Performance Analysis of Synthetic Control Chart

a thesis submitted in partial fulfillment of the requirements for a degree of

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in

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by

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National Institute of Technology
Rourkela

CERTIFICATE

This is to certify that the thesis entitled **"Performance Analysis of Synthetic Control Chart"** submitted by Mr. Rohit Rajan in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my guidance.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any University/Institute for the award of any Degree or Diploma.

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ABSTRACT

Control charts are mainly used for the sole purpose of detecting any assignable cause which may affect the process stability. They are used for monitoring and controlling manufacturing processes. In the present project a synthetic control chart which is based on the non-central chi-square statistic was analyzed and the ARL values for the chart were found out with the help of a MATLAB program. The values of the ARL for the varying mean shifts and process variances were thus obtained.

CONTENTS

CHAPTER 1

INTRODUCTION

Control charts are graphical tools which are used for monitoring the activity of an ongoing process. Control chart is sometimes referred to as Shewart control charts, because Walter.A.Shewart first proposed their general theory. The value of a quality characteristic which is first plotted on the vertical axis and the horizontal axis represents the samples or subgroups (in order of time), from which the quality characteristic is found. Samples of a certain size (say 4 or 5 observations) are selected, and the quality characteristic (say average length) is calculated based on the number of observations in the sample. These characteristics are then plotted in the order in which the samples were taken.

There are different examples of quality characteristic which include average length, average diameter average tensile strength, average resistance and average service time. These characteristics are variables and numerical values can be obtained for each. The term attribute applies to such quality characteristics as the proportion of non-conforming items, the number of non-conformities in a unit, and the number of demerits per unit.

The control chart is the tool used to find variation in a process and determine whether a process is “normal” or “unusual”. It associates actual variation in a production process with recognized variation limits based on the antique arithmetical performance of the process. Control charts specify when to regulate the process, when to leave it alone, and when to close it down.

And the decision whether to use them should be a financial one. If the control limits are outside the desired limits it may be difficult to control the process. Most processes tend to produce output that is concentrated around a performance value. Otherwise most of the value of a characteristic tends to group around a performance value, whatever it may be, and only a few away from it.

This can be represented on the basis of the bell shaped curve which is known as normal distribution .Control charts are understood based on the probabilities of the normal distribution. Control charts inform us whether the probability of producing a product using a given process is following a particular normal distribution. If not, the operator must alter the process. Hence, a control chart estimates changes in a process for normal distribution around a target (typical value) and control limits (acceptable variation from the target). When the variation is more than the probability of producing a product at a target value (0.27 percent or less), the cause of this variation appears to be identified, or is considered to be "assignable."

When the variation is in such a way that the probability of producing product at a target value is more than 0.27 %, the cause of the variation becomes difficult to regulate and is considered "random." In many cases control charts are produced at the request of a customer. This can boomerang. Another company may find the charts unsuccessful because they were not used properly. And employees will quickly tire of pursuing worthless data. Ideally, control charts should be used when it is monetary to control a process. (Thus, a control chart, properly used, can always become an economic tool.)Control charts must be used when a process has been properly branded in terms of its statistical variation.

All the assignable causes must be mitigated before using a control chart to monitor a process. The charts are established with assignable causes present in the process. If assignable causes are present, the process should be shut down regularly, and the chart will become a annoyance to management and ignored by operators. Hence control charts should be recognized when the process is under statistical control, that is, when only random causes are present.

There should not be problem should exist that could cause variation beyond the control limits. Control charts are divided into two categories: "variable" and "attribute." The type of chart is used is based on the type of data collected.

Example of variable data may include time, height, thickness, temperature, pressure, current, voltage, and power.

Variable-data charts can be used for processes which are complex and continuous in nature, whereas attribute-data charts are used to control the processes that are mechanical and less complex, such as assembly processes.

01.1 ELEMENTS OF A CONTROL CHART

A control chart consists of:

- A central line
- An upper control limit
- A lower control limit
- Process values plotted on the graph

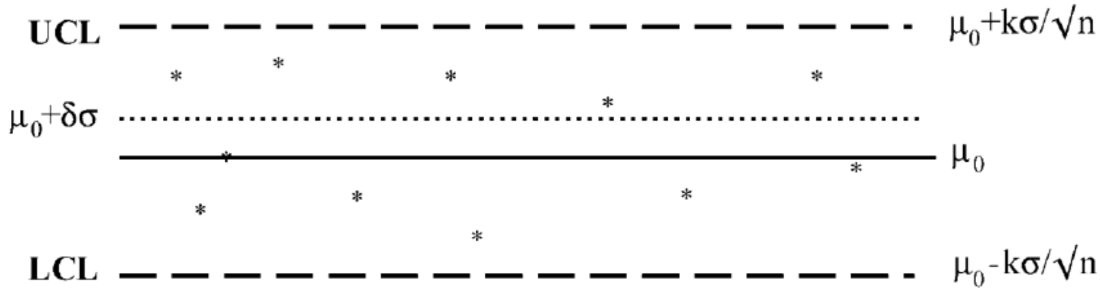

Fig 1: Control limits of the x bar control chart

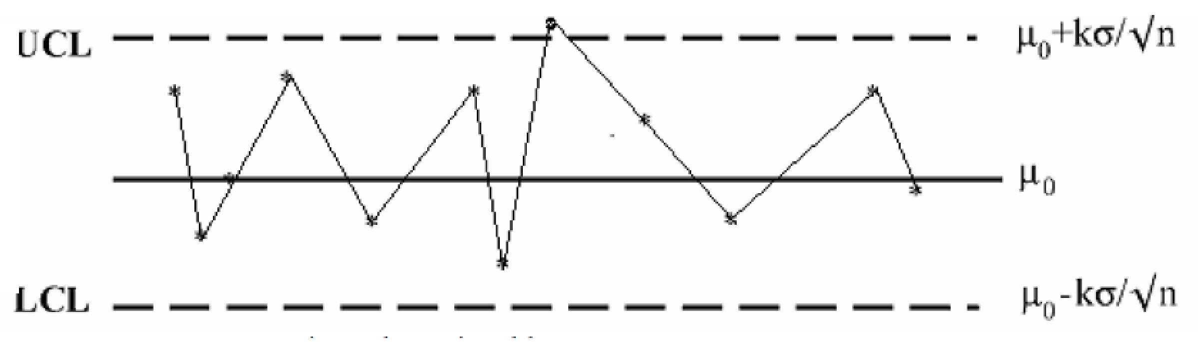

Fig 2: The assignable cause

01.2 DESIGNING A CONTROL CHART

All the process values are plotted on the chart. If it falls within the control limits it is said to be in in control and if the values fall outside the limits it is said to out of control.

01.3 TYPES OF CONTROL CHARTS

There are various types of control charts

01.3.1 Variables Chart

When the control chart is constructed by collecting data periodically and plotting it versus time such a chart is called variable chart.

If more than one data value is collected at the same time, statistics such as the mean, range, median, or standard deviation are plotted. Control limits are added to the plot to signal unusually large deviations from the center line, and run rules are employed to detect other unusual patterns.

01.3.2 X-bar and Range charts

These charts are used for variable data which is continuous as well as quantitative in measurement likely measured dimension or time. The X-bar charts monitor the process location over time which is based on a series of observations called a subgroup. The range chart is used for monitoring variations between observations in the subgroup over a period of time.

01.3.3 Attribute chart

Attribute charts normally focus on a more general idea which arises from a pass or fail testing.

The charts are normally used for plotting the rates or proportions.

When the sample size varies the control limits are dependent on the size of the samples that are taken into consideration.

Attribute charts are specifically designed for attribute data in a very general sense. It monitors the process location and the variation over time with the usage of a single chart.

The attribute chart family includes:

- **np chart:** It is used for monitoring the number of times a condition occurs, and the relative with respect to a constant sample size in such a way that each of the sample size has this condition or which do not have this condition.
- **C chart :**This type of chart basically is used for monitoring the number of times a condition occurs with respect to a constant sample size when each of the sample can have more than one instance of the condition. The C chart is based on the Poisson distribution.
- **P chart:** It is used for monitoring the percentage of the samples having the condition relative to either a fixed or varying sample size when each sample can have this condition or not. It is based on a binomial distribution.
- **U chart:** Monitoring the percentage of samples having the condition relative to a fixed or varying sample size when each of the samples can have more than one instance of the condition.

01.3.4 CUSUM Charts

A Cumulative sum chart uses all the information from all the prior samples by displaying the cumulative sum of the deviation of the sample values from a specified target value.Cumulative sum charts can model the proportion of non-conforming items, the number of non-conformities, the sample range, the sample standard deviation or the process means.

01.3.5 Time Weighted Charts

When the data is collected for one sample at a time and when it is plotted on an individual's chart, the control limits which we use are pretty large and this causes the chart to have a very poor capacity in detecting the out of control situations .Such a problem can be tackled by plotting a weighted average or the cumulative sum of the data and not just the recent observation. The ARL (average run length) that is calculated is usually less than that of the simple X chart.

01.3.6 Moving Average Charts

A moving average chart is that which is very effective in detecting shifts of small magnitude in the process mean. It can also be used for situations where the sample size is 1, or when the product characteristics are measured automatically or when the time to produce a unit is long.

01.3.7 Geometric Moving Average Charts

In order to detect a trend there is a need for us to weigh successive samples in order to form a moving average chart. This idea is a more generalized and easier way to say it. So instead of an arithmetic moving average we can compute a geometric moving average.

01.3.8 Regression Control Chart

This type of chart is used in circumstances that involve tool wear or die wear is expected to gradually increase or decrease. The center line of rather being horizontal will be sloping upwards or downwards. The allowable initial and final values of the process mean will be determined by the specification limits.

01.3.9 Synthetic Control Chart

A synthetic control chart is particularly based on the statistic T which is used for monitoring the mean as well as the variance. It is used in a process which normally combines the specified usage of a regular Shewart chart and the usage of a conforming run length chart. The control chart based on this statistic is a very good performance compared to other charts and in here the speed with which the causes are detected is pretty fast. Here we consider the shift in the process mean as well as an increase of the variability of the process or both cases where there is a mean shift as well as an increase in the variance of the process. The conforming run length chart is also an efficient method for detecting various assignable causes.

CHAPTER 2
LITERATURE REVIEW

02.1 General Purpose of a Control Chart

In most of the industries involving production planning, maintenance and quality control are the most basic parameters and they are interrelated as well. The main purpose of production planning is the determination of an optimal method which is used to reduce the production cost, the holding host, the setup cost and to guarantee that no stock outs occur during the production cycle. In most of the production processes what we do is monitoring the limit to which our products meet the specifications.

02.2 General Approach

The most common approach in order to monitor the quality online is a very transparent one .It simply extracts samples of a certain size from the running production process. Then line charts have been produced to check the variability in such samples and to consider the closeness of those target specifications. There has been a trend that emerges in those lines, or in such a way that if the samples are such that they fall outside the control limits we declare the process to be particularly out of control and the necessary actions have been taken to find the cause of such a problem

Synthetic control charts have been especially having interests in various fields related to the aspect of quality control. Particularly active surveys have been done by (Wu and Speeding 2000; Wu and Yeo 2001; Wu et al. 2001) Lot of practitioners prefers waiting till there is an occurrence of a second point beyond the control limits. Hence look for an assignable cause. Control charts have been specifically used in order to detect particular assignable causes that have been affecting the process stability.

There has always been a general policy to use the X chart if there is the purpose of a detecting the mean shifts and the R chart has been specifically used for the purpose when the causes result in an increase in the process variability. These cases are particularly common when the size of the sample is small (if we are considering sample size in the range of 2, 3, 4, 5, and 6) .There have been comprehensive studies on the combination usage of the X chart and the R chart. These have been extensively studied for the monitoring of processes that might be subjected to various types of assignable causes that shifts the mean and increases the variability (Jones and Case 1981; Saniga 1989; Costa 1993; Rahim 1989)

However the problem associated with these charts was that none of them were able to effectively detect the assignable causes that has been causing problem in the process. When in practice the most important consideration should be the speed of detection in the case of a control chart. Hence a more effective method was proposed in order to monitor the mean and the variance simultaneously.

The omnibus EMWA chart was proposed by Domangue and Patch (1991) which can be used for detection in the location and spread. Some of the most recent idea was proposed by Chen *et al.*2004 which has been using the idea of a single statistic to control the mean and the variance.

This statistic has been defined in a different way than the sample variance S^2 and it is defined $T^*=\sum(X_i-\mu_o)^2 /n^2$ and in order to improve the ability of the chart in detecting the assignable cause Costa and Rahim (2004) proposed a statistic $T =\sum[X_i-\mu_o +\xi\sigma_o]^2$ .In this case the target value of the process is considered to be of the process variance where σ_o^2 is the variance.

The control chart which is constructed on the statistic that has been defined as T is therefore very effective in detecting not only the mean shift but also the increase in the process variability.

The synthetic control chart that has been proposed by Wu and Speeding (2000) combines a regular Shewart chart with that of a Conforming Run Length chart.

The conforming run length chart was developed by Bourke (1991) for the purpose of attribute quality control.

02.3 Objective

The performance analysis of a synthetic control chart which utilizes the new statistic is to be used in this process. The ARL values shall be calculated for the chart with the help of a MATLAB program.

CHAPTER 3

Synthetic Control Chart

A synthetic control chart has been used to monitor the quality characteristic which is of primary interest .In this case let us consider that to be X.

We consider this to be normally distributed and it has a mean μ it has a standard deviation σ .It is such that way that the in control value of the mean is μ_0 and the in control value of the standard deviation is given by σ_0.The main of this chart is to monitor the process that is involved and the detection of any assignable causes that cause the shifting of the mean from μ_0 to μ_1 and similarly detect for assignable causes that shift the standard deviation from σ_0 to $\sigma_{1.}$

The mean shift $\mu_1= \mu_0+-\delta\sigma_0$ such that $\delta\neq0$ and also the standard deviation varies in such a way that $\sigma_1=\gamma\sigma_0$ where $\gamma\neq1$. Now if there is a shift in the σ there is a need for the detection of increases in σ, and this is because of the reason that an increase in σ decreases the quality of the product.

Let us assume products are produced in an industry in such a way that x_{ij}(where i=1,2,3,4,5…; j=1,2,3….n) are the measurements of the variable X which are arranged in the groups of size such a way that n>1 and the indexing group number is taken to be i.

Now $\overline{X}_i$= $(X_{i1}+X_{i2}+X_{i3}+\ldots\ldots\ldots X_{in})/n$ is the ith sample mean .We also have here $e_i= \overline{X}_i-\mu_0$ is the difference between the ith sample mean and the target value of the process mean

We now define a particular value:

ξ_i={d if d ≥ 0 or …………………………………(1)

$-d$ if $d \le 0$

Now we define the statistic as (Costa and Rahim,2004)

$T_i=\sum_{j=1}^{n}(X_{ij}-\mu_o+\xi_i \sigma_o)^2$,i=1,2… ………………….(2)

In this case d is a positive constant .When d is taken as 0 we have $T_i/(\gamma\sigma_{0)})^2$ can be distributed as a non-central chi-square distribution which is having n degrees of freedom and there is as well as a non-centrality parameter $\lambda=n\delta^2/\gamma^2$……………………….. (Costa and Rahim, 2004)

The use of this non central parameter has been used to control the process mean and variance.

The usage of the synthetic control chart has been based on the decision whether a sample is conforming or non- conforming and it is based on the conforming run length.

The CRL can be defined as the number of samples taken from the process until the occurrence of a non-conforming sample that is a sample linked to a point beyond the control limits. (Costa and Rahim, 2004). In order to determine the CRL it has been referred to the previous sample. If there is an absence of a previous sample the reference is the beginning of the surveillance. So we can say that for every point that is plotted that is beyond the control limits CRL is determined.

When the value of the CRL ≤ Where L is a specified positive integer, the process can thus concluded to be out of control and hence a signal is thus generated for the process.

In the synthetic chart that we have considered the value of the upper control limit is taken to be $k\sigma_0^2$ where k can be considered to be the value that is taken for the protection against false alarms.

We now have an equation for the probability of obtaining such a non -conforming sample

In a synthetic control chart

$P_s=Pr\ [T_i>k\sigma_0^2]$

$$=\frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{-\delta\sqrt{n}/ɣ} \Pr\{\chi n2 > [k/ɣ2 - (y + \sqrt{n}\frac{\delta - d}{ɣ})]\}e - 0.5ydy \quad + \frac{1}{\sqrt{2\Pi}}\int_{-\delta\sqrt{n}/ɣ}^{\infty} \Pr\{\chi n2 > [k/ɣ2 - (y + \sqrt{n}\frac{\delta + d}{ɣ})]\}e - 0.5ydy.$$

A control charts effectiveness can be found out by the calculation of the average run length .The average run length, may be said to be the expected number of samples drawn out when the process is out of control and the chart gives a signal. A synthetic control chart resembles a runs rules chart which has a head start feature .The most important issue is the chart performance in the steady state. The steady state ARL is used to measure the speed with which the control chart gives signals. If we consider this case it measures the effects of the head start that have faded away the steady state ARL has been calculated with the help of the Markov Chain Model.

This is one of the most general procedures that have been used in order to determine the values of the Average Run Length. This can be applied to the synthetic control chart which is based on the chi-square Statistic.

CHAPTER 4
MARKOV CHAIN MODEL

The Markov chain model requires a lot of understanding of the variety of skills which are required in order to understand the probability theory surrounding it.

With the introduction of discrete time and discrete space the stationary transition probabilities of the models have been discussed.

04.1 General Policies

The models which have been used in this course are mainly consisting of finite state space Markov chains. Generally, the subject matters that need to be taught in need of necessity to be rather simple and brief.

Techniques which have been used recursively for the generation of such a model with the difference in the equations are on the rise. A Markov chain is a random process evolving in time in accordance with the transition probabilities of the Markov chain. Students have to be made aware of the time element in a Markov chain. Some pictorial representations or diagrams may be helpful to students. Only two visual displays will be discussed in this paper. These visual displays are sample path diagram and transition graph. A sample path diagram is similar to a tree diagram that is usually taught in an

Introductory probability course. In this diagram, starting from an initial state all the possible

Sample paths of the Markov chain are drawn for a small value of the time parameter, n.

Usually n is taken to be four or five.

CHAPTER 5

DESIGN OF THE SYNTHETIC CHART

The parameters of the control chart that constitute the synthetic chart are the upper control limit and the lower limit L. It can be implemented as follows

1. The sample value of the quality characteristic is taken
2. The statistic has been calculated

   $T_i=\sum_{j=1}^{n}(X_{ij}-\mu_o+\xi_i\ \sigma_o)^2$,i=1,2…
3. If the value of the statistic calculated is more than the value of UCL .i.e

   if T_i>UCL the process is considered to be out of control
4. Now the value of the CRL is determined where the CRL is the number of non-conforming samples between the current and the last non-conforming samples plus one.

   If the value of the CRL ≥L the process is still considered to be in control and go back to step 1. Otherwise if the CRL>L then go to the next step
5. The process will be stopped for further investigation.

CHAPTER 6

Calculation of the ARL of the Synthetic Chart

The zero state ARL of the synthetic control chart is calculated on the basis of the process.

In this the process shift is assumed to take place either from the beginning or immediately after a non-conforming sample.

The probability that a synthetic control chart produces an out of control signal is given by

Q (δ) =Prob [(this sample is non-conforming) AND (CRL between the last sample and the non-conforming sample<L.

p (δ).P(δ).

Where p (δ) =prob (a sample is non-conforming)

=Prob (T>UCL)

$=1-\Phi\left(\frac{UCL-(\mu 0+\delta\sigma 0)}{\sigma 0}\right)$

P (δ) =Prob(a CRL between 2 non-conforming samples<L)

$= (1-(1-p(\delta))^{L-1}$

Where Φ ( ) is the cumulative probability function of the standard normal distribution. And the CRL is assumed to follow a geometric distribution. Since the synthetic chart is a Shewhart chart, the zero-state ARL zero for a given mean shift δ is calculated by (Wu and Spedding 2000.

$ARL_{zero}(\delta) = \frac{1}{Q(\delta)} = \frac{1}{p(\delta).P(\delta)}$

By usage of the above two equations we get

ARL_{zero} (δ) = $\frac{1}{p(\delta)(1-(1-p(\delta))L-1)}$.

The in control average run length ARL_0 is equal to ARL_{zero} (0).

CHAPTER 7

ANALYSIS OF THE SYNTHETIC CHART

Now we consider random 100 samples each of sample size 5 and with the mean of the sample to be taken as 3.148 and the standard deviation to be taken as 0.1238

Now the analysis is done on these samples and the observations of each of the sample is given in the tables below .The randomly generated sample using matlab are given below.

Using matlab 100 samples each of size 5 has been generated .Such a distribution is given below:

	1	2	3	4	5
1	3.1746	3.1432	3.1199	3.1646	3.0767
2	3.1840	3.0194	3.2605	3.2518	3.0811
3	3.0550	3.3848	3.1222	3.0869	2.8871
4	3.2677	3.2172	3.1244	3.2742	3.2034
5	3.2227	3.2128	2.8971	3.1601	3.4871
6	3.2154	3.1475	3.1435	3.1694	3.1418
7	3.2917	3.2844	3.1377	3.0597	3.2013
8	3.1542	3.0509	3.3128	3.1617	3.0331
9	2.9937	2.9820	3.0130	3.0873	2.9789
10	3.1342	3.2088	3.1581	3.0706	3.0786
11	3.1680	3.0223	3.2541	3.1277	3.1623
12	3.3106	3.1834	3.0857	3.2179	3.1565
13	3.1380	3.1357	3.0736	3.0315	3.0301
14	3.0250	3.1397	3.3374	3.1799	3.1239
15	3.2175	3.1217	3.0749	3.2887	3.1712
16	3.3237	3.1670	2.9402	3.1412	3.0470
17	3.2194	3.2152	3.3394	3.2737	3.0644
18	2.9782	2.9290	3.2012	3.1286	3.0886
19	3.2383	3.1381	3.1072	3.1656	3.0165
20	3.2316	2.9596	3.1241	2.9314	3.1340
21	2.9671	2.9812	3.1415	2.9997	3.0775
22	3.1563	3.0920	3.0926	3.0683	3.2369
23	3.2393	3.0995	3.2105	3.0659	3.0641
24	3.0871	3.4355	3.2677	3.0024	3.2081
25	3.2305	3.1947	3.1445	2.9237	3.1342
26	3.2446	3.1729	3.1073	2.8482	3.1742
27	3.2483	3.0659	3.1851	3.2521	3.0725
28	3.3300	3.1817	3.0842	3.1982	3.0734
29	2.8992	3.1796	3.1547	3.2870	3.0005
30	2.8975	3.0796	2.9793	3.2755	3.2026
31	3.1196	2.9434	3.1921	3.0301	3.0367
32	3.0807	3.1559	3.2375	3.2704	2.8658
33	3.1503	3.0225	3.2996	3.0204	3.2372
34	2.9778	3.0500	3.3282	3.2406	2.9431
35	3.4077	2.9610	3.2262	3.0973	3.1097
36	3.1451	3.0033	3.3299	3.2010	3.1643
37	3.0358	3.1382	3.2405	3.1545	2.9213
38	3.1919	3.1530	3.2154	3.2621	3.1602
39	2.9885	3.2803	3.1282	3.1367	3.1504
40	3.2100	3.0625	3.1547	2.9604	3.1759
41	3.1062	2.9238	3.0590	3.2286	3.0329
42	3.0277	3.1670	2.9850	3.2304	2.9563
43	3.1198	3.2045	3.1677	3.1877	3.2714
44	3.1145	3.1025	3.2838	3.0639	3.1226
45	3.2201	3.1341	3.0991	3.1450	2.9894
46	3.1184	3.3476	3.3369	3.0756	3.0091
47	3.1457	3.0835	3.1311	3.1277	3.1755
48	3.2860	2.9345	3.1202	3.2048	3.1442
49	3.1159	3.3253	3.3019	3.1812	3.0421
50	3.0934	3.0305	3.1695	2.9748	3.1142
51	3.1442	3.0186	3.2276	3.1843	3.2296
52	3.3266	3.1061	3.1004	3.2558	3.1833
53	3.3581	2.9372	3.1588	3.0993	3.1335
54	3.1196	3.1013	3.2285	3.2120	2.9440
55	3.2013	3.2531	3.2822	3.0795	3.0731
56	3.2922	3.2900	3.1095	2.9406	3.2100
57	3.0787	3.0885	3.0256	3.2173	3.1443
58	3.2009	3.2447	3.3634	3.0403	3.0875
59	3.0627	3.1390	3.2953	2.9721	3.3165

	1	2	3	4	5
60	3.1151	3.0915	3.2253	2.8202	3.0845
61	2.9823	3.2979	3.2964	3.1751	3.0542
62	3.1098	3.0721	3.1057	3.2572	3.1413
63	3.0826	2.9112	3.0134	3.0376	3.3707
64	3.1151	3.0999	3.1878	3.0010	3.2760
65	3.2028	3.1233	3.2217	3.1952	2.9957
66	3.0680	3.1396	2.9799	3.0764	3.1929
67	2.9766	3.0224	3.1606	3.0456	3.2476
68	2.9044	3.1380	3.1341	3.0035	3.0675
69	3.0306	3.1241	3.0894	3.2277	3.0570
70	3.3114	3.3771	3.2812	3.0933	3.0197
71	3.3272	3.1752	3.0789	3.1233	3.0367
72	3.0799	3.1421	3.0593	3.0677	2.8930
73	3.1038	3.0507	3.0267	3.4176	3.0500
74	3.1386	3.2648	3.1474	3.2260	3.2203
75	3.2461	3.0732	3.1157	3.0494	3.0745
76	3.1618	3.1915	3.1268	3.0562	3.3201
77	3.1429	3.3096	3.3418	3.2295	2.9752
78	3.2613	3.0617	3.2932	3.1453	3.1471
79	3.1416	3.0879	3.1625	3.0733	3.0692
80	2.9269	2.9698	3.3459	3.0081	3.1397
81	3.1624	3.0161	3.1043	3.2011	3.1147
82	3.1425	3.1913	2.8593	3.2174	3.0325
83	3.3710	3.1121	3.3097	3.0651	3.1223
84	2.9681	3.0255	3.1080	3.0951	3.2878
85	3.3293	3.1735	3.1057	2.8680	3.0608
86	3.2416	3.2549	3.2950	3.0382	3.1224
87	3.2799	3.3896	3.3525	3.0796	3.1921
88	3.1742	3.1905	2.9603	3.2191	3.2002
89	3.0100	3.2744	3.2412	3.1469	3.1217
90	3.0788	3.1312	2.9799	3.1629	3.2212
91	3.4194	3.0196	3.1063	3.1893	3.1860
92	3.0927	3.1625	3.2258	3.1327	3.2478
93	3.2820	3.4430	3.1853	3.0025	3.2356
94	3.1416	3.0990	3.2009	3.0183	3.2155
95	2.9751	3.1580	3.3085	3.1204	3.3440
96	3.1623	3.0888	3.0679	3.1703	3.1310
97	3.1742	2.8534	3.1103	3.1252	3.1741
98	3.1990	3.1029	3.1848	3.1249	3.0947
99	3.2233	3.0777	3.1638	3.2239	3.1819
100	3.2928	3.1740	3.2697	3.1220	3.1121

Table 1: MATLAB GENERATION OF 100 SAMPLES OF SIZE 5

Using these samples the statistic T was calculated and the value for the statistic was determined

Now the control limits were respectively calculated and the chart was simulated using matlab for various shifts .The respective mean and the standard deviation was also calculated using this procedure.

CHAPTER 8

MATLAB PROGRAM FOR THE ANALYSIS

The Matlab Program for the generation of the statistic is given as follows:

```
% function SYN=synthetic (sigma)
  clc;
  clear all;
  mu=3.1438;
  sigma=0.1238;
  n=5; m=100;
  d=0.7;
  for runs=1:100
    x=normrnd (mu,sigma,m,n);
    x_mean=mean(x,2);
    e (m)=0;
    for i=1: m
      if x_mean (i)>mu
        e (i) =d;
      else
        e (i) =-d;
      end;
    end;
    z (m)=0;
    t=z;
    for kp=1: m
      for p=1:n
        t(kp)=t(kp)+(x(kp,p)-mu+e(kp)*sigma)^2;
      end;
    end;
```

Matlab Program for the calculation of the mean shift

```
% ******************for new mean process--------***********
    shift=0:0.25:1.5;
    len_sh=length(shift);
    for i=1:len_sh
```

```
    mu_new(i)=mu+shift(i)*sigma;
  end;
  e_new(m,len_sh)=0;
  for k=1:len_sh
    for i=1:m
      if x_mean(i)>mu_new(k)
        e_new(i,k)=d;
      else
        e_new(i,k)=-d;
       end;
  end;
  end;

  t_new(runs,m,len_sh)=0;
  for i=1:len_sh
    for k=1:m
      for p=1:n
        t_new(runs,k,i)=t_new(runs,k,i)+(x(k,p)-mu_new(i)+e_new(k)*sigma)^2;
      end;
    end;
  end;
```

Matlab Program for calculating the ARL

```
    ucl=k*sigma^2;

    i=1;

    while t(i)<ucl

      i=i+1;

    end;

  %   value_before_RL(runs)=t(i-1);

  %   value_at_RL(runs)=t(i);

    RL(runs)=i;     %**************RUN Length**********%

     for pp=1:len_sh

      pk=1;

      while t_new(runs,pk,pp)<ucl

        pk=pk+1;
```

```
        end;
         new_run(runs,pp)=pk;
      end;
  %   end;
  end;
  RL_avg=mean(RL)
  new_run_avg=mean(new_run)
% return
```

CHAPTER 9

OBSERVATIONS

δ(Mean shift)	ARL(k=10.020) L=1	ARL(k=15.432) L=5	ARL(k=17.928) L=10	ARL(18.123) L=20
0	25.96	25.96	25.96	25.96
0.25	17.82	16.61	12.22	13.41
0.5	9.92	7.52	5.97	5.91
0.75	6.46	4.46	3.31	3.46
1.0	5.32	3.04	2.02	2.08
1.25	3.93	2.04	1.91	1.90
1.5	2.51	2.04	1.91	1.71

Table 2: For calculation of ARL Values when ɣ=1 and d=0.7

δ(Mean shift)	ARL(k=10.020) L=1	ARL(k=15.432) L=5	ARL(k=17.928) L=10	ARL(k=18.123) L=20
0	14.61	14.61	14.61	14.61
0.25	11.23	10.98	9.43	8.26
0.5	5.46	4.32	3.39	2.96
0.75	3.91	2.82	1.94	1.94
1.0	2.51	2.21	1.51	1.21
1.25	1.58	1.56	1.45	1.10
1.5	1.25	1.20	1.32	0.95

Table 3: For calculation of ARL values when ɣ=1 and d=0

δ(Mean shift)	ARL(d=0.0) k=13.596	ARL(d=0.5) k=18.503	ARL(d=0.7) k=21.416	ARL(d=1.0) k=26.704
0	**11.46**	13.04	13.97	15.64
0.25	**10.23**	10.45	11.00	11.86
0.5	7.43	**6.52**	6.57	6.75
0.75	4.92	3.94	3.87	**3.84**
1.0	3.21	2.65	2.54	**2.47**
1.25	2.44	1.84	1.85	**1.75**
1.5	1.63	1.47	1.45	**1.42**

Table 4:For calculation of ARL when L=10 and ɣ=1.3

09.1 Results and Discussion

In this project work for samples that have been generated the ARL values were calculated based upon different conditions like using mean shifts and changing the standard deviation using MATLAB.

From the above analysis we have found out that the steady state ARL values decrease as the value of L increases .We also found out that as L increases up to 20 the chart's speed in detecting the isolated process mean shifts has been decreased if the shift is moderate or large(δ>0.5)

CHAPTER 10

CONCLUSION

By using such an analysis of a synthetic control chart and from the observations and the results we got from the tables we can say that the Statistic T with the values of $d>0$ is much more effective in detecting mean shifts or process variances as the ARL decreases much more rapidly than the regular X chart and R chart.

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