Vibration and Buckling of Composite Twisted Panels Subjected to Hygrothermal Environment

A THESIS SUBMITTED IN PARTIAL FULFILMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF TECHNOLOGY

IN

CIVIL ENGINEERING

(STRUCTURAL ENGINEERING)

BY

SNIGDHA MISHRA

ROLL NO-210CE2031

DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

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UNDER THE GUIDANCE OF

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CERTIFICATE

This is to certify that the thesis entitled, **"VIBRATION AND BUCKLING OF COMPOSITE TWISTED PANELS SUBJECTED TO HYGROTHERMAL ENVIRONMENT"** submitted by **SNIGDHA MISHRA** bearing roll no. **210ce2031** in partial fulfillment of the requirements for the award of **Master of Technology** degree in **Civil Engineering** with specialization in **"Structural Engineering"** during 2010-2012 session at the National Institute of Technology, Rourkela is an authentic work carried out by her under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date: 25.05.12

Place: Rourkela

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CONTENTS

ABSTRACT

The twisted cantilever panels have significant applications in wide chord turbine blades, compressor blades, fan blades, aircraft or marine propellers, helicopter blades, and particularly in gas turbines. These structural members are often subjected to various environmental loads during their service life. The presence of temperature and moisture concentration may significantly reduce the stiffness and strength of the structures and may affect some design parameter such as vibration and stability characteristics of the structures. To avoid the typical problems caused by vibrations and stability, it is important to determine natural frequency, critical buckling load of the composite laminated pre twisted cantilever panels under hygrothermal conditions. Therefore the vibration and buckling behavior of laminated composite twisted panels subjected to hygrothermal loadings are studied in the present investigation.

The present study deals with the free vibration and buckling analysis of pre-twisted cantilever laminated composite panels subjected to hygrothermal loading using Finite element method. A computer program based on FEM in MATLAB environment is developed to perform all necessary computations. Here eight noded isoparametric quadratic shell element with five degrees of freedom per node is used based on FSDT theory with hygrothermal loading. The influences of various parameters such as angle of twist, aspect ratio and lamination parameters are examined on the vibration and buckling characteristics of laminated twisted panels subjected to hygrothermal loads. Numerical results are presented to show the effects of pre-twist angles, geometry and lamination details on the vibration and buckling characteristics of twisted plates.

NOMENCLATURE

The principal symbols used in this thesis are presented for easy reference.

English

a, b	Dimensions of the twisted panel
a/b	Aspect ratio of the twisted panel
A_{ij}, B_{ij}, D_{ij} and S_{ij}	Extensional, bending-stretching coupling, Bending and transverse shear stiffnesses
b/h	Width to thickness ratio of the twisted panel
[B]	Strain displacement matrix for the element
C, C_0	elevated and reference moisture concentrations
[D]	Stress-strain
dx, dy	Element length in x and y-direction
dV	Volume of the element
E_1, E_2	Young's moduli of a lamina along and across the fibers, respectively
G_{12}, G_{13}, G_{23}	Shear moduli of a lamina with respect to 1, 2 and 3 axes
$[K_e]$	The elastic stiffness matrix
$[K_{Ge}{}^{r}]$	The initial stress stiffness matrix
$[K_{Ge}{}^{a}]$	The geometric stiffness matrix due to applied in-plane loads
K_x, K_y, K_{xy}	Curvatures of the plate
M_x, M_y, M_{xy}	Internal moment resultants per unit length

$M_x^{i}, M_y^{i}, M_{xy}^{i}$	Initial internal moment resultants per unit length
$M_x^{N}, M_y^{N}, M_{xy}^{N}$	Non-mechanical moment resultants per unit length due to moisture and temperature
[N]	The shape function matrix
N_i	Shape function at a node i
N_x, N_y, N_{xy}	In-plane internal force resultants per unit length
$N_x^{a}, N_y^{a}, N_{xy}^{a}$	Applied in-plane forces per unit length
$N_x^{i}, N_y^{i}, N_{xy}^{i}$	In-plane initial internal force resultants per unit length
$N_x^{N}, N_y^{N}, N_{xy}^{N}$	In-plane non-mechanical force resultants per unit length due to moisture and temperature
N_{xx}	Critical buckling load in x-direction
$\{P_e\}$	The element load vector due to external transverse static load
$\{P_e^{N}\}$	The element load vector due to hygrothermal forces and moments
Q_x, Q_y	Transverse shear resultants.
Q_x^{i}, Q_y^{i}	Initial transverse shear resultants
t	Thickness of the plate
T, T_0	Elevated and reference temperatures
u, v	Displacements of the mid-plane along x and y axes, respectively
u_i, v_i, w_i	Displacements of node i along x, y and z axes, respectively
w	Displacement along z axis
x, y, z	System of co-ordinate axes

Z_k, Z_{k-1}	Bottom and top distance of lamina from mid-plane

Greek

α	Shear correction factor
α_1, α_2	Thermal coefficients along 1 and 2 axes of a lamina, respectively
β_1, β_2	Moisture coefficients along 1 and 2 axes of a lamina, respectively
ε_x ,ε_y ,γ_{xy}	In-plane strains of the mid-plane.
ε_x^N , ε_y^N , ε_{xy}^N	Non-mechanical strains due to moisture and temperature
θ	Fiber orientation in a lamina
θ_x, θ_y	Rotations of the plate about x and y axes
$\vartheta_{12}, \vartheta_{21}$	Poisson's ratios
$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$	Partial derivatives with respect to x and y
ρ	Mass density
$(\rho)_k$	Mass density of kth layer from mid-plane
η, ξ	Local natural co-ordinates of an element
η_i, ξ_i	Local natural co-ordinates of the element at i^{th} node
φ_x ,φ_y	Shear rotations in x-z and y-z planes, respectively
ω_n	Natural frequency
R_x, R_y	The radii of curvatures in the x and y direction
R_{xy}	Radius of twist of the twisted plate
ϕ	Angle of twist of the twisted panel

Mathematical Operators

$[\quad]^{-1}$	Inverse of the matrix
$[\quad]^{T}$	Transpose of the matrix

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Publications

Paper Presented in Conference

S. K. Sahu and S. Mishra: Hygrothermal effects on the free vibration of laminated composite twisted plates and shells, National Conference on Recent Advances in Mechanics and Materials, 25-26, Feb, 2012, VSSUT, Burla, Odisha.

CHAPTER 1

INTRODUCTION

Introduction

The laminated composite twisted cantilever panels have significant applications in wide chord turbine blades, compressor blades, fan blades, particularly in gas turbines. The varying environmental conditions due to moisture absorption and temperature seem to have an adverse effect is inevitable on the stiffness and strength of the structural composites. Hygrothermal expansions or contractions change the stress and strain distributions in the composite. So it is has tremendous technical importance to study and analyze the vibration and static characteristics of laminated composite shells under hygrothermal conditions.

1.1 Importance of the present study

In a weight sensitive application such as aircraft engine, turbomachinery twisted composite materials are advantageous because of their light weightness, stiffness and strength and excellent thermal characteristics. Structures used in the above fields are more often exposed to high temperature and moisture. The varying environmental condition due to moisture absorption and temperature seem to have an adverse effect on stiffness and strength of the twisted composites. The effect of temperature is known as the thermal effect and the effect of moisture is known as the hygroscopic effect. The combined effect of temperature and moisture is known as hygrothermal effect. Heat gets conducted into the laminate when subjected to rise in the temperature. The laminate absorbs moisture when subjected to the wet conditions. The swelling or expansion is more across the fibres of the lamina. Hygrothermal effects induce a dimensional change in the lamina. But due to the mismatch of the properties of the constituents of the laminate, its free movement is inhibited. As a result, deformations and corresponding stress conditions are induced. The induced hygrothermal stresses is referred as residual stresses. As the matrix is more susceptible to the hygrothermal condition than fibre the deformation is observed to be more in the transverse direction of the composite. The rise in temperature and moisture reduces the elastic moduli of the material and induces internal stresses, which may affect the stability as well as safety of the structure. So it is necessary to study and analyze the behavior such as buckling, natural frequencies of laminated composite twisted panels under hygrothermal conditions. Composite materials are being increasingly used in turbomachinery blades and sensitive aerospace applications, primarily because of the large values of specific strength and these can be tailored through the variation of fibre orientation and stacking sequence to obtain an efficient design. The optimum design of laminated structures demands an effective analytical procedure. But the presence of various coupling stiffnesses and hygrothermal loading complicates the problem of vibration and buckling analysis of twisted panels for obtaining a suitable theoretical solution. So a clear understanding about vibration and buckling characteristics of the composite twisted panels is of great importance.

A comprehensive analysis of the vibration problems of homogeneous turbomachinery blades, modelled as beams has been studied exhaustively. Some studies available on the untwisted plates subjected to hygrothermal loading and some studies are there on vibration aspects of laminated composite pre twisted cantilever panels. The vibration and buckling analysis of laminated composite pre twisted cantilever panels subjected to hygrothermal environment is current problem of interest. A thorough review of earlier works done in this field is an important requirement to arrive at the objective and scope of the present investigation.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The vast use of twisted panels leads to significant amount of research over the years. Due to its wide range of application in the practical field, it is important to understand the nature of deformation, vibration and stability behaviour of twisted plates. Though the present investigation is mainly focused on stability studies of twisted panels subjected to hygrothermal loading some relevant researches on vibration, static stability and dynamic stability of untwisted plates are also studied for completeness. The literature reviewed in this chapter are grouped into

- Twisted panels
- Composite plate

2.2 Reviews on twisted panels

The twisted panels have significant applications in wide chord turbine blades, compressor blades, fan blades, particularly in gas turbines. This range of practical applications demands a proper understanding of their vibration and buckling. Due to its significance, a large number of references deal with the free vibration of twisted plates.

2.2.1 Vibration and buckling of twisted panels subjected to hygrothermal load

With the continually increasing use of turbomachinery at higher performance levels, especially in aircraft, the study of vibration problems arising in twisted blades has become increasingly important. Free vibration frequencies and mode shapes are essential for the analysis of resonant response and flutter. Due to its significance in structural mechanics, many researchers have worked on the vibration characteristics of turbomachinery blades without considering the hygrothermal effect. Ansari (1975) evaluated the nonlinear modes of vibration of a pretwisted non uniform cantilevered blade of unsymmetrical cross-section mounted on the periphery of a rotating disk. Kirkhope and Wilson (1976) calculated the coupled vibration modes of a rotating blade-disc system by using finite element method. Rao and Banerjee (1977) developed a polynomial frequency equation method to determine the natural frequencies of a cantilever blade with an asymmetric cross-section mounted on a rotating disc. Considering the blade as a discrete system, generalised polynomial expressions for the slope, linear and angular deflections are derived, using Myklestad expressions with necessary modifications. Walker (1978) studied a conforming finite shell element suitable for the analysis of curved twisted fan blades and applied to a number of fan blade models. The element is assumed to be a doubly curved right helicoidal shell, in which the curvature is shallow with respect to the twisted base plane defining the helicoid. Sreenwasamurthy and Ramamurti (1980) investigated the effect of a tip mass on the natural frequencies of a rotating pre-twisted cantilever plate. Sreenwasamurthy and Ramamurti (1981) used a finite element technique to determine the natural frequency of a pre-twisted and tapered plate mounted on the periphery of a rotating disc. The pre-twisted plate has been idealised as an assemblage of three noded rectangular shell elements with six degrees of freedom at each node. Jensen (1982) analysed the free vibration of turbine blades by using general shell theory. Leissa *et al*. (1983) determined the Vibrational characteristic of doubly curved shallow shells having rectangular plan forms, clamped along one edge and free on the other three using Ritz method with algebraic polynomial trial function. Leissa *et al.* (1984) experimented on vibration of twisted cantilever plates and summarised the previous and current studies. Ramamurti and Kielb (1984) presented a detailed comparison of the eigen frequencies of twisted rotating plates as obtained by using two different shape functions. Fox (1985) examined the free vibration of rotating uniform radial cantilever beams of compact cross section with account taken of centrifugal coupling between motions in the principal elastic planes. Rao and Gupta (1987) analysed the free vibration characteristic of a rotating pre twisted small aspect ratio blade, mounted on a disc at a stagger angle, are determined using classical bending theory of thin shells. Qatu and Leissa (1991) presented the vibration studies for laminated composite twisted cantilever plates using Ritz method with algebraic polynomial displacement function.

Liew *et al*. (1994) presented a mathematical model to investigate the effects of initial twist on the vibratory characteristics of cantilever shallow conical shells. Pai and Nayfeh (1994) used a new approach to develop a geometrically exact non-linear beam model for naturally curved and twisted solid composite rotor blades undergoing large vibrations in three dimensional space. Lakhtakia (1995) studied wave propagation in a piezoelectric, continuously twisted, structurally chiral medium along the axis of spirality. Liew *et al*. (1995) presented a computational investigation into the objects of initial twist and thickness variation on the vibratory characteristics of cantilevered pretwisted thin shallow conical shells with generally varying thickness. Rand (1995) presented a experimental method for detecting the natural frequencies of helicopter-like thin-walled rotating composite blades, and a study of their tendency in lamination angles and in the rotor angular velocity. Rand & Barkai (1997) studied a nonlinear formulation for the structural behavior of initially twisted solid and thin walled composite blades is presented. The model is designed to handle arbitrary thick solid cross-sections or general thin-walled geometries, and includes three-dimensional out-of-plane warping. Parhi *et al*. (1999) investigated about the dynamic analysis of delaminated composite twisted plates based on a simple multiple delaminated model using finite element method. He *et al*. (2000) presented a paper about a computational method for characterizing the resonant frequency properties of cantilever pre-twisted plate composed of fibre-reinforced laminated composites. Chen and Chen (2001) employed a finite element model to investigate the mean square response and reliability of a rotating composite blade with external and internal damping under stationary or non stationary random excitation. The effects of transverse shear deformation and rotary inertia are considered. Hu and Tsuiji (2001) investigated the vibration analysis of laminated cylindrical thin panels with twist and curvature using Rayleigh-Ritz method. Choi and Chou (2001) proposed a modified differential quadrature method (MDQM) for vibration analysis of elastically supported turbomachinery blades. Yoo *et al.* (2001) derived the equations of motion for the vibration analysis of rotating pre-twisted blades, derived from a modelling method which employs hybrid deformation variables. Hu *et al.* (2002) studied a numerical method for free vibration of a rotating twisted and open conical shell by the energy method, based on a non-linear strain–displacement relationship of a non-rotating twisted and open conical shell on thin shell theory. Hu *et al.* (2002) investigated the vibration analysis of 7twisted conical shells with tapered thickness. Hu *et al*. (2002) studied a methodology for free vibration of a laminated composite conical shell with twist is proposed, in which a strain–displacement relationship of a twisted conical shell is given by considering the Green strain tensor on the general thin shell theory, the principle of virtual work is utilized, and the governing equation is formulated by the Rayleigh–Ritz procedure with algebraic polynomials in two elements as admissible displacement functions. Zhu *et al.* (2002) examined the modelling of torsional vibration induced by extension-twisting coupling of anisotropic composite laminates with piezoelectric actuators. Lee *et al.* (2002) studied the finite element method based on the Hellinger Reissner principle with independent strain is applied to the vibration problem of cantilevered twisted plates and cylindrical, conical laminated shells. Kuang, Hsu (2002) studied the eigenvalue problem of a tapered pre-twisted orthotropic composite blade is formulated by employing the differential quadrature method (DQM). The Euler–Bernoulli beam model is used to characterize the pre-twisted orthotropic composite blade. Lim (2003) presented a new approach in the bending analysis of helicoidal structures with a large non-linear pretwist and an external lateral loading. Yoo and Pierre (2003) investigated the modal characteristic of a rotating cantilever plate by using a dynamic modelling method for rectangular plates undergoing prescribed overall motion to derive the equations of motion. Oh *et al*. (2003) analysised the effects of pretwist and presetting on coupled bending vibrations of rotating thin-walled composite beams using refined dynamic theory of rotating blades modelled as anisotropic composite thin-walled beams, experiencing the flapping-lagging-transverse shear coupling. Lin *et al*. (2003) investigated about rotating no uniform pretwisted beams with an elastically restrained root and a tip mass Using Hamilton's principle derives the governing differential equations for the coupled bending–bending vibration of a rotating pretwisted beam with an elastically restrained root and a tip mass, subjected to the external transverse forces and rotating at a constant angular velocity. Chandiramani *et al.* (2003) examined the free and forced vibration of a rotating, pretwisted blade modelled as a laminated composite, hollow (single celled), and uniform box-beam. The structural model includes transverse shear flexibility, restrained warping and centrifugal and Coriolis effects. Nabi and Ganesan (2003) proposed the vibration characteristics of pre-twisted composite blades are analysed using a three-noded triangular cylindrical shell element. The specific example of glass fibre reinforced plastic material is analysed with its material damping. The effect of different parameters such as pre-twist, fibre orientation, skew angle, taper ratio and aspect ratio on natural frequency and system loss factor is investigated. Sakar and Sabuncu (2004) presented a finite element model for the static and dynamic stability study of a pretwisted aerofoil cross-section rotating blade subjected to an axial periodic force. Tsai (2004) studied the Rotating vibration behavior of the turbine blades with different groups of blades. Hu *et al.* (2004) studied vibration of twisted plate Based on general shell theory and the first order shear deformation theory, an accurate relationship between strains and displacements of a twisted plate is derived by the Green strain tensor. Kee and Kim (2004) derived a general formulation for an initially twisted rotating shell structures including the effect of centrifugal force and Coriolis acceleration to study the vibration characteristic of initially twisted rotating shell type composite blades. Hu *et al.* (2004) proposed vibration of an angle-ply laminated plate with twist considering transverse strain and rotary inertia, an analytical method by using Rayleigh-Ritz procedure. Dokainish and Rawtani (2005) used a finite element technique is to determine the natural frequencies and the mode shapes of a cantilever plate mounted on the periphery of a rotating disc. The plane of the plate is assumed to make any arbitrary angle with the plane of rotation of the disc. Chazly (2005) analysed the Static and dynamic analysis of wind turbine blades using the finite element method. Sahu *et al.* (2005) studied the vibration and stability behaviour of angle ply laminated twisted panels using Finite element method. Huang (2006) examined the effect of number of blades and distribution of cracks on vibration localization in a cracked pre-twisted blade system. Sahu *et al.* (2007) studied the buckling and vibration analysis of cross-ply laminated cantilever twisted plate using the finite element method with first order shear deformation theory. An eight noded isoparametric quadratic element is employed in the present analysis with five degrees of freedom per node. Choi *et al*. (2007) studied the bending vibration control of the pre-twisted rotating composite thin-walled beam is studied. The formulation is based on single cell composite beam including a warping function, centrifugal force, Coriolis acceleration, pre-twist angle and piezoelectric effect. Hashemi *et al.* (2009) studied a finite element formulation for vibration analysis of rotating thick plates is developed. Mindlin plate theory combined with second order strain–displacement assumptions are applied for plate modelling. Sinha and Turner (2011) proposed starting with the thin shell theory, the governing partial differential equation of motion for the transverse deflection of a rotating pre-twisted plate is derived to determine the natural frequencies of a twisted blade in a centrifugal field. Farhadi and Hasemi (2011) examined the aeroelastic behavior of a supersonic rotating rectangular plate in the air medium using the Mindlin first-order shear deformation plate theory along with Von Korman non linear terms.

2.3 Reviews on Plates

The behaviour of structures subjected to in-plane loads with hygrothermal load is less understood in comparison with structures under transverse loads. Some of the literature covering composite plates subjected to hygrothermal loading is presented here.

2.3.1 Vibration and buckling of composite panels subjected to hygrothermal load

The deformation and stress analysis of the laminated composite plates subjected to moisture and temperature has been the subject of research interest of many investigators. Flaggs and Vinson (1978) studied the combine effect of temperature and humidity in a general laminated composite plate buckling theory formulation based on the theory of minimum potential energy. Yoseph *et al.* (1987) developed an analytical method an analytical numerical method to predict the time-dependent mechanical behavior of polymer-dominated multi-material systems. The method takes into account the interdependence of stress, strain, moisture diffusion and heat conduction, and it provides stress, strain, moisture and temperature distributions through the system as functions of time, when the external hygrothermal-mechanical conditions are given. Chen and Chen (1988) performed a Finite element model for the free and forced vibration of the laminated rectangular composite plate exposed to a steady state hygrothermal environment .Sai Ram and Sinha (1990) presented the effects of moisture and temperature on the bending characteristics of laminated composite plates by finite element method using quadratic isoparametric plate bending element which takes transverse shear deformation into account. Birman and Bert (1990) examined the dynamic stability of thin, multi-layered composite shells reinforced by axial and ring stiffeners and subjected to pulsating loads acting in the axial direction is considering the thermal field. Sai Ram and Sinha (1992) investigated the effects of moisture and temperature on the free vibration of laminated composite plates, carried out by the finite element method with the quadratic isoparametric element, which takes transverse shear deformation into account. Sai Ram and Sinha (1992) proposed the effects of moisture and temperature on the static instability of laminated composite plates, carried out by the finite element method with the quadratic isoparametric element, which takes transverse shear deformation into account. Dawe and Ge (2000) studied the buckling of shear-deformable composite laminated plates by the spline finite strip method taking first order shear deformation in to account. Barut *et al.* (2000) presented the response of moderately thick laminated panels experiencing large displacements and rotations under non –uniform thermal loading through a non linear finite element analysis. Sarath Babu and Kant (2000) proposed two refined higher order theories, one that neglects and the other that takes into account the effect of transverse normal deformation, are used to develop two discrete finite element models for the thermal buckling analysis of composite laminates and sandwiches. Parhi *et al.* (2001) presented a quadratic isoparametric finite element formulation based on the first order shear deformation theory for the free vibration and transient response analysis of multiple delaminated doubly curved composite shells subjected to a hygrothermal environment. Rohwer *et al*. (2001) investigated higher theories for thermal stresses in layered plates. Cheng and Batra (2001) studied the effect of thermal loads on imperfectly bonded laminated composite shells, the interfacial imperfections necessitate that conditions requiring the continuity of surface tractions and displacements between adjoining faces be suitably modified, and interfacial damage properly accounted for. Shen and Shen (2001) examined the effect of hygrothermal conditions on the buckling and postbuckling of shear deformable laminated cylindrical shells subjected to combined loading of axial compression and external pressure is investigated using a micro-to macro-mechanical analytical model. Singha *et al.* (2001) analysed the thermal post buckling behaviour of graphite/epoxy multi-layered rectangular plates of various boundary conditions using the finite element method. Vel and Batra (2001) investigated the generalized plane strain quasi-static thermo elastic deformations of laminated anisotropic thick plates by using the Eshelby-Stroh formalism. Shen and Shen (2001) studied the influence of hygrothermal effects on the postbuckling of shear deformable laminated plates subjected to a uniaxial compression is investigated using a micro-to-macro-mechanical analytical model. Rutgerson and Bottega (2002) presented the buckling behavior of multilayer shells for composite structures subjected to combinations of uniform temperature change, applied external pressure, and applied and reactive circumferential edge loads. Wu and Chiu (2002) analysed on thermally induced dynamic instability of laminated composite conical shells is investigated by means of a perturbation method. The laminated composite conical shells are subjected to static and periodic thermal loads. Patel *et al.* (2002) studied the static and dynamic characteristics of thick composite laminates exposed to hygrothermal environment are studied using a realistic higher-order theory developed. Shen and Shen (2002) studied the effect of hygrothermal conditions on the buckling and postbuckling of shear deformable laminated cylindrical panels subjected to axial compression is investigated using a micro-to-macro-mechanical analytical model. Khare *et al*. (2003) examined the Closed-form formulations of 2D higher-order shear deformation theories for the thermo-mechanical analysis of simply supported doubly curved cross-ply laminated shells. Yang and Shen (2003) analysed the nonlinear bending analysis of shear deformable functionally graded plates subjected to thermo-mechanical loads and under various boundary conditions. Tounsi and Bedia (2003) presented some observations on the evolution of transversal hygroscopic stresses in laminated composites plates: effect of anisotropy. Raja *et al.* (2004) carried out the active stiffening and active compensation analyses to present the influence of active stiffness on the dynamic behaviour of piezo-hygro-thermo-elastic laminates. Rao and Sinha (2004) investigated the effects of temperature and moisture on the free vibration and transient response of multidirectional composites. A three dimensional finite element procedure is developed using 20 noded isoparametric quadratic elements. Huang *et al*. (2004) studied the hygrothermal effects on the nonlinear vibration and dynamic response of shear deformable laminated plates. The temperature field considered is assumed to be a uniform distribution over the plate surface and through the plate thickness. Wang *et al.* (2005) studied the Hygrothermal effect on the response histories and distribution of dynamic interlaminar stresses in laminated plates with piezoelectric actuator layers, under free vibration are studied, based on Hygrothermoelectrodynamic differential equations. Dawe *et al.* (2005) studied the thermo mechanical post buckling behaviour of composite laminated plates is studied with the aid of the B-spline finite strip method under the combination of temperature load and applied uniaxial mechanical stress. Naidu and Sinha (2005) studied the large deflection bending behaviour of composite cylindrical shell panels subjected to hygrothermal environments. The present finite element formulation considers doubly curved thick shells and includes large deformations with Green–Lagrange strains**.** Xiao and Chen (2005) established the nonlinear models of the elastic and elastic linear strain-hardening square plates with four immovably simply-supported edges are established by employing Hamilton’sVariational Principle in a uniform temperature field to study the dynamic and buckling analysis of a thin elastic-plastic square plate in a uniform temperature field. Liew *et al.* (2006) analysed the linear and nonlinear vibration analysis of a three-layer coating FGM substrate cylindrical panel with general boundary conditions and subjected to a temperature gradient across the thickness due to steady heat conduction. Bouazza *et al*. (2007) developed an analytical model based on the notion of the stress perturbation function and applied to study the effect of multiple cracks in aged cross-ply laminates on the stiffness of a laminated composite. Panda and Pradhan (2007) presented two sets of full three-dimensional thermo elastic finite element analyses of superimposed thermo-mechanically loaded FRP composite laminates with embedded interfacial elliptical delaminations, emphasizing the influence of residual thermal stresses and material anisotropy on the delamination fracture behavior characteristics. Naidu and Sinha (2007) investigated the nonlinear free vibration behaviour of laminated composite shells subjected to hygrothermal environment is investigated using the finite element method. Riberio and Jansen (2008) studied the geometrically non-linear vibrations of linear elastic composite laminated shallow shells under the simultaneous action of thermal fields and mechanical excitations are analysed. Cho ((2009) analysed dynamic responses of an orthotropic plate subjected to hygrothermal environments have been optimized. Non-gradient evolutionary genetic algorithm (GA) is employed to optimize dynamic behaviours of orthotropic composite. Kundu and Han (2009) also studied the hygrothermoelastic buckling behaviour of laminated composite shells are numerically simulated using geometrically nonlinear finite element method. Joshi and Patel (2010) examined nonlinear thermo elastic static response characteristics of laminated composite conical panels are studied employing finite element approach based on first-order shear deformation theory and field consistency principle. Mahato and Maiti (2010) investigated the aeroelastic performances of smart composite plates under aerodynamic loads are investigated in hygrothermal environment. Upadhyay *et al.* (2010) presented the nonlinear flexural response of the elastically supported moderately thick laminated composite rectangular plates subjected to hygro-thermo-mechanical loading. Lal *et al.* (2011) analysed the effect of random system properties on transverse nonlinear central deflection of laminated composite spherical shell panel subjected to hygro-thermo-mechanical loading. Lal *et al.* (2011) studied the effect of random system properties on the post buckling load of geometrically nonlinear laminated composite cylindrical shell panel subjected to hygrothermomechanical loading. Singh and Chakrabarti (2011) presented hygrothermal analysis of laminated composite plates has been done by using an efficient higher order shear deformation theory. In the present formulation, the plate model has been implemented with a computationally efficient C^0 finite element developed by using consistent strain field.

2.4 Aim and scope of the present studies

A Review of literature shows that a lot of work has been done on the vibration of laminated composite twisted cantilever panels. Some work has been done on static stability of laminated composite twisted cantilever panels. A lot of work has been done on the vibration and buckling of composite panels subjected to hygrothermal loading. However no study is available on vibration and buckling of laminated composite twisted cantilever panels subjected to hygrothermal loading. The present study is mainly aimed at filling some of the lacunae that exist in the understanding of the vibration and buckling characteristic of laminated composite twisted cantilever panels subjected to hygrothermal loading.

Based on the review of literature, the different problems identified for the present investigation are presented as follows.

- Effect of temperature and moisture on the vibration of laminated composite twisted cantilever panels subjected to hygrothermal loading.
- Effect of temperature and moisture on the buckling of laminated composite twisted cantilever panels subjected to hygrothermal loading.

The influence of various parameters such as angle of twist, curvature, number of layers, lamination sequence, and ply orientation, degree of orthotropy factors on the vibration and buckling behaviour of twisted panels are examined in detail.

CHAPTER 3

THEORY AND FORMULATON

3.1 The Basic Problem

The mathematical formulation for vibration and static stability analysis of the twisted plate and shell structures is presented here. The basic configuration of the problem considered here is a composite laminated twisted panel of sides ‘a’ and ‘b’ as shown in Figure. The twisted panel is modelled as a doubly curved panel with twisting curvature so that the analysis can be done for twisted plates, cylindrical and spherical configurations by changing the value of the curvature. The boundary conditions are taken to be that of a cantilever, that is fixed at the left end and free at the other edges. The basic composite twisted curved panel is considered to be composed of composite material laminates. ‘n’ denotes the number of layers of the laminated composite twisted panel.

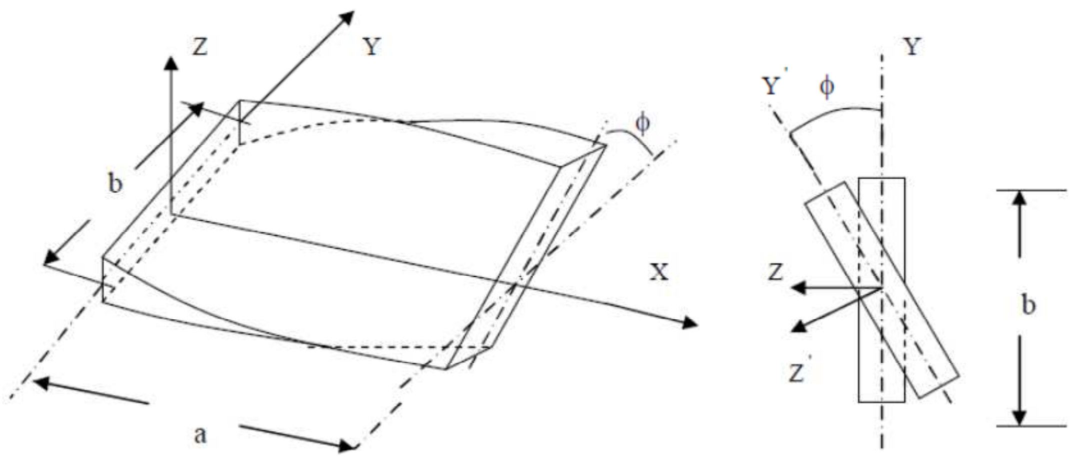

Fig.1 composite twisted panel

n
.
3
2
1

Fig.2 the lamination

3.2 Proposed Analysis

The governing equations for the vibration and buckling of laminated composite twisted panels/shells subjected to in-plane loading are developed. The presence of external in-plane loads induces a stress field in the structure. This necessitates the determination of the stress field as a prerequisite to the solution of problems like vibration and buckling behaviour of pretwisted plates and shells. As the thickness of the structure is relatively smaller, the determination of the stress field reduces to the solution of a plane stress problem. The equation of motion represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The development of the regions of instability arises from Floquet’s theory and the solution is obtained by Bolotin’s approach using finite element method. The governing differential equations have been developed using the first order shear deformation theory (FSDT). The assumptions made in the analysis are given below.

3.3 Assumptions of the analysis

1) The analysis is linear with a few exceptions. This implies both linear constitutive relations (generalized Hooke's law for the material and linear kinematics) and small displacement to accommodate small deformation theory.

2) The twisted panels are of various shapes with no initial imperfections. The considerations of imperfections are less important for dynamic loading.

3) The straight line that is perpendicular to the neutral surface before deformation remains straight but not normal after deformation (FSDT). The thickness of the twisted panel is small compared with the principal radii of curvature. Normal stress in the z-direction is neglected.

4) The loading considered is axial with a simple harmonic fluctuation with respect to time.

5) All damping effects are neglected.

3.4 Governing Equations

The governing differential equations, the strain energy due to loads, kinetic energy and formulation of the general dynamic problem are derived on the basis of the principle of potential energy and Lagrange's equation.

3.4.1 Governing Differential Equations

The governing differential equations for vibration of a shear deformable laminated composite plate in hygrothermal environment derived on the basis of first order shear deformation theory (FSDT) subjected to in-plane loads are:

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}-\frac{1}{2}\left(\frac{1}{R_y}-\frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial y}+\frac{Q_x}{R_x}+\frac{Q_y}{R_{xy}}=P_1\frac{\partial^2 u}{\partial t^2}+P_2\frac{\partial^2 \theta_x}{\partial t^2}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}+\frac{1}{2}\left(\frac{1}{R_y}-\frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial x}+\frac{Q_y}{R_y}+\frac{Q_x}{R_{xy}}=P_1\frac{\partial^2 v}{\partial t^2}+P_2\frac{\partial^2 \theta_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}-\frac{N_x}{R_x}-\frac{N_y}{R_y}-2\frac{N_{xy}}{R_{xy}}+N_x^0\frac{\partial^2 w}{\partial x^2}+N_y^0\frac{\partial^{2w}}{\partial y^2}=P_1\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=P_3\frac{\partial^2\theta_x}{\partial t^2}+P_2\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_x=P_3\frac{\partial^2\theta_y}{\partial t^2}+P_2\frac{\partial^2 v}{\partial t^2} \quad (1)$$

Where N_x, N_y and N_{xy} are the in-plane stress resultants,

M_x, M_y and M_{xy} are moment resultants and Q_x, Q_y are transverse shear stress resultants.

R_x, R_y and R_{xy} identify the radii of curvatures in the x and y direction and radius of twist.

$$(P_1,P_2,P_3)=\sum_{k=1}^{n}\int_{z_{k=1}}^{z_k}(\rho)_k(1,z,z^2)dz \quad (2)$$

Where n= number of layers of laminated composite curved pane $(\rho)_k$= mass density of kth layer from mid-plane.

3.4.2 Constitutive Relations

The constitutive relation for the twisted plate, when subjected to moisture and temperature, are given by

$$\{F\}=[D]\{\varepsilon\}-\{F^N\} \quad (3)$$

Where

$\{F\}=\{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^T$

$\{F^N\}=\{N_x^N, N_y^N, N_{xy}^N, M_x^N, M_y^N, M_{xy}^N, 0, 0\}^T$

$\{\varepsilon\}=\{\varepsilon_x, \varepsilon_y, \Upsilon_{xy}, K_x, K_y, K_{xy}, \varphi_x, \varphi_y,\}^T$

$$[D]=\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix}$$

N_x, N_y, N_{xy} = in-plane internal force resultants

M_x ,M_y ,M_{xy} = internal moment resultants.

Q_x ,Q_y = transverse shear resultants.

N_x^N,N_y^N,N_{xy}^N = in-plane non mechanical force resultants due to moisture and temperature.

M_x^N,M_y^N,M_{xy}^N = non-mechanical moment resultants due to moisture and temperature.

ε_x ,ε_y ,Υ_{xy} = in-plane strains of the mid-plane.

K_x ,K_y ,K_{xy} = curvature of the plate

φ_x ,φ_y = shear rotations in X-Z and Y-Z planes respectively.

A_{ij} , B_{ij}, D_{ij} and Sij are the extensional, bending-stretching coupling, bending and transverse shear stiffnesses. They may be defined as

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (Z_k - Z_{k-1})$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k ({Z^2}_k - {Z^2}_{k-1})$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k ({Z^3}_k - {Z^3}_{k-1}) \qquad \text{for } i,j = 1,2,6$$

$$S_{ij} = \alpha \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (Z_k - Z_{k-1}) \qquad \text{for } i,j = 4,5 \qquad (4)$$

α = shear correction factor

Z_k, Z_{k-1} = bottom and top distance of lamina from mid-plane

The non-mechanical force and moment resultants are expressed as

$$\{N_x{}^N, N_y{}^N, N_{xy}{}^N\}^T = \sum_{k=1}^{n} (\overline{Q_{IJ}})_k \{\varepsilon\}_k (Z_k - Z_{k-1})$$

$$\{M_x{}^N, M_y{}^N, M_{xy}{}^N\} = \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{IJ}})_k \{\varepsilon\}_k (Z^2{}_k - Z^2{}_{k-1}) \qquad \text{for i, j} = 1,2,6 \qquad (5)$$

Where

$$\{\varepsilon\}_k = \{\varepsilon_x{}^N, \varepsilon_y{}^N, \varepsilon_{xy}{}^N\}^T = [T]\{\beta_1, \beta_2\}_k{}^T (C - C_0) + [T]\{\alpha_1, \alpha_2\}_k{}^T (T - T_0),$$

in which

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta \\ \sin^2\theta & \cos^2\theta \\ \sin2\theta & -\sin2\theta \end{bmatrix}$$

$\varepsilon_x{}^N$, $\varepsilon_y{}^N$, $\varepsilon_{xy}{}^N$ = non-mechanical strains due to moisture and temperature

β_1, β_2 = moisture coefficients along 1 and 2 axes of a lamina, respectively

α_1, α_2 = thermal coefficients along 1 and 2 axes of a lamina, respectively

T, T_0 = elevated and reference temperatures

C, C_0 = elevated and reference moisture concentrations

$(\overline{Q_{IJ}})_k$ in equations (10) and (11) is defined as $(\overline{Q_{IJ}})_k = [T_1]^T [Q_{ij}]_k [T_1]$

$(\overline{Q_{IJ}})_k = [T_1]^T [Q_{ij}]_k [T_1]$ For i,j =1,2,6

$(\overline{Q_{IJ}})_k = [T_2]^T [Q_{ij}]_k [T_2]$ For i,j = 4,5……… (6)

Where

$$[T_1] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$[T_2] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$[Q_{ij}]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \qquad \text{For i,j =1,2,6}$$

$$[Q_{ij}]_k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \qquad \text{For i,j = 4,5}$$

$$Q_{11} = \frac{E_1}{(1 - \vartheta_{12}\vartheta_{21})}$$

$$Q_{12} = \frac{\vartheta_{12}E_1}{(1 - \vartheta_{12}\vartheta_{21})}$$

$$Q_{22} = \frac{E_2}{(1 - \vartheta_{12}\vartheta_{21})}$$

$$Q_{44} = G_{13}, Q_{55} = G_{23}$$

$E_{1,}E_2$ = Young's moduli of a lamina along and across the fibers, respectively

G_{12}, G_{13}, G_{23}= Shear moduli of a lamina with respect to 1, 2 and 3 axes.

$\vartheta_{12}, \vartheta_{21}$ = Poisson's ratios

3.4.3 Strain Displacement relations

Green-Lagrange's strain displacement relations are used throughout the structural analysis. The linear part of the strain is used to derive the elastic stiffness matrix and the nonlinear part of the strain is used to derive the geometric stiffness matrix. The total strain is given by

$$\{\varepsilon\}=\{\varepsilon_l\}+\{\varepsilon_{nl}\} \qquad (7)$$

The linear strain displacement relations for a twisted shell element are:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{R_x} + zk_x$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{w}{R_y} + zk_y$$

$$\gamma_{xyl} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} + zk_{xy}$$

$$\gamma_{xzl} = \frac{\partial w}{\partial x} + \theta_x - \frac{u}{R_x} - \frac{v}{R_{xy}}$$

$$\gamma_{xzl} = \frac{\partial w}{\partial y} + \theta_y - \frac{v}{R_y} - \frac{u}{R_{xy}} \qquad (8)$$

The bending strains k are expressed as,

$$k_x = \frac{\partial \theta_x}{\partial x} \qquad k_y = \frac{\partial \theta_y}{\partial y}$$

$$k_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad (9)$$

The nonlinear strain components are defined as follows:

$$\varepsilon_{xnl} = \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right]$$

$$\varepsilon_{ynl} = \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right]$$

$$\gamma_{xynl} = \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right) + \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)\left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right) + z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right]$$

$$\gamma_{xznl} = \left(\frac{\partial u}{\partial x}\right)\theta_y - \left(\frac{\partial v}{\partial x}\right)\theta_x + z\left[\left(\frac{\partial \theta_y}{\partial x}\right)\theta_y + \left(\frac{\partial \theta_x}{\partial x}\right)\theta_x\right]$$

$$\gamma_{yznl} = \left(\frac{\partial u}{\partial y}\right)\theta_y - \left(\frac{\partial v}{\partial y}\right)\theta_x + z\left[\left(\frac{\partial \theta_y}{\partial y}\right)\theta_y + \left(\frac{\partial \theta_x}{\partial y}\right)\theta_x\right] \tag{10}$$

u,v = displacements of the mid-plane along x and y axes, respectively

w = displacement along z axis

θ_x, θ_y = rotations of the plate about x and y axes.

3.5 Finite Element Formulation

For problems involving complex geometrical and boundary conditions, analytical methods are not easily adaptable and numerical methods like finite element methods (FEM) are preferred. The finite element formulation is developed hereby for the structural analysis of isotropic as well as composite twisted panels using a curved shear deformable shell theory.

3.5.1 The shell element

The plate is made up of perfectly bonded layers. Each lamina is considered to be homogeneous and orthotropic and made of unidirectional fiber-reinforced material. The orthotropic axes of symmetry in each lamina are oriented at an arbitrary angle to the plate

axes. An eight-noded isoparametric quadratic shell element is employed in the present analysis with five degrees of freedom u, v, w, θ_x andθ_y per node as shown in Figure. But the in-plane deformations u and v are considered for the initial plane stress analysis. The isoparametric element shall be oriented in the natural coordinate system and shall be transferred to the Cartesian coordinate system using the Jacobian matrix. In the analysis of thin shells, where the element is assumed to have mid-surface nodes, the shape function of the element is derived using the interpolation polynomial.

For problems involving complex in-plane loading and boundary conditions numerical methods like finite element method (FEM) are preferred. Eight-nodded isoperimetric element is used to the present free vibration problem. Five degrees of freedom u, v, w, θ_x and θ_y are considered at each node. The stiffness matrix, the geometric stiffness matrix due to residual stresses, geometric stiffness matrix due to applied in-plane loads and nodal load vector of the element are derived using the principle of minimum potential energy.

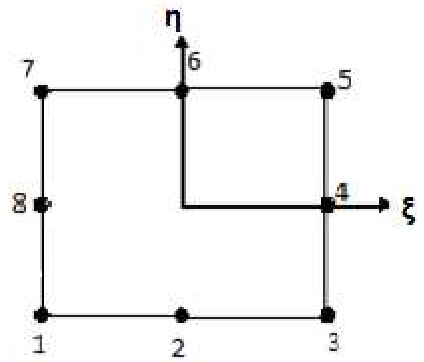

Fig.3 Eight nodded isoparametric element

The element displacements are expressed in terms of their nodal values by using the element shape functions and are given by

$$u = \sum_{i=1}^{8} N_i v_i \qquad v = \sum_{i=1}^{8} N_i v_i \qquad w = \sum_{i=1}^{8} N_i w_i$$

$$\theta_x = \sum_{i=1}^{8} N_i u_i \qquad \theta_y = \sum_{i=1}^{8} N_i \theta_{yi} \qquad (11)$$

N_i = Shape function at a node i

ξ ,η = Local natural co-ordinates of an element

3.5.2 Stiffness matrix

The linear strain matrix {ε} is obtained by substituting equations (11) into (9), and is expressed as

{ε} = [B]{δe}…………………. (12)

Where

$\{\delta e\} = \{u_1\, v_1 w_1 \theta_{x1}\ \theta_{y1} \dots \dots \dots \dots \dots \dots .\, u_8\, v_8 w_8 \theta_{x8}\ \theta_{y8}\}^T$

$$[B]=\begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & \frac{N_i}{R_x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & \frac{N_i}{R_x} & 0 & 0 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 2\frac{N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \\ 0 & 0 & \frac{\partial N_i}{\partial x} & N_i & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & N_i \end{bmatrix}$$

For i= 1, 2……….8

The elastic stiffness matrix is given by

$$[K_e] = \iint [B]^T[D][B]dxdy \qquad (13)$$

3.5.3 Geometric stiffness matrix $[\mathbf{K_{Ge}}^r]$

The non-linear strains, equations (10), are represented in matrix form as

$$\{\varepsilon_{nl}\} = \{\varepsilon_{xnl}, \varepsilon_{ynl}, \gamma_{xynl}, \gamma_{xynl}, \gamma_{yznl}\}^T = [R]\{d\}/2 \qquad (14)$$

Where

$$\{d\} = \left\{\left(\frac{\partial u}{\partial x}\right), \left(\frac{\partial u}{\partial y}\right), \left(\frac{\partial v}{\partial x}\right), \left(\frac{\partial v}{\partial y}\right), \left(\frac{\partial w}{\partial x}\right), \left(\frac{\partial w}{\partial y}\right), \left(\frac{\partial \theta_x}{\partial x}\right), \left(\frac{\partial \theta_x}{\partial y}\right), \left(\frac{\partial \theta_y}{\partial x}\right), \left(\frac{\partial \theta_y}{\partial y}\right), \theta_x, \theta_y\right\}^T$$

By using equations (11), {d} may be expressed as

{d} = [G]{δe}…………………………………….. (15)

Where

$$[G]=\begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 \\ \frac{\partial N_i}{\partial y} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial x} \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{For } i = 1, 2\ldots\ldots\ldots 8$$

The initial stress stiffness matrix is given by

$$[K_{Ge}{}^{r}] = \iint [G]^{T}\,[S][G]dxdy \qquad (16)$$

Where

$$[S]=\begin{bmatrix} S_{11} & & & & & & & & & & & \\ S_{21} & S_{22} & & & & & & & & & & \\ 0 & 0 & S_{33} & & & & & & & & & \\ 0 & 0 & S_{43} & S_{44} & & & & & & & & \\ 0 & 0 & 0 & 0 & S_{55} & & & & & & & \\ 0 & 0 & 0 & 0 & S_{65} & S_{66} & & & & & & \\ 0 & 0 & S_{73} & S_{74} & 0 & 0 & S_{77} & & & & & \\ 0 & 0 & S_{83} & S_{84} & 0 & 0 & S_{87} & S_{88} & & & & \\ S_{91} & S_{92} & 0 & 0 & 0 & 0 & 0 & 0 & S_{99} & & & \\ S_{101} & S_{102} & 0 & 0 & 0 & 0 & 0 & 0 & S_{109} & S_{1010} & & \\ 0 & 0 & S_{113} & S_{114} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ S_{121} & S_{122} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In which

$S_{11} = S_{33} = S_{55} = N_x{}^{i}$ $\qquad S_{22} = S_{44} = S_{66} = N_y{}^{i}$ $\qquad S_{21} = S_{43} = S_{65} = N_{xy}{}^{i}$

$S_{77} = S_{99} = N_x{}^{i}\,t^2/12$ $\qquad S_{88} = S_{1010} = N_y{}^{i}\,t^2/12$ $\qquad S_{877} = S_{109} = N_{xy}{}^{i}\,t^2/12$

$-S_{73} = S_{91} = M_x{}^i$ $\quad -S_{84} = S_{102} = M_y{}^i$ $\quad -S_{74} = -S_{83} = S_{92} = S_{101} = M_{xy}{}^i$

$-S_{113} = S_{121} = Q_x{}^i$ $\quad -S_{114} = S_{122} = Q_y{}^i$

$N_x{}^i, N_y{}^i, N_{xy}{}^i$= in-plane initial internal force resultants per unit length

$M_x{}^i, M_y{}^i, M_{xy}{}^i$ = initial internal moment resultants per unit length

$Q_x{}^i, Q_y{}^i$= initial transverse shear resultants

3.5.4 Geometric stiffness matrix$[\mathbf{K_{Ge}}^{\mathbf{a}}]$

The first three non-linear strains in eqns (11) are represented in a matrix form:

$\{\varepsilon_{xnl}\ \varepsilon_{ynl}\varepsilon_{xynl}\}^T = [U]\{f\}/2$

Where $\{f\} = \{\ \overline{u_x}\ \overline{u_y}\ \overline{v_x}\ \overline{v_y} w_x w_y \theta_{x,x}\theta_{x,y}\theta_{y,x,}\theta_{y,y}\}^T$ and [U] is obvious from the expressions for ε_{xnl} , $\varepsilon_{ynl}, \varepsilon_{xynl}$

Using eqns (12), {f} is expressed

$\{f\} = [H]\{\delta_e\}$

Where

$$[H]=\begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & 0 \\ \frac{\partial N_i}{\partial y} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial x} & 0 & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial x} \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} \end{bmatrix}$$

The geometric stiffness matrix due to applied in-plane loads is given by:

$$[K_{Ge}{}^{r}] = \iint [H]^T [P][H] dxdy \qquad (17)$$

Where

$$[P] = \begin{bmatrix} P_{11} & & & & & & & & & \\ P_{21} & P_{22} & & & & & & & & \\ 0 & 0 & P_{33} & & & & & & & \\ 0 & 0 & P_{43} & P_{44} & & & & & & \\ 0 & 0 & 0 & 0 & P_{55} & & & & & \\ 0 & 0 & 0 & 0 & P_{65} & P_{66} & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{77} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{87} & P_{88} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{99} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{109} & P_{1010} \end{bmatrix}$$

In which,

$P_{11} = P_{33} = P_{55} = N_x{}^a$

$P_{22} = P_{44} = P_{66} = N_y{}^a$

$P_{21} = P_{43} = P_{65} = N_{xy}{}^a$

$P_{77} = P_{99} = N_x{}^a\, t^2/12$

$P_{88} = P_{1010} = N_y{}^a\, t^2/12$

$P_{87} = P_{109} = N_{xy}{}^a\, t^2/12$

3.5.5 Element mass matrix

$$[M_e] = \iint [N]^T [P][N] dxdy \qquad (18)$$

Where the shape function matrix

$$[N]=\sum_{i=1}^{8}\begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}$$

$$[P]=\begin{bmatrix} P_1 & 0 & 0 & P_2 & 0 \\ 0 & P_1 & 0 & 0 & P_2 \\ 0 & 0 & P_1 & 0 & 0 \\ P_2 & 0 & 0 & P_3 & 0 \\ 0 & P_2 & 0 & 0 & P_3 \end{bmatrix}$$

In which

$$(P_1 P_2 P_3) = \sum_{k=1}^{n} \int_{zk-1}^{zk} (\rho)_k (1, z, z^2) dz$$

The element load vector due to external transverse static load q per unit area is given by

$$\{P_e\} = \iint N_i \begin{bmatrix} q \\ 0 \\ 0 \end{bmatrix} dxdy \qquad (19)$$

The element load vector due to hygrothermal forces and moments is given by

$$\{P_e{}^N\} = \iint [B]^T \{F^N\} \, dxdy \qquad (20)$$

3.6 Solution Process

The stiffness matrix, the initial stress stiffness matrix , the mass matrix and the load vectors of the element, given by equations (13) and (16)-(20), are evaluated by first expressing the integrals in local natural co-ordinates, of the element and then performing numerical integration by using Gaussian quadrature. Then the element matrices are assembled to obtain the respective global matrices [K], $[KG^r]$, $[KG^a]$, [M], {P} and $\{P^N\}$.The first part of the solution is to obtain the initial stress resultants induced by the external transverse static load and by moisture and temperature in static conditions.

$$[K]\{\delta^i\} = \{P\} + \{P^N\} \quad (21)$$

Then the initial stress resultants Nx^i , Ny^i ,Nxy^i, Mx^i, My^i, Mxy^i, Qx^i and Qy^i are obtained from equations (10) and (17).

The second part of the solution involves determination of natural frequencies from the condition.

$$[K] + [K_G{}^r] - \omega_n{}^2[K_G{}^a]\{\delta\} = 0$$

Where ω_n = natural frequency

3.7 Computer Program

A computer program is developed by using MATLAB environment to perform all the necessary computations. The element stiffness and mass matrices are derived using a standard procedure. Numerical integration technique by Gaussian quadrature is adopted for the element matrix. Since the stress field is non-uniform, plane stress analysis is carried out using the finite element techniques to determine the stresses and these stresses are used to formulate the geometric stiffness matrix. The overall matrices [K], [Kg], and [M] are obtained by assembling the corresponding element matrices, using skyline technique. The boundary conditions are imposed restraining the generalized displacements in different nodes of the discretized structure.

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Introduction

The present chapter deals with the results of the analysis of the vibration and buckling characteristics of homogeneous and laminated composite twisted cantilever panels subjected to hygrothermal loading using the formulation given in the previous chapter. As explained, the eight-node isoparametric quadratic shell element is used to develop the finite element procedure. The first order shear deformation theory is used to model the twisted panels considering the effects of transverse shear deformation. The vibration and buckling studies are carried out for laminated composite twisted panels subjected to hygrothermal loads to consider the effect of various parameters. The studies in this chapter are presented as follows:

- Convergence study
- Comparison with previous studies
- Numerical results

4.2 Boundary conditions

The clamped (C) boundary condition of the laminated composite twisted panel using the first order shear deformation theory is:

$u = v = w = \theta x = \theta y = 0$ at the left edge.

4.3 Vibration and buckling of twisted panels

- Convergence study
- Comparison with previous results
- Numerical results

4.3.1 Convergence study

The convergence study is done for non-dimensional frequencies of free vibration of cantilever square 4 layer symmetric cross ply and symmetric angle ply laminated composite plates for elevated temperature and moisture conditions for different mesh division as shown in Table 1 and table 2 for three angle of twist(ϕ =0°, 15°, 30°). This mesh is employed throughout free vibration and buckling analysis of laminated composite plates in hygrothermal environment. The study is further extended to buckling analysis of laminated composite twisted cantilever plates subjected to hygrothermal condition as presented in Table 3 and 4 and this mesh is employed throughout free vibration, buckling and dynamic stability analysis of laminated composite plates in hygrothermal environment.

Table 1: Convergence of non-dimensional free vibration frequencies for cantilever twisted plate for different ply orientations at 325K temperature

a/b=1, a/t=25, At T=300K, $E_1=130\text{x}10^9$, $E_2=9.5\text{x}10^9$, $G_{12}=6\text{x}10^9$, $G_{13}=G_{12}$, $G_{23}=0.5G_{12}$

ϑ_{12} = 0.3, $\alpha_1=-0.3\text{x}10^{-6}/^{\circ}K$, $\alpha_2 = 28.1\text{x}10^{-6}/^{\circ}K$

Non-dimensional frequency, $\varpi=\omega_n a^2\sqrt{\rho/E_2t^2}$

Mesh	Non- dimensional frequencies of free vibration for different angles of twist and ply orientation at 325K Temperature					
	0/90/90/0			45/-45/-45/45		
	0°	15°	30°	0°	15°	30°
4x4	0.8485	0.8303	0.7690	0.1596	0.1838	0.1166
8x8	0.8484	0.8302	0.7689	0.1527	0.1782	0.1084
10x10	0.8483	0.8302	0.7689	0.1517	0.1774	0.1072

Table 2: Convergence of non-dimensional free vibration frequencies for cantilever twisted plate for different ply orientations at 0.1% moisture concentration

a/b=1, a/t=25, At C=0.00, E_1=130x10^9, E_2=9.5x10^9, G_{12}=6x10^9, G_{13}=G_{12}, G_{23}=0.5G_{12}

ϑ_{12} =0.3, α_1= -0.3x10^{-6}/^{o} K, α_2 = 28.1x10^{-6}/^{o}K

Non-dimensional frequency, $\varpi=\omega_n a^2\sqrt{\rho/E_2 t^2}$

Mesh	Non- dimensional frequencies of free vibration for different angles of twist and ply orientation					
	0/90/90/0			45/-45/-45/45		
	0^o	15^o	30^o	0^o	15^o	30^o
4x4	0.8779	0.8612	0.8053	0.3045	0.3158	0.2869
8x8	0.8777	0.8611	0.8052	0.3007	0.3124	0.2835
10x10	0.8777	0.8611	0.8052	0.3001	0.3118	0.2830

Table 3: Convergence of non-dimensional critical loads for cantilever twisted plate for different ply orientations at 325K temperature

a/b=1, a/t=25, At T=300K, E_1=130x10^9, E_2=9.5x10^9, G_{12}=6x10^9, G_{13}=G_{12}, G_{23}=0.5G_{12}

ϑ_{12}=0.3, α_1= -0.3x10^{-6}/^{o} K, α_2 = 28.1x10^{-6}/^{o}K

Non-dimensional buckling load λ= $(N_x b^2)/E_2 h^3$

Mesh	Non- dimensional critical loads for different angles of twist and ply orientation at 325K Temperature					
	0/90/90/0			30/-30/-30/30		
	0^o	15^o	30^o	0^o	15^o	30^o
4x4	1.9561	1.8305	1.4589	0.7879	0.9158	0.8196
8x8	1.9558	1.8303	1.4587	0.7803	0.9099	0.8148
10x10	1.9558	1.8303	1.4586	0.7790	0.9090	0.8141

Table 4: Convergence of non-dimensional critical loads for cantilever twisted plate for different ply orientations at 0.1% moisture concentration

a/b=1, a/t=25, At C=0.00, E_1=130x10^9, E_2=9.5x10^9, G_{12}=6x10^9, G_{13}=G_{12}, G_{23}=0.5G_{12}

ϑ_{12} = 0.3, α_1 = -0.3x10^{-6}/ᴼ K, α_2 = 28.1x10^{-6}/ᴼK

Non-dimensional buckling load $\lambda= (N_x b^2)/E_2 h^3$

Mesh	Non- dimensional critical loads for different angles of twist and ply orientation					
	0/90/90/0			30/-30/-30/30		
	0ᴼ	15ᴼ	30ᴼ	0ᴼ	15ᴼ	30ᴼ
4x4	2.1049	1.9793	1.6077	0.8871	0.9886	0.8843
8x8	2.1046	1.9791	1.6075	0.8800	0.9828	0.8794
10x10	2.1046	1.9791	1.6075	0.8789	0.9820	0.8788

4.3.2 Comparison with previous studies

After the convergence study, the accuracy and efficiency of the present formulation are established through comparison with previous studies. The results obtained by this formulation are compared with the analytical, independent finite element and/or experimental results published by other investigators wherever possible for variety of problems on twisted plates and shells. Unless otherwise mentioned the mesh division used in this present analysis is 8x8 considering the whole plate/shell for almost all the geometrical configurations, based on convergence studies.

4.3.2.1 Vibration of composite plates and shells subjected to hygrothermal environment

The present finite element formulation is validated for free vibration analysis of laminated composite twisted panels as shown in table 5. The non dimensional frequency calculated by the present formulation is compared with non-dimensional free vibration frequencies for cantilever twisted plates for different ply orientations presented by Qatu and Leissa (1991) and He *et al.* (2000) in Table 5. This shows good agreement between the present study and those of Qatu and Leissa (1991) and He *et al.* (2000).

Table 5: Comparison of non-dimensional free vibration frequencies for cantilever twisted plates for different ply orientations

a/b=1, E_1=138x10^9, E_2=8.96x10^9, G_{12}=7.1x10^9, G_{13}=G_{12}= G_{23}

ϑ_{12} =0.3, α_1= -0.3x10^{-6}/° K, α_2 = 28.1x10^{-6}/°K, ϕ=15°

Non-dimensional frequency, $\varpi=\omega_n a^2\sqrt{\rho/E_2 t^2}$

Ply orient -ation θ	non-dimensional free vibration frequencies for cantilever twisted plates for different ply orientations					
	b/h=100			b/h=20		
	He *et al.*(2000)	Qatu and Leissa (1991)	Present study	He *et al.*(2000)	Qatu and Leissa (1991)	Present study
0	1.0034	1.0035	1.0029	1.0031	1.0031	0.9889
15	0.92938	0.9296	0.9285	0.89791	0.8981	0.8841
30	0.74573	0.7465	0.7449	0.68926	0.6899	0.6789
45	0.52724	0.5286	0.5270	0.47810	0.4790	0.4716
60	0.35344	0.3545	0.3539	0.33374	0.3343	0.3310
75	0.27208	0.2723	0.2724	0.26934	0.2696	0.2687
90	0.25544	0.2555	0.2554	0.25540	0.2554	0.2550

4.3.2.2 Buckling of composite plates and shells subjected to hygrothermal environment

The present finite element formulation is validated for buckling analysis of laminated composite twisted panels as shown in table 6. The non dimensional buckling load calculated by the present formulation is compared with non-dimensional buckling load for cantilever twisted plates for different ply orientations and different angle of twist presented by Sahu *et al.* (2005) in Table 6. This shows good agreement between the present studies.

Table 6: Comparison of non-dimensional buckling load for cantilever twisted plates for different ply orientations and angle of twist

a/b=1, b/t=100, b/R_y=0.25 $E_1=138\times10^9$, $E_2=8.96\times10^9$, $G_{12}=7.1\times10^9$, $G_{13}=G_{12}=G_{23}$

ϑ_{12} =0.3, $\alpha_1 = -0.3\times10^{-6}/^{o}$ K, $\alpha_2 = 28.1\times10^{-6}/^{o}$K

Non-dimensional buckling load $\lambda= (N_{xx}b^2)/E_2h^3$

Angle of twist	Ply orientation	Sahu *et al.* (2005)	Present study
0º	0	7.826	7.8231
	15	6.525	6.5229
	30	5.884	5.8825
	45	4.858	4.8621
	60	3.222	3.2255
	75	2.366	2.3664
	90	2.122	2.1223
30º	0	2.775	2.7755
	15	2.828	2.8258
	30	2.135	2.1373
	45	1.235	1.2372
	60	0.589	0.5899
	75	0.299	0.300
	90	0.215	0.2151

4.3.3 Numerical Results

After obtaining the convergence study and validating the formulation with the existing literature, the results for vibration and buckling studies for the twisted plate subjected to temperature and moisture are presented. The material properties used for the numerical study:

At T=300K, $E_1=130\times10^9$, $E_2=9.5\times10^9$, $G_{12}=6\times10^9$, $G_{13}=G_{12}$, $G_{23}=0.5G_{12}$

ϑ_{12}=0.3, $\alpha_1 = -0.3\times10^{-6}/^{o}$ K, $\alpha_2 = 28.1\times10^{-6}/^{o}$K, β_1=0, β_2=0.44

4.3.3.1 Vibration Results for plate

The variation of non-dimensional fundamental frequency of vibration of different pre-twisted cantilever plates for angle-ply laminates are shown in the figure 4. With increase in temperature the frequency decreases for different twisted cantilever plates. As the temperature increases from 300K to 350K the decrease in non dimensional frequency for ply orientations 0⁰ and 15⁰ is about 5.07% and 11.25% respectively while the decrease in frequency is more for the ply orientation 30⁰ which is about 18.25%.

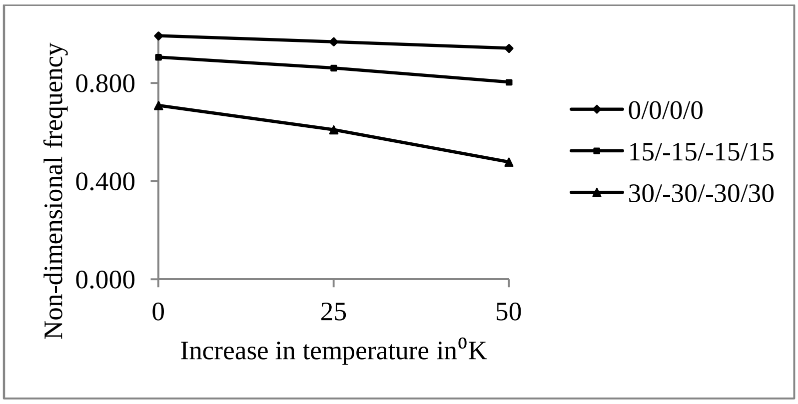

Fig 4: Effect of temperature on non-dimensional frequency for ϕ =15⁰ and for angle-ply laminated pre twisted cantilever plates (a /b=1, b/t=25,)

The variation of non-dimensional frequency of vibration of pre twisted cantilever laminated symmetric angle ply plates with moisture is presented in figure 5. As the moisture concentration increases from 0% to 0.3% the decrease in non-dimensional frequency is decreased about 9.93% for 0⁰ and about16.28% for 15⁰ ply orientations while for 30⁰ ply orientations the % decrease in the non dimensional frequency due to increase in moisture concentration is about 41.88%.

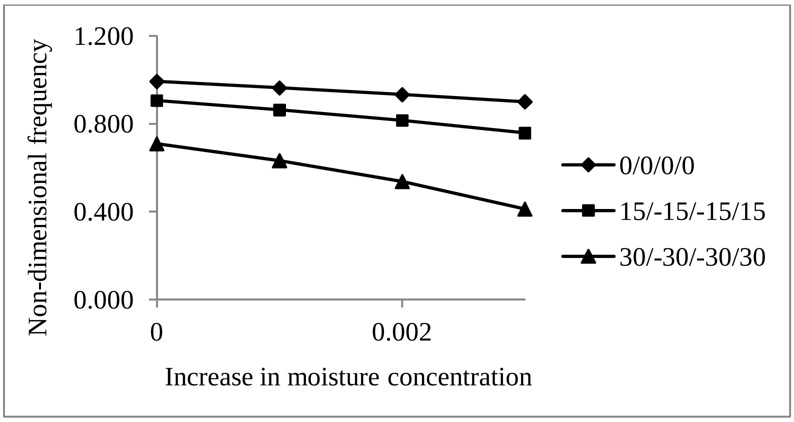

Fig 5: Effect of moisture on non-dimensional frequency for ϕ =15⁰ and for angle-ply laminated pre twisted cantilever plates (a /b=1, b/t=25,)

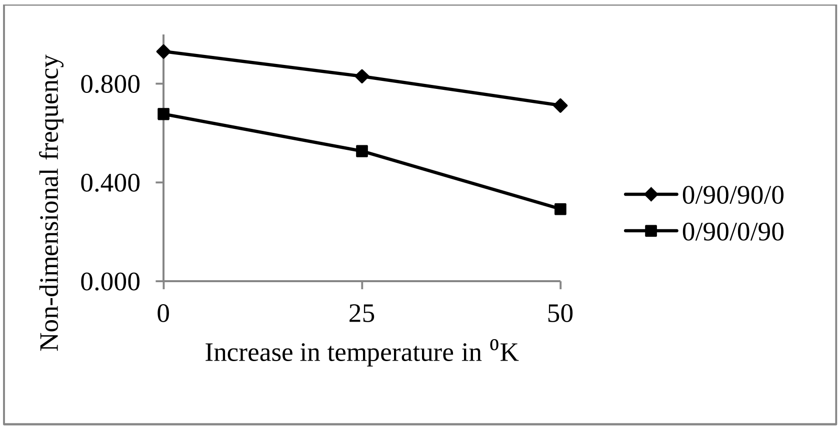

Fig 6: Effect of temperature on non-dimensional frequency for ϕ =15⁰ and for cross-ply laminated pre twisted cantilever plates (a /b=1, b/t=25,)

The variation of non-dimensional frequency of vibration of pre twisted cantilever laminated symmetric and anti symmetric cross ply plates with temperature is presented in figure 6. In case of cross-ply plates decrease in frequency for un-symmetric cross-ply plates is more than symmetric cross-ply plates. Decrease in frequency for un-symmetric cross-ply plate is more with increase in temperature than other angle-ply and cross-ply plates. The percentage decrease is about 56.93% for un symmetric cantilever cross-ply laminates.

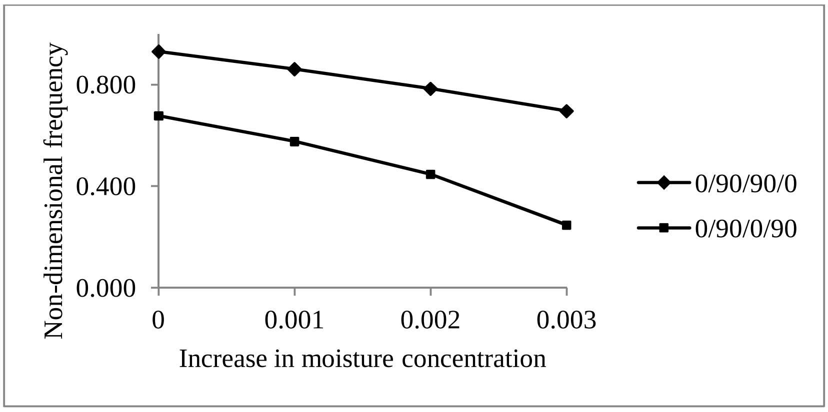

Fig 7: Effect of moisture on non-dimensional frequency for Φ =15⁰ and for cross ply laminated pre twisted cantilever plates (a /b=1, b/t=25,)

The variation of non-dimensional frequency of vibration of pre twisted cantilever laminated symmetric and anti symmetric cross ply plates with moisture is presented in figure 7. The percentage decrease is about 63.66% for un symmetric cantilever cross-ply laminates. The percentage decrease in non dimensional frequency for symmetric cross ply cantilever laminates is about 25.24%.

4.3.3.2 Effect of angle of twists on vibration of pre twisted cantilever plates

The variation of non dimensional frequency with increase in temperature for symmetric angle ply laminated twisted cantilever plate (15/-15/-15/15) for different angles of twist (ϕ) is shown in figure 8.When the temperature increases from 300K to 325K the decrease in frequency for angle of twist 0^{o}, is about 11.08%, when the temperature increases to 350K the decrease in frequency for angle of twist 0^{o} is about 74.83%. For twist angles 15^{o} and 30^{o} when the temperature rises to 350K the percentage decrease in non dimensional frequency is 11.25% and 10.6% respectively. For twist angle 45^{o} the percentage decrease in non-dimensional frequency due to increase in temperature is 14.72%.Which shows the % decrease in non dimensional frequency gradually increases when the temperature increases to 325K but when the temperature rises to 350K the decrease in frequency is more for the angle of twist 0^{o}.

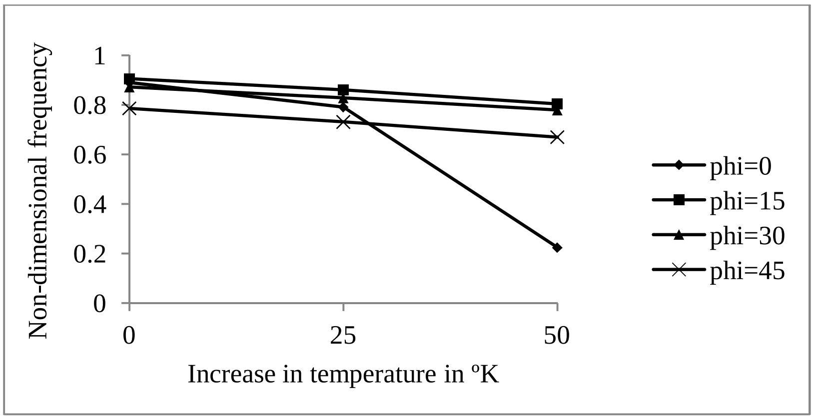

Fig 8: Effect of temperature on non-dimensional frequency for laminated angle ply pre twisted cantilever plate (15/-15/-15/15) for different angles of twist (a/b=1, b/t=25)

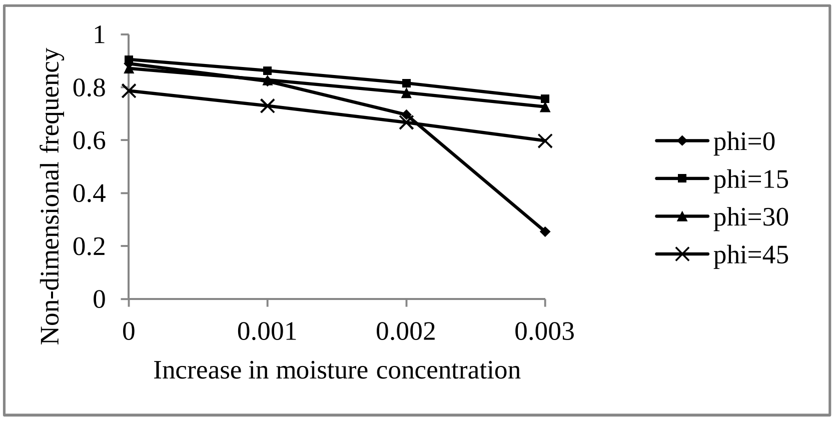

Fig 9: Effect of moisture on non-dimensional frequency for laminated angle ply pre twisted cantilever plate (15/-15/-15/15) for different angle of twist (a/b=1, b/t=25)

The variation of non dimensional frequency with increase in moisture for symmetric angle ply laminated twisted cantilever plate (15/-15/-15/15) for different angles of twist (ϕ) is shown in figure 9.When the moisture increases from 0% to 0.2% the decrease in frequency for angle of twist 0^{o} is about 28.92%, when the moisture increases to 0.3% the decrease in frequency for angle of twist 0^{o} is about 71.42%. For twist angles 15^{o} and 30^{o} when the moisture rises to 0.3% the percentage decrease in non dimensional frequency is 16.29% and 16.67% respectively. For twist angle 45^{o} the percentage decrease in non-dimensional frequency due to increase in moisture is 24.02%.Which shows the % decrease in non dimensional frequency gradually increases when the moisture increases to 0.2% but when the moisture rises to 0.3% the decrease in frequency is more for the angle of twist 0^{o}.

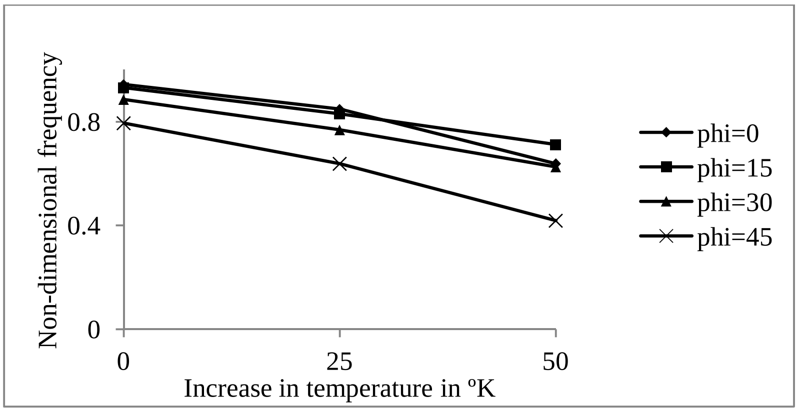

Fig 10: Effect of temperature on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/90/0) for different angle of twist (a/b=1, b/t=25)

The variation of non dimensional frequency with increase in temperature for symmetric cross ply laminated twisted cantilever plate (0/90/90/0) for different angles of twist (ϕ) is shown in figure 10. When the temperature increases from 300K to 325K the decrease in frequency for angle of twist 0° and 15° is almost same about 10.12% and 10.84% respectively, when the temperature increases to 350K the decrease in frequency for angle of twist 0° is more than the decrease for twist angle 15°. When the twist angle is 30° the percentage decrease in non-dimensional frequency is 29.32%, for twist angle 45° the percentage decrease in non-dimensional frequency due to increase in temperature is 47.30%.Which shows the % decrease in non dimensional frequency gradually increases when the temperature increases, as the angle of twist varies from 0° to 45°.

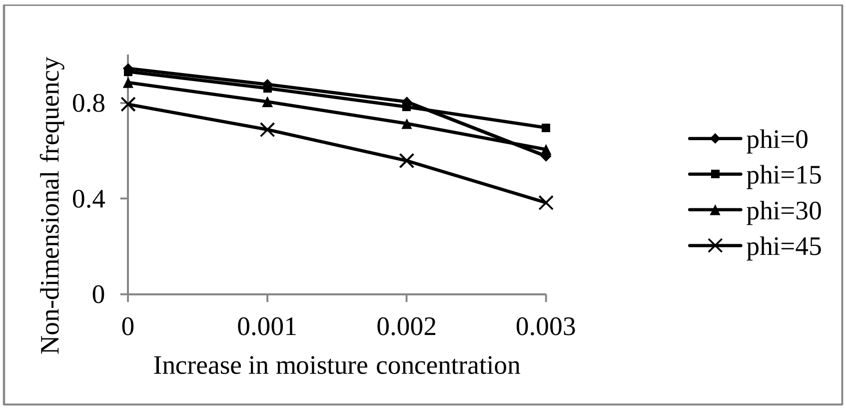

Fig 11: Effect of moisture on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/90/0) for different angles of twist (a/b=1, b/t=25)

The variation of non dimensional frequency with increase in moisture for symmetric cross ply twisted cantilever plate (0/90/90/0) for different angles of twist (ϕ) is shown in figure 11. When the moisture increases from 0% to 0.2% the decrease in frequency for angle of twist 0^o and 15^o is almost similar about 7.02% and 7.51% respectively, when the moisture increases to 0.3% the decrease in frequency for angle of twist 0^o is more than the decrease for twist angle 15^o. When the twist angle is 30^o the percentage decrease in non-dimensional frequency is 31.61%, for twist angle 45^o the percentage decrease in non-dimensional frequency due to increase in moisture is 51.87%. This shows the % decrease in non dimensional frequency gradually increases when the moisture increases, as the angle of twist varies from 0° to 45°.

The effect of temperature on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90) for different angle of twist (ϕ) is shown in figure 12. As the temperature increases from 300K to 340K the decrease in non dimensional frequency for

angle of twist 0^o is about 37.36%. As the temperature increases from 300K to 350K the decrease in non dimensional frequency for angle of twist 15^o and 30^o is 56.87% and 85.5% respectively. This result shows that antisymmetric cross-ply plates are more unstable towards the increase in angle of twist than symmetric cross-ply plates.

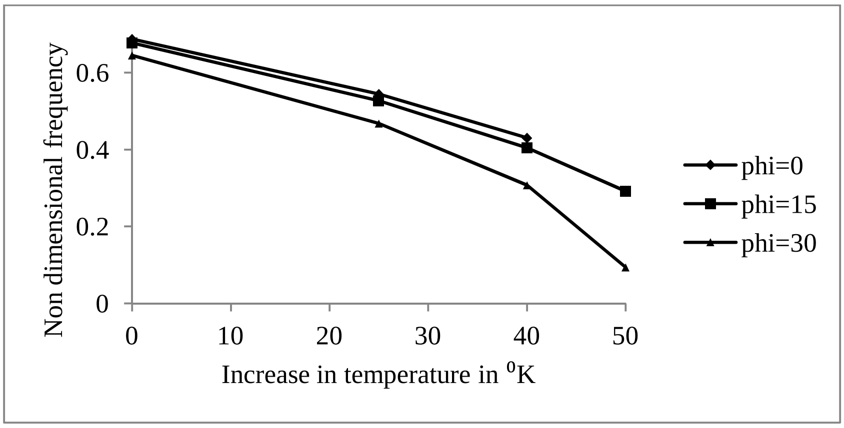

Fig 12: Effect of temperature on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90) for different angle of twist (a/b=1, b/t=25)

The effect of moisture on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90) for different angles of twist (ϕ) is shown in figure 13. The decrease in non dimensional frequency is almost similar for angle of twist 0° and 15° which is about 31.76% and 34.71% respectively. The decrease in non dimensional frequency for angle of twist 30° is about 43.47% and for twist angle 45° the decrease in frequency is 85.75%.

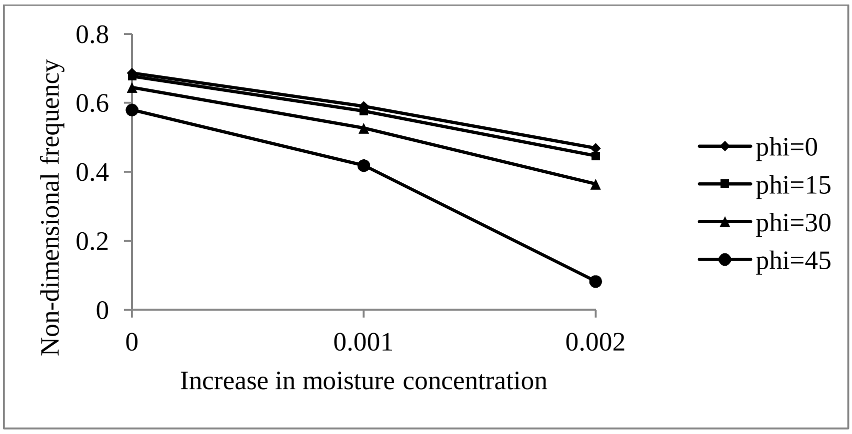

Fig 13: Effect of moisture on non-dimensional frequency for laminated cross ply pre twisted cantilever plate (0/90/0/90) for different angles of twist (a/b=1, b/t=25)

4.3.3.3 Vibration results for shells

The change in frequency of different angle ply twisted cantilever cylindrical shells (b/R_y = 0.2) while increasing the temperature is shown in figure 14.While the temperature increases from 300K to 350K the decrease in non dimensional frequency is almost similar for 0° and 15° ply orientations, for 30° ply orientation the % decrease in frequency is 20.15%.The variation is more for 45° ply orientation and the frequency is almost equal to zero at 340K temperature. The decrease in frequency is about 93.5% for 45/-45/-45/45 ply orientation.

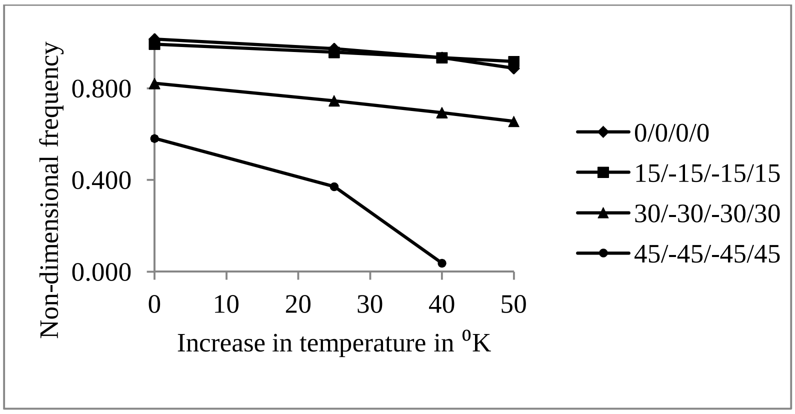

Fig 14: Effect of temperature on non-dimensional frequency for Φ =15⁰ and for laminated composite pre twisted cantilever cylindrical angle ply panels (a /b=1, b/t=25,b/R_y=0.2)

The change in frequency of different angle ply twisted cantilever cylindrical shells (b/R_y=.2) while increasing moisture concentration is shown in figure 15. While there is a increase in moisture from 0% to 0.3% the decrease in the frequency is almost same for 0º and 15º ply orientations and the decrease in frequency for 45º ply orientation is 63.47% when the moisture concentration increases from 0% to 0.2%. The non-dimensional frequency is zero between the increase in moisture concentration from 0.2% to 0.3%. So the 45/-45/-45/45 ply orientation is more vulnerable to increase in temperature and moisture in case of twisted angle-ply cantilever panels.

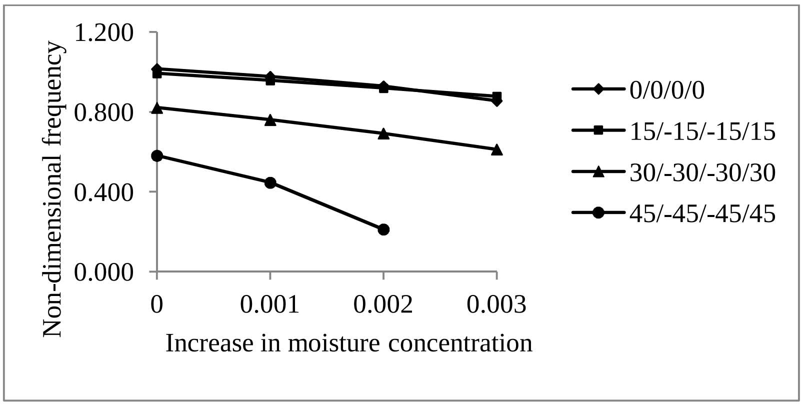

Fig 15: Effect of moisture on non-dimensional frequency for ϕ =15⁰ and for laminated composite pre twisted cantilever cylindrical angle ply panels (a /b=1, b/t=25,b/R_y=0.2)

The change in frequency of different cross ply twisted cantilever cylindrical shells (b/R_y=0.2) while increasing the temperature is shown in figure 16. While the temperature increases from

300K to 350K the percentage decrease in non dimensional frequency 24.73% for symmetric cross-ply laminates and for anti-symmetric cross-ply panels the decrease in frequency is 58.01%.

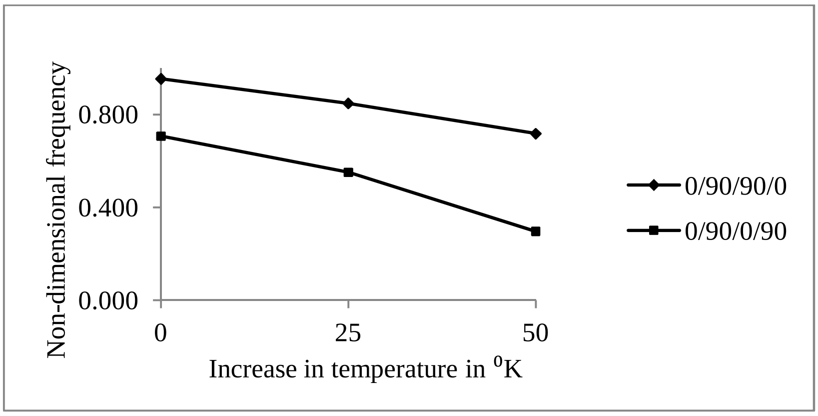

Fig 16: Effect of temperature on non-dimensional frequency for ϕ =15⁰ and for laminated composite pre twisted cantilever cylindrical cross ply panels (a /b=1, b/t=25,b/R_y=0.2)

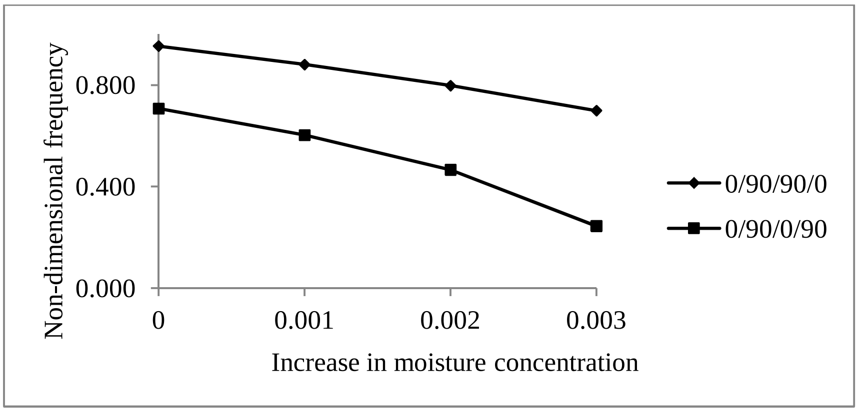

Fig 17: Effect of moisture on non-dimensional frequency for ϕ =15⁰ and for laminated composite pre twisted cantilever cylindrical cross ply panels (a /b=1, b/t=25, b/R_y=0.2)

The change in frequency of different cross ply twisted cantilever cylindrical shells (b/R_y=.2) while increasing moisture concentration is shown in figure 17. For cross-ply symmetric laminates the percentage decrease in non dimensional frequency 26.67%, for anti-symmetric

ross-ply panels the decrease in frequency is 65.40% when there is increase in moisture concentration from 0% to 0.3%. In figure 18, the variation in frequency with increase in temperature for the spherical cantilever twisted angle ply shells are shown. While the temperature increases from 300K to 325K the decrease in frequency for 0⁰ and 15⁰ ply orientations are almost similar about 3.92% and 4.02% respectively but when the temperature rises to 350K the decrease in frequency is more in 0^o than 15^o ply-orientation, which is almost same as frequency of 30^o at 350K. The percentage decrease in frequency for 30/-30/-30/30 spherical shells is 24.12%.

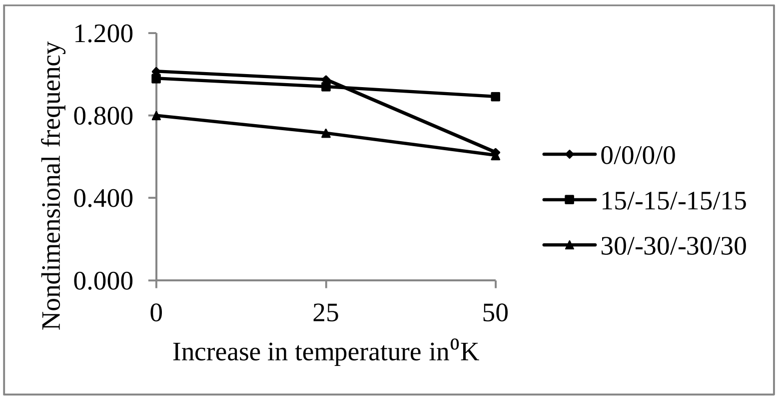

Fig 18: Effect of temperature on non-dimensional frequency for ϕ =15^o and for laminated composite pre twisted cantilever spherical angle ply panels (a =1, b/t=25, b/R_y=0.2, R_x=R_y)

In figure 19 the variation in frequency with increase in moisture concentration for the spherical cantilever twisted angle ply panels are shown. While the moisture increases from 0% to 0.2% the decrease in frequency for 0^o and 15^o ply orientations are almost similar about 8.23% and 8.15% respectively but when the moisture rises to 0.3% the decrease in frequency is more in 0^o than 15^o ply-orientations.

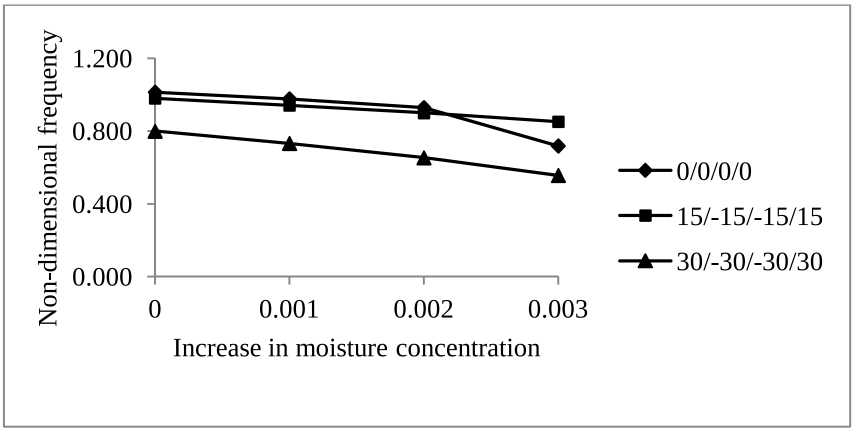

Fig 19: Effect of moisture on non-dimensional frequency for ϕ =15^o and for laminated composite pre twisted cantilever spherical angle ply panels (a /b=1, b/t=25, b/R_y=0.2,R_x=R_y)

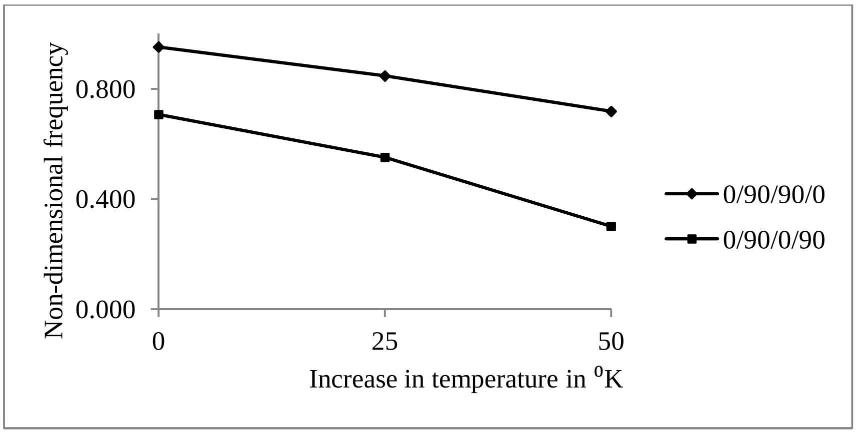

Fig 20: Effect of temperature on non-dimensional frequency for Φ =15⁰ and for laminated composite pre twisted cantilever spherical cross ply panels (a /b=1, b/t=25, b/R_y=0.2, R_x=R_y)

Effect of temperature on non-dimensional frequency for laminated composite pre twisted cantilever spherical cross ply panels is shown in figure 20. For cross-ply laminates the decrease in non-dimensional frequency is almost similar for both symmetric and anti-symmetric laminates, where the percentage decrease in frequency for symmetric cross-ply panels is 24.57% and for anti-symmetric laminates the decrease is 57.13%. Anti-symmetric cross-ply panels are more vulnerable towards the temperature increase. Effect of moisture on non-dimensional frequency for laminated composite pre twisted cantilever spherical cross ply panels are shown in figure 21. For symmetric cross-ply twisted spherical panels the initial frequency is almost same as 0⁰ and 15 ⁰ angle ply laminates. When the moisture increases 0.3% the decrease in frequency is same as 0⁰ angle-ply laminate which is about 26.5%. For anti-symmetric cross-ply panels the decrease in frequency for increase in moisture concentration 0% to 0.3% is about 64.76%.

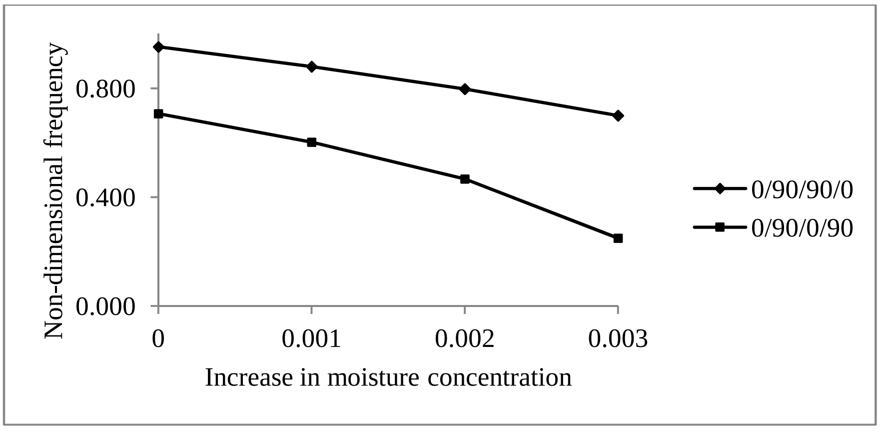

Fig 21: Effect of moisture on non-dimensional frequency for ϕ =15⁰ and for laminated composite pre twisted cantilever spherical cross ply panels (a /b=1, b/t=25, b/R_y=0.2, R_x=R_y)

4.3.3.4 Buckling results for plates

The variation of non dimensional critical load due to increase in temperature and moisture for laminated composite twisted cantilever cross ply and angle ply plates is presented here.

The effect of temperature on non dimensional critical load for angle ply laminated pre twisted cantilever plates is presented in figure 22. In this figure it is shown that as the temperature increases there is a decrease in the non dimensional buckling load for different angle ply plates. As the temperature increases the percentage decrease in non dimensional frequency is 10.81% for 0/0/0/0 ply orientation, 24.18% for 15/-15/-15/15 ply orientation and 57.58% for 30/-30/-30/30 ply orientation. This shows as the ply orientation increases the non dimensional buckling load decreases with increase in temperature for angle ply plates.

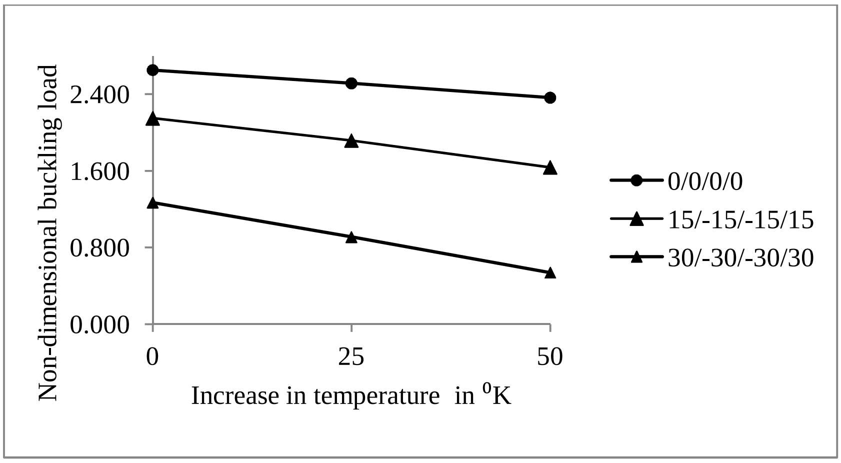

Fig 22: Effect of temperature on non-dimensional critical load for ϕ =15⁰ and for angle-ply laminated pre twisted cantilever plates (a /b=1, b/t=25,)

The effect of moisture on non dimensional critical load for angle ply laminated pre twisted cantilever plates is presented in figure 23. It is observed from the figure that as the moisture concentration increases there is a decrease in the non dimensional buckling load for different angle ply plates. As the moisture concentration increases the percentage decrease in non

dimensional frequency is 19.18% for 0/0/0/0 ply orientation, 32.74% for 15/-15/-15/15 ply orientation and 68.82% for 30/-30/-30/30 ply orientation. This shows as the ply orientation increases the non dimensional buckling load decreases with increase in moisture for angle ply plates.

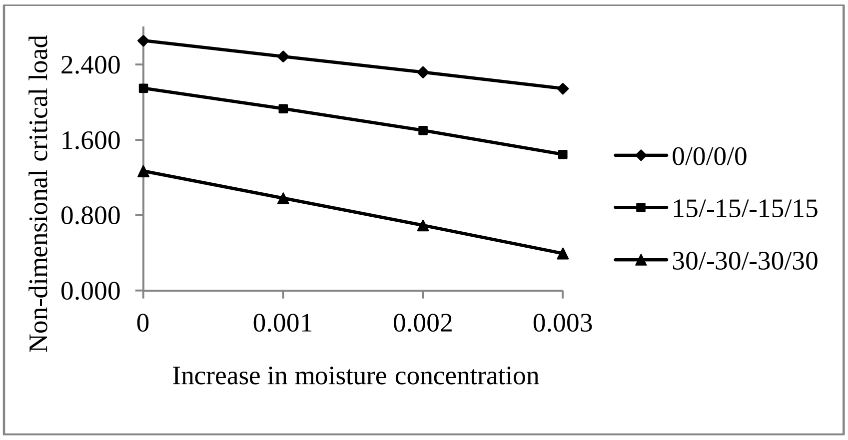

Fig 23: Effect of moisture on non-dimensional critical load for ϕ =15⁰ and for angle-ply laminated pre twisted cantilever plates (a /b=1, b/t=25,)

The variation of critical load with increase in temperature for cross ply laminated pre twisted cantilever plates is presented in figure 24. With increase in temperature the percentage decrease in non dimensional buckling load for symmetric and anti symmetric cross ply laminates are 43.52% and 82.53% respectively. This shows anti symmetric cross ply laminates are more vulnerable towards increase in temperature.

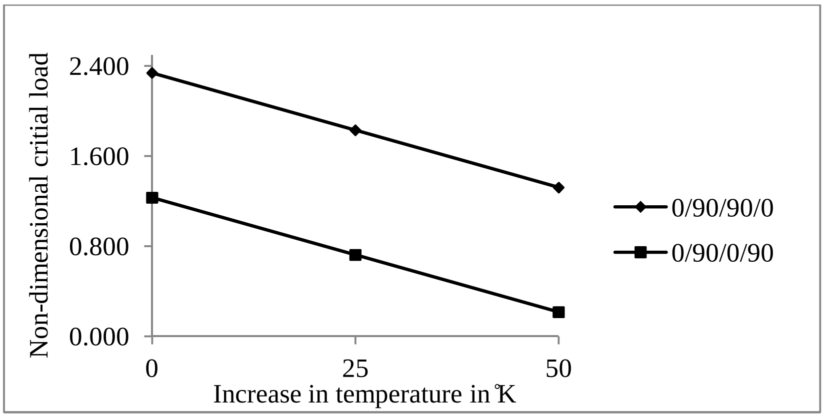

Fig 24: Effect of temperature on non-dimensional critical load for ϕ =15⁰ and for cross-ply laminated pre twisted cantilever plates (a /b=1, b/t=25,)

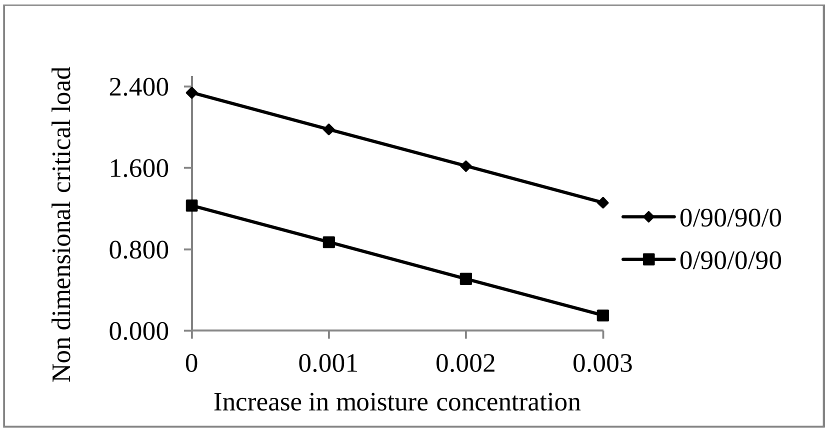

Fig 25: Effect of moisture on non-dimensional critical load for ϕ=15⁰ and for cross-ply laminated pre twisted cantilever plates (a /b=1, b/t=25,)

The variation of critical load with increase in moisture for cross ply laminated pre twisted cantilever plates is presented in figure 25. With increase in moisture concentration the percentage decrease in non dimensional buckling load for symmetric and anti symmetric cross ply laminates are 46.17% and 87.65% respectively. This shows anti symmetric cross ply laminates are more vulnerable towards increase in moisture concentration.

4.3.3.5 Effect of angle of twists on buckling of pre twisted cantilever plates

Figure 26 shows the variation of non dimensional buckling load with increase in temperature for angle ply 15/-15/-15/15 for different angles of twists (ϕ). The decrease in non dimensional buckling load for twist angle 0° while the temperature increases from 300K to 350 K is about 94.51%. This shows there is a sharp decrease in frequency for the plate when the temperature rises from 300K to 350K. The decrease in non dimensional buckling load for twist angle 15°, 30° and 45° while the temperature increases from 300K to 350 K is about 23.84%, 19.94% and 29.20% respectively.

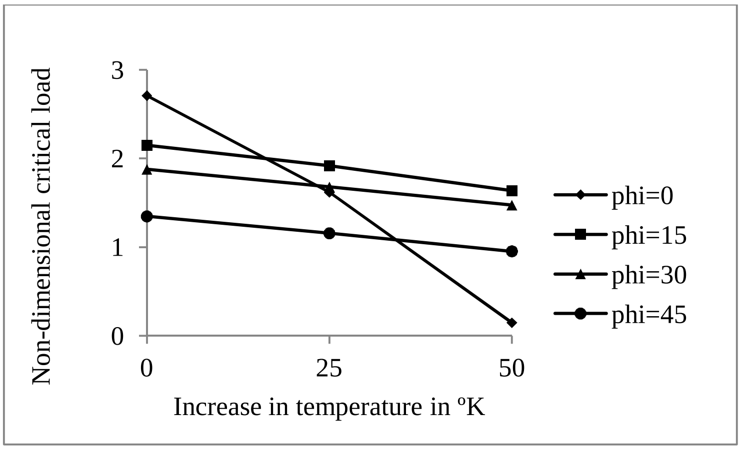

Fig 26: Effect of temperature on non-dimensional buckling load for laminated angle ply pre twisted cantilever plate (15/-15/-15/15) for different angles of twist (a/b=1, b/t=25)

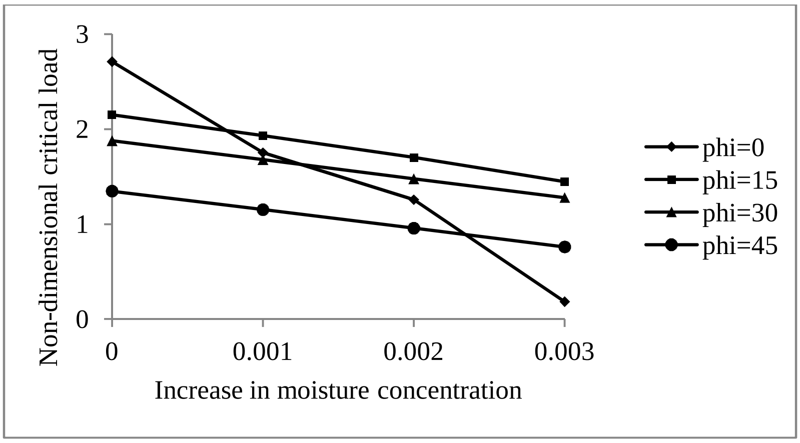

Fig 27: Effect of moisture on non-dimensional buckling load for laminated angle ply pre twisted cantilever plate (15/-15/-15/15) for different angles of twist (a/b=1, b/t=25)

Figure 27 shows the variation of non dimensional buckling load with increase in moisture concentration for angle ply 15/-15/-15/15 for different angles of twists (ϕ). The decrease in non dimensional buckling load for twist angle 0^o while the moisture concentration increases from 0.1% to 0.3% is about 93.21%. This shows there is a sharp decrease in critical load for the plate when the temperature rises from 0.1% to 0.3%. The decrease in non dimensional critical load for twist angle 15^o, 30^o and 45^o while the moisture concentration increases from 0.1% to 0.3%is about 32.71%, 31.90% and 43.57% respectively. Figure 28 shows the variation of non dimensional buckling load with increase in temperature for angle ply 0/90/90/0 for different angles of twists (ϕ). The decrease in non dimensional buckling load for twist angle 0^o while the temperature increases from 300K to 350 K is about 48.50%. The decrease in non dimensional critical load for twist angle 15^o, 30^o and 45^o while the temperature increases from 300K to 350 K is about 43.51%, 51.73% and 73.22% respectively. This shows the decrease in buckling increases with increase in angle of twist.

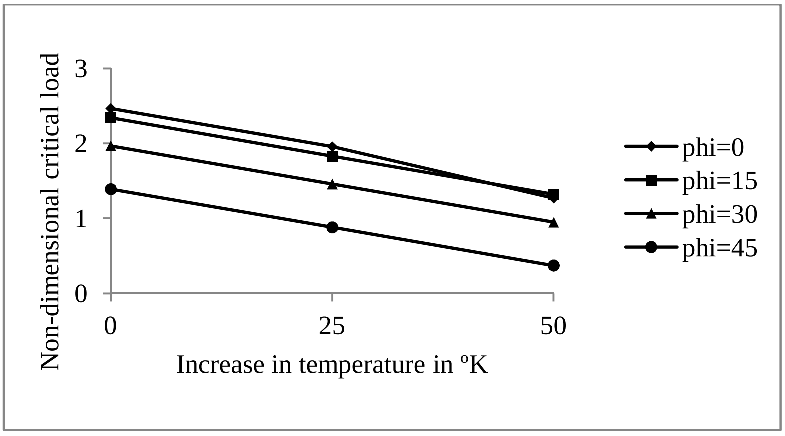

Fig 28: Effect of temperature on non-dimensional buckling load for laminated cross ply pre twisted cantilever plate (0/90/90/0) for different angles of twist (a/b=1, b/t=25)

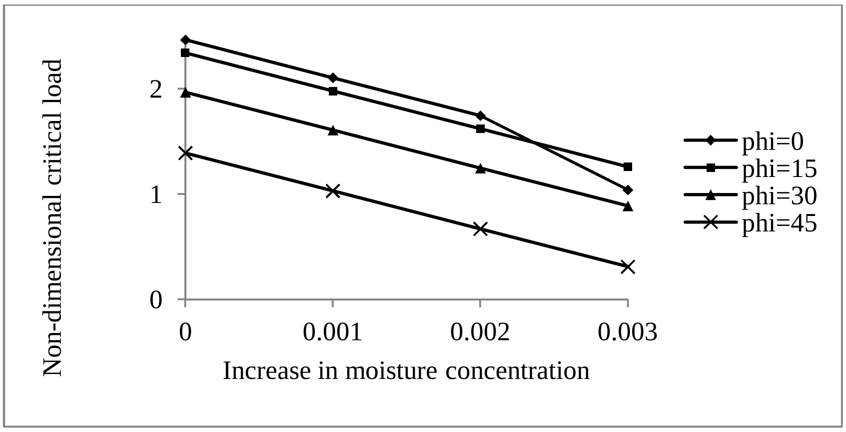

Fig 29: Effect of moisture on non-dimensional buckling load for laminated cross ply pre twisted cantilever plate (0/90/90/0) for different angles of twist (a/b=1, b/t=25)

Figure 29 shows the variation of non dimensional buckling load with increase in moisture concentration for angle ply 0/90/90/0 for different angles of twists (ϕ). The decrease in non dimensional buckling load for twist angle 0° while the moisture concentration increases from 0.1% to 0.3% is about 57.83%. The decrease in non dimensional critical load for twist angles 15°, 30° and 45°, while the moisture concentration increases from 0.1% to 0.3% is about 46.18%, 54.9% and 77.71% respectively. Figure 30 shows the variation of non dimensional buckling load with increase in temperature for angle ply 0/90/0/90 for different angles of twists (ϕ). The decrease in non dimensional buckling load for twist angle 0° while the temperature increases from 300K to 340 K is about 62.77%. The decrease in non dimensional critical load for twist angle 15° and 30° while the temperature increases from 300K to 350 K is about 82.57% and 98.02% respectively. This shows the decrease in buckling increases with increase in angle of twist.

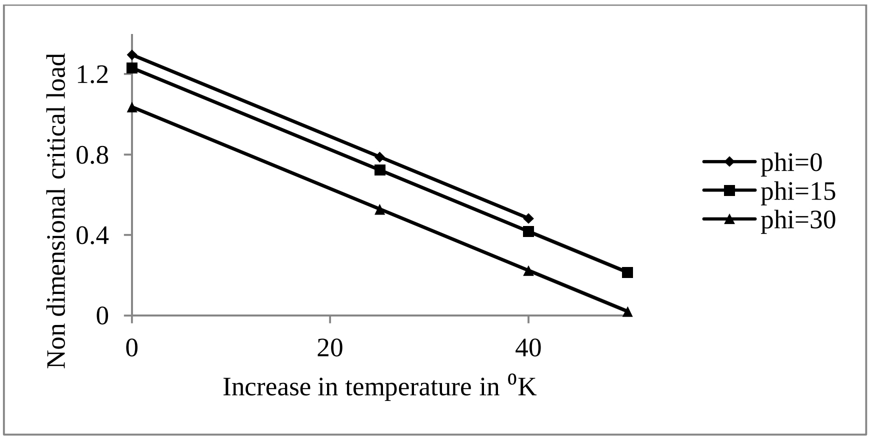

Fig 30: Effect of temperature on non-dimensional buckling load for laminated cross ply pre twisted cantilever plate (0/90/0/90) for different angles of twist (a/b=1, b/t=25)

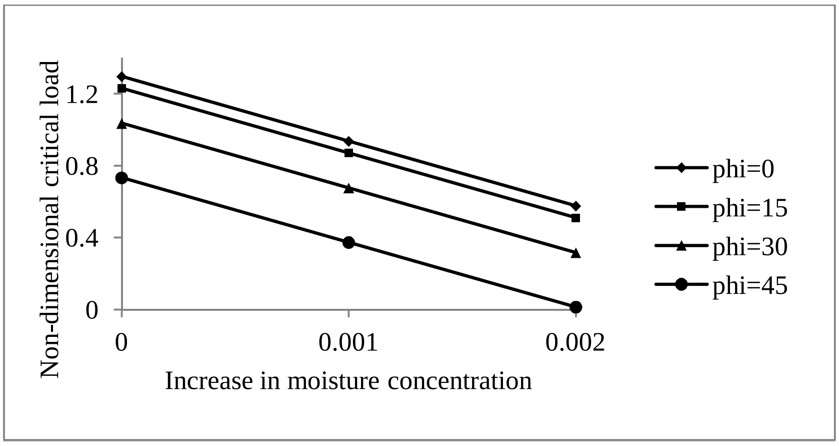

Fig 31: Effect of moisture on non-dimensional buckling load for laminated cross ply pre twisted cantilever plate (0/90/0/90) for different angles of twist (a/b=1, b/t=25)

Figure 31 shows the variation of non dimensional buckling load with increase in moisture concentration for angle ply 0/90/0/90 for different angles of twists (ϕ). The decrease in non dimensional buckling load for twist angle 0^{o} while the moisture concentration increases from 0.1% to 0.3% is about 55.51%. The decrease in non dimensional critical load for twist angle 15^{o}, 30^{o} and 45^{o} while the moisture concentration increases from 0.1% to 0.3%is about 58.44%, 69.38% and 98.06% respectively. This shows the non dimensional buckling load almost decreases to zero for angle of twist 45^{o}.

4.3.3.6 Buckling result for shells

The variation of non dimensional critical load due to increase in temperature and moisture for laminated composite twisted cantilever cross ply and angle ply cylindrical and spherical shells are presented here.

The effect of temperature on non-dimensional buckling load for laminated composite pre twisted cantilever cylindrical angle ply panels is shown in figure 32. The decrease in non dimensional buckling load due to increase in temperature from 300K to 350K for 0^o,15^o and 30^o are 22.94%,15.55% and 39.8% while for 45^o ply orientation the decrease in non

dimensional buckling load is almost 100% when the temperature is 340K and it reduces to almost zero.

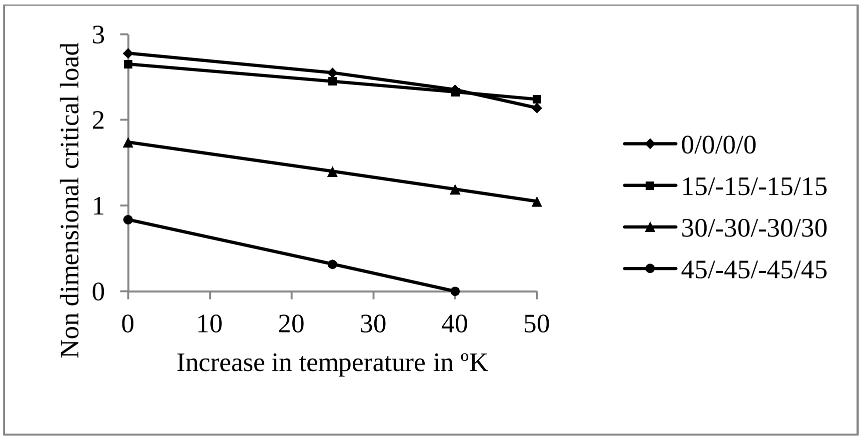

Fig 32: Effect of temperature on non-dimensional buckling load for ϕ =15ᵒ and for laminated composite pre twisted cantilever cylindrical angle ply panels (a /b=1, b/t=25, b/R_y=0.2)

The effect of moisture on non-dimensional buckling load for laminated composite pre twisted cantilever cylindrical angle ply panels is shown in figure 33. The decrease in non dimensional buckling load due to increase in moisture concentration from 0%to 0.3% for 0^o,15^o and

30^o are 22.13%, 19.15% and 39.77% while for 45^o ply orientation the decrease in non dimensional buckling load is almost 88.06% when the increase in moisture concentration is 0.2%.

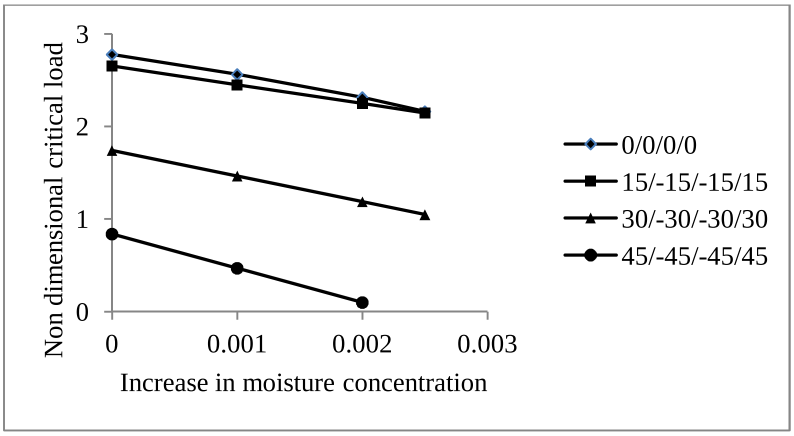

Fig 33: Effect of moisture on non-dimensional buckling load for ϕ =15⁰ and for laminated composite pre twisted cantilever cylindrical angle ply panels (a /b=1, b/t=25, b/R_y=0.2)

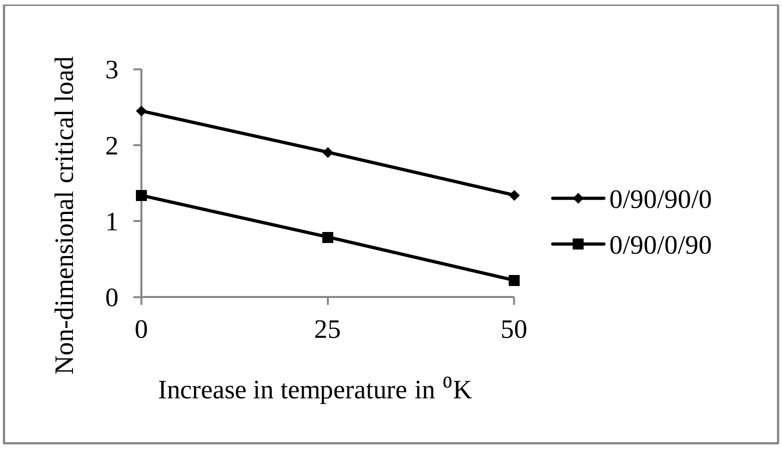

Fig 34: Effect of temperature on non-dimensional buckling load for ϕ =15⁰ and for laminated composite pre twisted cantilever cylindrical cross ply panels (a /b=1, b/t=25, b/R_y=0.2)

The effect of temperature on non-dimensional buckling load for laminated composite pre twisted cantilever cylindrical cross ply panels is shown in figure 34. The decrease in non dimensional buckling load due to increase in temperature from 300K to 350K for symmetrical cross-ply is 45.27% and for anti symmetric panels the decrease in buckling load is about 83.48%. So anti symmetric cross-ply panels are more unstable than symmetric cross ply panels.

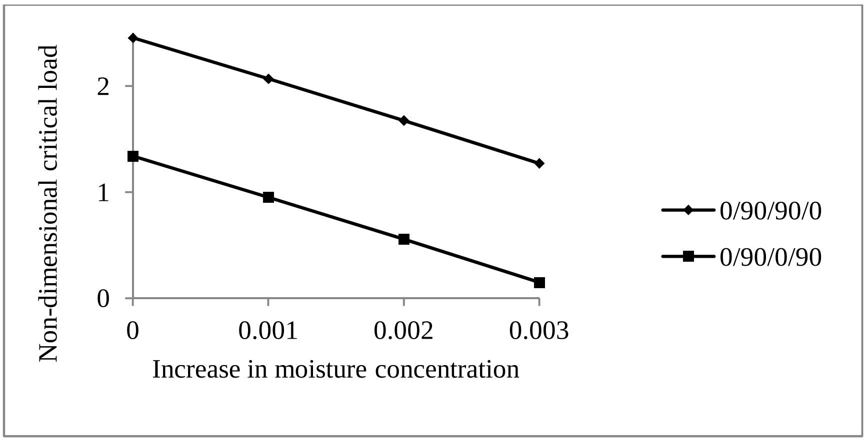

Fig 35: Effect of moisture on non-dimensional buckling load for $\phi = 15^o$ and for laminated composite pre twisted cantilever cylindrical cross ply panels (a /b=1, b/t=25, b/R_y=0.2)

The effect of moisture on non-dimensional buckling load for laminated composite pre twisted cantilever cylindrical cross ply panels is shown in figure 35. The decrease in non dimensional buckling load due to increase in moisture concentration from 0% to0.3% for symmetrical cross-ply is 48.17% and for anti symmetric panels the decrease in buckling load is about 88.83%. So anti symmetric cross-ply panels are more unstable than symmetric cross ply panels. The effect of temperature on non-dimensional buckling load for laminated composite pre twisted cantilever spherical angle ply panels is shown in figure 36. The decrease in non dimensional buckling loads due to increase in temperature from 300K to 350K for 0^o,15^o and 30^o are 53.53%, 20.13% and 46.82%.

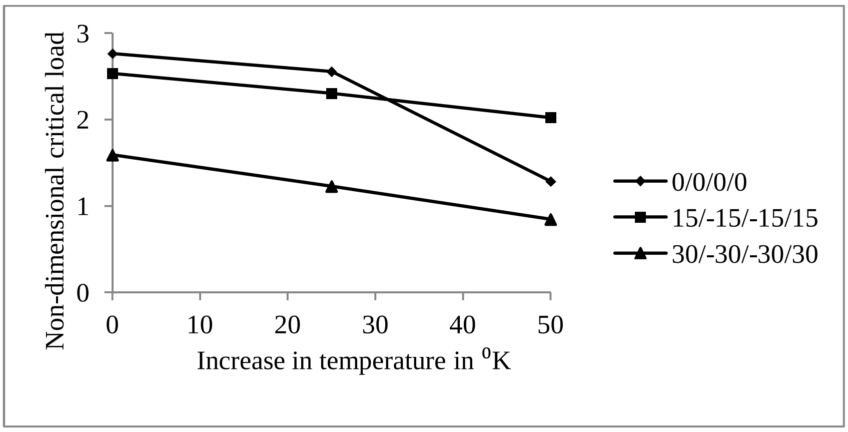

Fig 36: Effect of temperature on non-dimensional buckling load for $\phi = 15^o$ and for laminated composite pre twisted cantilever spherical angle ply panels (a /b=1, b/t=25, b/R_y=0.2)

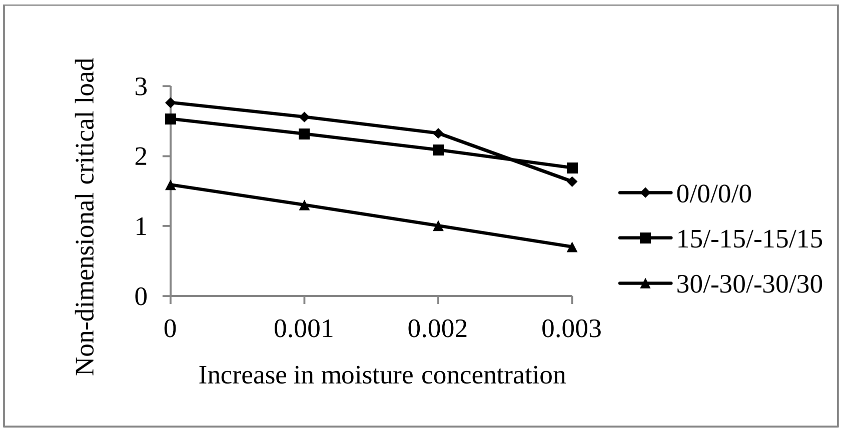

Fig 37: Effect of moisture on non-dimensional buckling load for ϕ = 15° and for laminated composite pre twisted cantilever spherical angle ply panels (a /b=1, b/t=25, b/R_y=0.2)

The effect of moisture on non-dimensional buckling load for laminated composite pre twisted cantilever spherical angle ply panels is shown in figure 37. The decrease in non dimensional buckling load due to increase in moisture concentration from 0%to 0.3% for 0°,15° and 30° are 40.75%, 27.63% and 55.77%.

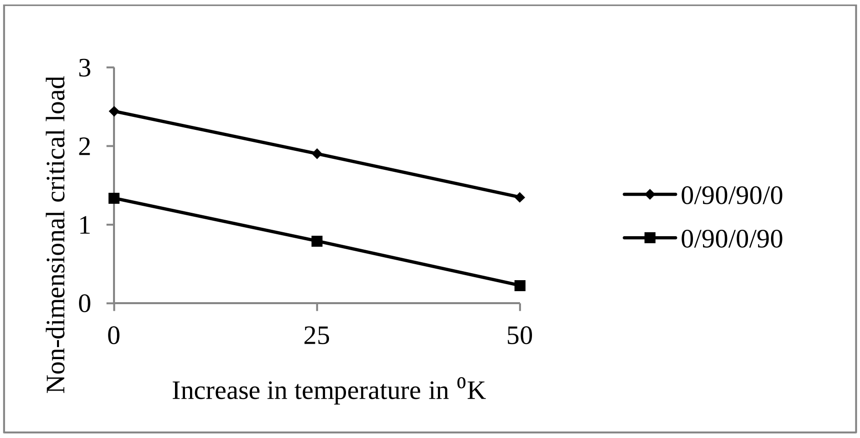

Fig 38: Effect of temperature on non-dimensional buckling load for ϕ = 15° and for laminated composite pre twisted cantilever spherical cross ply panels (a /b=1, b/t=25, b/R_y=0.2)

The effect of temperature on non-dimensional buckling load for laminated composite pre twisted cantilever spherical cross ply panels is shown in figure 38. The decrease in non dimensional buckling load due to increase in temperature from 300K to 350K for symmetrical cross-ply is 44.87% and for anti symmetric panels the decrease in buckling load is about 83.02%.

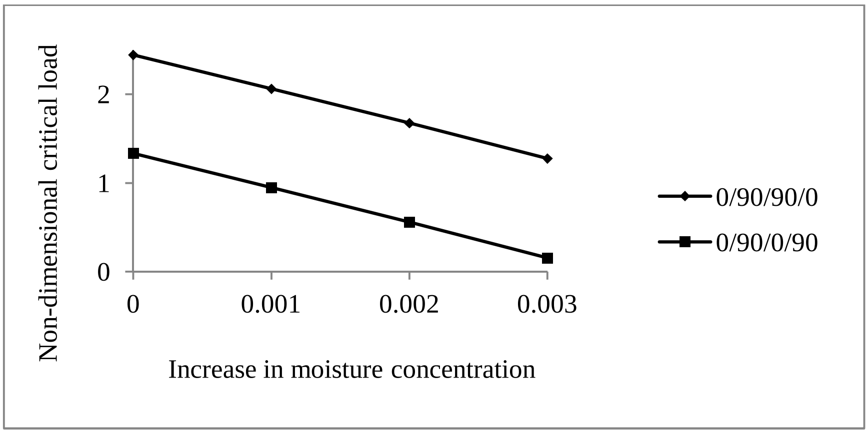

Fig 39: Effect of moisture on non-dimensional buckling load for ϕ = 15^o and for laminated composite pre twisted cantilever spherical cross ply panels (a /b=1, b/t=25, b/R_y=0.2)

The effect of moisture on non-dimensional buckling load for laminated composite pre twisted cantilever spherical cross ply panels is shown in figure 39. The decrease in non dimensional buckling load due to increase in moisture concentration from 0% to0.3% for symmetrical cross-ply is 47.75% and for anti symmetric panels the decrease in buckling load is about 88.37%. So anti symmetric cross-ply spherical panels are more unstable than symmetric cross ply panels.

CHAPTER 5

CONCLUSION

In the present study the conventional finite element formulation is modified to study the hygrothermal effects on the free vibration and stability of laminated composite twisted cantilever panels. An eight noded isoparametric shell element is used for this analysis with five degrees of freedom at each node. The numerical results for free vibration and buckling are presented and discussed above. The conclusion that can be made from this analysis is summarised below.

- The fundamental frequency decrease with increase in temperature and moisture concentration.
- For symmetric angle ply twisted cantilever panels as the temperature and moisture increases the frequency decreases with increase in ply orientation.
- The effect of hygrothermal conditions are much less in a twisted plate than without twist.
- For twisted cantilever cross-ply laminates the percentage decrease in frequency is more for anti symmetric laminates than symmetric laminates.
- The decrease in frequency is more significant when angle of twists are increased for symmetric and anti symmetric cross-ply twisted plates and 15^{o} angle-ply twisted plates with increase in temperature and moisture concentration.
- The frequencies of vibration increase with introduction of curvature in the laminated composite pretwisted plates.

- In case of plates and spherical shells, 45° ply orientations are more vulnerable towards the increase in temperature and moisture than cylindrical shells having same ply orientation.
- The critical load of angle ply and cross-ply twisted cantilever panel decreases as the temperature and moisture concentration increases.
- The buckling load increases with introduction of curvature in the twisted panels.
- The percentage decrease in non dimensional critical load is more for anti symmetric cross-ply twisted cantilever panels with increase in temperature and moisture concentration when angle of twist increases.

From the above study, it is concluded that the vibration and buckling behaviour of the laminated composite twisted cantilever plates and shells are greatly influenced by the geometry, lamination parameter, angle of twist and hygrothermal condition. This can be utilised while designing the structures of twisted cantilever panels in hygrothermal environment.

Scope for future work

The possible extensions to the present study are presented below:

- The present investigation can be extended to dynamic stability studies of cantilever twisted panels subjected to hygrothermal loading.
- There is a scope to study the vibration and buckling of twisted cantilever panels subjected to hygrothermal loading by experimental method.
- Material non linearity can be taken into account for further studies of twisted cantilever panels.

CHAPTER 6

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