and a manipulator introduces some kinematic redundancy. The aim of this paper is to investigate the use of dual quaternions for the motion generation of a mobile manipulator with a non-holonomic vehicle. In this preliminary work, only planar cases are considered. The description of the system in the planar quaternion space is developed and then simulation results are presented.

Aissaoui et al. [14] presented a study to investigate the accuracy of a new algorithm based on dual quaternion algebra for the estimation of the finite screw axis. The advantages of using finite screw axis to describe the kinematics of the knee cadaver specimens is that is rotation and translation around the screw axis are independent from the coordinate system. However, the orientation and the position of the finite screw axis depend on the coordinate system. In this study they used the geometric center of the femoral for the establishment of reference system. However, this is the first study that compares the use of different techniques to assess the finite screw axis. The Dual Quaternion algorithm is safe from any singularities and incorporate a way to deal with noisy data since it enable the simultaneous matrix of rotation and translation.

Pennestri and Stefanelli [15] presented the dual version of some classical linear algebra algorithms. These algorithms had been tested for the position analysis of the spatial mechanism and computational improvements over existing methods obtained basically of solution of a redundant system of nonlinear equations. Harry H. Cheng [16] introduced the dual plane along with dual meta numbers. The dual arithmetical and relational operations and dual functions are defined in the syntax of the C (H) programming language. With this algorithm numerical computations are handled in a more integrated way. Kavan and Zara [17] established techniques for blending of rotations to include all rigid transformations. It was shown that algorithms based on dual quaternion are computationally more efficient than previous solutions and have better properties like constant speed, shortest path and coordinate invariance.

Johan E. Mebius [18] proved the classical quaternion representation theorem for rotations in 4D Euclidean spaces which states that an arbitrary 4D rotation matrix is the product of a matrix representing left–multiplication by a unit quaternion and a matrix representing right–multiplication by a unit quaternion. This decomposition is unique up to sign of the pair of component matrices he also studied the behavior of its matrix formulation under a predetermined class of similarity transformations.

Ricardo and Paolo [19] defined the quaternion skew-field, algebraic properties of quaternion polynomials and investigated divisibility and coprimeness properties of these systems. Finally the tool developed was used to analyze stability of quaternionic linear systems in a behavioral framework.

Horn [20] presented a closed form solution of the least square problem by three or more points and derivation of the problem is simplified by using unit quaternion representing rotation. It provides the best rigid body transformation between two coordinate systems given measurements of the coordinates of a set of points that are not collinear. However the solution simplifies when there are only three points and it seems to be complex in nature.

Pernas [21] defined new operators in differential forms on quaternion manifold. He enlightened on the future holomorphic function theory. This theory led to acceptable results but failed to contain simple algebraic functions and the identity is not regular. Persa et al. [22] proposed a novel technique for the determination of the pose and the twist of rigid bodies using point-acceleration data. These data are collected from an accelerometer array, which is a kinematically redundant set of triaxial accelerometers.

The formulation developed is then utilized in the simulation analysis of two sample motions. The new algorithm can also be useful for estimation of the deflections of a structurally flexible robot from its end-effector pose and twist data and it can be used in real time applications.

Zhuang et al. [23] presented a linear solution that allows a simultaneous computation of the transformations from robot world to robot base and from robot tool to robot flange coordinate frames. The solution was applied to accurately locating a robot with respect to a reference frame and a robot sensor with respect to a robot end-effector. Quaternion algebra is applied to derive explicit linear solutions given three robot pose measurements. They found that the rewlting solution algorithm is computationally, non-iterative, fast and robust.

Perez and McCarthy [24] presented a dual quaternion methodology for the kinematic synthesis of constrained robotic systems from one or more serial chains such that each chain imposes at least one constraint on the movement of the work piece. The kinematics equations of the chain were transformed to successive screw displacements, and then written in dual quaternion form. These dual quaternion kinematics equations are evaluated at a finite set of task positions to yield design equations for the chain.

Dam et al. [25] studied the mathematical properties of quaternion and implemented quaternion as better choice than the well known matrix. They developed alternative interpolation method that is based on a set of objective constraints for an optimal interpolation curve and compared it with methods like Slerp and Squad. However the differential equations cannot be solved analytically. As an alternative, they proposed a numerical solution for the differential equations. The different interpolation methods are visualized and commented. A comprehensive treatment of quaternions, rotation with quaternion for series of rotations is also provided in their work.

Wheeler and Ikeuchi [26] derived a simple form for the gradient and Jacobian of rotation with respect to the quaternion. The new forms were used to predict numerical problems in gradient and Jacobian searches. They studied that normalizing or transforming the data or gradient to a canonical reference frame can ensure that the algorithm performs properly independent of the scale of the data. The relative scale of parameters greatly affects the convergence characteristics and computational expense of gradient based searches. However the data need to be appropriately scaled before applying search algorithm.

Perez and McCarthy [27] presented a dual quaternion methodology for the kinematic synthesis of constrained robotic systems from one or more serial chains. The workspace of a constrained serial robot was represented as the group of spatial transformations which in turn was represented by a subset of dual quaternion. The methodology is an extension of the kinematic synthesis of linkages which is based on finding the geometric constraints of the serial chain. They simplified the structure of the design equations for the spatial 2TPR robot by using dual quaternion. However it is applicable to robots having less than 6-DOFs.

Haetinger et al. [28] developed an articulate mechanical arm type robot, built from low cost materials, as a supporting tool for learning of Mathematics. The authors represented another form of the quaternions, simplifying the formulas of the control system for the direct and inverse kinematic models. This modeling proved to be simple and allows to the student a more dynamic and practical learning of the concepts developed in Linear Algebra, Numerical Methods, Computer Graphics and Programming Languages.

Agrawal [29] defined Hamilton operators and the algebra of dual-number-quatemions is developed by using these operators. Properties of Hamilton operators are then used to find some mathematical expressions for screw motion of a line and a point. The formulation presented establishes a relationship between dual-number-quaternions and its equivalent matrix algebra.  The present formulation provides a singularity free set of kinematic relations.

## 2.3 Kinematic Representation and Lie Algebra

Liang Gu [30] reviewed a number of conventional methods and an analysis based on lie algebra has been presented for exploring possible definition of three dimensional orientation vectors and unifying representations between position and orientation. Based on the study a particular angle is suggested for definition of orientation vector, but it has got a mathematical singularity.

Mladenova [31] discussed manipulator modeling and control through a nonstandard parameterization of rotation motions. The advantage of the method is the computational facilities arising at the kinematical level, provided with a Lie group structure. An idea of a group approach using Lie groups for describing a rigid body and an open loop rigid body's chain is suggested. A new method for kinematical and dynamical analysis and synthesis, as for the trajectory planning of MS, is developed. He proposed an approach for kinematical and dynamical description of manipulators through vector parameterization of the group SO(3). Evaluations of the computational cost of the suggested algorithms are done and found the new method to be efficient and feasible in real-time application.

Mladenova [32] derived basic important kinematical consequences by using vector-parameterization of the SO (3) Lie group with a composition law. The vector and matrix transformations are given in terms of the dual algebra and the screw geometry. The author treated the kinematical and dynamical problems in describing the Euclidean motions of rigid body systems and the rotation is expressed by defining its action on a vector, and the Lie group SO (3) is parameterized by vector parameters making a Lie group with a clear geometrical sense and a simple composition law. The paper proposed a useful interplay of screw geometry, dual algebra and vector and matrix transformations. The geometrical and kinematical models of a manipulator are expressed in a closed form using dual orthogonal matrices and dual vector-parameters.

Mladenova [33] reviewed problems concerning the modeling and control of rigid and elastic joint multibody mechanical systems, including some investigations into nonholonomic systems. The properties of the parameterization more or less influence the efficiency of the dynamics model. The vector parameter is used for parallel considerations of rigid body motion and of rigid and elastic joint multibody mechanical systems. It was found that the vector-parameter approach is efficient in its computational aspect and quite convenient for real time simulation and control.

Mladenova [34] presented that the simple composition law of vector parameters as well others of their nice properties reduce the computational burden in solving direct and inverse kinematic problems, both in dynamic modelling and full simulation of the motion of a manipulator system. He proved that the standard configurational space approach becomes stronger over the group manifold where all kinematical equations are pure algebraic and the differential equations of motion 'feel' the group structure. The suggested approach gives the opportunities for the powerful methods of Lie group theory to be involved in controllability and observability of the treatment of manipulator systems. The introduction of a dual vector parameter provides an interesting interplay of special geometrical considerations and special algebraic structures in this important area of the application of linear algebra.

Wayne [35] examined mathematical representations commonly used in modeling flexible arms and arms with flexible drives. He presented the design considerations directly arising from the flexible nature of the arm and discussed controls of joints for general and tip motion.

Martinez and Duffy [36] expressed the acceleration of a point in the end effector of the serial chain in terms of the direction and moments parts of the same screw coordinates. Their work is an extension of the screw theory into the acceleration analysis of linkages. The acceleration is written as relatively simple expression involving screw coordinates and lie products.

Fumiki Tanaka et al. [37] formulated the constraints involved in assembly model as groups of rigid body transformations and proposed a constraint reduction procedure based on Lie algebra. This approach is then applied to the constraint representation and reduction in an assembly model. The reduced kinematic model of assembly was derived by the proposed method for kinematic analysis.

Dai [38] demonstrated the development of the finite twist or the finite screw displacement in the field of theoretical kinematics and the proposed q-pitch with the tangent of half the rotation angle studied the rigid body displacements.  He reviewed the work from Chasles motion to Cayley’s formula and then to Hamilton’s quaternions and Rodriguez parameterization and relates the work to Clifford biquaternions and to Study dual angle proposed in the late 19th century. The review of the work from these mathematicians concentrates on the description and the representation of the displacement and transformation of a rigid body, and on the mathematical formulation and its progress.

Paulo and Urbano [39] illustrated the use of Lie algebra to control nonlinear systems, essentially in the framework of mobile robot control. The study of path following control of a mobile robot using an input-output feedback linearization controller is performed. The effectiveness of the nonlinear controller is illustrated with simulation examples.

Milos and Vijay Kumar [40] investigated methods for computing a smooth motion that interpolates a given set of positions and orientations. The position and orientation of a rigid body can be described with an element of the group of spatial rigid body displacements. They interpolated the orientations and the positions separately and then combined the resulting interpolating curves. They investigated several properties of the trajectories and systematically analyze the invariance of the interpolating curves with respect to the choice of the inertial and the body fixed frames and the smoothness properties of the trajectories.

Ozgoren [41] reviewed the mathematical properties associated with the exponential rotation matrices. By means of two typical mechanism examples, it is demonstrated that these properties constitute a versatile analytical tool, which can be used effectively in kinematic studies on spatial mechanical systems involving position, velocity, acceleration, and singularity analyses using lie algebra. The mechanism in the first example allows analytical solution for its joint variables, whereas the joint variables of the mechanism in the second example can be obtained only by a semi analytical solution. However the scope of these examples is limited with the position, velocity, acceleration, and singularity analyses. But the symbolic manipulation power of the exponential rotation matrices demonstrated by these examples can be used with equal benefit in the area of kinematic synthesis.

Nielsen and Roth [42] reviewed and summarized the three most useful solution techniques by studying the kinematic geometry of some systems. The solution techniques are polynomial continuation, Gröbner bases, and elimination. Then they discussed the results with these techniques in the solution of two basic problems, namely, the inverse kinematics for serial-chain manipulators, and the direct kinematics of in-parallel platform devices. It was pointed out that specialized series chains can have the advantage of being modeled by relatively low-degree characteristic polynomials, and yet they can have a relatively large number of possible configurations with the same endeffector pose. For the in-parallel platform mechanism, it was shown that the specializations due to merging spherical joint centers can cause large variations in the degree of the characteristic polynomial, and hence in the number of poses corresponding to a set of leg lengths. Specializations making the platforms planar, in general, cause surprisingly little change. They do halve the degree of the characteristic polynomial, due to symmetry, but they leave the number of configurations unchanged from the spatial case.

Karger [43] developed a theory which allows describing higher order singularities by using lie algebra properties of the screw space. He developed an algorithm, which determines the degree of a singularity from the knowledge of the actual configuration of axes of the robot-manipulator. For serial robot-manipulators with the number of degrees of freedom different from six he showed that up to certain exceptions singular configurations can be avoided by a small change of the motion of the end-effecter. The new algorithm allows determining equations of the singular set for any serial robot-manipulator.

From the above study it is clear that quaternion algebra and lie algebra has been used efficiently in many kinematic synthesis, dynamics and control to represent translation and rotation. However the mathematical concept has not been clearly understood and it has been not applied effectively for higher degree of freedom robots. The study promts us to carry out further research work in this area with an objective to understand and analyze systematically the geometrical significance and develop new algorithms which will be efficient and easily understood by robotics community.

### 2.4 Objectives of Present Work

The study and analysis of some of the important literatures in the area of the robot manipulator kinematics make the present work spell out the following precise objectives.

- To find out alternative representations of robot manipulator’s forward kinematics with the help of higher mathematical theories such as lie algebra and quaternion algebra.
- To test the capabilities of the developed representations for higher DOF manipulators.
- To carry out a comprehensive study of the developed methods with the existing one in terms of representation convenience and computational convenience.

## 2.5 Summary

The geometric design of a robot manipulator defines the topology and dimensions of the articulated system that provides the end-effector position and velocity performance needed for a specified set of applications.  This chapter presents the survey of the related work using three distinct types of approach viz. (i) homogeneous transformation, (ii) Quaternion algebra and (iii) Lie Algebra.  The study of various work resulted in defining the objective of the present work in a precise manner and the same is presented at the end of the chapter.

# Chapter-III

## DESCRIPTION OF METHODS

# CHAPTER 3

## Description of Methods

### 3.1 Overview

The numerical solution of kinematics for revolute robot is important in many areas of modern technology, ranging from computer animated film making to the development of aircraft and spacecraft simulators. For the various applications in robotics there is need to describe both position and orientation of the end effector of the robot manipulator. The most common method in the robotics community is based on homogeneous matrix transformation. This matrix is used to describe one coordinate system with respect to another one. This has been the basis of tracking the position of the end-of-arm tool since ages. On the other hand, such matrix method is highly redundant to represent six independent degrees of freedom. This redundancy can introduce numerical problems in storage space, and often increase the computational cost of algorithms. In parallel implementations, the extra data required to fetch the operands can also be a significant factor. As a result, alternative methods for relating non-inertial coordinates to inertial coordinates have been developed. Such methods should be compact and computationally more efficient for representations of spatial transformations.

Keeping these in mind, alternative methods such as Euler angle, Epsilon algebra, unit and dual quaternion algebra, lie algebra, screw transformation are being sought by various researchers [44, 45, 46] for representing the same and reducing the computational time to make the system fast responsive in a flexible environment. Low and Dubey [3] used rotational generalized coordinates and equivalent Euler parameters to the inverse- kinematics problem for a six-degree-of-freedom robot manipulator. Michael W. Walker [5] presented the position of a manipulator expressed as either in joint coordinates or in Cartesian coordinates. General least-square and adaptive identification techniques are commonly employed by Kinsey and Whitcomb to identify general linear maps from input-output data. Lee and Merlet [10] solved the geometric design problem of serial-link robot manipulators using an interval analysis method. Funda, Taylor and Paul [11] implemented three-dimensional modeling of rotations and translations in robot kinematics employing quater-nion/vector pairs as spatial operators. Nielsen and Roth [42] reviewed and summarized the three most useful solution techniques: polynomial continuation, Grobbner bases, and elimination. Mladenova [32] derived basic important kinematical consequences by using vector parameterization of the SO(3) Lie group, dual algebra and the screw geometry. Liang Gu [30] presented lie algebra for exploring possible definition of three dimensional orientation vectors.

From the above discussion it is clear that homogeneous matrix method is the established traditional method for representation of position and orientation of the end effector. This can be used as a bench mark for comparing other methods and their efficiency with homogeneous matrix method. From the aforementioned methods and the previous literature survey the quaternion algebra and lie algebra are the two methods extensively used in kinematic analysis of revolute robots. But the detail analysis and the theory behind application of these methods are still not clear in robotics community. So further study of quaternion algebra and lie algebra is required for detail understanding and analysis and implementation has to be done to prove its effectiveness over other methods.

### 3.2 Robot Kinematics and their Representation

Kinematics is the study of motion without considering the forces and moment that causes the motion. Robot kinematics deals with the analytic study of the motion of a robot arm with respect to a fixed reference coordinate system as a function of time.

#### 3.2.1 Euclidean space

The space of rigid body motion has different properties than the well-known three-dimensional Euclidean space we live in. A Euclidean space is a continuous set of points together with an extra structure which is able to describe orthogonality and to measure length. This extra structure is the scalar product. It is important to realize that since a point can be fixed or moving with respect to an observer, for a given Euclidean space we need to specify an observer which does not move within it. We suppose that there exists an inertial observer we consider as reference in which the points belonging to the Euclidean space are not moving. Isometry means that the motion does not change the distance between points and orientation preserving implies that the motion does not map a right-handed coordinate frame onto a left-handed frame.

#### 3.2.2 Description of geometric parameters

The D-H representation of a rigid link depends on four geometric parameters associated with each link. These four parameters completely describe any revolute or prismatic joints. These four parameters are as follows:

i) joint angle (θ)
ii) joint distance (d)
iii) link length (a)
iv) link twist angle (α)

The first two parameters are known as joint parameters and last two parameters are known as link parameters. The joint angle (θ) is the rotation about $z^{t-1}$ needed to make axis $x^{t-1}$ parallel with axis $x^t$ shown in fig. 3.1. The second joint parameter joint distance (d) is the translation along $z^{t-1}$ needed to make axis $x^{t-1}$ intersect with the axis $x^t$. The link parameters are link length and link twist angle as shown in fig. 3.2. Link length (a) is the translation along $x^t$ needed to make axis $z^{t-1}$ intersect with axis $z^t$. Link twist angle ($\alpha$) is the rotation about $x^t$ needed to make axis $z^{t-1}$ parallel with axis $z^t$.

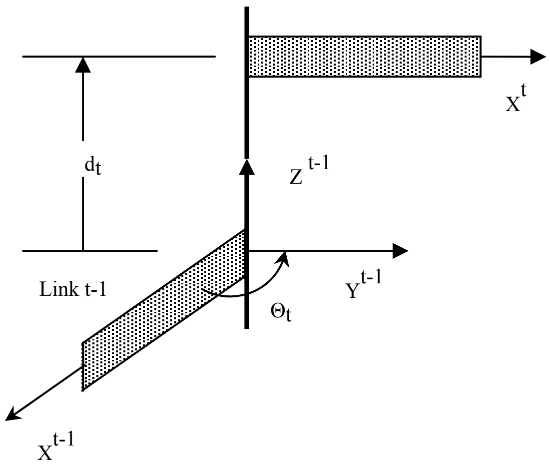



Figure 3.1: Joint angle ‘θ’ and joint distance ‘d’

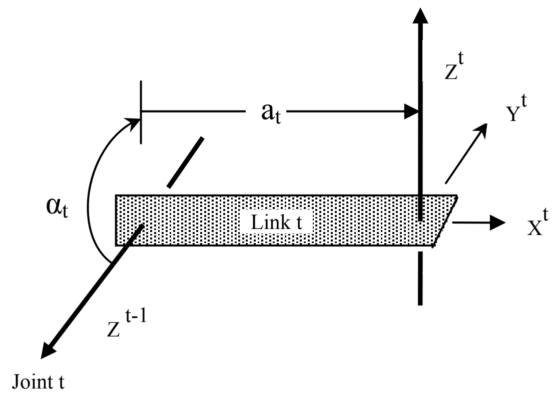



Figure 3.2: Link length ‘a’ and link twist angle ‘α’

For a revolute robot d, a, α are the joint parameters and these parameters remain constant whereas θ is the joint variable that changes with link t moving with respect to link t-1. Out of the four parameters, two link parameters always remain constant and are specified as a part of mechanical design. The list of kinematic parameters are given in table 3.1.

Table 3.1: kinematic parameters

| Parameters | Symbol | Revolute joint | Prismatic joint |
|---|---|---|---|
| Joint angle | $\theta$ | Variable | Fixed |
| Joint distance | D | Fixed | Variable |
| Link length | $a$ | Fixed | Fixed |
| Twist angle | $\alpha$ | Fixed | Fixed |

The transformation matrix ${}^{t-1}A_t$ in equation 3.7 is obtained for each t such that, $1 < t \leq m$. To find ${}^{t-1}A_t$ , the following four operations are performed in succession:

1. Translate by $d_t$ along the $z_t$ -axis.
2. Rotate $\theta_t$ counterclockwise by about the $z_t$ -axis.
3. Translate by $a_{t-1}$ along the $x_{t-1}$ -axis.
4. Rotate counterclockwise by $\alpha_{t-1}$ about the $x_{t-1}$ -axis.

To represent any position and orientation of $A_1$, it could be defined as a general rigid-body homogeneous transformation matrix in equation 3.7. If the first body is only capable of rotation via a revolute joint, then a simple convention is usually followed.

### 3.2.3 Denavit-Hartenberg representation

The position and orientation of the end effector of a serial chain are defined in terms of its joint parameters and physical dimensions by the kinematics equations. The Denavit–Hartenberg formulation is used to assign the local joint coordinate frames. Homogeneous transformation method is based on the traditional Denavit-Hartenberg's algorithm. In order to describe the translational and rotational relationship between adjacent links homogeneous matrix method was developed by Denavit and Hartenberg in 1955 [47]. In this system a coordinate system is established systematically by attaching a coordinate frame to each joint of the revolute robot with respect to certain rules as described below.

The following algorithm assigns an orthonormal coordinate system to each link of the robot arm according to the arm configurations. The labeling of the coordinate system begins from the supporting base to the end effector of the robot arm. The relation between the adjacent links can be represented by a 4X4 homogeneous transformation matrix.

**D-H algorithm**

1) Number the joints from 1 to n starting with the base and ending with the tool.

2) Assign a right handed orthonormal coordinate frame ($x_0$, $y_0$, $z_0$) to the base, so that $z^0$ aligns with axis of the joint 1, set t=1.

3) Align $z^t$ with the axis of the joint t+1.

4) Locate the origin of frame ($x_t$, $y_t$, $z_t$) at the intersection of $z^t$ and $z^{t-1}$ axes. If they do not intersect, the intersection of $z^t$ is used with a common normal between $z^t$ and $z^{t-1}$.

5) Select $x^t$ to be orthogonal to both $z^t$ and zt-1. If $z^t$ and $z^{t-1}$ are parallel, point $x^t$ away from $z^{t-1}$.

6) Select yt to form a right handed orthonormal coordinate frame ($x_t$, $y_t$, $z_t$).

7) Set t=t+1. If t<n, go to step3; else continue.

8) Set the origin of ($x_n$, $y_n$, $z_n$) at the tool tip. Align $z_n$ with the approach vectoryn with the sliding vector, and $x^n$ with the normal vector of the tool. Set k=1.

9) Locate point $b^t$ at the intersection of the $x^t$ and $z^{t-1}$ axes.If they don't intersect, use the intersection of $x^t$ with a common normal between $x^t$ and $z^{t-1}$.

10) Compute $\theta_t$ as the angle of rotation from $x^{t-1}$ to xt measured about $z^{t-1}$.

11) Compute $d_t$ as the distance from the origin of frame ($x_{t-1}$, $y_{t-1}$, $z_{t-1}$) to point $b^t$ measured along $x^t$.

12) Compute $\alpha_t$ as the distance from point $b^t$ to the origin of frame ($x_t$, $y_t$, $z_t$) measured along $x^t$.

13) Compute $\alpha_t$ as the angle of rotation from $z^{t-1}$ to $z^t$ measured about $x^t$.

14) Set t = t+1. If $t \leq n$, go to step 9; else stop.

Thus each link at the joint is expressed with respect to the previous joint coordinate system in terms of transformation matrix. Finally the end effector which is expressed in 'hand coordinates' can be transformed and expressed in the 'base coordinates' through sequential transformation. For a six dof revolute robot, as shown in figure 3.3, an orthonormal Cartesian coordinate frame ($x_t$, $y_t$, $z_t$) is established for each link at its joint axis, where i=1, 2, 3, where i=number of degree of freedom plus the base coordinate frame.

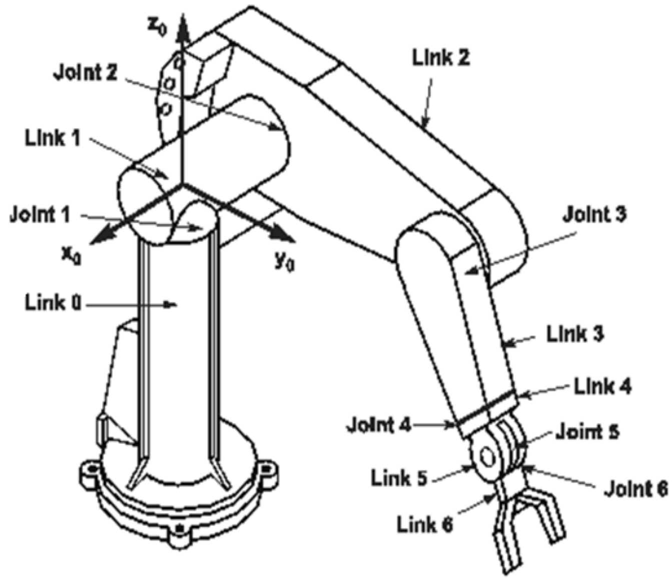



Figure 3.3: 6 dof revolute robot

Since a revolute joint has only one degree of freedom each ($x_t$, $y_t$, $z_t$) coordinate frame of a robot arm corresponds to joint t+1 and it is fixed in link t. When joint actuator activates joint t, link t will move with respect to link t-1. The base coordinates are defined as the $0^{th}$ coordinate frame ($x_0$, $y_0$, $z_0$) which is the initial coordinate frame of the robot arm. Thus, for a six axis PUMA robot arm, there are seven coordinate frames beginning from ($x_0$, $y_0$, $z_0$), ($x_1$, $y_1$, $z_1$)… ($x_6$, $y_6$, $z_6$). The location of the coordinate frame 0 is chosen anywhere in the supporting base such that $z_0$ axis lies along the axis of motion of the first joint. After establishing the D-H coordinate system for each link a homogeneous transformation matrix is developed relating to the t $^{th}$ coordinate frame to the (t-1) $^{th}$ coordinate frame.

### 3.3 Homogeneous Matrix Method

Homogeneous matrix method is the classical method to describe the relationship between two adjacent rigid mechanical links [48, 49]. These coordinate systems are established in a systematic manner following Denavit-Hartenberg's (D-H) algorithm.

Homogeneous matrices have the following advantages:

- Simple explicit expressions exist for many familiar transformations including rotation .
- These expressions are n-dimensional.
- There is no need for auxiliary transformations, as in vector methods for rotation.
- More general transformations can be represented (e.g. projections, translations).
- Directions (ideal points) can be used as parameters of the transformation, or as inputs.
- Many homogeneous transformation matrices display the duality between invariant axes and centers.

**Mathematical representation**

The first method based on homogeneous transformation is formulated by using D-H algorithm which depends upon already derived transformation operators such as matrices or vector. In most robotic application; spatial description of the end-effector of the manipulator with respect to a fixed reference coordinate system is required. To use homogeneous matrix method for displacement analysis of a spatial linkage we need to attach a coordinate frame to each link. These coordinate systems are established in a systematic manner following Denavit-Hartenberg's algorithm as explained in paragraph 3.3(c).

The basic rotation around x-axis is represented by

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \tag{3.1}$$

where, $\theta$ = angle of rotation

$R_x$ = rotation matrix about ox axis.

The resultant rotation matrix is given by multiplying the three basic rotation matrices.

$$R = R_{y,\phi} R_{z,\theta} R_{x,\alpha} \tag{3.2}$$

where,

$R_{y,\phi}$ = rotation about y-axis with an angle $\phi$

$R_{z,\theta}$ = rotation about z-axis with an angle $\theta$

$R_{x,\alpha}$ = rotation about x-axis with an angle $\alpha$

This can be written in short as;

$$R = \begin{pmatrix} c\phi c\theta & s\phi s\alpha - c\phi s\theta c\alpha & c\phi s\theta c\alpha + s\phi s\alpha \\ s\theta & c\theta c\alpha & -c\theta s\alpha \\ -s\phi c\theta & s\phi s\theta c\alpha & c\phi c\alpha - s\phi s\theta s\alpha \end{pmatrix} \tag{3.3}$$

The matrix representation for rotation of a rigid body simplifies many operations but it needs nine elements to completely specify the orientation of a rotating rigid body it does not lead directly to a complete set of generalized coordinates, such a set of generalized coordinate can describe the orientation of a rotating rigid body with respect to a reference coordinate frame. The 3X3 matrix does not give any provision for translation and scaling, so a fourth coordinate or component is introduced to a position vector p where,

$$p = (p_x, p_y, p_z) \tag{3.4}$$

Alternatively the orientation matrix can be represented through Euler angles. There are different sequences of Euler angle representation. Here the sequence of Euler angle followed is

$$R_{\phi,\theta,\psi} = R_{z,\phi} R_{y,\theta} R_{x,\psi} = \begin{pmatrix} c\phi c\theta & c\phi s\theta s\psi - s\phi c\psi & c\phi s\theta c\psi + s\phi s\psi \\ s\phi c\theta & s\phi s\theta s\psi + c\phi c\psi & s\phi s\theta c\psi - c\phi s\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{pmatrix} \quad (3.5)$$

More complicated joints can be modeled as a sequence of degenerate joints. For example, a spherical joint can be considered as a sequence of three zero-length revolute joints; the joints perform a roll, a pitch, and a yaw. Another option for more complicated joints is to abandon the D-H representation and directly develop the homogeneous transformation matrix. This might be needed to preserve topological properties that become important in this chapter.

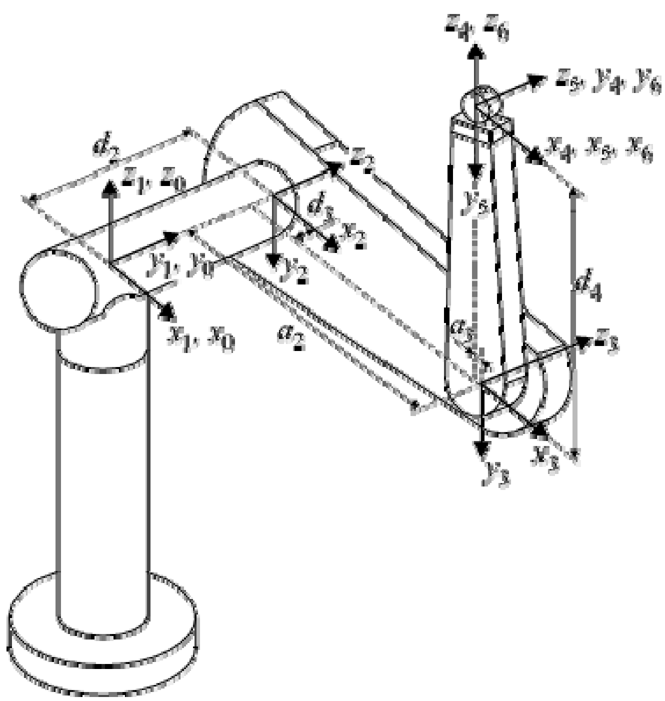



Figure 3.4: The Puma 560 is shown along with the DH parameters and body frames for each link in the chain.

The transformation $T_t$ for $t > 1$ gives the relationship between the body frame of A and the body frame of $A_{i-1}$. The position of a point (x,y,z) on $A_n$ is given by

$$T_1 T_2 \cdots T_n \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{3.6}$$

For each revolute joint, $\theta_t$ is treated as the only variable in $T_t$. Prismatic joints can be modeled by allowing $a_t$ to vary. For PUMA 560 robot the parameters are shown in table 3.2.

Table 3.2: The DH parameters are shown for substitution into each homogeneous transformation matrix

| Matrix | $\alpha_{t-1}$ | $a_{t-1}$ | $\theta_t$ | $d_t$ |
|---|---|---|---|---|
| $T_1(\theta_1)$ | 0 | 0 | $\theta_1$ | 0 |
| $T_2(\theta_2)$ | $-\pi/2$ | 0 | $\theta_2$ | $d_2$ |
| $T_3(\theta_3)$ | 0 | $a_2$ | $\theta_3$ | $d_3$ |
| $T_4(\theta_4)$ | $\pi/2$ | $a_3$ | $\theta_4$ | $d_4$ |
| $T_5(\theta_5)$ | $-\pi/2$ | 0 | $\theta_5$ | 0 |
| $T_6(\theta_6)$ | $\pi/2$ | 0 | $\theta_6$ | 0 |

Let the $a_0, \alpha_0$ parameters of $T_1$ be assigned as $a_0 = \alpha_0 = 0$ (there is no $z_0$-axis).

Here $a_3$ and $d_3$ are negative in this example, they are signed displacements, not as distances. This example demonstrates the 3D chain kinematics on a classic robot manipulator, the PUMA 560, shown in Figure 3.4. The procedure is to determine appropriate body frames to represent each of the links. The first three links allow the hand (called an end-effector) to make large movements in W, and the last three enable the hand to achieve a desired orientation. There are six degrees of freedom, each of which arises from a revolute joint. The parameters from Figure 3.4 may be substituted into the homogeneous transformation matrices to obtain equation 3.7.

The homogeneous transformation matrix is a 4X4 matrix which maps a position vector expressed in homogeneous coordinates from one coordinate system to another coordinate system. The basic homogeneous matrix is represented by

$$^{t-1}A_t = \begin{pmatrix} \cos\theta_t & -\cos\alpha_t \sin\theta_t & \sin\alpha_t \sin\theta_t & a_t \cos\theta_t \\ \sin\theta_t & \cos\alpha_t & -\sin\alpha_t \cos\theta_t & \sin\theta_t \\ 0 & \sin\alpha_t & \cos\alpha_t & d_t \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.7}$$

The body frames are shown in Figure 3.3, and the corresponding DH parameters are given in Table 3.2. Each transformation matrix can also be written as $T_t$ for simplification and it is a function of $\theta_t$; hence, it is written $T_t(\theta_t)$. The other parameters are fixed for this example. Only, $\theta_1$, $\theta_2$, $\theta_3$ . . ., are allowed to vary and transformation matrices can be derived from equation 3.7 as follows.

$$T_1(\theta_1) = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.8}$$

$$T_2(\theta_2) = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.9}$$

$$T_3(\theta_3) = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.10}$$

$$T_4(\theta_4) = \begin{pmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & a_3 \\ 0 & 0 & -1 & -d_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.11}$$

$$T_5(\theta_5) = \begin{pmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_5 & -\cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.12}$$

and $$T_6(\theta_6) = \begin{pmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.13}$$

In a kinematic chain the transformation matrix ${}^{t-1}A_t$ describes the local coordinate frame for $t$ th link of the manipulator with respect to the local frame of the previous link $t-1$.The forward kinematic equation of the manipulator can be developed by multiplying the above matrix ${}^{t-1}A_t$ calculated sequentially for each link. Using the ${}^{t-1}A_t$ matrix one can relate a point $p_t$ at rest in link $t$ and expressed in homogeneous coordinates with respect to the coordinate system $t-1$ established at link $t-1$ by $p_{t-1} = {}^{t-1}A_t\, p_t$. The orientation of a body in three dimensions is difficult to visualize and describe, so alternative methods have been tried. In the orientation matrix nine parameters are used to represent three degrees of freedom to specify the orientation of a body.

An algorithm is presented for the derivation of kinematic equation of a n-link robot which is based on homogeneous transformation.

i) Assignment of a local coordinate system to every link and a global one to the base of the robot.

ii) Determination of the kinematic parameters for the links 1 to n.

iii) Determination of the transformation matrices ${}^{t-1}T_t$ for $t = 1$ to $n$, describing each local coordinate system with respect to its previous one by

$$
{}^{t-1}T_t = \begin{bmatrix} \cos\theta_t & -\sin\theta_t & 0 & l_{t-1} \\ \sin\theta_t \cos\alpha_{t-1} & \cos\theta_t \cos\alpha_{t-1} & -\sin\alpha_{t-1} & -\sin\alpha_{t-1} d_t \\ \sin\theta_t \sin\alpha_{t-1} & \cos\theta_t \sin\alpha_{t-1} & \cos\alpha_{t-1} & \cos\alpha_{t-1} d_t \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.14}
$$

iv) Calculation of the final transformation matrix ${}^{0}T_n$ of the end effector coordinate system with respect to the base frame is done using following equation.

$$
{}^{0}T_n = T_1(\theta_1)T_2(\theta_2)T_3(\theta_3)T_4(\theta_4)T_5(\theta_5)T_6(\theta_6)\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{3.15}
$$

In general the homogeneous method of representation is highly redundant since it requires 12 numbers to completely represent six degree of freedom. The basic homogeneous matrix in equation 3.15 can be represented as

$$T = \begin{pmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.16}$$

where, n=normal vector of the hand

s=sliding vector of the hand

a=approach vector of the hand

p=position vector of the hand

The position of the end effector is given by the last column of the matrix and orientation is given by upper left 3 X 3 sub matrix. Thus the position and orientation can be computed by using equation 3.1 to equation 3.9. The foundation of this algorithm is formulation of the combined homogeneous matrix. Once this matrix is obtained subsequent matrices are calculated sequentially for higher order links. The final matrices are formulated by multiplying the transformation matrices representing simple rotation about the principal axis and translations along the principal direction of the local coordinate system of the links. Homogeneous matrix method thus provides a systematic way to understand and implement the algorithm step by step.

### 3.4 Quaternion Algebra Method

Quaternion algebra enunciated by Hamilton, has played a significant role recently in several areas of the physical science; namely, in differential geometry, in analysis and synthesis of mechanisms and machines, simulation of particle motion in molecular physics and quaternion formulation of equation of motion in theory of relativity[50,51]. In mathematics, a quaternion algebra over a field, F, is a particular kind of central simple algebra, A, over F, namely such an algebra that has dimension 4, and therefore becomes the 2×2 matrix algebra over some field extension of F, by extending scalars (i.e., tensoring with a field extension). The classical Hamilton quaternions are the case of F the real number field, and A is uniquely defined up to isomorphism by the condition that it is such a quaternion algebra that is not the 2×2 real matrix algebra.

Quaternion algebra therefore means something more general than the algebra of Hamilton's quaternions. When the coefficient field F does not have characteristic 2, any quaternion algebra over F is a slightly twisted form of the familiar quaternions with coefficients in F. It has a basis 1, i, j, and k such that

$$i^2 = a$$

$$j^2 = b$$

$$ij = k,\ ji = -k$$

where a and b are any nonzero elements of F, and a short calculation shows $k^2 = -ab$. (The Hamilton quaternions are the case when a and b both equal −1.) When F has characteristic 2, a different explicit description in terms of a basis of 4 elements is also possible, but in any event the definition of quaternion algebra over F as a 4-dimensional central simple algebra over F applies uniformly in all characteristics. Quaternion algebras are applied in number theory, particularly to quadratic forms.

#### 3.4.1 Application of quaternion algebra

Recently new development has been made regarding some identities associated with dual-number-quaternion for analysis of mechanisms and machines. After Hamilton, Clifford [52] found it very important to develop the concept of dual numbers and dual quaternion for his geometrical investigations. It appears that Blaschke was the first to recognize the importance of the concept of quatemions in kinematics. More recently, a unified presentation of spatial displacements in terms of a generalized screw displacement pair and its relation with different representations of spherical motion is done. The investigation includes applications of dual-number quaternions to screw motion of a line and a point. One of the major advantages of quaternion is that the equations governing the physical systems written in terms of quaternion are singularity free. Thus, it provides a numerically stable set of equations. From the above discussion and the references therein, it is clear that there are numerous application of quaternion in the area of physical science that employ quaternion as an analytical tool. However, the quaternion have not received wide publicity in the area of kinematics and dynamics of mechanisms because the algebra underlying the quaternions are quite involved and they are quite difficult to interpret in a three dimensional space. Observing the above fact, Wehage proceeded to present the quaternion treatments of real numbers using matrices and linear algebra. In this method two operators, which are related to the real quaternion, are defined and its properties are developed. These operators allow one to directly translate the language of quaternion into the matrices. This not only provides an elegant tool for matrix treatment of quaternion but also provides a compact formulation that can be translated easily into computer programs for numerical computations. The present investigation of dual quaternion algebra and Hamilton matrix operators associated with it reveal several new properties and identities of the matrices involved which otherwise cannot be visualized easily. It is also shown how these properties can be utilized to develop some kinematic relations for spatial motion of a body.

The second method in general is a line transformation method where step by step calculation of the transformation operator is done along with line vectors. Quaternions are a natural extension of complex numbers. Like complex numbers they have a real part, however, they have three imaginary parts, *i, j,* & *k*, with some special rules. Once again, quaternions have uses concerning fractals. Due to the fact that they have four components, these fractals are four dimensional.

### 3.4.2 Mathematical representation

In this section quaternion algebra is presented and its use to formulate the forward kinematic problem of the robot arm is discussed. Rotation quaternion can be used to calculate the rotated point from the original position of the point; this allows translation of points without using matrices [53, 54, 55]. Quaternion plays a vital role in the representation of rotations in computer graphics, primarily for animation. Interpolating the quaternion representation of a sequence of rotations is more natural than doing so for the familiar Euler angles, such as yaw, pitch, and roll. The quaternion occupies a smooth, seamless, isotropic space which is a generalization of the surface of a sphere.

A brief summary of dual numbers and dual quaternion is presented in this section for an ease of reference and to provide the necessary background for the mathematical formulations to be developed further. A dual number has the form q and εq' where q and εq' are real numbers and ε is the dual symbol subjected to the rules
$\varepsilon^2=0$, $0\varepsilon=0$, $\varepsilon 0=0$, $\varepsilon 1=1\varepsilon=\varepsilon$. Before going into detail analysis of quaternion and steps of formulation of kinematic equation for robot arm some properties of quaternion algebra is presented. Quaternion can be represented as;

$$q = w + ix + jy + kz \tag{3.17}$$

Where w, x, y, z are real numbers, and i, j, k, are quaternionic units which satisfy the non-commutative multiplication rules. Here w is the real part and x, y, z are imaginary parts. Each of these imaginary dimensions has a unit value of square root -1, all are mutually perpendicular to each other known as i, j, k.
If a quaternion q has length 1, we say that q is a unit quaternion. One of the major properties of quaternion is that they are anti commutative. Quaternion algebra can be

understood as an extension of complex number. As we know complex number consist of one real part and one imaginary part similarly quaternion has four dimensions i.e. one real part and three imaginary part.

i) Conjugate of quaternion:

The conjugate of a quaternion number is a quaternion with the same magnitudes but with the sign of the imaginary parts changed. So conjugate of $q=w+ix+jy+kz$ is

$$q' = w - ix - jy - kz \tag{3.18}$$

ii) Magnitude:

The magnitude of a quaternion $q = w + ix + jy + kz$ is $\sqrt{(w^2 + x^2 + y^2 + z^2)}$ (3.19)

iii) Norm:

Norm of a quaternion is defined by

|| $q$ || = square root of ( $q$ *conj( $q$ ))

$$= \sqrt{(w^2 + x^2 + y^2 + z^2)} \tag{3.20}$$

iv) Inverse:

The inverse of a quaternion refers to the multiplicative inverse $\frac{1}{q}$ and can be computed by

$$q^{-1} = q'/(q*q') \tag{3.21}$$

If a quaternion q has length 1, we say that q is a unit quaternion. The inverse of a unit quaternion is its conjugate, i.e. $q^{-1} = q' \; q^{-1}$. One of the major properties of quaternion is that they are anti commutative. Quaternion algebra can be understood as an extension of complex number. As we know complex number consist of one real part and one imaginary part similarly quaternion has four dimensions i.e. one real part and three imaginary part. From equation 3.17 to equation 3.21 basic quaternions and their properties are represented in mathematical form.

A dual number is shown in equation 3.22 and it can be defined as

$$q+\varepsilon q^{0} \tag{3.22}$$

where, $q$ and $q^{0}$ are real numbers with '$\varepsilon$' as a dual unit having definite property.

The above definitions need the following comments:

i) The norm of a dual-number-quaternion is, in general, not a real number but a dual number.

ii) The reciprocal of a dual quaternion'q' exists if and only if norm of that quaternion is not zero, although conjugate of the norm may be zero.

iii) For a unit quaternion the scalar product of q and q' must be zero. The third requirement is very similar to the requirement of a unit dual vector. The quaternion multiplication is, in general, not commutative. If quaternion can be used to represent rotation and quaternion multiplication can be used to get the result of subsequent rotation.

### 3.4.3 Rotation representation

Let $q_1$ and $q_2$ are unit quaternion representing two rotations. Then subsequent rotation can be done by, rotating first $q_1$ and then $q_2$. The composite rotation is represented by the quaternion $q_2 * q_1$.

Let $p_1$ = vector representing the initial position of a point being transformed.

$p_2$=vector representing the final position of the point after translation.

$$\begin{aligned} P_2 &= q_2 * (q_1 * P * q_1^{-1}) * q_2^{-1} \\ &= (q_2 * q_1) * P * (q_1^{-1} * q_2^{-1}) \\ &= (q_2 * q_1) * P * (q_2 * q_1)^{-1} \end{aligned} \quad (3.23)$$

The quaternion can represent 3D reflections, rotations and scaling, however a single quaternion operation cannot include translations with rotation. So for rotation, reflection or scaling around a point other than the origin, we would have to handle the translation part separately.

### 3.4.4 Representation of pure translation

The translation in quaternion algebra is done by using a quaternion operator and it is defined by

$$q = 1 + (x_1/2)i\varepsilon + (y_1/2)j\varepsilon + (z_1/2)k\varepsilon \quad (3.24)$$

where, $x_1, y_1, z_1$ are the translation carried out along x,y,z direction respectively.

Quaternion transformation is represented as

$$p_2 = q * p_1 * q' \quad (3.25)$$

where,

$p_1$ and $p_2$ are initial and final position of a point

$q$ =dual quaternion operator representing transform

$q'$ =conjugate of $q$ .

The rotation and translation representation in quaternion algebra is carried out using equation 3.23 to equation 3.25.

### 3.4.5 Mathematical formulation for 6-DOF revolute robot

Multiplication of quaternion numbers together behaves similarly to cross product of the unit basis vectors. Unit line vector is a vector which is constrained to lie on a definite line in figure 3.5, $u$ is a unit vector and $u^0 = r \times u$ is the moment vector, where $r$ is the position vector of an arbitrary point $P$ on the line. The vectors $u, u_0$ are often called plucker vectors.

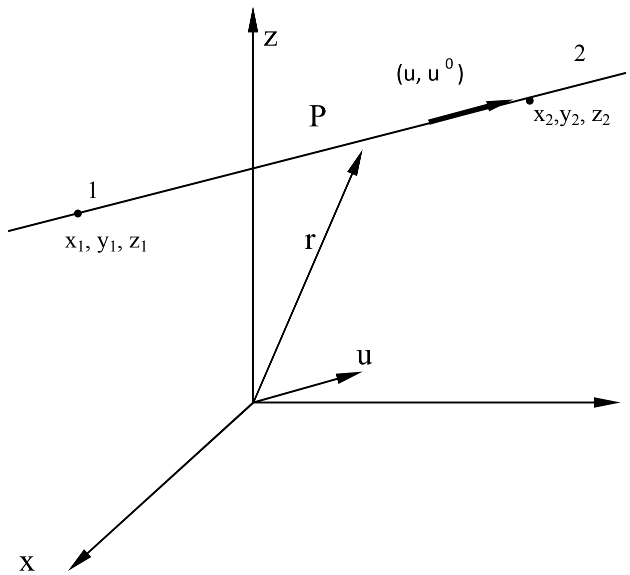



Figure 3.5: A unit line vector

A unit line vector in its dual form can be represented as;

$$\hat{u} = u + \varepsilon u^0 \tag{3.26}$$

where, u and $u^0$, respectively are real and dual quaternion components. The multiplication of quaternionic units with dual symbol is commutative; i.e. iε=εi and so on. Owing to the properties of the eight units that are four real quaternion and four dual part of the quaternion, equality, addition, and subtraction of dual-number quaternions are governed by the rules of ordinary algebra.

From the geometrical point of view a quaternion can also be represented as;

$$q = \cos\frac{\theta}{2} + u\sin\frac{\theta}{2} \tag{3.27}$$

where, $\cos\frac{\theta}{2}$, the first part of the equation is the real part and the second part $\sin\frac{\theta}{2}$ is the complex part. Quaternion can be thought of as an axis angle $u$ along which rotation is considered. It is quite difficult to give physical meaning to a quaternion but it forms an interesting mathematical system. A quaternion can also be written as

$$q = q_0 + q_x i + q_y j + q_z k \tag{3.28}$$

which is a combination of a scalar $q_0$, and $q_x i + q_y j + q_z k$ represents the vector component along three mutually perpendicular directions. A dual quaternion may be written as

$$\mathrm{Q} = (\mathrm{q}_0 + \varepsilon \mathrm{q}_0^0)1 + (\mathrm{q}_x + \varepsilon \mathrm{q}_x^0)\mathrm{i} + (\mathrm{q}_y + \varepsilon \mathrm{q}_y^0)\mathrm{j} + (\mathrm{q}_z + \varepsilon \mathrm{q}_z^0)\mathrm{k} \tag{3.29}$$

In this equation the first part is a real part and the second part associated with $\varepsilon$ is dual part. The combined rotation and translation can be represented by quaternion operator. The relationship between two non parallel and non intersecting unit line vector can be obtained as follows.

Let,

$\hat{u}$=A unit line vector.

$\hat{v}$= Unit line vector obtain by translation of $\hat{u}$ by a distance $d$, followed by rotation of a dual angle $\varphi = \varphi + \varepsilon d$.

The transformation of $\hat{u}$ into $\hat{v}$ is given by

$$\hat{v} = \hat{Q}\,\hat{u} \tag{3.30}$$

The transformation operator is defined as

$$\hat{Q} = \cos\,\hat{\varphi} + \hat{e}\sin\,\hat{\varphi} \tag{3.31}$$

which is also a dual quaternion. So by multiplying a unit line vector by the transformation operator $\hat{Q}$ , the image of that line is obtained in a new location defined by the parameters of this transformation operator.

A dual quaternion is the set of four dual numbers in a definite manner. Just like we extend 3 X 3 matrices to 4 X 4 matrices to allow them to translation in addition to rotation we extend quaternion to dual quaternion to allow them to represent translations in addition to rotation. The dual of a quaternion can model the movement of a solid object in 3 dimensions which can rotate and translate without changing the shape.

To formulate forward kinematic equation: first assignment of coordinate system to every link and to the base as in traditional method is done. The kinematic parameters for the links 1 to n are determined. Calculation of unit line vector is done which is coincident with the common normal between $t$ th and $(t+1)$ th axis and $s_t$ as the unit line vector along z axis of the $t$ th joint. The mathematical equations used to obtain kinematic equation are as follows.

The two unit line vectors are

$$a_{t,t+1=}\hat{Q}_t\,\hat{a}_{t-1,t} \tag{3.32}$$

and $\hat{s}_{t+1} = \hat{Q}_{t,t+1}\hat{s}_t$ (3.34)

For base frame unit vectors

$$a_{0,1} = i \text{ and } s_1 = k \text{ .} \tag{3.35}$$

The transformation operators are

$$\hat{Q}_t = \cos \hat{\theta}_t + s_t \sin \hat{\theta}_t \text{ and} \tag{3.36}$$

$$\hat{Q}_{t,t+1} = \cos \hat{\alpha}_{t,t+1} + \hat{a}_{t,t+1} \sin \hat{\alpha}_{t,t+1} \tag{3.37}$$

where $\hat{\alpha}_{t,t+1}$ the dual angle between $\hat{s}_t$ and $\hat{s}_{t+1}$ is defined as

$$\hat{\alpha}_{t,t+1} = \hat{\alpha}_t + \varepsilon L_t \tag{3.38}$$

and $\hat{\theta}_t$, the dual angle between $\hat{a}_{t-1,t}$ and $\hat{a}_{t,t+1}$ is defined as

$$\hat{\theta}_t = \theta_t + \varepsilon d_t \tag{3.39}$$

in terms of four D-H kinematic parameters.

The position vector of the end effector is given by

$$P_n = \sum_{t=1}^{n} (d_t s_t + L_t a_{t,t+1}) \tag{3.40}$$

The orientation matrix R of the end effector coordinate system by the three vectors

$$n_n = a_{n,n+1}$$

$$o_n = s_{n+1} \times a_{n,n+1}$$

$$a_n = s_{n+1} \text{ .} \tag{3.41}$$

Following the steps of the given algorithm and using equation 3.26 to equation 3.41 the kinematic equation of any spatial manipulator can be evaluated. The unit line vector $s_t$ defines the axis of the joint $t$ and $\hat{a}_{t,t+1}$ defines the common perpendicular to the axes of the $t$ and $t+1$ joints. The dual quaternion transforms the unit line vector $\hat{a}_{t-1,t}$ to $\hat{a}_{t,t+1}$. In other words, the operator translates the x-axis of the frame $t-1$ along the axis of the joint $t$ by an angle $\theta_t$. The dual quaternion $\hat{Q}_{t,t+1}$ is similar. It translates and rotates the joint axis along and about the common perpendicular to this joint axis and the next one. The vectors $s$ and $a$ are the unit vectors defining the orientation of the z and x axis respectively. The unit vector of the Y axis of the last coordinate system is calculated by the vector product of the $s$ and $a$ vectors.

### 3.5 Lie Algebra Method

In mathematics, a Lie algebra is an algebraic structure whose main use is in studying geometric objects such as Lie groups and differentiable manifolds. Lie algebras were introduced to study the concept of infinitesimal transformations. The term "Lie algebra" (was introduced by Hermann Weyl in the 1930s. Although Lie algebras are often studied in their own right, historically they arose as a means to study Lie groups. Given a Lie group, a Lie algebra can be associated to it either by endowing the tangent space to the identity with the differential of the adjoint map, or by considering the left-invariant vector fields as mentioned in the examples[56]. This association is functorial, meaning that homomorphism of Lie groups lift to homomorphism of Lie algebras, and various properties are satisfied by this lifting: it commutes with composition, it maps Lie subgroups, kernels, quotients and cokernels of Lie groups to sub algebras, kernels, quotients and cokernels of Lie algebras, respectively.

The functor which takes each Lie group to its Lie algebra and each homomorphism to its differential is a full and faithful exact functor. This functor is not invertible; different Lie groups may have the same Lie algebra, for example SO(3) and SU(2) have isomorphic Lie algebras. Even worse, some Lie algebras need not have *any* associated Lie group. Nevertheless, when the Lie algebra is finite-dimensional, there is always at least one Lie group whose Lie algebra is the one under discussion, and a preferred Lie group can be chosen. Any finite-dimensional connected Lie group has a universal cover. This group can be constructed as the image of the Lie algebra under the exponential map. More generally, we have that the Lie algebra is homomorphic to a neighborhood of the identity. But globally, if the Lie group is compact, the exponential will not be injective, and if the Lie group is not connected, simply connected or compact, the exponential map need not be surjective.

If the Lie algebra is infinite-dimensional, the issue is more subtle. In many instances, the exponential map is not even locally a homeomorphism. (for example, in Diff ($\mathbf{S}^1$), one may find diffeomorphisms arbitrarily close to the identity which are not in the image of exp)[57, 58]. Furthermore, some infinite-dimensional Lie algebras are not the Lie algebra of any group.

The correspondence between Lie algebras and Lie groups is used in several ways, including in the classification of Lie groups and the related matter of the representation theory of Lie groups. Every representation of a Lie algebra lifts uniquely to a representation of the corresponding connected, simply connected Lie group, and conversely every representation of any Lie group induces a representation of the group's Lie algebra; the representations are in one to one correspondence. Therefore, knowing the representations of a Lie algebra settles the question of representations of the group. As for classification, it can be shown that any connected Lie group with a given Lie algebra is isomorphic to the universal cover mod a discrete central subgroup. So classifying Lie groups becomes simply a matter of counting the discrete subgroups of the center, once the classification of Lie algebras is known.

A Lie Group G is a set of elements embodying simultaneously the properties of a group and a smooth manifold. By the group property, (1) composition of any two elements g1, g2 ϵ G is defined, yielding a composite element g that also belong to G; (2) identity: there exist a element e in G such that composition of e with any other element g yields the element g itself, and (3) for any element g in G there exist an inverse $g^{-1}$ such that the composition of g and $g^{-1}$ yields the identity e. By the manifold property, any two points in G can be connected by a smooth trajectory, and at any point g one can define a differential dg that is tangent to G. The inter relation between the lie group and its composition is shown in figure 3.6.

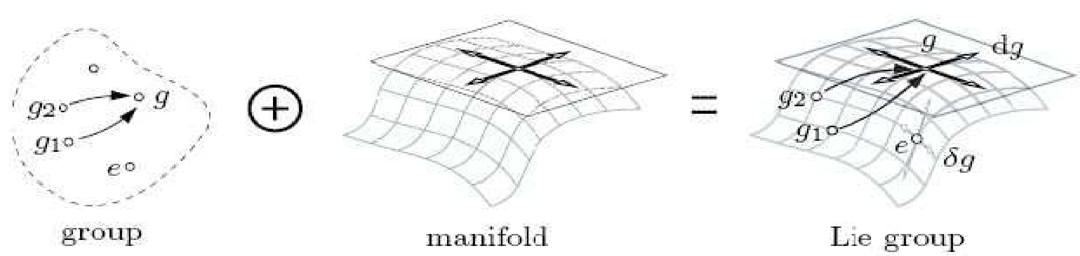



Figure 3.6: Lie group

The differential at the neutral element (identity) is particularly important. This tangent space at the identity is called the Lie algebra for that group. The Lie algebra along with a bilinear map called Lie bracket, forms a vector space.

The lie bracket [:; :], satisfies ;

1) Skew symmetry: [x; y] = -[x; y]:

2) Jacobi identity: [x; [y; z]] + [z; [x; y]] + [y; [z; x]] = 0

for every x,y,z in the associated Lie algebra. The primary connection between a Lie group and its Lie algebra is the exponential mapping. On matrix groups the exponential mapping corresponds to the ordinary matrix exponential, i.e., if A is an element of the Lie algebra, then exp(A) is an element of the Lie group.

### 3.5.1 History and application

A group that is a differentiable manifold is called a Lie group, which is a differentiable manifold. The name comes after the famous mathematician Sophus Lie. A lot of geometrical and analytical methods have been developed over the last three centuries to describe the changes of configuration of rigid bodies. The previously mentioned methods are all global, geometrical, well-defined methods. These methods are different from Euler angles which are only valid locally and they are not as powerful as the methods described in the following sections. As the complexity of multibody system increases, the need for more elegant formulation of the equations of motion becomes an issue of paramount importance. For example, it is desirable to have an explicit representation of the equations of motion that can be manipulated at a high-level and in which the kinematic and dynamic parameters of the system appear in a transparent manner. Moreover, many applications in robot control and planning require a set of dynamic equations that can be defined explicitly with respect to parameter of interest. It is also desirable to have the equations of motion in a form which can be implemented effectively using a computer for the purpose of simulation. In this work, the techniques of Lie group are used to derive kinematic synthesis of robot. These equations of motion can be arranged in a recursive form for serial link manipulators. The resulting recursive formulation is particularly suitable for computer implementation.

### 3.5.2 Mathematical representation

In this part translation and rotation are represented with Special Orthogonal and Euclidean groups, SO (3) and SE (3). Transformations on orientation in Euclidian space are accomplished by the use of the rotation matrix θ, which is an element of the special Orthogonal group, SO (3), within Lie group since it satisfies the properties of group and manifold mentioned above. SO (3) is a schematic of the group of rotations in three dimensions. Any rotation can be specified by a vector pointing along the axis of rotation, with a length equal to the amount of rotation; using this correspondence. The group is drawn as a sphere with a wedge removed to reveal the interior but the true topology identifies opposite points on the surface, which represent rotations of 180° around opposite axes. The overall effect is equivalent to moving the centre of the sphere around the group in a constant direction; the sphere always encompasses the entire group, but the particular element lying at the centre changes.
The rigid body transformation, cast in 4x4 homogeneous form, is also a Lie group referred as the special Euclidian group or SE (3) given in equation 3.34. Given the rotation θ ϵ SO (3) and translation b ϵ $R^3$, the SE (3) is giving as

$$\begin{bmatrix} \theta & b \\ 0 & 1 \end{bmatrix} \tag{3.42}$$

The Lie algebra associated with the Lie group SO(3), denoted by so(3), can be determined by evaluating the tangent vector to a smooth curve θ(*t*) on *SO*(3) where θ (0) = I , the identity. Differentiating both sides of θ (t) θ (t) T = I with respect to t and evaluating at t = 0 results in θ (0) + θ (0)*T* = 0.

Therefore the Lie algebra of SO (3), denoted by so (3), given in equation 3.43. It consists of a set of skew symmetric matrices on $R^{3x3}$ of the form;

$$[\omega] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{3.43}$$

where ω= ($\omega_x$, $\omega_y$, $\omega_z$ ) є $R^3$ and [.] is a cross operator which change the three dimensional vector into the associated skew symmetric matrix.

In a similar fashion, it can be shown in equation 3.44 that the Lie algebra associated with SE (3), denoted se (3), and consists of 4x4 matrix of the form;

$$\begin{bmatrix} \omega & v \\ 0 & 1 \end{bmatrix} \tag{3.44}$$

where ω є $R^3$ and $v$ є $R^3$.

### 3.5.3 Euclidean group

The displacement of a rigid body *B* can be described in reference frame {*A*}, by establishing a reference frame {*B*} on *B* and describing the position and orientation of {*B*} in {*A*} via a homogeneous transformation matrix:

$${}^{A}A_{B} = \begin{bmatrix} {}^{A}R_{B} & {}^{A}r^{o'} \\ 0_{1\times3} & 1 \end{bmatrix} \tag{3.45}$$

where ${}^{A}r^{O'}$ is the position vector of the origin *O'* of {*B*} in the reference frame {*A*}, and ${}^{A}R_{B}$ is a rotation matrix that transforms the components of vectors in {*B*} into components in {*A*}. As the composition of two displacements, from {*A*} to {*B*}, and from {*B*} {*C*}, is achieved by by matrix multiplication of ${}^{A}A_{B}$ and ${}^{B}A_{C}$. The set of all displacements or the set of all such matrices in Equation 3.45 with the composition rule is called SE (3), the special Euclidean group of rigid body displacements in three dimensions:

$$SE(3\{A\colon A = \begin{bmatrix} R & r \\ 0_{1\times3} & 1 \end{bmatrix}, R \in R^{3\times3}, r \in R^{3}, R^{T}R = RR^{T} = I \tag{3.46}$$

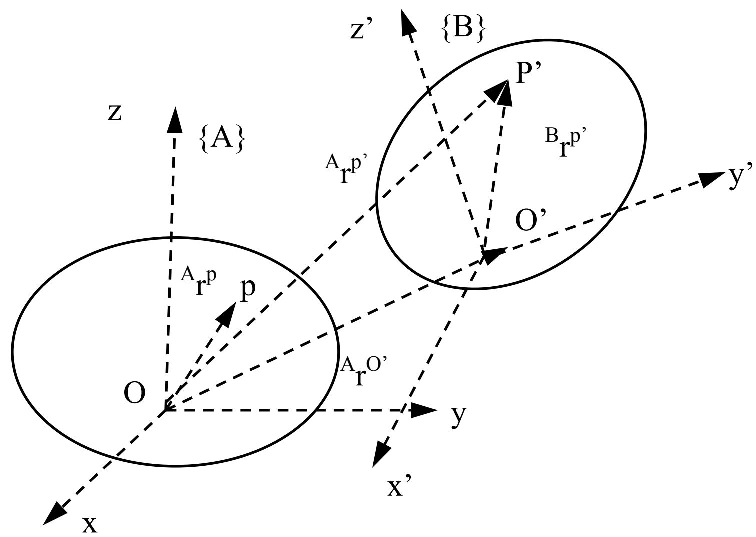



Figure 3.7: The rigid body displacement

In figure 3.7, the rigid body displacement from an initial position and orientation to a final position and orientation is shown. Here the body fixed reference frame is coincident with {*A*} in the initial position and orientation, and with {*B*} in its final position and orientation. The point *P* attached to the rigid body moves from P to P'. If we consider this set of matrices with the binary operation defined by matrix multiplication, it is easy to see that SE (3) satisfies the four axioms that must be satisfied by the elements of an algebraic group:

i) The set is closed under the binary operation. In other words, if A and B are any two matrices in SE(3), AB ϵ SE(3).

ii) The binary operation is associative. In other words, if A, B, and C are any three matrices ϵ SE(3), then (AB) C = A (BC).

iii) For every element A ϵ SE (3), there is an identity element given by the 4X4 identity matrix,

I ϵ SE (3), such that AI = A.

iv) For every element A ϵ SE (3) there is an identity inverse, $A^{-1}$ ϵ SE (3), such that A $A^{-1}$ = I.

Table 3.3: The important subgroups of SE (3)

| Subgroup | Notation | Definition | Significance |
|---|---|---|---|
| The group of rotations in three dimensions | SO(3) | The set of all proper orthogonal matrices. $SO(3) = \{R\ R \in R3\times3,\ RTR = RRT = I\}$ | All spherical displacements or the set of all displacements that can be generated by a spherical joint (S-pair). |
| Special Euclidean group in two dimensions | SE(2) | The set of all 3×3 matrices with the Structure $\begin{bmatrix} \cos\theta & \sin\theta & r_x \\ -\sin\theta & \cos\theta & r_y \\ 0 & 0 & 1 \end{bmatrix}$ where θ, rx, and ry are real numbers | All planar displacements or the set of displacements that can be generated by a planar pair (E pair). |
| The group of rotations in two dimensions | SO(2) | The set of all 2×2 proper orthogonal matrices. They have the structure $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ where θ is a real number. | All rotations in the plane, or the set of all displacements that can be generated by a single revolute joint (R-pair). |
| The group of translations in *n* dimensions. | T(n) | The set of all nX1 real vectors with vector addition as the binary operation. | All translations in *n* Dimensions, n = 2 indicates planar, n = 3 indicates spatial displacements. |
| The group of translations in one dimension. | T(1) | The set of all real numbers with addition as the binary operation. | All translations parallel to oneaxis, or the set of all displacements that can be generated by a single prismatic joint (P-pair). |
| The group of cylindrical displacements | SO(2)XT(1) | The Cartesian product of SO(2) and T(1) | All rotations in the plane and translations along an axis perpendicular to the plane, or the set of all displacements that can be generated by a cylindrical joint(C-pair). |
| The group of Screw displacement | H(1) | A one-parameter subgroup of SE(3) | All displacements that can be generated by a helical joint (H-pair). |

The product of any two elements in SE (3) is a continuous function of the two elements and the inverse of any element in SE (3) is a continuous function of that element. Thus SE (3) is a continuous group and any open set of elements of SE (3) has a 1-1 map onto an open set of $R^6$. Since SE (3) is a Lie group, it has many interesting properties that are of interest in screw system theory. In addition to the special Euclidean group in three dimensions, there are many other groups that are of interest in rigid body kinematics. They are all subgroups of SE (3).

A subgroup of a group consists of a collection of elements of the group which themselves form a group with the same binary operation. Some important subgroups are listed in table 3.3 and their significance in kinematics along with their properties are described below.

### 3.5.4 The group of rotations

A rigid body *B* is said to rotate relative to another rigid body *A*, when a point of *B* is always fixed in {A}. Let the frame {B} is attached so that its origin O' is at the fixed point. The vector ${}^{A}\mathbf{r}^{O'}$ is equal to zero in the homogeneous transformation in equation 3.7. The set of all such displacements, also called spherical displacements, can be easily seen to form a subgroup of SE (3). Only the 3×3 sub matrix of the homogeneous transformation matrix plays a role in describing rotations. Further, the binary operation of multiplying 4×4 homogeneous transformation matrices reduces to the binary operation of multiplying the corresponding 3×3 sub matrices. Thus 3×3 rotation matrices can be used to represent spherical displacements as in equation 3.47. This sub group, is called the special orthogonal group in three dimensions represented as

$$SO(3) = \left\{ \mathrm{R} \;\; \mathrm{R} \in R^{3\times 3}, \mathrm{R}^{T}\mathrm{R} = \mathrm{R}\mathrm{R}^{T} = \mathrm{I} \right\} \qquad (3.47)$$

The orthogonal matrices whose determinants are negative are excluded. It is well known that any rotation can be decomposed into three finite successive rotations, each about a different axis than the preceding rotation. The three rotation angles, called Euler angles, completely describe the given rotation. The basic idea is as follows. If we consider any two reference frames $\{A\}$ and $\{B\}$, and the rotation matrix ${}^{A}R_{B}$, we can construct two intermediate reference frames $\{M\}$ and $\{N\}$, so that

1. The rotation from {A} to {M} is a rotation about the *x* axis (of {A}) through $\psi$;
2. The rotation from {M} to {N} is a rotation about the *y* axis (of {M}) through $\phi$; and
3. The rotation from {N} to {B} is a rotation about the *z* axis (of {N}) through $\theta$.

Thus any rotation can be viewed as a composition of these three elemental rotations except for rotations at which the Euler angle representation is singular. These singularities are easily found out explicitly and identifying points at which the Euler angles are not unique. Note that we have chosen the so-called x-y-z representation for Euler angles, in which the first rotation is about the *x*-axis, the second about the *y*-axis and third about the *z*-axis. There are eleven other choices of Euler angle representations which can be derived by choosing different axes for the three elemental rotations. For any rotation, it is always possible to find a suitable non singular Euler angle representation.

This in turn means all rotations in an open neighborhood in SO(3) can be described by three real numbers. It can be shown that there is a 1-1, continuous map from SO (3) onto an open set in $R^3$. This gives SO (3) the structure of a three-dimensional differentiable manifold, and therefore a Lie group. The rotations in the plane, or more precisely rotations about axes that are perpendicular to a plane, form a subgroup of SO(3), and therefore sub group of SE(3). For the evidence let us take an example. In figure 3.7, the rigid bodies A and B are connected with a revolute joint whose axis is along the z axis. The homogeneous transformation matrix has the form:

$$ {}^{A}A_{B} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.48} $$

where $\theta$ is the angle of rotation. The equation 3.40 is composed of two such rotations, ${}^{A}\mathbf{A}_{B}$ and ${}^{B}\mathbf{A}_{C}$, through $\theta_1$ and $\theta_2$ respectively, the product is given by:

$$ {}^{A}A_B \times {}^{B}A_C = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & 0 & 0 \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.49} $$

All matrices in the equation 3.49 are the same periodic function of one real variable 'θ' given by:

$$ R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.50} $$

The equation 3.50 is similar as basic rotation matrix as discussed earlier. This subgroup is called SO (2). Further, since $\mathbf{R}(\theta_1) \times \mathbf{R}(\theta_2) = \mathbf{R}(\theta_1 + \theta_2)$, we can think of the subgroup as being locally isomorphic to $R^1$ with the binary operation being addition.

### 3.5.5 The group of translations

A rigid body *B* is to translated relative to another rigid body A by attaching reference frames to A and to B that are always parallel. The rotation matrix ${}^{A}R_B$ equals the identity in the homogeneous transformation in Equation 3.45.

The translation in group theory is represented as

$$ {}^{A}A_{B} = \begin{bmatrix} I_{3\times 3} & {}^{A}r^{o'} \\ 0_{1\times 3} & 1 \end{bmatrix} \tag{3.51} $$

The set of all such homogeneous transformation matrices is the group of translations in three dimensions and is denoted by T(3) . The composition of two translations ${}^{A}\mathbf{A}_{B}$ and ${}^{B}\mathbf{A}_{C}$, the product is given by:

$$ {}^{A}A_{B} \times {}^{B}A_{C} = \begin{bmatrix} I_{3\times 3} & {}^{A}r^{O'} \\ 0_{1\times 3} & 1 \end{bmatrix} \times \begin{bmatrix} I_{3\times 3} & {}^{B}r^{O''} \\ 0_{1\times 3} & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} I_{3\times 3} & {}^{A}r^{O'} + {}^{B}r^{O''} \\ 0_{1\times 3} & 1 \end{bmatrix} \tag{3.52} $$

Equation 3.51 and equation 3.52 represent the group of rotations in lie algebra. Only the $3\times 1$ vector part of the homogeneous transformation matrix plays a role in describing translations. Thus a element of T(3) is considered as a simple $3\times 1$ vector, ${}^{A}r^{O'}$. Since the composition of two translations is captured by simply adding the two corresponding $3\times 1$ vectors, ${}^{A}rO'$ and ${}^{B}r^{O''}$, we can define the subgroup T(3), as the real vector space $R^3$ with the binary operation being vector addition. Similarly, we can describe the two subgroups of T(3), T(1) and T(2), the group of translations in one and two dimensions respectively. Because they are subgroups of *T*(3), they are also subgroups of *T*(3). It is worth noting that T(1) consists of all translations along an axis and this is exactly the set of displacements that can be generated by connecting A and B with a single prismatic joint.

In figure 3.8, the rigid bodies A and B are connected by a revolute joint with the axis l and u is a unit vector along the axis and P is a point on the axis. O-x-y-z is the reference frame {A}. Thus the velocity of any point attached to *B* in frame {A} can be calculated by using the velocity operator. It consists of the angular velocity matrix of {*B*} and the velocity of the point o, both as seen in frame {*A*}.

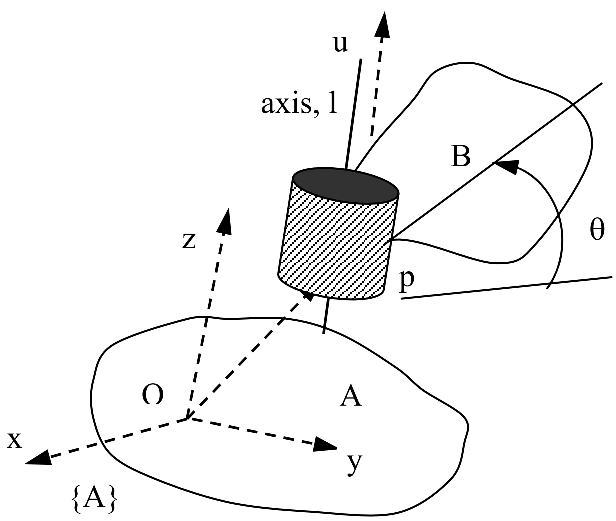



Figure 3.8: The rigid bodies *A* and *B* are connected by a revolute joint with the axis *l*.

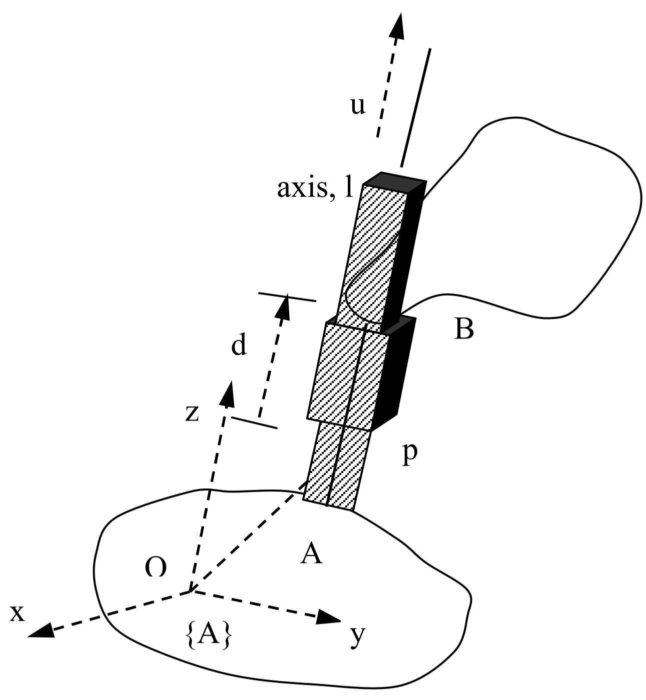



Figure 3.9: The rigid bodies A and *B* are connected by a prismatic joint with the axis l.

In Figure 3.9, the two rigid bodies A and B, are connected by a prismatic joint with an axis parallel to the line l. u is a unit vector along the axis and *P* is a point on the axis. O-xy- z is the reference frame {A}.

### 3.5.6 Lie algebra formulation for 6-DOF revolute robot

The Lie algebra of the SO (3) of the orientation matrices is the vector space of skew - symmetric 3X3 matrices. By using Lie algebra and group theory, a 3-D orientation vector can be represented satisfying the following criteria.

i) It has a one-to-one correspondence relation with the orthogonal orientation matrix ;

ii) It requires finite computations in all related transformations.

The orientation vector also known as the Rodriguez parameters is given by

$$s = \tan\frac{\psi}{2} n \tag{3.53}$$

where 'n' is the unit vector of the rotation axis and Ψ is the rotation angle. This orientation vector offers an almost full range of angle using quite simple computations. The vector is on the line which does not change when the orientation operates to a body in 3-D space, in other words the vector is an invariant under the change of orientation by so it is the unit eigenvector of the matrix R. The cross-product operator of a vector s= $[a, b, c]^T$ is the skew-symmetric matrix S given by

$$S = \vec{s} = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix} \tag{3.54}$$

All 3X3 skew-symmetric matrices over the real field span a linear space, so a Lie algebra can be defined in such a set and is referred as sk(3).

The Lie algebra of all real 3X3 skew-symmetric matrices can be transformed onto the group SO(3) and vice-versa, by

$$S = \frac{R - R^T}{1 + trR}$$

and

$$R = (I + S)(I - S)^{-1} \quad (3.55)$$

$$R(\vec{s}) = \frac{(1 - \vec{s}^2)I + 2\vec{s}.\vec{s} + 2\vec{s}}{1 + \vec{s}^2}$$

Being orthogonal, the matrix R satisfies the following relations:

$$1 + trR = \frac{4}{1 + \vec{s}^2} \quad (3.56)$$

$$R - R^T = \frac{4\vec{s}}{1 + \vec{s}^2} \quad (3.57)$$

The set of orientation vectors form a Lie group defined by a product '$\circ$' which is given by

$$\vec{s} = s_1 \circ s_2 = \frac{s_1 + s_2 + s_1 \times s_2}{1 - s_1.s_2} \quad (3.58)$$

The calculation of the orientation vector of the end-effector with respect to the base coordinate system is presented as following. The above formula will be used to transform the position vector, which points to the origin of a link frame with respect to its previous frame, to the base frame in order to determine the position of the end-effector.

The orientation of the frame'$t$' with respect to the frame (t-1) is composed by two simple rotations:

i) One rotation around the z-axis of the frame'$t$' by an angle $\theta_t$ is given by the equation

$$s_\theta = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tan \frac{\theta_t}{2} \tag{3.59}$$

ii) One rotation around the x-axis of the frame t-1 by an angle $\alpha_{(t-1)}$ is given by the equation

$$s_\alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \tan \frac{\alpha_{t-1}}{2} \tag{3.60}$$

In Lie algebra representation formulation of the kinematic equations of a n-link robot is based on assignment of a local coordinate system to every link and to the base of the robot identical with the afore mentioned homogeneous matrix method. After determination of kinematic parameters for the links 1 to n the vectors representing the orientation of the $t^{th}$ frame with respect to the $t$-$1^{th}$ frame is obtained using the group composition law which is given as follows.

$${}^{t-1}s_t = s_\alpha s_\theta$$

$${}^{t-1}s_t = \begin{bmatrix} \tan \frac{\alpha_{t-1}}{2} \\ \tan \frac{\theta_t}{2} \tan \frac{\alpha_{t-1}}{2} \\ \tan \frac{\theta_t}{2} \end{bmatrix} \tag{3.61}$$

Computation of the orientation vector ${}^{0}s_{t}$ can be done for 't' = 1to n of every link coordinate system to the base system, using the relation

$$ {}^{0}s_{t} = {}^{0}s_{1} \circ {}^{1}s_{2} \circ {}^{2}s_{3} \circ \ldots {}^{t-1}s_{t} \tag{3.62} $$

Where '$\circ$' lie group product operator defined in equation 3.58.

Transformation of the position vector ${}^{t}P_{t+1}$ representing the position of the frame t+1 with respect to frame t , for t=1 to t-1 into the base coordinate system, given by

$$ R({}^{0}s_{t})\ {}^{t}P_{t+1} \tag{3.63} $$

The position vector of the gripper $r_t$ is computed by following relation

$$ r_{t} = {}^{0}P_{1} + \sum_{t=1}^{n-1} (R^{0}s_{t}\ {}^{t}P_{t+1}) \tag{3.64} $$

where, ${}^{t}P_{t+1}$ is the fourth column of the homogeneous matrix given by equation 3.8 if the index t is replaced t+1. By using equation 3.53 to equation 3.64we can compute the position and orientation of any serial chain manipulator. The core of this lie algebra method is the determination of the vector given by equation 3.61 which plays an important role in kinematic synthesis of serial chain robots. The method offers a compact way to represent the orientation of the end-effector of a manipulator with respect to the base frame. However, the position of the end-effector has to be calculated through above formula given by equation 3.64, borrowing the determination of the position vector from the first method based on the homogeneous transformation.

### 3.6 Application of the New Methods in a 6-DOF Puma Robot

To illustrate the application of the algorithms and for better understanding of the physical significance of the transformation parameters, the kinematic equations for a 6-DOF revolute robot have been formulated. The local coordinate system and the parameters of each link have been defined according to D–H notation. The algorithms explained in above methods are implemented to a 6-DOF Puma robot. The schematic diagram with of the Puma robot with its joints is shown in figure.3.4 having joint parameters as given in table 3.4.

Table 3.4: Kinematic parameters of 6 DOF robot

| Kinematic parameters of 6 DOF robot | | | | |
|---|---|---|---|---|
| $t$ | $\theta_t$ | $\alpha_{t,t+1}$ | $L_{t,t-1}$ | $d_t$ |
| 1 | 90 | -90 | 0 | 0 |
| 2 | 0 | 0 | 431.8mm | 149.09mm |
| 3 | 90 | 90 | -20.32 | 0 |
| 4 | 0 | -90 | 0 | 433.07 |
| 5 | 0 | 90 | 0 | 0 |
| 6 | 0 | 0 | 0 | 56.25 |

Applying homogeneous matrix transformation matrix, the forward kinematic equation for the transformation of the frame 0 to 1 is given by;

$$ {}^{t-1}T_t = \begin{bmatrix} \cos\theta_t & 0 & \sin\alpha_t \sin\theta_t & a_t \cos\theta_t \\ \sin\theta_t & \cos\alpha_t \cos\theta_t & \sin\alpha_t \cos\theta_t & a_t \sin\theta_t \\ 0 & \sin\alpha_t & \cos\alpha_t & d_t \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_1 = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}T_2 = \begin{bmatrix} 1 & 0 & 0 & 431.8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 149.09 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}T_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -20.32 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Similarly for frames 4,5 and 6 transformation matrices can be calculated. The position and the orientation of the 6DOF Puma robot can be given as

$$ \mathrm{T}={}^{0}\mathrm{T}_1\times{}^{1}\mathrm{T}_2\times{}^{2}\mathrm{T}_3\times{}^{3}\mathrm{T}_4\times{}^{4}\mathrm{T}_5\times{}^{5}\mathrm{T}_6 $$

$$ = \begin{pmatrix} 0 & -1 & 0 & -149.09 \\ 0 & 0 & 1 & 921.12 \\ -1 & 0 & 0 & 20.32 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

In quaternion algebra method the calculation can be made by using equation 3.26 to equation 3.40, as follows.

For base coordinate,

$$t = 0$$

$$a_{01} = i$$

$$s_t = k$$

i) For joint 1 i.e. t=1

$$Q_1 = C_1 + S_1 k$$

$$a_{1,2} = C_1 i + S_1 j$$

$$Q_{1,2} = -a_{1,2}$$

$$s_2 = -S_1 i + C_1 j$$

$$P_1 = d_1 s_1 + L_1 a_{1,2} = 0$$

$$n_1 = a_{1,2} = j$$

$$o_1 = s_2 \times a_{1,2} = -k$$

$$a_1 = s_2 = -i$$

ii) For joint 2 i.e t=2,

Position of the end-effector is calculated by using equation 3.41.

$$P_2 = -149.09 i + 431.8 j$$

$$n_2 = a_{2,3} = j$$

$$o_2 = s_3 a_{2,3} = -k$$

$$a_2 = s_3 = -i$$

iii) For joint 3 i.e. t=3,

The position of the end effecter is calculated as

$P_3 = -149.09i + 431.8j + 20.32k$

Similarly the position of the end-effector is calculated for 4th, 5th and 6th joint as follows.

$$P_4 = -149.09i + 864.87j + 20.32k$$

$$P_5 = -149.09i + 864.87j + 20.32k$$

$$P_6 = -149.09i + 921.12j + 20.32k$$

Here it is observed that the position of the end effector is represented as last column of the homogeneous matrix for 6-DOF Puma robot is exactly same with that obtained in quaternion algebra method.

Following the third method that is the lie algebra , the formulation of the kinematic equations of the six degree of freedom robot starts by the calculation of the ${}^{t-1}s_t$ vectors by replacing the link parameters from table 3.4 in equation 61. Since the first axis is rotational the vector ${}^{t-1}s_t$ is calculated for joints t=1 to 6.

$$ {}^{0}s_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} $$

$$ {}^{1}s_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$
{}^{2}s_{3} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
$$

$$
{}^{3}s_{4} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}
$$

$$
{}^{4}s_{5} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

$$
{}^{5}s_{6} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

Using the Lie group product defined in equation 58, the orientation vector of the end-effector with respect to the base frame is calculated as follows.

$$
{}^{0}s_{5} = {}^{0}s_{1} \circ {}^{1}s_{2} \circ {}^{2}s_{3} \circ {}^{3}s_{4} \circ {}^{4}s_{5} \circ {}^{5}s_{6}
$$

The orientation matrix of the end-effector frame with respect to the base frame is obtained by equation 3.55.

$$
{}^{0}R_{6} = \begin{pmatrix} c\phi c\theta & s\phi s\alpha - c\phi s\theta c\alpha & c\phi s\theta c\alpha + s\phi s\alpha \\ s\theta & c\theta c\alpha & -c\theta s\alpha \\ -s\phi c\theta & s\phi s\theta c\alpha & c\phi c\alpha - s\phi s\theta s\alpha \end{pmatrix}
$$

In order to determine the position of the end effector, first the position vectors for t=1 to 6, are determined by replacing the link characteristics from table 3.4 as given below.

$$
{}^{t}P_{t+1} = \begin{bmatrix} L_t \\ -\sin\alpha_t d_{t+1} \\ \cos\alpha_t \end{bmatrix}
$$

Then by using equation 3.64, the coordinates of the origin of the end effector frame are determined.

### 3.7 Summary

In this chapter homogeneous algebra, quaternion algebra and lie algebra are extensively discussed along with their history, development and application. The mathematical models were developed and equations are presented for kinematic representation of the robot arm. An example problem of 6-DOF revolute robot was considered and solved by using aforementioned methods. The results were calculated and presented for each method by manually as well as using MATLAB software.

# Chapter-IV

# RESULTS AND DISCUSSIONS

# CHAPTER 4

## Results and Discussions

### 4.1 General

The formulation of the problem using the three different methods viz. i) homogeneous transform, ii) quaternion algebra, and iii) lie algebra were made as described in Chapter 3. The solutions to the problems were made through programs developed in MATLAB and the programs are listed in Appendix. The results in terms of figures for the position and orientation of the robot manipulator are noted and the computation time along with the storage requirement for the computational operations is observed.

### 4.2 Results of Homogeneous Transformation Method

The following comments on the storage and computational time requirements may help to choose the most appropriate method for his or her problem. A comparative study of the presented methods is done and illustrated in this section. The position and orientations were computed by using all the three methods and results were presented.

From section 3.7, the final position and orientation of 6-dof puma robot is obtained as;

$$T={}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_3 \times {}^{3}T_4 \times {}^{4}T_5 \times {}^{5}T_6 = \begin{pmatrix} 0 & -1 & 0 & -149.09 \\ 0 & 0 & 1 & 921.12 \\ -1 & 0 & 0 & 20.32 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In the homogeneous transformation method, four trigonometric function calls and six multiplications are required for calculation of the transformation matrix ${}^{t-1}T_{t}$. The multiplication of two 4 X 4 transformation matrices needs 48 multiplications and 36 additions and subtraction, since the elements of the last row of the matrix are constants. In n-link robot arm the number of transformation matrices is n, so n-1 number of matrix products is required in order to determine the total transformation matrix.

### 4.3 Results of Quaternion Algebra Method

From section 3.7, by using quaternion algebra formulation the final position of 6 dof puma robot is obtained as;

$$P_6 = -149.09i + 921.12j + 20.32k$$

The orientation is obtained from equation 3.41 as; -1,1,-1 which exactly matches with the homogeneous method.

### 4.4 Results of Lie Algebra Method

The results derived by using lie algebra method from equation 3.64 is

x = -149.09

y = 921.12

z = 20.32 and orientation is also same with the above two methods.

### 4.5 Discussion

In both homogeneous transformation and Lie algebra algorithms, there is a need for storing the transformation matrix or the orientation vector of every coordinate system with respect to its previous one from the beginning. In screw theory via the dual quaternions method, the storage cost is minimum because it is not necessary to store all the transformation quaternions from the beginning. Quaternions are calculated from the unit line vectors in an iterative way (see chapter 3). The dual quaternion requires eight memory locations, while the orientation vector requires three and the homogeneous matrix twelve memory locations. However, there is an inherent redundancy in line transformation methods since the Plucker coordinates of a line are six, while the independent coordinates for a line definition are four. The storage requirement affects the computational time because the cost of fetching an operand from memory exceeds the cost of performing a basic arithmetic operation. The complexity of the geometric design problem increases with the number of structural parameters. Four independent parameters define the axis of a revolute joint and two define a prismatic joint.

In the homogeneous transformation method for the determination of the end effector position and orientation needs (48(n-1) + 6n) multiplications and 36(n-1) additions and subtractions, while only the orientation needs 31(n-1) multiplications and 18(n-1) additions and subtractions, where ‘n’ is the number of degree of freedom. In quaternion representation the total number of addition and subtraction required is 22n+3 and the total number of multiplication and division required is 39n-12.

Table 4.1: Computational operations for addition

| DOF | No of additions | | |
|---|---|---|---|
| | Homogeneous matrix | Quaternion algebra | Lie algebra |
| 3 | 72 | 69 | 81 |
| 4 | 108 | 91 | 114 |
| 5 | 144 | 113 | 147 |
| 6 | 180 | 135 | 180 |
| 7 | 216 | 157 | 213 |
| 8 | 252 | 179 | 246 |
| 9 | 288 | 201 | 378 |
| 10 | 324 | 223 | 312 |

Table 4.1 presents the number of addition operations required in the three methods considered for computation of the desired parameters. This can be used to make an analysis for the computational cost of all the three methods with respect to addition and/or subtraction. The above data are shown in figure 4.1 which clearly speaks about the efficiency of each method.

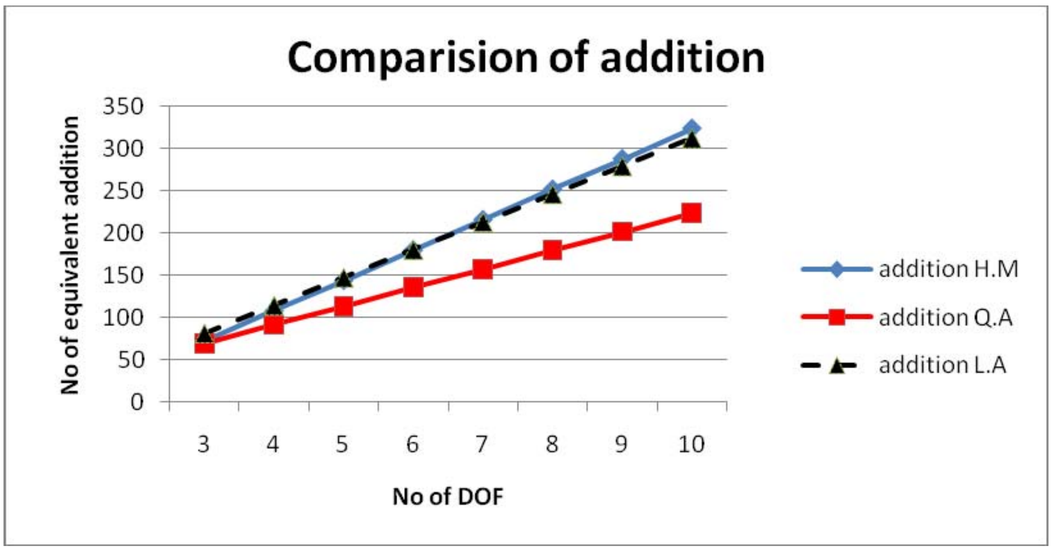



Figure 4.1: Comparison of addition

Table 4.2: Computation cost for multiplication

| DOF | Number of multiplication | | |
|---|---|---|---|
| | Homogeneous matrix | Quaternion algebra | Lie algebra |
| 3 | 114 | 105 | 126 |
| 4 | 168 | 144 | 162 |
| 5 | 222 | 183 | 198 |
| 6 | 276 | 222 | 234 |
| 7 | 330 | 261 | 270 |
| 8 | 384 | 300 | 306 |
| 9 | 438 | 339 | 342 |
| 10 | 492 | 378 | 378 |

Table 4.2 presents the number of multiplication operations required in the three methods considered for computation of the desired parameters. For the case of 6DOF robot (n=6), 276 multiplications and 180 additions and subtractions are necessary for homogeneous matrix method. In summary, (22n + 3) additions and subtractions and (36n + 6) multiplications are required in order to determine the position and orientation of the end-effector by the quaternion method. For the case of 6-DOF manipulator 222 multiplications and 135 additions and subtractions are necessary.

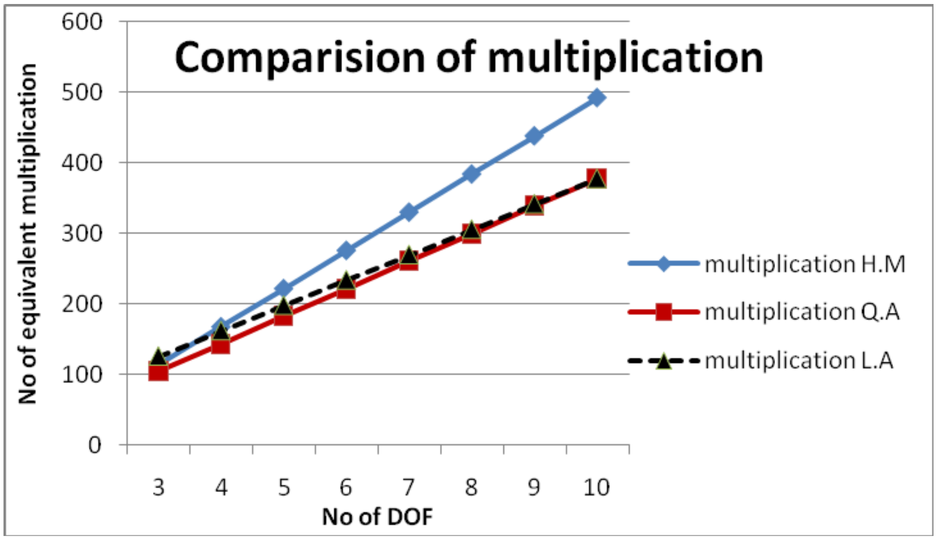



Figure 4.2: Comparison of multiplication

This speaks about the number of mathematical operations required for computing the homogeneous matrix and hence the time needed for the same. In lie algebra representation the number of addition and subtraction is 33n-18 and number of multiplication and division is 36n+18. For 6 DOF (n=6), 234 (table 4.1) number of equivalent multiplications and 180 additions and subtractions are necessary. So no of addition is same in homogeneous and lie algebra methods. This is evident in the figure 4.2, but again when the degree of freedom increases lie algebra method requires less no of additional computation as seen in the downward deviation of the line from the homogeneous line in the graph.

The observation made from the models and the subsequent solution prompt a comparison of the three methods. From the algorithm, mentioned in section 3.5, it can be seen that all the necessary unit vectors $a_{t,t+1}$ and $s_t$ are determined successively in a loop with $t$ = 1 to $n$. In every step of this loop two main operations are performed. The first one is the determination of the transformation quaternion $\hat{Q}_t$ and $\hat{Q}_{t+1}$ using equation 3.36 and equation 3.37 respectively. For determination of each of these 3 multiplications are needed. The second operation is the quaternion product used to determine the unit line vectors $a_{t,t+1}$ and $s_t$ from equation 3.34 and equation 3.38. Here the vector s and a are known as quaternions with zero scalar part. The quaternion product needs 8 additions and subtractions and 12 multiplications. After the ending of this loop, the position vector of the end-effector is determined by adding the *n* position vectors of every joint in the open kinematic chain of the robot as represented in equation 3.37. To determine the position vector of every joint and to add it to the previous one, 6 additions and subtractions and 6 multiplications are needed. The first and last column of the orientation matrix of the end-effector is known from the unit vectors $\mathrm{a}_{t,t+1}$ and $\mathrm{s}_{t+1}$ . The determination of the second column of the orientation matrix, which is the cross product of the other two columns, needs 3 additions and subtractions and 6 multiplications.

To facilitate the comparison, it is supposed that the computational time to perform an addition is half of the time required for one multiplication. A comparison of the number of mathematical operations required for computation of end-effector position using the two methods is presented. It is clear from the above comparison along with the graphical illustration, that the number of operations is almost same with manipulators having less DOF. As the number of DOF goes on increasing and the complicacies of computations increase, the quaternion method scores significantly better than the homogeneous matrix method and lie algebra method as is evident in the graph presented in Figure 4.2. It is obvious that for manipulators with more than three degrees of freedom, the quaternion theory based algorithm requires less computational time than the traditional homogeneous algorithms. Quaternions require eight memory locations for the representation of position while three memory locations for orientation. But the homogeneous method requires 16 memory locations for both position and orientation. The storage requirement affects the computational time as the cost of fetching an operand from memory exceeds the cost of performing a basic arithmetic operation. Therefore, it can be concluded that for manipulators with higher number of DOF the dual quaternion theory method is more cost effective than the homogeneous transformation and lie algebra method. Further, in dual quaternion method the storage cost is low because it is not necessary to store all the transformation quaternions from the beginning.

The majority of applications of quaternions involve pure rotations, for this we restrict the quaternions to those with unit magnitude and we use only multiplications and not addition to represent a combination of different rotations. When quaternions are normalised in this way, together with the multiplication operation to combine rotations, form a mathematical group, in this case SU(2).One can use this to do lots of operations which are required in practical applications such as, combining subsequent rotations (and equivalently orientations), interpolating between them, etc.

When quaternions are used in this way one can think of them as being similar to axis-angle except that real part is equal to cos(angle/2) and the complex part is made up of the axis vector times sin(angle/2). It is quite difficult to give a physical meaning to a quaternion, and many people find this similarity to axis-angle as the most intuitive way to think about it, others may just prefer to think of quaternions as an interesting mathematical system which has the same properties as 3D rotations.

Given a tiny rotation one can represent it as three Euler angles a, b, c, all of which are tiny. Considering a, b and c as forming a vector [a,b,c], apart from an even smaller error, multiplication of rotations becomes ordinary addition of vectors and the order of rotations isn't significant. But if one chooses not to ignore this small error it is seen that a rotation represented by u and a rotation represented by v don't quite commute and the order does matter. The size of this error is measured by the cross product of u and v. This is intuitively plausible, one would expect that rotations defined by vectors in a similar direction would be closer to commuting, and this is reflected in the fact that the cross product is zero for parallel vectors.

## 4.6 Summary

The vector parameterization through the Lie algebra facilitates the definition the robot tool orientation in a task-oriented manner. The Euler angles parameterization is not convenient, since the definition of the orientation is sequential, so it is very difficult to define the orientation or more difficult to interpolate between two successive orientations of the robot end-effector. Since quaternion represents explicitly the axis of rotation, by using quaternions the axis of rotation is constant and the angle of rotation interpolated in a linear mode. Among the difficulties in representation of the orientation of a body is the one due to the fact that the angular velocity is not the time derivative of some vector representing the orientation of the body, as happens in the case of position definition. The vector parameter representation of the orientation proved very convenient for the definition of the error and time derivative of the orientation of a robot end-effector. In some tasks such as painting or arc welding, it seems natural and geometrically easy to define the desired orientation and position of the axis of the robot tool with respect to the spatial path, by simply defining a line. Therefore, the line transformation methods have to be investigated for trajectory generation in such cases. Generally speaking, despite the fact that a line presentation needs an extra degree of freedom compared to the point presentation, it is often useful to analyze spatial manipulators in terms of lines since lines present physically rotational axis of robot tools. Using screw theory, the position/velocity of the end-effector can be defined by a motor and the determination of the Jacobean matrix is straightforward.

Despite the advantages of the line-oriented methods are not well-suited for efficient kinematic computations or real-time control applications mainly because of the fact that computational cost advantages and important details of key algorithms are too complex to be well understood by the robotics community. One of the aims of the present work is to contribute in the clarification of these methods, in order to help in wider utilization of these methods in the robotics community.

# Chapter-V

# CONCLUSION AND FUTURE SCOPE

# CHAPTER 5

## Conclusion and Future Scope

### 5.1 General

In the present work homogeneous transformation, quaternion algebra, lie algebra and their geometrical significance has been studied. The mathematical formulations for the forward kinematic solution of open link revolute robot having six joints are done corresponding to all the methods. This work introduces a new formulation for the kinematic synthesis of open link robots having six joints. The standard kinematic equations of the chain are transformed into successive quaternion transformation and then expressed using dual quaternion. It is evident from the results that a matrix product requires many more operations compared to quaternion product. A lot of time can be saved and at the same time more numerical accuracy can be preserved with quaternion than with matrices. From the example mentioned, it is clear that quaternion algebra provides a very effective and efficient method over other two methods for representation of forward kinematics equation. Further, the method is cost effective as compared to matrix method and lie algebra method as it requires less computer memory and saves lot of time by reducing the number of mathematical calculations.

Further, the method is cost effective as it requires less computer memory and saves lot of time by reducing the number of mathematical calculations. Comparing the quality of results obtained from the two methods, it is observed that the quaternion method gives exactly same result as that of homogeneous method. This is a general method applied specifically to robot manipulator in the present work. However this can also be extended to any other open kinematic chain for the purpose of kinematic analysis. Therefore this can be used as a powerful tool in the solution of kinematic problems in general.

The screw theory and Lie algebra-based methods offer a more compact and consistent way for the definition of the end-effector than the homogeneous transformation one. The explanation of the physical meaning of the parameters and operations in screw theory and Lie algebra shows that the intuitive understanding of the orientation definition it is not a hard task. Therefore, the wider use of these methods into the robotics community has to be considered.

It is worth to perform, in the future, a comparative study of these methods in the velocity and dynamic analysis of the robot manipulators. In addition, the advantages of the trajectory generation based on screw theory have to be investigated.

## 5.2 Remarks on Formulation, Convenience and Applicability

The homogeneous matrix method is widely used and accepted for robotics application. But quaternion algebra becomes an effective method for robots having DOF more than four. The efficiency of Lie algebra is midway between homogeneous matrix method and quaternion algebra method and again the mathematical formulation is difficult than the quaternion algebra method. In summary it can be concluded that quaternion algebra method can be used as most effectively way for higher DOF robots. Presently there are many quaternion applications in the area of aerospace sequence, spherical trigonometry, calculus for kinematics and dynamics, rotation in phase space etc.

## 5.3 Scope for Future Work

The formulation done in this work is limited to robot kinematics only but it can be also extended into robot dynamics involving computation of velocity, acceleration and force related to joints. This can be also applicable for the kinematics of simple mechanisms. The methods discussed in this work can be compared with other standard methods like neural network, genetic algorithm for suitability of application.

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# APPENDIX

# Appendix

### A) Computer Program for representation of position and orientation of 6-dof revolute robot, using homogeneous matrix method

```
A=[-160 -54.4 160;-225 135 45;-45 45 225;-110 -16.7 170;-100 -33.34 100;-266 -88.7 266];
[m,n]=size(A);
C=[];
S=[];
a=[0 431.8 -20.32 0 0 0];
d=[0 149.09 0 433.07 0 56.25];
for i=1:n
   A1=A(:,i);
   for j=1:m
      c1(j)=cos(A1(j));
      s1(j)=sin(A1(j));
      a1=A1(2);
      a2=A1(3);
      C23(i)=cos(a1+a2);
      S23(i)=sin(a1+a2);
   end
   C=[C;c1];
   S=[S;s1];
end
T=[];
for i=1:n
   c=C(i,:);
   c23=C23(i);
   s23=S23(i);
   s=S(i,:);
   nx(i)=c(1)*[c23*(c(4)*c(5)*c(6)-s(4)*s(6))-s23*s(5)*c(6)]-s(1)*((s(4)*c(5)*c(6)+c(4)*s(6)));
   ny(i)=s(1)*[c23*(c(4)*c(5)*c(6)-s(4)*s(6))-s23*s(5)*c(6)]+c(1)*(s(4)*c(5)*c(6)+c(4)*s(6));
   nz(i)=-s23*[c(4)*c(5)*c(6)-s(4)*s(6)]-c23*s(5)*c(6);
   sx(i)=c(1)*[-c23*(c(4)*c(5)*c(6)+s(4)*c(6))+s23*s(5)*s(6)]-s(1)*(-s(4)*c(5)*s(6)+c(4)*c(6));
   sy(i)=s(1)*[-c23*(c(4)*c(5)*s(6)+s(4)*c(6))+s23*s(5)*s(6)]+c(1)*(-s(4)*c(5)*s(6)+c(4)*c(6));
   sz(i)=s23*(c(4)*c(5)*s(6)+s(4)*c(6))+c23*s(5)*s(6);
   ax(i)=c(1)*(c23*c(4)*s(5)+s23*c(5))-s(1)*s(4)*s(5);
   ay(i)=s(1)*(c23*c(4)*s(5)+s23*c(5))+c(1)*s(4)*s(5);
   az(i)=-s23*c(4)*s(5)+c23*c(5);
   px(i)=c(1)*[d(6)*(c23*c(4)*s(5)+s23*c(5))+s23*d(4)+a(3)*c23+a(2)*c(2)]-
s(1)*(d(6)*s(4)*s(5)+d(2));

py(i)=s(1)*[d(6)*(c23*c(4)*s(5)+s23*c(5))+s23*d(4)+a(3)*c23+a(2)*c(2)]+c(1)*(d(6)*s(4)*s(5)+d(2))
;
   pz(i)=d(6)*(c23*c(5)-s23*c(4)*s(5))+c23*d(4)-a(3)*s23-a(2)*s(2);
   T_1=[nx(i) sx(i) ax(i) px(i);ny(i) sy(i) ay(i) py(i);nz(i) sz(i) az(i) pz(i);0 0 0 1];
   T=[T T_1];
end
```

```
    T1=T(:,1:4);
    T2=T(:,5:8);
    T3=T(:,9:end);
```

**B) Computer Program for representation of position and orientation of 6-dof revolute robot, using quaternion algebra method**

```
%%%%%%% position vector%%%%
clc
clear all
close all
d = [0 149.09 0 433.07 0 56.25];
for n = 1:6
    theta(n) = input('Enter the joint angle between in radian:');
end
twistangle = [-90 0 90 -90 90 0]*pi/180;
link_length = [0 431.8 -20.32 0 0 0];
a = cell(1,7);
q = cell(1,6);
p = cell(1,6);
a{1} = [1 0 0];
s{1} = [0 0 1];
q{1} = [0 0 1];
for m = 1:6
    if abs(cos(theta(m))) < 0.01
        theta1 = round(cos(theta(m)));
    else
        theta1 = cos(theta(m));
    end
    if abs(sin(theta(m))) < 0.01
        theta2 = round((sin(theta(m))));
    else
        theta2= sin(theta(m));
    end
    if abs(cos(twistangle(m)))< 0.01
        twist1 = round(cos(twistangle(m)));
    else
        twist1 = cos(twistangle(m));
    end
    if abs(sin(twistangle(m)))< 0.01
        twist2 = round(sin(twistangle(m)));
    else
        twist2 = sin(twistangle(m));
    end
    a{m+1} = theta1*a{m}+ theta2*cross(s{m},a{m});
    s{m+1} = twist1*s{m}+twist2*cross(a{m+1},s{m});
    p{m} = d(m)*s{m}+ link_length(m)*a{m+1};
    if m>1
%       iter = m;
%       while iter > 1
          p{m} = p{m}+p{m-1};
%           iter = iter -1;
```

```
%        end
   end
end

%%% finding orientation vector pn%
 Pn = p{1}+p{2}+p{3}+p{4}+p{5}+p{6};
```