HYPERBOLIC AXIAL DISPERSION MODEL

A THESIS SUBMITTED IN THE PARTIAL FULFILMENT OF THE REQUIRMENTS FOR THE DEGREE OF

MASTER OF TECHNOLOGY

In

MECHANICAL ENGINEERING

[Specialization: Thermal Engineering]

By

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National Institute of Technology, Rourkela
Odisha – 769008

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CERTIFICATE

This is to certify that the thesis entitled "**Hyperbolic Axial Dispersion Model**", being submitted by Mr. Radharaman Dalai, roll no. 210ME3327 in the partial fulfillment of the requirement for the award of the degree of M. Tech in Mechanical Engineering, is a research carried out by him at the Department of Mechanical Engineering, National Institute of Technology Rourkela, under my guidance and supervision.

The results presented in this thesis has not been, to the best of my knowledge, submitted to any other University or Institute for the award of any degree.

The thesis, in my opinion, has reached the standards fulfilling the requirement for the award of the degree of **Master of technology** in accordance with regulations of the Institute.

Prof. R. K. Sahoo
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Date:-

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I am extremely fortunate to be involved in an exciting and challenging research project like **“HYPERBOLIC AXIAL DISPERSION MODEL”**. It has enriched my life, giving me an opportunity to work in a new environment of heat and mass transfer. This project increased my thinking and understanding capability and after the completion of this project, I experienced the feeling of achievement and satisfaction.

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CONTENTS

ABSTRACT

Axial dispersion model is most reliable tool for analyzing the transient response of flow system inside a bundle of pipes. The proposed hyperbolic model considers propagation velocity of flow disturbance to be finite and even of the order of flow velocity to describe flow maldistribution. The traditional parabolic model is included as a special case under the hyperbolic model. Both backmixing and forward flow are considered to model the hyperbolic dispersion equation. This model is proposed for a flow system having multiple pipes with a pulse tracer input. The expressions for system outlet response using residence time distribution have been derived. It will help researchers to determine dispersion coefficient with higher accuracy than the parabolic model.

LIST OF FIGURES

NOMENCLATURE

a	dimensionless parameter, = Pe/2
b	dimensionless parameter defined by equation (16a,16b)
c	dimensionless concentration
ψ	dimension less flux
$\bar{c}$	dimensionless concentration in the Laplace domain
$\bar{\psi}$	dimensionless flux in the Laplace domain
C	concentration (kg m^{-3})
ξ	mass flux caused by dispersion (kg m^{-2} s)
C_1, C2	dimensionless constants
D	dispersion coefficient of the system (m^2s^{-1})
E	residence time distribution function (s^{-1})
F	dimensionless inlet concentration function
$\bar{f}$	inlet concentration function in Laplace domain
i	$\sqrt{-1}$
L	Laplace transform function
s	Laplace parameter (s^{-1})

θ	dimensionless time
U	velocity (m s^{-1})
x	dimensionless length
X	flow length (m)
Z	flow length of test section (m)
δ	Dirac impulse function
t	time (s)
$\bar{t}_R$	mean system residence time (s)
θ_R	dimensionless mean system residence time
w	parameter defined by $w = \sqrt{s^2M^2 + a^2 + 2as}$
j	run parameter

CHAPTER 1

INTRODUCTION

Introduction

Literature Survey

Objective of Work

1.1 INTRODUCTION

Mathematical analysis of flow through pipes in heat exchangers has been a center of attention for the researchers for quite a sometime. From design point of view two type of analysis can be done in the heat exchangers, one is steady-state analysis and the other is transient analysis. Steady state analysis have stressed the need for theoretical and experimental evaluation of heat exchanger performance during normal operation while transient analysis have brought out the response feature due to off normal behavior which are of huge importance in the designing of heat exchanger. With advancement of computing technology it has been possible to completely simulate the heat exchanger, still understanding of transport phenomena is important for design and control of heat exchanger.

1.1.1 What is Dispersion?

Mass spreading of a non-reactive tracer as it moves with fluid has been traditionally ascribed to three different mechanisms: molecular diffusion, hydrodynamic dispersion, and heterogeneous advection .This description of mass spreading has served well in relatively homogenous unconsolidated media because usually one mechanism dominates over the other two, depending upon the length scale of investigation. At the pore scale, for example, diffusion is the most important mechanism of mass spreading, at local scales (1-10 m) hydrodynamic dispersion is usually most significant, and at larger scales (>10-100 m) spreading is largely controlled by the variable advection with varying hydraulic conductivities, often called heterogeneous advection.

➢ MOLECULAR DIFFUSION

Molecular diffusion is the most straightforward spreading mechanism to discern as it is independent of fluid velocity. As diffusion is caused by the random kinetic motion of jostling water molecules, it is truly isotropic. Compared to most natural water velocities in permeable media, molecular diffusion works very slowly.

➢ HYDRODYNAMIC DISPERSION

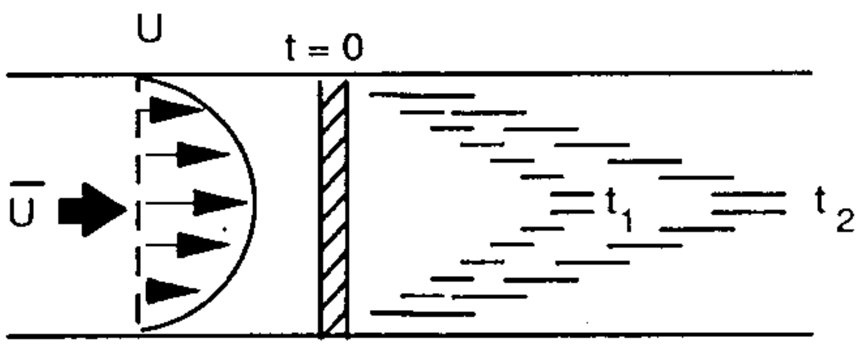

Fig. 1 Dispersion profile

The term hydrodynamic dispersion is used primarily in association with the advection-dispersion equation. It accounts for diffusion-like "Fickian" spreading of mass during transport. The coefficient of hydrodynamic dispersion is generally expressed as the combination of spreading in the direction of average linear velocity (longitudinal component) and perpendicular to the direction of average linear velocity (transverse component). Although there is some evidence that small amounts of dispersion can occur upstream in fluids, hydrodynamic dispersion acts primarily in the direction of flow which is known as axial dispersion. This is a fundamental difference between hydrodynamic dispersion and molecular diffusion, as diffusion acts in all directions simultaneously.

➢ HETEROGENEOUS ADVECTION

Spreading of dissolved mass in fluid can occur at macro-scales, i.e. at scales well beyond the pore scale. Whereas the characteristic length scale is normally around a meter, macro-dispersion is expected to have a length scale on the order of tens of meters. At this scale, mass spreading is not easily conceptualized as a “mixing” phenomenon. Rather, it is most often depicted as the summed effect of mass traveling through multiple layers of varying hydraulic conductivity. Unless these layers can be individually monitored downstream, mass arrival is dispersed by the staggered arrival of multiple mass transport pathways.

In a heat exchanger of finite length dispersion prevails over other mode of transport mechanism as stated earlier. So axial dispersion model is popularly used in modeling of non-ideal flow in case of pipe flow. The concept of axial dispersion model first proposed by Taylor (1954) [1,2], significantly simplifies the transport phenomena inside a pipe. Dispersion phenomena may be caused by eddies, recirculating currents dead zones of fluid flow, turbulent impulses and other non-uniformity of fluid flow, in brief by any deviation from plug flow. With time different models has been proposed to explain the behavior of fluid inside pipe. One is parabolic axial dispersion model which depicts the propagation velocity of a tracer inside a fluid is infinitely larger than flow velocity. For simplicity this has been received well by everyone for quite a long time as it provides information only about dispersion co-efficient. But with age advancement people are becoming more and more concerned about the efficiency of the model. This has been shown that propagation velocity in parabolic model can be of the order of flow velocity, which brings another term Mach number into picture.

1.1.2 MACH NUMBER

Mach number is the ratio of fluid flow velocity to the propagation velocity of disturbance and is a flow property. In parabolic axial dispersion models it is considered as zero assuming the propagation velocity is much higher than the fluid flow velocity. This assumption normally holds good for normal flow conditions, but maldistribution effects will propagate with a finite velocity which has the order of flow velocity. In this case the value of Mach number cannot be zero rather more than zero. For a multi pipe bundle the velocities may be different from pipe to pipes, even in some pipes it is negative, so the value of M may become fractional. Due to the difference in velocities in different pipes fluid may return through other pipe after exiting from one pipe. This is known as back mixing. In this case the value of M will have a value between 0 and 1. This region is known as subsonic zone. If there is forward flow then the value of M will be more than one which constitute supersonic zone. For sonic region the value of M is unity, which means there is flow stagnation in the pipe bundle. In the present study it is a weighted parameter, but it has a broader meaning of ‘third sound wave Mach number’ [3,4].

Our main aim of modeling axial dispersion is to find out the dispersion coefficients in fluid flow through pipes using residence time distribution(RTD).

1.1.3 RESIDENCE TIME DISTRIBUTION:

If an element enters a system then it takes some time to come out from the system. The time it spends inside the particular system is known as its residence time (t). Residence time distribution function (E (t)) describes in a quantitative manner how much time different elements have spent in the system.

1.1.3.1 Moments:

Moments are used to define the residence distribution function in terms of different parameters like mean residence time, spread in residence time(variance),skewness of distribution,etc.

The moments are defined as

$$MO_k = \int_0^\infty t^k E(t)dt \text{ , where } MO_k \text{ is the } k^{th} \text{ moment around the origin.} \quad (1)$$

In dimension less form

$$MR_k = \int_0^\infty \theta^k E(\theta)d\theta \text{ , where } MR_k \text{ is the reduced } k^{th} \text{ moment around the origin.} \quad (2)$$

$$= \lim_{s\to 0}(-1)^k \frac{d^k\overline{E}}{ds^k} \text{ , in Laplace domain.} \quad (3)$$

1.1.3.2 Mean Residence Time:

.Mean residence time is the average amount of time that a substance spends in a system. This measurement varies directly with the amount of substance that is present in the system. Residence time may also be defined as the ratio of the capacity of a system to hold a substance and the rate of flow of substance through the system.

Mathematically mean residence time can be expressed as

$$\overline{t} = MO_1 = \text{First reduced moment of distribution } = \int_0^\infty tE(t)dt \quad (4a)$$

Or, in dimensionless form

$$\overline{\theta} = MR_1 = \text{First reduced moment of distribution } = \int_0^\infty \theta E(\theta)d\theta \quad (4b)$$

1.1.3.3 Variance:

It is the probability distribution of substance that spreads out in a system.

Mathematically it is defined as

$$\sigma_t^2 = \int_0^\infty (t-\overline{t})^2 E(t)dt = MO_2 - MO_1^2 \tag{5a}$$

In dimensionless form

$$\sigma_\theta^2 = \int_0^\infty (\theta-\overline{\theta})^2 E(\theta)d\theta = MR_2 - MR_1^2 \tag{5b}$$

1.1.3.4 Skewness:

It is defined as a measurement of asymmetry in a probability distribution. A positive skew indicates that the tail in the right side is longer than the tail in the left side in a probability distribution and most amount of values lie in the left side of mean. A negative skew indicates that the tail in the left side is longer than the tail in the right side of a probability distribution function and the bulk amount values lie to the right of the mean.

Mathematically it can be expressed as

$$sk_t^3 = \int_0^\infty (t-\overline{t})^3 E(t)dt = MO_3 - 3MO_1MO_2 + 2MO_1^3 \tag{6a}$$

In dimensionless form

$$sk_\theta^3 = \int_0^\infty (\theta-\overline{\theta})^3 E(\theta)d\theta = MR_3 - 3MR_1MR_2 + 2MR_1^3 \tag{6b}$$

1.2 LITERATURE SURVEY

The concept of axial dispersion in mass transfer was first introduced by S.G. Taylor [1,2]. He introduced brine solution to the inlet of a water tube and observed that the solution spreads almost symmetrically due to the action of molecular diffusion and variation of velocity over a cross-section. This is due to the fact of axial dispersion as later he described.

Danckwerts [5] first established the boundary conditions for a parabolic axial dispersion model, which was a significant breakthrough in the field of axial dispersion. Those boundary conditions still holds good for dispersion models having larger propagation velocity.

Mao Zaisha and Chen Jiayong [6] investigated different types of dispersion model systematically with different set of boundaries like closed-closed, closed-open, open-open. They established the solutions of the dispersion models in a very systematic way using previous researches.

Wilfried Roetzel and Frank Balzereit [7] introduced residence time measurement as a transient measurement technique to effectively determine the axial dispersion coefficients in plate heat exchangers. First of all they mathematically modeled parabolic axial dispersion model by solving the general dispersion equation with the help of Danckwerts [5] boundary conditions. Then the curves (concentration vs residence time) were plotted. In the inlet of the heat exchanger a salt solution pulse was injected into the steady flow stream of water which propagates with dispersion to the outlet of the heat exchanger. The concentration profiles were plotted at the inlet and outlet of the heat exchanger. After comparing the analytical and experimental data axial dispersion coefficient can be evaluated. This technique guarantees fast measurement of residence time as happens in heat exchangers.

Wilfried Roetzel and Chakkrit Na Ranong [3] numerically calculated the axial temperature profiles in a shell and tube heat exchanger for given maldistribution in the tube side. The same maldistributions are solved with both parabolic and hyperbolic dispersion model for the purpose of comparison with fitted values for the axial dispersion coefficient and third sound wave velocity. The results clearly demonstrated that the hyperbolic dispersion model is better suited to describe steady state axial temperature profiles.

Ranjit Kumar Sahoo and Wilfried Roetzel [4,8] proposed the boundary conditions for hyperbolic axial dispersion model. They derived the hyperbolic axial dispersion model for a shell and tube heat exchanger with pure maldistribution in tube side flow and plug flow on the shell side by using the axial temperature profiles of heat exchangers. They derived the boundary conditions and Mach number and compared with the previous researches. Then they observed that hyperbolic axial dispersion model is the best compared to the previous model as it suits well with the actual calculations.

Wilfried Roetzel, Xing Luo, Yimin Xuan [9] proposed a periodic transient test technique based on the axial dispersion model to determine heat transfer coefficients and axial dispersion coefficients in a heat exchanger. The model uses a parameter (Peclet number) to determine the deviation of flow pattern from ideal plug flow. They have taken axial dispersion in fluid and axial conduction in wall into account and solved analytically by complex Fourier transform. Finally they proposed that axial dispersion has a significant role in dynamic temperature response of heat exchanger.

1.3 OBJECTIVE OF WORK

- Investigation of Parabolic axial dispersion model
- Mathematical modeling of both subsonic and supersonic zone of hyperbolic axial dispersion model
- Comparison of all models

CHAPTER 2

MATHEMATICAL MODELLING

General Hyperbolic Axial Dispersion Equation

Boundary Conditions

Solution to General Equation

Parameters of RTD

HYPEBOLIC AXIAL DISPERSION MODELING

2.1 General Equation

The general dispersion equation can be formed from one dimensional diffusion equation for constant fluid total density and diffusivity by replacing diffusivity for molecular diffusion with dispersion coefficient D. Diffusion is assumed to be negligible in the present case of dispersive flow.

Hence the dispersion equation:

$$-\frac{d\xi}{dX} - u\frac{dC}{dX} = \frac{dC}{dt} \tag{7}$$

Introducing the following dimensionless parameters:

Dispersion coefficient $Pe = \frac{uZ}{D}$

Flow length $x = \frac{X}{Z}$

Time $\theta = t\frac{u}{Z} = t\frac{\dot{V}}{V}$

And concentration $c = \frac{C}{C^*}$ with $C^* = \frac{\int_0^\infty C_{out}\,dt}{\bar{t}_R}$

we obtained the governing equation for axial dispersion as:

$$-\frac{1}{Pe}\frac{d\psi}{dx} - \frac{dc}{dx} = \frac{dc}{d\theta} \tag{8}$$

The flux equation for hyperbolic flow is proposed elsewhere[] as:

$$\psi + \frac{M^2}{Pe}\frac{d\psi}{dx} + \frac{M^2}{Pe}\frac{d\psi}{d\theta} = -\frac{dc}{dx} \tag{9}$$

This equation is derived in analogous with Chester's law of conduction. Equation (9) takes the form analogous to Fourier's law of heat conduction if M=0, resulting in the classical parabolic dispersion model. The existence of hyperbolic model largely depends on the evaluation of Mach number and Peclet number. In the present work the concept mainly revolves around determination of Mach number and Peclet number by residence time distribution.

Taking Laplace transformation we have

$$-\frac{1}{Pe}\frac{d\overline{\psi}}{dx} - \frac{d\overline{c}}{dx} = s\overline{c} \tag{10}$$

$$\overline{\psi} + \frac{M^2}{Pe}\frac{d\overline{\psi}}{dx} + \frac{M^2}{Pe}s\overline{\psi} = -\frac{d\overline{c}}{dx} \tag{11}$$

Rearranging the above two equations we have

$$\frac{d\overline{\psi}}{dx} = -\left(\frac{sM^2 + Pe}{M^2 - 1}\right)\overline{\psi} + \left(\frac{sPe}{M^2 - 1}\right)\overline{c} \tag{12}$$

$$\frac{d\overline{c}}{dx} = \frac{1}{Pe}\left(\frac{sM^2 + Pe}{M^2 - 1}\right)\overline{\psi} - \left(\frac{sM^2}{M^2 - 1}\right)\overline{c} \tag{13}$$

Equations 12 and 13 can be rewritten combinedly in a matrix form for simplicity of solving the set of equations as follows

$$\begin{bmatrix} \frac{d\overline{\psi}}{dx} \\ \frac{d\overline{c}}{dx} \end{bmatrix} = \begin{bmatrix} -\left(\frac{sM^2+Pe}{M^2-1}\right) & \left(\frac{sPe}{M^2-1}\right) \\ \frac{1}{Pe}\left(\frac{sM^2+Pe}{M^2-1}\right) & -\left(\frac{sM^2}{M^2-1}\right) \end{bmatrix} \begin{bmatrix} \overline{\psi} \\ \overline{c} \end{bmatrix} \quad (14)$$

The roots of the equations called as Eigen values are

$$\chi_1 = \frac{a+sM^2+w}{1-M^2} \text{ and } \chi_2 = \frac{a+sM^2-w}{1-M^2} \quad (15)$$

Now the eigen vectors for each eigen values of equation 15 are

$$[v_1] = \begin{bmatrix} -\left(\frac{a+w}{sM^2+2a}\right)2a \\ 1 \end{bmatrix} \text{ and } [v_2] = \begin{bmatrix} -\left(\frac{a-w}{sM^2+2a}\right)2a \\ 1 \end{bmatrix} \text{ respectively.}$$

Where a=Pe/2 and $w = \sqrt{s^2M^2 + a^2 + 2as}$

Now the general solution for the equations 10 and 11 can be written as

$$\begin{bmatrix} \overline{\psi} \\ \overline{c} \end{bmatrix} = Ae^{\chi_1 x}[v_1] + Be^{\chi_2 x}[v_2] \quad (16)$$

Where A and B are constants.

After simplification

$$\overline{c} = \left[C_1 \cosh\left(\frac{w}{1-M^2}x\right) + C_2 \sinh\left(\frac{w}{1-M^2}x\right)\right] \exp\left(\frac{sM^2+a}{1-M^2}x\right) \quad (17)$$

$$\overline{\psi} = -2a\left[\frac{C_1a+C_2w}{sM^2+2a}\cosh\left(\frac{w}{1-M^2}x\right) + \frac{C_2a+C_1w}{sM^2+2a}\sinh\left(\frac{w}{1-M^2}x\right)\right]\exp\left(\frac{sM^2+a}{1-M^2}x\right) \quad (18)$$

Where $C_1\left(=\frac{A+B}{2}\right)$ and $C_2\left(=\frac{A-B}{2}\right)$ are constants.

2.2 Boundary conditions:-

The initial boundary conditions for hyperbolic axial dispersion model have been proposed by Sahoo and Roetzel[4], where Danckwerts[5] boundary condition happens to be a special case as M=0. The system through which the fluid flows is assumed to be a closed boundary system, i.e., no dispersion outside the boundary layer.

Subsonic Zone (0≤M<1):-

In the subsonic zone both the forward and backward fluid flows are present. This is the region of pure backmixing and backflow.

The inlet boundary condition is

x=0: (time domain) $$c+\frac{1}{Pe}\psi=f(\theta) \tag{19a}$$

x=0: (Laplace domain) $$\bar{c}+\frac{1}{Pe}\bar{\psi}=\bar{f}(s)=1 \tag{19b}$$

The rate at which mass transfer occurs across the plane at x=0 by combined convective flow and dispersion must be equal to the rate at which mass is transported to the system. The result is a sudden drop of concentration at the inlet even at a steady state mass transport to the system, hence $f(\theta)$ =C*. This is characteristic of a closed system with negligible boundaries, and is obviously a realistic condition for heat exchangers connected to tubes with plug flow.

` The function $f(\theta)$ can be any transient function for the concentration at the system inlet.

In the experiments a pulse of tracer substance is injected into the fluid flow, which can be regarded as a Dirac pulse at the inlet boundary x=0: $f(\theta) = \delta(\theta)$ and hence we get: $\bar{f}(s) = 1$.

And the exit boundary condition is

x = 1: (time domain) $\psi = 0$ (20a)

x = 1: (Laplace domain) $\bar{\psi} = 0$ (20b)

Supersonic Zone(M>1):-

In this zone the forward flow in the tubes gives M>1. This is the zone of pure maldistribution. Here the two boundary conditions are to be specified at the inlet.

Hence the boundary conditions are

x = 0: (time domain) $c = f(\theta)$ (21a)

x = 0: (Laplace domain) $\bar{c} + \frac{1}{Pe}\bar{\psi} = \bar{f}(s) = 1$ (21b)

x = 0: (time domain) $\psi = 0$ (22a)

x = 0: (Laplace domain) $\bar{\psi} = 0$ (22b)

2.3 Solution of the hyperbolic axial dispersion equation

Subsonic Zone ($0 \le M < 1$):-

Solving boundary condition (19b), we have

$$C_1 - \frac{C_1 a + C_2 w}{sM^2 + 2a} = 1 \tag{23}$$

And hence

$$C_1\left(1 - \frac{a}{sM^2 + 2a} - \frac{C_2}{C_1}\frac{w}{sM^2 + 2a}\right) = 1 \tag{24}$$

Now solving boundary condition (20b), we have

$$\frac{C_1 a + C_2 w}{sM^2 + 2a}\cosh\frac{w}{1 - M^2} + \frac{C_1 w + C_2 a}{sM^2 + 2a}\sinh\frac{w}{1 - M^2} = 0 \tag{25}$$

This after simplification gives

$$\frac{C_2}{C_1} = -\left[\frac{ia\cos\frac{iw}{1 - M^2} + w\sin\frac{iw}{1 - M^2}}{iw\cos\frac{iw}{1 - M^2} + a\sin\frac{iw}{1 - M^2}}\right] \tag{26}$$

After equating equations (24) and (26) the values of C_1 and C_2 can be found out as follows

$$C_1 = \frac{a\sin\frac{b}{1 - M^2} + b\cos\frac{b}{1 - M^2}}{\frac{sM^2}{sM^2 + 2a}\left(a\sin\frac{b}{1 - M^2} + b\cos\frac{b}{1 - M^2}\right) + \frac{a^2 - b^2}{sM^2 + 2a}\sin\frac{b}{1 - M^2} + \frac{2ab}{sM^2 + 2a}\cos\frac{b}{1 - M^2}} \tag{27}$$

$$C_2 = i\left[\frac{a\cos\frac{b}{1-M^2} - b\sin\frac{b}{1-M^2}}{\frac{sM^2}{sM^2+2a}\left(a\sin\frac{b}{1-M^2} + b\cos\frac{b}{1-M^2}\right) + \frac{a^2-b^2}{sM^2+2a}\sin\frac{b}{1-M^2} + \frac{2ab}{sM^2+2a}\cos\frac{b}{1-M^2}}\right] \tag{28}$$

Where b=-iw, b ϵ R

As $w = \sqrt{s^2M^2 + a^2 + 2as}$

So we have $-b^2 = a^2 + M^2s^2 + 2as$ (29a)

$$s = \frac{-a \pm \sqrt{a^2 - M^2(a^2+b^2)}}{M^2} \tag{29b}$$

$$sM^2 + 2a = -\left(\frac{a^2+b^2}{s}\right) \tag{29c}$$

Using equations (27),(28),(29a),(29b)and (29c) the values of C_1 and C_2 are simplified to

$$C_1 = \frac{a\sin\frac{b}{1-M^2} + b\cos\frac{b}{1-M^2}}{(a+s)\sin\frac{b}{1-M^2} + b\cos\frac{b}{1-M^2}} \tag{30}$$

$$C_2 = i\left[\frac{a\cos\frac{b}{1-M^2} - b\sin\frac{b}{1-M^2}}{(a+s)\sin\frac{b}{1-M^2} + b\cos\frac{b}{1-M^2}}\right] \tag{31}$$

The solution (17) to the general equations (10) and (11) after substituting the values C_1 and C_2 from equations (30) and (31) can be rewritten in a more generalized a way as

$$\overline{c}(x,s)=\frac{a\sin\left(\frac{b}{1-M^2}(1-x)\right)+b\cos\left(\frac{b}{1-M^2}(1-x)\right)}{(a+s)\sin\frac{b}{1-M^2}+b\cos\frac{b}{1-M^2}}\exp\left(\frac{sM^2+a}{1-M^2}x\right) \qquad (32)$$

The time domain c(x, θ) is obtained by the method of residues for inversion of the transform c(x,s):

$$c(x,\theta)=L^{-1}\left\{\overline{c}(x,s)\right\}=\sum_{j=1}^{n}\operatorname{Res}\ \overline{c}(s_j)\exp\left(s_j\theta\right)$$

(33)

To calculate the residues the singularities of $\overline{c}(x,s)$ must be determined. Let $b_j(1,2,\ldots\infty)$ be the j^{th} positive real root of the denominator

$$(a+s)\sin\frac{b}{1-M^2}+b\cos\frac{b}{1-M^2}=0 \quad \text{Or} \quad \tan\left(\frac{b_j}{1-M^2}\right)=-\left(\frac{b_j}{a+s_j}\right) \qquad (34)$$

And $$s_j=\frac{-a\pm\sqrt{a^2-M^2(a^2+b_j{}^2)}}{M^2} \qquad (35)$$

The residues of a function having simple poles are determined by

$$\operatorname{Res}\ \overline{c}(s_j)\exp\left(s_j\theta\right)=\frac{N(s_j)}{D'(s_j)} \qquad (36)$$

Where N(sj) = Numerator of equation (32) and D(sj) = Denominator of equation(32)

After calculating the residues according to the formula (36) at each pole and applying the results to equation (33), the solution of the dispersion equation in the time domain becomes:

$$c(x,\theta)=\sum_{j=1}^{\infty}\left[\frac{b_j^2\left(-s_j\right)\left[\cos\frac{b_j x}{1-M^2}+\frac{s_j M^2+a}{b_j}\sin\frac{b_j x}{1-M^2}\right]}{b_j^2\left(1+\frac{s_j M^2+a}{1-M^2}\right)+\frac{\left(s_j M^2+a\right)\left(1+a+s_j-M^2\right)\left(s_j+a\right)}{1-M^2}}\exp\left(\frac{s_j M^2+a}{1-M^2}x+s_j\theta\right)\right] \quad (37)$$

The system outlet response (x=1) to the Dirac pulse input at time θ=0 represents the residence time distribution (RTD) of the fluid [10].

$$c(x,\theta)=\sum_{j=1}^{\infty}\left[\frac{b_j^2\left(-s_j\right)\left[\cos\frac{b_j}{1-M^2}+\frac{2s_j M^2+Pe}{2b_j}\sin\frac{b_j}{1-M^2}\right]}{b_j^2\left(1+\frac{2s_j M^2+Pe}{2(1-M^2)}\right)+\frac{\left(2s_j M^2+Pe\right)\left(2s_j+Pe\right)\left\{Pe+2s_j+2(1-M^2)\right\}}{8(1-M^2)}}\exp\left(\frac{2s_j M^2+Pe}{2(1-M^2)}+s_j\theta\right)\right] \quad (38)$$

Supersonic zone (M>1):

Solving boundary conditions (21b) and (22b), we have

C_1 = 1 and C_2 = $-\frac{a}{w}$

The solution (17) to the general equations (10) and (11) after substituting the values of C_1 and C_2 can be written as:

$$\overline{c}(x,s)=\left[\cosh\left(\frac{w}{m^2-1}x\right)+\frac{a}{w}\sinh\left(\frac{w}{m^2-1}x\right)\right]\exp\left(\frac{sM^2+a}{1-M^2}x\right)$$

$$=\frac{w-a}{2w}\exp\left(-\frac{sM^2+a+w}{M^2-1}x\right)+\frac{w+a}{2w}\exp\left(-\frac{sM^2+a-w}{M^2-1}x\right) \tag{39}$$

The solution for dispersive flux in Laplace domain can be found out by using the values of C_1 and C_2 in the equation (18) as

$$\overline{\psi}(x,s)=2a\frac{s}{w}\sinh\left(\frac{w}{M^2-1}x\right)\exp\left(-\frac{sM^2+a}{M^2-1}x\right) \tag{40}$$

Expanding the above equation:

$$\overline{c}(x,s)=\begin{bmatrix}\left(\frac{1}{2}-\frac{a}{2M\sqrt{(s+\beta)^2+\alpha^2}}\right)\exp\left(-k\sqrt{(s+\beta)^2+\alpha^2}\right)\\+\left(\frac{1}{2}+\frac{a}{2M\sqrt{(s+\beta)^2+\alpha^2}}\right)\exp\left(k\sqrt{(s+\beta)^2+\alpha^2}\right)\end{bmatrix}\exp\left(-kM(s+\beta)\right) \tag{41}$$

$$\overline{\psi}(x,s)=2a\frac{s}{2M\sqrt{(s+\beta)^2+\alpha^2}}\begin{pmatrix}\exp\left(k\sqrt{(s+\beta)^2+\alpha^2}\right)\\-\exp\left(-k\sqrt{(s+\beta)^2+\alpha^2}\right)\end{pmatrix}\exp\left(-\frac{sM^2+a}{M^2-1}x\right) \tag{42}$$

Where $k = \frac{Mx}{M^2 - 1}$

$\beta = \frac{a}{M^2}$

$\alpha = \beta\sqrt{M^2 - 1}$

To make this closed-open condition, closed-closed exit condition[3] is to be applied as follows

$$E(x,s) = c(x,s) + \frac{1}{Pe}\psi(x,s)$$

The Laplace inversion of the above equation has been found out numerically.

CHAPTER 3
PERFORMANCE ANALYSIS

Analysis Using Parameters of RTD

Analysis using RTD Curves

Determination of M and Pe

PERFORMANCE ANALYSIS

In the present work two types of methods are being used to analyze and compare the response of system in subsonic and supersonic case of dispersion. The RTD is a very essential tool for analyzing this type of transient models. These are as follows

3.1 Analysis using Parameters of Residence time distribution

It is noteworthy that the parameters of residence time distribution for a parabolic axial dispersion model as found from earlier research works [7] are exactly same as that of subsonic hyperbolic dispersion model with M=0. Again if we simplify outlet response equation of the subsonic hyperbolic axial dispersion model with M=0, we can see that the resulting equation is exactly same as the one proposed by Roetzel and Balzereit [7]. So we can safely conclude that the parabolic axial dispersion model is a subset of hyperbolic axial dispersion model with M=0.

Again if we consider the supersonic hyperbolic axial dispersion model without exit condition, we can find that the system will behave like an closed open case of axial dispersion model with mean residence time more than one(i.e., equal to $1+\frac{1}{\mathrm{Pe}}$). This case will arise if we consider a larger length of pipe in consideration.

In the present case of study mean residence time of one indicates that all the particles will travel through the system and after some time there will be no traces of particle inside the system. This means the particles that enter to the system will go out of the system after a certain time. Dependency of Peclet number and the output response curve can be verified by the fact that with increase in Peclet number variance decreases suggesting the more smoother the distribution curve becomes. This means the distribution will lie in the vicinity of mean residence time. In both the cases of subsonic and supersonic zone it is found that the skewness is always positive. So we can conclude that that distribution curve is heavily loaded towards left of mean residence time and the tail of the distribution curve is longer towards the right side.

3.2 Analysis using output response curves

Different sets of curves have been plotted for subsonic and supersonic hyperbolic axial dispersion model which will justify the existence of the model. All the graphs are plotted by taking dimensionless residence time in X-axis and dimensionless concentration in the Y-axis.

Let's consider the subsonic case of hyperbolic axial dispersion model. The curves in fig(2 left) and fig(2 right) are the cases of parabolic axial dispersion model and subsonic hyperbolic axial dispersion model (with M=0) for different Peclet numbers. This shows that both the set of curves are exactly same and undistinguishable from each other.

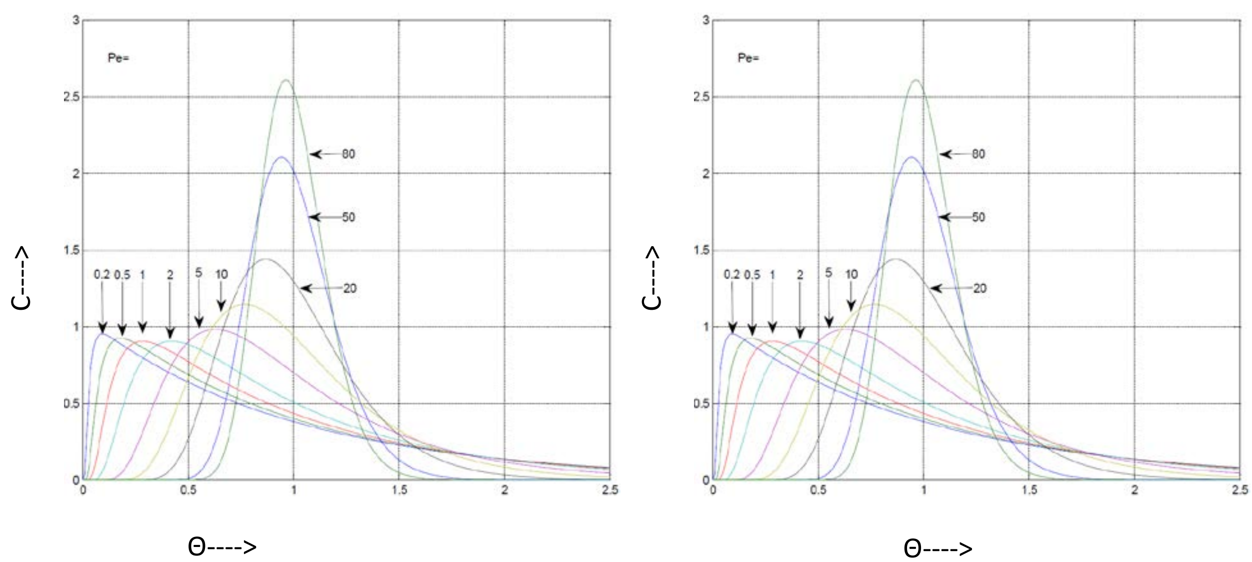

Fig.2. The outlet response curve .Left: Parabolic axial dispersion model. Right: Subsonic hyperbolic axial dispersion model with M=0.

While considering both subsonic and supersonic zone of hyperbolic axial model if we examine the behavior of the outlet response with respect to Mach number we can clearly see in fig(3) that the response curve squeezes inwards with increase in Mach number. This happens due to the fact that with increase in Mach number concentration flux velocity increases as compared to fluid velocity which results in clustering of particles and they will exit nearer to mean residence time

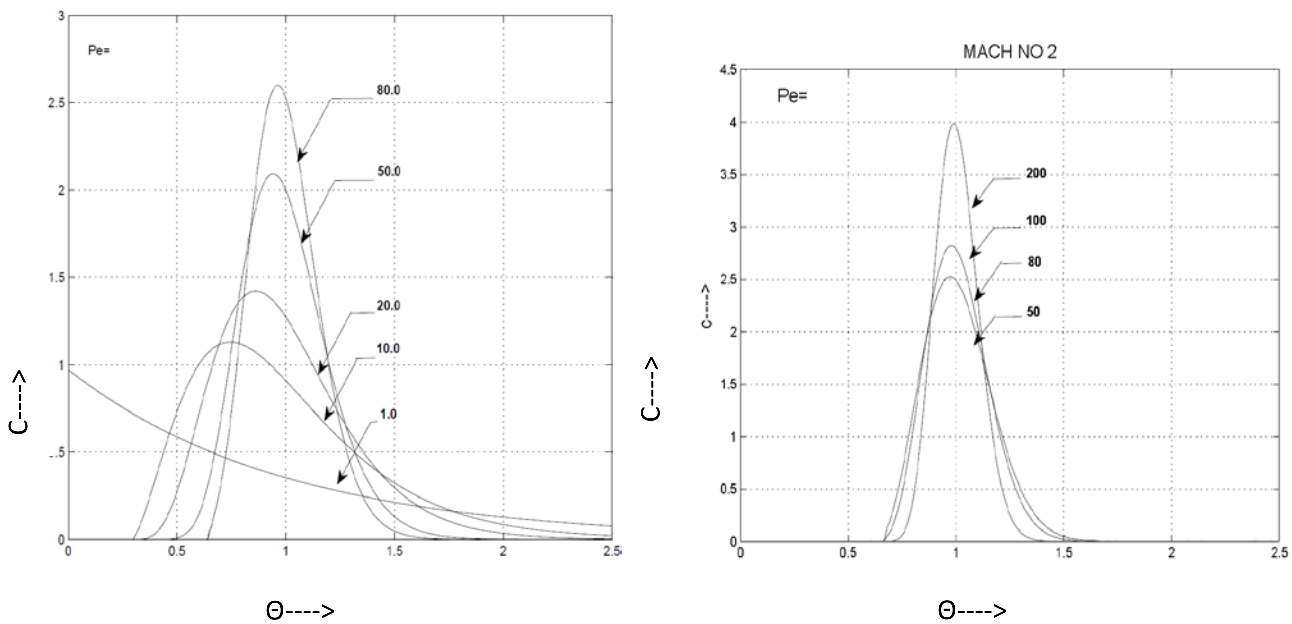

Fig.3. System outlet concentration profile for different Peclet numbers. Left: Subsonic zone with M=0.5 Right: Supersonic zone with M=2

In another case with increase in Peclet number the variance and skewness of the outlet response curve decreases. This is primarily due to the fact that as Peclet number increases dispersion decreases hence impact on tracer particles to spread will be less and they will move out from the system nearer to mean residence time. But in case of lower Peclet number the particles leaving later from the system have been subjected to a longer period of dispersion and hence the curve is more positive skewed as shown in fig(3 left) and fig(3 right)

3.3 Determination of Mach number and Peclet number

From an experimental data the outlet response curve can be found out. The response curve has to be then compared with the theoretical response curves. For lower Peclet numbers the curves of subsonic and supersonic zone can be easily distinguished. Then they can be matched with the theoretical one to find out the Peclet number and hence the hyperbolic axial dispersion coefficient of the flow. However for higher Peclet numbers the curves of subsonic and supersonic hyperbolic axial dispersion model are very much similar. So the Peclet number can be found out without knowing the Mach number simply by comparing the curves with the theoretical one.

CHAPTER 4

CONCLUSION AND FUTURE WORKS

Conclusion

Future Works

4.1 CONCLUSION

In the present work of hyperbolic axial dispersion model, subsonic and supersonic zone of the model are solved to find the exact response curve of the tracer at the outlet. This gives a proper idea about the Mach number and Peclet number of a particular flow system. For a compact heat exchanger it is very difficult to find out the effectiveness of the design as it is impractical to know what is happening inside the heat exchanger. So this model will help to find out the values of Mach number and Peclet number which are significant in relation to the design of heat exchanger. Present research is a significant boost to the researchers as it can provide more accurate value of axial dispersion coefficient as compared to the earlier works.

4.2 FUTURE WORKS

- Validating the hyperbolic axial dispersion model with experimental data.
- Extending hyperbolic axial dispersion model from mass transfer to heat and/or mass transfer.
- Validating experimental value with theoretical one.
- Establishing hyperbolic model as the best suitable axial dispersion model for design of tube/heat exchanger.

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