PAPR Reduction of OFDM Signals Using Selected Mapping Technique

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by

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Certificate

This is to certify that the work in the thesis entitled ***PAPR Reduction of OFDM Signals Using Selected Mapping Technique*** by ***Himanshu Bhusan Mishra*** is a record of an original research work carried out by him during 2011 - 2012 under my supervision and guidance in partial fulfillment of the requirements for the award of the degree of Master of Technology in Electronics and Communication Engineering (Communication and Signal Processing), National Institute of Technology, Rourkela. Neither this thesis nor any part of it has been submitted for any degree or diploma elsewhere.

Place: NIT Rourkela

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Dr. Sarat Kumar Patra

Professor

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Abstract

According to the demand of advance communication field there should be high data rate in addition to both power efficiency and lower bit error rate. This demand of high data rate can be fulfilled by the single carrier modulation with compromising the trade off between the power efficiency and bit error rate. Again in the presence of frequency selective fading environment, it is very difficult to achieve high data rate for this single carrier modulation with a lower bit error rate performance. With considering an advance step towards the multi carrier modulation scheme it is possible to get high data rate in this multipath fading channel without degrading the bit error rate performance. To achieve better performance using multi carrier modulation we should make the subcarriers to be orthogonal to each other i.e. known as the Orthogonal Frequency Division Multiplexing (OFDM) technique.

But the great disadvantage of the OFDM technique is its high Peak to Average Power Ratio (PAPR). As we are using the linear power amplifier at the transmitter side so it's operating point will go to the saturation region due to the high PAPR which leads to in-band distortion and out-band radiation. This can be avoided with increasing the dynamic range of power amplifier which leads to high cost and high consumption of power at the base station.

This report presents an efficient technique i.e the Selected Mapping which reduces the PAPR. Also the analysis of bit error rate performance and the computational complexity for this technique are being discussed here. In additions to the above analysis one important analysis of the mutual independence between the alternative

OFDM signals generated using this technique, also being presented.

One scheme proposed here which satisfies the PAPR reduction criteria with reducing the computational complexity. Also this new scheme has an important advantage of avoiding the extra bits along with the transmitted OFDM signal. This scheme can also be applied for the multiple transmitting antenna cases.

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Chapter 1

Introduction

1.1 Introduction

The demand of high data rate services has been increasing very rapidly and there is no slowdown in sight. We know that the data transmission includes both wired and wireless medium. Often, these services require very reliable data transmission over very harsh environment. Most of these transmission systems experience much degradation such as large attenuation, noise, multipath, interference, time variance, nonlinearities and must meet the finite constraints like power limitation and cost factor. One physical layer technique that has gained a lot of popularities due to its robustness in dealing with these impairments is multi-carrier modulation technique. In multi-carrier modulation, the most commonly used technique is Orthogonal Frequency Division Multiplexing (OFDM); it has recently become very popular in wireless communication.

Unfortunately the major drawback of OFDM transmission is its large envelope fluctuation which is quantified as Peak to Average Power Ratio (PAPR). Since power amplifier is used at the transmitter, so as to operate in a perfectly linear region the operating power must lies below the available power. For reduction of this PAPR lot of algorithms have been developed. All of the techniques has some sort of advantages and disadvantages [1]. Clipping and Filtering is one of the basic technique in which some part of transmitted signal undergoes into distortion. Also the Coding scheme reduces the data rate which is undesirable. If we consider Tone Reservation (TR) technique it also allows the data rate loss with more probable of increasing power. Again the techniques like Tone Injection (TI) and the Active Constellation Extension (ACE) having a criteria of increasing power will be undesirable in case of power constraint environment. If we go for the Partial Transmit Sequence (PTS) and Selected Mapping (SLM) technique, the PTS technique has more complexity than that of SLM technique.

This Selected Mapping is one of the promising technique due to its simplicity for implementation which introduces no distortion in the transmitted signal. It has been described first in [2] i.e. to be known as the classical SLM technique. This technique has one of the disadvantage of sending the extra Side Information (SI) index along with the transmitted OFDM signal. Which can be avoided using a special technique described in [3].

The concentration of this thesis work is specially upon the Selected Mapping technique. Here the three important analysis of this technique has been done. Out of them one is, how to avoid the transmission of extra information along with the OFDM signal which will be discussed in the section *Avoiding the SI index Transmission*. Another one important analysis of this technique is how to reduce the computational complexity. Also one important analysis is to be done about the mutual independence between the alternative phase vectors used in this technique. One technique also being proposed which has an advantage of reducing the PAPR simultaneously reducing the computational complexity in comparison to that of the Classical SLM. In addition to this the proposed technique also avoids the sending of extra SI index.

1.2 Digital Communication System

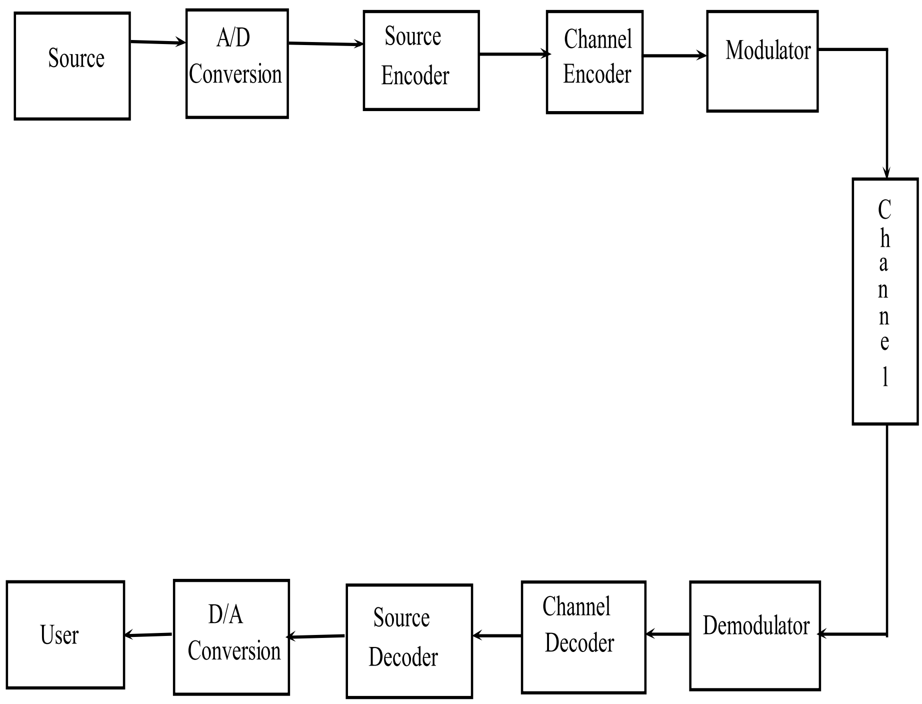

Figure 1.1: Block Diagram a General Digital Communication System

The figure 1.1 describes about a general digital communication system blocks. The A/D converter being used to convert the analog source to the digital i.e. in the form of binary sequences. The source encoding takes place to compress the transmitted digital data up to an extent such that it can be received with out any loss. There are some basic source coding techniques are available like the Hoffman coding and Shannon-Fano coding. The objective of source encoding is to remove redundancy from the source. The sequence of binary digits from the source encoder also known as information sequence, is passed to the channel encoder. The channel encoder add redundant bits to the information sequence from the received signal for the reliable communication. The channel encoder maps k information bits into a unique n bit sequence called codeword. The ratio $\frac{n}{k}$ is a measure of the redundancy introduced by the channel encoder and the reciprocal of this ratio is called code rate. The output of the channel encoder is passed to the digital modulator.

The digital modulator maps the binary information sequence into signal waveforms. The modulation may be binary or $m-$ary. In binary modulation two distinct waveforms are used to represent the binary digits 0 and 1 whereas in $m-$ary modulation $m = 2^b$ distinct waveforms are used to represent a binary word of b bits. The modulated wave form is being transmitted from the transmitter to the receiver through channel. In the channel due to addition of noise the transmitted signal becomes corrupted. The sources of noise are thermal noise, atmospheric noise, man-made noise etc., which are random in nature and generally unpredictable. At the receiving end the digital demodulator consists of matched filter type detector or correlator type detector converts the received signal waveforms into binary sequence, which represent the estimated word. The output from the demodulator is passed to the channel decoder, that recovers the information sequence from the knowledge of the code.

The average probability of a bit-error at the output of the decoder is a measure of the performance of the demodulator decoder combination. However the probability of error is a channel characteristics, coding , modulation, demodulation and decoding techniques.Finally the source decoder reconstructs the output from the source that was transmitted. The reconstructed signal is an approximation of the source output as the encoders and decoders have introduced errors and distortion to the signal. The difference is a measure of the distortion introduced by the digital communication system. This encoded signal will be passed through the digital to analog converter and finally received by the user.

1.3 Multipath Channels

The transmitted signal faces various obstacles and surfaces of reflection, as a result of which the received signals from the same source reach at different times. This gives rise to the formation of echoes which affect the other incoming signals. Dielectric constants, permeability, conductivity and thickness are the main factors affecting the system. Multipath channel propagation is devised in such a manner that there will be a minimized effect of the echoes in the system in an indoor environment. Measures are needed to be taken in order to minimize echo in order to avoid ISI (Inter Symbol Interference). The figure 1.2 shows the scenario for multipath propagation.

1.4 Muticarrier Transmission Schemes

In a single carrier system, a single fade causes the whole data stream to under go into the distortion i.e known as the frequency selective fading. To overcome the frequency selectivity of the wideband channel experienced by single-carrier transmission, multiple carriers can be used for high rate data transmission. In multicarrier transmission [4], a single data stream is transmitted over a number of lower rate subcarriers. The figure 1.3 shows the basic structure and concept of a multicarrier transmission system.

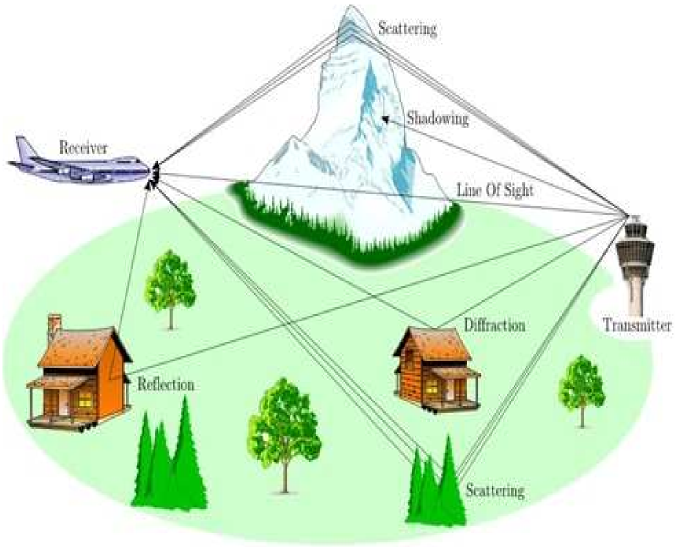

Figure 1.2: Multipath Propagation

Using this multicarrier transmission the frequency-selective wideband channel can be approximated by multiple frequency-flat narrowband channels. Let the wideband be divided into N narrowband subchannels, which have the subcarrier frequency of f_k, $k = 0, 1, ..., N - 1$. Orthogonality among the subchannels should be maintained to suppress the ICI (Inter Carrier Interference) which leads to the distortionless transmission. So in this transmission scheme the different symbols are transmitted with orthogonal subchannels in parallel form. If the oscillators are being used to generate the subcarriers for each subchannel, the implementation of this transmission scheme becomes complex. To avoid this complexity one important transmission scheme comes into picture that is the OFDM (Orthogonal Frequency Division Multiplexing).

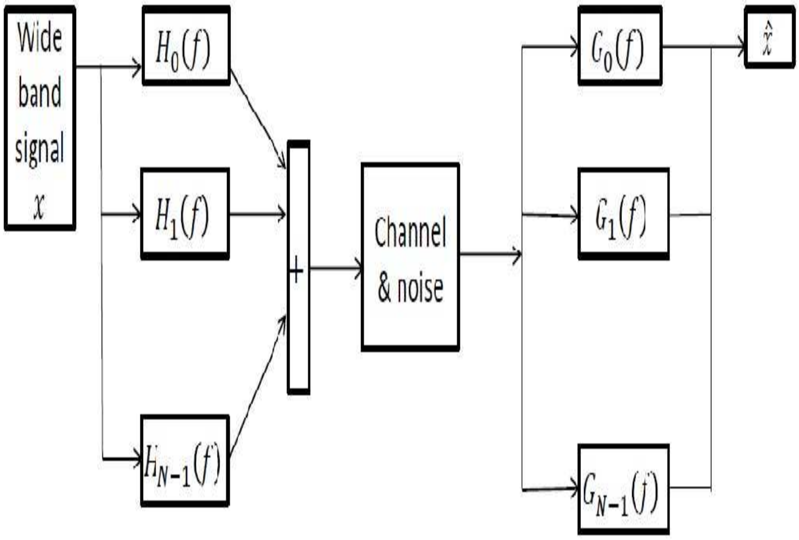

Figure 1.3: Multicarrier Transmission

1.5 OFDM Transmission Scheme

Orthogonal frequency division multiplexing (OFDM) [5],[6] transmission scheme is a type of multichannel system which avoids the usages of the oscillators and band-limited filters for each subchannel. The OFDM technology was first conceptualized in the 1960s and 1970s. The main idea behind the OFDM is that since low-rate modulations are less sensitive to multipath, the better way is to send a number of low rate streams in parallel than sending one high rate waveform. It divides the frequency spectrum into sub-bands small enough so that the channel effects are constant (flat) over a given sub-band. Then a classical IQ (In phase Quadrature phase) modulation (BPSK, QPSK, M-QAM, etc) is sent over the sub-band. If it designed correctly, all the fast changing effects of the channel disappear as they are now occurring during the transmission of a single symbol and are thus treated as flat fading at the receiver.

A large number of closely spaced orthogonal subcarriers are used to carry data. The data is divided into several parallel data streams or channels, one for each subcarrier. Each subcarrier is modulated with a conventional modulation scheme such as Quadrature Amplitude Modulation (QAM) or Phase Shift Keying (PSK) at a low symbol rate. The total data rate is to be maintained similar to that of the conventional single carrier modulation scheme with the same bandwidth. Orthogonal Frequency Division Multiplexing (OFDM) is a promising technique for achieving high data rate and combating multipath fading in Wireless Communications. Orthogonal Frequency Division Multiplexing is a special form of multicarrier modulation which is particularly suited for transmission over a dispersive channel. Here the different carriers are orthogonal to each other, that is, they are totally independent of one another. This is achieved by placing the carrier exactly at the nulls in the modulation spectra of each other as shown in figure 1.4.

The orthogonality of the carriers means that each carrier has an integer number of cycles over a symbol period. Due to this integer number of cycles, the spectrum of each carrier has a null at the center frequency of each of the other carriers in the system that results in no interference between the carriers, allowing them to be spaced as close as possible. The problem of overhead carrier spacing required in Frequency Division Multiplexing (FDM) can be recovered. So this multicarrier transmission scheme allows the overlapping of the spectra of subcarriers for bandwidth efficiency [7]. The OFDM transmission scheme is being shown in the figure 1.5.

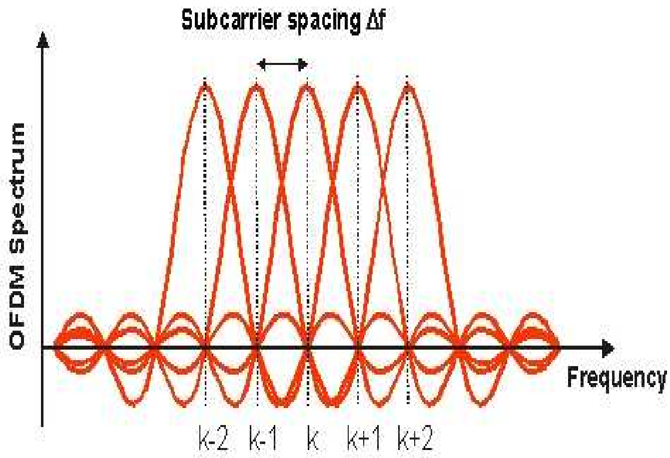

Figure 1.4: OFDM Spectrum

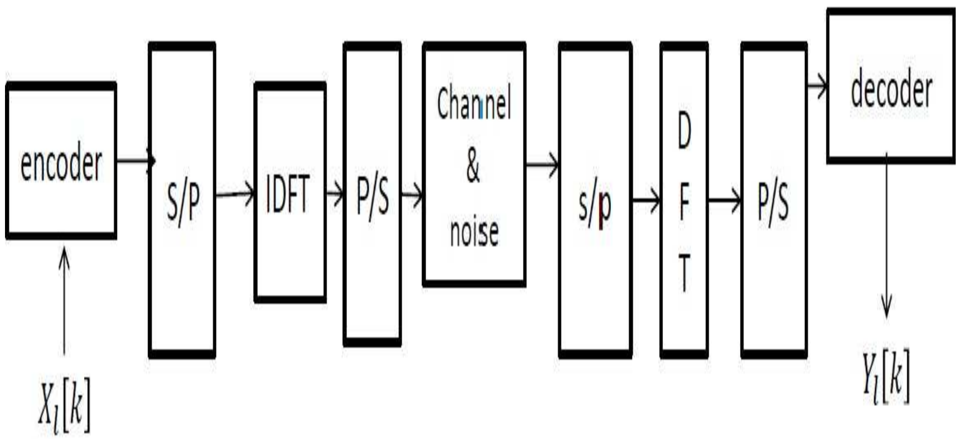

Figure 1.5: OFDM Transmission Scheme

1.5.1 Inter Symbol Interference

Inter symbol interference (ISI) is a form of distortion of a signal in which one symbol interferes with subsequent symbols. This is an unwanted phenomenon as the previous symbols have similar effect as noise, which makes the communication as some sort of unreliable. It is usually caused by multipath propagation or the inherent nonlinear frequency response of a channel causing successive symbols to blur together. The presence of ISI in the system introduces error in the decision device at the receiver output. Therefore, in the design of the transmitting and receiving filters, the objective is to minimize the effects of ISI and thereby deliver the digital data to its destination with the smallest error rate possible.

1.5.2 Inter Carrier Interference

Presence of Doppler shifts and frequency and phase offsets in an OFDM system causes loss in orthogonality of the sub-carriers. As a result, interference is observed between sub-carriers. This phenomenon is known as inter - carrier interference (ICI).

1.5.3 Cyclic Prefix

The Cyclic Prefix or Guard Interval is a periodic extension of the last part of an OFDM symbol that is added to the front of the symbol in the transmitter, and is removed at the receiver before demodulation. According to the figure 1.5 the addition of Cyclic Prefix (CP) takes place after the parallel to serial conversion and being removed at the receiver side before the DFT operation. The OFDM symbol with considering the Cyclic Prefix is shown in figure 1.6.

1.5.4 Advantages of OFDM

The Orthogonal Frequency Division Multiplexing (OFDM) transmission scheme has the following key advantages.

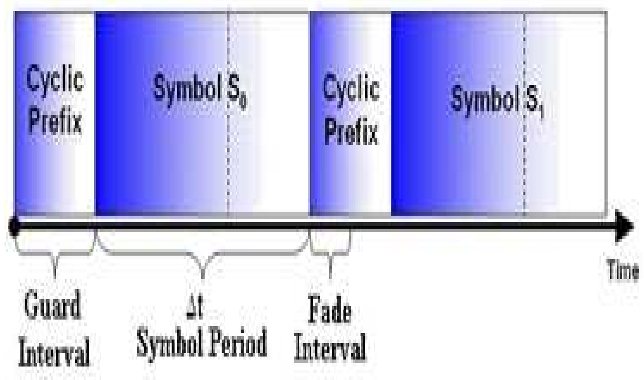

Figure 1.6: Cyclic Prefix

- OFDM is computationally efficient by using FFT techniques to implement the modulation and demodulation functions.
- By dividing the channel into narrowband flat fading sub channels, OFDM is more resistant to frequency selective fading than single carrier systems.
- By using adequate channel coding and interleaving, the symbols lost can be recovered, due to the frequency selectivity of the channel.
- OFDM is a bandwidth efficient modulation scheme and has the advantage of mitigating ISI in frequency selective fading channels.
- Channel equalization becomes simpler than by using adaptive equalization techniques with single carrier systems.
- In conjunction with differential modulation, there is no need to implement a

channel estimator.

- Provides good protection against co-channel interference and impulsive parasitic noise.
- OFDM can easily adapt to severe channel conditions without complex time-domain equalization.
- It eliminates Inter Symbol Interference (ISI) through the use of a cyclic prefix.
- OFDM is less sensitive to sample timing offsets than the single carrier systems.
- OFDM provides greater immunity to multipath fading and impulse noise.
- OFDM makes efficient use of the spectrum by allowing overlap.
- OFDM eliminates the need for equalizers.

1.5.5 Disadvantages of OFDM

The Orthogonal Frequency Division Multiplexing (OFDM) transmission scheme is an attractive technology but has the following disadvantages:

- OFDM is more sensitive to carrier frequency offset and drift than single carrier systems, due to leakage of the Discrete Fourier Transform (DFT).
- OFDM is sensitive to frequency synchronization problems.
- It is sensitive to Doppler Shift.
- The OFDM signal has a noise like amplitude with a very large dynamic range; therefore it requires RF power amplifiers with a high Peak-to-Average Power Ratio (PAPR).

- The high PAPR increases the complexity of the Analog-to-Digital (A/D) and Digital-to-Analog (D/A) converters.

- The high PAPR also lowers the efficiency of power amplifiers.

1.6 Outline of Thesis

An overview of multipath transmission scheme basically the OFDM transmission scheme is given in this chapter.

Chapter 2 introduces about the Peak to Average Power Ratio (PAPR) of the OFDM signal and its reduction techniques. This chapter specially concentrates upon the Selected Mapping (SLM) technique. Also a lot of analysis like computational complexity reduction, analysis of covariance for this technique is being described here with presenting some simulation works. A proposed scheme also being presented here having a criteria of reduced complexity in addition to the PAPR reduction.

Chapter 3 presents the method to apply this Selected Mapping technique to the MIMO-OFDM system with presenting some simulation results. Also the PAPR reduction performance of the proposed technique to this MIMO-OFDM system being shown in this chapter.

Chapter 4 concludes the present work and predicts some work to be done in future.

Chapter 2

Peak to Average Power Ratio

2.1 Peak to Average Power Ratio

It is defined as the ratio between the maximum power and the average power for the envelope of a baseband complex signal $\tilde{s}(t)$ i.e.

$$PAPR\left\{\tilde{s}\left(t\right)\right\} = \frac{\max\left|\tilde{s}\left(t\right)\right|^2}{E\left|\tilde{s}\left(t\right)\right|^2} \tag{2.1}$$

Also we can write this PAPR equation for the complex passband signal $s(t)$ as

$$PAPR\left\{s\left(t\right)\right\} = \frac{\max\left|s(t)\right|^2}{E\left\{\left|s(t)\right|^2\right\}} \tag{2.2}$$

2.1.1 Effect of High PAPR

The linear power amplifiers are being used in the transmitter so the Q-point must be in the linear region. Due to the high PAPR the Q-point moves to the saturation region hence the clipping of signal peaks takes place which generates in-band and out-of-band distortion. So to keep the Q-point in the linear region the dynamic range of the power amplifier should be increased which again reduces its efficiency and enhances the cost. Hence a trade-off exists between nonlinearity and efficiency [1]. And also with the increasing of this dynamic range the cost of power amplifier increases. As a communication engineer our objective should be to reduce this PAPR.

2.1.2 PAPR Reduction Techniques

A lot of techniques presents for the reduction of this PAPR [1]. About some of the reduction techniques like Clipping and Filtering, Coding, Partial Transmit Sequence, Selected Mapping, Tone Reservation, Tone Injection, Active Constellation Extension are briefly described here.

Clipping and Filtering

This is a simplest technique used for PAPR reduction. Clipping [8] means the amplitude clipping which limits the peak envelope of the input signal to a predetermined value.Let $x[n]$ denote the pass band signal and $x_c[n]$ denote the clipped version of $x[n]$,which can be expressed as

$$x_c\left[n\right] = \begin{cases} -A & x\left[n\right] \ \leq -A \\ x\left[n\right] & \left|x\left[n\right]\ \right| < A \\ A & x\left[n\right] \ \geq A \end{cases} \tag{2.3}$$

where A is the pre-specified clipping level. However this technique has the following drawbacks:

- Clipping causes in-band signal distortion,resulting in Bit Error Rate performance degradation.
- It also causes out-of-band radiation,which imposes out-of-band interference signals to adjacent channels. This out-of-band radiation can be reduced by filtering.
- This filtering of the clipped signal leads to the peak regrowth. That means the

signal after filtering operation may exceed the clipping level specified for the clipping operation.

So we came to know that this clipping and filtering [9] technique has some sort of distortion during the transmission of data.

Coding

The coding technique [10] is used to select such codewords that minimize or reduce the PAPR.It causes no distortion and creates no out-of-band radiation, but it suffers from bandwidth efficiency as the code rate is reduced. It also suffers from complexity to find the best codes and to store large lookup tables for encoding and decoding, especially for a large number of sub carriers.

Partial Transmit Sequence

In the Partial Transmit Sequence(PTS) [11] technique, an input data block of N symbols is partitioned into disjoint sub blocks. The sub-carriers in each sub-block are weighted by a phase factor for that sub-block. The phase factors are selected such that the PAPR of the combined signal is minimized. But by using this technique there will be data rate loss.

Tone Reservation

According to this technique the transmitter does not send data on a small subset of subcarriers that are optimized for PAPR reduction. Here the objective is to find the time domain signal to be added to the original time domain signal such that the PAPR is reduced. Here the data rate loss will be take place also probability of power increase is more.

Tone Injection Technique

The basic idea used in this technique is to increase the constellation size so that each symbol in the data block can be mapped into one of the several equivalent constellation points, these extra degrees of freedom can be exploited for PAPR reduction. Here the transmitted power increases.

Active Constellation Extension (ACE) Technique

This technique for PAPR reduction is similar to Tone Injection technique. According to this technique [12], some of the outer signal constellation points in the data block are dynamically extended towards the outside of the original constellation such that PAPR of the data block is reduced. In this case also there will be increase of transmitted power take place.

Selected Mapping (SLM) Technique

The basic idea of this technique is first generate a number of alternative OFDM signals from the original data block and then transmit the OFDM signal having minimum PAPR. But data rate loss and complexity at the transmitter side are two basic disadvantages for this technique. This technique has been described exhaustively in the section 2.2.

The performance comparison for all the PAPR reduction techniques described above are being shown in the table 2.1.

2.1.3 Analysis of PAPR using CCDF

CDF stands for Cumulative Distribution Function. If Y is a random variable then the CDF of y is defined as the probability of the event $\{Y \leq y\}$. So the Comple-

Techniques	Distortion	Power increase	Data rate loss
Clipping and Filtering	Yes	No	No
Coding	No	No	Yes
Partial Transmit Sequence	No	No	Yes
Tone Reservation	No	Yes	Yes
Tone Injection	No	Yes	No
ACE	No	Yes	No
Selected Mapping	No	No	Yes

Table 2.1: Performance Comparison between the Techniques

mentary Cumulative Distribution Function(CCDF) is defined as the probability of the event $\{Y > y\}$. With using this density function it is easy to analyze the PAPR reduction performance. Let us consider x is the transmitted OFDM signal then from [7] we got the theoretical CCDF of PAPR i.e. to find the probability of the event $\{PAPR\{x\} > \gamma\}$ which is given as

$$\Pr\left(PAPR\{x\} > \gamma\right) = 1 - \left(1 - e^{-\gamma}\right)^{N} \tag{2.4}$$

where N is the number of subcarriers. However the PAPR for the discrete-time baseband signal $x[n]$ may not be same as that for the continuous-time baseband signal $x(t)$. In fact, the PAPR for $x[n]$ is lower than that for $x(t)$,simply because $x[n]$ may not have all the peaks of $x(t)$ [1]. In practice the PAPR for the continuous-time baseband signal can be measured only after implementing the actual hardware,including digital-to-analog converter (DAC). So there is some way of estimating the PAPR from the discrete-time signal $x[n]$. It is known that $x[n]$ can show almost the same PAPR as $x(t)$ if it is V-times interpolated(oversampled) where $V \geq 4$ [1]. According to [7] the approximate value of the CCDF for the oversampled signal is given as

$$\Pr\left(PAPR\{x\} > \gamma\right) = 1 - \left(1 - e^{-\gamma}\right)^{\alpha N} \tag{2.5}$$

where α has to be determined by fitting the theoretical CCDF into the actual one.

2.2 Selected Mapping Technique

This is an effective and distortion less technique used for the PAPR reduction in OFDM. The name of this technique indicates that one sequence has to be selected out of a number of sequences. According to the concept of discrete time OFDM transmission we should make a data block considering N number of symbols from the constellation plot. Where N is the number of subcarriers to be used. Then using that data block U number of independent candidate vectors are to be generated with the multiplication of independent phase vectors. Let us consider X is the data block with $X(k)$ as the mapped sub symbol(i.e. the symbol from the constellation). Where $k = \{0, 1, 2, ...N - 1\}$. Let the u^{th} phase vector is denoted as $B^{(u)}$,where $u = \{1, 2, ...U\}$. The u^{th} candidate vector that is generated by the multiplication of data block with the phase vector is denoted as $X^{(u)}$. So we can write the equation to get the k^{th} element of u^{th} candidate vector as

$$X^{(u)}(k) = X(k)\ B^{(u)}(k) \tag{2.6}$$

Then by doing IFFT operation to each candidate vector we will obtain U number of alternative OFDM signals, so the n^{th} symbol of u^{th} alternative OFDM signal can be written mathematically as

$$x^{(u)}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X^{(u)}(k)\ e^{j\left(\frac{2\pi nk}{N}\right)} \tag{2.7}$$

So out of the U number of alternative OFDM signals the signal having minimum PAPR is to be selected for transmission. Let that selected OFDM signal is denoted as $x^{(\tilde{u})}(k)$. This selected mapping(SLM) technique is known as the classical SLM [2]. The block diagram for this technique is shown in figure 2.1.

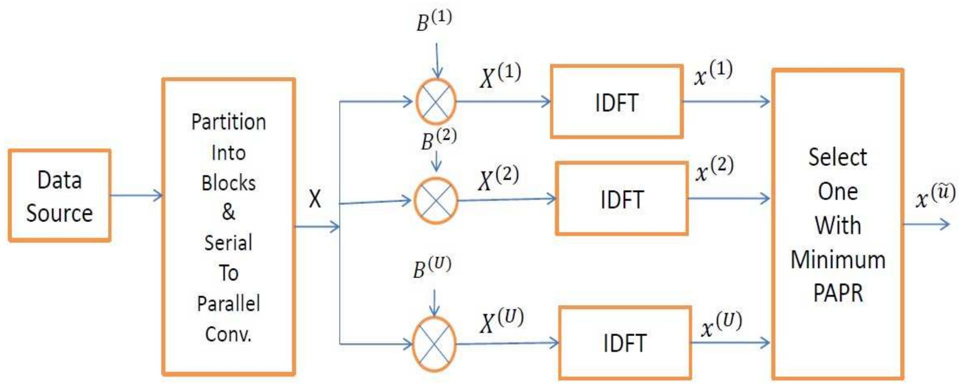

Figure 2.1: Block Diagram of SLM

So in this technique for generation of alternative OFDM symbols the independent phase vectors has to generate. We get from the equation 2.6, the k^{th} value of u^{th} phase vector is denoted as $B^{(u)}(k)$ and can be found by

$$B^{(u)}(k) = e^{j\phi^{(u)}(k)} \tag{2.8}$$

where $\phi(k)$ is the random phase value. So from the equation 4 we get that $X^{(u)}(k)$ be a phase rotated version of $X(k)$. From [13] we came to know that two phase vectors $B^{(m)}$ and $B^{(l)}$ is dependent if any joint cumulant between them is nonzero. So the condition of mutual independence between $b^{(m)}(n)$ and $b^{(l)}(n)$ is given as

$$E\left[e^{j\phi}\right] = 0 \tag{2.9}$$

To make satisfy the above condition ϕ should be uniformly distributed in $[0, 2\pi)$. According to this selection criteria of ϕ the variation of the PAPR reduction performance will be shown in the next subsection.

The figure 2.1 provides description about the transmitter side of the SLM technique. This selected OFDM signal at transmitter side has to be detected at the receiver. So the receiver must have the information about the perfect phase phase vector that has been multiplied to generate that selected OFDM signal. Hence to fulfill the requirement of the receiver some side information(SI) has to be transmitted along with the selected OFDM signal.This SI index is generally transmitted as a set of $\lceil \log_2 U \rceil$ bits. For the efficient transmission of these extra bits channel coding technique should be required. If any SI index can not be detected perfectly then that total recovered transmitted block will be in error. So we should follow a new SLM technique [3] which avoids the sending of SI index. This technique is discussed briefly in the following sections.

2.2.1 Analysis of PAPR using CCDF

As discussed above the analysis of the performance of PAPR reduction is very easy through the CCDF. This performance using the classical SLM technique is shown in figure 2.2. If we consider all the candidate vectors in a matrix form then without following the oversampling concept the dimension of that matrix will be $U \times N$ and with following the oversampling concept the dimension becomes $U \times VN$. Here the number of subcarriers used to be $N = 128$ and the oversampling factor $V = 4$.

So this figure 2.2 describes the performance criteria of the classical SLM technique on the basis of PAPR reduction performance. Another PAPR analysis also being

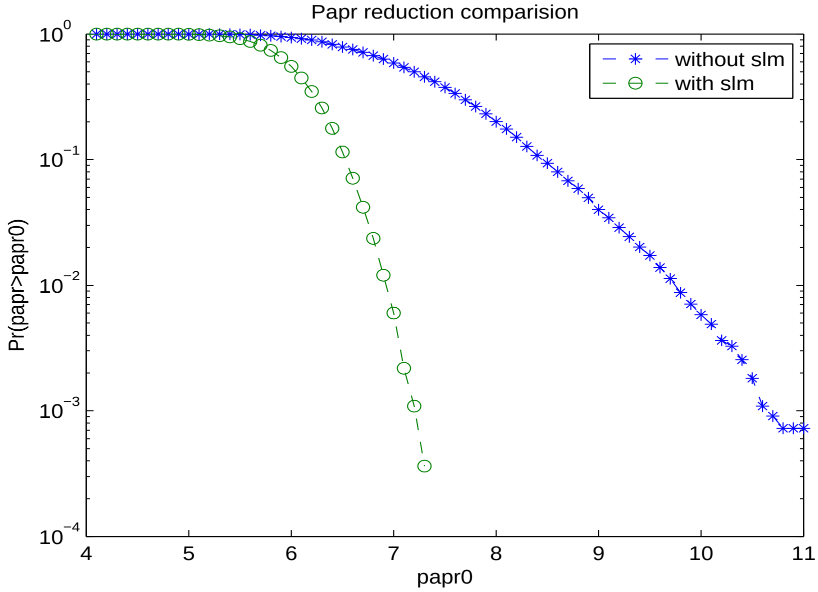

Figure 2.2: PAPR comparison for classical slm

done here on the basis of different phase vectors to satisfy the equation 2.9.With considering the case 1 as the ϕ takes on values 0 and π with equal probability.In case 2,ϕ is uniformly distributed in $\left[0, \frac{\pi}{2}\right)$. Obviously the equation 8 is satisfied for case 1 but is violated for case 2. Hence due to mutual dependency of alternative phase vectors in case 2 the CCDF plot for PAPR reduction moves away from the theoretical plot. The expression of theoretical PAPR [13] for the classical SLM is given by

$$\Pr\left(PAPR\left\{x\right\} > \gamma\right) = \left(1 - \left(1 - e^{-\gamma}\right)^{N}\right)^{U} \tag{2.10}$$

where U is the number of alternative vectors. This comparison plot is shown in figure 2.3. Results of this simulation is shown without using any oversampling factor.

2.2.2 Avoiding the SI index Transmission

As we have discussed before that to recover the original data block a SI index is required to transmit along with the selected OFDM signal which leads to data rate loss.

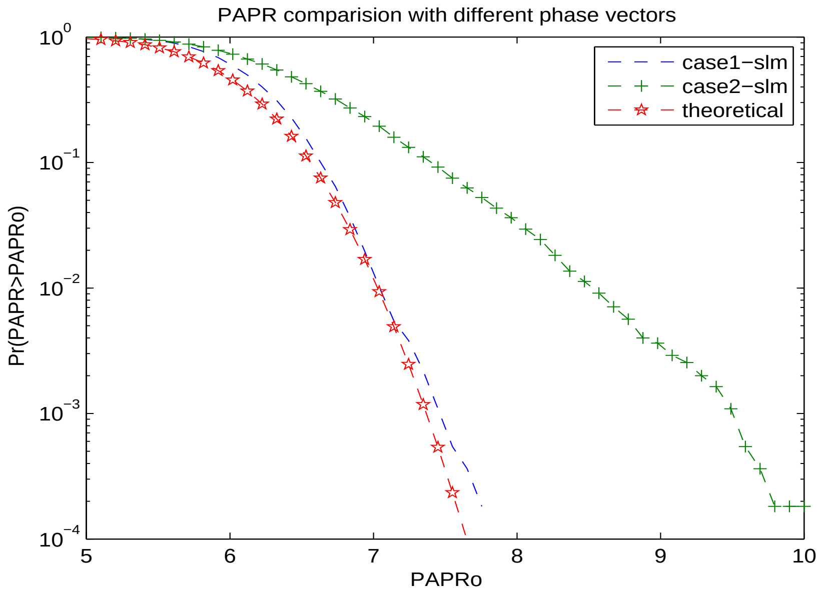

Figure 2.3: PAPR comparison using different phase sets

According to the classical SLM the magnitude of each value in the phase vector will be $\left|B^{(u)}\left(k\right)\right| = 1$. But according to the new SLM technique [3] a different methodology should be followed to construct the phase vector. To construct this phase vector first divide that vector into subvectors of length M, where M is the smallest possible integer satisfying the condition $\begin{pmatrix} M \\ p \end{pmatrix} \geq U$,where p will be any integer smaller than M. So if the length of vector is N then $\frac{N}{M}$ number of subvectors presents. For the each vector B^u one subvector will be constructed where out of M number of places in p number of places the magnitude of $B^u\left(k\right)$ taken as a constant C, which can also be known as the extension factor with value must be greater than 1. Then simply place that subvector for $\frac{N}{M}$ times to construct one phase vector of length N. Like this we should construct one subvector for each alternative phase vector by varying the positions which should occupy C value. With following this construction criteria of the alternative phase vectors we can detect the perfect SI index without transmitting this along with the selected OFDM signal. Also by using this technique the PAPR performance will not be changed. Here we have shown three simulation plots one for PAPR performance i.e. in figure 2.4. Here the simulation result is compared for the two different subcarriers of the WiMAX standard. Another plot in figure 2.5 shows the variation of error in detecting the SI index with respect to the value of C and also here the same two subcarriers used for simulation work. Finally the figure 2.6 shows the bit error rate performance. Here the bit error rate analysis is done using QPSK and 16−QAM modulation schemes. For each modulation scheme two plots are compared, one with assuming the availability of perfect SI index at the receiver and the another one with detecting the SI index at the receiver by sub-optimal algorithm.

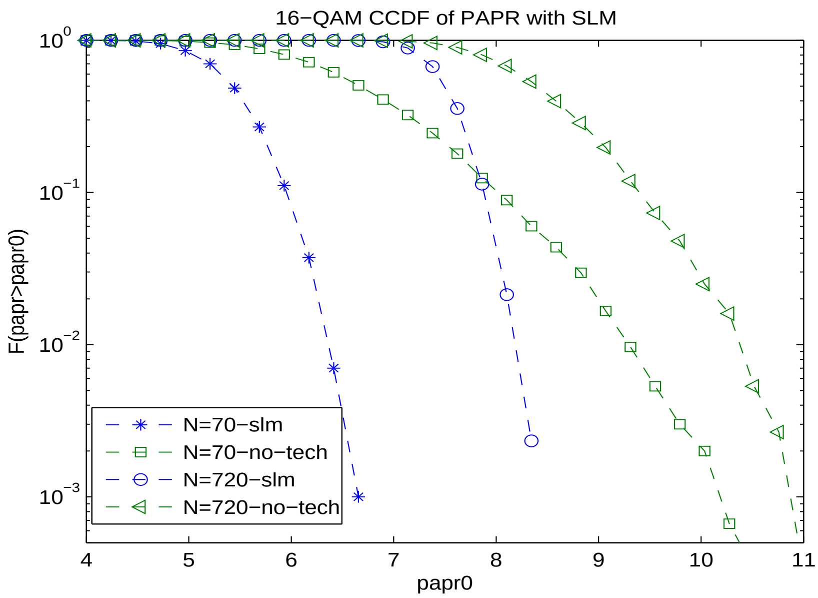

Figure 2.4: PAPR comparison using different sub-carriers of WiMAX

Also one important thing is we have to send the information about the generation of random matrix at the transmitter side to the receiver side. So to avoid this sending of information to the receiver we can use the Riemann matrix [14],[15] at the

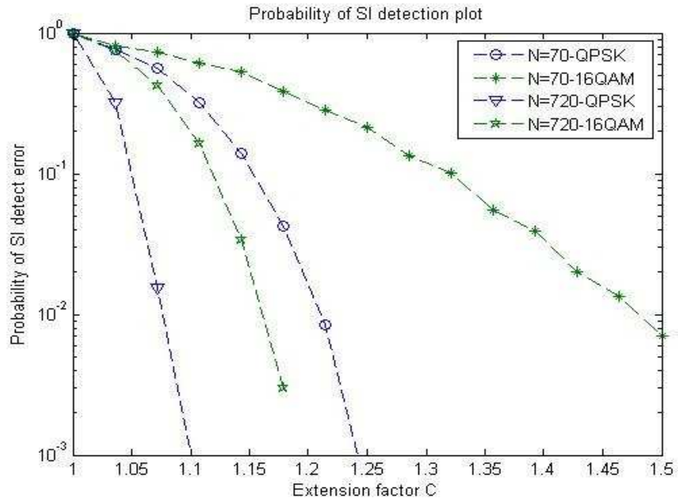

Figure 2.5: Probability of error in detecting SI index w.r.t. C

transmitter with replacing that random matrix. The Riemann matrix $\mathbf{R}$ is obtained by removing the first row and first column of the matrix $\mathbf{A}$, where

$$A\left(m,n\right)=\left\{\begin{array}{cc} m-1 & if\,m\,divides\,n \\ -1 & otherwise \end{array}\right\} \tag{2.11}$$

This Riemann matrix also improves the PAPR reduction performance. With considering 64 number of subcarriers and the oversampling factor $V=4$ the performance of PAPR reduction has been verified in the figure 2.7. According to this figure one plot is with consideration of that random matrix in classical SLM technique and another one with considering the Riemann matrix.

Another important technique used for further reduction of PAPR is that of introducing the new phase sequences based on the rows of centering matrix [16]. The

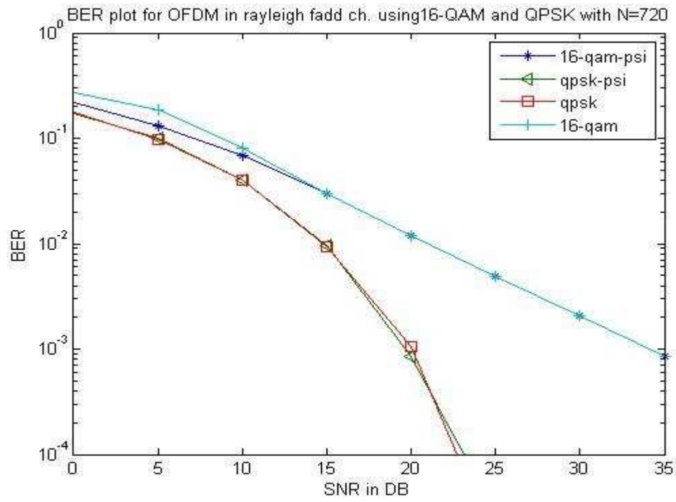

Figure 2.6: Probability of error in detecting SI index w.r.t. Extension factor

structure of centering matrix is given by

$$C = I - \frac{O}{n} \tag{2.12}$$

where I is the identity matrix of size n and O is an $n-$by$-n$ matrix of all 1^s. The verification of the PAPR reduction is done in the figure 2.8 with consideration of $N = 64$,$V = 4$ and 16$-$QAM modulation scheme. So according to this figure the reduction of PAPR is found to be good. In this case also not required to send the information about the generation of matrix to the receiver.

2.2.3 Analysis of Complexity

In this SLM technique one of the vital point is with increasing the number of alternative OFDM signals (i.e. increasing the value of U) the PAPR is being reduced. Figure 2.9 shows this analysis with consideration of 64 number of subcarriers and oversampling factor $V = 4$. However the number of alternative OFDM signals is same as the

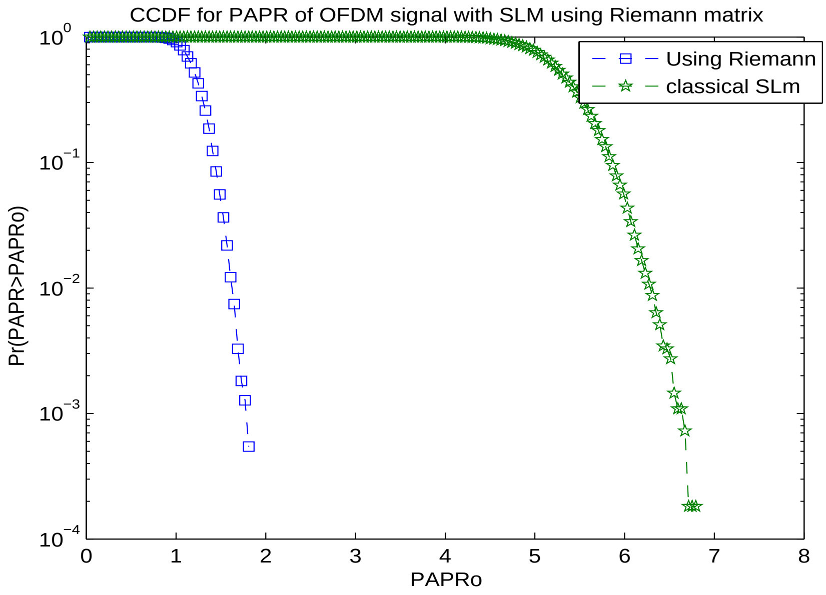

Figure 2.7: PAPR reduction performance using the Riemann matrix

number of IFFT blocks. As we know that for N point IFFT the number of complex multiplications and additions are $\frac{N}{2}\log_2 N$ and $N\log_2 N$ respectively. Hence for U number of alternative OFDM signals the number of complex multiplications becomes $\frac{UN}{2}\log_2 N$ and the number of complex additions becomes $UN\log_2 N$. So increasing of U leads to increasing of computational complexity. We proposed one technique that has been discussed in next section which reduces the computational complexity and also simultaneously reduces the PAPR. There is also one existing SLM technique [17] which reduces the computational complexity without affecting the PAPR reduction performance. To acquire knowledge about this technique we have to know about two techniques, one is Bit Based Selected Mapping (BSLM) [18] and the another is Partial Bit Based Selected Mapping (PBISLM) [18].

According to the classical SLM technique the rotation of phases of QAM (Quadrature Amplitude Modulated) symbols takes place in the frequency domain but the mag-

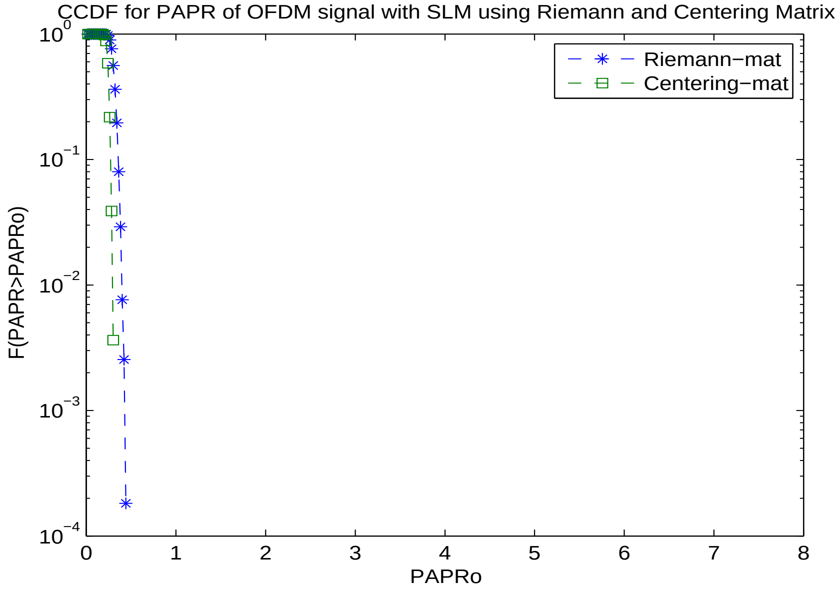

Figure 2.8: PAPR reduction performance using the Centering matrix

nitude part remains same. But In case of both the BSLM and PBISLM technique the changing of simultaneous phase and magnitude of the symbols take place.

Bitwise SLM (BSLM) generates the alternative symbol sequences through multiplying a binary data sequence by binary data phase sequences with the same length as that of the binary data sequence. An input symbol sequence $\mathbf{A}$ of length N with $M-$QAM modulated can be expressed as the following binary sequence of length $Nlog_2M$.

$$\mathbf{A_B} = [\mathbf{A_{0,0}}, \mathbf{A_{0,1}}.....\mathbf{A_{0,log_2M-1}}.....\mathbf{A_{N-1,log_2M-1}}] \tag{2.13}$$

where $A_{k,b} \in \{\pm 1\}$ and $A_{k,b}$ denotes the bth bit of the kth $M-$QAM symbol. If a phase sequence $P^{(u)}$ (i.e the uth row of the phase matrix P) is a binary sequence composed of $\{\pm 1\}$ with length $Nlog_2M$. The uth alternative binary sequence is generated by multiplying the input symbol sequence in the binary form by the uth binary phase sequence before mapped to $M-$QAM symbols as

$$X_B^{(u)} = \left[A_{0,0}P_0^{(u)}....A_{0,log_2M-1}P_{log_2M-1}^{(u)}...A_{N-1,log_2M-1}P_{Nlog_2M-1}^{(u)}\right] \tag{2.14}$$

Then these alternative binary sequences are mapped to $M-$QAM symbols and the alternative symbol sequences X^u are generated. After doing the IFFT to these alternative symbol sequences the OFDM signal sequence x^u are generated. Then the v^{th} OFDM signal sequence with minimum PAPR is selected for transmission, where

$$v = argmin_{0\leq u<U} PAPR\left(x^u\right) \tag{2.15}$$

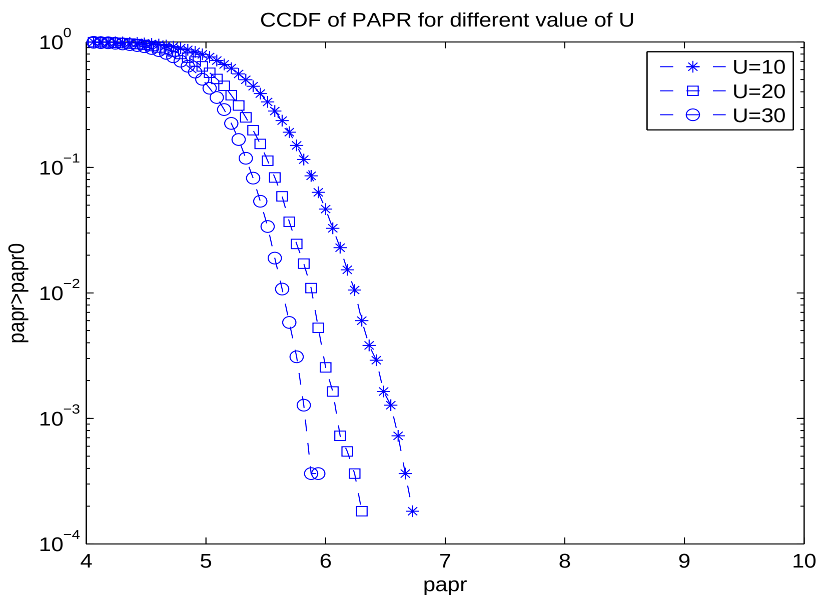

Figure 2.9: PAPR comparison considering different number of rows

Another one important technique is Partial Bit Inverted SLM (PBISLM). According to this scheme the alternative symbol sequences are generated by multiplying some preselected bits of each $M-$QAM symbol A_k by $P_k^{(u)}$ to generate alternative symbol

$X_k^{(u)}$,where $P_k^{(u)} \in \{\pm 1\}$. As $M-$QAM symbol consists of $log_2 M$ number of bits with denoting a set of bit indices $T = [0, 1, ..., log_2 M - 1]$. Let $R = [b_0, b_1, ..., b_{S-1}]$ be the subset of T. The tth bit $X_{k,t}^{(u)}$ of the kth symbol in the binary form of the uth alternative symbol sequence can be written as

$$X_{k,t}^{(u)} = \begin{Bmatrix} A_{k,t} P_k^{(u)}, \ t \in R \\ \\ A_{k,t}\,, \quad t \in R^c \end{Bmatrix} \tag{2.16}$$

If $P_k^{(u)}$ is -1, the S number of bits of A_k corresponding to R are inverted and we will get the mapped version of $A_k^{(u)}$ as another $M-$QAM symbol $X_k^{(u)}$. Then by doing IFFT the OFDM signal sequences are being generated and the signal with minimum PAPR is selected for transmission. The figure 2.10 shows the generation of mapped QAM symbols with considering the change of all bits. The PAPR reduction performance and the covariance analysis of this BSLM and PBISLM are to be shown in the next section. One technique [17] specially depends upon the PBISLM for the reduction of complexity is being described as follows.

If we consider one alternative phase sequence consists of elements are -1 and also allow all bits under go into change then we will found the alternative symbols according to the figure 2.10. So according to this figure we can write the kth alternative symbol of the uth alternative symbol sequence for the phase sequence $P_k^{(u)} \in \{\pm 1\}$ can be

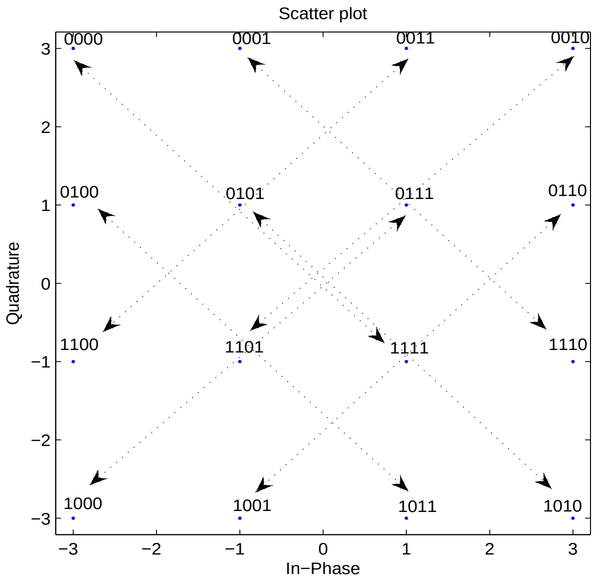

Figure 2.10: An example of PBISLM with 16 QAM

expressed as

$$X_k^{(u)} = A_k + D_k^{(u)}$$
$$= \begin{cases} A_k - D(M), & if\ A_k \in Q^{(1)}\ and\ P_k^{(u)} = -1 \\ A_k + D(M)^*, & if\ A_k \in Q^{(2)}\ and\ P_k^{(u)} = -1 \\ A_k + D(M), & if\ A_k \in Q^{(3)}\ and\ P_k^{(u)} = -1 \\ A_k - D(M)^*, & if\ A_k \in Q^{(4)}\ and\ P_k^{(u)} = -1 \\ A_{k,} & if\ P_k^{(u)} = 1 \end{cases} \tag{2.17}$$

where $D(M) = d(1+j)\sqrt{M}/2$ and $D(M)^*$ denotes the complex conjugate of $D(M)$. Here d represents the smallest distance between $M-$QAM symbols, and $Q^{(l)}$ is the set of symbols belonging to the lth quadrant of the 2-dimensional signal constellation. So the uth additive mapping sequence $D^{(u)} = \left[D_0^{(u)}, D_1^{(u)}, ..., D_{N-1}^{(u)},\right]^T$ can be found according to the equation 2.17, which depends upon the uth phase sequence $P^{(u)}$ and the position of $M-$QAM symbol sequence. Let us consider two different phase sequence $P^{(-1)}$ (i.e. the sequence consists all are -1) and $P^{(m)}$ (i.e a random phase sequence $\in \{\pm 1\}$). Where $1 \leq m \leq V$ and for the present example consider $V = 1$. v is the extra number of random phase sequences except the first one i.e. $P^{(-1)}$. This V can be generalized as $V = \left\lceil \frac{(U-4)}{12} \right\rceil$ being discussed later and $\lceil y \rceil$ is the smallest integer not less than y. So according to these phase sequences the additive mapping sequences $D^{(-1)}$ and $D^{(m)}$ can be generated. We can represent the additive mapping sequence $D^{(-1)}$ as

$$D^{(-1)} = D_I^{(-1)} + jD_Q^{(-1)} \tag{2.18}$$

where $D_I^{(-1)}$ and $D_Q^{(-1)}$ denote in-phase and quadrature components respectively of $D^{(-1)}$. Hence we also can get

$$D^{(m)} = D_I^{(m)} + jD_Q^{(m)} \tag{2.19}$$

Then find the IFFT of the additive mapping sequences i.e.

$$IFFT\left(D^{(-1)}\right) = d_I^{(-1)} + jd_Q^{(-1)} \tag{2.20}$$

$$IFFT\left(D^{(m)}\right) = d_I^{(m)} + jd_Q^{(m)} \tag{2.21}$$

So now we have four number of additive mapping sequences that are $d_I^{(-1)}$,$jd_Q^{(-1)}$,$d_I^{(m)}$ and $jd_Q^{(m)}$. Then we can generate 16 number of alternative signal sequences according to the equation given by

$$x^{(u)} = a + b^{(u)} \tag{2.22}$$

where $a = IFFT(A)$ and $b^{(u)}$ be the 16 different combinations of the above four additive mapping sequences as shown below

$b^{(1)} = 0$

$b^{(2)} = d_I^{(-1)}$

$b^{(3)} = jd_Q^{(-1)}$

$b^{(4)} = d_I^{(-1)} + jd_Q^{(-1)}$

$b^{(5)} = d_I^{(m)}$

$b^{(6)} = jd_Q^{(m)}$

$b^{(7)} = d_I^{(m)} + jd_Q^{(m)}$

$b^{(8)} = d_I^{(-1)} - d_I^{(m)}$

$b^{(9)} = d_I^{(-1)} + jd_Q^{(m)}$

$b^{(10)} = d_I^{(-1)} - d_I^{(m)} + jd_Q^{(m)}$

$b^{(11)} = jd_Q^{(-1)} + d_I^{(m)}$

$b^{(12)} = jd_Q^{(-1)} - jd_Q^{(m)}$

$b^{(13)} = jd_Q^{(-1)} + d_I^{(m)} - jd_Q^{(m)}$

$b^{(14)} = d_I^{(-1)} + jd_Q^{(-1)} - d_I^{(m)}$

$b^{(15)} = d_I^{(-1)} + jd_Q^{(-1)} - jd_Q^{(m)}$

$b^{(16)} = d_I^{(-1)} + jd_Q^{(-1)} - d_I^{(m)} - jd_Q^{(m)}$

According to the 16 number of alternative additive mapping sequences i.e. $b^{(1)}$ to $b^{(16)}$ the 16 different signal sequences are to be generated by following the equation 2.22. The computational complexity of two real $N-$point IFFTs is equivalent to that of one complex $N-$point IFFT and $N-2$ complex additions. [19]. If L being considered as the oversampling factor then the IFFT length will become LN. For this technique to get the first 4 alternative signal sequences it is required to generate a, $d_I^{(-1)}$ and $d_Q^{(-1)}$. To generate $d_I^{(-1)}$ and $d_Q^{(-1)}$ two real IFFTs are required which is equivalent to one complex IFFT and $LN-2$ complex additions. So out of U number of alternative signal sequences 4 numbers already being generated. Hence to generate extra $U-4$ number of alternative signals it required to consider V number of alternative phase sequences i.e. $P^{(m)}$, where $1 \leq m \leq V$ and the V can be calculated as $V = \left\lceil \frac{(U-4)}{12} \right\rceil$. The number of The calculation of the total number of complex multiplications and complex additions are given below.

- **Number of Complex Multiplications**

  The total number of complex multiplications to generate U number of alternative signal sequences is given as

$$\begin{aligned} & 2\tfrac{LN}{2}\log_2(LN) + V\tfrac{JN}{2}\log_2(LN) \\ & = (2+V)\tfrac{LN}{2}\log_2(LN) \end{aligned} \tag{2.23}$$

- **Number of Complex Additions**

  In addition to the IFFT operations LN number of complex additions will be required to generate each alternative signal except the first one. So the total number of complex additions will be found to be

$$\begin{aligned} & 2LN\log_2(LN) + VLN\log_2(LN) \\ & \quad + (V+1)(LN-2) + (12V+3)LN \\ & = (2+V)LN\log_2(LN) + (13V+4)LN - 2(V+1) \end{aligned} \tag{2.24}$$

This technique used for reduction of complexity without affecting the PAPR reduction performance. The figure 2.11 represents the performance of the PAPR reduction of this modified technique with comparing to that of the conventional SLM using 64 number of subcarriers and 16-QAM modulation scheme. Also the plot for number of complex additions with respect to the different number of rows of phase matrix being shown in figure 2.12 and also about the number of complex multiplication shown in figure 2.13.

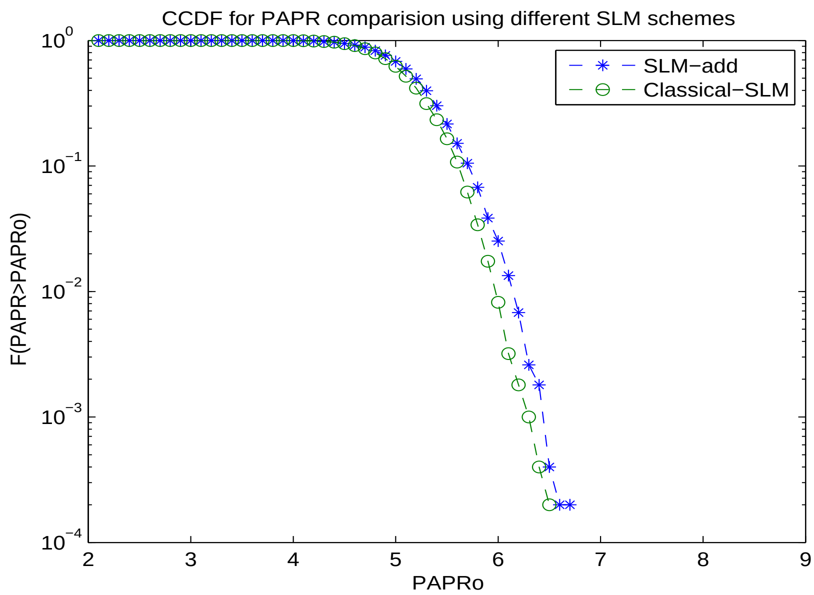

Figure 2.11: PAPR Reduction Performance of this Modified technique

2.2.4 Analysis of Covariance

If we consider two random variables X and Y then the covariance between these two random variables is defined as

$$\begin{aligned} C_{xy} &= E\left[(X-\mu_x)(Y-\mu_y)\right] \\ &= E\left[XY\right] - E\left[X\right]E\left[Y\right] \end{aligned} \tag{2.25}$$

where $E[X] = \mu_x$ (mean of X) and $E[Y] = \mu_y$ (mean of Y). As we are using the random phase vectors to generate the alternative OFDM signals in case of this SLM

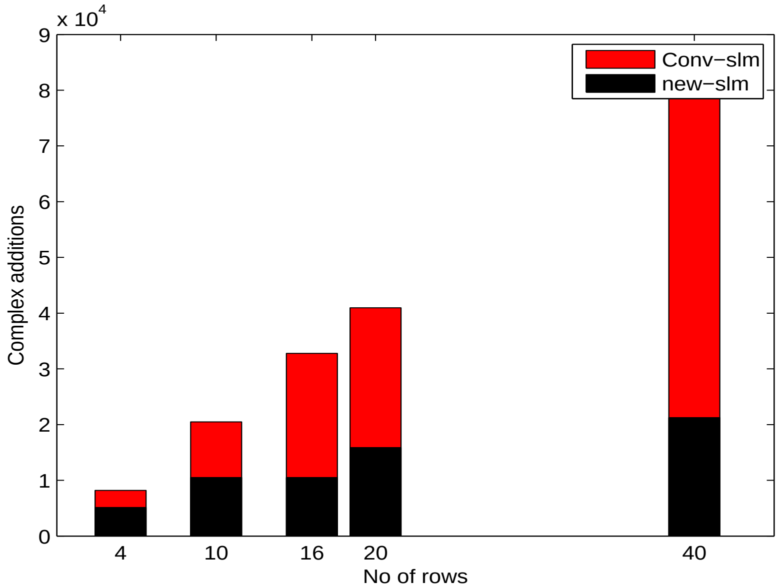

Figure 2.12: No. of complex additions w.r.t. the no. of rows

technique so it is important to analyze the covariance between the two alternative phase vectors. It is because we know that the PAPR reduction performance improves if the alternative OFDM signals are mutually independent [13]. When the number of subcarriers N for the OFDM signal is large then according to the central limit theorem [20] the time domain samples have a Gaussian distribution.

If the OFDM signal sequences are complex Gaussian distributed then the zero covariance of two alternative OFDM signals guarantee the mutual independency between them. But if OFDM signal sequences are not complex Gaussian distributed then the zero covariance of two alternative OFDM signals does not guarantee the mutual independency between them. So according to the central limit theorem if the N value is not very large then we can not consider the time domain samples of the OFDM signal as Gaussian distribution. For this case of OFDM signal sequences which are not Gaussian distributed, the zero covariance between two alternative sig-

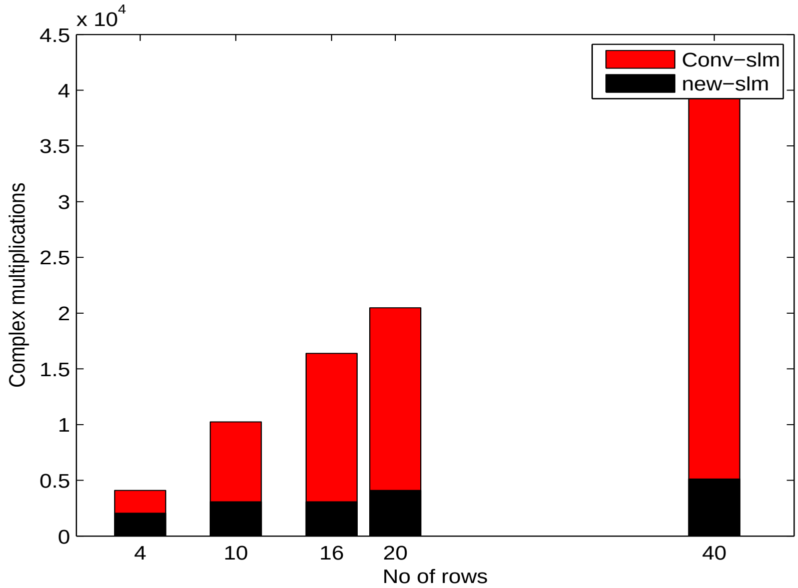

Figure 2.13: No. of complex multiplications w.r.t. the no. of rows

nals does not guarantee the mutual independency between them. In this case instead of covariance, we consider the property of joint cumulants of alternative OFDM signals. If the joint cumulants of all orders are equal to zero then the two alternative OFDM signal sequences are mutually independent [18]. Let us consider the lth and mth alternative OFDM signals $x_n^{(l)}$ and $x_n^{(m)}$. According to [18] the fourth order joint cumulant between these two alternative sequences can be given as

$$\begin{aligned} & cum\left(x_n^{(l)},\ x_n^{(l)*}, x_n^{(m)}, x_n^{(m)*}\right) \\ & = E\left[x_n^{(l)} x_n^{(l)*} x_n^{(m)} x_n^{(m)*}\right] - E\left[x_n^{(l)} x_n^{(l)*}\right] E\left[x_n^{(m)} x_n^{(m)*}\right] \\ & = E\left[\left(\frac{1}{N}\sum_{k=0}^{N-1}\left|A_k\gamma_k^{(l)}\right|^2\right)\left(\frac{1}{N}\sum_{k=0}^{N-1}\left|A_k\gamma_k^{(m)}\right|^2\right)\right] \\ & \quad -E\left[\frac{1}{N}\sum_{k=0}^{N-1}\left|A_k\gamma_k^{(l)}\right|^2\right] E\left[\frac{1}{N}\sum_{k=0}^{N-1}\left|A_k\gamma_k^{(m)}\right|^2\right] \end{aligned} \tag{2.26}$$

Here γ_k is the gain of kth $M-$QAM symbol. If the equation 2.9 is being satisfied then we will get the fourth order joint cumulant as the above equation 2.26. We can write the average symbol power of uth symbol sequence as

$$\bar{P}^{(u)} = \frac{1}{N} \sum_{k=0}^{N-1} \left| A_k \gamma_k^{(u)} \right|^2 \tag{2.27}$$

According to the equation 2.25 the covariance between the average symbol power of the lth and mth alternative symbol sequences will be

$$\operatorname{cov}\left(\bar{P}^{(l)}, \bar{P}^{(m)}\right) = E\left[\left(\bar{P}^{(l)} - E\left[\bar{P}^{(l)}\right]\right)\left(\bar{P}^{(m)} - E\left[\bar{P}^{(m)}\right]\right)\right] \tag{2.28}$$

With verifying the equation 2.26,2.27 and 2.28 the conclusion can be drawn as the fourth order joint cumulant is equivalent to the covariance of average symbol powers of alternative symbol sequences. By simplifying the equation 2.26 and 2.28 the covariance between the average symbol power of the lth and mth alternative symbol sequences can be found as

$$\operatorname{cov}\left(\bar{P}^{(l)}, \bar{P}^{(m)}\right) = \frac{1}{N}\left(E\left[|A_k|^4 \left|\gamma_k^{(u)}\right|^2 \left|\gamma_k^{(m)}\right|^2\right] - 1\right) \tag{2.29}$$

If the equation 2.29 becomes zero then $X^{(l)}$ and $X^{(m)}$ are mutually independent, that is, two alternative symbol sequences are generated independently. However in case of the conventional SLM $\left|\gamma_k^{(u)}\right| = 1$ but $E\left[|A_k|^4\right] \neq 1$ which makes the value of covariance in equation 2.29 as non zero even if the phase sequences satisfy the optimality conditions. It concludes that the mutually independent alternative OFDM signal sequences cannot be generated by using the conventional SLM scheme. Therefore, we have to design the scheme such that the amplitude gain $\gamma_k^{(u)}$ can be changed. This can be possible with using two SLM schemes that are BSLM and PBISLM. As we have discussed in case of the PBISLM technique the pre selected bits of the symbol are going to be changed. For our simulation work we have considered PBISLM type-I as some preselected bits of the $M-$QAM symbols will be considered and in case of PBISLM type-II all the bits are to be considered.

The figure 2.14 shows the covariance plot with respect to the number of subcarriers for the conventional SLM,BSLM and the PBISLM with considering 16-QAM and 64-QAM modulation scheme. Considering 128 number of subcarriers and 16-QAM

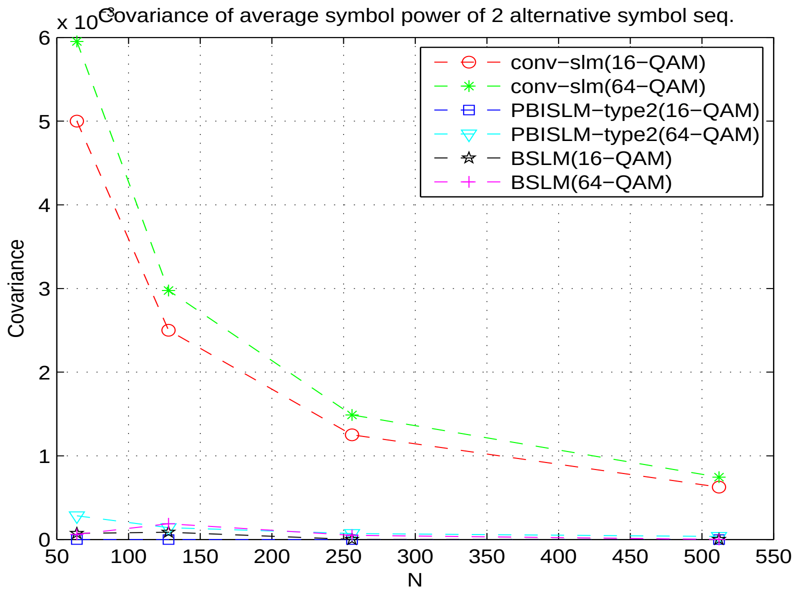

Figure 2.14: Covariance plot for different SLM techniques

modulation scheme the figure 2.15 shows the comparison of the theoretical PAPR reduction plot with that of the BSLM technique and the conventional SLM technique. As we know that the covariance of average symbol powers of alternative symbol sequences for the conventional SLM is non zero, hence the PAPR reduction plot moves away from the theoretical plot. But for BSLM it follows the theoretical plot. Similarly the figure 2.16 shows the analysis of PAPR reduction for the PBISLM, BSLM, Conventional SLM and theoretical plot with considering 64-QAM modulation scheme. Here the PBISLM type-I means the bit positions are taken to be $R = [0, 2, 3, 5]$ in case of the 64-QAM modulation scheme and PBISLM type-II means all bits are to be considered.

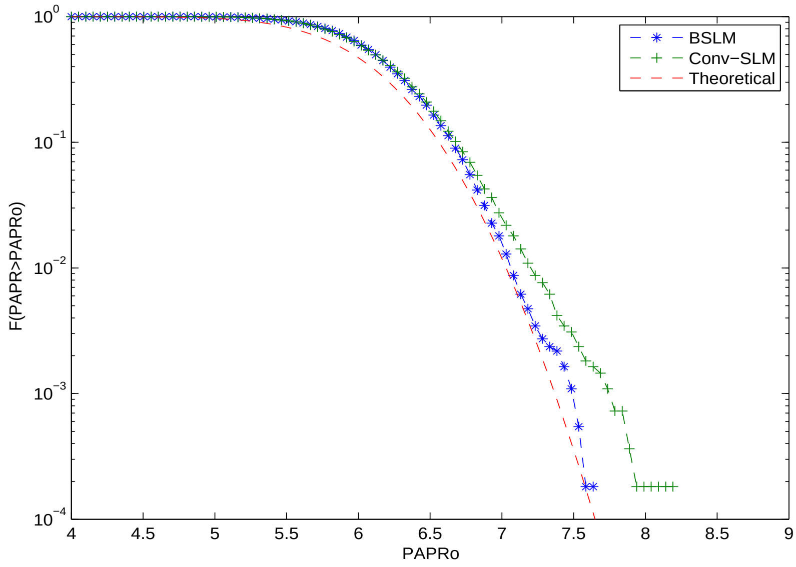

Figure 2.15: PAPR comparison with the theoretical plot

2.3 New Scheme For Reduced Complexity

According to the idea in SLM the original data block will be converted into several independent signals and the signal having lowest PAPR is going to be transmitted. To get back the original data block it must be required to send side information as a set of bits along with the selected signal. The erroneous detection of this side information will give arise to loss of the whole data block. So this is one of the disadvantages of SLM technique. Another disadvantage of this technique is its high complexity due to presence of a lot of IFFT blocks before selecting a particular OFDM signal. Here a method being being proposed to generate a random matrix from the existing phase matrix of the classical SLM technique which fullfils the criteria that the new matrix has less number of rows than that of the existing matrix. According to the discussion

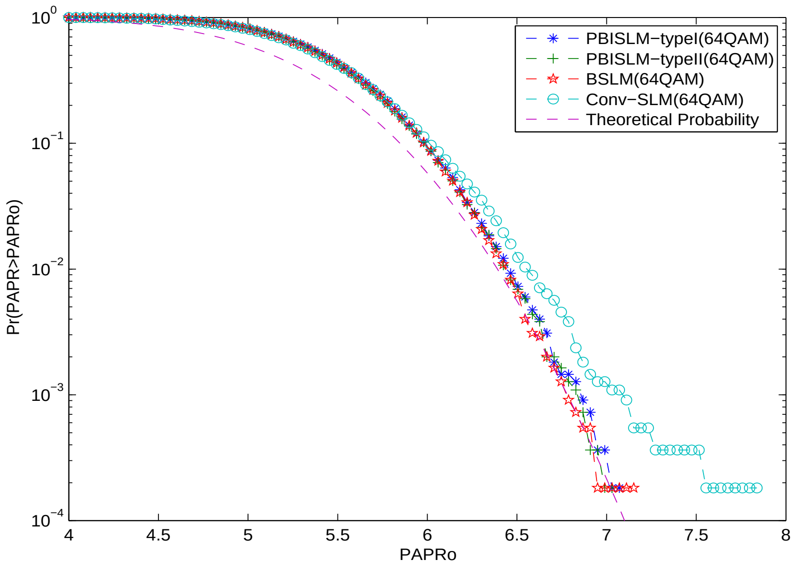

Figure 2.16: PAPR comparison with the theoretical plot

in the previous section that this reduction of number of rows leads to the reduction in computational complexity. Also by using this new technique the original data block can be detected without sending any side information along with the selected signal. But for detecting the information about the row of the random matrix that has been multiplied at the transmitter side here we have applied the sub-optimal algorithm [3]. The PAPR of this technique is also being reduced than that of the classical SLM technique. The alternative phase vectors that are used in the classical SLM technique can be considered as a $U \times N$ matrix, where U denotes the total number of alternative signals and N denotes the number of subcarriers. According to the classical SLM technique we should generate the random matrix on the basis of the criteria described in [13]. So we got the matrix $\mathbf{B}$, where $B\left(u,n\right)$ is the nth value of the uth row. Then generate a new matrix B_1 having $\frac{U}{2}$ number of rows according to the following steps.

- Find each row of the new matrix by doing the following additions

$$\begin{aligned} B_1(1) &= B(1) + B(2) \\ B_1(2) &= B(3) + B(4) \\ &. \\ &. \\ B_1\left(\tfrac{U}{2}\right) &= B(U-1) + B(U) \end{aligned} \tag{2.30}$$

- Then we may get some elements of the matrix as zero. So we should apply the following condition.

  If $B_1(u,n) = 0$ then put $B_1(u) = 1$ or we can also put $B_1(u) = -1$.

Each row of the matrix B is a set of random variables. Let us consider two random variables X and Y with variances σ_x^2 and σ_y^2 respectively. The sum of two random variables will be another random variable i.e. $Z = X + Y$ having variance σ_z^2 defined as

$$\begin{aligned} \sigma_z^2 &= E\left[(Z-\mu_z)^2\right] \\ &= E\left\{\left[(X-\mu_x)+(Y-\mu_y)\right]^2\right\} \end{aligned} \tag{2.31}$$

Where μ_x,μ_y and μ_z are the mean of X, Y and Z respectively i.e. $E\{X\} = \mu_x$,$E\{Y\} = \mu_y$ and $E\{Z\} = \mu_z$. As $Z = X + Y$ so $\mu_z = \mu_x + \mu_y$. After simplifying the above equation 2.31 we will get that

$$\sigma_z^2 = \sigma_x^2 + 2\rho_{xy}\sigma_x\sigma_y + \sigma_y^2 \tag{2.32}$$

Where ρ_{xy} is known as the correlation coefficient i.e. defined as the ratio between the covariance of two random variables X and Y to the product of their standard deviations.

$$\rho_{xy} = \frac{C_{xy}}{\sigma_x \sigma_y} \tag{2.33}$$

As the rows of this random matrix are mutually independent to each other so the covariance will be zero hence $\sigma_z^2 = \sigma_x^2 + \sigma_y^2$.

Hence from these analysis it is to be known that by adding to random variables the variance of the resulting random variable increases. So the variance of each row of the matrix B_1 will be more than that of the matrix B which leads to the further reduction of PAPR compared to the classical SLM technique. The number of complex additions and multiplications for N point DFT using FFT algorithm will be $Nlog_2N$ and $\frac{N}{2}log_2N$ respectively.

For the classical SLM technique:

- Number of complex additions $= UNlog_2N$
- Number of complex multiplications $= \frac{UN}{2}log_2N$

For the proposed technique:

- Number of complex additions $= \frac{U}{2}Nlog_2N$
- Number of complex multiplications $= \frac{U}{2}\frac{N}{2}log_2N$

So by using this technique 50 percent complexity for computations has been reduced. The simulation plot for PAPR reduction performance with considering 128 number of subcarriers and with over sampling factor of 4 is shown in the figure 2.17. According to this figure the proposed scheme i.e. the new SLM technique has better PAPR reduction than the conventional one. The plot for reducing the number of complex

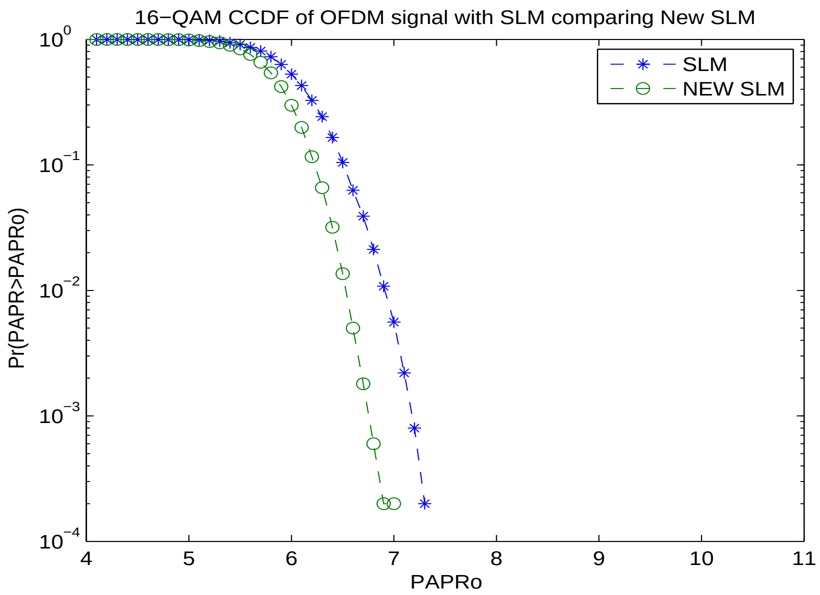

Figure 2.17: PAPR comparison with the Conventional SLM technique

additions with respect to different number of rows being shown in the figure 2.18 and the plot for that of the number of complex multiplications shown in the figure 2.19.

Also with using this technique the extra side information is not required to send along with the selected OFDM signal. This side information can be detected with using the sub-optimal algorithm [3]. The bit error rate plot performance in figure 2.20 shows between the two cases, one with considering the detection of perfect SI (Side Information) index at the receiver side another one with applying sub-optimal algorithm at the receiver side.

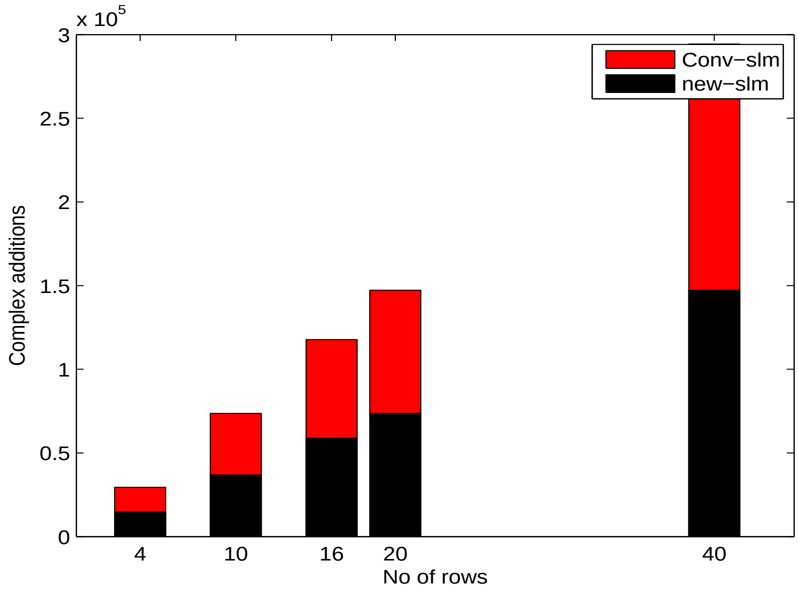

Figure 2.18: No.of complex additions w.r.t. the no. of rows

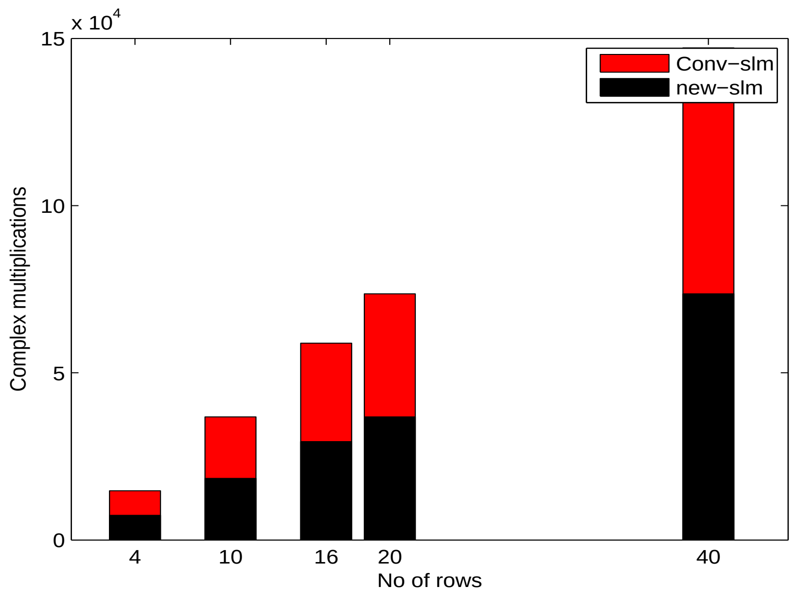

Figure 2.19: No.of complex multiplications w.r.t. the no. of rows

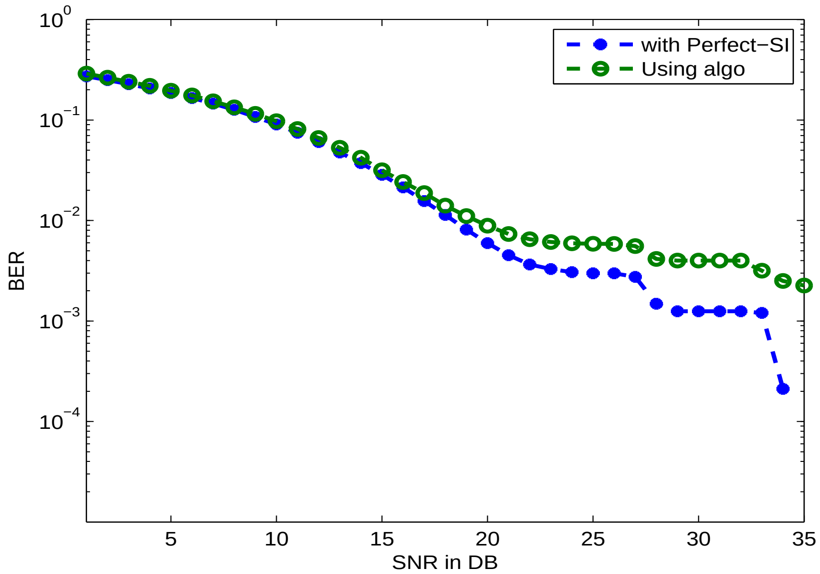

Figure 2.20: Bit Error Rate Performance

Chapter 3

Application to MIMO-OFDM

3.1 Introduction to MIMO-OFDM

The antenna diversity is a technique which combats the effect of frequency selective multipath fading channel. If at the base station multiple antennas are used and at the remote unit only one antenna is used then i.e. called the transmit diversity. We can also call it as Multiple Input Single Output (MISO) case. This diversity technique is very economical. If at the transmitter side we use single antenna and at the receiver side multiple antenna then that will be known as receiver diversity or SIMO (Single Input Multi Output) system. If we use multiple antennas at both transmitter and receiver side then that will be known as MIMO (Multi Input Multi Output) system. As we are using OFDM technique before transmitting the message through the antenna hence it will be called as MIMO-OFDM Technique.

3.2 Application of SLM technique

Here the application of Selected Mapping technique [21] has done on the transmit diversity case especially for the case of two transmitting antenna and one receiving antenna (i.e. 2×1 MISO). So to transmit a signal from these two antennas we should

Time	antenna1	antenna2
time t	x_1	x_2
time $t+T$	$-x_2^*$	$-x_1^*$

Table 3.1: STBC for transmit diversity

have to follow some transmit diversity technique. Here the simulation works are being analyzed with considering a well known transmit diversity scheme i.e. known as the Alamouti coding scheme [22]. According to this diversity technique there will be two encoding schemes that to be used at the transmitter side, one is the Space Time Block Coding (STBC) scheme and the another one is Space Frequency Block Coding (SFBC) scheme shown below.

Space Time Block Coding (STBC)

In this case at a given symbol period, two signals are simultaneously transmitted from the two antennas. The signal transmitted from antenna one is denoted by x_1 and from antenna two by x_2. Then during the next symbol period the antenna one transmits the signal $-x_2^*$ and signal $-x_1^*$ is transmitted from antenna two where x^* is the complex conjugate operation. This encoding scheme is shown in the table 3.1. According to the table encoding is done in space and time. The coding also can be done in space and frequency i.e. described below.

Space Frequency Block Coding (SFBC)

Here instead of two adjacent symbol periods, two adjacent carriers can be used. Let us consider the original OFDM frame as X then the two vectors X_1 and X_2 will be generated using this SFBC as follows

$$\begin{pmatrix} X_1\,(2k) & X_1\,(2k+1) \\ X_2\,(2k) & X_2\,(2k+1) \end{pmatrix} = \begin{pmatrix} X\,(2k) & X\,(2k+1) \\ X^*\,(2k+1) & -X^*\,(2k) \end{pmatrix} \tag{3.1}$$

where $k = 0, 1, ..., \frac{N_c}{2} - 1$. The block diagram shown in figure 3.1 describes the way of applying this SLM technique into MIMO-OFDM system.

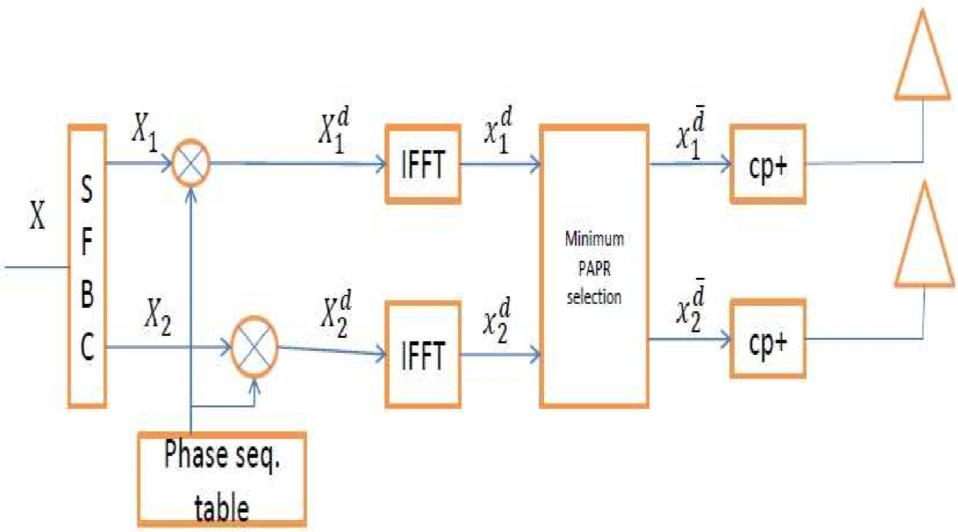

Figure 3.1: Block Diagram for application of SLM to MIMO

For the simulation studies the SFBC scheme has been used. According to the figure 3.1 the same phase sequence will be multiplied to the two different signals that are X_1 and X_2. Then do the IFFT of these signals for one antenna and choose the OFDM signal with minimum PAPR and also the same thing will be done for the another antenna. Then to find out the Complementary Cumulative Distribution Function plot for the performance analysis of PAPR the maximum PAPR value will be considered out of two different minimum PAPR value from that of two antennas. So with considering 64 number of subcarriers and oversampling factor of 4 the PAPR reduction performance has been shown in figure 3.2. Also with considering Riemann matrix the simulation for PAPR reduction being shown in figure 3.3.

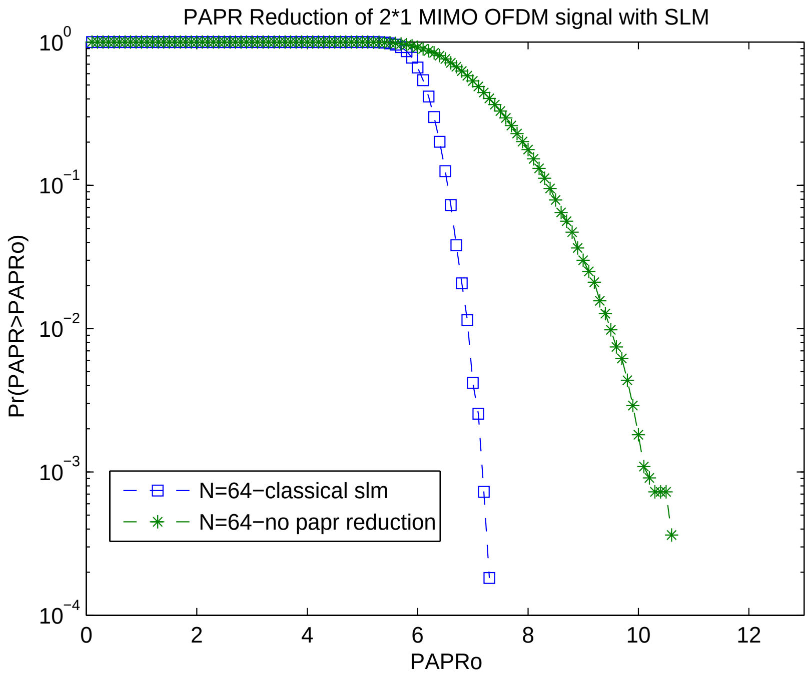

Figure 3.2: PAPR Reduction of 2*1 MIMO OFDM signal with SLM

Also the application of the proposed scheme has done for this 2×1 transmit diversity case with consideration of 64 number of subcarriers and over sampling factor of 4 which is shown in figure 3.4

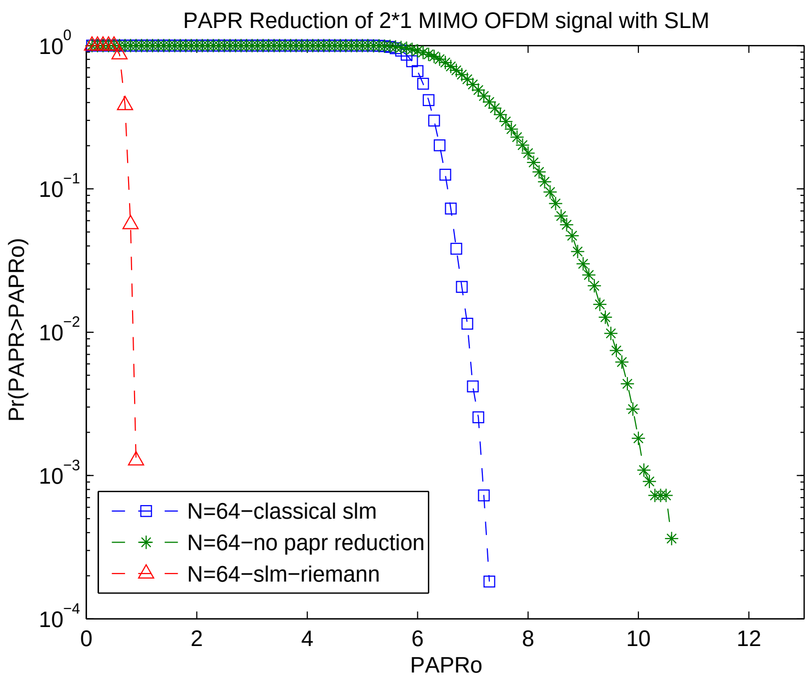

Figure 3.3: PAPR Reduction of 2*1 MIMO OFDM signal with Riemann Matrix

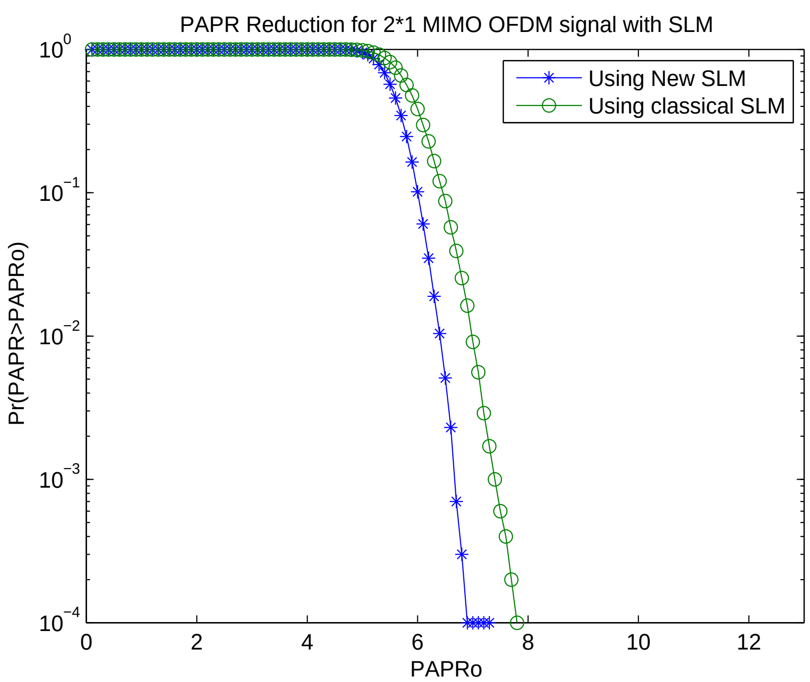

Figure 3.4: PAPR Reduction of 2*1 MIMO OFDM signal with Proposed Scheme

Chapter 4

Conclusion and Future Work

4.1 Conclusion

Here various types of Selected Mapping technique have been verified for the PAPR reduction performance. Some techniques also being there which avoids the sending of Side Information (SI) index along with the selected OFDM signal. One technique also being described with low computational complexity having same PAPR reduction criteria as that of the classical SLM. Also some techniques are presented here which satisfy the criteria of the mutual independence between the alternative phase sequences that leads to better PAPR reduction.

The proposed scheme also being presented here which has better PAPR reduction performance than that of the classical SLM. Moreover it also fulfills the criteria of low computational complexity. But this amount in reduction of complexity is not better than that of the technique depending upon the PBISLM. This proposed scheme has an additional advantage of avoiding the extra SI index along with the OFDM signal.

Also verification of this technique has been done for the MIMO-OFDM system. It is much more required to reduce the computational complexity in case of transmit diversity case than that of SISO (Single Input Single Output) case. Which can be achieved with using this proposed technique and also the technique of SLM with using the additive mapping sequences.

4.2 Future Work

The application of this Selected Mapping technique also can be verified in the OFDMA system. Analysis for avoiding the sending of SI index in case of the Riemann matrix should be done.

Also further reduction of the computational complexity for the proposed technique can be predicted.

Publication

- Mishra, Himanshu Bhusan; Mishra, Madhusmita; Patra, Sarat Kumar, "**Selected mapping based PAPR reduction in WiMAX without sending the side information**," *IEEE. Conf. RAIT*, 2012., pp. 182-184.

Bibliography

[1] Seung Hee Han and Jae Hong Lee. An overview of peak-to-average power ratio reduction techniques for multicarrier transmission. *IEEE Wireless Communications*, 12(2):56 – 65, 2005.

[2] R.W. Bauml, R.F.H. Fischer, and J.B. Huber. Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping. *Electronics Letters*, 32(22):2056 – 7, 1996/10/24.

[3] S.Y. Le Goff, S.S. Al-Samahi, Boon Kien Khoo, C.C. Tsimenidis, and B.S. Sharif. Selected mapping without side information for papr reduction in ofdm. *IEEE Transactions on Wireless Communications*, 8(7):3320 – 5, 2009/07/.

[4] B. R. Saltzberg. Performance of an efficient parallel data transmission system. *IEEE Trans. Commun.*, 15(6):80511, Dec 1967.

[5] S.B. Weinstein and P.M. Ebert. Data transmission by frequency-division multiplexing using the discrete fourier transform. *IEEE Transactions on Communication Technology*, CM-19(5):628 – 34, 1971/10/.

[6] Jr. L. J. Cimini. Analysis and simulation of a digital mobile channel using orthogonal frequency division multplexing. *IEEE Trans. Commun.*, 33(7):66575, July 1985.

[7] Won Young Yang Chung-Gu Kang Yong Soo Cho, Jaekwon Kim. *MIMO-OFDM Wireless Communications With Matlab*. John Wiley & Sons, illustrated edition, 2010.

[8] R. O'Neill and L.B. Lopes. Envelope variations and spectral splatter in clipped multicarrier signals. volume 1, pages 71 – 75, Toronto, Can, 1995.

[9] Xiaodong Li and Jr. Cimini, L.J. Effects of clipping and filtering on the performance of ofdm. *IEEE Communications Letters*, 2(5):131 – 3, 1998/05/.

[10] A.E. Jones, T.A. Wilkinson, and S.K. Barton. Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission schemes. *Electronics Letters*, 30(25):2098 – 9, 1994/12/08.

[11] S. H. Mller and J. B. Huber. Ofdm with reduced peaktoaverage power ratio by optimum combination of partial transmit sequences. *Elect. Lett.*, 33(5):36869, Feb 1997.

[12] B.S. Krongold and D.L. Jones. Par reduction in ofdm via active constellation extension. volume vol.4, pages 525 – 8, 2003//.

[13] G.T. Zhou and Liang Peng. Optimality condition for selected mapping in ofdm. *IEEE Transactions on Signal Processing*, 54(8):3159 – 65, 2006/08/.

[14] F Roesler. Riemanns hypothesis as an eigenvalue problem. *Linear Algebra Appl*, 81:153198, 1986.

[15] N.V. Irukulapati, V.K. Chakka, and A. Jain. Slm based papr reduction of ofdm signal using new phase sequence. *Electronics Letters*, 45(24):1231 – 2, 2009/11/19.

[16] Thitapha Chanpokapaiboon, Potchara Puttawanchai, and Prapun Suksompong. Enhancing papr performance of mimo-ofdm systems using slm technique with centering phase sequence matrix. pages 405 – 408, Khon Kaen, Thailand, 2011.

[17] Hyun-Bae Jeon, Jong-Seon No, and Dong-Joon Shin. A low-complexity slm scheme using additive mapping sequences for papr reduction of ofdm signals. *IEEE Transactions on Broadcasting*, 57(4):866 – 75, 2011/12/.

[18] Hyun-Bae Jeon, Kyu-Hong Kim, Jong-Seon No, and Dong-Joon Shin. Bit-based slm schemes for papr reduction in qam modulated ofdm signals. *IEEE Transactions on Broadcasting*, 55(3):679 – 85, Sept. 2009.

[19] H.V. Sorensen, D.L. Jones, M.T. Heideman, and C.S. Burrus. Real-valued fast fourier transform algorithms. *IEEE Transactions on Acoustics, Speech and Signal Processing*, ASSP-35(6):849 – 63, 1987/06/.

[20] S Pillai Athanasios Papoulis. *Probability, Random Variables And Stochastic Processes*. Tata Mcgraw Hill Education Private Limited, 4thedition edition, 2002.

[21] M.F. Naeiny and F. Marvasti. Selected mapping algorithm for papr reduction of space-frequency coded ofdm systems without side information. *IEEE Transactions on Vehicular Technology*, 60(3):1211 – 16, 2011/03/.

[22] S.M. Alamouti. A simple transmit diversity technique for wireless communications. *IEEE Journal on Selected Areas in Communications*, 16(8):1451 – 8, 1998/10/.