DECISION-MAKING IN FUZZY ENVIRONMENT

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In

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By

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Certificate of Approval

This is to certify that the thesis entitled **DECISION-MAKING IN FUZZY ENVIRONMENT** submitted by ***Sri Chitrasen Samantra*** has been carried out under my supervision in partial fulfillment of the requirements for the Degree of ***Master of Technology*** in ***Production Engineering*** at National Institute of Technology, NIT Rourkela, and this work has not been submitted elsewhere before for any other academic degree/diploma.

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Abstract

Decision-making is a logical human judgment process for identifying and choosing alternatives based on the values and preferences of the decision maker that mostly applied in the managerial level of the concerned department of the organization/ supply chain. Recently, decision-making has gained immense popularity in industries because of their global competitiveness and to survive successfully in respective marketplace. Therefore, decision-making plays a vital role especially in purchase department for reducing material costs, minimizing production time as well as improving the quality of product or service. But, in today's real life problems, decision-makers generally face lot of confusions, ambiguity due to the involvement of uncertainty and subjectivity in complex evaluating criterions of alternatives. To deal such kind of vagueness in human thought the title 'Decision-Making in Fuzzy Environment' has focused into the emerging area of research associated with decision sciences. Multiple and conflicting objectives such as 'minimize cost' and 'maximize quality of service' are the real stuff of the decision-makers' daily concerns. Keeping this in mind, this thesis introduces innovative decision aid methodologies for an evaluation cum selection policy analysis, based on theory of multi-criteria decision-making tools and fuzzy set theory.

In the supplier selection policy, emphasis is placed on compromise solution towards the selection of best supplier among a set of alternative candidate suppliers. The nature of supplier selection process is a complex multi-attribute group decision making (MAGDM) problem which deals with both quantitative and qualitative factors may be conflicting in nature as well as contain incomplete and uncertain information. Therefore, an application of VIKOR method combined with fuzzy logic has been reported as an efficient approach to support decision-making in supplier selection problems.

This dissertation also proposes an integrated model for industrial robot selection considering both objective and subjective criteria's. The concept of Interval-Valued Fuzzy Numbers (IVFNs) combined with VIKOR method has been adapted in this analysis.

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Chapter 1- Introduction

1.1 Overview

Decision making is the cognitive process generally used in upstream of both industries and academia resulting in the selection of a course of action among a set of alternative scenario. In other words, decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Analysis of individual decision is concerned with the logic of decision making (or reasoning) which can be rational or irrational on the basis of explicit assumptions. Logical decision making is an important part of all science based professions, where specialists apply their knowledge in a given area to make informed decisions. However, it has been proved that the decision made collectively tend to be more effective than decision made by an individual. Therefore group decision making is a collective decision making process in which individuals' decisions are grouped together to solve a particular problem. But sometimes, when individuals make decisions as part of a group, there may be a tendency to exhibit biasness towards discussing shared information, as opposed to unshared information. To overcome such kind of error in decision making process, highly experience, dynamic and brilliant experts or practitioners are indeed required to participate and they should have much knowledge in the concerned area of judgment. Moreover, decision making is a nonlinear and recursive process because most of decisions are made by moving back and forth between the choice of criteria and the identification of alternatives. Every decision is made within a decision environment, which is defined as the collection of information, alternatives, values, and preferences *available at the time of the decision.* Since both information and alternatives are constrained because the time and effort to gain information or identify alternatives are limited. In fact decisions must be made within this constrained environment. Today, the *major challenge of decision making is uncertainty*, and a major goal of decision analysis is to reduce uncertainty. Recent robust decision efforts have formally integrated uncertainty and criterion subjectivity into the decision making process. Due to such kind of uncertainty and subjectivity involved in evaluative criterion, fuzziness has come into the picture. To deal with the kind of qualitative, imprecise and incomplete information decision problems, **Zadeh (1965)** suggested employing the fuzzy set theory as a modeling tool for complex systems. Fuzziness is a type of imprecision which is associated with the use of fuzzy sets that is, the classes in which there is no sharp transition from membership to non-membership **(Zimmermann, 1991)**. The term 'decision-making in fuzzy environment' means a decision making process in which the goals and/or the constraints, but not necessarily the system under control, are fuzzy in nature. This means that the goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined **(Bellman and Zadeh, 1970)**.

A major part of decision making involves the analysis of a finite set of alternatives described in terms of some evaluative criteria. These criteria may be *benefit* or *cost* in nature. Then the problem seeks to rank these alternatives in terms of their appropriateness to the decision maker(s); when all the criteria are considered simultaneously. Another goal is to find the best alternative or to determine the relative total priority of each alternative. Solving such problems is the focus of Multi-Criteria Decision Making (MCDM) in decision and information sciences. Decision making in presence of multiple, generally conflicting as well as non-commensurable criteria is simply called multi-criteria decision making. Multiple and confliting objectives, for example, 'minimize cost' and 'maximize quality of service' are the real stuff of the decision makers' or managers' daily concerns.

Moreover, in some situations the criterions may be tangible and intangible in nature and invites uncertainty in decision making process. In a real-world decision making situation, the application of the classic MCDM methods faces serious practical constraints, because of inherent imprecision or vagueness present in the criteria information. In order to tackle such kind of problems, **Bellman and Zadeh (1970)** introduced fuzzy sets contributed to the field of MCDM and called fuzzy Multi-Criteria Decision Making (FMCDM) approach. Now-a-days, it has been observed that, FMCDM has gained immense popularity in the real life applications. The following five important applications of FMCDM have been found in various fields like:

a) Evaluation of weapon systems

b) A project maturity evaluation system

c) Technology transfer strategy selection in biotechnology

d) Aggregation of market research data

e) Supply chain management and many others.

The area of decision making has attracted the interest of many researchers and management practitioners, is still highly debated as there are many MCDM methods which may yield different results when they are applied on exactly the same data. This leads to a decision making inconsistency.

1.2 Research Background

In the literature, there are two crucial approaches to multi-criteria decision making problems: multiple attribute decision making (MADM) and multiple objective decision making (MODM). The main difference between the MADM and MODM approaches is that MODM concentrates on continuous decision space aimed at the realization of the best solution, in which several objective functions are to be achieved simultaneously. The decision processes involve searching for the best solution, given a set a conflicting objectives. In fact, a MODM problem is associated with the problem of design for optimal solutions through mathematical programming. Conversely, MADM refers towards making decisions in the discrete decision spaces and focuses on how to select or to rank different predetermined alternatives. Accordingly, a MADM problem can be associated with a problem of choice or ranking of the existing alternatives **(Zimmermann, 1987)**. The following important methods such as analytical hierarchy process (AHP), analytical network process (ANP), technique for order performance by similarity to ideal solution (TOPSIS), outranking methods (e.g. ELECTRE, PROMETHEE, ORESTE) and multi attribute utility theory (MAUT) etc. are mainly involved in the category of MADM. Similarly some of the mathematical programming techniques such as linear programming (LP), genetic programming (GP) and mixed integer programming (MIP) are typically associated with MODM approaches.

The classic MADM methods generally assume that all criteria and their respective weights are expressed in crisp values and, thus, the appropriateness rating and the ranking of the alternatives can be carried out without any difficulty. In a real world decision situation, the application of the classic MADM method may face serious practical constraints from the criteria perhaps containing uncertainty, incompleteness, imprecision or vagueness in the data. In many cases, performance of the criteria can only be expressed qualitatively or by using linguistic terms, which certainly demands a more appropriate method to tackle with. Classical MADM methods cannot handle such linguistic data effectively due the involvement of fuzziness or imprecision arise in the decision making process. In the contrary, the application of the fuzzy set theory in the field of MADM is well justified when the intended goals (attributes) or their attainment cannot be defined crisply but only as fuzzy sets **(Zimmermann, 1987)**.

Following literature survey depicts some of the extensive works carried out in the field of MCDM under fuzzy environment. **Bellman and Zadeh (1970)** introduced the approach regarding decision making in a fuzzy environment. **Baas and Kwakernaak (1977)** applied the most classic work on the fuzzy MADM method and it was used as a benchmark for other similar fuzzy decision models. Their approach consisted of both phases of MADM, the rating of criteria and the ranking of multiple aspect alternatives using fuzzy sets. **Kickert (1978)** summarized the fuzzy set theory applications in MADM problems. **Dubois and Prade (1980), Zimmermann (1987), Chen and Hwang (1992), and Ribeiro (1996)** differentiated the family of fuzzy MADM methods into two main phases. The first phase is generally known as the rating process, dealing with the measurement of performance ratings or the degree of satisfaction with respect to all attributes of each alternative. The aggregate rating, indicating the global performance of each alternative, which can be obtained through the accomplishment of suitable aggregation operations of all criteria involved in the decision. The second phase, the ranking of alternatives that is carried out by ordering the existing alternatives according to the resulted aggregated performance ratings obtained from the first phase.

Chang and Chen (1994) proposed a fuzzy MCDM method for technology transfer strategy selection in biotechnology by using linguistic variables and triangular fuzzy numbers. The selection and ranking of alternative was done on the concept of the index of optimism. **Cheng and Mon (2003)** applied analytical hierarchy process (AHP) to multi-criteria decision making for the evaluation of weapons system based on the fuzzy scales. In this paper, the evaluation criteria’s was generally multiple and conflict, and the descriptions of the weapon systems are usually linguistic and vague. **Altrock and Krause (1994)** presented a fuzzy multi-criteria decision-making system for optimizing the design process of truck components, such as gear boxes, axels or steering. They considered both objective data based on the number of design change in last month and subjective data such as maturity of parts of a component and finally optimization was carried by fuzzy data analysis for the optimum design effort to be required until completion of project. Their hierarchically defined system (using the commercial fuzzy logic design tool fuzzyTECH) is now in use at Mercedes-Benz in Germany. **Fan et al. (2002)** proposed a new approach to solve the MADM problem, where the decision makers were instructed to give his/her preference on alternatives in a fuzzy relation. To reflect the decision makers' preference information, an optimization model was constructed to assess the attribute weights and then to select the most desirable alternatives. **Omero et al. (2005)** dealt with the problem of assessing the performance of a set of production units, simultaneously considering different kinds of information, yielded by Data Envelopment Analysis (DEA), a qualitative data analysis, and an expert assessment. **Hua et al. (2005)** developed a fuzzy multiple attribute decision making (FMADM) method with a three level hierarchical decision making model to evaluate the aggregate risk for green manufacturing projects. **Ling (2006)** presented a fuzzy MADM method in which the attribute weights and decision matrix elements (attribute values) were fuzzy variables. The author used some fuzzy arithmetic operations and the expected value operator of fuzzy variables to solve the FMADM problem. **Xu and Chen (2007)** developed an interactive method for multiple attribute group decision making in a fuzzy environment. The method could be used in situations where the information about attribute weights were partly known, the weights of decision makers were expressed in exact numerical values or triangular fuzzy numbers, and the attribute values were triangular fuzzy numbers. **Wu et al. (2006)**

developed a new approximate algorithm for solving fuzzy multiple objective linear programming (FMOLP) problems involving fuzzy parameters in any form of membership functions in both objective functions and constraints.

1.3 Motivation of the Present Work

Lots of fuzzy MCDM techniques are readily available in the literature of various fields; an analyst can get confused in determining which technique is to be employed when confronted in a decision-making cum selection problem. This ambiguity can lead to inappropriate selection, resulting in a misleading solution and incorrect conclusions. If this made casually, the entire design may proceed down a poor path, resulting in a weak solution. This in turn results waste of time, money, resources, and energy. Though all the criterions correspond to qualitative and vague information in general decision making practice, a robust, accurate MCDM technique is indeed required for the best compromise solution. All the methods that have been described globally presented; the most effective one is difficult to infer. For example axioms are the easy technique based on mathematical approach but it loses some flexibility in the system. In other hand MCDM somewhat deals with sensitivity analysis approach which is basically computer oriented, but sensitivity analysis does not provide by how much what items were changed and does not provide limitations of algorithm. Therefore, the applicability of most accurate and appropriate method in right direction has become a challenging job for today's researchers. Trying to point the best method doesn't always mean to get the most accurate method, sometimes designers are allowed to approximate solutions to certain extend. Hence, the best method could be the one that provide them with the cheapest solution or the fastest method.

Introducing a technique with lots of weights and matrix calculation could be too much time consuming and would require a in-depth skills from the designers so that the process would use its relative ease of use.

The objective of the current work is to provide a robust, quantified MCDM monitor of the *level-of-satisfaction* among the decision makers and capability to tackle vague-incomplete information and uncertainty in real life application followed by two case studies viz.

1. *Supplier selection*
2. *Industrial robot selection*

1.4 Organization of the Thesis

The entire thesis has been organized in four chapters. **Chapter 1** presents the concept of decision making in fuzzy environment and theory of MCDM followed by its category of classification and field of application. An extensive literature survey also depicts the applicability of fuzzy sets in MCDM and also covers a section highlighting motivation of the current research. **Chapter 2** covers presentation of necessary mathematical background on fuzzy sets and related conceptual definitions of some used MCDM methods. In this chapter, readers may get a clear understanding with root mathematical concept of fuzzy sets and importance of linguistic variables in the course of multiple conflicting decision making problems. **Chapter 3** and **Chapter 4** illustrate the applicability of recent methodologies in supplier selection and industrial robot selection respectively under fuzzy environment as a two case studies. Moreover, a brief survey of some literatures on the field of supplier selection and robot selection has also been provided separately. Finally, concluding remarks of this dissertation have been presented in subsequent chapter end.

Finally, the outcome of the present research work has been furnished in terms of publications of international standard.

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Chapter 2: Mathematical Background

2.1 Concepts of Fuzzy based MCDM

Multi-criteria decision making (MCDM) has become a most focusing area of research because of the involvement of a set of conflict objectives in real life problems. Introduction of mathematical concepts in to decision making science was first found in late-nineteenth-century welfare economics, in the works of *Edgeworth and Pareto*. A mathematical model of MCDM can be shortly presented here as follows **(Kahraman, 2008)**:

$$\underset{S}{Min}\ z = [z_1(x), z_2(x), \ldots, z_K(x)]^T \tag{2.1}$$

Here, $S = \{x \in X \mid Ax \leq b, x \in R^n, x \geq 0\}$

Also $Z(x) = C\,x$ is the K-dimensional vector of objective functions and C is the vector of cost corresponding to each objective function,

S is the feasible region that is bounded by the given set of constraints,

A is the matrix of technical coefficients of the left-hand side of constraints,

b is the right-hand side of constraints (i.e., the available resources),

x is the n-dimensional vector decision variables.

When the objective functions and constraints are linear, than the model is a linear multi-objective optimization problem (LMOOP). But, if any objective function and/or constraints are nonlinear, then the problem is described as a nonlinear multi-objective optimization problem (NLMOOP). MCDM model can be treated as a deterministic model.

But, in real world situations, the input information to model (shown by **Eq. 2.1**) may be vague, means the technical coefficient matrix (A) and/or the available resource values (b) and/or the coefficients of objective functions (C) are may be vague in nature. Apart from this, vagueness may exist due to the aspiration levels of goals $(Z_i(x))$ and the preference information during the interactive process. For the above case only fuzzy multi-criteria model has come into existence and this can be written as follows:

$$\underset{S}{Min}\ z \cong [z_1(x), z_2(x), ..., z_K(x)]^T \tag{2.2}$$

Here, $S = \left\{ x \in X \middle| \tilde{A}x \lesssim \tilde{b}, x \in R^n, x \geq 0 \right\}$

This fuzzy model has been transformed into crisp (deterministic) by using an appropriate membership function. As like model (shown in **Eq. 2.1**), this model can also be classified into two classes. If any of the objective functions, constraints, and membership functions are linear, then the model will be LFMOOP. But, if any of the objective functions and/or constraints and/or membership functions is nonlinear, then the model is described as NLFMOOP. Different approaches can handle the solution of fuzzy multi-criteria problems, (i.e., model shown in **Eq. 2.2**). All of these approaches depend on transforming problem (refer **Eq. 2.2**) from fuzzy model to crisp model by using an appropriate membership function which is the foundation of fuzzy programming **(Abd El-Wahed, 2008)**.

In fact, a group multiple-criteria decision-making (GMCDM) problem, which may be described by means of the following, sets **(Chen et al., 2006)**:

(i) a set of K decision-makers called $E = \{D_1, D_2, ..., D_K\}$;

(ii) a set of m possible alternatives called $A = \{A_1, A_2, ..., A_m\}$;

(iii) a set of n criteria, $C = \{C_1, C_2, ..., C_n\}$;

(iv) a set of performance ratings of $A_i\,(i = 1, 2, ..., m)$ with respect to criteria $C_j\,(j = 1, 2, ..., n)$,

called $X = \{x_{ij}, i = 1, 2, ..., m, j = 1, 2, ..., n\}$.

2.2 Fuzzy Set Theory

To deal with vagueness in human thought, **Lotfi A. Zadeh (1965)** first introduced the fuzzy set theory, which has the capability to represent/manipulate data and information possessing based on nonstatistical uncertainties. Moreover fuzzy set theory has been designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision inherent to decision making problems. Some basic definitions of fuzzy sets, fuzzy numbers and linguistic variables are reviewed from **Zadeh (1975), Buckley (1985), Negi (1989), Kaufmann and Gupta (1991)**. The basic definitions and notations below will be used throughout this thesis until otherwise stated.

2.2.1 Definitions of fuzzy sets:

Definition 1. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval $[0,1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ **(Kaufmann and Gupta, 1991)**.

Definition 2. A fuzzy set $\tilde{A}$ in a universe of discourse X is convex if and only if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right) \qquad \textbf{(2.3)}$$

For all x_1, x_2 in X and all $\lambda \in [0,1]$, where $\min$ denotes the minimum operator **(Klir and Yuan, 1995).**

Definition 3. The height of a fuzzy set is the largest membership grade attained by any element in that set. A fuzzy set $\tilde{A}$ in the universe of discourse X is called normalized when the height of $\tilde{A}$ is equal to 1 **(Klir and Yuan, 1995)**.

2.2.2 Definitions of fuzzy numbers:

Definition 1. A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal. **Fig. 2.1** shows a fuzzy number $\tilde{n}$ in the universe of discourse X that conforms to this definition **(Kaufmann and Gupta, 1991)**.

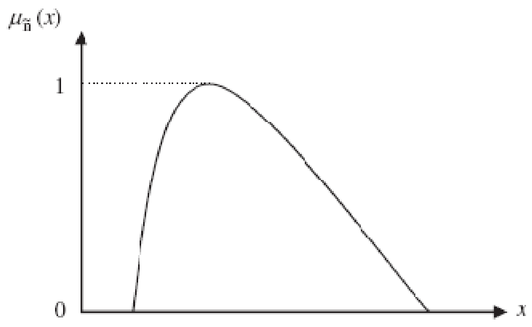

Fig. 2.1. A fuzzy number $\tilde{n}$

Definition 2. The α-cut of fuzzy number $\tilde{n}$ is defined as:

$$\tilde{n}^{\alpha} = \{x_i : \mu_{\tilde{n}}(x_i) \geq \alpha, x_i \in X\}, \qquad \textbf{(2.4)}$$

Here, $\alpha \in [0,1]$.

The symbol $\tilde{n}^{\alpha}$ represents a non-empty bounded interval contained in X, which can be denoted by $\tilde{n}^{\alpha} = [n_l^{\alpha}, n_u^{\alpha}]$, n_l^{α} and n_u^{α} are the lower and upper bounds of the closed interval, respectively **(Kaufmann and Gupta, 1991; Zimmermann, 1991)**. For a fuzzy number $\tilde{n}$, if $n_l^{\alpha} > 0$ and $n_u^{\alpha} \leq 1$ for all $\alpha \in [0,1]$, then $\tilde{n}$ is called a standardized (normalized) positive fuzzy number **(Negi, 1989)**.

Definition 3. Suppose, a positive triangular fuzzy number (PTFN) is $\tilde{A}$ and that can be defined as (a,b,c) shown in **Fig. 2.2**. The membership function $\mu_{\tilde{n}}(x)$ is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & if\ a \leq x \leq b, \\ (c-x)/(c-b), & if\ b \leq x \leq c, \\ 0, & otherwise, \end{cases} \qquad \textbf{(2.5)}$$

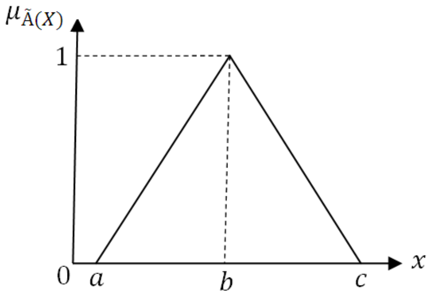

Fig. 2.2 A triangular fuzzy number $\tilde{A}$

Based on extension principle, the fuzzy sum $\oplus$ and fuzzy subtraction Θ of any two triangular fuzzy numbers are also triangular fuzzy numbers; but the multiplication $\otimes$ of any two triangular fuzzy numbers is only approximate triangular fuzzy number **(Zadeh, 1975)**. Let's have a two positive triangular fuzzy numbers, such as $\tilde{A}_1 = (a_{1,} b_1, c_1)$, and $\tilde{A}_2 = (a_2, b_2, c_2)$, and a positive real number $r = (r, r, r)$, some algebraic operations can be expressed as follows:

$$\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \quad \textbf{(2.6)}$$

$$\tilde{A}_1 \Theta \tilde{A}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2), \quad \textbf{(2.7)}$$

$$\tilde{A}_1 \otimes \tilde{A}_2 = (a_1 a_2, b_1 b_2, c_1 c_2), \quad \textbf{(2.8)}$$

$$r \otimes \tilde{A}_1 = (ra_1, rb_1, rc_1), \quad \textbf{(2.9)}$$

$$\tilde{A}_1 \varnothing \tilde{A}_2 = (a_1/c_2, b_1/b_2, c_1/a_2), \quad \textbf{(2.10)}$$

The operations of $\vee$ (max) and $\wedge$ (min) are defined as:

$$\tilde{A}_1 (\vee) \tilde{A}_2 = (a_1 \vee a_2, b_1 \vee b_2, c_1 \vee c_2), \quad \textbf{(2.11)}$$

$$\tilde{A}_1 (\wedge) \tilde{A}_2 = (a_1 \wedge a_2, b_1 \wedge b_2, c_1 \wedge c_2), \quad \textbf{(2.12)}$$

Here, $r > 0$, and $a_1, b_1, c_1 > 0$,

Also the crisp value of triangular fuzzy number set $\tilde{A}_1$ can be determined by defuzzification which locates the Best Non-fuzzy Performance (BNP) value. Thus, the BNP values of fuzzy number are calculated by using the center of area (COA) method as follows: **(Moeinzadeh and Hajfathaliha, 2010)**

BNP$_i$ = $\frac{[(c-a)+(b-a)]}{3}+a$, $\forall_i$, **(2.13)**

Definition 4. A matrix $\tilde{\mathbf{D}}$ is called a fuzzy matrix if at least one element is a fuzzy number **(Buckley, 1985)**.

2.2.3 Linguistic variable:

Definition 1. A linguistic variable is the variable whose values are not expressed in numbers but words or sentences in a natural or artificial language, i.e., in terms of linguistic **(Zadeh, 1975)**. The concept of a linguistic variable is very useful in dealing with situations, which are too complex or not well defined to be reasonably described in conventional quantitative expressions **(Zimmermann, 1991)**. For example, 'weight' is a linguistic variable whose values are 'very low', 'low', 'medium', 'high', 'very high', etc. Fuzzy numbers can also represent these linguistic values.

2.2.4 The concept of generalized trapezoidal fuzzy numbers

By the definition given by **(Chen, 1985)**, a generalized trapezoidal fuzzy number can be defined as $\tilde{A}=(a_1,a_2,a_3,a_4;w_{\tilde{A}})$, as shown in **Fig. 2.3**.

and the membership function $\mu_{\tilde{A}}(x):R\rightarrow[0,1]$ is defined as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-a_1}{a_2-a_1}\times w_{\tilde{A}}, & x\in(a_1,a_2)\\ w_{\tilde{A}}, & x\in(a_2,a_3)\\ \frac{x-a_4}{a_3-a_4}\times w_{\tilde{A}}, & x\in(a_3,a_4)\\ 0, & x\in(-\infty,a_1)\cup(a_4,\infty)\end{cases}$$ **(2.14)**

Here, $a_1\le a_2\le a_3\le a_4$ and $w_{\tilde{A}}\in[0,1]$

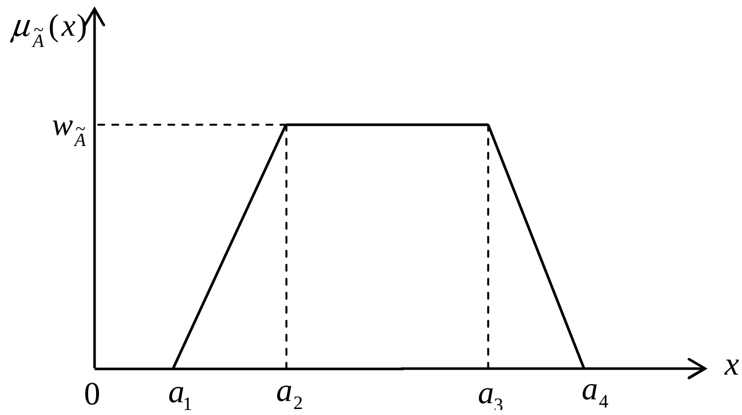

Fig. 2.3 Trapezoidal fuzzy number $\tilde{A}$

The elements of the generalized trapezoidal fuzzy numbers $x \in R$ are real numbers, and its membership function $\mu_{\tilde{A}}(x)$ is the regularly and continuous convex function, it shows that the membership degree to the fuzzy sets. If $-1 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$, then $\tilde{A}$ is called the normalized trapezoidal fuzzy number. Especially, if $w_{\tilde{A}} = 1$, then $\tilde{A}$ is called trapezoidal fuzzy number (a_1, a_2, a_3, a_4); if $a_1 < a_2 = a_3 < a_4$, then $\tilde{A}$ is reduced to a triangular fuzzy number. If $a_1 = a_2 = a_3 = a_4$, then $\tilde{A}$ is reduced to a real number.

Suppose that $\tilde{a} = (a_1, a_2, a_3, a_4; w_{\tilde{a}})$ and $\tilde{b} = (b_1, b_2, b_3, b_4; w_{\tilde{b}})$ are two generalized trapezoidal fuzzy numbers, then the operational rules of the generalized trapezoidal fuzzy numbers $\tilde{a}$ and $\tilde{b}$ are shown as follows **(Chen and Chen, 2009)**:

$$\tilde{a} \oplus \tilde{b} = (a_1, a_2, a_3, a_4; w_{\tilde{a}}) \oplus (b_1, b_2, b_3, b_4; w_{\tilde{b}}) =$$

$$(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(w_{\tilde{a}}, w_{\tilde{b}})) \quad \textbf{(2.15)}$$

$$\tilde{a} - \tilde{b} = (a_1, a_2, a_3, a_4; w_{\tilde{a}}) - (b_1, b_2, b_3, b_4; w_{\tilde{b}}) =$$

$$(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1; \min(w_{\tilde{a}}, w_{\tilde{b}})) \quad \textbf{(2.16)}$$

$$\tilde{a} \otimes \tilde{b} = (a_1, a_2, a_3, a_4; w_{\tilde{a}}) \otimes (b_1, b_2, b_3, b_4; w_{\tilde{b}}) =$$

$$\left(a, b, c, d; \min\left(w_{\tilde{a}}, w_{\tilde{b}}\right)\right) \quad \textbf{(2.17)}$$

Here,

$$a = \min\left(a_1 \times b_1, a_1 \times b_4, a_4 \times b_1, a_4 \times b_4\right)$$

$$b = \min\left(a_2 \times b_2, a_2 \times b_3, a_3 \times b_2, a_3 \times b_3\right)$$

$$c = \max\left(a_2 \times b_2, a_2 \times b_3, a_3 \times b_2, a_3 \times b_3\right)$$

$$d = \max\left(a_1 \times b_1, a_1 \times b_4, a_4 \times b_1, a_4 \times b_4\right)$$

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ are real numbers, then

$$\tilde{a} \otimes \tilde{b} = \left(a1 \times b1, a2 \times b2, a3 \times b3, a4 \times b4; \min\left(w_{\tilde{a}}, w_{\tilde{b}}\right)\right)$$

$$\tilde{a} / \tilde{b} = {\left(a_1, a_2, a_3, a_4; w_{\tilde{a}}\right)} \big/ {\left(b_1, b_2, b_3, b_4; w_{\tilde{b}}\right)}$$

$$= \left(a_1 / b_4, a_2 / b_3, a_3 / b_2, a_4 / b_1; \min\left(w_{\tilde{a}}, w_{\tilde{b}}\right)\right) \quad \textbf{(2.18)}$$

Chen and Chen (2003) proposed the concept of COG point of generalized trapezoidal fuzzy numbers, and suppose that the COG point of the generalized trapezoidal fuzzy number $\tilde{a} = \left(a_1, a_2, a_3, a_4; w_{\tilde{a}}\right)$ is $\left(x_{\tilde{a}}, y_{\tilde{a}}\right)$, then:

$$y_{\tilde{a}} = \begin{cases} \dfrac{w_{\tilde{a}} \times \left(\dfrac{a_3 - a_2}{a_4 - a_1} + 2\right)}{6}, & if\ a_1 \neq a_4 \\ \dfrac{w_{\tilde{a}}}{2}, & if\ a_1 = a_4 \end{cases} \quad \textbf{(2.19)}$$

$$x_{\tilde{a}} = \frac{y_{\tilde{a}} \times \left(a_2 + a_3\right) + \left(a_1 + a_4\right) \times \left(w_{\tilde{a}} - y_{\tilde{a}}\right)}{2 \times w_{\tilde{a}}} \quad \textbf{(2.20)}$$

2.3 Theory of Interval-Valued Fuzzy Sets (IVFS)

In fuzzy set theory, it is often difficult for an expert to exactly quantify his/ her opinion as a number in interval $[0,1]$. Therefore, it is more suitable to represent this degree of certainty by an interval. **Sambuc (1975)** and **Grattan (1975)** noted that the presentation of a linguistic expression in the form of fuzzy sets is not enough. Interval-valued fuzzy sets (IVFS) were suggested for the first time by **Gorzlczany (1987)**. Also **Corneils et al. (2006)** and **Karnik and Mendel (2001)** noted that the main reason for proposing this new concept is the fact that, in the linguistic modeling of a phenomenon, the presentation of the linguistic expression in the form of ordinary fuzzy sets is not clear enough. **Wang and Li (1998)** defined IVFNs and gave their extended operations. Based on definition of IVFS in **Gorzlczany (1987)**, an IVFS as defined on $(-\infty,+\infty)$ is given by:

$$A = \{(x, [\mu_A^L(x), \mu_A^U(x)])\} \quad \textbf{(2.21)}$$

$$\mu_A^L, \mu_A^U : X \to [0,1] \qquad \forall x \in X, \qquad \mu_A^L \le \mu_A^U$$

$$\overline{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)]$$

$$A = \{(x, \overline{\mu}_A(x))\}, x \in (-\infty, +\infty)$$

Here, $\mu_A^L(x)$ is the lower limit of the degree of membership and $\mu_A^U(x)$ is the upper limit of the degree of membership.

Let, two IVFNs $N_x = [N_x^-; N_x^+]$ and $M_y = [M_y^-; M_y^+]$, according to **(Gorzlczany, 1987)**, we have:

Definition 1: If $. \in (+,-,\times,\div)$, then $N.M(x.y) = [N_x^-.M_y^-; N_x^+.M_y^+]$, for a positive nonfuzzy number (υ), and $\upsilon.M(x,y) = [\upsilon.M_y^-; \upsilon.M_y^+]$.

Definition 2: The intersection of two IVFS **(Gorzlczany, 1987)** is defined as the minimum of their respective lower and upper bounds of their membership intervals. Given two intervals of $[0,1]$ and $N_x = [N_x^-; N_x^+] \subset [0,1]$, $M_y = [M_y^-; M_y^+] \subset [0,1]$, the minimum of both intervals is an interval $K = MIN(N_x, M_y) = [MIN(N_x^-, M_y^-), MIN(N_x^+, M_y^+)]$.

Definition 3: The union of two IVFS **(Gorzlczany, 1987)** is defined as the maximum of their respective lower and upper bounds of their membership intervals. Given two intervals of $[0,1]$ and $N_x = [N_x^-; N_x^+] \subset [0,1], M_y = [M_y^-; M_y^+] \subset [0,1]$, the maximum of both intervals is an interval $K = MAX(N_x, M_y) = [MAX(N_x^-, M_y^-), MAX(N_x^+, M_y^+)]$.

2.4 Interval-Valued Fuzzy Numbers (IVFNs)

Wang and Li (2001) represented the interval-valued trapezoidal fuzzy numbers as follows:

$$\tilde{\tilde{A}} = \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U}\right)\right]$$

$$\text{Here, } \begin{array}{l} 0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L \le 1, \\ 0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U \le 1, \text{ and } \tilde{\tilde{A}}^L \subset \tilde{\tilde{A}}^U. \\ 0 \le w_{\tilde{\tilde{A}}^L} \le w_{\tilde{\tilde{A}}^U} \end{array}$$

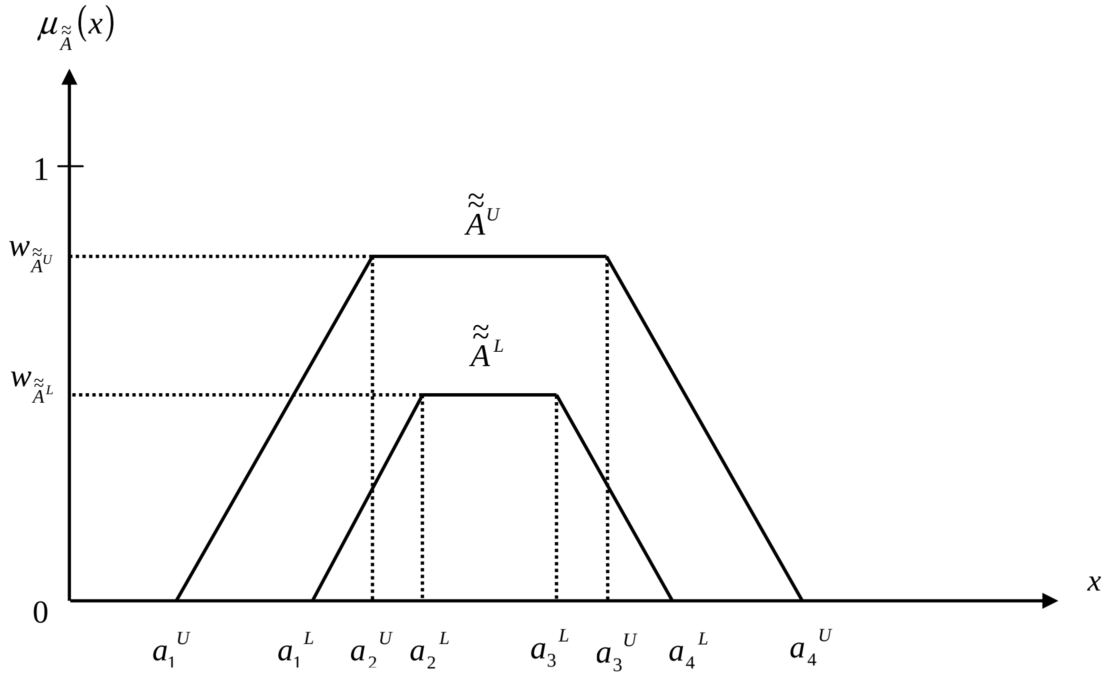

Fig. 2.4 Interval-valued trapezoidal fuzzy numbers **(Liu and Wang, 2011)**

From **Fig. 2.4**, it can be concluded that interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$ consists of two level of values such as, lower values of interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}^L$ and the upper values of interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}^U$ **(Liu and Wang, 2011)**. The operation rules of interval-valued trapezoidal fuzzy numbers as given by **Wei and Chen (2009)** have been reproduced below.

Suppose that,

$\tilde{\tilde{A}} = \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U}\right)\right]$ and

$\tilde{\tilde{B}} = \left[\tilde{\tilde{B}}^L, \tilde{\tilde{B}}^U\right] = \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U}\right)\right]$ are the two interval-valued trapezoidal fuzzy numbers, where,

$$\begin{aligned}
&0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L \le 1, \\
&0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U \le 1, \\
&0 \le w_{\tilde{\tilde{A}}^L} \le w_{\tilde{\tilde{A}}^U} \le 1, \quad \tilde{\tilde{A}}^L \subset \tilde{\tilde{A}}^U
\end{aligned}$$

$$\begin{aligned}
&0 \le b_1^L \le b_2^L \le b_3^L \le b_4^L \le 1, \\
&0 \le b_1^U \le b_2^U \le b_3^U \le b_4^U \le 1, \\
&0 \le w_{\tilde{\tilde{B}}^L} \le w_{\tilde{\tilde{B}}^U} \le 1, \quad \tilde{\tilde{B}}^L \subset \tilde{\tilde{B}}^U
\end{aligned}$$

1.The sum of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}} \oplus \tilde{\tilde{B}}$:

$$\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U}\right)\right] \oplus \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U}\right)\right]$$

$$= \left[\left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min\left(w_{\tilde{\tilde{A}}^L}, w_{\tilde{\tilde{B}}^L}\right)\right), \left(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; \min\left(w_{\tilde{\tilde{A}}^U}, w_{\tilde{\tilde{B}}^U}\right)\right)\right]$$

(2.22)

2.The difference of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}} - \tilde{\tilde{B}}$:

$$\tilde{\tilde{A}} - \tilde{\tilde{B}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U}\right)\right] - \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U}\right)\right]$$

$$= \left[\left(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L; \min\left(w_{\tilde{\tilde{A}}^L}, w_{\tilde{\tilde{B}}^L}\right)\right), \left(a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U; \min\left(w_{\tilde{\tilde{A}}^U}, w_{\tilde{\tilde{B}}^U}\right)\right)\right]$$

(2.23)

3.The product of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}} \otimes \tilde{\tilde{B}}$:

$$\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U}\right)\right] \otimes \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U}\right)\right]$$

$$= \left[\left(a_1^L \times b_1^L, a_2^L \times b_2^L, a_3^L \times b_3^L, a_4^L \times b_4^L; \min\left(w_{\tilde{\tilde{A}}^L}, w_{\tilde{\tilde{B}}^L}\right)\right), \left(a_1^U \times b_1^U, a_2^U \times b_2^U, a_3^U \times b_3^U, a_4^U \times b_4^U; \min\left(w_{\tilde{\tilde{A}}^U}, w_{\tilde{\tilde{B}}^U}\right)\right)\right] \tag{2.24}$$

4.The product between an interval-valued trapezoidal fuzzy number and a constant $\lambda\tilde{\tilde{A}}$:

$$\lambda\tilde{\tilde{A}} = \lambda \times \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U}\right)\right]$$

$$= \left[\left(\lambda a_1^L, \lambda a_2^L, \lambda a_3^L, \lambda a_4^L; w_{\tilde{\tilde{A}}^L}\right), \left(\lambda a_1^U, \lambda a_2^U, \lambda a_3^U, \lambda a_4^U; w_{\tilde{\tilde{A}}^U}\right)\right], \quad \lambda > 0. \tag{2.25}$$

2.5 Division Operator Ø for IVFNs

Wei and Chen (2009) proposed a new division operator for interval-valued trapezoidal fuzzy numbers for fuzzy risk analysis. According to them, given for two fuzzy numbers:

Let $\tilde{\tilde{A}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; \hat{w}_{\tilde{\tilde{A}}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; \hat{w}_{\tilde{\tilde{A}}}^U\right)\right]$, $\tilde{\tilde{B}} = \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; \hat{w}_{\tilde{\tilde{B}}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; \hat{w}_{\tilde{\tilde{B}}}^U\right)\right]$,

$$U^L = \left\{ a_1^L / b_1^L, a_2^L / b_2^L, a_3^L / b_3^L, a_4^L / b_4^L \right\}, U^U = \left\{ a_1^U / b_1^U, a_2^U / b_2^U, a_3^U / b_3^U, a_4^U / b_4^U \right\}, \tag{2.26}$$

$x^L = \min(U^L), x^U = \min(U^U), y^L = \max(U^L), y^U = \max(U^U)$, where

$0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L \le 1,$
$0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U \le 1,$

$0 \le b_1^L \le b_2^L \le b_3^L \le b_4^L \le 1,$
$0 \le b_1^U \le b_2^U \le b_3^U \le b_4^U \le 1.$

The division operator Ø proposed by **(Wei and Chen, 2009)** between interval-valued trapezoidal fuzzy numbers has been presented follows:

$$\tilde{\tilde{A}} \varnothing \tilde{\tilde{B}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; \hat{w}_{\tilde{\tilde{A}}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; \hat{w}_{\tilde{\tilde{A}}}^U\right)\right] \varnothing \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; \hat{w}_{\tilde{\tilde{B}}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; \hat{w}_{\tilde{\tilde{B}}}^U\right)\right]$$

$$= \begin{bmatrix} \left(\min\left(U^L\right), \min\left(U^L - x^L\right), \max\left(U^L - y^L\right), \max\left(U^L\right); \min\left(\hat{w}_{\tilde{\tilde{A}}}^L, \hat{w}_{\tilde{\tilde{B}}}^L\right)\right), \\ \left(\min\left(U^U\right), \min\left(U^U - x^U\right), \max\left(U^U - y^U\right), \max\left(U^U\right); \min\left(\hat{w}_{\tilde{\tilde{A}}}^U, \hat{w}_{\tilde{\tilde{B}}}^U\right)\right) \end{bmatrix} \tag{2.27}$$

Here $\left(U^L - x^L\right)$ denotes deleting the element x^L from the set U^L, $\left(U^U - x^U\right)$ denotes deleting the element x^U from the set U^U, $\left(U^L - y^L\right)$ denotes deleting the element y^L from the set U^L, $\left(U^U - y^U\right)$ denotes deleting the element y^U from the set U^U.

2.5.1 *Evaluating concepts of COG points for Interval-valued trapezoidal fuzzy numbers*

The coordinate of COG points $\left(x_{\tilde{\tilde{A}}^L}, y_{\tilde{\tilde{A}}^L}\right), \left(x_{\tilde{\tilde{A}}^U}, y_{\tilde{\tilde{A}}^U}\right), \left(x_{\tilde{\tilde{B}}^L}, y_{\tilde{\tilde{B}}^L}\right), \left(x_{\tilde{\tilde{B}}^U}, y_{\tilde{\tilde{B}}^U}\right)$ which belongs to the generalized interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U, \tilde{\tilde{B}}^L, \tilde{\tilde{B}}^U$; can be computed as follows **(Wei and Chen, 2009)**:

$$x_{\tilde{\tilde{A}}^L} = \frac{y_{\tilde{\tilde{A}}^L}\left(a_3^L + a_2^L\right) + \left(a_4^L + a_1^L\right)\left(\hat{w}_{\tilde{\tilde{A}}^L} - y_{\tilde{\tilde{A}}^L}\right)}{2\hat{w}_{\tilde{\tilde{A}}^L}} \tag{2.28}$$

$$y_{\tilde{\tilde{A}}^L} = \begin{cases} \dfrac{\hat{w}_{\tilde{\tilde{A}}^L}\left(\dfrac{a_3^L - a_2^L}{a_4^L - a_1^L} + 2\right)}{6}, & if \;\; a_1^L \neq a_4^L \;\; and \; 0 < \hat{w}_{\tilde{\tilde{A}}^L} \leq 1, \\ \dfrac{\hat{w}_{\tilde{\tilde{A}}^L}}{2}, & if \;\; a_1^L = a_4^L \;\; and \; 0 < \hat{w}_{\tilde{\tilde{A}}^L} \leq 1. \end{cases} \tag{2.29}$$

$$x_{\tilde{\tilde{A}}^U} = \frac{y_{\tilde{\tilde{A}}^U}\left(a_3^U + a_2^U\right) + \left(a_4^U + a_1^U\right)\left(\hat{w}_{\tilde{\tilde{A}}^U} - y_{\tilde{\tilde{A}}^U}\right)}{2\hat{w}_{\tilde{\tilde{A}}^U}} \tag{2.30}$$

$$y_{\tilde{\tilde{A}}^U} = \begin{cases} \dfrac{\hat{w}_{\tilde{\tilde{A}}^U}\left(\dfrac{a_3^U - a_2^U}{a_4^U - a_1^U} + 2\right)}{6}, & if\ \ a_1^U \neq a_4^U \ \ and\ 0 < \hat{w}_{\tilde{\tilde{A}}^U} \leq 1, \\ \dfrac{\hat{w}_{\tilde{\tilde{A}}^U}}{2}, & if\ \ a_1^U = a_4^U \ \ and\ 0 < \hat{w}_{\tilde{\tilde{A}}^U} \leq 1. \end{cases} \quad \textbf{(2.31)}$$

$$x_{\tilde{\tilde{B}}^L} = \frac{y_{\tilde{\tilde{B}}^L}\left(b_3^L + b_2^L\right) + \left(b_4^L + b_1^L\right)\left(\hat{w}_{\tilde{\tilde{B}}^L} - y_{\tilde{\tilde{B}}^L}\right)}{2\hat{w}_{\tilde{\tilde{B}}^L}} \quad \textbf{(2.32)}$$

$$y_{\tilde{\tilde{B}}^L} = \begin{cases} \dfrac{\hat{w}_{\tilde{\tilde{B}}^L}\left(\dfrac{b_3^L - b_2^L}{b_4^L - b_1^L} + 2\right)}{6}, & if\ \ b_1^L \neq b_4^L \ \ and\ 0 < \hat{w}_{\tilde{\tilde{B}}^L} \leq 1, \\ \dfrac{\hat{w}_{\tilde{\tilde{B}}^L}}{2}, & if\ \ b_1^L = b_4^L \ \ and\ 0 < \hat{w}_{\tilde{\tilde{B}}^L} \leq 1. \end{cases} \quad \textbf{(2.33)}$$

$$x_{\tilde{\tilde{B}}^U} = \frac{y_{\tilde{\tilde{B}}^U}\left(b_3^U + b_2^U\right) + \left(b_4^U + b_1^U\right)\left(\hat{w}_{\tilde{\tilde{B}}^U} - y_{\tilde{\tilde{B}}^U}\right)}{2\hat{w}_{\tilde{\tilde{B}}^U}} \quad \textbf{(2.34)}$$

$$y_{\tilde{\tilde{B}}^U} = \begin{cases} \dfrac{\hat{w}_{\tilde{\tilde{B}}^U}\left(\dfrac{b_3^U - b_2^U}{b_4^U - b_1^U} + 2\right)}{6}, & if\ \ b_1^U \neq b_4^U \ \ and\ 0 < \hat{w}_{\tilde{\tilde{B}}^U} \leq 1, \\ \dfrac{\hat{w}_{\tilde{\tilde{B}}^U}}{2}, & if\ \ b_1^U = b_4^U \ \ and\ 0 < \hat{w}_{\tilde{\tilde{B}}^U} \leq 1. \end{cases} \quad \textbf{(2.35)}$$

2.5.2 ***Evaluating the distance of two Interval-valued trapezoidal fuzzy numbers***

Suppose that,

$\tilde{\tilde{A}} = \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}^L}\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}^U}\right)\right]$ and

$\tilde{\tilde{B}} = \left[\tilde{\tilde{B}}^L, \tilde{\tilde{B}}^U\right] = \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}^L}\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}^U}\right)\right]$ are the two generalized interval-valued trapezoidal fuzzy numbers, then the distance of two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ is computed by the following steps:

a. Utilize **Eqs. 2.28-2.35** to calculate the coordinate of COG points $\left(x_{\tilde{\tilde{A}}^L}, y_{\tilde{\tilde{A}}^L}\right), \left(x_{\tilde{\tilde{A}}^U}, y_{\tilde{\tilde{A}}^U}\right), \left(x_{\tilde{\tilde{B}}^L}, y_{\tilde{\tilde{B}}^L}\right), \left(x_{\tilde{\tilde{B}}^U}, y_{\tilde{\tilde{B}}^U}\right)$ which belong to the generalized interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U, \tilde{\tilde{B}}^L, \tilde{\tilde{B}}^U$.

b. The distance of two interval-valued trapezoidal fuzzy number is:

$$d\left(\tilde{\tilde{A}}, \tilde{\tilde{B}}\right) = \frac{1}{2}\sqrt{\left\{\left(y_{\tilde{\tilde{A}}^L} - y_{\tilde{\tilde{B}}^L}\right)^2 + \left(x_{\tilde{\tilde{A}}^L} - x_{\tilde{\tilde{B}}^L}\right)^2 + \left(y_{\tilde{\tilde{A}}^U} - y_{\tilde{\tilde{B}}^U}\right)^2 + \left(x_{\tilde{\tilde{A}}^U} - x_{\tilde{\tilde{B}}^U}\right)^2\right\}} \quad \textbf{(2.36)}$$

Here, $d\left(\tilde{\tilde{A}}, \tilde{\tilde{B}}\right)$ satisfies the following properties:

(i) If $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are normalized interval-valued trapezoidal fuzzy numbers, then $0 \leq d\left(\tilde{\tilde{A}}, \tilde{\tilde{B}}\right) \leq 1$.

(ii) $\tilde{\tilde{A}} = \tilde{\tilde{B}} \Rightarrow d\left(\tilde{\tilde{A}}, \tilde{\tilde{B}}\right) = 0$

(iii) $d\left(\tilde{\tilde{A}}, \tilde{\tilde{B}}\right) = d\left(\tilde{\tilde{B}}, \tilde{\tilde{A}}\right)$

(iv) $d\left(\tilde{\tilde{A}}, \tilde{\tilde{C}}\right) + d\left(\tilde{\tilde{C}}, \tilde{\tilde{B}}\right) \geq d\left(\tilde{\tilde{A}}, \tilde{\tilde{B}}\right)$

In the real decision making, it is difficult to get the form of generalized interval-valued trapezoidal fuzzy numbers for the attribute values and weights directly by the decision makers. So the form of linguistic terms is usually adopted.

2.6 VIKOR Method

The Serbian name VIKOR stands for '*VlseKriterijumska Optimizacija I Kompromisno Resenje*', means multi-criteria optimization and compromise solution was developed by Opricovic in late 1998 **(Opricovic and Tzeng, 2004)**. This method concentrates on ranking and selecting the best solution from a set of alternatives, which are associated with multi-conflicting criteria. Moreover, it makes easy to the decision makers to reach the final decision by finding the compromise solution (closest to the ideal) of a problem. The basic principle of VIKOR is to determine the positive-ideal solution as well as negative-ideal (anti-ideal) solution in the search place **(Wu and Liu, 2011)**. The positive-ideal solution is the best value of alternatives under measurement criteria, and the negative-ideal solution is the worst value of alternatives under measurement criteria. At the end, arranging the precedence of the schemes is based on the closeness of the alternatives assessed value to the ideal scheme. Therefore, VIKOR method is popularly known as multi-criteria decision making method based on ideal point technique **(Opricovic and Tzeng, 2007)**. For compromise ranking of multi-criteria measurement, VIKOR adopted a following form of L_P-metric aggregate function **(Yu, 1973)**:

$$L_{Pi} = \left\{ \sum_{j=1}^{n} \left[w_j \left(f_j^* - f_{ij} \right) / \left(f_j^* - f_j^- \right)^P \right] \right\}^{1/P}, \tag{2.37}$$

Here, $1 \le P \le \infty;\ j = 1,...,n,$ with respect to criteria and the variable $i = 1,...,m,$ represent the number of alternatives such as $A_{1,} A_{2,} .....A_m$. For alternative A_i, the evaluated value of the j th criterion is denoted by f_{ij}, and n is the number of criteria. The measure L_{Pi} shows the distance between alternative A_i and positive-ideal solution. Within the VIKOR method L_{1i} (as S_i in Eq. (2.40)) and $L_{\infty i}$ (as R_i in Eq. (2.41)) has been used to formulate ranking measure. The value obtained by minimum S_i is with a maximum group utility (‘majority’ rule) and the solution obtained by minimum R_i is with a minimum individual regret of the ‘opponent’ **(Sanayei et al., 2010)**. Then the compromise ranking algorithm of the traditional VIKOR method has following steps **(Chang, 2010)**:

Step 1. Compute the positive-ideal solutions (best) value f_j^* and negative-ideal solutions (worst) value f_j^- for all criterion ratings **(Wu and Liu, 2011; Kannan et al., 2009)**:

$$f_j^* = \begin{cases} \max\limits_{i=1,\dots,m} f_{ij}, & j \in C_1 \\ \min\limits_{i=1,\dots,m} f_{ij}, & J \in C_2 \end{cases} \tag{2.38}$$

$$f_j^- = \begin{cases} \min\limits_{i=1,\dots,m} f_{ij}, & j \in C_1 \\ \max\limits_{i=1,\dots,m} f_{ij}, & J \in C_2 \end{cases} \tag{2.39}$$

Here, $j = 1,\dots,n$ and C_1 is a benefit type criteria set, C_2 is a cost type criteria set.

Step 2. Compute the values of S_i and R_i $(i = 1,\dots,m)$, by using the relations:

$$S_i = \sum_{j=1}^{n} w_j \left(f_j^* - f_{ij}\right) / \left(f_j^* - f_j^-\right), \tag{2.40}$$

$$R_i = \max_{j=1,\dots,n} \left[w_j \left(f_j^* - f_{ij}\right) / \left(f_j^* - f_j^-\right)\right] \tag{2.41}$$

Here, S_i is the aggregated value of i^{th} alternatives with a maximum group utility and R_i is the aggregated value of i^{th} alternatives with a minimum individual regret of 'opponent'. w_j is the fuzzy weighted average of each criterion.

Step 3. Compute the values Q_i for $i = 1,\dots,m$ with the relation,

$$Q_i = v\left(S_i - S^*\right) / \left(S^- - S^*\right) + (1 - v)\left(R_i - R^*\right) / \left(R^- - R^*\right) \tag{2.42}$$

Here, $S^* = \min\limits_{i=1,\dots,m} S_i, S^- = \max\limits_{i=1,\dots,m} S_i, R^* = \min\limits_{i=1,\dots,m} R_i, R^- = \max\limits_{i=1,\dots,m} R_i$ and v is a weight for strategy of maximum group utility, and $v = 0.5$ where as $1 - v$ is the weight of individual regret. The compromise can be selected with 'voting by majority' ($v > 0.5$), with 'consensus' ($v = 0.5$), with 'veto' ($v < 0.5$).

Step 4. Rank the alternatives by sorting each $S, R,$ and Q values in ascending order.

Step 5. If following two conditions are satisfied simultaneously, then the scheme with minimum value of Q in ranking is considered the optimal compromise solution. Such as,

C1. The alternative $Q\left(A^{(1)}\right)$ has an *acceptable advantage*; in other words,

$Q\left(A^{(2)}\right) - Q\left(A^{(1)}\right) \geq 1/(m-1)$.

Here, $A^{(2)}$ is the alternative with second position in the ranking list by and m is the number of alternatives.

C2. The alternative $Q(A^{(1)})$ is *stable within the decision making process*; in other words, it is also best ranked in S_i and R_i .

If condition C1 is not satisfied, that means $Q(A^{(m)}) - Q(A^{(1)}) \langle 1/(m-1)$, then alternatives $A^{(1)}$, $A^{(2)}$......$A^{(m)}$ all are the same compromise solution, there is no comparative advantage of $A^{(1)}$ from others. But for the case of maximum value, the corresponding alternative is the compromise (closeness) solution. If condition C2 is not satisfied, the stability in decision making is deficient while $A^{(1)}$ has comparative advantage. Therefore, $A^{(1)}$ and $A^{(2)}$ has same compromise solution.

Step 6. Select the best alternative by choosing $Q(A^{(m)})$ as a best compromise solution with minimum value of Q_i and must have to satisfy with the above conditions **(Park et al., 2011)**.

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Chapter 3: Supplier Selection

3.1 Coverage

In today's competitive global markets, selection of a potential supplier plays an important role to cut production costs as well as material costs of the company. This leads to successful survival and sustainability in competitive marketplace. Therefore, an evaluation and selection of an appropriate supplier has become an important part of supply chain management. The nature of supplier selection process is a complex multi-attribute group decision making (MAGDM) problem which deals with both quantitative and qualitative factors may be conflicting in nature as well as contain incomplete and uncertain information. In order to solve such kind of MAGDM problems, development of an effective supplier selection model is evidently desirable. In this chapter, an application of VIKOR method combined with fuzzy logic has been used to solve supplier selection problems with conflicting and non-commensurable (different units) criteria, assuming that compromising is acceptable for conflict resolution. The decision maker wants a solution, which must be closest to the ideal, and the alternatives are evaluated according to all established criteria. Linguistic values are used to assess the ratings and weights for conflicting factors. These linguistic ratings can be expressed in triangular fuzzy numbers. Then, a hierarchy MAGDM model based on fuzzy sets theory and VIKOR method has been proposed to deal with the supplier selection problems in the supply chain system. A case study has been illustrated an application of the proposed model.

3.2 Introduction and State of Art

In today's' competitive business scenario, supplier selection has become a major concern for every organizations. Supplier selection requires wide conceptual and experimental framework to be carried out by the purchasing managers in a supply chain management. Therefore, it is being considered to be one of the most important responsibilities in the philosophy of any organizational purchase management. In the literature survey, an extensive work was found that made by previous researchers in the area of supplier selection and they solved a variety of supplier selection problems using different multi-criteria decision making (MCDM) methods like Performance Value Analysis (PVA), Analytical Hierarchy Process (AHP), Analytical Network Process (ANP), Fuzzy logic, and TOPSIS approach. Apart from this, some hybrid and innovative approaches such as AHP-LP, ANP-TOPSIS and fuzzy-QFD are also being used to find a more precise decision towards selection of a best alternative supplier from among the set of feasible alternatives. But this is still limited to extent because as there are many multi-attribute group decision making (MAGDM) methods which may yield very different results when they are applied on exactly the same data. MAGDM problems are one of the important phases of multi-criteria decision making (MCDM) process in which three or more decision makers have been grouped together for ranking and selecting the best alternative in decision making process. Literature depicts some extensive work has been made in MCDM area as follows.

Roodhooft and Konings (1996) proposed an Activity Based Costing (ABC) approach for vendor selection and evaluation. This system helped to compute the total cost caused by supplier in production process, thereby increasing the objectivity in the selection process. **Weber et al. (1998)** developed a theory and methodology of non-cooperative negotiation strategies for vendor selection. **Ghodsypour and ÓBrien (1998)** proposed an integration of Analytical Hierarchy Process and Linear Programming (AHP-LP) to consider both tangible and intangible factors in selecting the best vendor. **Altinoz and Winchester (2001)** focused on the implementation of the rule-based supplier selection methodology using fuzzy logic concepts. **Tsai et al. (2003)** applied grey relational analysis to the vendor selection model. Overall performance for each candidate vendor was evaluated; based on that, optimum decision was taken. **Kumar et al. (2004)** developed a fuzzy goal programming approach to deal with the effect of vagueness and imprecision statement in the objectives of the vendor selection process and also highlighted how the quota allocation of vendors was changed with the uncertainty. **Saghafian and Hejazi (2005)** presented a modified Fuzzy TOPSIS Technique (Order Performance by Similarity to Ideal Solution) for the Multi-Criteria Decision Making (MCDM) problem when there was a group of decision makers. **Kubat and Yuce (2006)** applied an integrated Fuzzy AHP and Genetic Algorithm (GA) approach to select the best supplier among the set of multiple suppliers deals with both subjective and objective criteria. **Bashiri and Badri (2011)** presented a new group decision making tool when decision data were not crisp and the decision maker wanted to rank the alternatives during fuzzy interactive linear programming process. Because of existence of linguistic terms in the decision matrix and the weight of each criterion which could be expressed in trapezoidal fuzzy numbers; an interactive method was proposed for ranking alternative with the best weight for each criterion. **Sanayei et al. (2008)** proposed an integrated approach of multi-attribute utility theory (MAUT) and linear programming (LP) for rating and choosing the best suppliers and defining the optimum order quantities among selected ones in order to maximize total additive utility. **Shahanaghi and Yazdian (2009)** proposed fuzzy group TOPSIS approach to make more realistic decisions for vendor selection in fuzzy multi-criteria decision making environment.

From literature review, it has been observed that, choosing a suitable and efficient methodology to solve a multi-criteria decision making problem and selecting the best alternative is a great challenge to the researchers as well as management practitioners due to the existence of conflicting and non-commensurable criteria associated with supplier selection problem. The selection is based on a group decision making process which is involved with uncertainty and imperfect information processing to some extent, such as randomicity and fuzzy **(Wu and Liu, 2011)**.

In order to tackle this kind of uncertainty in decision-making process, in the present work, a fuzzy based VIKOR approach has been attempted to evaluate the best supplier under multi-criteria decision making situations. The concept of fuzzy set theory has been applied here to express decision-makers viewpoint in linguistic terms to overcome uncertainty on estimation of qualitative factors. Linguistic judgment has been transformed to corresponding fuzzy number. Then, a hierarchy MCDM model based on fuzzy sets theory and VIKOR has been used to deal with a supplier selection problem. The VIKOR method, a recently introduced new MCDM method developed to solve multiple criteria decision making (MCDM) problems with conflicting and non-commensurable criteria, may provide the basis for developing supplier selection models that can effectively deal with characteristics of this problem **(Opricovic, 1998)**.

Opricovic and Tzeng (2004) conducted a comparative analysis of VIKOR and TOPSIS, based on an aggregating function representing closeness to the reference point and provide the compromise solution by MCDM methods. **Huang et al. (2009)** developed a VIKOR model for MCDM which was used to determine the preference ranking from a set of alternatives in the presence of conflicting criteria. **Chang (2010)** proposed a modified VIKOR method to solve multiple criteria decision making (MCDM) problems with contradicting and non-commensurable criteria. **Moeinzadeh and Hajfathaliha (2010)** presented a supply chain risk assessment approach based on the analytic network process (ANP) and VIKOR method under the fuzzy environment where the vagueness and subjectivity were handled with linguistic terms parameterized by triangular fuzzy numbers. **Sanayei et al. (2010)** studied a group decision making process for supplier selection with VIKOR method under fuzzy environment. They selected a suitable supplier out of a set of five suppliers associated with multi-conflicting criteria and the evaluation process was carried out using trapezoidal fuzzy membership functions. **Kuo and Liang (2011)** proposed an effective approach by combining VIKOR with GRA techniques for evaluating service quality of Northeast-Asian international airports by conducting customer surveys under fuzzy environment. This model was solved by an effective algorithm, which incorporated the decision-makers attitude and/or preference for customers' assessments on weights and performance ratings of each criterion.

3.3 Methodology Applied

Based on concept of fuzzy set theory and VIKOR method, the proposed fuzzy VIKOR method has been applied to find the best compromise solution under multi-person multi-criteria decision making supplier selection problem. Usually, decision making problems are dealing with some alternatives which can be ranked, with respect to the distinct criteria. Ratings of the alternatives and the weights of each criterion are the two most significant data which can affect on the results of decision making problems. Therefore, the proposed methodology has been used here, to calculate the definite weight of criteria and ranking of the alternatives. In this chapter, the importance weights of various criteria and ratings of qualitative criteria are measured as linguistic variables, because linguistic assessment can only have a capability to approximate the subjective judgment through decision maker's opinion. Moreover, linear triangular membership functions are considered for capturing the vagueness of these linguistic assessments. The definition of triangular fuzzy membership functions and its corresponding fuzzy numbers with operational rules have been described in **Section 2.2** of **Chapter 2**. The proposed algorithm consists of following steps:

Step 1. *Make a list of feasible alternatives, find the evaluation criteria, and constitute a group of decision makers.* Suppose, there are k decision makers $\left(D_t, t=1,...,k\right)$, whom are responsible for assessing m alternatives $\left(A_i, i = 1,..., m\right)$, with respect to the importance of each of the n criteria, $(C_j, = 1,..., n)$ **(Bashiri and Badri, 2011)**.

Step 2. *Identify appropriate linguistic variables and their positive triangular fuzzy numbers.* Linguistic variables are used to calculate the importance weights of criteria and the ratings of the alternatives with respect to distinct criteria. For example, linguistic variable "Very High (VH)" which can be defined by a triangular fuzzy number (0.75; 1; 1).

Step 3. *Construct a fuzzy decision matrix by pulling the decision makers' opinions to get the aggregated fuzzy weight of criteria, and the aggregated fuzzy rating of alternatives.* Let k is the number of decision makers in a group and, the aggregated fuzzy weight ($\tilde{w}_j$) with respect to each criterion can be calculated as **(Chen, 2000)**:

$$\tilde{w}_j = \frac{1}{k}\left[\tilde{w}_{j1} \oplus \tilde{w}_{j2} \oplus .... \oplus \tilde{w}_{jk}\right]. \qquad \textbf{(3.1)}$$

And also the aggregated fuzzy ratings ($\tilde{x}_{ij}$) of alternatives with respect to each criterion can be calculated as:

$$\tilde{x}_{ij} = \frac{1}{k}\left[\tilde{x}_{ij1} \oplus \tilde{x}_{ij2} \oplus .... \oplus \tilde{x}_{ijk}\right]. \qquad \textbf{(3.2)}$$

In supplier selection problem, the value of aggregated weights and ratings are expressed in matrix format as follows:

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \qquad \tilde{W} = \left[\tilde{w}_1 \quad \tilde{w}_2 \quad \cdots \quad \tilde{w}_n\right],$$

$i = 1,..., m$ for alternatives, and $j = 1,..., n,$ for criteria

Step 4. *Defuzzify the fuzzy decision matrix and fuzzy weight* of each criterion in to crisp values using the relation BNP_i based on COA defuzzification method proposed in **Chapter 2** by **(Eq. 2.13)**.

Step 5. Determine best crisp value $\left(f_j^*\right)$ and worst crisp value $\left(f_j^-\right)$ for all criterion ratings, $(j=1,...,n)$ by using the relations:

$$\left(f_j^*\right)=\max_i \tilde{x}_{ij}\ , \quad . \left(f_j^-\right)=\min_i \tilde{x}_{ij}\ , \qquad \textbf{(3.3)}$$

Step 6. Compute the values S_i and R_i using **Eqs. 2.40-2.41** as described in **Chapter 2** respectively.

Step 7. Compute the values Q_i using **Eq. 2.42** as described in **Chapter 2**.

Step 8. Rank the alternatives by sorting each $S, R,$ and Q values in ascending order.

Step 9. Select the best alternatives as a compromise solution by referring ***Step 5*** of **Chapter 2**.

3.4 Case Study

Supplier selection is an important part of the business as well as production strategy for industrial organizations. Selection of best supplier enhances the quality and economic growth of enterprise but, still it is being a difficult task to select an appropriate supplier. Therefore, the proposed model has been used to evaluate and select the most suitable supplier of a computer manufacturing industry in southern part of India. The proposed supplier selection approach has been made in following steps:

Step 1: Some key components and accessories of computers have to be purchased for the production of new product of the company. Therefore, company needs to select a suitable supplier. There are five suppliers such as, S_1, S_2, S_3, S_4, and S_5 participating in the selection process. These are the six qualitative criteria used to evaluate the suppliers:

C_1 : On time delivery of goods, C_2 : Quality of products,

C_3 : Response to correspondence, C_4 : flexibility,

C_5 : Services contract performance, C_6 : Cost/Price.

Three decision makers D_1, D_2 and D_3 have been grouped to resolve the problems of entire selection process.

Step 2: Decision makers have used the five linguistic variables for weighting as shown in **Fig. 3.1** and also five linguistic variables for rating of suppliers which are shown in **Fig. 3.2**. The corresponding fuzzy numbers of linguistic variables for weights and ratings are shown in **Table 3.1** and **Table 3.2** respectively. Then, the decision maker's use the linguistic weighting variables to assess the importance weight of each criterion are shown in **Table 3.3**. Also they have been used the linguistic ratings to rate the alternatives is presented in **Table 3.4**. Next, the calculated fuzzy numbers of importance weights and ratings are tabulated in **Table 3.5** and **3.6** respectively.

Step 3: The aggregated fuzzy weight ($\tilde{w}_j$) of each criterion and aggregated fuzzy ratings $\left(\tilde{x}_{ij}\right)$ of each criterion with respect to the suppliers are calculated by using **Eqs. 3.1-3.2** respectively. Then, construct a fuzzy decision matrix by putting these aforesaid data and shown in **Table 3.7**.

Step 4: Compute the crisp values of decision matrix and weight of each criterion and presented in **Table 3.8**.

Step 5: The best and worst values of all criterion ratings are determined using **Eq. 3.3** and listed below:

$$f_1^* = 0.86, \quad f_2^* = 0.86, \quad f_3^* = 0.72, \quad f_4^* = 0.80, \quad f_5^* = 0.80, \quad f_6^* = 0.86$$

$$f_1^- = 0.58, \quad f_2^- = 0.58, \quad f_3^- = 0.50, \quad f_4^- = 0.58, \quad f_5^- = 0.42, \quad f_6^- = 0.72$$

Step 6: Compute the values of S, R and Q for all suppliers and presented in **Table 3.9**.

Step 7: Ranking of suppliers by S, R and Q in ascending order are shown in **Table 3.10**.

Step 8: From **Table 3.9**, it has been shown that, the suppliers S_5 is best ranked by Q and also both C_1 and C_2 conditions are satisfied, means $(Q_{S4} - Q_{S5}) \geq \frac{1}{(5-1)}$ and S_5 is best ranked by R and S also. Therefore, S_5 is the best selected suppliers for the best compromise solution.

3.5 Concluding Remarks

Supplier selection is a part of supply chain management which is used in upstream of the production process and affecting all the areas of an organization. In this chapter an efficient method has been proposed to solve the supplier selection problems and select the best supplier through multi criteria group decision making process under fuzzy environment. In decision making process, the decision makers are unable to express their opinions exactly in numerical values, due to the imprecision in subjective judgment of decision-makers. In order to deal with such problems fuzzy set theory has been implemented and the evaluations are expressed in linguistic terms. In this research an efficient MDCM approach, VIKOR under fuzzy environment has been implemented to deal with both qualitative and quantitative criteria and a suitable supplier has been selected successfully. The outranking order of suppliers and rating of suppliers both can easily be determined by using this method. Finally, the proposed method has been seemed simple, flexible and systematic approach and can be applied in different types of decision making problems.

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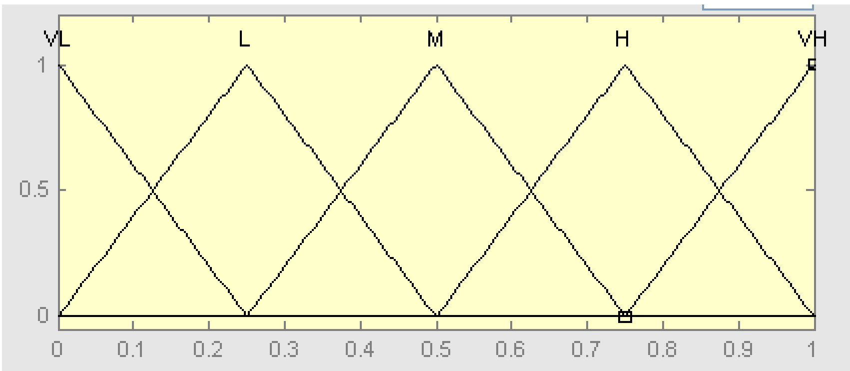

Fig. 3.1. Linguistic variables for importance weight of each criteria

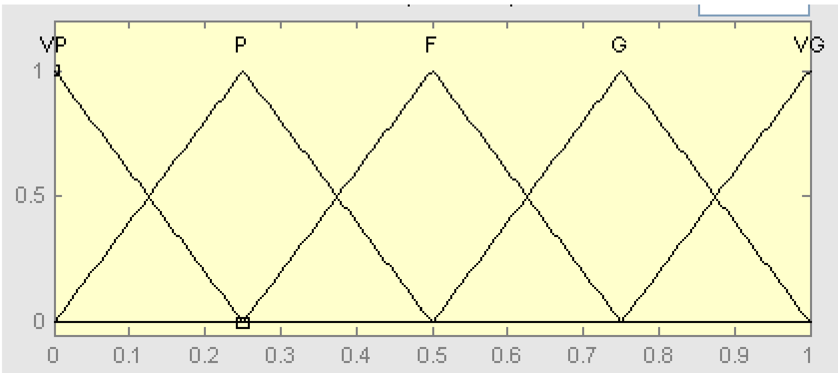

Fig. 3.2. Linguistic variables for ratings

Table 3.1: Linguistic variables for weights

Very Low (VL)	(0, 0, 0.25)
Low (L)	(0, 0.25, 0.5)
Medium (M)	(0.25, 0.5, 0.75)
High (H)	(0.5, 0.75, 1)
Very High (VH)	(0.75, 1, 1)

Table 3.2: Linguistic variables for ratings

Very Poor (VP)	(0, 0, 0.25)
Poor (P)	(0, 0.25, 0.5)
Fair (F)	(0.25, 0.5, 0.75)
Good (G)	(0.5, 0.75, 1)
Very Good (VG)	(0.75, 1, 1)

Table 3.3: Importance weight of criteria from three decision makers

Criteria	Decision makers (DMs).		
	D_1	D_2	D_3
C_1	VH	H	VH
C_2	H	VH	H
C_3	M	VH	VH
C_4	VH	M	M
C_5	H	M	H
C_6	VH	VH	VH

Table 3.4: Ratings of five suppliers under each criterion in terms of linguistic variable determined by DMs

DMs	Suppliers	Criteria					
		C_1	C_2	C_3	C_4	C_5	C_6
D_1	S_1	F	VG	F	G	G	VG
	S_2	G	VG	G	F	G	VG
	S_3	VG	G	F	G	F	F
	S_4	VG	F	G	G	P	VG
	S_5	VG	G	F	F	G	G
D_2	S_1	G	F	F	F	G	G
	S_2	F	VG	F	G	G	G
	S_3	F	G	G	VG	F	VG
	S_4	VG	G	F	G	G	VG
	S_5	F	G	F	G	VG	VG
D_3	S_1	F	G	F	G	VG	VG
	S_2	VG	G	G	F	F	F
	S_3	F	F	VG	G	G	G
	S_4	G	F	G	G	P	F
	S_5	VG	VG	G	G	F	G

Table 3.5: Importance weights of criteria in terms of fuzzy numbers of each criterion

	C_1	C_2	C_3	C_4	C_5	C_6
D_1	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.75,1.00,1.00)
D_2	(0.50,0.75,1.00)	(0.75,1.00,1.00)	(0.75,1.00,1.00)	(0.25,0.50,0.75)	(0.25,0.50,0.75)	(0.75,1.00,1.00)
D_3	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.75,1.00,1.00)	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.75,1.00,1.00)

Table 3.6: Rating of each supplier under each criterion in terms of fuzzy numbers

Supplier	S_1					
Criteria	C_1	C_2	C_3	C_4	C_5	C_6
D_1	(0.25,0.50,0.75)	(0.75,1.00,1.00)	(0.25,0.5,0.75)	(0.50,0.75,1.00)	(0.50,0.75,1.00)	(0.75,1.00,1.00)
D_2	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.25,0.5,0.75)	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.50,0.75,1.00)
D_3	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.25,0.5,0.75)	(0.50,0.75,1.00)	(0.75,1.00,1.00)	(0.75,1.00,1.00)
	S_2					
D_1	(0.50,0.75,1.00)	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.75,1.00,1.00)
D_2	(0.25,0.50,0.75)	(0.75,1.00,1.00)	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.50,0.75,1.00)	(0.50,0.75,1.00)
D_3	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.25,0.50,0.75)	(0.25,0.50,0.75)
	S_3					
D_1	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.25,0.50,0.75)
D_2	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.50,0.75,1.00)	(0.75,1.00,1.00)	(0.25,0.50,0.75)	(0.75,1.00,1.00)
D_3	(0.25,0.50,0.75)	(0.25,0.50,0.75)	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.50,0.75,1.00)	(0.50,0.75,1.00)
	S_4					
D_1	(0.75,1.00,1.00)	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.50,0.75,1.00)	(0.00,0.25,0.50)	(0.75,1.00,1.00)
D_2	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.50,0.75,1.00)	(0.75,1.00,1.00)
D_3	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.5,0.75,1.00)	(0.50,0.75,1.00)	(0.00,0.25,0.50)	(0.25,0.50,0.75)
	S_5					
D_1	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.50,0.75,1.00)
D_2	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.50,0.75,1.00)	(0.75,1.00,1.00)	(0.75,1.00,1.00)
D_3	(0.75,1.00,1.00)	(0.75,1.00,1.00)	(0.50,0.75,1.00)	(0.50,0.75,1.00)	(0.25,0.50,0.75)	(0.50,0.75,1.00)

Table 3.7: Fuzzy decision matrix

	Criteria.					
	C_1	C_2	C_3	C_4	C_5	C_6
Weight	(0.67,0.92,1.00)	(0.58,0.83,1.00)	(0.58,0.83,0.92)	(0.42,0.67,0.83)	(0.42,0.67,0.92)	(0.75,1.00,1.00)
S_1	(0.33,0.58,0.83)	(0.50,0.75,0.92)	(0.25,0.50,0.75)	(0.42,0.67,0.92)	(0.58,0.83,1.00)	(0.67,0.92,1.00)
S_2	(0.50,0.75,0.92)	(0.67,0.92,1.00)	(0.42,0.67,0.92)	(0.33,0.58,0.83)	(0.42,0.67,0.92)	(0.50,0.75,0.92)
S_3	(0.42,0.67,0.83)	(0.42,0.67,0.92)	(0.50,0.75,0.92)	(0.58,0.83,1.00)	(0.33,0.58,0.83)	(0.50,0.75,0.92)
S_4	(0.67,0.92,1.00)	(0.33,0.58,0.83)	(0.42,0.67,0.92)	(0.50,0.75,1.00)	(0.16,0.42,0.67)	(0.58,0.83,0.92)
S_5	(0.58,0.83,0.92)	(0.58,0.83,1.00)	(0.33,0.58,0.83)	(0.42,0.67,0.92)	(0.50,0.75,0.92)	(0.58,0.83,1.00)

Table 3.8: Crisp values for decision matrix and weight of each criterion

	Criteria					
	C_1	C_2	C_3	C_4	C_5	C_6
Weight	0.86	0.80	0.78	0.64	0.67	0.92
S_1	0.58	0.72	0.50	0.67	0.80	0.86
S_2	0.72	0.86	0.67	0.58	0.67	0.72
S_3	0.64	0.67	0.72	0.80	0.58	0.72
S_4	0.86	0.58	0.67	0.75	0.42	0.78
S_5	0.78	0.80	0.58	0.67	0.72	0.80

Table 3.9: The values of S, R and Q for all suppliers

	Suppliers				
	S_1	S_2	S_3	S_4	S_5
S	2.42	2.40	2.52	2.32	1.83
R	0.86	0.92	0.92	0.80	0.39
Q	0.87	0.91	1.00	0.74	0.00

Table 3.10: The ranking of the suppliers by S, R and Q in ascending order

	Ranking suppliers				
	1	2	3	4	5
By S	S_5	S_4	S_2	S_1	S_3
By R	S_5	S_4	S_1	S_2	S_3
By Q	S_5	S_4	S_1	S_2	S_3

Chapter 4: Selection of Industrial Robot

4.1 Coverage

A robot is a mechanical or virtual intelligent agent which can perform tasks on its own, or with guidance. In practice, a robot is usually an electro-mechanical machine which is guided by computer as well as electronic programming. Industrial robot is an automatically controlled, reprogrammable, multipurpose, manipulator programmable in three or more axes, which may be either fixed in place or mobile for use in industrial automation applications. Recently industrial robots are being immensely applied in almost every manufacturing or production industries for improvement of quality as well as productivity. Depending on the nature of the job to be performed, appropriate robot selection has become an important as well as challenging task for an automated manufacturing cell. Several criteria attributes are assumed to be responsible towards performance of a particular robot. Hence, a strong multi-attribute decision support model is indeed required to facilitate this evaluation and selection process. To address this issue, present work explores the concept of interval-valued trapezoidal fuzzy numbers set integrated with VIKOR method to help such a decision-making problem.

4.2 Background and Motivation

An industrial robot is defined as an *automatically controlled, reprogrammable, multipurpose manipulator programmable in three or more axes*. A typical robot will have several, or possibly all, of the following characteristics. It is an electric machine which has some ability to interact with physical objects and to be given electronic programming to do a specific task or to do a whole range of tasks or actions. It may also have some ability to perceive and absorb data on physical objects, or on its local physical environment, or to process data, or to respond to various stimuli. This is in contrast to a simple mechanical device such as a gear or a hydraulic press or any other item which has no processing ability and which does tasks through purely mechanical processes and motion. Typical applications of robots include welding, painting, assembly, pick and place (such as packaging, palletizing and SMT), product inspection, and testing; all accomplished with high endurance, speed, and precision. Defining parameters for an industrial robot include: number of axes, degrees of freedom, working envelope, kinematics, carrying capacity (payload), speed and acceleration, accuracy, and repeatability.

In order to improve product quality and to enhance productivity, robot selection has always been an issue of major concern for manufacturing industries. Many potential robot selection criteria (or attributes), e.g. cost, load capacity, man–machine interface, availability of diagnostic software, programming flexibility, positioning accuracy etc. must be considered for the performance evaluation as well as selection of a particular robot **(Huang and Ghandforoush, 1984; Jones et al., 1985; Offodile, 1987; Offodile and Johnson, 1990; Liang and Wang, 1993)**. **Goh et al. (1996)** presented a revised weighted sum model that incorporated the values assigned by a group of experts on different factors in selecting robots. The model reduced the impact of any decision maker, with a vastly different opinion, on the overall decision. Using this model, the highest and lowest experts' values on the weights and the subjective factors were eliminated. **Kahraman et al. (2007)** proposed a fuzzy hierarchical TOPSIS model for the multi-criteria evaluation of the industrial robotic systems. **Koulouriotis and Ketipi (2011)** proposed a fuzzy digraph method for robot selection which associated with various industrial applications. **Karsak (2008)** applied Quality Function Deployment (QFD) and fuzzy linear regression and developed a decision model for robot selection. **Chatterjee et al. (2010)** attempted to solve the robot selection problem using two multi-criteria decision-making (MCDM) methods (VIKOR and ELECTRE) and compared their relative performance for a given industrial application. However, they considered only subjective criteria in influencing robot selection. **Athawalea and Chakraborty (2011)** considered most popular MCDM methods and compared their relative performance with respect to the rankings of the alternative robots as engaged in some industrial operation. It was observed that all these methods produced almost same ranking pattern of the alternative robots, although the performance of WPM, TOPSIS and GRA methods were experienced slightly better than the others.

Many precision-based methods for robot selection have been developed **(Huang and Ghandforoush, 1984; Jones et al., 1985; Offodile et al., 1987; Offodile and Johnson, 1990; Knott and Getto, 1982; Imang and Schlesinger, 1989)**. **Chu and Lin (2003)** noted that all the above methods were developed based on the concepts of accurate measurement and crisp evaluation, i.e. the measuring values must be exact. However, in real life, measures of subjective attributes, e.g. man–machine interface and programming flexibility etc., may not be precisely defined by decision-makers. Moreover, the evaluation of robot suitability versus subjective criteria and the weights of the criteria are usually expressed in linguistic terms **(Zadeh, 1975, 1976)**. To overcome this, **Liang and Wang (1993)** proposed a fuzzy multi-criteria decision-making (MCDM) approach for robot selection; however, the method had various limitations as highlighted by **Chu and Lin (2003)**.

To solve these limitations, in the present work an interval-valued trapezoidal fuzzy numbers set combined with VIKOR method has been presented to model a decision-support system for appropriate robot selection. It has been found that previous researchers utilized fuzzy theory for decision modeling towards industrial robot selection by adapting mainly triangular fuzzy numbers **(Chu and Lin, 2003; Kahraman et al., 2007; Koulouriotis and Ketipi, 2011)**. Trapezoidal fuzzy membership functions (MF) were rarely attempted by previous researchers in this particular area. However, it is felt that conventional fuzzy set theory is not accurate enough in dealing with subjective judgment of decision-makers (DMs) individual perceptions. **Corneils et al. (2006)** and **Karnik and Mendel (2001)** noted that the main reason for proposing Interval-Valued Fuzzy Set (IVFS) theory is the fact that, in the linguistic modeling of a phenomenon, the presentation of the linguistic expression in the form of ordinary fuzzy sets is not clear enough.

In this context, Interval-Valued Fuzzy Set (IVFS) theory (with trapezoidal MFs) has been proposed here in combination with *VIKOR* (*VIsekriterijumsko KOmpromisno Rangiranje*) method to develop a logical and systematic approach for industrial robot selection for a manufacturing organization. Instead of using triangular IVF numbers **(Devi, 2011)**; this study utilizes Interval-Valued-trapezoidal fuzzy numbers for analyzing the decision-making procedure (shown in **Fig. 2.4**; in **Section 2.4** of **Chapter 2**). The theory of interval valued fuzzy sets and the operational rules of two interval-valued trapezoidal fuzzy numbers have been described in **Chapter 2**. Moreover the concept of locating the COG points to determine the distance of two interval-valued trapezoidal fuzzy numbers has been illustrated clearly in **Section 2.5** of **Chapter 2**. The methodology presented here seems to improve the degree of reliability as well as accuracy in decision-making over existing conventional ordinary fuzzy based approaches.

4.3 The IVF-VIKOR (Liu and Wang, 2011; Devi, 2011)

In fuzzy MCDM problems, performance values and criteria weights are usually characterized by fuzzy numbers. A fuzzy number is a convex fuzzy set, defined by a given interval of real numbers, each with a membership value between 0 and 1. Considering the fact that, in some cases, determining this value precisely is difficult, the membership value can be expressed as an interval, consisting real numbers. In this reporting, criteria values as well as attribute weights are considered as linguistic variables. The concept of linguistic variable is very useful in dealing with situations that are two complex or ill-defined to be reasonably described in conventional quantitative expressions **(Zadeh, 1975, 1976)**. These linguistic variables can be converted to trapezoidal IVFNs. These linguistic variables can be converted to trapezoidal IVFN as depicted in **Table 4.1**.

a. Formulation of the decision-making problem

Let $E = \{e_1, e_2, ..., e_q\}$ be the set of decision-makers in the group decision making process.

$A = \{A_1, A_2, ..., A_m\}$ be the set of alternatives, and

$C = \{C_1, C_2, ..., C_n\}$ be the set of criteria-attributes.

Suppose that

$\tilde{\tilde{a}}_{ijk} = \left[\left(a^L_{ijk1}, a^L_{ijk2}, a^L_{ijk3}, a^L_{ijk4}; w^L_{ijk}\right), \left(a^U_{ijk1}, a^U_{ijk2}, a^U_{ijk3}, a^U_{ijk4}; w^U_{ijk}\right)\right]$ is the attribute value given by the decision-maker e_k, where $\tilde{\tilde{a}}_{ijk}$ is an interval-valued trapezoidal fuzzy number for the alternative A_i with respect to the attribute C_j.

Let $\tilde{\tilde{w}}_{kj} = \left[\left(w^L_{kj1}, w^L_{kj2}, w^L_{kj3}, w^L_{kj4}; \eta^L_{kj}\right), \left(w^U_{kj1}, w^U_{kj2}, w^U_{kj3}, w^U_{kj4}; \eta^U_{kj}\right)\right]$ be the attribute weight given by the decision-maker e_k, where $\tilde{\tilde{w}}_{kj}$ is also an interval-valued trapezoidal fuzzy number. Let $\lambda = (\lambda_1, \lambda_2, ..., \lambda_q)$ be the vector of decision makers, where λ_k is a real number, and $\sum_{k=1}^{q} \lambda_K = 1$.

Then we use the attribute weights, the decision-makers' weights, and the attribute values to rank the order of preference for the alternatives.

b. Normalization of decision-making information

In order to eliminate the impact of different physical dimension to the decision making result, the decision-making information is to be normalized. Consider that there are generally benefit attributes (I_1) and cost attributes (I_2). The normalizing methods are shown as follows:

$\tilde{\tilde{x}}_{ijk} = \left[\left(x^L_{ijk1}, x^L_{ijk2}, x^L_{ijk3}, x^L_{ijk4}; w^L_{ijk}\right), \left(x^U_{ijk1}, x^U_{ijk2}, x^U_{ijk3}, x^U_{ijk4}; w^U_{ijk}\right)\right] =$

$$\left[\left(\frac{a_{ijk1}^{L}}{m_{jk}},\frac{a_{ijk2}^{L}}{m_{jk}},\frac{a_{ijk3}^{L}}{m_{jk}},\frac{a_{ijk4}^{L}}{m_{jk}};w_{ijk}^{L}\right),\left(\frac{a_{ijk1}^{U}}{m_{jk}},\frac{a_{ijk2}^{U}}{m_{jk}},\frac{a_{ijk3}^{U}}{m_{jk}},\frac{a_{ijk4}^{U}}{m_{jk}};w_{ijk}^{L}\right)\right] \quad \textbf{(4.1)}$$

For benefit attributes, where $m_{jk}=\max_{i}\left(a_{ijk4}^{U}\right)$.

$$\tilde{\tilde{x}}_{ijk}=\left[\left(x_{ijk1}^{L},x_{ijk2}^{L},x_{ijk3}^{L},x_{ijk4}^{L};w_{ijk}^{L}\right),\left(x_{ijk1}^{U},x_{ijk2}^{U},x_{ijk3}^{U},x_{ijk4}^{U};w_{ijk}^{U}\right)\right]=$$

$$\left[\left(\frac{n_{jk}}{a_{ijk1}^{L}},\frac{n_{jk}}{a_{ijk2}^{L}},\frac{n_{jk}}{a_{ijk3}^{L}},\frac{n_{jk}}{a_{ijk4}^{L}},w_{ijk}^{L}\right),\left(\frac{n_{jk}}{a_{ijk1}^{U}},\frac{n_{jk}}{a_{ijk2}^{U}},\frac{n_{jk}}{a_{ijk3}^{U}},\frac{n_{jk}}{a_{ijk4}^{U}},w_{ijk}^{U}\right)\right] \quad \textbf{(4.2)}$$

For cost attributes, where $n_{jk}=\min_{i}\left(a_{ijk1}^{L}\right)$.

The normalization method mentioned above is to preserve the property that the ranges of normalized interval-valued trapezoidal fuzzy numbers belong to [0, 1]. In this reporting, to avoid these computations and to make it more easier practical procedure, all fuzzy numbers that simply define in the interval of [0, 1] to omit the need of normalization method **(Saghafian and Hejazi, 2005)**. Therefore, it is necessary to construct the fuzzy numbers scalable in the ranges closed to interval [0, 1] then, only to avoid calculations shown in **Eqs. 4.1-4.2**.

In contrast, the MCDM methods like TOPSIS and VIKOR often may require normalization operation to eliminate the units of the criterion functions but, the normalization techniques are somewhat different. VIKOR method uses linear normalization whereas TOPSIS method uses vector normalization. The normalized value in the VIKOR method does not depend on the evaluation unit of criterion function, whereas the normalized values by vector normalization in the TOPSIS method may depend on the evaluation unit **(Chu et al., 2007)**.

c. Aggregation of evaluation information of each decision-maker

According to the different alternatives' attribute values and weights given by different experts, the collective attribute values and weights are calculated as follows:

$$\tilde{\tilde{x}}_{ij} = \left[\left(x_{ij1}^{L}, x_{ij2}^{L}, x_{ij3}^{L}, x_{ij4}^{L}; w_{ij}^{L}\right), \left(x_{ij1}^{U}, x_{ij2}^{U}, x_{ij3}^{U}, x_{ij4}^{U}; w_{ij}^{U}\right)\right] =$$

$$\sum_{k=1}^{q}\left(\lambda_k . \tilde{\tilde{x}}_{ijk}\right) = \sum_{k=1}^{q}\left\{\lambda_k \left[\left(x_{ijk1}^{L}, x_{ijk2}^{L}, x_{ijk3}^{L}, x_{ijk4}^{L}; w_{ijk}^{L}\right), \left(x_{ijk1}^{U}, x_{ijk2}^{U}, x_{ijk3}^{U}, x_{ijk4}^{U}; w_{ijk}^{U}\right)\right]\right\} =$$

$$\left[\begin{array}{l} \left(\sum_{k=1}^{q}\left(\lambda_k . x_{ijk1}^{L}\right), \sum_{k=1}^{q}\left(\lambda_k . x_{ijk2}^{L}\right), \sum_{k=1}^{q}\left(\lambda_k . x_{ijk3}^{L}\right), \sum_{k=1}^{q}\left(\lambda_k . x_{ijk4}^{L}\right); \min_k\left(w_{ijk}^{L}\right)\right), \\ \left(\sum_{k=1}^{q}\left(\lambda_k . x_{ijk1}^{U}\right), \sum_{k=1}^{q}\left(\lambda_k . x_{ijk2}^{U}\right), \sum_{k=1}^{q}\left(\lambda_k . x_{ijk3}^{U}\right), \sum_{k=1}^{q}\left(\lambda_k . x_{ijk4}^{U}\right); \min_k\left(w_{ijk}^{U}\right)\right) \end{array}\right] \quad \textbf{(4.3)}$$

$$\tilde{\tilde{\varpi}}_{j} = \left[\left(\varpi_{j1}^{L}, \varpi_{j2}^{L}, \varpi_{j3}^{L}, \varpi_{j4}^{L}; \eta_{j}^{L}\right), \left(\varpi_{j1}^{U}, \varpi_{j2}^{U}, \varpi_{j3}^{U}, \varpi_{j4}^{U}; \eta_{j}^{U}\right)\right] =$$

$$\sum_{k=1}^{q}\left(\lambda_k . \tilde{\tilde{\varpi}}_{jk}\right) = \sum_{k=1}^{q}\left\{\lambda_k \left[\left(\varpi_{jk1}^{L}, \varpi_{jk2}^{L}, \varpi_{jk3}^{L}, \varpi_{jk4}^{L}; \eta_{jk}^{L}\right), \left(\varpi_{jk1}^{U}, \varpi_{jk2}^{U}, \varpi_{jk3}^{U}, \varpi_{jk4}^{U}; \eta_{jk}^{U}\right)\right]\right\} =$$

$$\left[\begin{array}{l} \left(\sum_{k=1}^{q}\left(\lambda_k . \varpi_{jk1}^{L}\right), \sum_{k=1}^{q}\left(\lambda_k . \varpi_{jk2}^{L}\right), \sum_{k=1}^{q}\left(\lambda_k . \varpi_{jk3}^{L}\right), \sum_{k=1}^{q}\left(\lambda_k . \varpi_{jk4}^{L}\right); \min_k\left(\eta_{jk}^{L}\right)\right), \\ \left(\sum_{k=1}^{q}\left(\lambda_k . \varpi_{jk1}^{U}\right), \sum_{k=1}^{q}\left(\lambda_k . \varpi_{jk2}^{U}\right), \sum_{k=1}^{q}\left(\lambda_k . \varpi_{jk3}^{U}\right), \sum_{k=1}^{q}\left(\lambda_k . \varpi_{jk4}^{U}\right); \min_k\left(\eta_{jk}^{U}\right)\right) \end{array}\right] \quad \textbf{(4.4)}$$

d. Construction of weighted decision-making matrix

Let $\tilde{\tilde{V}} = \left[\tilde{\tilde{v}}_{ij}\right]_{m \times n}$ be the weighted matrix, then:

$$\tilde{\tilde{v}}_{ij} = \left[\left(v_{ij1}^{L}, v_{ij2}^{L}, v_{ij3}^{L}, v_{ij4}^{L}; \varpi_{ij}^{L}\right), \left(v_{ij1}^{U}, v_{ij2}^{U}, v_{ij3}^{U}, v_{ij4}^{U}; \varpi_{ij}^{U}\right)\right] = \tilde{\tilde{x}}_{ij} \otimes \tilde{\tilde{\varpi}}_{j} =$$

$$\left[\begin{array}{l} \left(x_{ij1}^{L}\varpi_{j1}^{L}, x_{ij2}^{L}\varpi_{j2}^{L}, x_{ij3}^{L}\varpi_{j3}^{L}, x_{ij4}^{L}\varpi_{j4}^{L}; \min\left(w_{ij}^{L}, \eta_{j}^{L}\right)\right), \\ \left(x_{ij1}^{U}\varpi_{j1}^{U}, x_{ij2}^{U}\varpi_{j2}^{U}, x_{ij3}^{U}\varpi_{j3}^{U}, x_{ij4}^{U}\varpi_{j4}^{U}; \min\left(w_{ij}^{U}, \eta_{j}^{U}\right)\right) \end{array}\right] \quad \textbf{(4.5)}$$

e. Decision-making based on VIKOR concept

1. Determine the positive ideal solution and the negative ideal solution of the evaluation objects. Suppose that the positive ideal solution and the negative ideal solution are:

For benefit attributes:

$\tilde{\tilde{V}}^{+} = \left[\tilde{\tilde{v}}_{j}^{+}\right]_{1 \times n}, \tilde{\tilde{V}}^{-} = \left[\tilde{\tilde{v}}_{j}^{-}\right]_{1 \times n}$ then,

$$\tilde{\tilde{v}}_j^+ = \left[\left(v_{j1}^{L+}, v_{j2}^{L+}, v_{j3}^{L+}, v_{j4}^{L+}; \varpi_j^{L+}\right), \left(v_{j1}^{U+}, v_{j2}^{U+}, v_{j3}^{U+}, v_{j4}^{U+}; \varpi_j^{U+}\right)\right] =$$

$$\begin{bmatrix} \left(\max_i \left(v_{ij1}^L\right), \max_i \left(v_{ij2}^L\right), \max_i \left(v_{ij3}^L\right), \max_i \left(v_{ij4}^L\right); \max_i \left(\varpi_{ij}^L\right)\right), \\ \left(\max_i \left(v_{ij1}^U\right), \max_i \left(v_{ij2}^U\right), \max_i \left(v_{ij3}^U\right), \max_i \left(v_{ij4}^U\right); \max_i \left(\varpi_{ij}^U\right)\right) \end{bmatrix} \tag{4.6}$$

$$\tilde{\tilde{v}}_j^- = \left[\left(v_{j1}^{L-}, v_{j2}^{L-}, v_{j3}^{L-}, v_{j4}^{L-}; \varpi_j^{L-}\right), \left(v_{j1}^{U-}, v_{j2}^{U-}, v_{j3}^{U-}, v_{j4}^{U-}; \varpi_j^{U-}\right)\right] =$$

$$\begin{bmatrix} \left(\min_i \left(v_{ij1}^L\right), \min_i \left(v_{ij2}^L\right), \min_i \left(v_{ij3}^L\right), \min_i \left(v_{ij4}^L\right); \min_i \left(\varpi_{ij}^L\right)\right), \\ \left(\min_i \left(v_{ij1}^U\right), \min_i \left(v_{ij2}^U\right), \min_i \left(v_{ij3}^U\right), \min_i \left(v_{ij4}^U\right); \min_i \left(\varpi_{ij}^U\right)\right) \end{bmatrix} \tag{4.7}$$

For cost attributes: $\tilde{\tilde{V}}^+ = \left[\tilde{\tilde{v}}_j^-\right]_{1\times n}, \tilde{\tilde{V}}^- = \left[\tilde{\tilde{v}}_j^+\right]_{1\times n}$

Benefit attribute means the larger the rating, the greater the preference. Conversely, cost attribute means, the smaller the rating, the greater the preference **(Wadhwa et al., 2009; Park et al., 2011; Kannan et al., 2009)**.

2. Calculate the weighted matrix and the COG of each attributes with respect to the positive ideal solution and the negative ideal solution.
3. Compute the values $S_i, R_i \; (i = 1, 2, ..., m)$ by the relations,

$$S_i = \sum_{j=1}^{n} \frac{d\left(\tilde{\tilde{v}}_j^+, \tilde{\tilde{v}}_{ij}\right)}{d\left(\tilde{\tilde{v}}_j^+, \tilde{\tilde{v}}_j^-\right)} \tag{4.8}$$

$$R_i = \max_j \left[\frac{d\left(\tilde{\tilde{v}}_j^+, \tilde{\tilde{v}}_{ij}\right)}{d\left(\tilde{\tilde{v}}_j^+, \tilde{\tilde{v}}_j^-\right)}\right] \tag{4.9}$$

4. Compute the values $Q_i, (i = 1, 2, ..., m)$ by the following relation:

$$Q_i = \nu \frac{\left(S_i - S^*\right)}{\left(S^- - S^*\right)} + (1-\nu) \frac{\left(R_i - R^*\right)}{\left(R^- - R^*\right)} \tag{4.10}$$

Here,

$$S^* = \min_i S_i, \; S^- = \max_i S_i \tag{4.11}$$

$$R^* = \min_i R_i, \; R^- = \max_i R_i \tag{4.12}$$

ν is introduced as weight of the strategy of 'the majority' of criteria (or 'the maximum group utility'), here, ν =0.5.

5. Rank the alternatives, sorting by the values S , R and Q , in ascending order.

4.4 Case Study

The set of criteria for robot selection has been adopted from the literature **(Chu and Lin, 2003; Tahriri and Taha, 2011)**. This has been used here to demonstrate the computational procedure of the fuzzy based VIKOR method, presented in previous section. A manufacturing unit requires a robot to perform a particular material-handling task. The said model has been applied towards decision-making for selection of industrial robot carried out by the production unit of a famous manufacturing industry in India. After initial selection, four alternative robots A1, A2, A3 and A4 have been chosen for further evaluation. To select the most suitable robot, a committee of four decision makers, DM1, DM2, DM3 and DM4 has been formed from academicians, manager of production unit and his team. The following set of criteria has been considered shown as follows:

[1] Speediness (C_1)

[2] Payload Capacity (C_2)

[3] Repeatability (C_3)

[4] Purchase Cost (C_4)

[5] Extent of manipulator reach (C_5)

[6] Extent of reliability (C_6)

[7] Programming flexibility (C_7)

[8] Positioning accuracy (C_8)

[9] Man-Machine interface (C_9)

The proposed IVF-VIKOR method has been applied to solve this problem; the computational procedure is summarized as follows:

Step 1: The number of the committee members is four, leveled as DM_1; DM_2, DM_3 and DM_4 respectively. The linguistic scale chosen to assign criteria weight as well as appropriateness rating has been shown in **Table 4.1**. Each DM presents his/her assessment based on linguistic variable for importance weight of each criterion **(Table 4.2)** as well as rating the performance criteria as depicted in **Table 4.3**. The final judgment (collective attribute values and weights by combining the individual evaluation information of each decision maker) of the DMs thus obtained and shown as follows **(Step 2)**.

Step 2: Combine the individual preferences of all DMs in order to obtain a collective preference value for each alternative are shown in the next page.

$$\left[\tilde{\tilde{x}}_{ij}\right]_{4\times 9} = \left[\begin{matrix} \left[(0.687,0.712,0.790,0.820;0.8),(0.687,0.712,0.790,0.820;1)\right], & \left[(0.582,0.620,0.740,0.785;0.8),(0.582,0.620,0.740,0.785;1)\right], \\ \left[(0.620,0.687,0.835,0.890;0.8),(0.620,0.687,0.835,0.890;1)\right], & \left[(0.672,0.737,0.855,0.897;0.8),(0.672,0.737,0.855,0.897;1)\right], \\ \left[(0.790,0.842,0.930,0.957;0.8),(0.790,0.842,0.930,0.957;1)\right], & \left[(0.652,0.702,0.780,0.820;0.8),(0.652,0.702,0.780,0.820;1)\right], \\ \left[(0.825,0.880,0.960,0.985;0.8),(0.825,0.880,0.960,0.985;1)\right], & \left[(0.877,0.930,0.980,0.992;0.8),(0.877,0.930,0.980,0.992;1)\right], \end{matrix}\right.$$

$$\begin{matrix} \left[(0.552,0.602,0.715,0.760;0.8),(0.552,0.602,0.715,0.760;1)\right], & \left[(0.652,0.695,0.800,0.840;0.8),(0.652,0.695,0.800,0.840;1)\right], \\ \left[(0.447,0.510,0.665,0.725;0.8),(0.447,0.510,0.665,0.725;1)\right], & \left[(0.620,0.687,0.835,0.890;0.8),(0.620,0.687,0.835,0.890;1)\right], \\ \left[(0.737,0.792,0.910,0.950;0.8),(0.737,0.792,0.910,0.950;1)\right], & \left[(0.725,0.787,0.875,0.905;0.8),(0.725,0.787,0.875,0.905;1)\right], \\ \left[(0.877,0.930,0.980,0.992;0.8),(0.877,0.930,0.980,0.992;1)\right], & \left[(0.725,0.787,0.875,0.905;0.8),(0.725,0.787,0.875,0.905;1)\right], \end{matrix}$$

$$\begin{matrix} \left[(0.542,0.555,0.595,0.622;0.8),(0.542,0.555,0.595,0.622;1)\right], & \left[(0.702,0.755,0.880,0.922;0.8),(0.702,0.755,0.880,0.922;1)\right], \\ \left[(0.550,0.612,0.775,0.835;0.8),(0.550,0.612,0.775,0.835;1)\right], & \left[(0.687,0.740,0.820,0.847;0.8),(0.687,0.740,0.820,0.847;1)\right], \\ \left[(0.702,0.755,0.880,0.922;0.8),(0.702,0.755,0.880,0.922;1)\right], & \left[(0.842,0.885,0.960,0.985;0.8),(0.842,0.885,0.960,0.985;1)\right], \\ \left[(0.725,0.787,0.875,0.905;0.8),(0.725,0.787,0.875,0.905;1)\right], & \left[(0.672,0.737,0.855,0.897;0.8),(0.672,0.737,0.855,0.897;1)\right], \end{matrix}$$

$$\begin{matrix} \left[(0.550,0.612,0.775,0.835;0.8),(0.550,0.612,0.775,0.835;1)\right], & \left[(0.582,0.640,0.780,0.832;0.8),(0.582,0.640,0.780,0.832;1)\right], \\ \left[(0.687,0.740,0.820,0.847;0.8),(0.687,0.740,0.820,0.847;1)\right], & \left[(0.742,0.792,0.875,0.905;0.8),(0.742,0.792,0.875,0.905;1)\right], \\ \left[(0.842,0.885,0.960,0.985;0.8),(0.842,0.885,0.960,0.985;1)\right], & \left[(0.672,0.737,0.855,0.897;0.8),(0.672,0.737,0.855,0.897;1)\right], \\ \left[(0.725,0.787,0.875,0.905;0.8),(0.725,0.787,0.875,0.905;1)\right], & \left[(0.860,0.890,0.960,0.985;0.8),(0.860,0.890,0.960,0.985;1)\right], \end{matrix}$$

$$\left.\begin{matrix} \left[(0.755,0.797,0.910,0.950;0.8),(0.755,0.797,0.910,0.950;1)\right] \\ \left[(0.685,0.742,0.890,0.942;0.8),(0.685,0.742,0.890,0.942;1)\right] \\ \left[(0.825,0.880,0.960,0.985;0.8),(0.825,0.880,0.960,0.985;1)\right] \\ \left[(0.702,0.755,0.880,0.922;0.8),(0.702,0.755,0.880,0.922;1)\right] \end{matrix}\right]$$

$$
\begin{aligned}
\left[\tilde{\tilde{\omega}}_j\right]_{9} = \ & [[(0.807,0.847,0.930,0.957;0.8),(0.807,0.847,0.930,0.957;1)],[(0.860,0.890,0.960,0.985;0.8),(0.860,0.890,0.960,0.985;1)],\\
& [(0.737,0.792,0.910,0.950;0.8),(0.737,0.792,0.910,0.950;1)],[(0.807,0.847,0.930,0.957;0.8),(0.807,0.847,0.930,0.957;1)],\\
& [(0.672,0.737,0.855,0.897;0.8),(0.672,0.737,0.855,0.897;1)],[(0.585,0.650,0.805,0.862;0.8),(0.585,0.650,0.805,0.862;1)],\\
& [(0.615,0.667,0.830,0.887;0.8),(0.615,0.667,0.830,0.887;1)],[(0.685,0.742,0.890,0.942;0.8),(0.685,0.742,0.890,0.942;1)],\\
& [(0.615,0.667,0.830,0.887;0.8),(0.615,0.667,0.830,0.887;1)]]
\end{aligned}
$$

Step 3: Computation of weighted decision making matrix.

$$
\left[\tilde{\tilde{v}}_{ij}\right]_{4\times 9} = \left[\begin{array}{l}
[(0.554,0.603,0.734,0.784;0.8),(0.554,0.603,0.734,0.784;1)],[(0.500,0.552,0.710,0.773;0.8),(0.500,0.552,0.710,0.773;1)],\\
[(0.500,0.582,0.776,0.851;0.8),(0.500,0.582,0.776,0.851;1)],[(0.578,0.656,0.821,0.883;0.8),(0.578,0.656,0.821,0.883;1)],\\
[(0.637,0.713,0.865,0.916;0.8),(0.637,0.713,0.865,0.916;1)],[(0.560,0.624,0.749,0.807;0.8),(0.560,0.624,0.749,0.807;1)],\\
[(0.665,0.745,0.892,0.942;0.8),(0.665,0.745,0.892,0.942;1)],[(0.754,0.827,0.941,0.977;0.8),(0.754,0.827,0.941,0.977;1)],
\end{array}\right.
$$

$$
\begin{array}{l}
[(0.407,0.476,0.650,0.722;0.8),(0.407,0.476,0.650,0.722;1)],[(0.526,0.588,0.744,0.804;0.8),(0.526,0.588,0.744,0.804;1)],\\
[(0.329,0.404,0.605,0.688;0.8),(0.329,0.404,0.605,0.688;1)],[(0.500,0.582,0.776,0.851;0.8),(0.500,0.582,0.776,0.851;1)],\\
[(0.543,0.627,0.828,0.902;0.8),(0.543,0.627,0.828,0.902;1)],[(0.585,0.666,0.813,0.866;0.8),(0.585,0.666,0.813,0.866;1)],\\
[(0.646,0.736,0.892,0.942;0.8),(0.646,0.736,0.892,0.942;1)],[(0.585,0.666,0.813,0.866;0.8),(0.585,0.666,0.813,0.866;1)],
\end{array}
$$

$$
\begin{array}{l}
[(0.364,0.409,0.508,0.558;0.8),(0.364,0.409,0.508,0.558;1)],[(0.410,0.491,0.708,0.794;0.8),(0.410,0.491,0.708,0.794;1)],\\
[(0.369,0.451,0.662,0.749;0.8),(0.369,0.451,0.662,0.749;1)],[(0.402,0.481,0.660,0.730;0.8),(0.402,0.481,0.660,0.730;1)],\\
[(0.472,0.556,0.752,0.827;0.8),(0.472,0.556,0.752,0.827;1)],[(0.492,0.575,0.773,0.849;0.8),(0.492,0.575,0.773,0.849;1)],\\
[(0.487,0.580,0.748,0.812;0.8),(0.487,0.580,0.748,0.812;1)],[(0.393,0.479,0.688,0.773;0.8),(0.393,0.479,0.688,0.773;1)],
\end{array}
$$

$$
\begin{array}{l}
[(0.338,0.408,0.643,0.740;0.8),(0.338,0.408,0.643,0.740;1)],[(0.398,0.475,0.694,0.784;0.8),(0.398,0.475,0.694,0.784;1)],\\
[(0.422,0.493,0.680,0.751;0.8),(0.422,0.493,0.680,0.751;1)],[(0.508,0.587,0.778,0.852;0.8),(0.508,0.587,0.778,0.852;1)],\\
[(0.518,0.590,0.797,0.873;0.8),(0.518,0.590,0.797,0.873;1)],[(0.460,0.547,0.761,0.845;0.8),(0.460,0.547,0.761,0.845;1)],\\
[(0.446,0.525,0.726,0.803;0.8),(0.446,0.525,0.726,0.803;1)],[(0.589,0.660,0.854,0.928;0.8),(0.589,0.660,0.854,0.928;1)],
\end{array}
$$

$$\begin{bmatrix} \cdots & [(0.464,0.531,0.755,0.842;0.8),(0.464,0.531,0.755,0.842;1)] \\ \cdots & [(0.421,0.495,0.738,0.835;0.8),(0.421,0.495,0.738,0.835;1)] \\ \cdots & [(0.507,0.587,0.796,0.873;0.8),(0.507,0.587,0.796,0.873;1)] \\ \cdots & [(0.432,0.503,0.730,0.818;0.8),(0.432,0.503,0.730,0.818;1)] \end{bmatrix}$$

Step 4: Calculation of positive ideal solution and negative ideal solution.

$$\begin{aligned} \tilde{\tilde{V}}^{+} = {} & [[(0.665,0.745,0.892,0.942;0.8),(0.665,0.745,0.892,0.942;1)],[(0.754,0.827,0.941,0.977;0.8),(0.754,0.827,0.941,0.977;1)], \\ & [(0.646,0.736,0.892,0.942;0.8),(0.646,0.736,0.892,0.942;1)],[(0.500,0.582,0.776,0.851;0.8),(0.500,0.582,0.776,0.851;1)], \\ & [(0.487,0.580,0.748,0.812;0.8),(0.487,0.580,0.748,0.812;1)],[(0.492,0.575,0.773,0.849;0.8),(0.492,0.575,0.773,0.849;1)], \\ & [(0.518,0.590,0.797,0.873;0.8),(0.518,0.590,0.797,0.873;1)],[(0.589,0.660,0.854,0.928;0.8),(0.589,0.660,0.854,0.928;1)], \\ & [(0.507,0.587,0.796,0.873;0.8),(0.507,0.587,0.796,0.873;1)]] \end{aligned}$$

$$\begin{aligned} \tilde{\tilde{V}}^{-} = {} & [[(0.500,0.582,0.776,0.851;0.8),(0.500,0.582,0.776,0.851;1)],[(0.500,0.552,0.710,0.773;0.8),(0.500,0.552,0.710,0.773;1)], \\ & [(0.329,0.404,0.605,0.688;0.8),(0.329,0.404,0.605,0.688;1)],[(0.585,0.666,0.813,0.866;0.8),(0.585,0.666,0.813,0.866;1)], \\ & [(0.364,0.409,0.508,0.558;0.8),(0.364,0.409,0.508,0.558;1)],[(0.393,0.479,0.688,0.773;0.8),(0.393,0.479,0.688,0.773;1)], \\ & [(0.338,0.408,0.643,0.740;0.8),(0.338,0.408,0.643,0.740;1)],[(0.398,0.475,0.694,0.784;0.8),(0.398,0.475,0.694,0.784;1)], \\ & [(0.421,0.495,0.738,0.835;0.8),(0.421,0.495,0.738,0.835;1)]] \end{aligned}$$

Step 5: Calculation for the weighted matrix and the COG of each attributes with respect to the positive ideal solution and the negative ideal solution (y, x) by using **Eqs. 2.28-2.35** of **Chapter 2**.

$$
[(y,x)_v]_{4\times 9} = \left[\begin{array}{l}
[(0.3426,0.6688),(0.4282,0.6688)],[(0.3438,0.6341),(0.4298,0.6341)],[(0.3403,0.5638),(0.4254,0.5638)],\\
[(0.3403,0.6770),(0.4254,0.6770)],[(0.3388,0.7338),(0.4235,0.7338)],[(0.3413,0.5068),(0.4266,0.5068)],\\
[(0.3393,0.7818),(0.4241,0.7818)],[(0.3341,0.6847),(0.4176,0.6847)],[(0.3413,0.7246),(0.4266,0.7246)],\\
[(0.3374,0.8098),(0.4218,0.8098)],[(0.3348,0.8732),(0.4185,0.8732)],[(0.3369,0.8024),(0.4212,0.8024)],
\end{array}\right.
$$

$$
\begin{array}{l}
[(0.3415,0.6654),(0.4268,0.6654)],[(0.3347,0.4599),(0.4184,0.4599)],[(0.3420,0.6009),(0.4275,0.6009)],\\
[(0.3403,0.6770),(0.4254,0.6770)],[(0.3407,0.5579),(0.4258,0.5579)],[(0.3394,0.5679),(0.4243,0.5679)],\\
[(0.3364,0.7314),(0.4205,0.7314)],[(0.3403,0.6514),(0.4253,0.6514)],[(0.3406,0.6720),(0.4257,0.6720)],\\
[(0.3364,0.7314),(0.4205,0.7314)],[(0.3356,0.6556),(0.4195,0.6556)],[(0.3400,0.5832),(0.4250,0.5832)],
\end{array}
$$

$$
\left.\begin{array}{l}
[(0.3446,0.5332),(0.4307,0.5332)],[(0.3423,0.5882),(0.4279,0.5882)],[(0.3457,0.6487),(0.4321,0.6487)]\\
[(0.3424,0.5865),(0.4280,0.5865)],[(0.3407,0.6810),(0.4258,0.6810)],[(0.3449,0.6230),(0.4311,0.6230)]\\
[(0.3444,0.6946),(0.4305,0.6946)],[(0.3408,0.6531),(0.4259,0.6531)],[(0.3428,0.6906),(0.4285,0.6906)]\\
[(0.3417,0.6249),(0.4272,0.6249)],[(0.3429,0.7578),(0.4287,0.7578)],[(0.3451,0.6213),(0.4313,0.6213)]
\end{array}\right]
$$

$$[(y,x)_{V^+}]_9 = \begin{matrix} [[(0.3374, 0.8098), (0.4218, 0.8098)], [(0.3348, 0.8732), (0.4185, 0.8732)], [(0.3369, 0.8024), (0.4212, 0.8024)], \\ [(0.3403, 0.6770), (0.4254, 0.6770)], [(0.3356, 0.6556), (0.4195, 0.6556)], [(0.3406, 0.6720), (0.4257, 0.6720)], \\ [(0.3444, 0.6946), (0.4305, 0.6946)], [(0.3429, 0.7578), (0.4287, 0.7578)], [(0.3428, 0.6906), (0.4285, 0.6906)]] \end{matrix}$$

$$[(y,x)_{V^-}]_9 = \begin{matrix} [[(0.3403, 0.6770), (0.4254, 0.6770)], [(0.3438, 0.6341), (0.4298, 0.6341)], [(0.3413, 0.5068), (0.4266, 0.5068)], \\ [(0.3364, 0.7314), (0.4205, 0.7314)], [(0.3347, 0.4599), (0.4184, 0.4599)], [(0.3400, 0.5832), (0.4250, 0.5832)], \\ [(0.3446, 0.5332), (0.4307, 0.5332)], [(0.3423, 0.5882), (0.4279, 0.5882)], [(0.3449, 0.6230), (0.4311, 0.6230)]] \end{matrix}$$

Step 6: Compute the values of S_i and R_i, $(i = 1, 2, ..., m)$ using **Eqs. 4.8-4.9** respectively.

$S_1 = 7.573, \quad S_2 = 6.378, \quad S_3 = 2.915, \quad S_4 = 3.457$

$R_1 = 1.067, \quad R_2 = 1.172, \quad R_3 = 1.000, \quad R_4 = 1.025$

Step 7: The values of Q_i, $(i = 1, 2, ..., m)$ for all alternatives are calculated using **Eqs. 4.10-4.12**.

$Q_1 = 1.000, \quad Q_2 = 0.872, \quad Q_3 = 0, \quad Q_4 = 0.131$

Step 8: Finally, the ranking of the alternatives has been made by sorting the value Q_i in ascending order, the position of the front is better than in the behind. Thus,

$A_3 > A_4 > A_2 > A_1$

4.5 Concluding Remarks

Selection of an industrial robot for a specific industrial application is one of the most challenging problems in real world manufacturing context. It has become highly complicated due to incorporation of advanced features and facilities that are continuously being adopted and automated into the robotic system by different manufacturers. Presently, different types of industrial robots with diverse capabilities, advanced features, flexibility in facilities and specifications are readily available in the global marketplace. Manufacturing environment, product design, and production system, functional aspects at workstation and cost involved in are some of the major influencing parameters that seem directly or indirectly affect the decision-making process for appropriate robot selection. The decision makers need to identify and select the best suited reliable robot in order to achieve desired level of output associated with high degree of accuracy at an economic cost and specific application capability. The aforesaid work attempts to develop such a decision-making procedural hierarchy.

4.6 Bibliography

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Table 4.1: Definitions of linguistic variables for criteria ratings (A-9 member interval linguistic term set)

Linguistic terms (Attribute/criteria ratings)	Linguistic terms (Priority weights)	Generalized interval-valued trapezoidal fuzzy numbers
Absolutely Poor (AP)	Absolutely Low (AL)	[(0, 0, 0, 0; 0.8), (0, 0, 0, 0; 1)]
Very Poor (VP)	Very Low (VL)	[(0, 0, 0.02, 0.07; 0.8), (0, 0, 0.02, 0.07; 1)]
Poor (P)	Low (L)	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1)]
Medium Poor (MP)	Medium Low (ML)	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1)]
Medium (M)	Medium (M)	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1)]
Medium Good (MG)	Medium High (MH)	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1)]
Good (G)	High (H)	[(0.72, 0.78, 0.92, 0.97; 0.8), (0.72, 0.78, 0.92, 0.97; 1)]
Very Good (VG)	Very High (VH)	[(0.93, 0.98, 1, 1; 0.8), (0.93, 0.98, 1, 1; 1)]
Absolutely Good (AG)	Absolutely High (AH)	[(1, 1, 1, 1; 0.8), (1, 1, 1, 1; 1)]

Table 4.2: The priority weight of criterion

C_i	DM1	DM2	DM3	DM4
C_1	VH	MH	AH	H
C_2	AH	AH	H	H
C_3	H	H	VH	MH
C_4	H	VH	MH	AH
C_5	VH	M	H	H
C_6	H	M	MH	H
C_7	MH	MH	H	MH
C_8	H	H	H	MH
C_9	H	MH	MH	MH

Table 4.3: DMs assessment on each criterion rating

Criterion	Alternatives	Decision-makers perception			
		DM1	DM2	DM3	DM4
C_1	A_1	AG	MP	MG	AG
	A_2	G	M	G	G
	A_3	VG	MG	VG	G
	A_4	VG	G	VG	G
C_2	A_1	MG	AG	MP	MG
	A_2	VG	G	M	G
	A_3	VG	MG	MP	VG
	A_4	VG	VG	G	VG
C_3	A_1	MP	M	G	AG
	A_2	M	MP	G	MG
	A_3	G	G	VG	MG
	A_4	G	VG	VG	VG
C_4	A_1	G	MP	G	AG
	A_2	G	M	G	G
	A_3	VG	M	G	VG
	A_4	VG	G	VG	M
C_5	A_1	AG	AG	VP	MP
	A_2	G	MG	MG	M
	A_3	VG	MG	G	MG
	A_4	M	VG	VG	G
C_6	A_1	MG	VG	G	MG
	A_2	G	MP	VG	VG
	A_3	G	G	AG	VG
	A_4	G	G	M	VG

C_7	A_1	G	MG	M	MG
	A_2	VG	G	MP	VG
	A_3	AG	G	G	VG
	A_4	M	G	VG	VG
C_8	A_1	G	G	G	MP
	A_2	VG	G	AG	M
	A_3	VG	G	G	M
	A_4	G	AG	AG	G
C_9	A_1	AG	G	G	MG
	A_2	MG	G	G	G
	A_3	VG	G	G	VG
	A_4	VG	G	MG	MG

Glossary

Alternatives

Alternatives are objects or options to be assessed or evaluated in a decision making process. An alternative may be assessed or evaluated based on its **attributes**.

Attributes

An attribute is an asset, quality or features of an alternative. To evaluate an alternative, a criterion is set up for each of its attributes and the attribute is examined against the criterion. Because of the one to one correspondence between an attribute and a criterion, sometimes attributes are also referred to as criteria. In the context of MCDM, the word attributes and criteria are used interchangeably.

Basically, attributes are two types such as **quantitative** and **qualitative**. Moreover attributes may break down further in to one or more levels of sub-attributes to build a **hierarchy structure**.

Criteria

See **attributes.**

Decision Matrix

Suppose there are m alternatives in a MCDM problem and each alternative has n attributes values.

A decision matrix is a $m \times n$ matrix whose element x_{ij} represents the preference of i^{th} alternative with respect to its j^{th} attribute/criteria.

MCDM, MCDA, MADM and MADA

These words are the acronyms for Multiple Criteria Decision Making, Multiple Criteria Decision Analysis, Multiple Attribute Decision Making and Multiple Attribute Decision Analysis. They are different terms for same mania and can be used interchangeably. The last two terms are mostly used for assessment problems with a finite number of alternatives.

Publications

1. **Chitrasen Samantra**, Saurav Datta, Siba Sankar Mahapatra, 2011, "*Selection of Industrial Robot using Interval-Valued Trapezoidal Fuzzy Numbers Set Combined with VIKOR Method*", **International Journal of Technology Intelligence and Planning (IJTIP)**, Volume 7, Number 4, pp. 344-360, **Inderscience Publications**, **Switzerland**.

2. **Chitrasen Samantra**, Saurav Datta, Siba Sankar Mahapatra, "*Application of Fuzzy based VIKOR Approach for Multi-Attribute Group Decision Making (MAGDM): A Case Study in Supplier Selection*", **Decision Making in Manufacturing and Services**, **A Journal of AGH University of Science and Technology**, **Krakow**, **Poland**. (**Accepted for Publication**)