

Damping of composite material structures with bolted joints

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UNDER THE GUIDANCE OF

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CERTIFICATE

This is to certify that the thesis titled “**Damping of composite material structures with bolted joint**” being submitted to the National Institute of Technology, Rourkela by **Mr. Sudhir Kumar Panigrahi** for the award of the degree of **B. Tech in Mechanical Engineering** is a record of bonafide research work carried out by him under my supervision and guidance. His work has reached the standard fulfilling the requirements and regulations for the degree. To the best of my information, the work incorporated in this thesis has not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

Place:

Prof. B.K. Nanda

Department of Mechanical Engineering

Date:

National Institute of Technology, Rourkela

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ABSTRACT

In the present work, the damping in composite structures with bolted joint has been extensively studied by the use of finite element analysis model. The various operations viz. modal, harmonic response and transient dynamic analyses are stimulated to find out different results in ANSYS and calculations were done as per the results from ANSYS to find further results. The parameters related to damping in composite bolted joints are found. It has been inferred from the paper that damping in composite bolted joint is very high. Rayleigh's damping constants and loss factor in the structure has also been calculated. A detailed mathematical and analytical model has been presented for understanding damping better. This topic of research is beneficial for various upcoming and ongoing products in the market.

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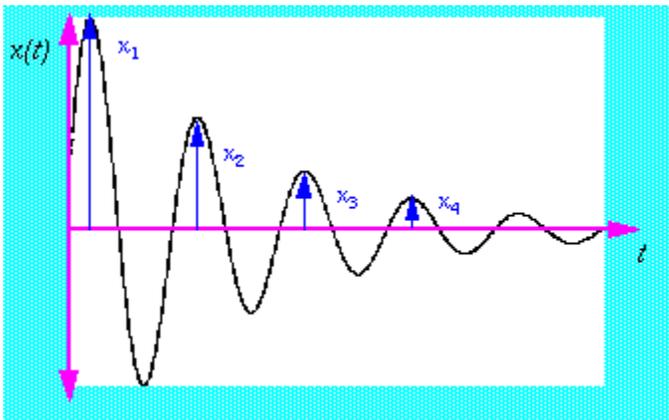
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CHAPTER 1: INTRODUCTION

1.1 Damping

Damping is a process on which vibrational energies are converted into some other energy form, usually heat. The kinds of vibration dealt with are either mechanical vibrations or acoustic vibrations. At the basic fundamental level, both of these vibrations are really the same and can be treated in a similar kind of way. It is certain to have both kinds of vibrations present at the same time. The vibrations we see are movements of the component itself rather than just internal vibrations of the molecules. For instance, a bullet hits a kevlar, it creates such an impulse on hitting that the soldier is pushed away but the kevlar damps all the vibrational energies in no time.

We all realize that eventually things stop vibrating as will the many parts made of composite materials which can often vibrate. The objectives of this paper are to examine the effects of vibration on composite structure with bolted joints, the damping effects of the assembly, and how a bolted joint helps structures to be more stable.



1.2 The Importance of Damping

Vibrations are essential and desirable, such as in a guitar or drum where the vibrations create proper sound. In most cases, however, vibrations are not needed and needs to be controlled by effective measures. Four examples cited in which vibrations can be considered disadvantageous and therefore we need damping. These are:

Wear and Damage

Vibrations can damage the part itself or mechanisms attached to the vibrating part. An evident example is an airplane part that breaks because of these vibrations. Even the breaking of a part from an impact can have vital implications in damping. Fatigue failure is another example of vibration caused damage.

Deformation and Misalignment

Vibrations can cause a part to move or to deform in an awkward way. The telescopes are a prime example where they need to be avoided to take perfect pictures.

Discomfort

Some vibrations are uncomfortable if not controlled like the vibrations that might be sensed from a driver seat in a car or the whack of a tennis racket. The economic and research potential for these applications are huge.

Noise

It is a form of energy that is unavoidable in any kind of vibratory or hit conditions of parts. There is a need for perfect material selection for the purpose.

1.3 Fundamental Mathematical Concepts

Mathematically, damped vibration of a structural model can be expressed differentially as,

$$m\ddot{q} + c\dot{q} + kq = 0$$

where m is the mass

c is the damping coefficient

k is the stiffness

Dynamic Modulus is the ratio of stress to strain under vibratory conditions. It is a property of viscoelastic materials. Viscoelasticity is studied using dynamic mechanical analysis where an oscillatory force (stress) is applied to a material and the resulting displacement (strain) is measured. Viscoelastic materials exhibit behavior somewhere in between that of a purely elastic and viscous, i.e. having some phase lag in strain to stress. Stress and strain in a viscoelastic material can be represented as,

$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$\sigma = \sigma_0 \sin(\omega t + \delta)$$

Storage and loss moduli in viscoelastic solids measure the stored energy (elastic portion) and the dissipated energy (viscous portion)

Tensile storage and loss moduli are defined as,

- Storage, $E' = \frac{\sigma_0}{\varepsilon_0} \cos \delta$
- Loss, $E'' = \frac{\sigma_0}{\varepsilon_0} \sin \delta$

Similarly, we can define shear storage and loss moduli, G' and G'' .

Stiffness (k) is a function of E' and a geometric factor.

Co-efficient of viscosity (c) is a function of E'' and a geometric factor.

1.4 Damping in Composites

The amount of damping in a part is dependent on:

- The materials out of which the part is made,
- The design of the model/experimental prototype and
- An element that might be added to the part which could include specific damping treatments and whose effect on damping is more complicated.

Composites have better damping properties than structural metals. Normally, the range of composite damping begins where the best damped structural metals end. The damping in composites is controlled by the matrix properties, the fiber properties, the interaction between the fibers and the matrix, laminar stacking sequence, and embedded viscoelastic layers.

1.4.1 Matrix properties

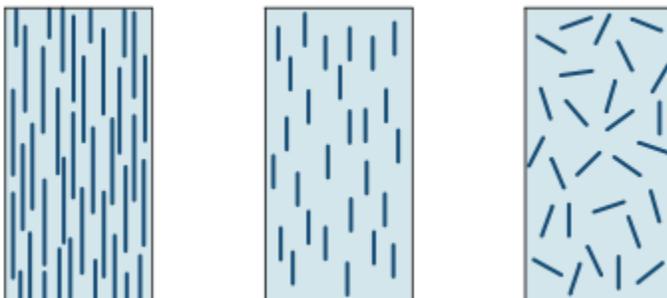
The matrix properties which relate to damping are E' and E'' , as has already been discussed earlier. Matrix materials with high amounts of internal molecular motion have higher E'' . Therefore the use of thermoplastic composite is desirable, such as those that can be made by injection molding, will improve damping. Of course, other considerations, such as overall strength and stiffness, may prohibit the use of these thermoplastic composites in some applications.

1.4.2 Fiber properties

The fiber properties which relate to damping are also E' and E'' . Generally, the amount of E'' in these fibers is quite small and so their internal energy loss is not usually considered to contribute directly to damping, at least in comparison to the matrix. In aramids, however, the values of E'' can be important and should be taken care of.

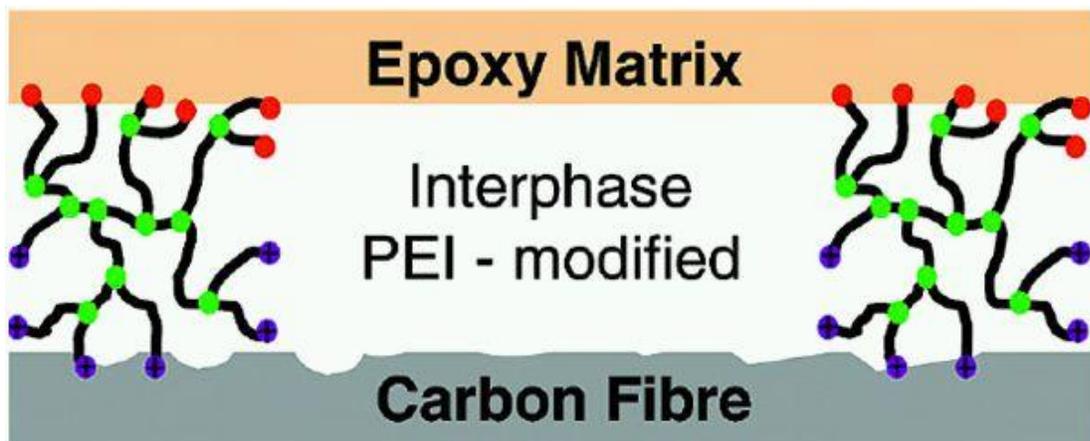
1.4.3 Interactions between fibers and matrix

The damping arising from the interactions between fibers and matrix can be very large and are quite complex because so many aspects of composites affect the interactions. For example, the damping arising from the interactions can be affected by fiber length, fiber orientation, and interface effects. Within almost any reasonable fiber length range, the effect of length on damping is very small and can be neglected. What little effect there is, seems to suggest that shorter fibers give slightly better damping, probably because there are more ends and, therefore, more interactions with the matrix. Fillers can be thought of as very short fibers. Therefore, the possibility exists that fillers can improve damping. However, perfectly spherical fillers show no damping improvement. Therefore, there are fillers to cause significant damping, they must be non-spherical but not very long, perhaps like short whiskers. Damping is increased when the orientation of the fibers is off-axis by 5 to 30°, with carbon fiber being generally in the higher end of that angular range. Generally the stiffer the carbon fiber is, the smaller the angle for maximum damping. Going all the way to 90° results in a composite where the damping is principally controlled by the matrix and the fiber has only a small contribution. Random fiber orientation will, in general, result in higher damping than would occur with aligned fibers.



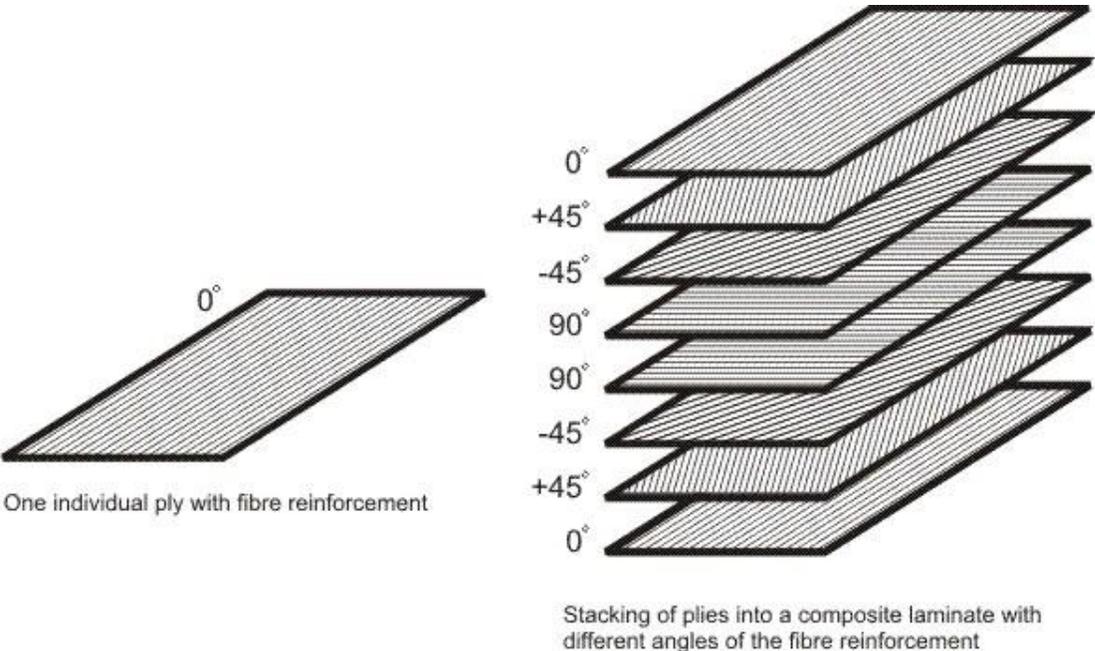
1.4.4 Fiber-matrix interface or interphase region

It is an area where energy can be converted into heat and thus, it is a region of potentially high damping. Factors which would tend to increase the energy loss in this area are: poor fiber-matrix adhesion, low modulus of the interphase itself, high molecular motion within the interphase, and a high total volume associated with the interface or interphase. Unfortunately, although these would all give higher damping, they would also result in decreased stiffness and, perhaps, lower strength and other normally desirable properties. Because of the effort to improve interfacial bonding, the damping effects of the interface are usually small. However, some circumstances could arise, such as the absorption of water at the interface or intentional poor bonding as in composite armor, which would cause the interface effects to be quite large.



1.4.5 Interlaminar stacking sequence effects

In the stacking sequence, we put the plies with the most damping (that is, those which are off-axis) in areas where they would experience the most strain, damping is raised. For instance, in bending, the outer parts of the composite would have the most strain and would be the places where the off-axis layers should be placed to maximize damping. Sadly, that type of placement will also decrease the overall stiffness of the laminate. We must, therefore, deal with tradeoffs. When a laminate is stuck on its surface, the arrangement of stacking can affect the amount of energy that is transmitted from one fiber layer to another. The maximum amount of energy transmission occurs when the layers are aligned, and the minimum energy transmission occurs when the layers are orthogonal. When the energy is not transmitted, it is either converted to heat in fiber layers interface (thus causing damping) or it is switched into a sideways delamination. Therefore, the fiber sequences that improve damping are the same as those that make delamination worse. The best overall state might be attained, therefore, by using sequence in which the angles between the layers are more than 0° but less than 90° .



Chapter 2 Literature review

2.1 Introduction

The widespread use of composite structures in aerospace applications has stimulated many researchers to study various aspects of their structural performance. These materials are particularly widely used in situations where a large strength : weight ratio is required. Similarly to isotropic materials, composite materials are subjected to various types of damage, mostly cracks and delamination. These result in local changes of the stiffness of elements for such materials and consequently their dynamic characteristics are altered.

Most engineering structures are built up by connecting structural components through mechanical connections. Such assembled structures need sufficient damping to limit excessive vibrations under dynamic loads. Damping in such structures mainly originates from two sources. One is the internal or material damping which is inherently low [1] and the other one is the structural damping due to joints [2].

The latter one offers an excellent source of energy dissipation, thereby adequately compensating the low material damping of structures. But, this is only in case of metallic structures and not in composites. It is estimated that metallic structures consisting of bolted or riveted members contribute about 90% of the damping through the joints. The internal damping or material damping in case of composites is generally more, when compared to material damping in metallic structures. Often, damping in composites starts when the best damped metal stops. For this very reason, damping in composites is of recent interest and many researches are being done.

2.2 Overview on damping

The three essential parameters that determine the dynamic responses of a structure and its sound transmission characteristics are mass, stiffness and damping. Mass and stiffness are associated with storage of energy. Damping results in the dissipation of energy by a vibration system. For a linear system, if the forcing frequency is the same as the natural frequency of the system, the response is very large and can easily cause dangerous consequences. In the frequency domain, the response near the natural frequency is "damping controlled". Higher damping can help to reduce the amplitude at resonance of structures. Increased damping also results in faster decay of free vibration, reduced dynamic stresses, lower structural response to sound, and increased sound transmission loss above the critical frequency. A lot of literatures have been published on vibration damping. ASME published a collection of papers on structural damping in 1959 [6]. Lazan's book published in 1968 gave a very good review on damping research work, discussed different mechanisms and forms of damping, and studied damping at both the microscopic and macroscopic levels [7]. Lazan conducted comprehensive studies into the general nature of material damping and presented damping results data for almost 2000 materials and test conditions. Lazan's results show that the logarithmic decrement values increase with dynamic stress, i.e., with vibration amplitude, where material damping is the dominant mechanism. This book is also valuable as a handbook because it contains more than 50 pages of data on damping properties of various materials, including metals, alloys polymers, composites, glass, stone, natural crystals, particle-type materials, and fluids. About 20 years later, Nashif, Jones and Henderson published another comprehensive book on vibration damping [8]. Jones himself wrote a handbook especially on viscoelastic damping 15 years later [9]. Sun and Lu's book published in 1995 presents recent research accomplishments on vibration damping in beams,

plates, rings, and shells [10]. Finite element models on damping treatment are also summarized in this book. There is also other good literature available on vibration damping [11-13]. Damping in vibrating mechanical systems has been subdivided into two classes: Material damping and system damping, depending on the main routes of energy dissipation. Coulomb (1784) postulated that material damping arises due to interfacial friction between the grain boundaries of the material under dynamic condition. Further studies on material damping have been made by Robertson and Yorgiadis (1946), Demer (1956), Lazan (1968) and Birchak (1977). System damping arises from slip and other boundary shear effects at mating surfaces, interfaces or joints between distinguishable parts. Murty (1971) established that the energy dissipated at the support is very small compared to material damping.

2.3 Review on Research done in Damping of Composite materials

Bert [14] and Nashif et al.[15] had done survey on the damping capacity of fibre reinforced composites and found out that composite materials generally exhibit higher damping than structural metallic materials. Chandra et al. [16] has done research on damping in fiber-reinforced composite materials.

Composite damping mechanisms and methodology applicable to damping analysis is described and had presented damping studies involving macro-mechanical, micromechanical and Viscoelastic approaches. Gibson et al.[17] and Sun et al.[18,19] assumed viscoelasticity to describe the behaviour of material damping of composites.

The concept of specific damping capacity (SDC) was adopted in the damped vibration analysis by Adams and his co-workers [20-21], Morison [22] and Kinra et al [23].

The concept of damping in terms of strain energy was apparently first introduced by Ungar et.al [24] and was later applied to finite element analysis by Johnson et.al [25]. Gibson et.al [26] has developed a technique for measuring material damping in specimens under forced flexural vibration. Suarez et al [27] has used Random and Impulse Techniques for Measurement of Damping in Composite Materials. The random and impulse techniques utilize the frequency-domain transfer function of a material specimen under random and impulsive excitation. Gibson et al [28] used the modal vibration response measurements to characterize, quickly and accurately the mechanical properties of fiber-reinforced composite materials and structures.

Lin et al. [29] predicted SDC in composites under flexural vibration using finite element method based on modal strain energy (MSE) method considering only two inter laminar stresses and neglecting transverse stress.

Koo KN et al. [30] studied the effects of transverse shear deformation on the modal loss factors as well as the natural frequencies of composite laminated plates by using the finite element method based on the shear deformable plate theory.

SINGH S. P et al. [31] analysed damped free vibrations of composite shells using a first order shear deformation theory in which one assumes a uniform distribution of the transverse shear across the thickness, compensated with a correction factor.

Polymeric materials are widely used for sound and vibration damping. One of the more notable properties of these materials, besides the high damping ability, is the strong frequency dependence of dynamic properties; both the dynamic modulus of elasticity and the damping characterized by the loss factor [30-35].

Mycklestad [32] was one of the pioneering scientists into the investigation of complex modulus behaviour of viscoelastic materials (Jones, 2001, Sun, 1995). Viscoelastic material properties are

generally modelled in the complex domain because of the nature of visco-elasticity. Viscoelastic materials possess both elastic and viscous properties. The typical behaviour is that the dynamic modulus increases monotonically with the increase of frequency and the loss factor exhibits a wide peak [8, 33].

It is rare that the loss factor peak, plotted against logarithmic frequency, is symmetrical with respect to the peak maximum, especially if a wide frequency range is considered. The experiments usually reveal that the peak broadens at high frequencies. In addition to this, the experimental data on some polymeric damping materials at very high frequencies, far from the peak centre, show that the loss factor–frequency curve “flattens” and seems to approach a limit value, while the dynamic modulus exhibits a weak monotonic increase at these frequencies [34-38]. These phenomena can be seen in the experimental data published by Madigosky and Lee [34], Rogers [35] and Capps [36] for polyurethanes, and moreover by Fowler [37], Nashif and Lewis [38] for other polymeric damping materials.

The computerized methods of acoustical and vibration calculus require the mathematical form of frequency dependences of dynamic properties. A reasonable method of describing the frequency dependences is to find a good material model fitting the experimental data.

Chapter 3 Viscoelastic Damping

3.1 Important viscoelastic behaviors that affect in damping are

- Creep under constant stress
- Relaxation under constant strain
- Hysteresis loop due to cyclical stress
- Strain rate dependency on strain rate curve

These behaviors are discussed in the later sections of the chapter. This paper describes the damping behavior of carbon epoxy composite with bolted joint. The bolt, nut and washer used are of structural steel.

3.2 Material Used

The composite material used in the analysis Carbon Fiber Composite Materials, Fiber / Epoxy resin (120°C Cure).

Mechanical properties

- Fibers @ 0° (UD), 0/90° (fabric) to loading axis, Dry, Room Temperature, $V_f = 60\%$ (UD), 50% (fabric)
- Epoxy resin and Standard CF Fabric

	Symbol	Units	Std CF Fabric
Young's Modulus 0°	E1	GPa	70
Young's Modulus 90°	E2	GPa	70
In-plane Shear Modulus	G12	GPa	5
Major Poisson's Ratio	v12		0.10
Ult. Tensile Strength 0°	Xt	MPa	600
Ult. Comp. Strength 0°	Xc	MPa	570
Ult. Tensile Strength 90°	Yt	MPa	600
Ult. Comp. Strength 90°	Yc	MPa	570
Ult. In-plane Shear Stren.	S	MPa	90
Ult. Tensile Strain 0°	ext	%	0.85
Ult. Comp. Strain 0°	exc	%	0.80
Ult. Tensile Strain 90°	eyt	%	0.85
Ult. Comp. Strain 90°	eyc	%	0.80
Ult. In-plane shear strain	es	%	1.80
Thermal Exp. Co-ef. 0°	Alpha1	Strain/K	2.10
Thermal Exp. Co-ef. 90°	Alpha2	Strain/K	2.10
Moisture Exp. Co-ef 0°	Beta1	Strain/K	0.03
Moisture Exp. Co-ef 90°	Beta2	Strain/K	0.03
Density		g/cc	1.60

3.3 Viscous Damping

When mechanical systems vibrates in a fluid medium, the resistance offered by the fluid to the moving body causes energy to be released. The amount of released energy depends on many factors, such as the size and shape of the vibrating body, the viscosity of the fluid, the frequency of vibration, and the velocity of the vibrating body. In viscous damping, the damping force is proportionate to the velocity of the vibrating body. Viscous damping force can be expressed by the equation,

$$F = -c\dot{x}$$

Where, c is a constant of proportionality and the velocity of the mass shown in Figure.

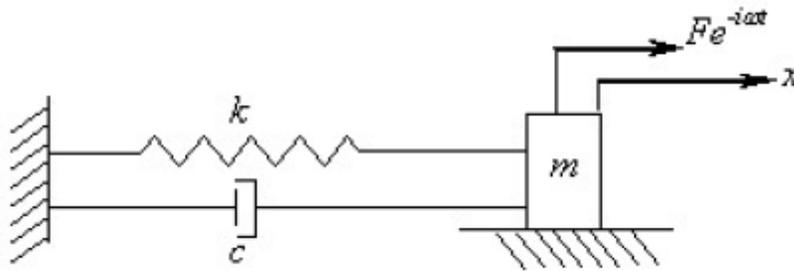


Figure 1 – Simple Spring Viscous Damping

When the single spring mass system undergoes free vibration, the equation of motion becomes

$$m\ddot{x} + c\dot{x} + kx = 0$$

Assuming a solution of the form, we have the eigen or the characteristic equation of the system as,

$$ms^2 + cs + k = 0$$

The solution of the equation is

$$x = e^{-\frac{c}{2m}t} \left(A e^{\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}t} + B e^{-\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}t} \right)$$

Where, A and B are arbitrary constants depending on how the motion is started.

It is observed that the behavior of the damped system depends on the numerical value of the radical in the exponential of equation. As a reference quality, a critical damping C_c is defined which reduces this radical to zero

$$\left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} = 0 \quad \text{or} \quad c_c = 2\sqrt{km} = 2m\omega_n$$

where ω_n is the natural circular frequency of the system. $\omega_n = \sqrt{k/m}$

An important parameter to describe the properties of the damping is damping ratio ζ , which is a non-dimensional ratio as

$$\zeta = \frac{c}{c_c} = \frac{c}{2m\omega_n}$$

Based on the value of damping ratio, the motion of the mass in Figure 1 can be divided into the following three cases:

- (1) Oscillatory motion when $0.1 < \zeta$;
- (2) Non Oscillatory motion when $0.1 > \zeta$ and
- (3) Critical damped motion when $0.1 = \zeta$. In last case, the general solution of the system is

$$x = (A + Bt)e^{-\omega_n t}.$$

Viscous damping can be used whatever may be the form of the excitation. The viscous damping is the Rayleigh-type damping given by

$$c = \alpha m + \beta k$$

3.4 Coulomb or Frictional Damping

Coulomb damping results from the sliding of two dry surfaces. The damping force is equal to the product of the normal force and the coefficient of friction μ and is assumed to be independent of the velocity, once the motion is initiated.

Because the sign of the damping force is always opposite to that of the velocity, the differential equation of motion for each sign is valid only for half-cycle intervals.

$$m\ddot{x} + kx = -\mu N$$

This is a second order nonhomogeneous differential equation. The solution can be expressed as

$$x(t) = A \cos \sqrt{\frac{k}{m}}t + B \sin \sqrt{\frac{k}{m}}t - \frac{\mu N}{k}$$

3.5 Hysteretic or Structural Damping

When the materials are deformed, energy is absorbed and dissipated by the material itself. The effect is due to friction between the internal planes, which slip or slide as the deformations take place. When a structure having material damping is subjected to vibration, the stress-strain diagram shows a hysteresis loop. Therefore, the structural damping is also called hysteretic damping. The area of this loop denotes the energy lost per unit volume of the body per cycle due to the damping.

To explain the hysteretic damping, the relationship between the response x and excitation force for viscous damping. For a harmonic motion, the relationship between them behaves as

$$F(t) = (-ic\omega + k)x$$

Equation gives the energy dissipated in one vibration cycle which is the area of the loop above

$$\Delta W = \oint \text{Im}(F) d \text{Im}(x) = \int_0^{2\pi/\omega} (cX\omega \cos \omega t + kX \sin \omega t)(X\omega \cos \omega t) dt = \pi \omega c X^2$$

where “Im” is the imaginary symbol.

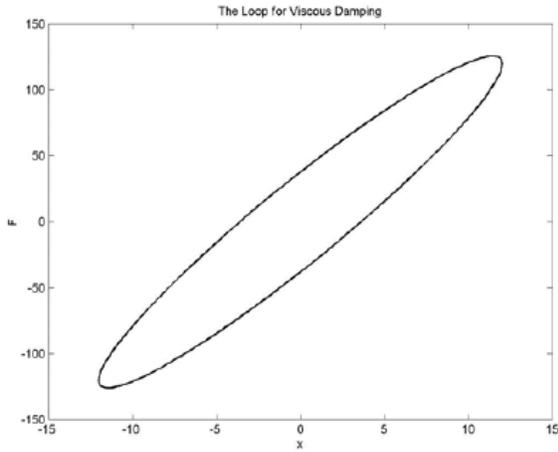


Figure 2 - The Loop for Viscous Damping

For the hysteretic damping, similarly, there is a hysteresis loop to be formed in the stress-strain or force-displacement curve in one loading and unloading cycle. It has been found experimentally that the energy loss per cycle due to internal friction is independent of the frequency for most structural metals, but approximately proportional to the square of the amplitude. In order to achieve the observed behavior from the equation above, the equivalent damping coefficient C_{eq} is assumed to be inversely proportional to the frequency as

$$c_{eq} = \frac{h}{\omega}$$

where h is a hysteretic damping coefficient.

After Substitution, the results in the energy dissipated by the hysteretic damping in a cycle of motion.

$$\Delta W = \pi h X^2$$

Chapter 4 Mathematical Model of Damping Properties

4.1 Structural damping factor γ

The viscous damping coefficient c , hysteretic damping coefficient h and the damping ratio ζ . There is another very vital factor, structural damping factor γ , to describe the property of the damping material.

The forced motion equation of a single spring mass system with a hysteretic damper is

$$m\ddot{x} + c_{eq}\dot{x} + kx = f(t)$$

For a harmonic problem, it becomes

$$-\omega^2 mx + k\left(1 - i2\frac{\omega}{\omega_n}\zeta_{eq}\right)x = f(t) \quad \text{where } \zeta_{eq} = \frac{c_{eq}}{c_c} = \frac{h}{2m\omega_n\omega}.$$

For the modal damping, $\omega = \omega_n$, therefore, we have

$$m\ddot{x} + k(1 - i\gamma)x = f(t)$$

Where, $\gamma = 2\zeta_{eq} = h/k$ is called the structural damping factor or modal damping ratio.

For the viscous damping, similarly, the viscous damping factor is $\gamma=2\zeta$.

4.2 Complex Stiffness

The effect of polymer material on the damping of the whole structure is influenced by the material stiffness as well as by its damping. These two properties are conveniently quantified by the complex Young's modulus or the complex shear modulus and $E\eta$ are usually assumed to be equal for a given material.

When the material is subjected to cyclic stress and strain with amplitude σ_0 and ϵ_0 , the maximum energy stored and dissipated per cycle in a unit volume are as

$$\text{Maximum energy stored per cycle} = E\epsilon_0^2 / 2$$

$$\text{Energy dissipated per cycle} = \pi E\eta \epsilon_0^2$$

A physical description of the loss factor can be found as follows. The energy dissipated per cycle for a structural damped system is,

$$\Delta W = \pi hX^2 = \pi \eta kX^2 = 2\pi \eta \times \frac{1}{2} kX^2 = 2\pi \eta U_m$$

Where, U_m is the maximum strain energy stored. Therefore, we have energy strain maximum cycle per dissipated energy

$$\eta = \frac{1}{2\pi} \frac{\Delta W}{U_m} = \frac{1}{2\pi} \frac{\text{energy dissipated per cycle}}{\text{maximum strain energy}}$$

From the equation, it is found that the loss factor is a way to compare the damping of one material to another. It is a ratio of the amount of energy dissipated by the system at a certain frequency to the amount of the energy that remains in this system at the same frequency. The more damping a material has, the higher the loss factor will be. The method of representing the structural damping should only be used for frequency domain analysis (modal) where the excitation is harmonic.

Chapter 5 Modeling and Analysis of the Composite Bolted Joint

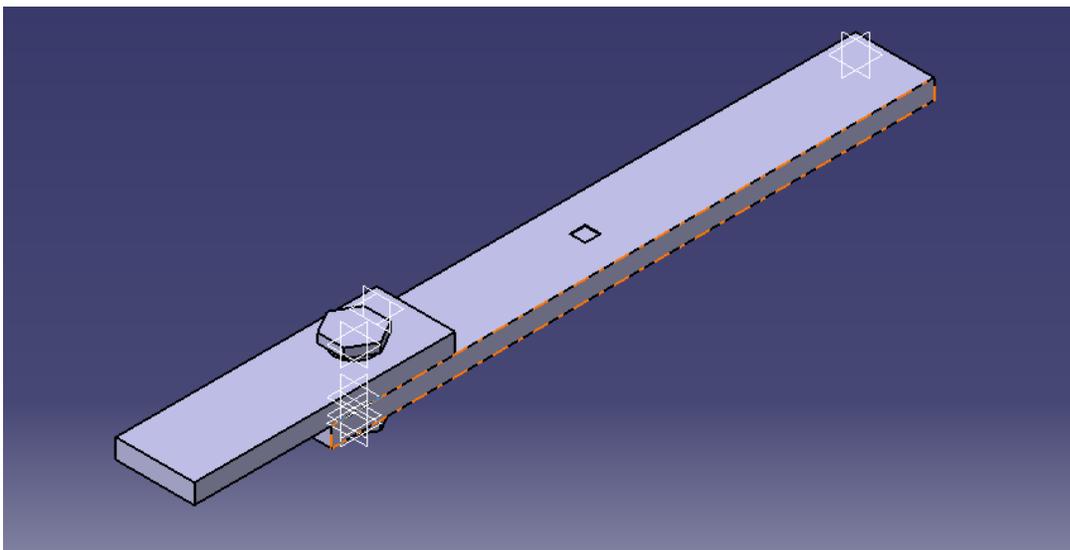
5.1 Modeling

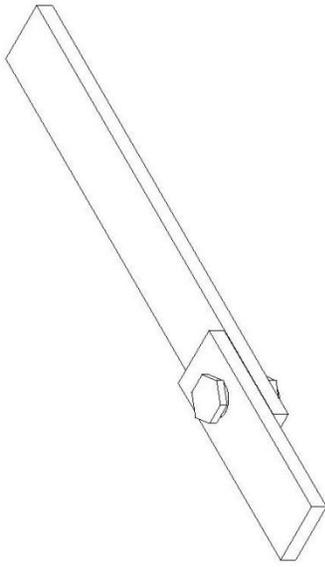
As discussed earlier, the geometry or the structure of the material matter in effective reduction in damping. The model prepared in this paper discusses a standard case in which bolts are generally used. The model is prepared in a workstation of CATIA V5R17.

Four parts of the joint

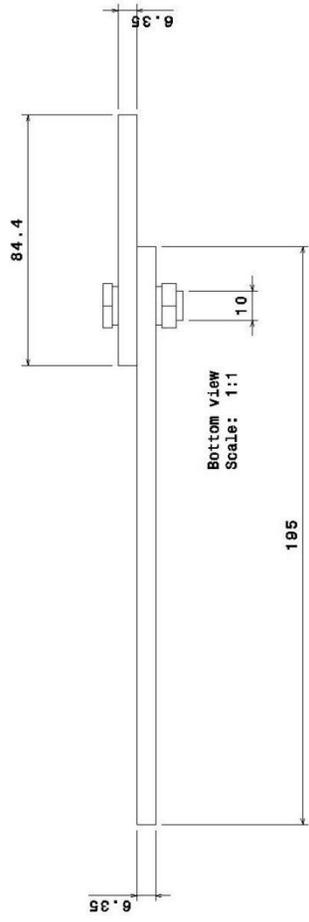
- Laminate 1 (generally larger in length)
- Laminate 2 (shorter)
- Bolt
- Nut
- 2 Washers

These parts are created individually and then assembled. The format of the saved assembly must be IGES for later analysis in ANSYS.





Isometric view
Scale: 1:1



Bottom view
Scale: 1:1



Front view
Scale: 1:1

5.2 Analysis of the model in ANSYS

In this paper, the consideration of vibration damping using software ANSYS for harmonic and modal analysis along with transient response and dynamic explicit modeling has been addressed. Several key points will be deduced after the analysis of the prepared model.

5.2.1 General Overview of Damping in ANSYS

The damping matrix \mathbf{C} in ANSYS may be used in harmonic, damped modal and transient analysis as well as substructure generation. In its most general form, it is:

$$[\mathbf{C}] = \alpha[\mathbf{M}] + \beta[\mathbf{K}] + \sum_{j=1}^{N_{sw}} \beta_j [\mathbf{K}_j] + \beta_c [\mathbf{K}] + [\mathbf{C}_\zeta] + \sum_{k=1}^{N_{sk}} [\mathbf{C}_k]$$

Where,

α constant mass matrix multiplier

β constant stiffness matrix multiplier

β_j constant stiffness matrix multiplier

β_c variable stiffness matrix multiplier

$$\beta_c = \frac{\zeta}{\pi f} = \frac{2\zeta}{\omega} = \frac{\eta}{\omega}$$

ζ constant damping ratio, the damping ratio ζ should be 2η where η is the loss factor.

f frequency in the range between f_b (beginning frequency) and f_e (end frequency);

$[\mathbf{C}_\zeta]$ frequency-dependent damping matrix

$[C_k]$ may be calculated from the stated ζ_r (damping ratio for mode shape r) and is never clearly computed.

$$\{u_r\}^T [C_r] \{u_r\} = 4\pi f_r \zeta_r$$

$\{u_r\}$ is the r^{th} mode shape

f_r frequency associated with mode shape r

$$\zeta_r = \zeta + \zeta_{mr}$$

ζ constant damping ratio

ζ_{mr} modal damping ratio for mode shape r

$[C_k]$ element damping matrix

5.2.2 Rayleigh Damping α and β

The most common form of damping is the so-called Rayleigh type damping $[C] = \alpha[M] + \beta[K]$.

The advantage of this representation is that the matrix becomes in modal coordinates

$$\bar{C} = \alpha \mathbf{I} + \beta \Lambda$$

\bar{C} is diagonal, so for the r^{th} mode, the equation of motion can be uncoupled. Each one is of the form

$$\ddot{q}_r + (\alpha + \beta\omega_r^2)\dot{q}_r + \omega_r^2 q_r = Q_r$$

Let $2\zeta_{mr}\omega_r = (\alpha + \beta\omega_r^2)$

The equation reduces to

$$\ddot{q}_r + 2\zeta_{mr}\omega_r\dot{q}_r + \omega_r^2q_r = Q_r$$

Where, ζ_{mr} is the r^{th} modal damping ratio.

The values of α and β are not known directly, but are calculated from modal damping ratios, ζ_{mr} .

It is the ratio of actual damping to critical damping for a particular mode of vibration, r . From the above equation, we have

$$\zeta_{mr} = \frac{\alpha}{2\omega_r} + \frac{\beta}{2}\omega_r$$

In many practical structural problems, the α mass proportional damping represents frictional damping may be ignored ($\alpha = 0$). In such case, the β damping can be estimated from known values of ζ_{mr} and ω_r which represents material structural damping. It is noted that only one value of β can be input in a load step, so we should select the most dominant frequency active in that load step to compute β .

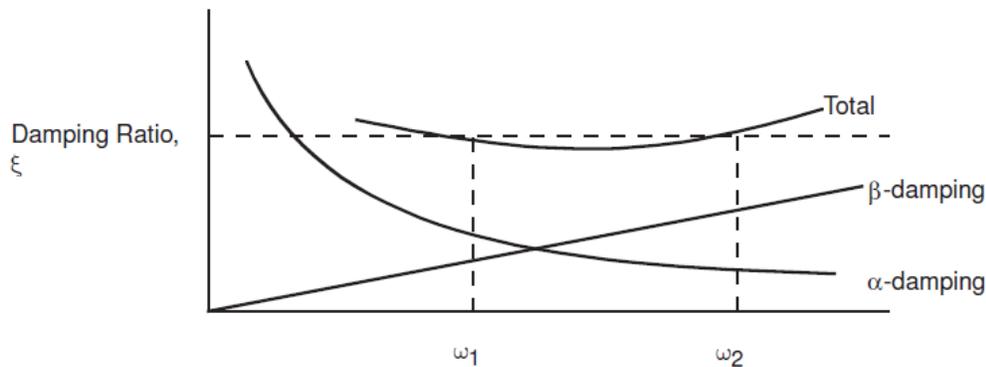


Figure3 - Rayleigh Damping

Chapter 6 Analysis of the model

In this paper, I have done various structural analysis of the previously prepared model in ANSYS.

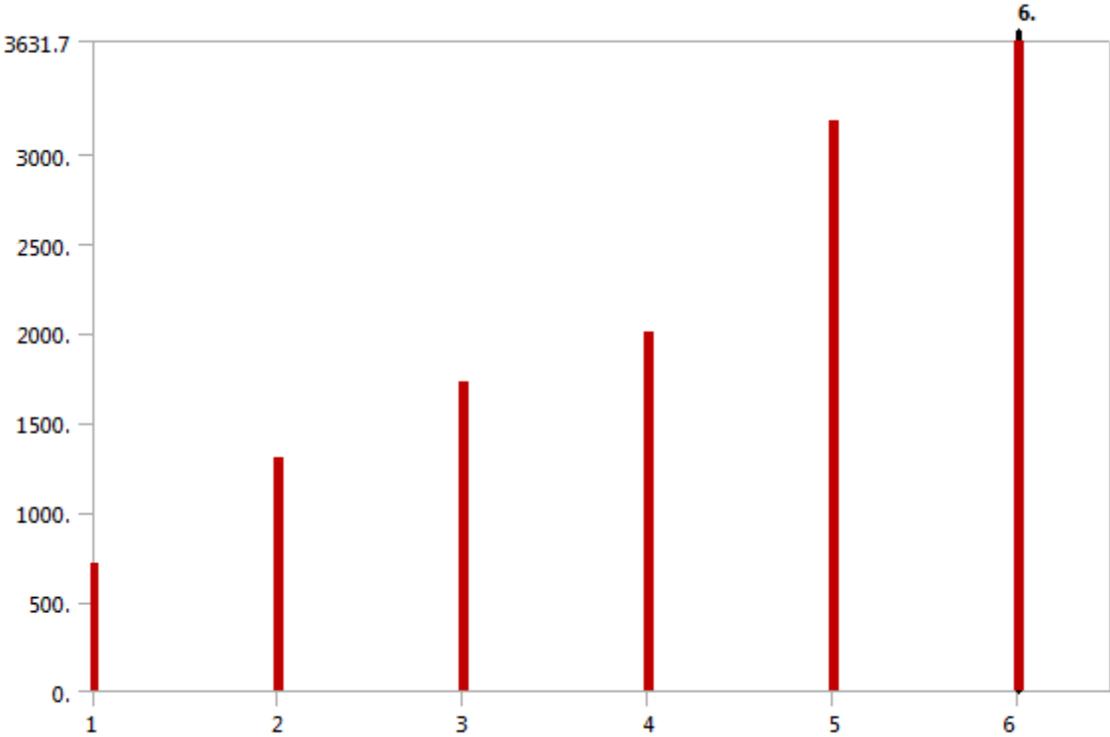


To start with,

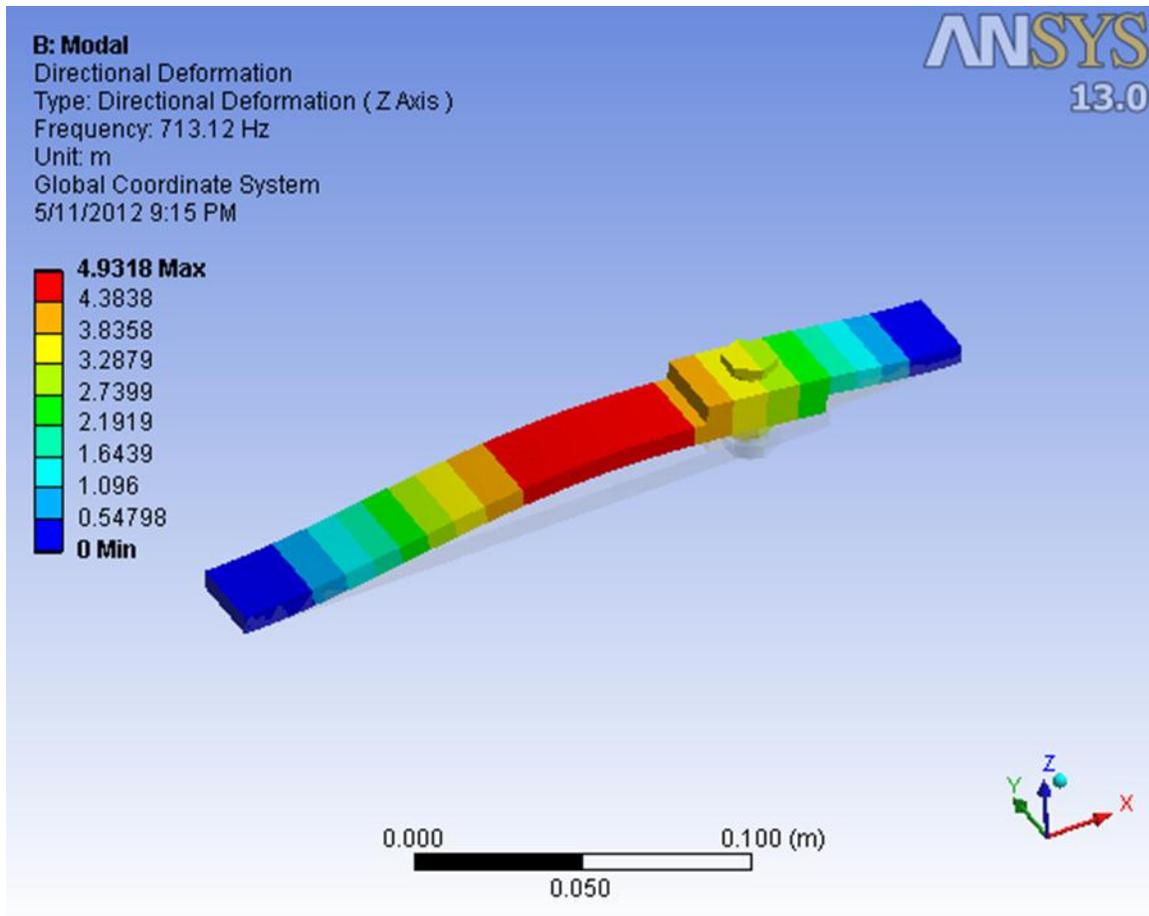
6.1 Modal Analysis

Modal analysis determines the natural frequency and mode shape of a structure. The natural frequency and mode shape are important parameters in the design of a structure for dynamic loading conditions and can be used in spectrum analysis or a mode superposition harmonic or transient analysis.

Mode	Frequency [Hz]
1.	713.12
2.	1304.5
3.	1731.1
4.	2003.9
5.	3188.7
6.	3631.7

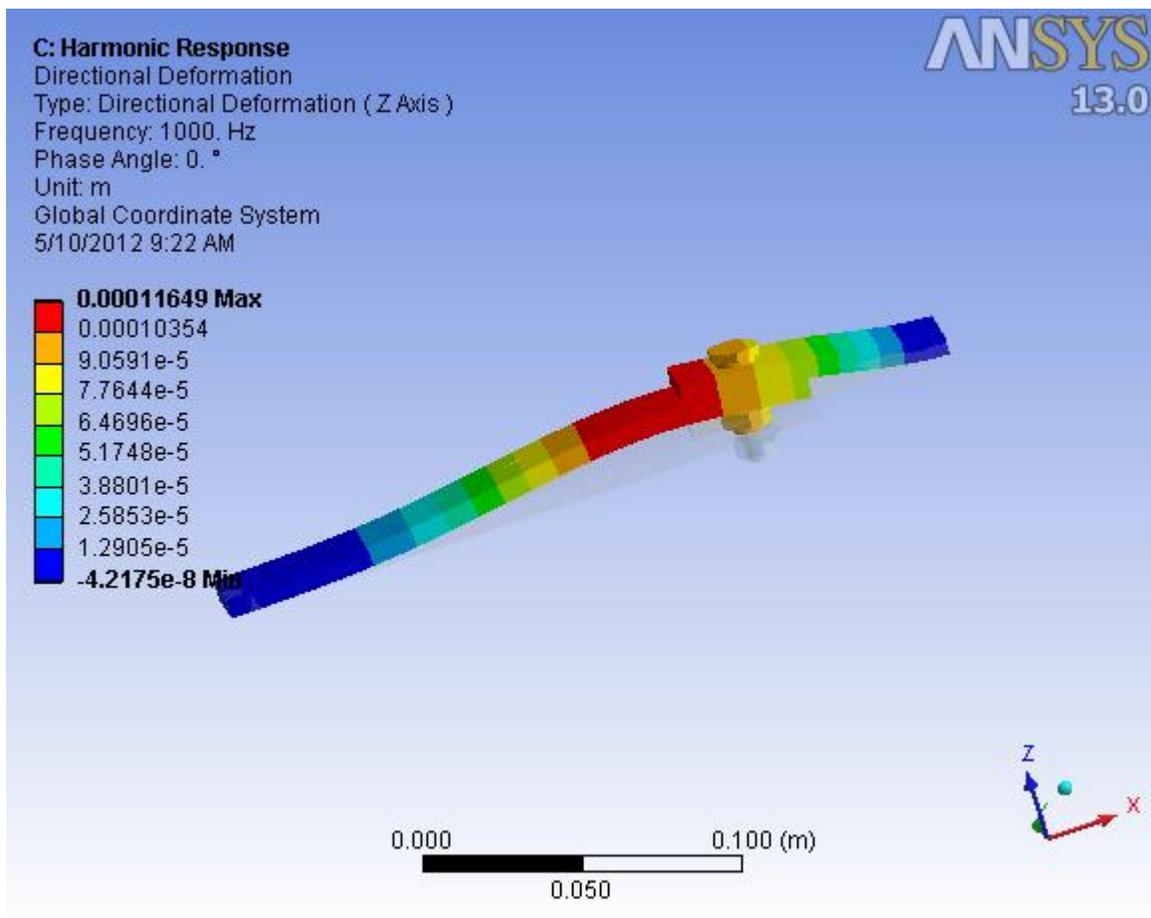
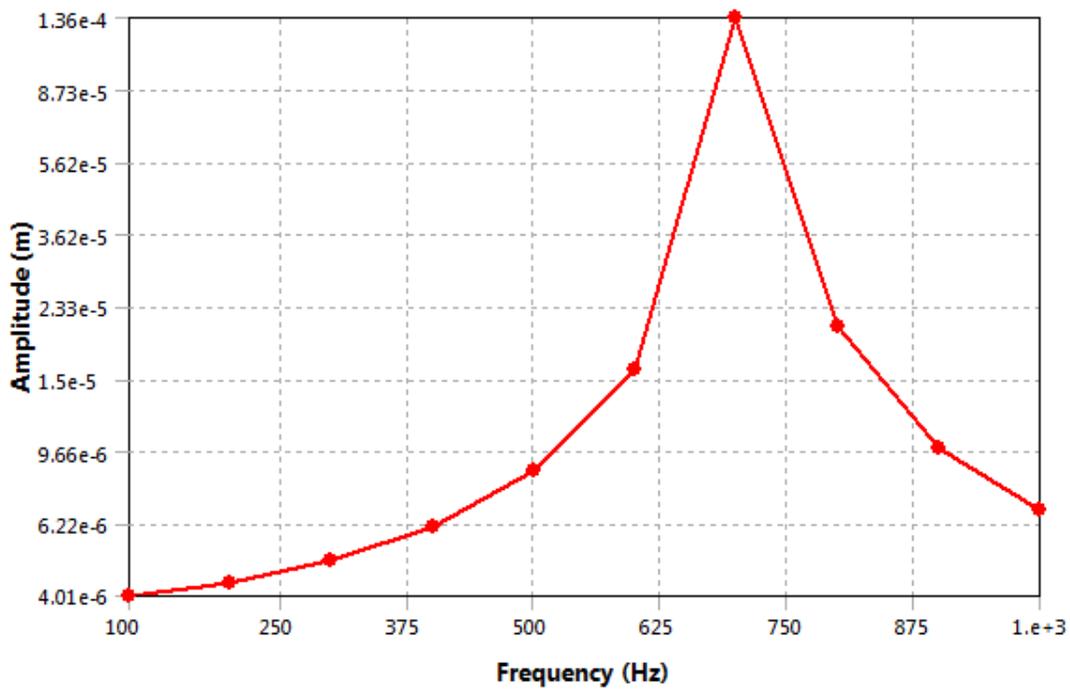


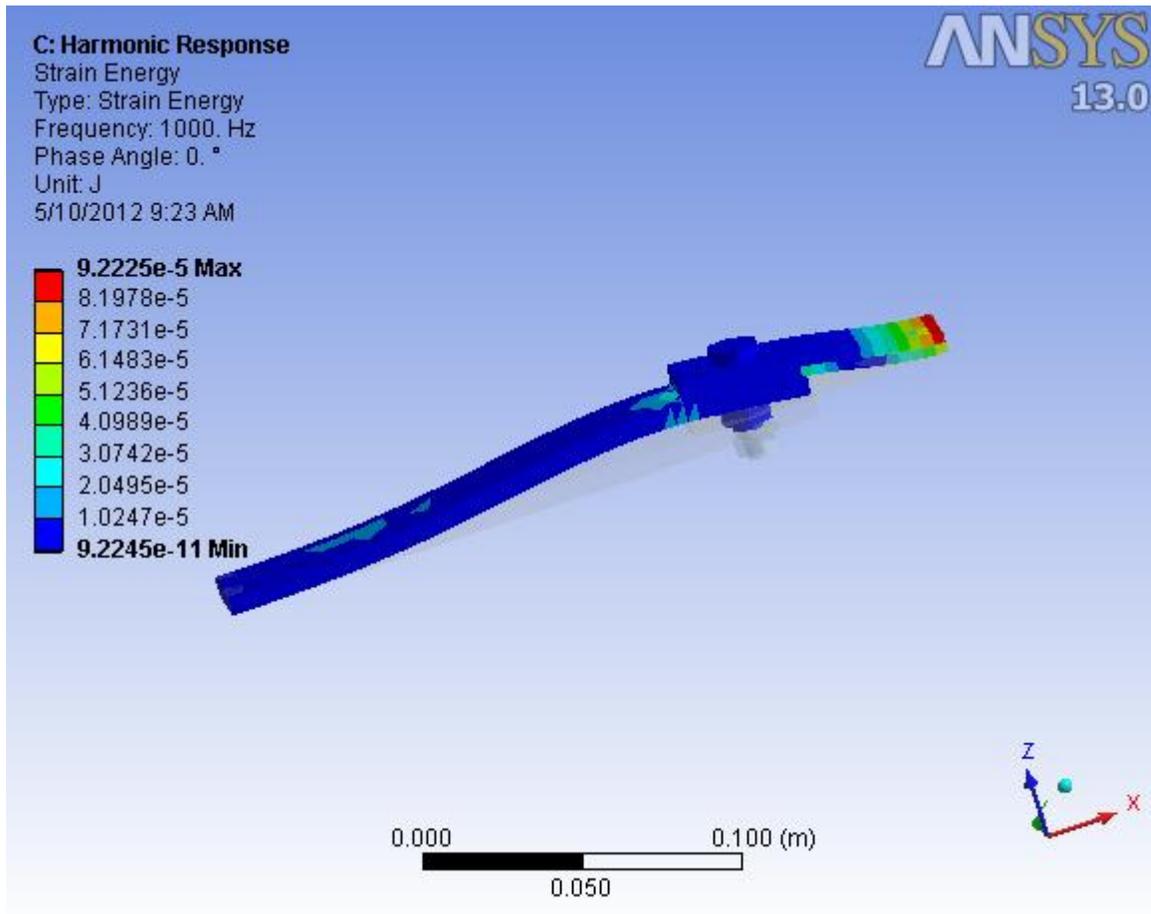
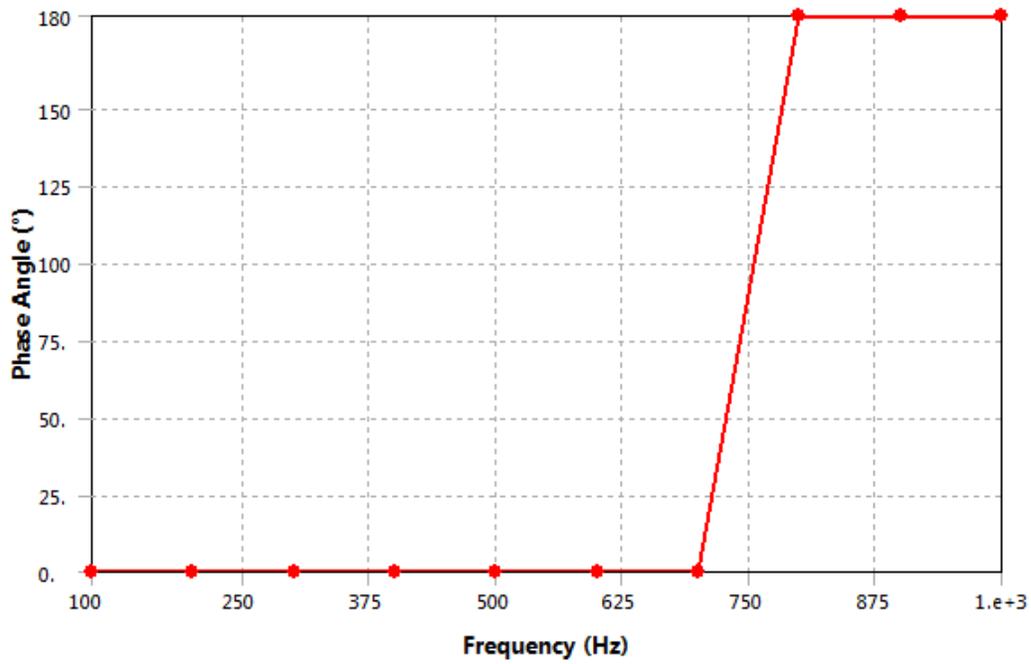
Reported Frequency = 713.12 Hz



6.2 Harmonic Response Analysis

It is a technique used to determine the steady state response of a linear structure to loads that vary sinusoidally with time. The mode superposition method calculations factored mode shapes (eigenvectors) from modal analysis to calculate the structures response. Hence it is known as harmonic response analysis.





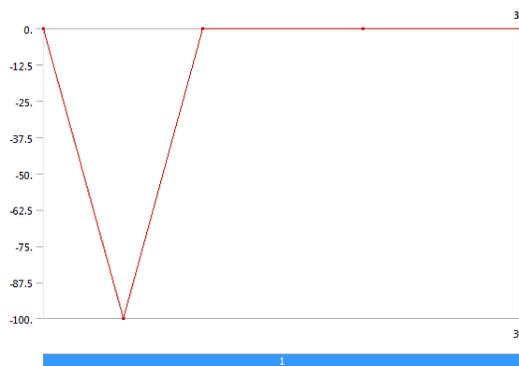
6.3 Transient Dynamic Analysis

It is also called time history analysis. It is the technique used to determine the dynamic response of a system under the action of any time dependent load.

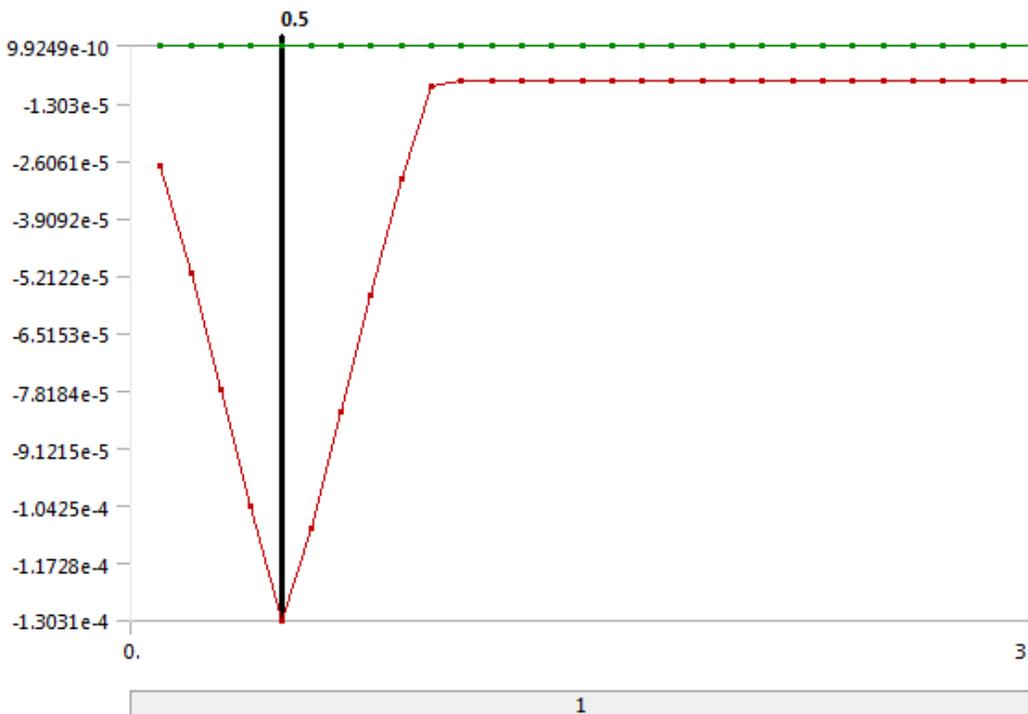
The basic equation of motion solved by a transient dynamic analysis is

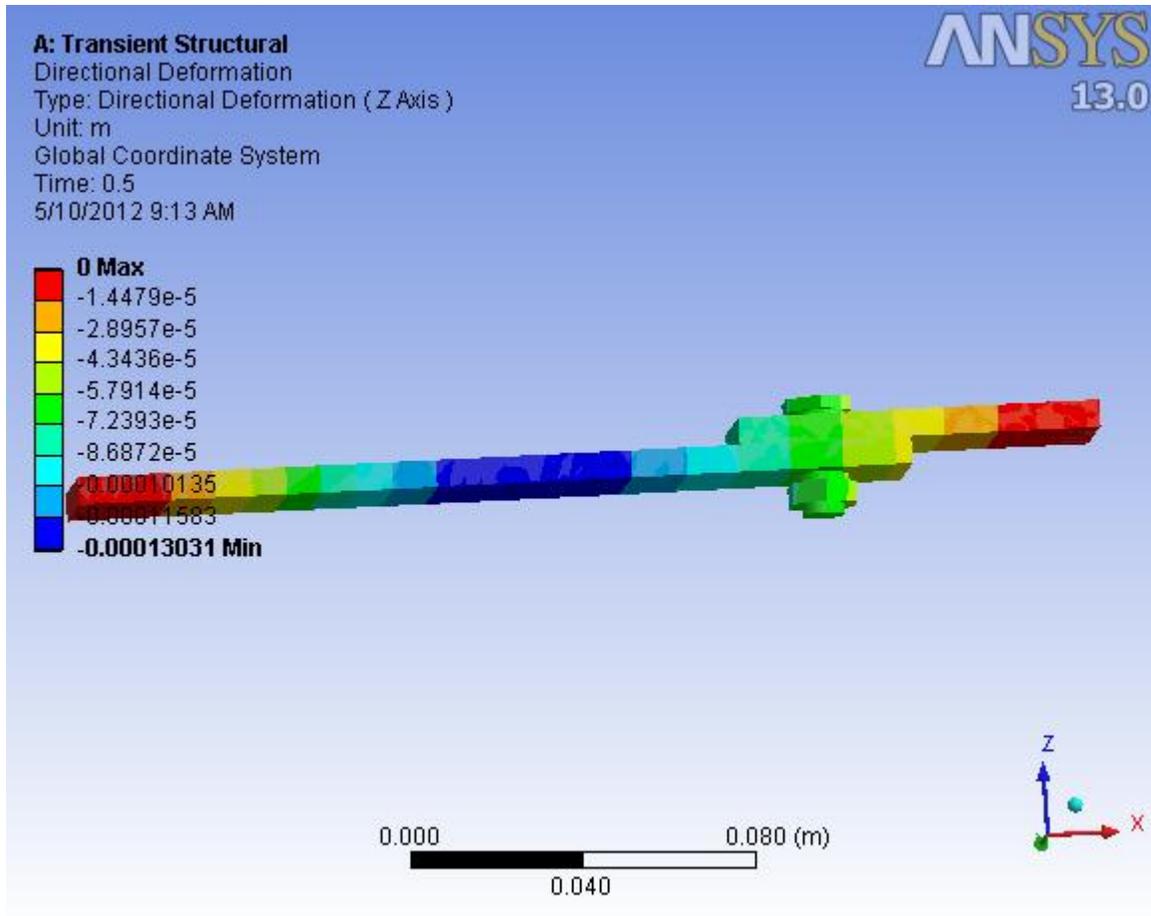
$$(M)\{u''\}+(C)\{u'\}+(K)\{u\}=\{f(t)\}$$

Impulsive Load Input



Steps	Time [s]	Force [N]
1	0.	0.
	0.5	-100.
	1.	0.
	2.	
	3.	





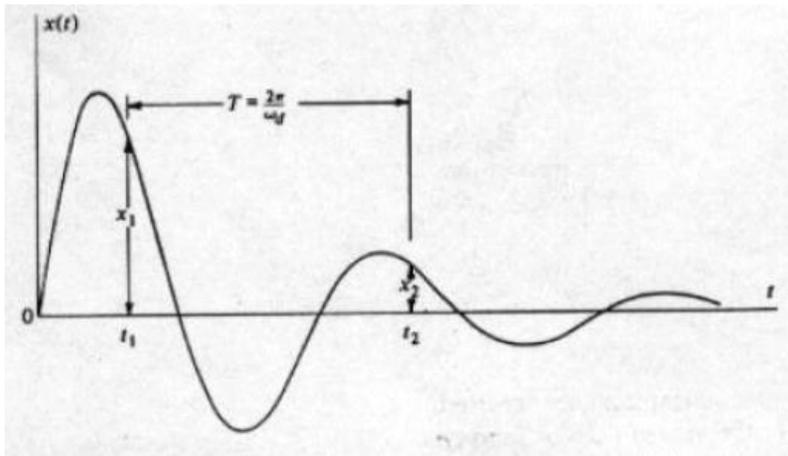
Directional deformation along z axis in tabular form

Time [s]	Minimum [m]	Maximum [m]
0.1	-2.7366e-005	0.
0.2	-5.1385e-005	
0.3	-7.7853e-005	
0.4	-1.0416e-004	
0.5	-1.3031e-004	
0.6	-1.0937e-004	
0.7	-8.3124e-005	
0.8	-5.6704e-005	
0.9	-3.0119e-005	
1.	-9.3484e-006	
1.1	-7.9237e-006	9.7777e-010
1.2	-7.9264e-006	9.0432e-010
1.3	-7.9231e-006	9.9249e-010
1.4	-7.9263e-006	9.0784e-010
1.5	-7.9233e-006	9.8753e-010

1.6	-7.9261e-006	9.138e-010
1.7	-7.9235e-006	9.811e-010
1.8	-7.9258e-006	9.2034e-010
1.9	-7.9238e-006	9.7471e-010
2.	-7.9256e-006	9.2641e-010
2.1	-7.924e-006	9.6904e-010
2.2	-7.9254e-006	9.3162e-010
2.3	-7.9242e-006	9.6432e-010
2.4	-7.9252e-006	9.3587e-010
2.5	-7.9243e-006	9.6054e-010
2.6	-7.9251e-006	9.3922e-010
2.7	-7.9244e-006	9.5759e-010
2.8	-7.925e-006	9.418e-010
2.9	-7.9245e-006	9.5533e-010
3.	-7.9249e-006	9.4375e-010

Chapter 7 Results and Conclusions

- From modal analysis reported modal frequency = **713.12 Hz**
- Harmonic response confirms the modal frequency and has a maximum deformation of **$1.36 \times 10^{-4} \text{ m}$** at the modal frequency of 713.12 Hz.
- From harmonic response model, Maximum strain energy = **$9.22 \times 10^{-5} \text{ J}$** .
- In transient analysis, the directional deformation along z axis with an impulsive force of **100 N** applied, the values of maximum deformation fluctuate and tend to converges to **9.5×10^{-10}** .
- $\omega = 2\pi f = \mathbf{4478.4 \text{ rad/sec}}$.
- logarithmic decrement, δ , as follows:



X_1 and X_2 are two consecutive displacements one cycle apart

$\delta = \ln(x_1/x_2) = \mathbf{0.005}$, X_1 and X_2 are taken from the values of the table

•

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

$\zeta = \mathbf{0.0025}$.

- $\alpha = 2\zeta\omega = 3.655 \text{ s}^{-1}$ and $\beta = 2\zeta/\omega = 6.8 \times 10^{-6} \text{ s}$
- energy dissipated $= \pi c\omega x^2 = 2.5 \times 10^{-5} \text{ J}$.
- Loss factor (η) $= 1/2\pi$ (energy dissipated per cycle / maximum strain energy) = **0.0437**.

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