COMPARATIVE STUDY OF VARIOUS BEAMS UNDER DIFFERENT LOADING CONDITION USING FINITE ELEMENT METHOD

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CERTIFICATE

This is to certify that this thesis entitled, "**COMPARATIVE STUDY OF VARIOUS BEAMS UNDER DIFFERENT LOADING USING FINITE ELEMENT METHOD**" submitted by Mr. RUDRANARAYAN KANDI & Mr. TANMAYA KUMAR NAYAK in partial fulfillment for the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at National Institute of Technology, Rourkela is an authentic work carried out by him under my guidance.

To the best of our knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma

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ABSTRACT

Beam is a horizontal structure element which can withstand the load by resisting the bending which use in various industrial application, architectural application, automobile application for supporting the loads and reliability. So it is very much essential to know property of beam and response of beam in various cases. In this article we studied some of the response of beam by using finite element method (FEM) and MATLAB. By using boundary condition, results for Timoshenko beam and Euler-Bernoulli's beam in different cases varies in stiffness matrix, mass matrix and graphs .According to old theory many assumption has been taken place which is different from the practical situation and new theory tells the practical one. By the finite element method beam can be analyzed very thoroughly. So that strength of beam can be manipulated and applied at the proper place. The comparison between the Timoshenko and Euler-Bernoulli beam has been studied here.

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CHAPTER-01

INTRODUCTION

There are three basic types of beams

(1) Simply supported beams (support at both end)

(2) Cantilever beam (support at one end and other end is free)

(3) Continuous beam (supported at more than two points)

Generally for the observation propose the beam is classified by two types

(i) Euler-Bernoulli's beam: Only translation mass & bending stiffness have been considered.

(ii) Raleigh Beam: Here the effect of rotary inertia has been taken care.

(iii) Timoshenko beam: Here both the rotary inertia and transverse shear deformation have been considered.

By the classical theory of Euler-Bernoulli's beam it assume that

(i) The cross-sectional plane perpendicular to the axis of the beam remains plane after deformation (assumption of a rigid cross-sectional plane).

(ii) The deformed cross-sectional plane is still perpendicular to the axis after deformation.

(iii) The classical theory of beam neglects transverse shearing deformation where the transverse shear stress is determined by the equations of equilibrium.

Below two assumptions are applicable to a thin beam. For a beam with short effective length or composite beams, plates and shells, it is inapplicable to neglect the transverse shear deformation.

In 1921, Timoshenko presented a revised beam theory considering shear deformation which retains the first assumption and satisfies the stress-strain relation of shear. In actual case the beam (deep beam) which cross-sectional area relatively high as compared to its length shear stresses are relatively high at the neutral axis as compared to the two other ends and for study propose it has taken that the cross-section remain plain during bending.

Deformation property of any structure can be easily analyzed by beam theory for different loading conditions. Also by inspecting the dimensions of the structure we can use the different beam theory.

Again analysis of beam with finite element method is very much essential. FEM is a numerical method of finding approximate solutions of partial differential equation as well as integral equation. The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution .By this method we divide a beam in to number of small elements and calculate the response for each small elements and finally added all the response to get global value. Stiffness matrix and mass matrix is calculate for each of the discretized element and at last all have to combine to get the global stiffness matrix and mass matrix. The shape function gives the shape of the beam element at any point along longitudinal direction. This shape function also calculated by finite element method. Both potential and kinetic energy of beam depends upon the shape function. To obtain stiffness matrix potential energy due to deflection and to obtain mass matrix kinetic energy due to application of sudden load are use. So it can be say that potential and kinetic energy of the beam depends upon shape function of beam obtain by FEM method.

LITERATURE REVIEW

Mainly, beams are of two kinds taking into consideration of shearing deformation, thickness & length of the beam. Those are Euler-Bernoulli beam & Timoshenko Beam. The comparative study of both the beam applying various boundary conditions has been studied by many scientists. The review consists of papers of different journals which are mentioned in at adequate place.

Gavin [7] has described the formation of stiffness matrix & mass matrix for structural elements such as truss bars, beam, and plates. For the formulation purpose, he used the gradient of kinetic energy & potential energy function with respect to a set of coordinates defining the displacement at the end or nodes of the element. The kinetic energy & potential energy were written in terms of these nodal displacements. He calculated both stiffness matrix & mass matrix for Euler-Bernoulli beam (excluding shearing deformation) & Timoshenko beam (including shearing deformation & rotational inertia).

Augared [3] has conducted a study on generation of shape function for straight beam element. For the formulation, he used the hermite polynomials & derived shape function from the Lagrangian interpolating polynomials.

Davis, Hensbell & Warburton [12] has conducted a study on derivation of stiffness & mass matrix for Timoshenko beam. They explained the convergent tests for simply supported & cantilever beam.

Thomas, Wilson & Wilson [4] has conducted a study on both Timoshenko element (having two degrees of freedom at each node) & complex Timoshenko element (having more than 2 degrees of freedom at each node & more than 4 degrees of freedom at 2 nodes). In this study, the element derived in this has two nodes with three degree of freedom at each node. The nodal variables were transverse displacement, cross sectional rotation (ϴ) & shear (Φ).

Falsone & Settineri [6] has conducted a study of a new finite approach for the solution of the Timoshenko beam.

Bazone & Khuslief [2] has conducted a study on derivation of shape function of 3D-timoshenko beam element. They used the hermitian polynomials & putting the boundary condition, they derived the shape function Timoshenko beam.

OBJECTIVE

1. To study the different beam equation for both Euler beam & Timoshenko beam.
2. To study the difference in shape function, stiffness matrix & mass matrix for both Euler beam & Timoshenko beam.
3. Study of the characteristics curves of beam using MATLAB code .The characteristics curves are plotted among Mode shape, Response, Frequency, Length/Diameter (L/D) ratio.

CHAPTER-02

THEORY

(1) Mathematical Formulation:

(1a) *Euler-Bernoulli beam*:

Euler–Bernoulli beam theory is a simplification of the linear theory of elasticity which provides a means of calculating the load-carrying and deflection characteristics of beams. This is also known as engineer's beam theory, classical beam theory or just beams theory.

The Euler-Bernoulli equation describes the relationship between the beam's deflection and the applied load.

$$\boldsymbol{q = EI(\frac{d^4w}{dx^4})}$$

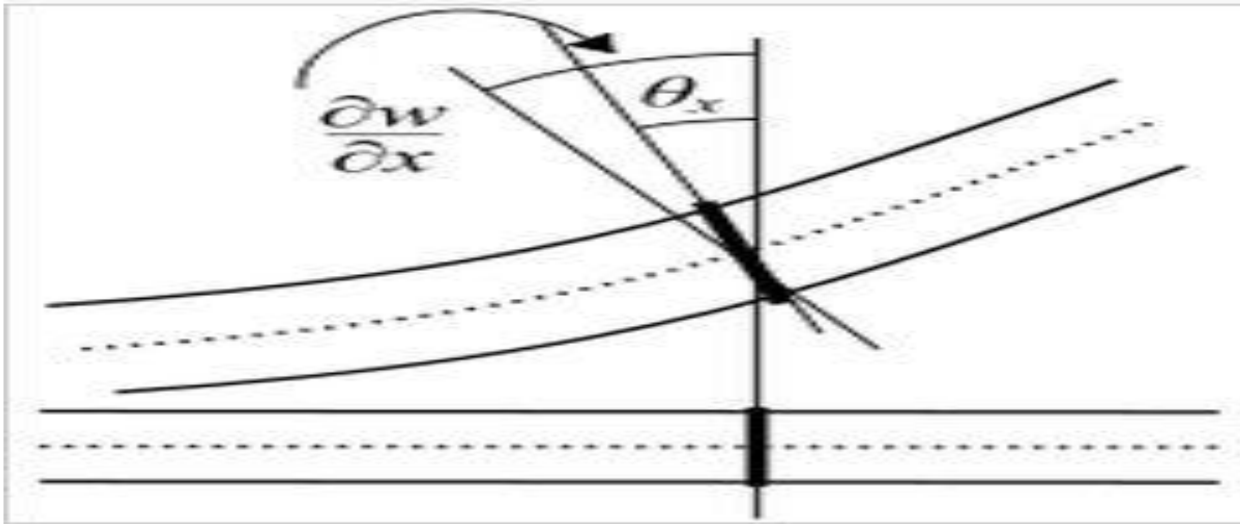
Where, **q** is a force per unit length.

E is the elastic modulus.

I *is* the second moment of area.

(1b) Timoshenko beam :

A Timoshenko beam takes into account shear deformation and rotational inertia effects, making it suitable for describing the behavior of short beams, sandwich composite beams or beams subject to high-frequency excitation when the wavelength approaches the thickness of the beam. The resulting equation is of 4^{th} order, but unlike ordinary beam theory - i.e. Bernoulli-Euler theory - there is also a second order spatial derivative present.



(Fig. 1: Deformation in Timoshenko Beam element)

In static Timoshenko beam theory without axial effects, the displacements of the beam are assumed to be given by

$$u_x(x, y, z) = -z\varphi(x);\ u_y = 0;\ u_z = w(x)$$

Where (x,y,z) are the coordinates of a point in the beam , u_x, u_y, u_z are the components of the displacement vector in the three coordinate directions, φ is the angle of rotation of the normal to the mid-surface of the beam, and ω is the displacement of the mid-surface in z-direction. The governing equations are the following uncoupled system of ordinary differential equations is:

$$\frac{dw}{dx} = \varphi - \frac{1}{kAG}\frac{d}{dx}(EI\frac{d\varphi}{dx})$$

Where k called is the Timoshenko shear coefficient, depends on the geometry.

G is called shear modulus.

E is the elastic modulus.

I is the second moment inertia.

A is the area of cross section.

The Timoshenko beam theory for the static case is equivalent to the Euler-Bernoulli theory when the last term above is neglected, an approximation that is valid when

$$\frac{\boldsymbol{EI}}{\boldsymbol{LAkG}} \ll \mathbf{1}$$

Where L is the length of the beam and H is the maximum deflection.

Stiffness Matrix:

In the finite element method and in analysis of spring systems, a stiffness matrix, K, is a symmetric positive semi index matrix that generalizes the stiffness of Hook’s law to a matrix, describing the stiffness of between all of the degrees of freedom so that

$$\boldsymbol{F} = -\boldsymbol{Kx}$$

Where **F** and **x** are the force and the displacement vectors, and

$$\boldsymbol{U} = \frac{\mathbf{1}}{\mathbf{2}} * \boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{Kx}$$

Is the system's total potential energy.

Mass Matrix:

A mass matrix is a generalization of the concept of mass to generalized bodies. For static condition mass matrix does not exist, but in case of dynamic case mass matrix is used to study the behavior of the beam element. When load is suddenly applied or loads are variable nature, mass & acceleration comes into the picture.

(2)Finite element Formulation :

(2a) Shape Function:

Beam represents fundamental structural components in many engineering applications & shape functions are essential for the final element discretisation of structures. Also, the shape function describes the shape of the beam element at any point along longitudinal direction. In this project basically hermite & modified hermite shape functions are used to formulate the stiffness & mass matrix for Euler-Bernoulli beam & Timoshenko beam respectively.

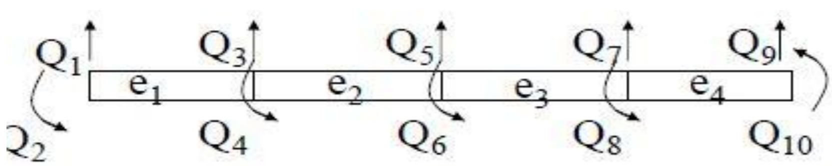
(2b)Formulation of Hermite shape function:

Beam is divided in to element. Each node has two degrees of

Freedom.

Degrees of freedom of node j are Q2j-1 and Q2j

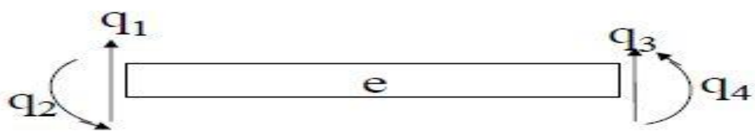
Q2j-1 is transverse displacement and Q2j is slope or rotation.



$Q = [Q_1\ Q_2 Q_3 ... Q_{10}]^T$

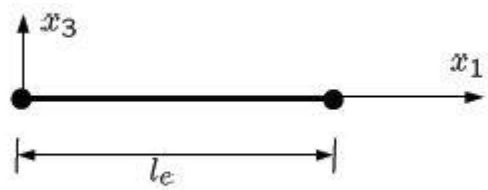
Q is the global displacement vector.

Local coordinates:

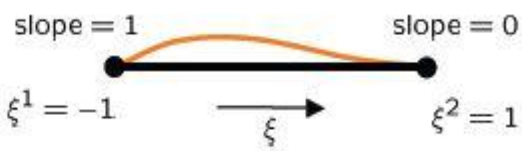


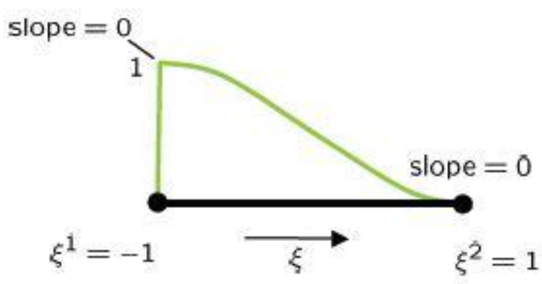
$$q = [q_1, q_2, q_3, q_4]^T$$
$$= [v_1, v_1', v_2, v_2']$$

Hermite shape function for an element of length le.

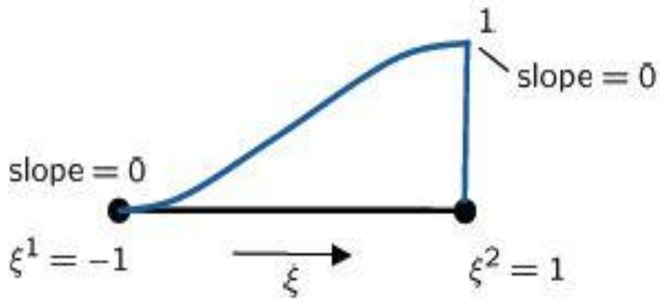


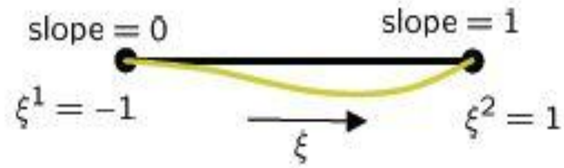
Shape function of node 1:





Shape function of node 2:





Each Hermite shape function is of cubic order represented by-

$$H = a_i + b_i\zeta + c_i\zeta^2 + d_i\zeta^3 \qquad i = 1, 2, 3, 4$$

The condition in the given table must be satisfied.

	H_1	H'_1	H_2	H'_2	H_3	H'_3	H_4	H'_4
ζ=-1	1	0	0	1	0	0	0	0
ζ=1	0	0	0	0	1	0	0	1

Putting the value of ζ in the above equation & simplifying

$$H1 = \frac{[(1-\zeta)^2 * (2+\zeta)]}{4}; \qquad H2 = \frac{[(\llbracket 1-\zeta)\rrbracket^2(\zeta+1)]}{4};$$

$$H3 = [(\llbracket 1+\zeta)\rrbracket^2 * (2+\zeta)]/4; \qquad H4 = \frac{[(\llbracket 1+\zeta)\rrbracket^2(\zeta-1)]}{4}$$

Now hermite function can be used to write v in the form:

$$v(\zeta) = H1v1 + H2\left(\frac{dv}{d\zeta}\right)1 + H3v3 + H4\left(\frac{dv}{d\zeta}\right)2$$

The coordinates transform by relationship:

$$x = \left(\frac{1-\zeta}{2}\right)x1 + \left(\frac{1+\zeta}{2}\right)x2$$

$$= \left(\frac{x1+x2}{2}\right) + \left(\frac{x2-x1}{2}\right) * \zeta$$

$$dx = \frac{le}{2} d\zeta \quad ;$$

$(x2 - x1)$ is the length of the element **le**.

$$\frac{dv}{d\zeta} = \frac{le}{2}\frac{dv}{dx}$$

Therefore, $v(\zeta) = H1q1 + \frac{le}{2}H2q2 + H3q3 + \frac{le}{2}H4q4$

i.e. $v = Hq$

Where the hermite shape matrix is $H = \left[H1, \frac{le}{2}H2, H3, \frac{le}{2}H4\right]$.

***(2c)Stiffness matrix [K]$_e$ for Euler- Bernoulli beam*:**

By the potential energy system,

$$U = \frac{EI}{2}\int_e \left(\frac{d^2v}{dx^2}\right) dx$$

Also, $\frac{dv}{dx} = \frac{2}{le}\frac{dv}{d\zeta}$ & $\frac{d^2v}{dx^2} = \frac{4}{le^2\left(\frac{d^2v}{d\zeta^2}\right)}$

Taking square of the both sides

$$\left(\frac{d^2v}{dx^2}\right)^2 = \frac{16}{le^4}\left(\frac{d^2Hq}{d\zeta^2}\right)^2$$

Now,

$$\frac{d^2H}{d\zeta^2} = \left[\frac{3}{2}\zeta, \frac{-1+3\zeta}{2} * \frac{le}{2}, -\frac{3}{2}\zeta, \frac{1-3\zeta}{2} * \frac{le}{2}\right]$$

On substituting $dx = \frac{le}{2}d\zeta$ in potential energy expression will be

$$U = q^T [K]^e q$$

Where the element matrix [K] is given by

$$[K]_e = \frac{EI}{le^3}\begin{bmatrix} 12 & 6le & -12 & 6le \\ 6le & 4le^2 & -6le & 2le^2 \\ -12 & -6le & 12 & -6le \\ 6le & 2le^2 & -6le & 4le^2 \end{bmatrix}$$

(2d)Mass matrix $[M]_e$ for Euler-Bernoulli beam:

The kinetic energy expressed in the degrees of freedom of the beam element becomes:

$$T = \frac{1}{2}\int_V \rho u T u dv$$

Where, ρ is the density of the material.

$\dot{u}$ is the velocity at the point x1 with the components $\dot{u}, \dot{v}$ & $\dot{w}$.

In finite element method we divide the element & in each element we express u in terms of the displacement q using shape function H.

Thus u=Hq

So the velocity vector is given by $\dot{u} = H\dot{q}$.

Putting the expression for the velocity $\dot{u}$ in the kinetic energy expression we get

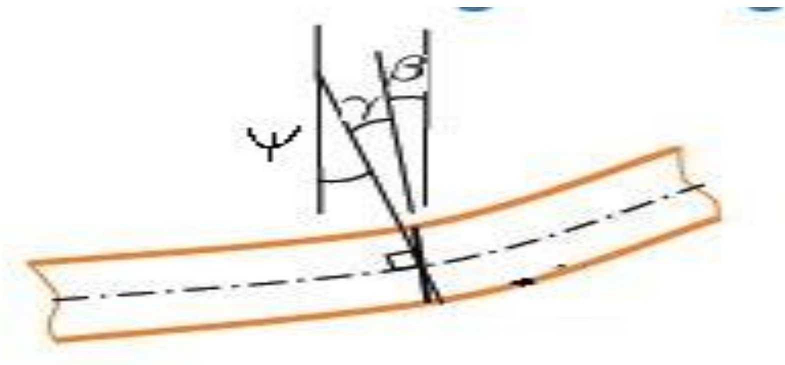
$$T = qT \int_e [[\rho] H^T H dv] \dot{q}$$

Where mass matrix element is $\quad \mathbf{[M]_e} = \int_e \rho H^T H dv$

Using the hermite shape function H & $dx = \frac{le}{2}\zeta$ the mass matrix will be in the form

$$[M]_e=\frac{\rho Aele}{420}\begin{bmatrix} 156 & 22le & 54 & -13le \\ 22le & 4 & 13le & -3le^2 \\ 54 & 13le & 156 & -22le \\ -13le & -3le^3 & -22le & 4le^2 \end{bmatrix}$$

***(2e) Formulation of modified hermite shape function*:**



For the Euler consideration the neutral axis is always perpendicular to the area of cross section but when we consider the neutral axis is not perpendicular to the cross section the angle between neutral axis & area of cross section be **γ.**

β, the bending angle exist due to bending of the beam. **γ,** the shear angle exists due to the shear deformation.

$$\psi = \beta + \gamma = \frac{dv}{dx}$$

Where ,v is the displacement.

$$V = \frac{dM}{dx}$$

Where V=shear force;

M=bending moment.

$$M = EI\frac{d\beta}{dx}$$

Putting the value of M in the expression for V, we get, $V = EI\frac{d^2\beta}{dx^2}$

Again we know $V = GAk\gamma$

Where G=modulus of rigidity,

A=area of the cross section,

K=shear factor,(shear force is not constant throughout the area of cross section so we are using shear correction factor)

The value of shear correction factor varies in different cross sections.

Where $\gamma = \frac{V}{GAk} = \frac{EI}{GAk}\frac{d^2\beta}{dx^2}$

Now let $\beta = \frac{dv}{dx}$ (only considering the bending)

In case of Euler beam both the bending moment & shear force are related to each other but in case of Timoshenko both are independent to each other.

Putting the value of β in the expression for γ ,we get

$$\gamma = \frac{EI}{GAk}\frac{d^3v}{dx^3}$$

The polynomial solution of $\overline{v} = a1x^3 + a2x^2 + a3x + a4$

Where $a1, a2, a3, a4$ are polynomial constants.

$$\frac{d^3\bar{v}}{dx^3} = 6a1$$

Putting the value of $\frac{d^3\overline{v}}{dx^3}$ at γ, we find

$\gamma = \frac{EI}{GAk} 6a1 = 6a1g$, where $g = \frac{EI}{GAk}$

Again; $\beta = \frac{dv}{dx} = 3a1x^2 + 2a2x + a3$

; $\psi = \beta + \gamma = 3a1x^2 + 2a2x + a3 + 6a1g$;

$\psi = (3x^2 + 6g)a1 + a3$ $+2a2x$;

From previous we know $\psi = \frac{dv}{dx}$

So $v = \int \psi \, dx$

$\Rightarrow v = a1x^3 + a2x^2 + (a3 + 6ga1)x + a4$

Now putting the boundary conditions for both ψ & v :

When x=0; v (0) =a4=d1;

$\psi(0) = a3 + 6ga1 = \phi 1$;

When $x = l$; $v(L) = a1L^3 + a2L^2 + a3L + a4 = d2$;

$\psi(L) = 2a2L + a3 + (3L + 6g)a1 = \phi 2$;

Putting all the boundary condition in the expression for v , we can get the value of constant coefficients in terms $d1\ \Phi1\ d2\ \Phi2$.

$$a1 = \frac{2d1 + L\phi1 - 2d2 + L\phi2}{(L^2 + 12g)L};$$

$$a2 = \frac{-3Ld1 - (2L^2 + 6g)\phi1 + 3Ld2 - (L^2 - 6g)\phi2}{(L^2 + 12g)L};$$

$$a3 = \frac{-12gd1 + (L^3 + 6gL)\phi1 + 12gd2 - 6gL\phi2}{(L^2 + 12g)L};$$

$$a4 = d1;$$

Putting the value $of\ a1\ a2\ a3\ \&\ a4$ in the expression for v , we get

$$v = \frac{1}{1+\lambda}[1 + 2\zeta^3 - 3\zeta^2 + \lambda(1-\zeta)]d1 + \frac{L}{1+\lambda}\left[\zeta + \zeta^3 - 2\zeta^2 + \frac{\lambda}{2}(\zeta - \zeta^2)\right]\phi1 + \frac{1}{1+\lambda}[-2\zeta^3 + 3\zeta^2$$

Where $\zeta = \frac{x}{L}\ \&\ \lambda = \frac{12g}{L^2}$

From the above expression we get the shape function due to bending as

$$N11 = \frac{1}{1+\lambda}[1 + 2\zeta^3 - 3\zeta^2 + \lambda(1-\zeta)];$$

$$N12 = \frac{L}{1+\lambda}\left[\zeta + \zeta^3 - 2\zeta^2 + \frac{\lambda}{2}(\zeta - \zeta^2)\right];$$

$$N13 = \frac{1}{1+\lambda}[-2\zeta^3 + 3\zeta^2 + \lambda\zeta];$$

$$N14 = \frac{L}{1+\lambda}\left[\zeta^3 - \zeta^2 + \frac{\lambda}{2}(\zeta^2 - \zeta)\right].$$

Similarly putting the values of $a1, a2, a3, a4$ in the expression ψ :

$\psi = (3x^2 + 6g)a1 + a3$ **+2a2x**, shape function due to rotation will be

$$N21 = \frac{6\,(\zeta - \zeta^2)}{(1+\lambda)L};$$

$$N22 = \frac{[3\zeta^2 - 4\zeta + 1 + \lambda(1-\zeta)]}{1+\lambda};$$

$$N23 = \frac{[6\zeta - 6\zeta^2]}{(1+\lambda)L};$$

$$N24 = \frac{[3\zeta^2 - 2\zeta + \lambda\zeta]}{1+\lambda};$$

Now we calculate the shape function for the shear angle γ

We know the equation for $\gamma = \psi - \frac{dv}{dx}$

So the shape functions for γ , it will be

$$N3 = N2 - \frac{dN1}{dx};$$

$$N31 = \frac{1}{1+\lambda}[\frac{1}{L}(6\zeta^2 - 6\zeta) - (6\zeta^2 - 6\zeta - \lambda)];$$

$$N32 = \frac{1}{1+\lambda}[\{3\zeta^2 - 4\zeta + 1 + \lambda(1-\zeta)\} - L\{3\zeta^2 - 4\zeta + 1 + \frac{\lambda}{2}(1-2\zeta)\}];$$

$$N33 = \frac{1}{1+\lambda}\left[\frac{1}{L} * (6\zeta - 6\zeta^2) - (6\zeta - 6\zeta^2 + \lambda)\right];$$

$$N34 = \frac{1}{1+\lambda}[(3\zeta^2 - 2\zeta + \lambda\zeta) - L(3\zeta^2 - 2\zeta + \frac{\lambda}{2} * (2\zeta - 1))].$$

(2f)Formulation of stiffness matrix for Timoshenko beam:

Due to the bending & shear deformation the potential energy is stored at the beam. We can write the potential energy as

$$U = \frac{EI}{2l} * \int_0^1 q^T \frac{dN2^T}{dx}\frac{dN2}{dx} q\,d\zeta + \frac{6EI}{\psi l}\int_0^1 q^T \frac{dN3^T}{dx}\frac{dN3}{dx} q\,d\zeta$$

$$\Rightarrow U = \frac{EI}{2l} * \int_0^1 q^T K_b q\,d\zeta + \frac{6EI}{\psi l}\int_0^1 q^T K_s q\,d\zeta$$

Where K_b is the bending stiffness matrix

K_s is the shear stiffness matrix.

Therefore, $K = K_b + K_s$

$$\Rightarrow K = \frac{EI}{l^3(1+\lambda)} * \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & (4+\lambda)l^2 & -6l & (2-\lambda)l^2 \\ -12 & -6l & 12 & -6l \\ 6l & (2-\lambda)l^2 & -6l & (4+\lambda)l^2 \end{bmatrix}$$

(2g)Formulation of mass matrix for Timoshenko beam:

Static analysis holds when the loads are slowly applied. When the loads are suddenly applied or when loads are of a variable in nature the mass & acceleration effects comes into the picture. The kinetic energy expression for the beam undergoing deformation can be written in the following form:

$$dY = \frac{1}{2} * \rho A(\dot{u}_x^2) + \frac{1}{2} * \rho I(\dot{\psi}_x^2)$$

Where, $\frac{1}{2} * \rho A\left(\dot{u}_x^2\right)$ is the translational kinetic energy,

$\frac{1}{2} * \rho I\left(\dot{\psi}_x^2\right)$ is the rotational kinetic energy.

Also $u_x = Nq$

Putting the value of u_x in the expression for dY,we will get

$$dY = \frac{1}{2} * \rho A(\dot{q}_x^T N_1^T N_1 \dot{q}_x) + \frac{1}{2} * \rho I(\dot{q}_x^T N_2^T N_2 \dot{q}_x)$$

Integrating the above w.r.to ζ from 0 to 1,we get

$$Y = \rho A l \int_0^1 \dot{q}_x^T N_1^T N_1 \dot{q}_x \, d\zeta + \frac{1}{2} * \rho I l \int_0^1 \dot{q}_x^T N_2^T N_2 \dot{q}_x \, d\zeta$$

$$Y = \frac{1}{2} * \dot{q}_x^T M_T \dot{q}_x + \frac{1}{2} * \dot{q}_x^T M_R \dot{q}_x$$

Solving these above equations we get

$$M_{T]} = \frac{\rho AL}{420(1+\lambda)^2} \begin{bmatrix} m_1 & lm_2^2 & m_3 & -lm_4 \\ lm_2^2 & l^2 m_5 & lm_4 & -l^2 m_6 \\ m_3 & lm_4 & m_1 & -lm_2 \\ -lm_4 & -l^2 m_6 & -lm_2 & l^2 m_5 \end{bmatrix}$$

& $[M_R] = \frac{\rho I}{30l(1+\lambda)^2} \begin{bmatrix} m_7 & lm_8 & -m_7 & lm_8 \\ lm_8 & l^2 m_9 & -lm_8 & -l^2 m_{10} \\ -m_7 & -lm_8 & m_7 & -lm_8 \\ lm_8 & -l^2 m_{10} & -lm_8 & l^2 m_8 \end{bmatrix}$

Where $m_1 = 156 + 294\lambda + 140\lambda^2$;

$m_2 = 22 + 38.5\lambda + 17.5\lambda^2$;

$m_3 = 54 + 126\lambda + 70\lambda^2$

$m_4 = 13 + 31.5\lambda + 17.5\lambda^2$;

$m_5 = 4 + 7\lambda + 3.5\lambda^2$;

$m_6 = 3 + 7\lambda + 3.5\lambda^2$;

$m_7 = 36$;

$m_8 = 3 - 15\lambda$;

$m_9 = 4 + 15\lambda + 10\lambda^2$;

$m_{10} = 1 + 5\lambda - 5\lambda^2$;

Now the total mass matrix [M]for the Timoshenko beam will be the summation of both $[M_T]$& $[M_R]$.

Therefore, $[M_T] = [M_T] + [M_R]$.

(2h) The Equation of motion of the beam:

The equation of motion for a multiple degree of freedom undamped structural system is represented as follows

$$[M]\{\ddot{y}\} + [K]\{y\} = \{F(t)\}$$

Where $\bar{y}$ and y are the respective acceleration and displacement vectors for the whole structure and {F(t)} is the external force vector.

CHAPTER-03

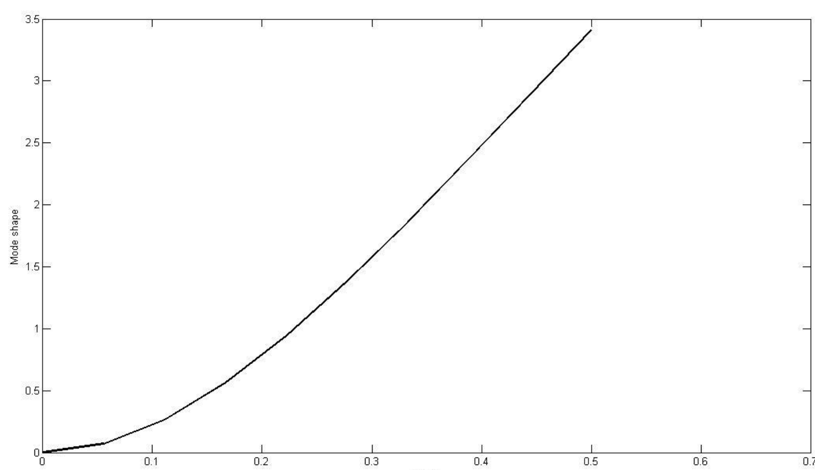
RESULT & DISCUSSION:

In this section the numerical results for cantilevered beam have been represented. The finite element modelling of beam is based on both Euler-Bernouli and Timoshenko beam theory. Here we have taken the cantilever beam & used different conditions in MATLAB code to get the behavior of the beam through graphs. For this discussion we have used Aluminium (Al) material having modulus of elasticity (E) 7.03 e 10 Pa & density ρ 2750kg/m3.

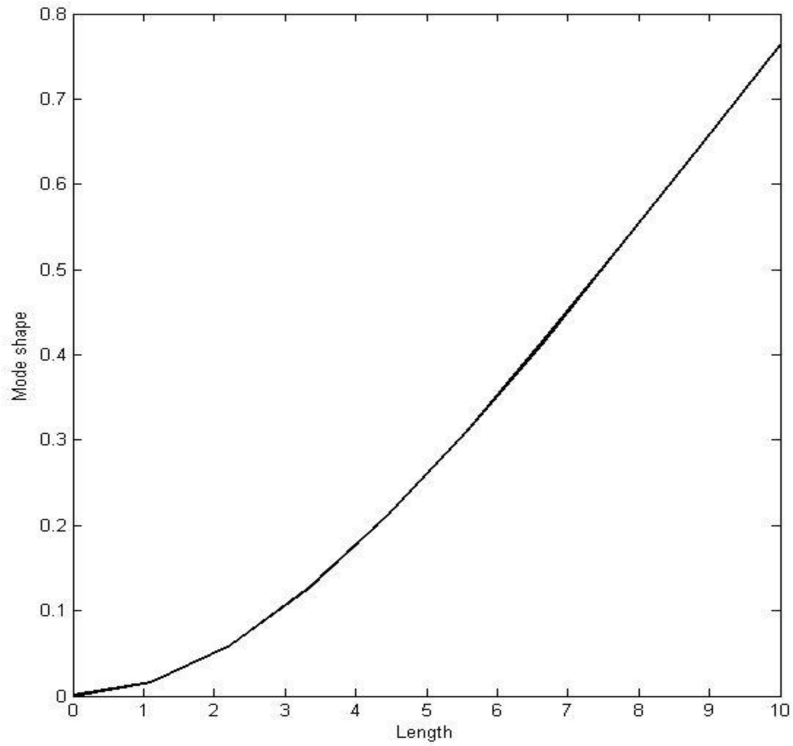
3.1 The beam is modeled by Euler-Bernouli beam theory

Case - I: Modal analysis

Figure 2 and figure 3 show that the first mode shape of the cantilebered beam for two different length (L = 0.5m and L = 10 m). The mode shape is independent of the length. The results are satisfactory. Which shows the correctness of the MATLAB code.



(Fig.2: First Mode shape (L = 0.5 m)



(Fig.3: First Mode shape (L = 10 m)

Case-II: **Frequency response**

Figure 4, figure 5 and figure 6 show that tip response of the cantilevered beam with different lengths when the beam is excited by harmonic excitation. When the excitation frequency matches with natural frequency, the resonance occour and consequently it will subject to severe vibration. The peaks corresponds to excitation frequency in the figures indicate the natural frequency for various modes. With increasing the length the beam stiffness will decrease as wel as fall in natural frequency. So more peaks are found when the length is high.

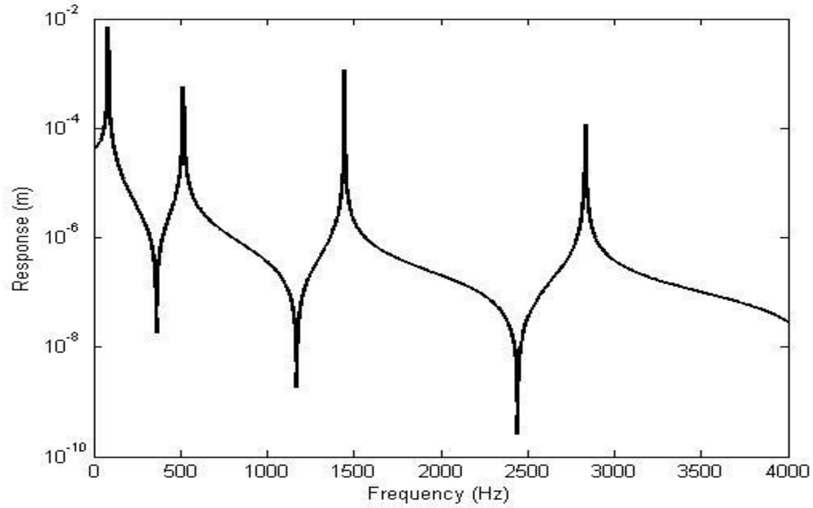


Fig.4: Response vs. Frequency for Euler Beam(L=0.5 m)

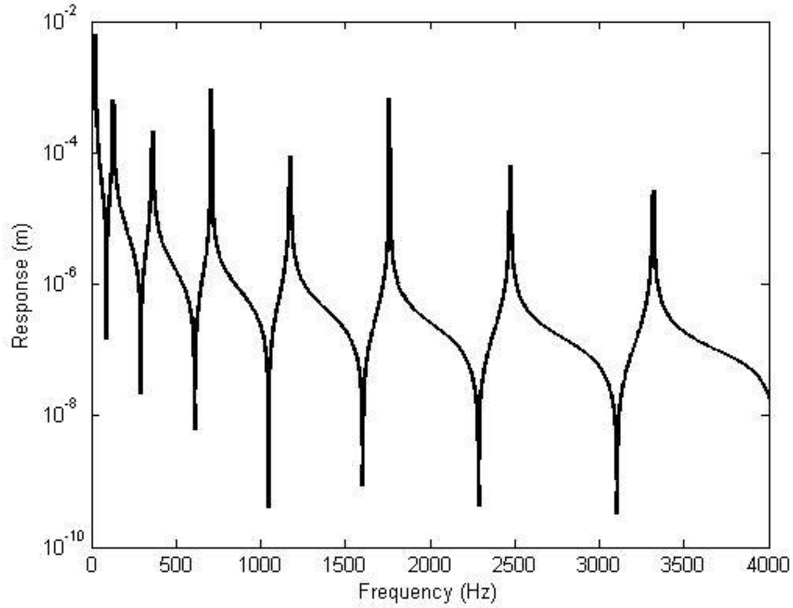


Fig.5: Response vs. Frequency for Euler Beam(L=1 m)

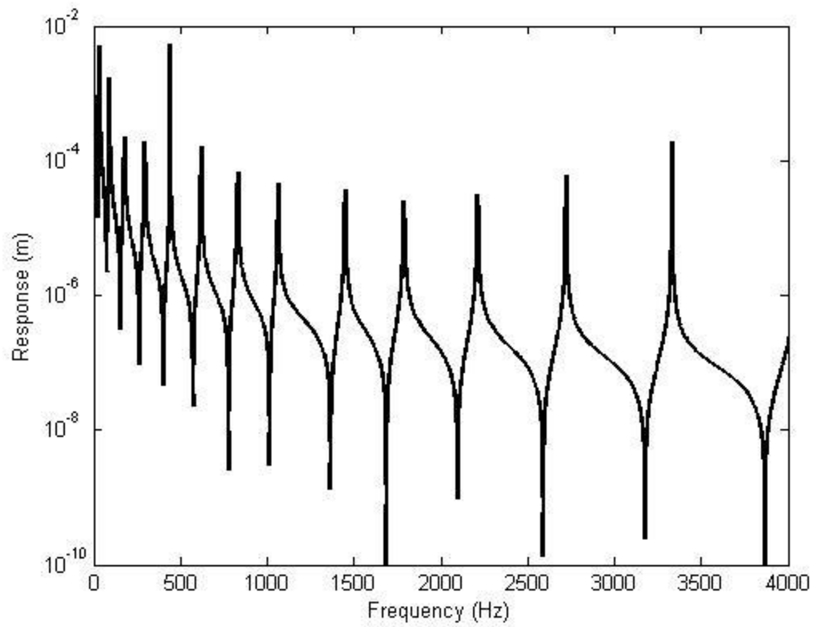
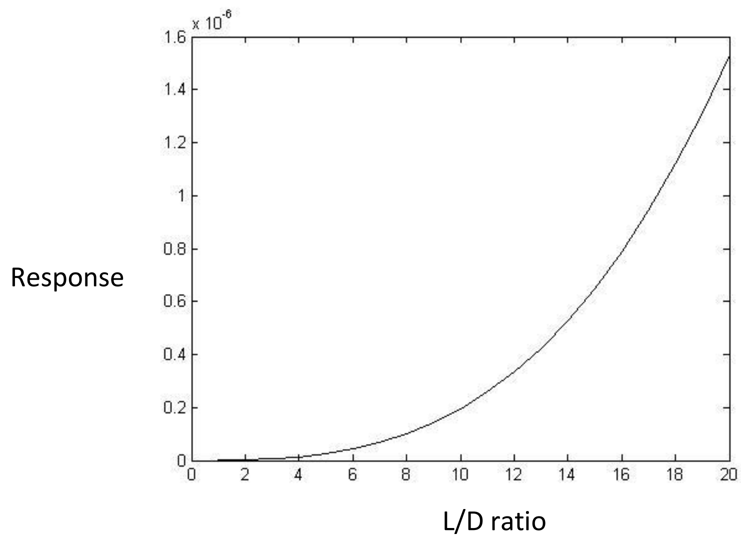


Fig.6: Response vs. Frequency for Euler Beam(L=2 m)

3.2 The beam is modeled by Timosenko beam theory

Case I: Static response

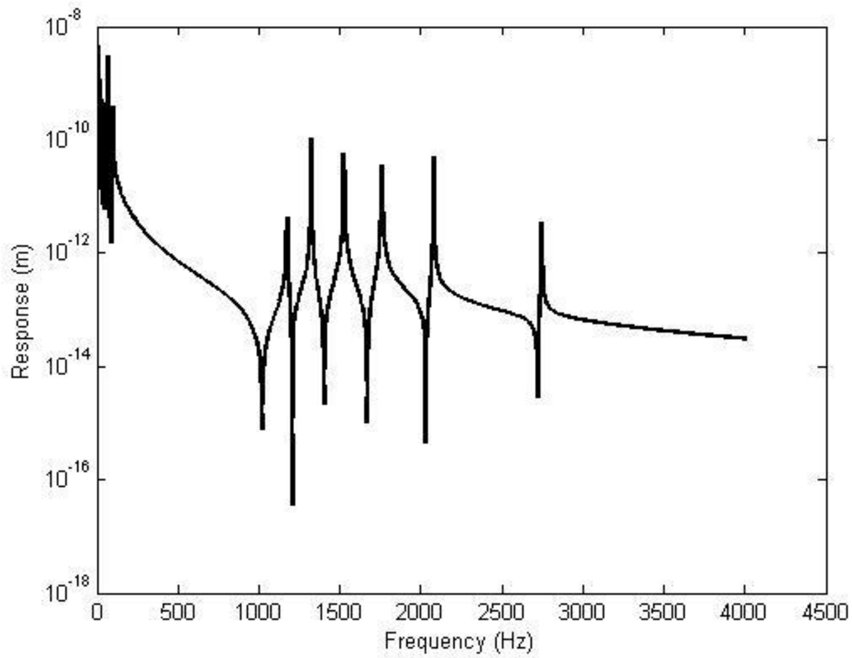
Figure 7 shows the static response of the cantilevered beam at the tip. The beam having a circular cross section with L = 0.5 m. With decreasing the diameter the response increases due to decrease of stiffness.



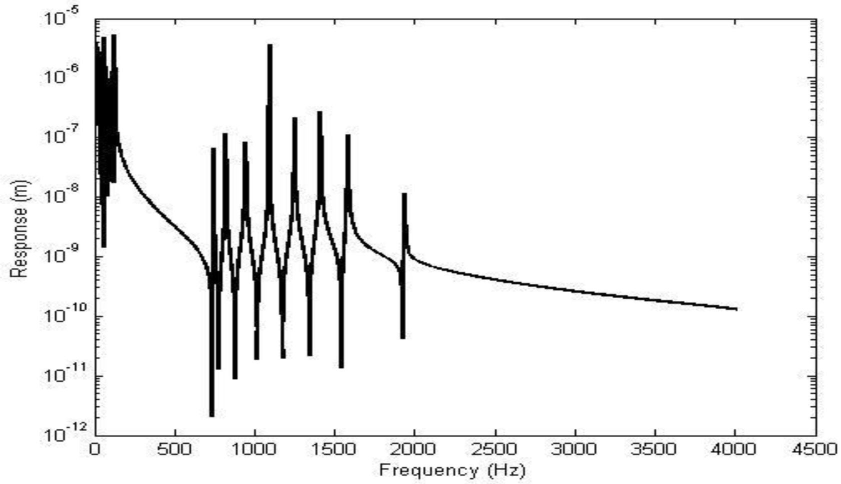
(Fig.7: Response vs. L/D ratio for Timoshenko Beam)

Case - II: Frequency response

Figure 8 shows the tip response for the cantilevered beam under sinusoidal excitation. The length of the beam is 1 m. In this case more peaks are found as compared to Euler-Bernouli case. The Timosenko beam theory considers the transverse shear deformation. Due to this, there is a chance in the fall in stiffness as wel as fall in natural frequency. figure 9 shows the frequency response of the same cantilever beam for rectangular cross section.



(Fig.8: Response vs. Frequency



(Fig.9: Response vs. Frequency

CHAPTER-04

CONCLUSIONS:

In the present analysis the finite element formulation for transversely loaded beam have been done. The beam is modeled by both Euler-Bernouli and Timosenko beam theoriy. The behavior of Timoshenko beam is same as that of Euler-Bernoulli beam when the shear factor is neglected excluding the shear deformation.

Using FEM analysis, we get different shape functions for both Euler-Bernoulli beam & Timoshenko beam. As the shape functions differ for both of the beam, so that the stiffness matrix & mass matrix for both of the beam are also different.

The mode shape for both Euler-Bernoulli beam & Timoshenko beam is independent of geometric dimensions like length, width, height.

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