PREDICTIVE STATE FEEDBACK CONTROL OF NETWORK CONTROL SYSTEMS

A THESIS

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IN

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CERTIFICATE

This is to certify that the thesis entitled "**Predictive State Feedback Control of Network Control Systems**" submitted by **Miss Aradhana Nayak**, in partial fulfilment of the requirements for the award of Bachelor of Technology in the Department of Electrical Engineering, at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by her under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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Thank you all

-Aradhana Nayak

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CONTENTS

ABSTRACT

Networked control systems have gained attention in the recent years due to their widespread applications to various real time systems. Controlling these systems poses several challenges which are currently still being investigated. A study of these issues is provided along with recent proceedings in technology to counter such issues like limited bandwidth, time delays and packet drop-outs. This thesis focuses on the problem of time delays in network control system which can cause instability of closed loop operation of these systems. A guaranteed cost approach is employed to achieve stability along with achieving a certain level of performance as defined by the cost function. A state feedback controller is used and along with it, a predictive control scheme is implemented to design variable gains of the feedback controller depending on the number of packets missed (packet drop-outs) and time delays of the received input sample or state of the plant, both of which can be random but bounded for a given communication channel. The controllers are connected to the plant via the network. They generate the appropriate input for the plant so that delays in the channel will not instabilize the system and thus they comprise the network delay compensator. The controller gains and the observer gain are determined by formulating a linear matrix inequality (LMI) problem and solving this problem by using the Robust Control Toolbox in MATLAB. Further, this technique is implemented on a fictitious system by modelling the networked system with constant delay in SIMULINK and the observer states as well as the plant output are shown to be stable.

LIST OF FIGURES:

Chapter 1

Introduction

1.1 NETWORK CONTROL SYSTEMS

A system or a group of spatially distributed systems that exchange information (input data or output data or control signal) with (a) controller(s) via a shared communication channel are network control systems. In simple terms, in a NCS the communication between the sensor and controller and (or) the controller and actuator occurs via a network. A systems biology viewpoint would be neurons, muscles, neural pathways, and the cerebral cortex. The importance of research on NCS can be estimated by the broad range of area it has found use in such as mobile sensor networks, remote surgery with collaboration over the Internet, and automated highway systems and unmanned aerial vehicles, multi-agent traffic control, military, surgical and emergency medical applications. The greatest commercial impact of NCS has been in the industrial sector, however, research suggests that with significant technical challenges in new applications such as co-ordinated groups of mobile robot agents and UAVs, these systems will have great potential.

However, its interdisciplinary nature has raised fundamental questions on combined across communications, information processing and control- dealing with the relationship between network and quality of overall system's operation. Traditionally, control theory focuses on the study of interconnected dynamical systems linked through "ideal channels", whereas communication theory studies the transmission of information over "imperfect channels". A combination of these two frameworks is needed to model NCS.

A number of design methods have been developed to control these systems such as optimal stochastic control which models time delays as Linear Quadratic Gaussian problem, H_∞ control problem, generalized predictive control problem and robust control problems.

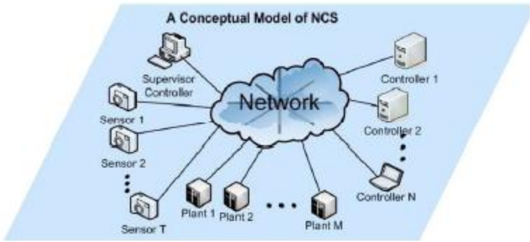
1.2 BACKGROUND

Networked control systems research lies primarily at the intersection of three research areas: control systems communication networks and information theory, and computer science. Networked control systems research can greatly benefit from theoretical developments in information theory and computer science. But, the main difficulty in merging results from these different fields is that studies have been the differences in emphasis in research so far. In information theory, delays in the transmitted information are not of central concern, as it is more important to transmit the message accurately- even though this may involve sometimes significant delays in transmission. In contrast, in control systems delays are of primary concern. Delays are much more important than the accuracy of the transmitted information due to the fact that feedback control systems are quite robust to such inaccuracies.

1.2.1 CHALLENGES IN CONTROL OF NETWORKED SYSTEMS

The basic challenges in networked systems occur due to **sharing** of a band limited digital communication network (internet,ethernet, wireless networks, fieldbus('88)), shared by other applications.

Fig 1: conceptual model of NCS



The following outline the key issues in designing a feedback controller through a network along with the respective research progress. Other issues being addressed by current research are actuator constraints, reliability, fault detection and isolation, graceful degradation under failure, reconfigurable control and ways to build increased degrees of autonomy into the system.

1.2.1.1 BAND LIMITED CHANNELS

Any communication network can only carry a finite amount of information per unit of time. In many applications, this limitation poses significant constraints on the operation of NCSs. In most digital networks, data is transmitted in atomic units called ***packets*** and sending a single bit or several hundred bits consumes the same amount of network resources.

Fundamental research involving **minimum bit rate** necessary to stabilize a LTI system have been derived. **Average bit rate** is a measure on how infrequent feedback information is needed (in digital networks) to guarantee that the system remains stable.

Intermittent feedback is another way in which the open loop is closed for certain fixed or time-varying periods, leading to opportunistic situations where sensor sends bursts of information when network is available. This helps in taxing the network less.

If **quantized feedback** is provided (in digital system implementation of NCS), and if the open loop system is unstable, only then can we determine the minimum average bit rate to process feedback information. Further research on communication constrained feedback channels is establishing a connection between stabilizability and an inequality relating feedback channel data to open loop eigen values.

The data rate theorem is a breakthrough in data rate requirement for a stable system over a network. It says that for any LTI plant having open-loop poles $a_{1,,....}$.., a_k in the right half-plane, a quantized feedback law can be designed to produce a bounded response if and only if the data-rate R around the closed feedback loop satisfies the data-rate

$$R > \log_2 e \sum \Re(a_i).$$

That is, the larger the magnitude of the unstable poles, the larger the required data rate through the feedback loop.

1.2.1.2 SAMPLING AND DELAY

To transmit a continuous-time signal over a network, the signal must be sampled, encoded in digital format, transmitted over the network, (see fig. Above) and finally the data must be decoded at the receiver side. This process is significantly different from the usual periodic sampling in digital control. The overall *delay* between sampling and eventual decoding at the receiver can be highly variable because both the network access delays (i.e., the time it takes for a shared network to accept data) and the transmission delays (i.e., the time during which data are in transit inside the network) depend on highly variable network conditions such as congestion and channel quality.

In some NCSs, the data transmitted are time stamped, which means that the receiver may have an estimate of the delay's duration and take appropriate corrective action

A significant number of results have attempted to characterize a maximum upper-bound on the sampling interval for which stability can be guaranteed. These results implicitly attempt to minimize the packet rate/ bit rate that is needed to stabilize a system through feedback (above).

1.2.1.3 PACKET DROP-OUT

Another significant difference between NCSs and standard digital control is the possibility **that data may be lost** while in transit through the network. Typically, *packet drop-outs* from transmission errors in physical network links delays sometimes result in **packet re-ordering**, which essentially amounts to a packet dropout if the receiver discards **"outdated" arrivals**.

Reliable transmission protocols, such as TCP, guarantee the eventual delivery of packets. However, these protocols are not appropriate for NCSs since the re-transmission of old data is generally not very useful.

1.2.1.4 NETWORK DELAY EFFECT

The network can introduce **unreliable/nondeterministic** levels of service in terms of **delays, jitter, and losses**. **REAL TIME ISSUE:** In time sensitive NCSs, if the delay time exceeds the specified tolerable time limit, the plant or the device can either be damaged or have a degraded performance. Time-sensitive applications can be either **hard real time** or **soft real time**. In hard real-time systems, the task must be completed before the **hard deadline.**

The limits to performane in NCSs are **caused primarily by delays and dropped packets**.

1.2.2 SYSTEM ARCHITECTURE

The configuration of network control system or the manner in which plant is connected to the network can vary. Modelling of a system is very important as it will change the control strategies differ with different configurations. The hierarchical form is a hybrid system and can be used to study inter-connection of different plants, whereas the direct form is a stand-alone control application. The later in a simpler forms the single loop feedback NCS, which context represents all the basic constraints in a NCS and is used in this thesis.

1.2.2.1 DIRECT FORM

The NCS in the direct structure is composed of a controller and a remote system containing a physical plant, sensors and actuators and linked by a data network to perform closed loop operation.

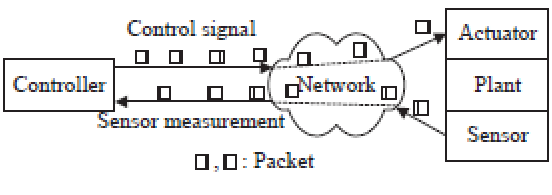


Fig2: Direct form I

Or,

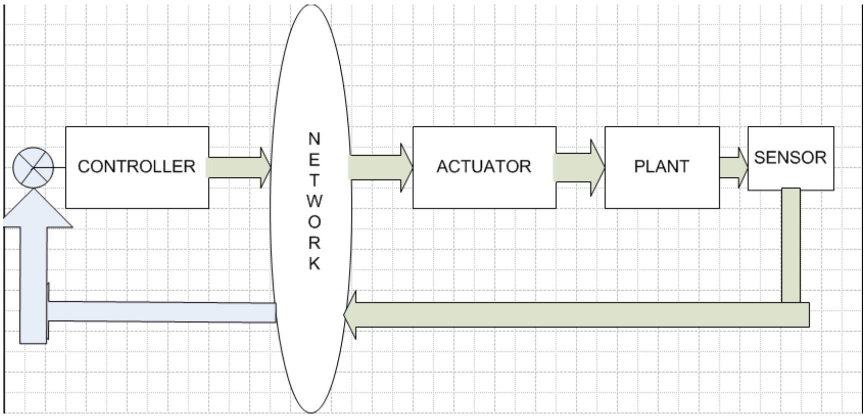


Fig 3: Direct form II

The single loop NCS shown in the figure above is sufficient to study the effect of sampling and delays in NCS as it captures the important features. Three different control architectures are covered by the single feedback loop depending on the presence and absence of delays and packet drop-outs in different channels .

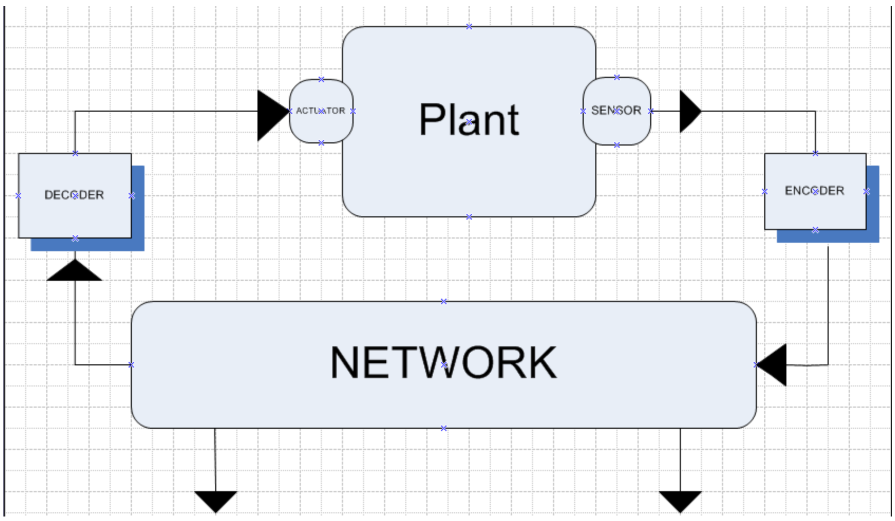


Fig4: single loop feedback NCS

1.2.2.2 HIERARCHICAL FORM

The basic hierarchical structure consists of a main controller and a remote closed loop system as depicted in Fig.5. The main controller computes and sends the reference signal in a frame or a packet via a network to the remote system and the remote system then processes the reference signal to perform local closed-loop control and returns to the sensor measurement to the main controller for networked closed-loop control.

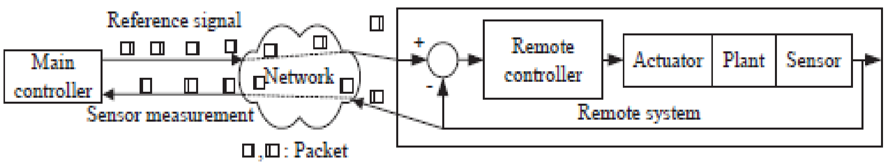


Fig5: Hierarchical Form

CHAPTER 2

STUDY OF DELAYS IN NETWORK CONTROL SYSTEMS

2.1 CLASSIFICATION AND TYPES OF DELAY IN NCS

The data transfers between the controller and the remote system introduce network delays in addition to the time taken by the controller- processing delay. Fig. 6 shows network delays in the control loop.

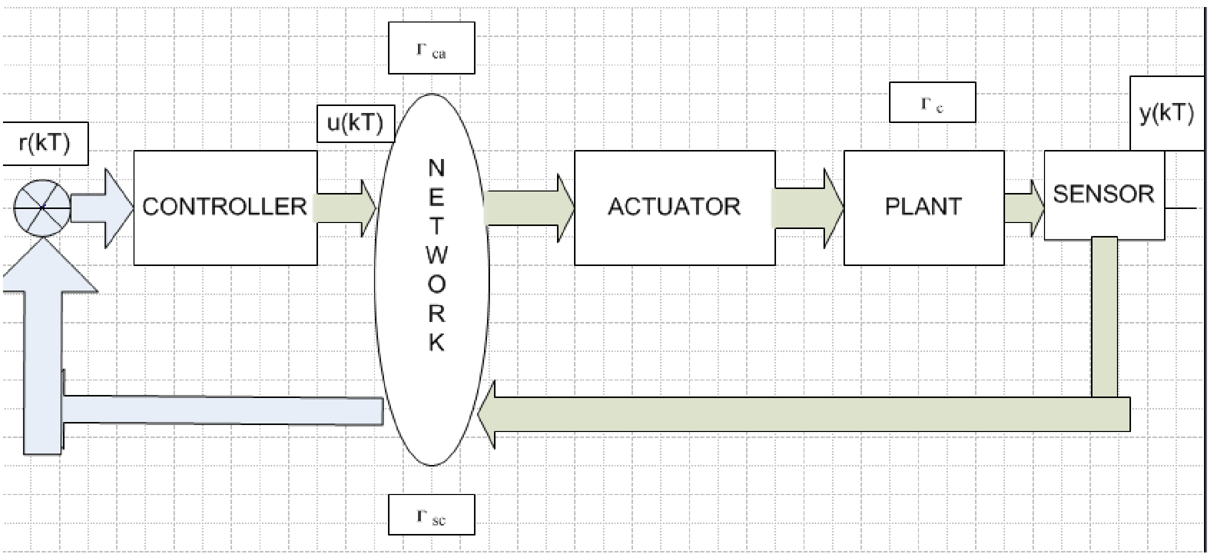


Fig 6: Schematic representation of network delays in closed loop NCS.

Here r is the reference signal, u is the control signal, y is the output signal, k is the time index and T is the sampling period.

Network delays in an NCS are categorized as:

1) sensor-to-controller data transfer delay = r_{sc}
2) controller-to-actuator data transfer delay = r_{ca}
3) computation delay = r_c

- The output at instant KT is delayed by r_{sc} by the time it reaches the controller from the sensor; the time is KT+ r_{sc} when the controller receives the signal.
- Now the controller takes processing time r_c to calculate the feedback signal.
- When the feedback signal (in the form of packet or in a frame) is sent to the actuator, the time is KT+ r_{sc} + r_c
- On reaching the actuator the global time is KT + r_{sc} + r_c + r_{ca}

So the total delay T' = $r_{sc} + r_c + r_{ca}$

These delays in input packet and output state of plant due to r_{ca} and r_{sc} respectively for a ZOH discrete system can be realized as shown in the Fig.7 that follows:

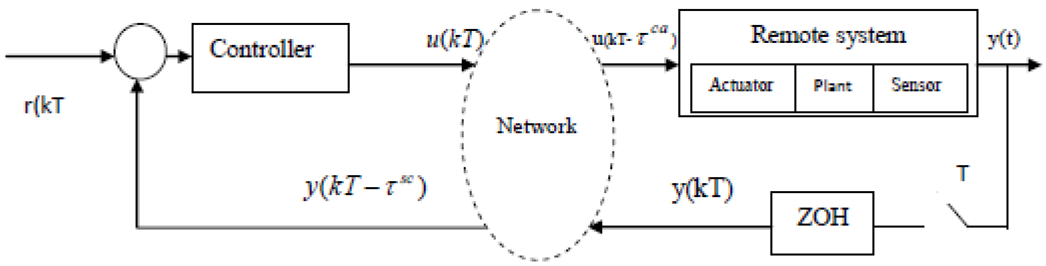


Fig. 7 Time delays in NCS

Further, the network delays (r_{sc} & r_{ca}) are classified into -

- Waiting time delay r_w -The waiting time delay is the delay, of which a source (the main controller or the remote system) has to wait for queuing and network availability before actually sending out a frame or a packet
- Frame time delay r_f - The frame time delay is the delay during the moment that the source is placing a frame or a packet on the network.
- Propagation delay r_p - The propagation delay is the delay for a frame or a packet travelling through a physical media. The propagation delay depends on the speed of signal transmission and the distance between the source and destination.

A timing diagram for a discrete time system with sampling time T, at two instants- kT and (k+1)T is shown below. The classification of delays can also be seen in this diagram. It shows the network delays for control input signal u(k) and actual plant output signal y(k) in Fig 8 below:

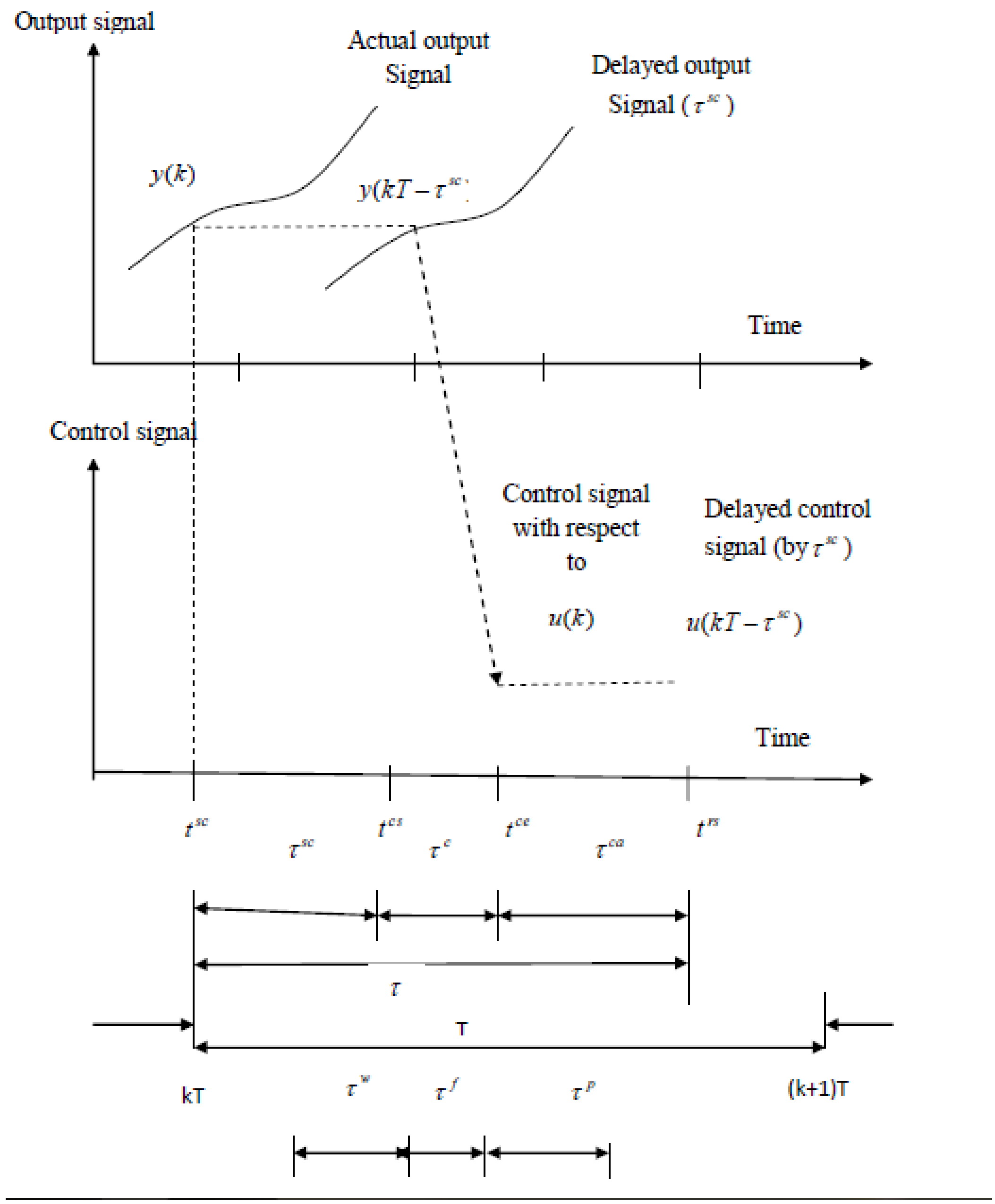


Fig. 8 Timing Diagram of Network Delay Compensation

2.2 EFFECT OF DELAYS ON CLOSED LOOP BEHAVIOUR OF NCS

One of the most important problems of NCS is the delay in data transmission between sensor and actuator and controller units leading to data packets spoilt or completely getting lost. So the end result is weak signals. The network induced delay appears mainly from sensor-controller and controller-actuator. The control systems designed without taking into account these delays have low performance and reliability. The delay in the control loop thus degrades system performance and destabilization of closed loop networked system

2.3 MODELLING OF TIME DELAYS IN NCS

- Constant delay is modelled as time buffer .
- Modelling of Delay with known probability distribution governed by Markov Chain Model can be thus modelled.
- Independent random delays modelling.
- End to end delay dynamics for internet can be modelled using system identification tools.

2.4 DELAY COMPENSATION TECHNIQUES

The following delay compensation techniques have been implemented with necessary assumptions to limit the destabilizing effect of delays on network control systems and obtain conditions for stable closed loop operation of NCS.

1. **Optimal stocastic method**
    - To control NCS on random delay networks
    - LQG problem is formulated based on network delay statistics and optimal control is used to find feedback gain.
    - But, this case requires the past information of output and input {y (0).... y (k), u (0)... u (k)} in conjunction with the past information of the delay.

2. **Queing and buffering**

Network delays become deterministic and hence, It transforms NCS into a time invariant system for both linear and non-linear plants

3. **Robust control Method**

   - Delays are considered as multiplicative perturbations on the system and the perturbation effects are minimized under the assumption of no observation noise.
   - Controller is designed in the frequency domain, without prior knowledge of probability distribution of delays.

4. **Non-linear and perturbation theory**

   - Network delays are modelled as the vanishing perturbation of a continuous-time system under the assumption that there is no observation noise
   - This methodology can be applied on an NCS on periodic delay networks and random delay networks at the sensor-to controller transmission.

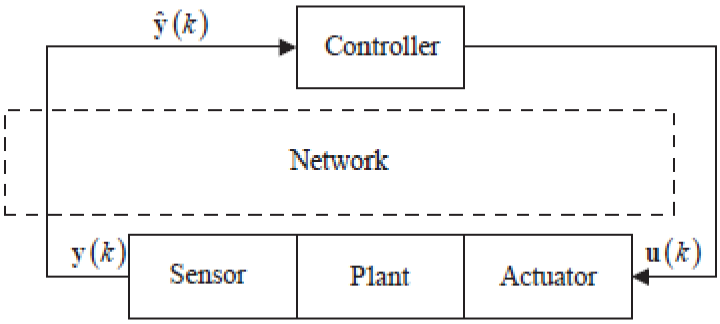


Fig 9: configuration of NCS in perturbation methodology

5. **Robust memory-less H_∞ controller** for uncertain NCS to combat effects of both network delay and data drop-out.

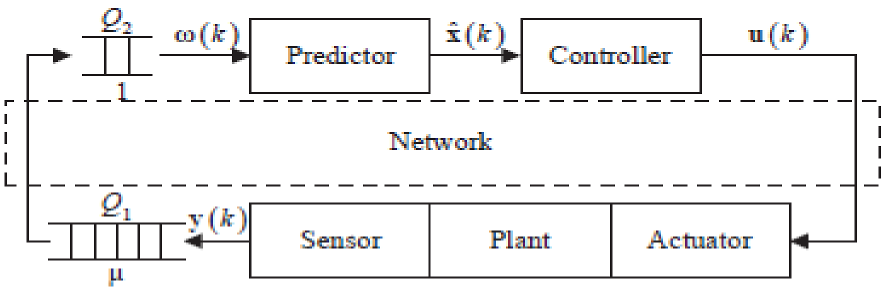
6. Multimode systems

To stabilize these systems, the proper Lyapunov–Krasovskii functionals are chosen and using a descriptor model transformation of the system, derived linear matrix inequality (LMI)- based sufficient conditions for stability are determined.

7. Probabilistic predictor based delay compensation

- The method utilizes probabilistic information along with the number of packets in a queue to improve state prediction. (Similar to queuing and buffering)
- The configuration of the NCS in probabilistic predictor-based delay compensation methodology is illustrated in Fig 10.

Fig. 10: probabilistic predictor based delay compensation



8. Sampling time scheduling

- A sampling time is selected such that network delays do not affect control system performance
- Multiple NCSs are connected on a single delay network and individual network delay < sampling interval

- The sampling times of all M NCS on the network are calculated from the sampling time of the most sensitive NCS based on the general frequency domain analysis on its worst-case delay bound.

9. event based methodology

- The system motion (reference) has to be a non-decreasing function of time in order to guarantee the system stability
- Because the overall system is not based on time, network delays will not destabilize the system.

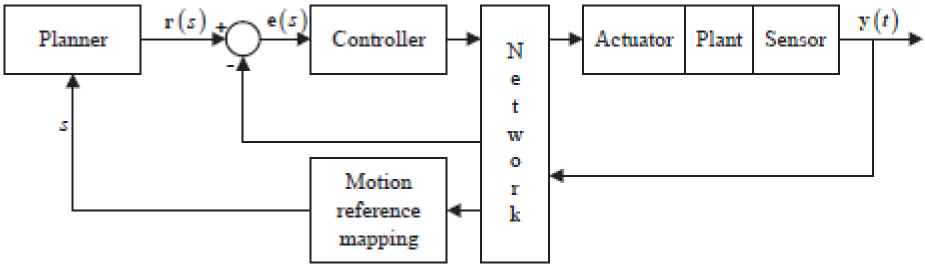


Fig 11. Configuration of NCS in event based methodology

10. Fuzzy logic modulation

The fuzzy logic modulator is used to modify the controller output to compensate the network delay effects based on fuzzy logic.

Method used in this thesis is a combination of probabilistic predictor method (7.) as we use a generalized predictive control scheme for the state feedback controller and we use the Lyapunov functional of the augmented system (containing possible input and plant states) to formulate the LMI, hence determining the gain of feedback controller.

CHAPTER 3

MODEL PREDICTIVE CONTROL

3.1 INTRODUCTION

Model predictive control (MPC), also referred to as moving horizon control or receding horizon control, is an attractive feedback strategy, especially for linear processes. Linear MPC refers to a family of MPC schemes in which linear models are used to predict the system dynamics, even though the dynamics of the closed-loop system is nonlinear due to the presence of constraints. Linear MPC approaches have found successful applications, especially in the process industries. By now, linear MPC theory is quite mature with more than 2200 applications in a very wide range from chemicals to aerospace industries are summarized. Important issues such as online computation, the interplay between modelling/identification and control and system theoretic issues like stability are well addressed today.

Many systems are, however, in general inherently nonlinear. This, together with higher product quality specifications and increasing productivity demands, tighter environmental regulations and demanding economical considerations in the process industry require operating systems closer to the boundary of the admissible operating region. In these cases, linear models are often inadequate to describe the process dynamics and nonlinear models have to be used. This requires the use of nonlinear model predictive control.

3.2 PRINCIPLE OF MODEL PREDICTIVE CONTROL (MPC)

In general, the model predictive control problem is formulated as solving **on-line** a finite horizon **open-loop optimal control** problem **subject to system dynamics and constraints** involving states and controls. Figure below shows the basic principle of model predictive control. Based on measurements obtained at time t, the controller predicts the future dynamic behaviour of the system over a **prediction horizon *Tp*** and determines (over **a control horizon *Tc*** $\leq Tp$) the input such that a predetermined open-loop performance objective functional is optimized. *If* there were no disturbances and no model-plant mismatch, and *if* the optimization problem could be solved for infinite horizons, then one could apply the input function found at time $t = 0$ to the system for all times $t \geq 0$. However, this is not possible in general. Due to disturbances and model-plant mismatch, the true system behaviour is different from the predicted behaviour. In order to incorporate some feedback mechanism, the open-loop manipulated input function obtained will be implemented only until the next measurement becomes available. The time difference between the recalculation and measurements can vary, however often it is assumed to be fixed, that is, the measurement will take place **every** δ **sampling time units**. Using the new measurement at time $t + \delta$, the whole procedure – prediction and optimization – is repeated to find a new input function with the control and prediction horizons moving forward. In the Figure below the input is depicted as arbitrary function of time. For numerical solutions of the open-loop optimal control problem it is often necessary to parameterize the input in an appropriate way. This is normally done by approximating the input could as piecewise constant over the sampling time δ. The calculation of the applied input based on the predicted system behaviour allows the inclusion of constraints on states and inputs as well as the optimization of a given cost function.

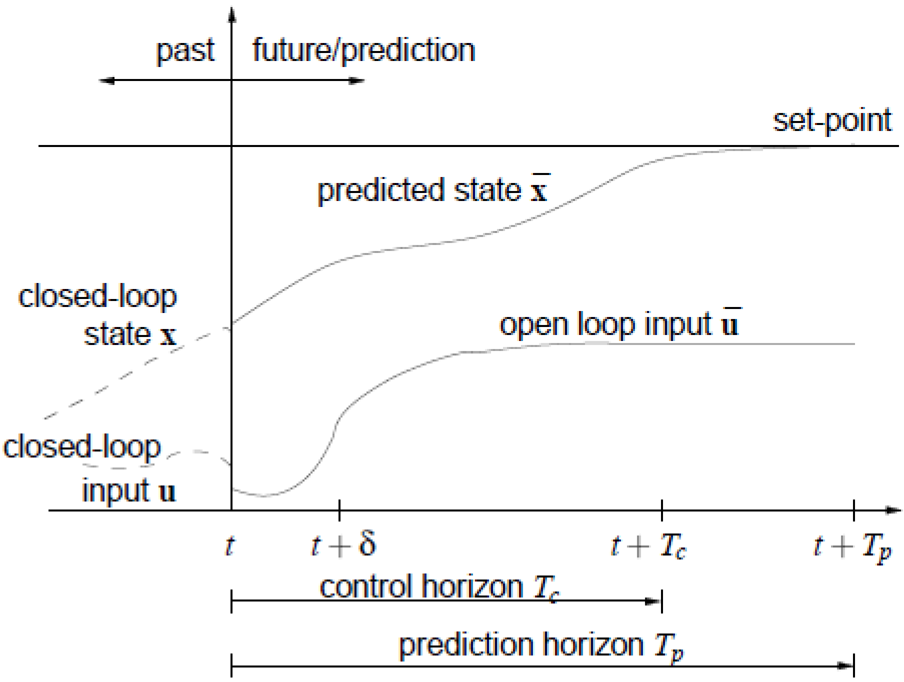


Fig.12. Principle of Model Predictive Control

3.3 ALGORITHM & KEY FEATURES OF MPC

Thus, the main idea of MPC is to use a model of the process to be controlled, in order to repeatedly solve an optimization problem, based on the measurement provided by the plant. Hence, it is an active control strategy. Then, only the first piece of trajectory is implemented and the problem is re-solved with the new measurement. At the recalculation times *ti € π*, *x* (t_i) is measured, and the following Optimal Control Problem (OCP) is solved

$$
\begin{aligned}
&\min_{\overline{u}(\cdot)} \int_{t_i}^{t_i+T_p} F(\overline{x}(\tau),\overline{u}(\tau))d\tau + E(\overline{x}(t_i+T_p)),\\
\text{s.t.}\quad &\dot{\overline{x}}(t) = f(\overline{x}(t),\overline{u}(t)), \quad \overline{x}(t_i) = x(t_i),\\
&\overline{u}(t) \in U, \quad t \in [t_i, t_{i+1}),\\
&\overline{x}(t) \in X,\\
&\overline{x}(t_i+T_p) \in \mathscr{E},
\end{aligned}
$$

Where, bar denotes the controller internal variables. The solution of the OCP is an optimal control signal *u* € (*t* ; *x*(*ti*)), for *t* € [*ti; ti*+*Tp*], where *Tp* represents the finite prediction horizon. The control input is then implemented for the time-span [*ti; ti*+ δ), i.e.

$$u(t) = u^*(t; x(t_i)),\ \text{for } t \in [t_i, t_i + \delta)$$

Where, δ interval between two consecutive recalculation times, i.e.

$$\delta = (t_{i+1} - t_i), \forall t_i, t_{i+1} \in \pi.$$

The closed loop system stability under the MPC can be achieved by properly choosing the cost functional *F*(*x*,*u*), the terminal cost *E*(*x*), the terminal region *E* € *X*, and the prediction horizon *Tp*.

The basic NMPC loop is as follows-

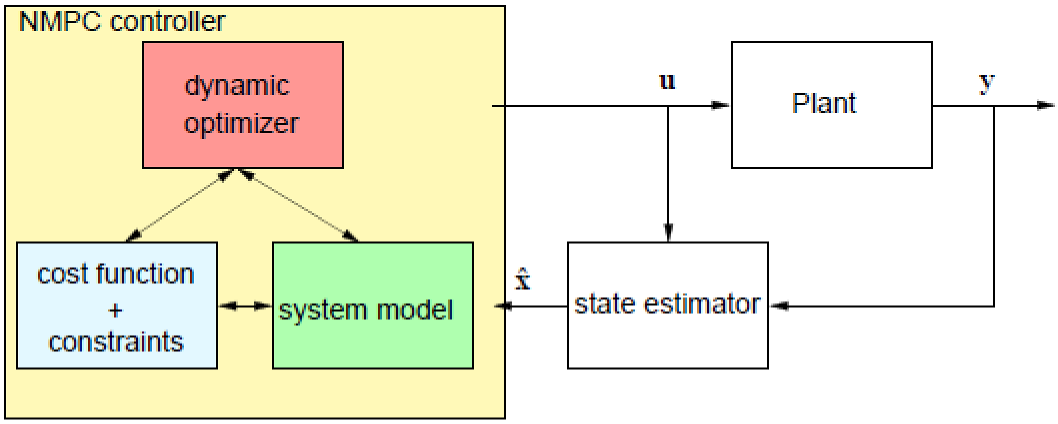


Fig.13. Basic NMPC loop

It is necessary to estimate plant states with the help of an Estimator as shown above.

Summarizing, the basic MPC scheme works as follows:

1. Obtain measurements/estimates of the states of the system
2. Compute an optimal input signal by **minimizing** a given **cost function** over a certain **prediction horizon** in the future using a **model of the system**
3. **Implement the first part of the optimal input signal** until new measurements/estimates of the state are available; then continue with 1.

The following are the key features of MPC:

1. In MPC a specified performance criteria is minimized on-line.
2. In MPC the predicted behaviour is in general different from the closed loop behaviour.
3. The on-line solution of an open-loop optimal control problem is necessary for the application of MPC.
4. To perform the prediction the system states must be measured or estimated.

CHAPTER 4

PROBLEM STATEMENT AND GUARANTEED COST CONTROL

4.1 PROBLEM STATEMENT

In this thesis * denotes symmetrical block in a symmetric matrix, I denotes the identity matrix and the trace of a matrix is denoted by *tr*(.) The NCS is shown below (Fig 14) and forward and backward channel delays are denoted by f_t and k_t respectively.

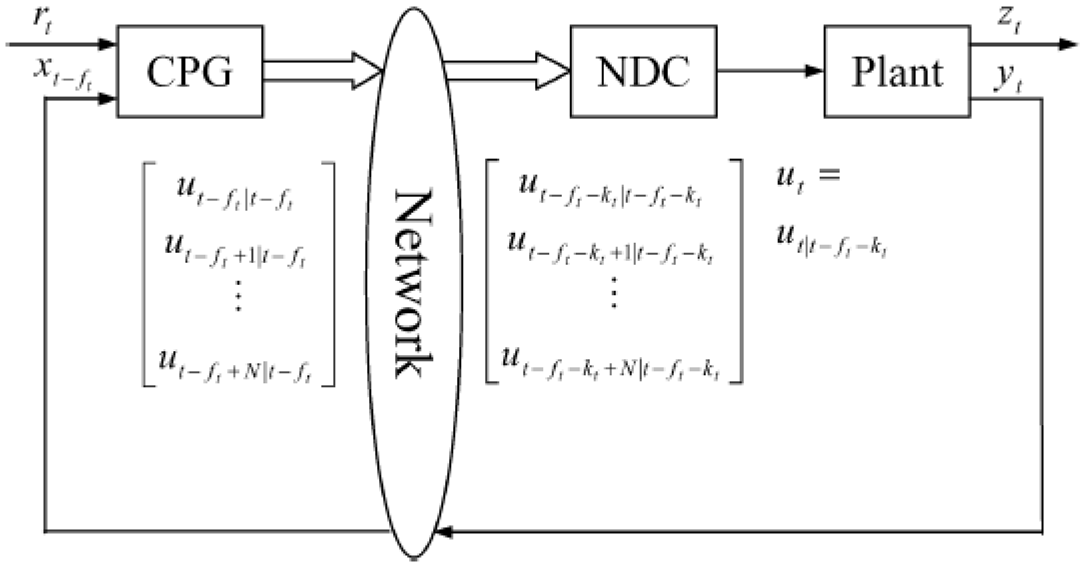


Fig. 14 Predictive control scheme for NCS; CPG- Control Prediction Generator; NDC- Network Delay compensator

4.2 STATE SPACE FORMULATION

4.2.1 SYSTEM MODEL

The plant is modelled in the following discrete-time space form:

$$x_{t+1} = Ax_t + Bu_t \;;\; y_t = Cx_t \qquad (1.1)$$

Where- $x_t \in R^n$, $u_t \in R^m$, $y_t \in R^p$ denote the state vector, control input and controlled output respectively. In order to measure the time delay occurring in any packet sent through the network; a time stamp is attached or transmitted together with control predictions or control sequence generated via the predictive controller. Although computer communication protocols may not have this feature, time triggered protocols like Flexray can support a time delay measurement. The guaranteed cost function associated with system (1.1) is:

$$J = \sum_{t=0}^{+\infty}((x_t)'Qx_t + (u_t)'Ru_t) \quad (1.2)$$

Where Q and R are positive definite weighted matrices having dimensions $n \times n$ and $m \times m$ respectively. Associated with the cost function (1.2), the guaranteed cost controller is defined as follows-

4.2.2 CONTROL STRATEGY

Definition 1: Considering system (1.1) and cost function (1.2), if there exists a control law u_t^* and a positive scalar J^* such that for all admissible uncertainties, the closed loop system is asymptotically stable and the value of the cost function satisfies a bound- $J \le J^*$ then, J^* is said to be guaranteed cost and u_t^* is said to be the guaranteed cost law.

We assume that-

1. The upper bounds of the time-varying network delays k_t in the forward channel and f_t in the feedback channel are not greater than N_1 and N_2 respectively, where N_1 and N_2 are positive integers, i.e. $k_t \in \{0,1.....,N_1\}$ and $f_t \in \{0,1.....,N_2\}$ where $t = 0,1,2....$ denotes the sampling instant.
2. The number of consecutive data drop-outs in the forward channel and the feedback channel are less than L_1 and L_2 respectively, both of which are positive integers. So, the upper bound of the consecutive data drop-outs and network delay is equal to $N = N_1 + N_2 + L_1 + L_2$

4.2.3 DESIGN OF OBSERVER FOR PREDICTION OF FUTURE CONTROL SEQUENCE

The state vector x is not available in our case due to time delay due and as state feedback control is to be employed, hence we have to design a state observer from our knowledge of the system parameters. It is defined as-

$$\hat{x}_{t+1} = A\hat{x}_t + Bu_t + L(y_t - C\hat{x}_t) \quad \text{.........(1.3)}$$

Where, $\hat{x}_t \in R^n$ is the observed state and $u_t \in R^m$ is the input of the observer at time t, respectively, L is the observer gain to be designed later.

For a system without delay, the state feedback controller is given as-

$$u_t = K_0 \hat{x}_t \quad \text{.........(1.4)}$$

Where K_0 is the $m \times n$ control matrix to be determined. But, when there are time varying delay and data drop-out in the **feedback channel**, the predictive controlled from time $t - f_t + 1$ to t is constructed as-

$$\begin{aligned} u_{t-f_t+1|t-f_t} &= K_1 \hat{x}_{t-f_t} \\ u_{t-f_t+2|t-f_t} &= K_2 \hat{x}_{t-f_t} \\ &\vdots \\ u_{t|t-f_t} &= K_{f_t} \hat{x}_{t-f_t} \end{aligned}$$

Where, $f_t = 0,1.....,N_2 + L_2$.

When time varying delay and data drop-out in the **forward channel**, the predictive controlled from time $t+1$ to $t+k_t$ is constructed as-

$$
\begin{aligned}
u_{t+1|t-f_t} &= K_{f_t+1}\hat{x}_{t-f_t} \\
u_{t+2|t-f_t} &= K_{f_t+2}\hat{x}_{t-f_t} \\
&\vdots \\
u_{t+k|t-f_t} &= K_{f_t+k_t}\hat{x}_{t-f_t}
\end{aligned}
$$

Where, $k_t = 0,1.....,N_1 + L_1$

Thus the overall state feedback controller can be given as-

$$u_{t\text{I}t-f_t-k_t} = K_{f_t+k_t}\hat{x}_{t-f_t-k_t} \quad \text{......} (1.5)$$

Therefore, the observer can be written as,

$$\hat{x}_{t+1} = (A-LC)\hat{x}_t + BK\hat{x}_{t-i} + LC\hat{x}_t, \quad i = 0,1,.....N. \quad \text{......}(1.6)$$

The closed loop system of (1.1) can be now written as-

$$x_{t+1} = Ax_t + BK_i\hat{x}_{t-i}, \quad i = 0,1,.....N \quad \text{......}(1.7)$$

4.2.4 AUGMENTED STATE SPACE SYSTEM REPRESENTATION

So, the augmented system becomes,

$$X_{t+1} = \Lambda_i X_t \quad \text{......}(1.8)$$

Where,

X_t has order $(2N+2)n \times 1$; comprising all possible states of plant [total of $(N+1)n$ entities] and observed state of plant [total of $(N+1)n$ entities] within the total delay and packet drop-out frame.

Λ_i has order $(2N+2)n \times (2N+2)n$ describing the system dynamics.

$$X_t = \begin{bmatrix} x_t^T \\ \vdots \\ x_{t-i}^T \\ \vdots \\ x_{t-N}^T \\ \hat{x}_t^T \\ \vdots \\ \hat{x}_{t-i}^T \\ \vdots \\ \hat{x}_{t-N}^T \end{bmatrix} ; \Lambda_i = \begin{bmatrix} \hat{\Pi} & \mathrm{E}_i \\ \Psi & \Gamma_i \end{bmatrix} ; \hat{\Pi} = \begin{bmatrix} A & 0_{n \times Nn} \\ I_{Nn} & 0_{Nn \times n} \end{bmatrix} ; \mathrm{E}i = \begin{bmatrix} 0_{n \times in} & BK_i & 0_{n \times (N-i)n} \\ 0_{(N+1)n \times in} & 0_{(N+1)n \times n} & 0_{(N+1)n \times (N-i)n} \end{bmatrix}$$

$$\Psi = \begin{bmatrix} LC & 0_{n \times Nn} \\ 0_{Nn \times n} & 0_{Nn \times Nn} \end{bmatrix}$$
$$\Gamma_i = \begin{bmatrix} A - LC & 0_{n \times (i-1)n} & BK_i & 0_{n \times (N-i)n} \\ I_n & I_{(i-1)n} & I_{(N-i)n} & 0_{Nn \times n} \end{bmatrix}$$

The above equations are derived using equations (1.6) and (1.7) only and they represent the delayed system dynamics.

4.3 LMI FORMULATION

4.3.1 INTRODUCTION TO LMIs

It has been seen in several referenced papers that for optimal control involving the Lyapunov functional or the Algebraic Riccatti inequalities or linear and quadratic inequalities, these inequalities are converted to ‘Linear Matrix Inequalities’ or LMIs, the solution of which require the use of algorithms and tools which are mathematically complex. Hence a control engineer resorts to use of off-the shelf software and in this case LMI solver provided in Robust Control Toolbox in MATLAB is used, with the help of which the controller gains (in the previous problem) and the observer gain matrix are determined.

A linear matrix inequality (LMI) is a convex con-straint. Linear inequalities, convex quadratic inequalities, matrix norm inequalities, and various constraints from control theory such as Lyapunov and Riccati inequalities can all be written as LMIs. Further, multiple LMIs can always be written as a single LMI of larger dimension. Thus, LMIs are a useful tool for solving a wide variety of optimization and control problems. Most control problems of interest that cannot be written in terms of an LMI can be written in terms of a more general form known as a bilinear matrix inequality (BMI). Computations over BMI constraints are fundamentally more difficult than those over LMI constraints, and there does not exist off-the-shelf algorithms for solving BMI problems.

A linear matrix inequality (LMI) has the form:

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i > 0$$

Where, $x \in \mathcal{R}^m, F_i \in \mathcal{R}^{n \times n}$ and *F(x)* is a positive definite matrix.

The above is an example of a strict LMI as it requires *F(x)* to be positive definite. Requiring only that *F(x)* be positive semi-definite is referred to as a non-strict LMI. The strict LMI is feasible if the set $\{x|F(x) > 0\}$ is nonempty (a similar definition applies to non-strict LMIs). Any feasible non-strict LMI can be reduced to an equivalent strict LMI that is feasible by eliminating implicit equality constraints and then reducing the resulting LMI by removing any constant null-space. Hence the basic requirement of an LMI is its feasibility and if an LMI is feasible, it can be solved by available software.

4.3.2 LMI FORMULATION IN GIVEN PROBLEM

Theorem 1:

For the augmented system given by (1.8) and the cost function (1.2); if there exists a positive definite matrix $P > 0$ such that

$$\Lambda_i^T P \Lambda_i - P + \overline{Q} + \overline{R} < 0$$ ....(1.9)where-

$$\overline{R} = \begin{bmatrix} K_i^T R K_i & 0_{n \times (2N+1)n} \\ 0_{(2N+1)n \times (2N+2)n} & \end{bmatrix}$$

$$\bar{Q} = \begin{bmatrix} Q & 0_{n \times (2N+1)n} \\ 0_{(2N+1)n \times (2N+2)n} & \end{bmatrix}$$

Then, the system (1.8) with controllers (1.5) is asymptotically stable and the cost function (1.2) satisfies the specified performance bound; $J \le X_0^T P X_0$ (1.10); where X_0 is the initial augmented state matrix

Proof:

The Lyapunov function defining energy of system at any time t is given by- $V_t = X_t^T P X_t$. Where P is appositive definite matrix of the order $(2N+2)n \times (2N+2)n$

For the dynamics to be stable, we have-

$\Delta V \le 0$ (has to be less than zero)

Or, $V_{t+1} - V_t \leq 0$

So, $X_t^T(\Lambda_i^T P \Lambda_i - P)X \leq 0$ Now, as the cost function $J = X_t^T(\overline{Q} + \overline{R})X_t \geq 0$ or, is always positive; we can modify our inequality above to include the later term. Hence, we prove Theorem 1.

The inequalities in Theorem 1 are now converted to matrix inequalities using Schur's Complement Lemma as follows-

Expressing $\overline{R}$ as $\overline{R} = \overline{I}^T K_i^T R K_i \overline{I}$; where, $\overline{I}$ is $[I\,0......0]$ of order $n \times (2N+2)n$; I is of nth order. We can obtain the following by applying Schur's complement in 2 steps. The matrix inequality is-

$$\begin{bmatrix} -P + \overline{Q} & \Lambda_i^T & \overline{I}^T K_i^T \\ * & -P^{-1} & 0 \\ * & * & -R^{-1} \end{bmatrix} < 0 \quad \text{...(1.11)}$$

4.3.3 SIMPLIFICATION OF INEQUALITY

The LMI conditions for guaranteed cost controller in (1.11) are difficult to solve because K_i and L are both present in the Λ_i^T term and both are to be determined. So, we further break down the Λ_i^T term as follows and separate the two unknown gains by defining new matrices B_1, B_2, $\overline{C}, I_i, \tilde{I}, I_0, \overline{A}, I_1$ which were previously combined along with K_i and L in Λ_i^T term are now separated to $\overline{A_i}$

Or, $\overline{A_i} = \overline{A} + B_1 K_i I_i + \tilde{I}\, L C + B_2 K_i I_i$ ....(1.12)

Where,

$$B_1 = \begin{bmatrix} B \\ 0_{(2N+1)n\times m} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0_{(N+1)n\times m} \\ B \\ 0_{Nn\times m} \end{bmatrix}$$

$$\tilde{I} = \begin{bmatrix} 0_{(N+1)n\times n} \\ I_n \\ 0_{Nn\times n} \end{bmatrix}$$

$$\bar{C} = [C\ 0_{p\times Nn}\ \ -C\ 0_{p\times Nn}]$$

$$I_0 = \begin{bmatrix} 0_{n\times(N+1)n}\ I_n\ 0_{n\times Nn} \end{bmatrix}$$

$$I_1 = \begin{bmatrix} 0_{n\times(N+2)n}\ I_n\ 0_{n\times(N-1)n} \end{bmatrix}, \ldots,$$

$$I_i = \begin{bmatrix} 0_{n\times(N+i+1)n}\ I_n\ 0_{n\times(N-i)n} \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} \Pi & 0_{(N+1)n\times(N+1)n} \\ 0_{(N+1)n\times(N+1)n} & \Pi \end{bmatrix}$$

The inequality (1.11) now becomes,

$$\begin{bmatrix} -P+\overline{Q} & \overline{A}+B_1K_iI_i+\tilde{I}\,LC+B_2K_iI_i & \overline{I}^T K_i^T \\ * & -P^{-1} & 0 \\ * & * & -R^{-1} \end{bmatrix} < 0 \text{...}(1.13)$$

4.3.4 LMI ALGORITHM

The inequality (1.13) is not an LMI due to presence of both P and P^{-1} terms. It can however, be solved by a cone complimentary linearization algorithm which converts the non-convex optimization problem to a LMI based minimization problem. This algorithm proposes 2 LMIs besides 1.13 which frame the minimization problem.

$$\begin{bmatrix} P & I \\ I & W \end{bmatrix} \geq 0 \quad (1.14) \qquad \text{and,} \qquad \begin{bmatrix} -\gamma & X_0^T \\ X_0 & -W \end{bmatrix} < 0 \quad (1.15)$$

4.4 ALGORITHM

The steps to be followed are:

1. Sufficiently large initial value of γ is chosen such that a feasible solution exists to inequalities (1.13), (1.16) and (1.15).
2. A feasible solution is determined for P, W, K_i and L. Set j=0
3. Using these feasible solutions for the jth round; i.e. P_j and W_j obtained above, the following minimization problem is solved-

   Minimize $tr(P_jW+P\,W_j)$ subject to LMIs (1.13), (1.17) and (1.15).
4. Condition (1.13) is used as a stopping criterion and if it is satisfied, γ is decreased to some extent and steps 1. to 4. Are repeated. Else the loop is terminated after a specific number of iterations.

CHAPTER 5

IMPLEMENTATION OF CONTROL STRATEGY

5.1 SAMPLE SYSTEM DESCRIPTION

We take,

$$A = \begin{bmatrix} 1.01 & 0.2710 & -0.4880 \\ 0.4820 & 0.1 & 0.24 \\ 0.0020 & 0.3681 & 0.7070 \end{bmatrix}; \qquad B = \begin{bmatrix} 5 & 5 \\ 3 & -2 \\ 5 & 4 \end{bmatrix}; \qquad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 1 \end{bmatrix}$$

It is assumed that upper bounds of the network delays in forward channel k_t , are not greater than 1and that of feedback channel f_t are not greater than 2. So, N=n=3 and m=p=2.

5.2 SOLUTION OF LMI PROBLEM

Taking γ as -0.01 and making maximum iterations to 40, we find a feasible solution to the feedback gains of the taken system. The various K_i values are obtained as-

K0=

-0.0128 0.0155 -0.0066

-0.0039 -0.0183 0.0197

K1=

0.0160 0.0158 0.0165

-0.0052 -0.0043 -0.0046

K2=

0.0133 0.0132 0.0132

-0.0041 -0.0040 -0.0040

K3=

0.0021 0.0021 0.0021

-0.0012 -0.0012 -0.0012

The observer gain is obtained as-

L =

-0.1355	0.1583
0.0078	0.0508
0.1208	-0.0203

5.3 SIMULATION BY DEVELOPING A NCS MODEL

The model of network control system cannot be exactly prepared. In practice, network laboratories are used for the purpose of simulation of control methods developed. So, in order to implement the controller developed in this thesis, a constant delay model is constructed in SIMULINK. It is assumed that the network induces a constant delay and/or packet drop-out of 1. The state matrices remain the same and the differential equations representing dynamics of the plant, observer and controller are implemented in the model as shown below:

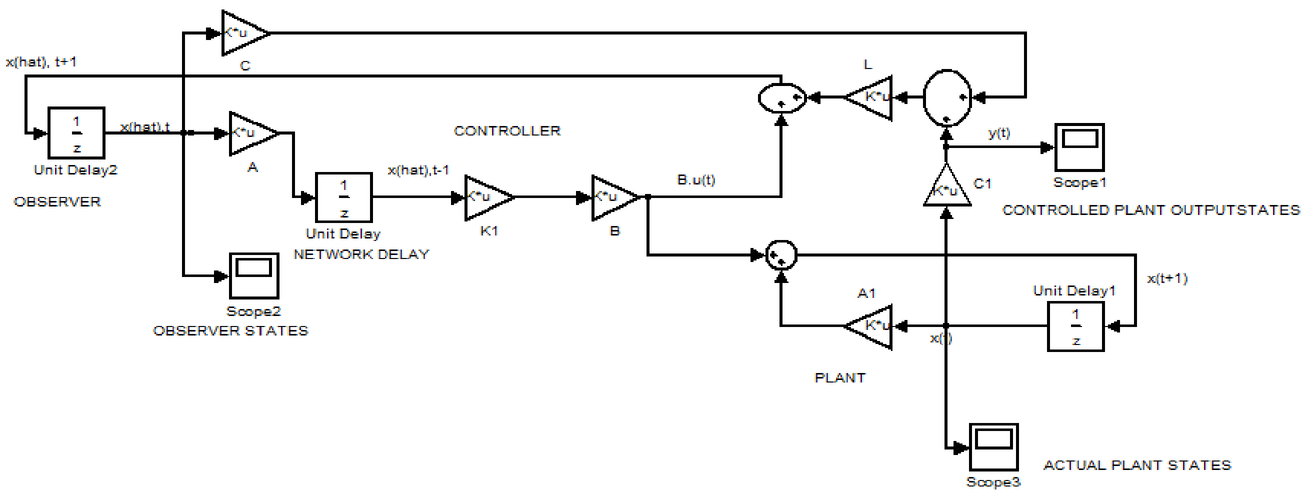


Fig 15: constant delay simulation of NCS

Taking Simulation time = 100; Step size =1; Solver: Fixed type, discrete (no continuous states); Initial conditions = [0.5 0.5 0.5]'

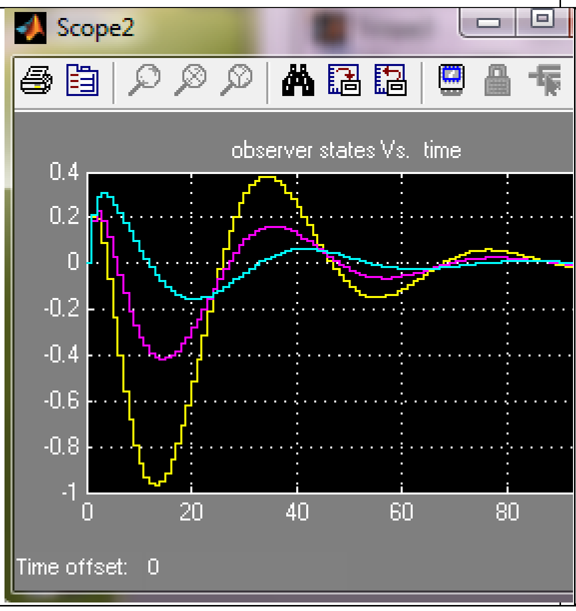
5.4 RESULTS

We solve the LMIs for i=1 as the above is a unit delay system and obtain γ as 0.8. Further, the controller gain K1 and observer gain L are:

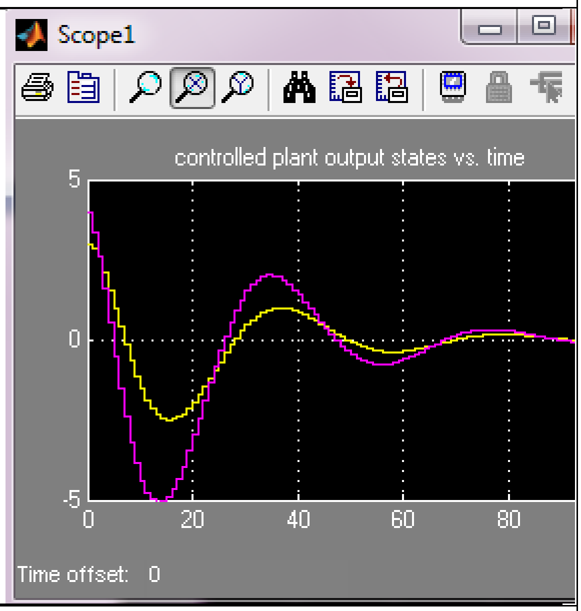
$$K1 = \begin{matrix} -0.0139 & -0.0058 & -0.0028 \\ 0.0034 & 0.0030 & 0.0019 \end{matrix}$$

$$L = \begin{matrix} -0.1470 & 0.1605 \\ -0.0025 & 0.0480 \\ 0.1127 & -0.0325 \end{matrix}$$

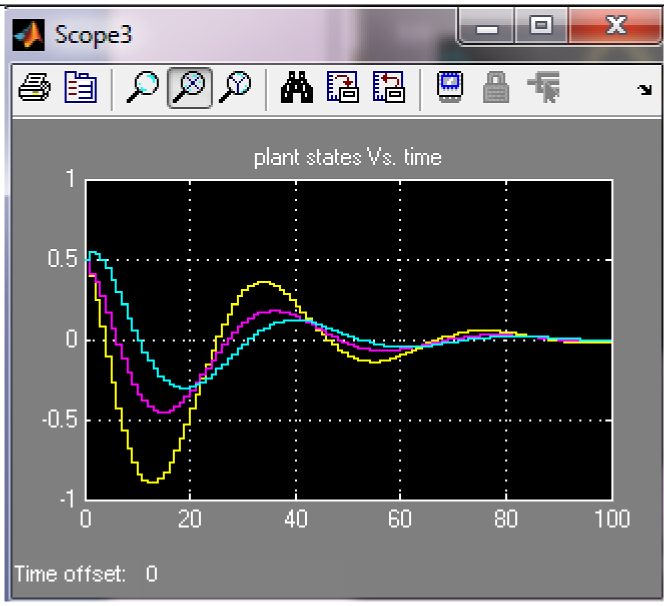
OBSERVER STATES



OUTPUT STATES OF PLANT



ACTUAL STATES OF PLANT



5.5 CONCLUSION

It is seen that the eigen values of system with the above state feedback controller is always negative, suggesting that the system is stable. It is also supported by the constant delay simulation as shown above. The states of observer and plant are stable. A network laboratory in which a plant (E.g. a servo motor) is connected to the predictive state feedback guaranteed cost controller via a network can be used to simulate the results in real-time and verify the effectivity of this method. This control scheme stabilizes the system in lesser time as compared to fixed gain state feedback controller. Hence, besides stabilizing a NCS, it also satisfies a certain guaranteed performance criteria.

APPENDIX-I

MATLAB CODE FOR SAMPLE SYSTEM & NCS MODEL IN SIMULINK

```
% define the state matrices
A = [1.01 0.2710 -.4880; .4820 .1 .24; .0020 .3681 .7070];
B = [5 5; 3 -2; 5 4];
C = [1 2 3;4 3 1];
Q = 0.2 * eye(3);
R = 0.1 * eye(2);

Y = inv(R);
pi = [A zeros(3,9); eye(9) zeros(9,3)];
Atilde = [pi zeros(12); zeros(12) pi];
B1 = [B; zeros(21,2)];
B2 = [zeros(12,2); B; zeros(9,2)];
Itilde = [zeros(12,3) ; eye(3); zeros(9,3)];
Cbar = [C zeros(2,9) -C zeros(2,9)];
I0 = [zeros(3,12) eye(3) zeros(3,9)];
Ii = [zeros(3,(5)*3) eye(3) zeros(3,(2)*3)];
Ibar = [eye(3) zeros(3,21)];
Qbar = [ Q zeros(3,21); zeros(21,24)];
m = [ ones(4,1);zeros(20,1)];

%define the LMI variables or unkown matrices
setlmis([])
P = lmivar(1,[24,1]);
K0 = lmivar(2,[2,3]); %K1=K0=Ki, state feedback gain
L = lmivar(2,[3,2]);    %observer gain
W = lmivar(1,[24,1]);


%Define the 1st LMI
lmiterm([1 1 1 P],-1,1);
lmiterm([1 1 1 0],Qbar);
lmiterm([1 1 2 0],Atilde');
lmiterm([1 1 2 -K0],Ii',B1');
lmiterm([1 1 2 -L],Cbar',Itilde');
lmiterm([1 1 2 -K0],Ii',B2');
lmiterm([1 1 3 -K0],Ibar',1);
lmiterm([1 2 2 W],-1,1);
lmiterm([1 3 3 0],-Y);

%Define the 2nd LMI
lmiterm([-2 1 1 P],1,1);
lmiterm([-2 1 2 0],eye(24));
lmiterm([-2 2 2 W],1,1);

%define the 3rd LMI
lmiterm([3 1 1 0],-0.8);
lmiterm([3 1 2 0],m');
lmiterm([3 2 2 W],-1,1);

%Find a feasible solution to the set of LMIs
lmisys = getlmis;
```

```
[tmin,xfeas] = feasp(lmisys);
w = dec2mat(lmisys,xfeas,W);
k = dec2mat(lmisys,xfeas,K0);
p = dec2mat(lmisys,xfeas, P);
l = dec2mat(lmisys,xfeas,L);

%Frame the minimization problem
c = zeros(612,1);
for j=1:612,
[Pj,Wj] = defcx(lmisys,j,P,W);
c(j) = trace(Pj*w + p*Wj);
end
[copt,xopt] = mincx(lmisys,c);

%values of all unknown matrices
Pnew = dec2mat(lmisys,xopt,P);
Wnew = dec2mat(lmisys,xopt,W);

%print the values of K1 and L obtained
K0new = dec2mat(lmisys,xopt,K0)
Lnew = dec2mat(lmisys,xopt,L)

%check if the stopping criterion is satisfied
if([(-Pnew+ Qbar) (Atilde + B1*K0new*Ii + Itilde*Lnew*Cbar +B2*K0new*Ii)'
(Ibar'*K0new');
            (Atilde + B1*K0new*Ii + Itilde*Lnew*Cbar +B2*K0new*Ii) (-Wnew)
zeros(24,2);
            (K0new*Ibar) zeros(2,24) -Y]<0)
        k=1;
else
    k=0;
end
k
```

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