

# **Multi-Objective Optimization Problem using Grey Taguchi Method**

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**CERTIFICATE**

This is to certify that the project entitled “**Multi Objective Optimization Problem using Grey Taguchi Method**” submitted by Rohit Anil Pathak in partial fulfillment of the requirements for the awards of Bachelor of Technology, NIT Rourkela (Deemed university) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge the matter embodied in the project has not been submitted to any Institute/University for the award of any degree or diploma.

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***Multi Objective Optimization Problem using Grey Taguchi Method***

## ***Abstract***

This study investigated the optimization of CNC turning operation parameters for Copper using the Grey relational analysis method. In turning process parameters such as cutting tool geometry and materials, depth of cut, feed rates, cutting speeds as well as the use of cutting fluids will impact the MRR and machining properties like surface roughness. The controllable input parameters were Speed (RPM), Feed (mm/rev) and Depth of Cut (mm). Twenty Seven experimental runs based on an orthogonal array of Taguchi method were performed. The properties of Surface Finish and Material Removal Rate were selected as the quality targets or the response variables. An optimal parameter combination of the turning operation was obtained via Grey relational analysis. By analyzing the Grey relational grade matrix, the degree of influence for each controllable process factor onto individual quality targets can be found. The optimal parameter combination is then tested for accuracy of conclusion with a test run using the same parameters.

## ***Introduction***

**Six Sigma** is a business management strategy, initially implemented by Motorola, which today enjoys widespread application in many sectors of industry.

Six Sigma seeks to improve the quality of process outputs by identifying and removing the causes of defects (errors) and variation in manufacturing and business processes. It uses a set of quality management methods, including statistical methods, and creates a special infrastructure of people within the organization ("Black Belts" etc.) who are experts in these methods. Each Six Sigma project carried out within an organization follows a defined sequence of steps and has quantified financial targets (cost reduction or profit increase).

Six Sigma was originally developed as a set of practices designed to improve manufacturing processes and eliminate defects, but its application was subsequently extended to other types of business processes as well. In Six Sigma, a defect is defined as anything that could lead to customer dissatisfaction.

The particulars of the methodology were first formulated by Bill Smith at Motorola in 1986. Six Sigma was heavily inspired by six preceding decades of quality improvement methodologies such as quality control, TQM, and Zero Defects, based on the work of pioneers such as Shewhart, Deming, Juran, Ishikawa, Taguchi and others.

Like its predecessors, Six Sigma asserts that –

- Continuous efforts to achieve stable and predictable process results (i.e. reduce process variation) are of vital importance to business success.
- Manufacturing and business processes have characteristics that can be measured, analyzed, improved and controlled.
- Achieving sustained quality improvement requires commitment from the entire organization, particularly from top-level management.

Features that set Six Sigma apart from previous quality improvement initiatives include –

- A clear focus on achieving measurable and quantifiable financial returns from any Six Sigma project.
- An increased emphasis on strong and passionate management leadership and support.
- A special infrastructure of "Champions," "Master Black Belts," "Black Belts," etc. to lead and implement the Six Sigma approach.
- A clear commitment to making decisions on the basis of verifiable data, rather than assumptions and guesswork.

The term "Six Sigma" is derived from a field of statistics known as process capability studies.

Originally, it referred to the ability of manufacturing processes to produce a very high proportion of output within specification. Processes that operate with "six sigma quality" over the short term are assumed to produce long-term defect levels below 3.4 defects per million

opportunities (DPMO). Six Sigma's implicit goal is to improve all processes to that level of quality or better.

Six Sigma is a registered service mark and trademark of Motorola, Inc. Motorola has reported over US\$17 billion in savings from Six Sigma as of 2006.

Other early adopters of Six Sigma who achieved well-publicized success include Honeywell (previously known as AlliedSignal) and General Electric, where the method was introduced by Jack Welch. By the late 1990s, about two-thirds of the Fortune 500 organizations had begun Six Sigma initiatives with the aim of reducing costs and improving quality.

In recent years, Six Sigma has sometimes been combined with lean manufacturing to yield a methodology named Lean Six Sigma.

Short-term sigma levels correspond to the following long-term DPMO values (one-sided):

- 1 sigma = 690,000 DPMO = 31% efficiency
- 2 sigma = 308,000 DPMO = 69.2% efficiency
- 3 sigma = 66,800 DPMO = 93.32% efficiency
- 4 sigma = 6,210 DPMO = 99.379% efficiency
- 5 sigma = 230 DPMO = 99.977% efficiency
- 6 sigma = 3.4 DPMO = 99.9997% efficiency

These figures assume that the process mean will shift by 1.5 sigma towards the side with the critical specification limit some time after the initial study determining the short-term sigma level. The figure given for 1 sigma, for example, assumes that the long-term process mean will be 0.5 sigma *beyond* the specification limit, rather than 1 sigma *within* it, as it was in the short-term study.

Six Sigma has two key methods: DMAIC and DMADV, both inspired by Deming's Plan-Do-Check-Act Cycle. DMAIC is used to improve an existing business process; DMADV is used to create new product or process designs.

## **DMAIC**

The basic method consists of the following five steps:

- *Define* high-level project goals and the current process.
- *Measure* key aspects of the current process and collect relevant data.
- *Analyze* the data to verify cause-and-effect relationships. Determine what the relationships are, and attempt to ensure that all factors have been considered.
- *Improve* or optimize the process based upon data analysis using techniques like Design of experiments.
- *Control* to ensure that any deviations from target are corrected before they result in defects. Set up pilot runs to establish process capability, move on to production, set up control mechanisms and continuously monitor the process.

## **DMADV**

The basic method consists of the following five steps:

- *Define* design goals that are consistent with customer demands and the enterprise strategy.
- *Measure* and identify CTQs (characteristics that are **Critical To Quality**), product capabilities, production process capability, and risks.
- *Analyze* to develop and design alternatives, create a high-level design and evaluate design capability to select the best design.
- *Design* details, optimize the design, and plan for design verification. This phase may require simulations.
- *Verify* the design, set up pilot runs, implement the production process and hand it over to the process owners.

DMADV is also known as DFSS, an abbreviation of "**D**esign **F**or **S**ix **S**igma".

### ***Multi-Objective Optimization Problem***

In modern industry the goal is to manufacture low cost, high quality products in a short time.

Automated and flexible manufacturing systems for that purpose along with computer numerical control machines are capable of achieving very high accuracy and very low processing time. Furthermore, in order to produce any product with desired quality by machining, cutting parameters should be selected properly.

In turning process parameters such as cutting tool geometry and materials, the materials, the depth of cut, feed rates, cutting speeds as well as the use of cutting fluids will impact the MRR and machining qualities like surface roughness.

Planning the experiments through the Taguchi method has been quite successfully implemented in process optimization. Therefore the study intends to apply the Taguchi method to plan the experiments on a turning operation.

The study is to investigate the optimization of CNC turning operation parameters using the *Grey Relational Analysis* method. The optimum parameters to obtain the best surface finish need to be ascertained for this specific turning operation.

Here we shall employ statistical methods to a turning operation. There are a lot of different parameters affecting surface finish and material removal rate like speed, feed and depth of cut. The parameters have to be controlled in a way such that DPMO (Defect per million output) is around *three* and the six sigma process is obtained. We therefore will find out the best parameter settings in a CNC turning operation.

The surface properties of roughness average and material removal rate (MRR) are selected as quality targets or the response variables.

The controlled parameters are Speed, feed and depth of cut. The response variables with respect to the controlled parameters behave as indicated below.

Surface finish increases if a.) Feed increases b.) Depth of cut decreases c.) Speed decreases.

Material Removal Rate increases if a.) Feed increases b.) Depth of cut increases c.) Surface finish increases, and vice versa.

An optimal parameter combination of the turning operation will be found out via *the Grey Relational Analysis*. By analyzing the *Grey* relational matrix, the degree of influence for each controllable factor onto each quality target can also be ascertained.

### ***Grey Relational Analysis***

#### 1. Data Preprocessing

Grey data processing must be performed before Grey correlation coefficients can be calculated.

A series of various units must be transformed to be dimensionless. Usually, each series is normalized by dividing the data in the original series by their average.

Let the original reference sequence and sequence for comparison be represented as  $x_0(k)$  and  $x_i(k)$ ,  $i=1, 2, \dots, m$ ;  $k=1, 2, \dots, n$ , respectively, where  $m$  is the total number of experiment to be considered, and  $n$  is the total number of observation data. Data preprocessing converts the original sequence to a comparable sequence. Several methodologies of preprocessing data can be used in Grey relation analysis, depending on the characteristics of the original sequence (Deng, 1989; Gau et al., 2006; You et al., 2007).

If the target value of the original sequence is “the-larger-the-better”, then the original sequence is normalized as follows.

$$x_i^*(k) = \frac{x_i^{(O)}(k) - \min . x_i^{(O)}(k)}{\max . x_i^{(O)}(k) - \min . x_i^{(O)}(k)}$$

If the purpose is “the-smaller-the-better”, then the original sequence is normalized as follows.

$$x_i^*(k) = \frac{\max . x_i^{(O)}(k) - x_i^{(O)}(k)}{\max . x_i^{(O)}(k) - \min . x_i^{(O)}(k)}$$

However, if there is “a specific target value”, then the original sequence is normalized using,

$$x_i^*(k) = 1 - \frac{|x_i^{(O)}(k) - OB|}{\max . \left\{ \max . x_i^{(O)}(k) - OB, OB - \min . x_i^{(O)}(k) \right\}}$$

Alternatively, the original sequence can be normalized using the simplest methodology that is the values of the original sequence can be divided by the first value of the sequence,  $x_i^{(O)}(1)$  (k)

$$x_i^*(k) = \frac{x_i^{(O)}(k)}{x_i^{(O)}(1)}$$

Where,  $x_i^{(o)}(k)$  is the original sequence,  $x_i^*(k)$ , the sequence after data preprocessing,  $\max, x_i^{(o)}(k)$ , the largest value of  $x_i^{(o)}(k)$  and  $\min, x_i^{(o)}(k)$ , the smallest value of  $x_i^{(o)}(k)$ .

## 2. Grey Relational Coefficients and Grey Relational Grades

Following the data preprocessing, a Grey relational coefficient can be calculated using the preprocessed sequences. The Grey relational coefficient is defined as follows.

$$\gamma(x_0^*(k), x_i^*(k)) = \frac{\Delta_{\min.} + \zeta \Delta_{\max.}}{\Delta_{0i}(k) + \zeta \Delta_{\max.}} \quad 0 < \gamma(x_0^*(k), x_i^*(k)) \leq 1$$

$\Delta_{0i}(k)$  is the deviation sequence of reference sequence  $x_0^*(k)$  and  $x_i^*(k)$  namely,

$$\Delta_{0i}(k) = |x_0^*(k) - x_i^*(k)|$$

$$\Delta_{\max.} = \max_{j \in i} \max_k |x_0^*(k) - x_j^*(k)|$$

$$\Delta_{\min.} = \min_{j \in i} \min_k |x_0^*(k) - x_j^*(k)|$$

A Grey relational grade is a weighted sum of the Grey Relational Coefficients, and is defined as follows,

$$\gamma(x_0^*, x_i^*) = \sum_{k=1}^n \beta_k \gamma(x_0^*(k), x_i^*(k)) \quad , \quad \sum_{k=1}^n \beta_k = 1$$

Here, the Grey relational grade represents the level of correlation between the reference and comparability sequences. If the two sequences are identical, then the value of the Grey relational grade equals to one. The Grey relational grade also indicates the degree of influence exerted by the comparability sequence on the reference sequence.

Consequently, if a particular comparability sequence is more important to the reference sequence than other comparability sequences, the Grey relational grade for that comparability sequence and the reference sequence will exceed that for other Grey relational grades. The Grey relational analysis is actually a measurement of the absolute value of data difference between the sequences, and can be used to approximate the correlation between the sequences.

## ***Experimental Procedure and test results***

### *Materials*

Copper is easily worked, being both ductile and malleable. The ease with which it can be drawn into wire makes it useful for electrical work in addition to its excellent electrical properties.

Copper can be machined, although it is usually necessary to use an alloy for intricate parts, such as threaded components, to get really good machinability characteristics. Good thermal conduction makes it useful for heat sinks and in heat exchangers. Yield stress of copper is 55-330 MPa and a Young's Modulus of 110-128 GPa and a hardness of 40 HRC. In this study, to properly control the depth of cut, the diameter of the work pieces has been fixed at 20mm for copper bars. Kerosene was used as a coolant for the operation.

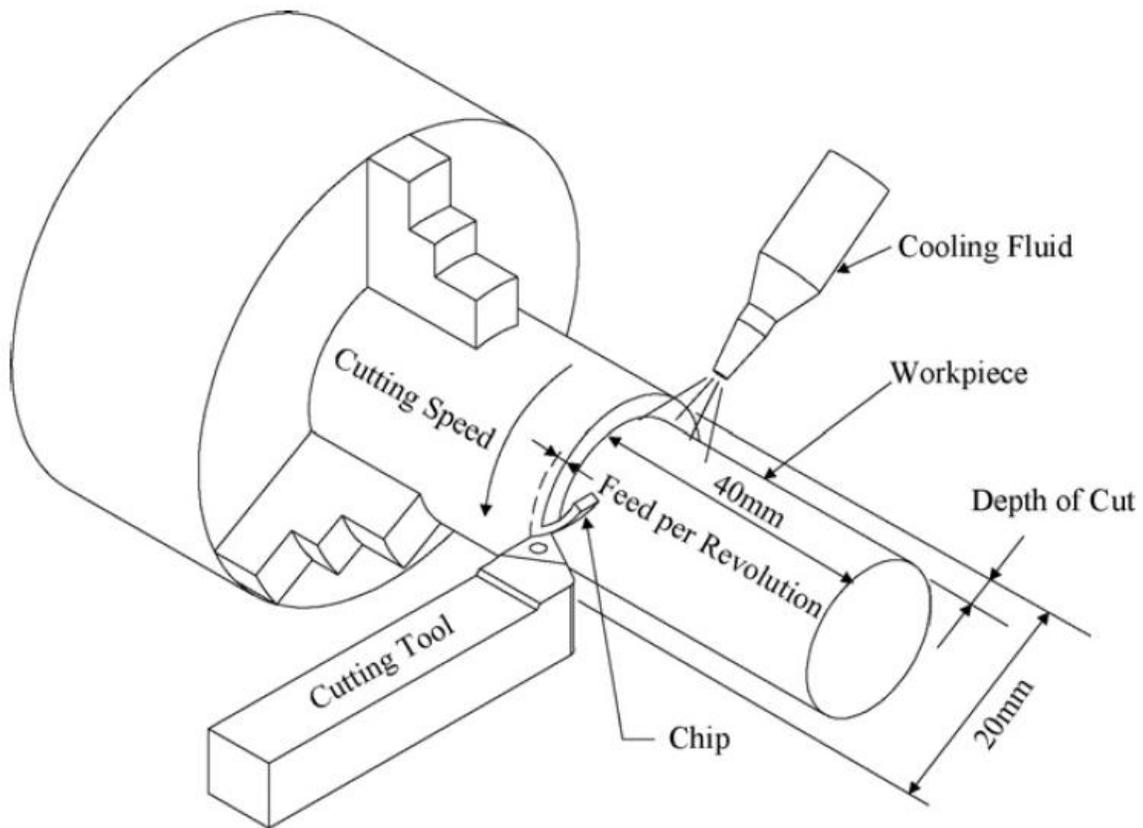
### *Schematic of machining*

In this study, the experiments were carried out on a rigid CNC turning center with a 2 hp spindle motor at 4200rpm (machine type of Pinacho | |). Fig. 1 shows the turning operation and the cutting length of work piece is 50 mm. At the same time, the cutter tool is K-20 HSS.

Furthermore, the cutting speed (m/min), the feed rate (mm/rev), and the depth of cut (mm) are regulated in this experiment of turning operations.

### *Experimental Parameters and Design*

In this study processing procedure for MRR and surface finish is investigated with respect to controllable factors. The cutting fluid mixture ratio is a constant and hence we have three controllable three level factors thus generating an  $L_9$  orthogonal array ( $3^3$ ) for twenty seven experimental runs. The quality targets chosen are measured using the apparatus as described below.

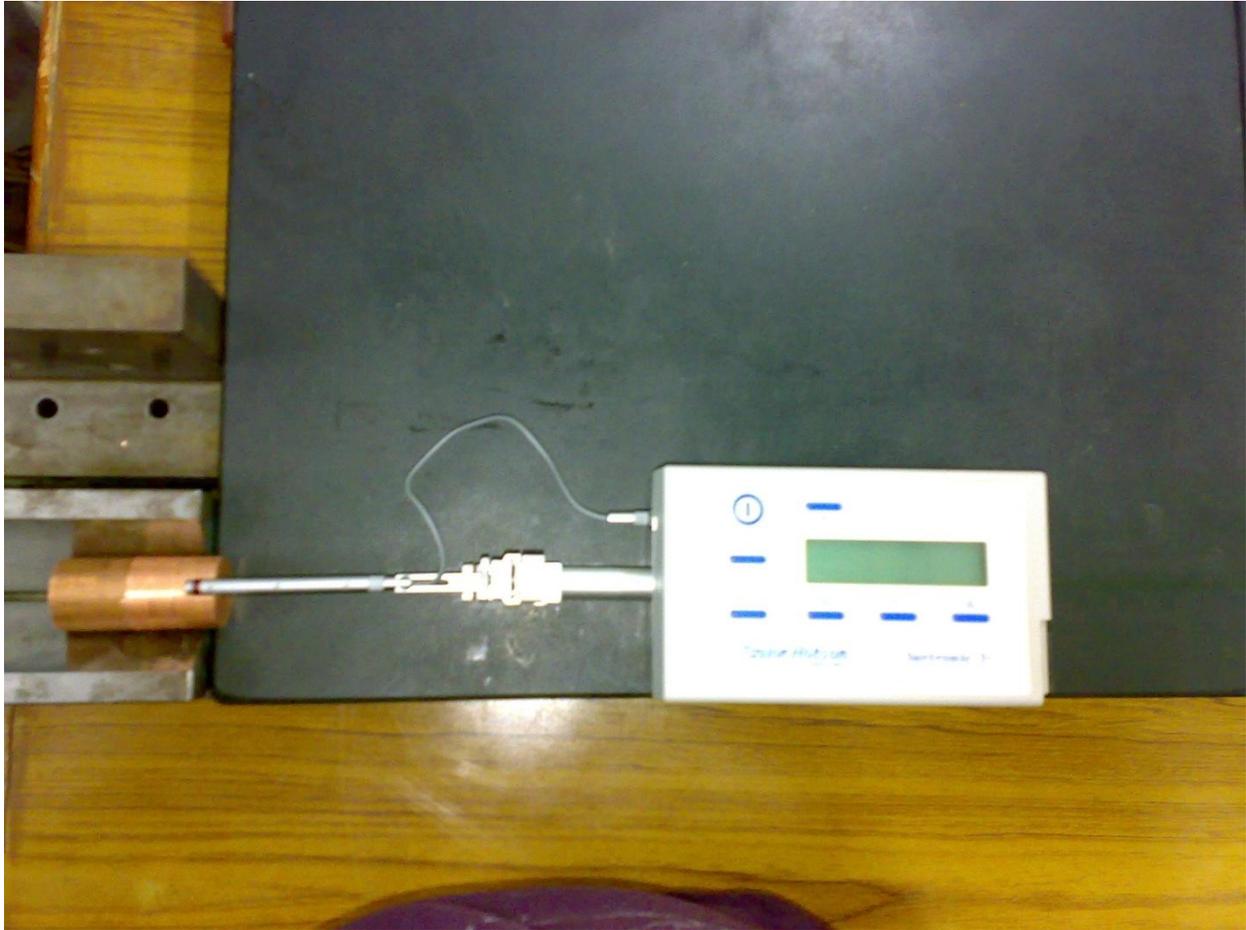




CNC Lathe Machine used for the turning operations.

### *Measuring apparatus*

The material removal rate was measured using a calibrated weighing machine by subtracting initial and final weight for each run of the turning operation and timing it accordingly. The surface finish was measured using a surface analyzer of the form Talysurf50.



Taylor and Hobson Surface Roughness and Roundness Average measuring equipment.

Manufacturer: Taylor and Hobson, UK

Least count: 0.18  $\mu\text{m}$

### ***The experimental runs***

The controllable input parameters were

Speed (RPM): 360, 530, and 860

Feed: 0.052, 0.066, and 0.083

Depth of Cut: 0.5, 0.35 and 0.2

The response variables were Material Removal Rate and Surface Finish. The tabulation for the same is indicated.

No	W/P	Speed (RPM)	Feed (mm/rev)	Depth of Cut	MRR (g/sec)	Surface Finish ( $\mu\text{m}$ )
1	4	360	0.052	0.5	0.0496	5.12
2		360	0.052	0.35	0.0548	4.54
3		360	0.052	0.2	0.0245	4.12
4	5	360	0.066	0.5	0.05	5.64
5		360	0.066	0.35	0.0413	4.69
6		360	0.066	0.2	0.022	4.61
7	6	360	0.083	0.5	0.0673	5.35
8		360	0.083	0.35	0.054	4.64
9		360	0.083	0.2	0.0211	4.21
10	7	530	0.052	0.5	0.124	4.64
11		530	0.052	0.35	0.063	4.77
12		530	0.052	0.2	0.034	5.14
13	8	530	0.066	0.5	0.126	5.64
14		530	0.066	0.35	0.0629	4.12
15		530	0.066	0.2	0.0324	4.04
16	9	530	0.083	0.5	0.152	4.08
17		530	0.083	0.35	0.062	3.98
18		530	0.083	0.2	0.034	3.42
19	3	860	0.052	0.5	0.0946	3.31
20		860	0.052	0.35	0.086	3.08
21		860	0.052	0.2	0.034	2.88
22	2	860	0.066	0.5	0.153	3.23
23		860	0.066	0.35	0.063	3.04
24		860	0.066	0.2	0.029	2.97
25	1	860	0.082	0.5	0.324	3.31
26		860	0.082	0.35	0.266	3.22
27		860	0.082	0.2	0.331	2.85

The relational coefficient for both Surface finish and Material Removal Rate were obtained using the Taguchi “larger-the-better” methodology.

Scheme	Surface Finish		MRR	
	C.S	R.C	C.S	R.C
1	0.81	0.78	0.09	0.93
2	0.6	0.82	0.1	0.93
3	0.45	0.86	0.01	0.98
4	1	0.74	0.09	0.91
5	0.65	0.78	0.06	0.97
6	0.63	0.81	0.002	0.96
7	0.89	0.76	0.14	0.84
8	0.64	0.81	0.1	0.9
9	0.48	0.85	0	0.96
10	0.64	0.81	0.33	0.63
11	0.68	0.8	0.13	0.85
12	0.82	0.77	0.04	0.95
13	1	0.74	0.33	0.63
14	0.45	0.86	0.13	0.85
15	0.42	0.87	0.03	0.93
16	0.44	0.86	0.42	0.57
17	0.4	0.87	0.13	0.85
18	0.2	0.93	0.04	0.95
19	0.16	0.94	0.23	0.73
20	0.08	0.97	0.2	0.76
21	0.01	0.99	0.04	0.95
22	0.13	0.95	0.42	0.57
23	0.06	0.97	0.13	0.85
24	0.04	1	0.02	1
25	0.16	0.94	0.97	0.35
26	0.13	0.95	0.79	0.4
27	0	0.98	1	0.34

C.S – Comparability Sequence

R.C – Relational Coefficient

The Grey Relational Grades for the schemes were then calculated. The tabulation is as below.

No.	<u>Surface Finish</u>	<u>Material Removal Rate</u>	<u>Grey Relational Grade</u>
	<u>R.C</u>	<u>R.C</u>	
1	0.78	0.93	0.855
2	0.82	0.93	0.875
3	0.86	0.98	0.92
4	0.74	0.91	0.825
5	0.78	0.97	0.875
6	0.81	0.96	0.885
7	0.76	0.84	0.8
8	0.81	0.9	0.855
9	0.85	0.96	0.908
10	0.81	0.63	0.72
11	0.8	0.85	0.825
12	0.77	0.95	0.86
13	0.74	0.63	0.685
14	0.86	0.85	0.855
15	0.87	0.93	0.9
16	0.86	0.57	0.715
17	0.87	0.85	0.86
18	0.93	0.95	0.94
19	0.94	0.73	0.835
20	0.97	0.76	0.865
21	0.99	0.95	0.97
22	0.95	0.57	0.76
23	0.97	0.85	0.91
24	1	1	1
25	0.94	0.35	0.645
26	0.95	0.4	0.675
27	0.98	0.34	0.66

The data sequences have a the-larger-the-better characteristic, the “larger-the-better” methodology, i.e. Eq. (2), was employed for data preprocessing. Moreover, the results of twenty seven experiments were the comparability sequences. The distinguishing coefficient can be substituted for the Grey relational coefficient in Eq. (5). If all the process parameters have equal weighting,  $\zeta$  is set to be 0.5. The table listed the Grey relational coefficients and the grade for all twenty seven comparability sequences. This investigation employs the response table of the Taguchi method to calculate the average Grey relational grades for each factor level. Since the Grey relational grades represented the level of correlation between the reference and the comparability sequences, the larger Grey relational grade means the comparability sequence exhibiting a stronger correlation with the reference sequence. Based on this study, one can select a combination of the levels that provide the largest average response. In the above table, from the Grey Relational Grade, the optimum parameter combination shows the largest value of the Grey Relational Grade. Thus a speed of 860 RPM, feed of 0.066 and a depth of cut of 0.2 is found to be the optimum parameter combination.

### **Confirmation Test / Conclusion**

After identifying the most influential parameters, the final phase is to verify the Speed, Feed and Depth of Cut by conducting the confirmation experiments. The *scheme 27* is an optimal parameter combination of the turning process via the Grey relational analysis. Therefore, the condition *scheme 27* of the optimal parameter combination of the turning process was treated as a confirmation test. If the optimal setting with a cutting speed of 860 RPM, 0.066 mm/rev of the feed rate, a cut depth of 0.2 mm, and the surface roughness obtained is 2.97  $\mu\text{m}$  and the MRR obtained is 0.029 g/sec.

## References

- Aslan, E., Camuscu, N., Birgoren, B., 2007. Design optimization of cutting parameters when turning hardened AISI 4140 steel (63 HRC) with Al<sub>2</sub>O<sub>3</sub> +TiCN mixed ceramic tool. *Mater. Des.* 28.
- Chen, D.C., Chen, C.F., 2007. Use of Taguchi method to study a robust design for the sectioned beams curvature during rolling. *J. Mater. Process. Technol.* 190, 130–137.
- Chiang, K.T., Chang, F.P., 2006. Optimization of the WEDM process of particle-reinforced material with multiple performance characteristics using Grey relational analysis. *J. Mater. Process. Technol.* 108, 96–101.
- Davim, J.P., 2003. Design of optimization of cutting parameters for turning metal matrix composites based on the orthogonal arrays. *J. Mater. Process. Technol.* 132, 340–344.
- Deng, J.L., 1989. Introduction to Grey system theory. *El Baradie, M.A., 1996. Cutting fluids: part I characterization.*
- Fung, C.P., Kang, P.C., 2005. Multi-response optimization in friction properties of PBT composites using Taguchi method and principle component analysis.
- Fung, C.P., Huang, C.H., Doong, J.L., 2003. The study on the optimization of injection molding process parameters with Gray relational analysis.
- Kalpakjian, S., Schmid, S.R., 2001. *Manufacturing Engineering and Technology, International, Fourth ed.* Prentice Hall Co., New Jersey, pp. 536–681.
- Lin, C.L., 2004. Use of the Taguchi method and Grey relational analysis to optimize turning operations with multiple performance characteristics. *Mater. Manuf. Process.* 19 (2), 209–220.

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