AN ANALYSIS OF FREQUENCY RECOGNITION ALGORITHMS AND IMPLEMENTATION IN REALTIME

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology
In
Telematics and Signal Processing

By

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Rourkela

2007-2008

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CERTIFICATE

This is to certify that the thesis entitled, "**An Analysis of Frequency Recognition Algorithms and Implementation in Real-Time**" submitted by **Ch.Vamshi** in partial fulfillment of the requirements for the award of Master of Technology Degree in **Electronics & communication Engineering** with specialization in "**Telematics and Signal Processing**" at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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ABSTRACT

Frequency recognition is an important task in many engineering fields, such as audio signal processing and telecommunications engineering. There are numerous applications where frequency recognition is absolutely necessary like in Dual-Tone Multi-Frequency (DTMF) detection or the recognition of the carrier frequency of a Global Positioning System (GPS) signal. Furthermore, frequency recognition has entered many other engineering disciplines such as sonar and radar technology, spectral analysis of astronomic data, seismography, acoustics and consumer electronics.

Listening to electronic music and playing electronic musical instruments is becoming more and more popular, not only among young musicians. This dissertation details background information and a preliminary analysis of a musical system, the Generic Musical Instrument System (GMIS), which allows composers to experiment with electronic instruments without actually, learning how to play them.

This dissertation gives background information about frequency recognition algorithms implemented in real time. It analyses state-of-the-art techniques, such as Dual- Tone Multiple-Frequency (DTMF) implementations and MIDI-based musical systems, in order to work out their similarities. The key idea is to adapt well-proven frequency recognition algorithms of DTMF systems, which are successfully and widely used in telephony. The investigations will show to what extent these principles and algorithms can be applied to a musical system like the GMIS.

This dissertation presents results of investigations into frequency recognition algorithms implemented on a Texas Instruments (TI) TMS320C6713 Digital Signal Processor (DSP) core, in order to estimate the frequency of an audio signal in real time. The algorithms are evaluated using selected criteria in terms of speed and accuracy with accomplishing over 9600 single measurements. The evaluations are made with simple sinusoids and musical notes played by instruments as input signals which allows a solid decision, which of these frequency recognition algorithms is appropriate for audio signal processing and for the constraints of the GMIS in real time.

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1. INTRODUCTION

Frequency recognition is an important task, not only in many scientific disciplines such as astronomy, physics or engineering but also in everyday life like in telephony, medical applications or consumer electronics. Frequency recognition is used in many applications,
for example spectrum analyzers and seismographs to analyze earth quakes which make life both more convenient and secure.
In consumer electronics, musical systems have found a broad distribution and have a remarkable market potential. Playing and listening to electronic music is becoming more and more important as a leisure activity, for young and old alike. Therefore, the design of a Generic Musical Instrument System (GMIS), allows musicians the chance to experiment with other musical instrument sounds without actually having to learn them.
One very widely spread application of frequency recognition is used in telephony and is called Dual-Tone Multiple-Frequency (DTMF). Its advantageous, well-proven algorithms could be adapted to musical systems which show similarities to DTMF systems. These comparable properties of DTMF systems have to be analyzed and their usability has to be evaluated with respect to musical systems such as the GMIS.
Therefore, four different Fourier transform-based frequency recognition algorithms are subject to an analysis: the Discrete Fourier Transform (DFT) taken as a baseline to evaluate all algorithms, the Fast Fourier Transform (FFT) which is the workhorse in many engineering applications, the Goertzel algorithm and the Non-Uniform Discrete Fourier Transform (NDFT) which are both successfully used in DTMF systems.
These frequency recognition algorithms are implemented on a Texas Instruments (TI) TMS320C6713 digital signal processor (DSP) core in order to estimate the frequency of the audio signal in real time. These frequency recognition algorithms are evaluated by selected criteria in terms of speed and accuracy by using both simple sinusoids and musical notes played by instruments as input

signals. This analysis allows a solid conclusion to be drawn regarding the application of the frequency recognition algorithms in musical systems such as the GMIS.

1.1 Background

1.1.1 Dual-Tone Multi-Frequency (DTMF) Systems:

Dual-Tone Multi-Frequency (DTMF) is used for remote mono-directional user- to machine communication in telephony, service selection in Intelligent Networks and serinteractive phone services such as telephone banking to obtain a desired service. The addressed machine is controlled by a unique mixture of two standardized sinusoids.

F in Hz		**High group**			
		1209	**1336**	**1477**	**1633**
Low group	**697**	1	2 abc	3 def	**A**
	770	4 ghi	5 jkl	6 mno	**B**
	852	7 pqrs	8 tuv	9 wxyz	**C**
	941	*	0	#	**D**

Figure 1: DTMF frequencies according to ITU-T Q.24

For each column and row, one sinusoid of a standardized frequency is allocated. When a button is pressed, the mixture of two of these frequencies is sent to the exchange. Therefore, in order for the exchange to determine the key being pressed, accurate frequency recognition is required to separate the two tones. The accuracy of this recognition is very high in order to comply with the ITU-T specifications.

DTMF systems are well-proven and have several advantages which make them convenient for signal processing and frequency recognition. For instance, DTMF systems operate on a low sampling frequency of $f_s = 8000Hz$. Consequently, compared to other systems with higher sampling rates, the number of sampling points per second of a signal is smaller than sampled.

Another feature that DTMF systems have that can be used to simplify the processing is the use of simple pure sinusoidal signals only, because these signals are meant to control machines and so these signals have to be unique. The frequency recognition algorithms used to determine these signals, simply have to perform a simple peak detection after having estimated the spectrum and do not have to take the shape of the spectrum into account.

One very important and requirement for the design of frequency recognition algorithms in DTMF systems is the fact that the tolerance of the frequency recognition is only 1.5%. This might not sound much, but actually regarding the lowest frequency of 697Hz, the absolute tolerance is 10.46Hz. This is, anticipatory, a relatively big tolerance, compared to other systems like musical systems as described in the following system.

Summing up, DTMF systems benefits from the following characteristics:

*→Low sampling rate of $f_s = 8KHz$.

*→8 standardized simple sinusoidal signals, known in advance.

*→Limited bandwidth: 697Hz - 1633Hz

*→Minimum absolute tolerance: 10.46Hz (1.5%).

These advantages are taken into account for DTMF detectors and are used to reduce computational requirements and increase accuracy when developing frequency recognition algorithms for the purpose of DTMF detection.

1.1.2 Musical Instrument Digital Interface (MIDI) in Musical Systems:

The Musical Instrument Digital Interface (MIDI) version 1.0 was defined by a consortium of musical instrument manufacturers, The International MIDI Association in 1983. The main purpose was to set up a standard interface in order to make electronic musical instruments of different manufacturers compatible among each other. The communication of these musical devices increased the sales quantities but what is more, this standard also caused a boom in the composition, development and recording of electronic music among musicians who cannot afford a professional recording studio.

MIDI notes and their corresponding frequencies are the basis for the following investigations. An electronic keyboard has 88 keys, starting with MIDI note #21 until MIDI note #108, and has its origin in the theory of the equal tempered piano. Based on note A4 (440Hz), each neighbored note is one 12th part of an octave distant. This distance is also called tempered semitone. Between two octaves, the notes' frequencies double. Taking these principles into account, the corresponding frequency of each MIDI note is

$$f(n_{MIDI}) = 440.2^{\frac{n_{MIDI}-69}{12}}\ Hz \qquad for\, 21 \le n_{MIDI} \le 108 \qquad (1)$$

Where n_{MIDI} is the MIDI note number.

In terms of digital signal processing, the advantages of the Musical Instrument Digital Interface (MIDI) used in musical systems (in particular of a keyboard), are worked out in this section.

One major important issue is the limited bandwidth with a minimum frequency of 27.5Hz and a maximum frequency of 4186.01Hz. Frequency recognition algorithms have to be designed with respect to this bandwidth which limitation reduces their complexity immensely. Also, since all 88 frequencies of interest are standardized, the algorithms can be developed by referring to these expected frequencies.

There are some considerable disadvantages of musical systems, however, which have to be faced when developing frequency recognition algorithms. Mostly, musical systems use a high sampling frequency of $f_s = 44100Hz$ to meet the Nyquist-Shannon theorem. The Nyquist- Shannon states that the sampling rate f_s have to be greater than the twice highest frequency in the signal, in order to be able reconstruct this signal correctly. If this condition is not fulfilled, all the frequencies above the half the sampling rate, i.e. the Nyquist frequency, will appear as lower frequencies in the reconstructed signal which is
called “aliasing”. Since the audible range of human is within 0Hz and 20000Hz and is therefore below the Nyquist frequency of $f_s / 2 = 22050Hz$, the Nyquist-Shannon sampling theorem is fulfilled using a sampling rate $f_s = 44100Hz$.

Recapitulating, the properties of musical systems as analyzed above are listed as follows:

*→ Sampling rate of $f_s = 44.1kHz$

* →88 standardized MIDI frequencies

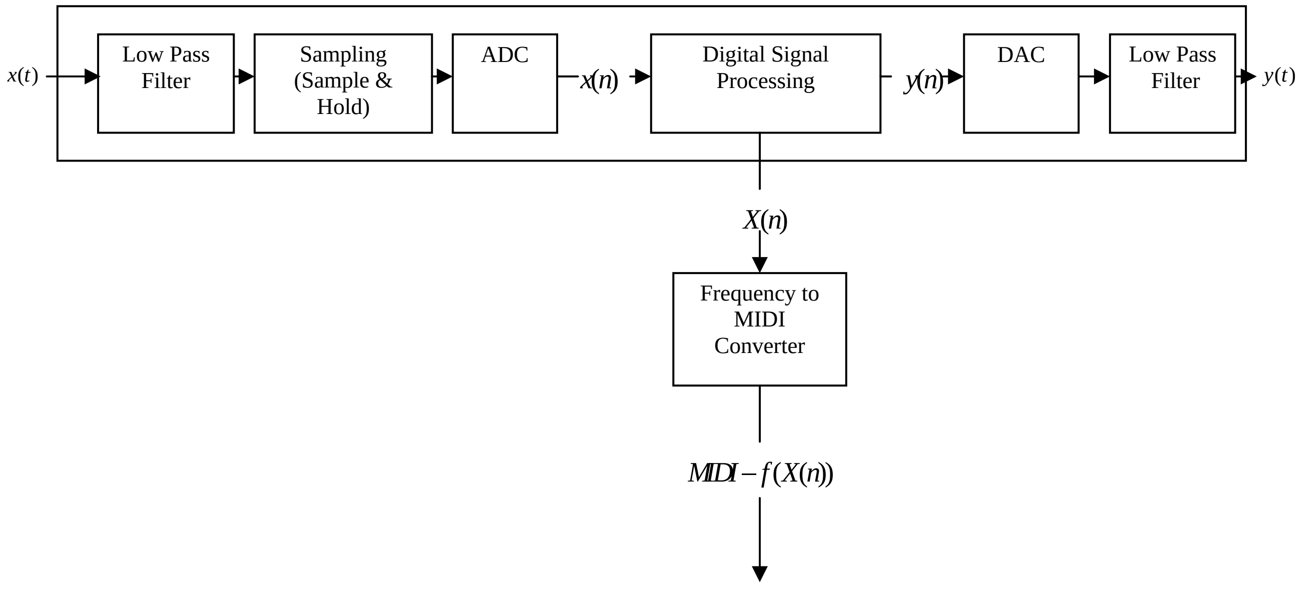
* →Limited bandwidth: 27.5Hz - 4189.01Hz

* →Minimum absolute tolerance: 0.82Hz (2.81%).

1.2 The Generic Musical Instrument System (GMIS)

Among young musicians, playing electronic instruments has become more and more popular. At the same time, the attraction towards learning classical instruments (for example flute, saxophone, etc) has decreased despite their importance in music composition. The design of a Generic Musical Instrument System (GMIS), therefore, allows musicians the opportunity to experiment with other musical instrument sounds without actually having to learn how to play them.

The GMIS is a system which can make any instrument sound like any other instrument. In order to attain maximum benefits, the system should operate in real time. The advantage of real time behavior is the musician’s chance to listen to the result immediately and actually compose by ear.



The Generic Musical Instrumental System (GMIS)

Figure 2: The Generic Musical Instrument System (GMIS)

The GMIS consists of a common Digital Signal Processor (DSP) system which is used to recognize the frequencies of the incoming audio data's input in real time, as shown in the figure2. The time and amplitude continuous audio data passes an anti-aliasing band-limiting low pass filter, before a sample and hold unit samples the signals in time. There is still continuous amplitude whose infinite values have to be quantized by the analogue to digital converter (ADC). The time and value discretized data can now be processed by the digital signal processor (DSP). After the processing the digital to analogue converter (DAC) converts the digital data into an analogue form. Before the audio data is output, another low pass filter smoothes the signal by removing the high frequency components which are an undesired by-product of the converting process.

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This dissertation analyses the frequency recognition part only and does not carry out
further investigations on the frequency to MIDI conversion.

2. Fourier transform-based Frequency Recognition Algorithms

In 1807, Jean Baptiste Joseph Fourier (1798 - 1830) developed the theory about the Fourier series but was rejected by his supervisors Lagrange, Laplace and Legrendre. Finally, in 1822, he published his work in his book “Théorie analytique de la chaleur”. More or less as a side product of this work, he derived the so-called Fourier Series, where he proved that any periodic signal consists of an infinite number of sinusoids and a constant. His theories have revolutionized science and are indispensable in many technical applications.
In this section, after a brief definition of the Fourier Series and the Fourier Transform, four Fourier transform-based algorithms are going to be introduced. These algorithms are the Discrete Fourier Transform (DFT) and its faster

implementation Fast Fourier Transform (FFT), the Goertzel algorithm, and the Non-Uniform Discrete Fourier Transform (NDFT).

2.1 The Fourier Series:

With the Fourier Series, it is possible to create any periodic signal, $x(t)$, in the time domain from the sum of an infinite number of sinusoids, i.e. sine and cosine functions, which are integer multiples of the fundamental frequency, f_0. The Fourier Series is defined as:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n.\cos(n.w_0.t) + b_n.\sin(n.w_0.t)) \qquad n = 0,1,.......\infty \qquad (2)$$

Where a_0 is the amplitude of the direct current component, n is the current number of the sinusoidal component, a_n and b_n the amplitude of the n^{th} sine and nth cosine function respectively, t the representative of the time domain, w_0 the angular frequency, with , $w_0 = 2.\pi.f_0$ where f_0 is the fundamental frequency.

$$a_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} x(t).\cos(n.w_0.t).dt \qquad (3)$$

$$b_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{+\frac{T_0}{2}} x(t).\sin(n.w_0.t).dt \qquad (4)$$

Where T_0 is the period of fundamental frequency, f_0, of the signal, with $w_0 = 2.\pi.f_0$.

2.2 The Fourier Transform:

The Fourier Series is a special case of the Fourier integral and is valid for periodic signals only. For any non-periodic signal, the Fourier Series cannot be applied anymore and the Fourier Transform has to be taken into account and is defined as:

$$X(f) = \int_{-\infty}^{+\infty} x(t).e^{-j.w_0.t}dt \tag{5}$$

where t and f stand for the time and frequency domain respectively; x(t) represents the continuous time signal and X(f) its spectrum in the frequency domain. The indicator for the imaginary part $j = \sqrt{-1}$ and w_0 is the angular frequency and is defined for the Fourier Transform as:

$$w_0 = 2.\pi.f \tag{6}$$

The Fourier Transform can finally be written as:

$$X(f) = \int_{-\infty}^{+\infty} x(t).e^{-j.2.\pi.f.t}dt \tag{7}$$

Since the Fourier Transform is valid for infinite continuous signals only and a numerical implementation is just possible with finite discrete signals, due to limited memory and computation time, the Discrete Fourier Transform (DFT) can be derived.

2.3 The Discrete Fourier Transform (DFT):

The easiest and most direct way to obtain the discrete spectrum of a signal is the Discrete Fourier Transform (DFT). It is also the basis for the four Fourier transform-based frequency recognition algorithms described in section 5.4.2.2. Because the DFT is the slowest algorithm, it is also taken as a baseline to evaluate all the other investigated algorithms.

$$t \rightarrow n.T \qquad n = 0,1,......,N-1 \tag{8}$$

$$f \rightarrow k.\frac{f_s}{N} \qquad k = 0,1,....N-1 \tag{9}$$

Where n represents the sample index for discrete time domain signal values and k the discrete spectral index. The sampling frequency f_s and the accordant period T_s are linked via the relation

$$T_s = \frac{1}{f_s} \tag{10}$$

The DFT is finally formulated more convenient as

$$X[k] = \sum_{n=0}^{N-1} x[n].e^{-j.\frac{2\pi}{N}.k.n} \qquad k = 0,1,....N-1 \qquad (11)$$

Equation (11) is the definition of the Discrete Fourier Transform (DFT), with x[n] as the
sampled time signal and X[k] as the representative of the discrete frequency spectrum.

2.4 The Fast Fourier Transform (FFT):

The Fast Fourier Transform (FFT) is not an independent time to frequency domain transform but an effective recursive algorithm to calculate the DFT and was developed by Cooley and Tukey in 1965. It has revolutionized digital signal processing and is the basis for many real time applications whenever a transform of signals from time to frequency domain is required.

2.4.1 Decimation of the DFT in Time (DIT):

The FFT takes advantage of reducing redundant calculation of DFT coefficients, also called twiddle factors. A prerequisite, therefore, is that the number of samples or the buffer size, N, is a power of two. If this condition is not given, zero-padding (in other words, adding zeros at the end of the input buffer) has to be applied. Then, according to the principle of divide and conquer, the input samples have to be re-ordered N times until N/2 2-point DFTs can be calculated. This process is also known as bit reversal. Starting with the calculation of the 2-point DFTs, this procedure follows $\log_2(N)$ times with $N/4$ 4-point DFTs, then with $N/8$ 8-point DFTs until the (N/N) N-point DFT is reached. This computational part is also known as butterfly computation.
The whole algorithm as described above and is summed up in table 1.

Table 1: The FFT algorithm (Decimation in Time)

Steps#	Action	Comment
0	Zeros padding (add zeros to the buffer), if the number of samples N is not to the power of two.	The FFT requires $N = 2^m$ (with m as an integer number) Input samples.
1	Bit-reverse input samples until N/2 input sample pairs for N/2 2-point DFT are reached.	This stage requires N steps
2	Apply butterfly computations recursively	This stage requires $\log_2(N)$ steps.

Following the principle of divide and conquer, the DFT can be split up into two half size N-point DFT, the even and the odd part as shown in equation (12) and (13) respectively.

$$X[k] = X[k]_{even} + X[k]_{odd} \qquad k = 0,1,........,N-1 \qquad (12)$$

$$X[k] = \sum_{n=0}^{N/2-1} x[2.n].W_{N/2}^{n.k} + W_N^k . \sum_{n=0}^{N/2-1} x[2.n+1].W_{N/2}^{n.k} \qquad k = 0,1,........,N-1 \qquad (13)$$

Finally, it can be concluded that

$$X[k] = X_{11}[k] + W_N^k X_{12}[k] \qquad k = 0,1,........,N-1 \qquad (14)$$

This divide and conquer process continues until a basis of $N/2$ 2-point DFTs is reached. The principle remains always the same: Two DFTs of equal length have to be calculated at the same time, whereas the DFT with the odd indices needs to be multiplied with $W_N^k = e^{-j.\frac{2\pi}{N}.k}$. The advantage of having twiddle factors is that they have to be calculated times $\log_2(N)$ only, instead of N^2 as it is with a common DFT.

2.4.2 Bit Reversal:

As figured out in the previous section, the key to the success of the FFT is the use of the butterfly network, which takes $\log_2(N)$ stages. A preliminary for using the butterfly network is the re-ordering of the input data in the time domain. This preparation of the data can be achieved by a bit reversal of the samples' physical addresses as shown in figure 3. This step of the algorithm takes N stages.

2.4.3 The Butterfly Network:

The bit reversal is a preparation of the input data for the butterfly network which is a recursive application of N DFTs. Starting with $N/2$ 2-point DFTs, which are the basis for $N/4$ 4-point DFTs, the recursive computation will be applied until the (N/N) N-point DFT is reached. The whole process of applying the butterfly network takes $\log_2(N)$ stages.

In figure 3, a basic butterfly for a 2-point DFT is shown. Taking the bit reversed input data as a basis, the twiddle factors are multiplied with the input sample with the odd index, whereas the odd indexed spectral component is additionally multiplied by -1.

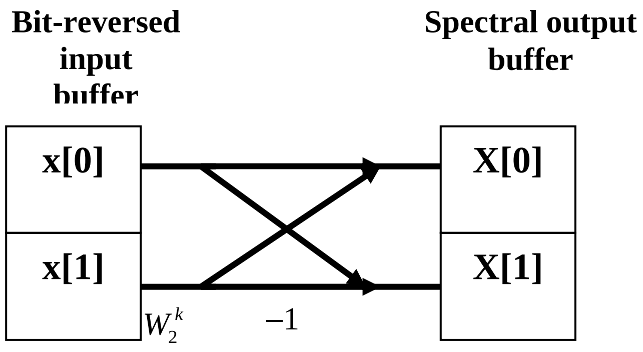


Figure 3: A basic butterfly for a 2-point DFT

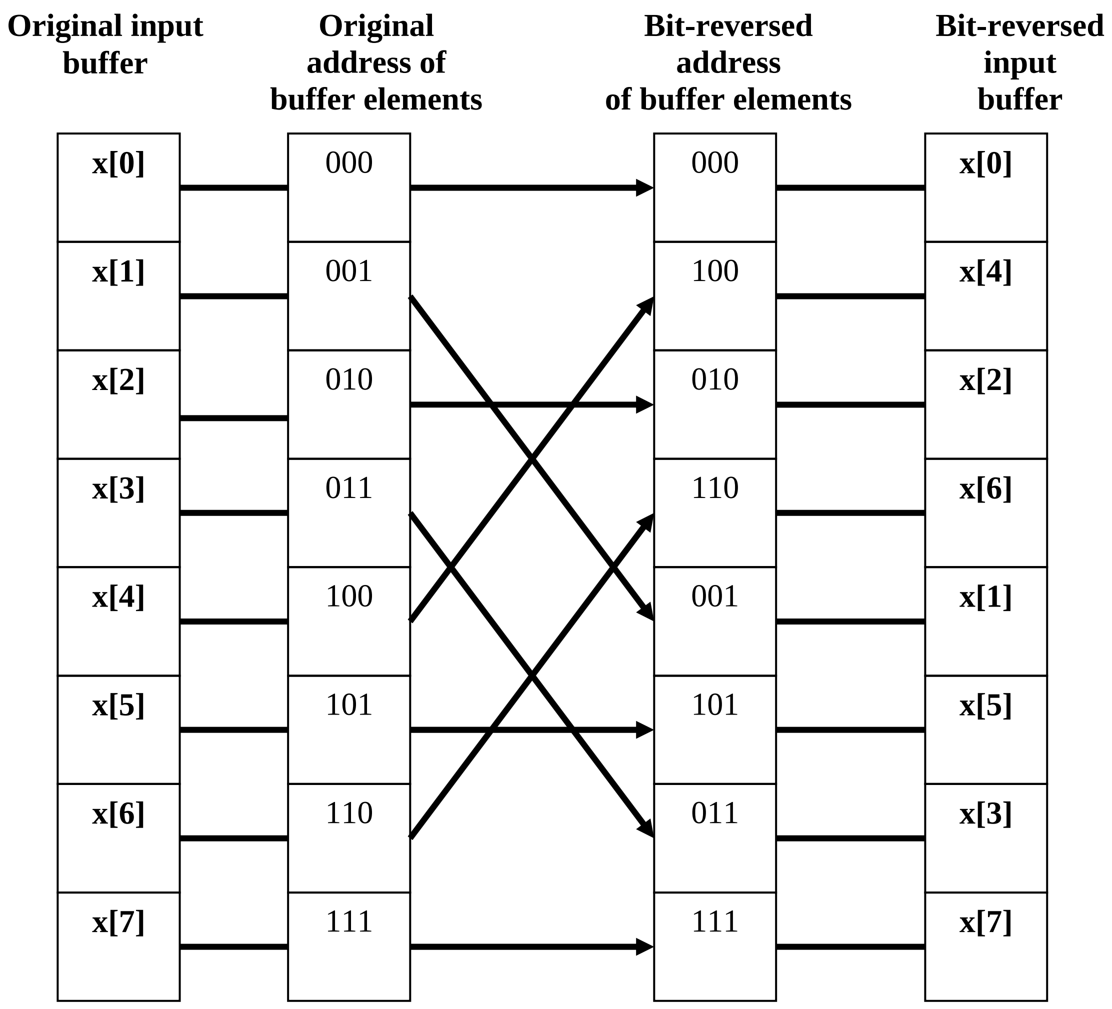


Figure 4: Bit reversal of the input data

2.5 The Non-Uniform Discrete Fourier Transform (NDFT):

The disadvantage of the DFT and FFT is the use of an evenly spaced frequency range which leads into a transform of the whole frequency spectrum for the sampling rate f_s as a constraint. With the Non-Uniform Fourier Transform (NDFT) it is possible, to analyze arbitrary frequency ranges with irregular intervals. Therefore, an increase of accuracy is possible by the application of a well-conditioned frequency vector.

Considering equation (11) of the DFT and taking the equidistant sampled frequency domain in equation (9) into account with

$$\frac{f_k}{f_s} = \frac{k}{N} \qquad k = 0,1,........,N-1 \qquad (14)$$

Where f_s is the sampling frequency, N the number of samples and $f(k) = f_k = [f_0, f_1, ..., f_{N-1}]$ the arbitrary chosen frequency at k, the DFT can be rewritten as a NDFT:

$$X[k] = \sum_{n=0}^{N-1} x[n].e^{-j\frac{2\pi}{f_s}.f_k.n} \qquad k = 0,1,........,N-1 \qquad (15)$$

The NDFT has still a complexity of N^2 but due to the vector of arbitrarily chosen frequencies, it is more accurate within the desired range. Everything outside that range will be of a lesser accuracy but as this range is not required and can be ignored. The vector itself holds the relevant frequencies of interest with well-conditioned arbitrarily chosen frequencies in between these frequency points.
The interpretation of the NDFT's results is raised by assigning the discrete non equidistant spectral index value k directly to the arbitrarily chosen frequency vector f_k (16) after having estimated the spectrum.

$$f(k) = f_k = [f_0, f_1, ..., f_{N-1}] \qquad k = 0,1,........,N-1 \qquad (16)$$

2.6 The Goertzel Algorithm:

Another effective derivative of the DFT is the Goertzel algorithm which found its earliest formulation in 1958. It is a widely-used algorithm used for DTMF applications. Different from DFT and FFT, the Goertzel algorithm does not regard the whole frequency spectrum. These advantages can be used in the adaptation of the Goertzel algorithm for musical systems, since the frequency range is also known from 27.5Hz to 4186.01Hz as well as the number of expected frequencies of interest which is 88. Therefore, it is only necessary to perform an analysis over this range hence fewer points are required for the computation.

The Goertzel algorithm is a filter bench consisting of recursive second order Infinite Impulse Response (IIR) filters. An example of these filters is depicted in figure 6. Its system function can be deduced from its structure and is stated in equation (17).

$$H(z) = \frac{b_0.z^{-0} + b_1.z^{-1}}{1 + a_1.z^{-1} + a_2.z^{-2}} \qquad (17)$$

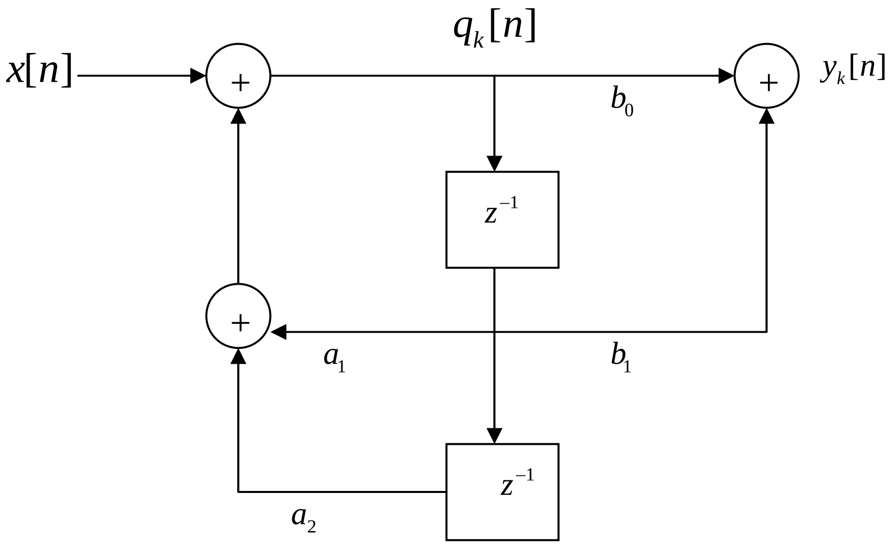


Figure 5: Filter structure of the Goertzel algorithm

Table 2: Filter coefficients for the Goertzel algorithm

Feed forward section	**Recursive section**
$b_0 = 1$	$a_1 = 2.\cos(\frac{2\pi}{N}.k)$
$b_1 = -e^{-j\frac{2.\pi}{N}.k}$	$a_2 = 1$

Inserting these coefficients into the system function and putting the value of $k = \frac{f_k}{f_s}.N$, the system function can be rewritten as:

$$H(k,z) = \frac{1 - e^{-j.w_k}.z^{-1}}{1 - 2.\cos(w_k).z^{-1} + z^{-2}} \qquad k = 0,1,.....88 \qquad (18)$$

The recursively calculated output signal finally is:

$$y_k[k] = a_k.q_k[n-1] - q_k[n-2] + x[n] - e^{-j.w_k}.q_k[n-1] \qquad k = 0,1,.....88 \quad (19)$$

3. Implementation

3.1 The Digital Signal Processor (TI TMS320C6713 DSP):

The Texas Instruments (TI) TMS320C6713 digital signal processor (DSP) is a 32 Bit floating point DSP of the C6000 series and runs at a frequency of 225MHz and is optimized for audio applications. The DSP core contains two exclusively fixed point Arithmetic-Logic Units (ALU), four both fixed and floating point ALU and two both fixed and floating point multipliers. It can perform both in single and double precision.

As in many commercially used processors, the TMS320C6713 has an 8KB first level and a second level cache of 256KB as internal memory. The reason for this unbalanced distribution of internal memory is that level 1 cache is on the other hand fast and therefore contains both program and data cache, but is more expensive compared to the level 2 cache on the other hand. Additionally, the TMS320C6713 also has access to peripheral 16MB external Synchronous Dynamic Random Access Memory (SDRAM) which can directly be accessed.

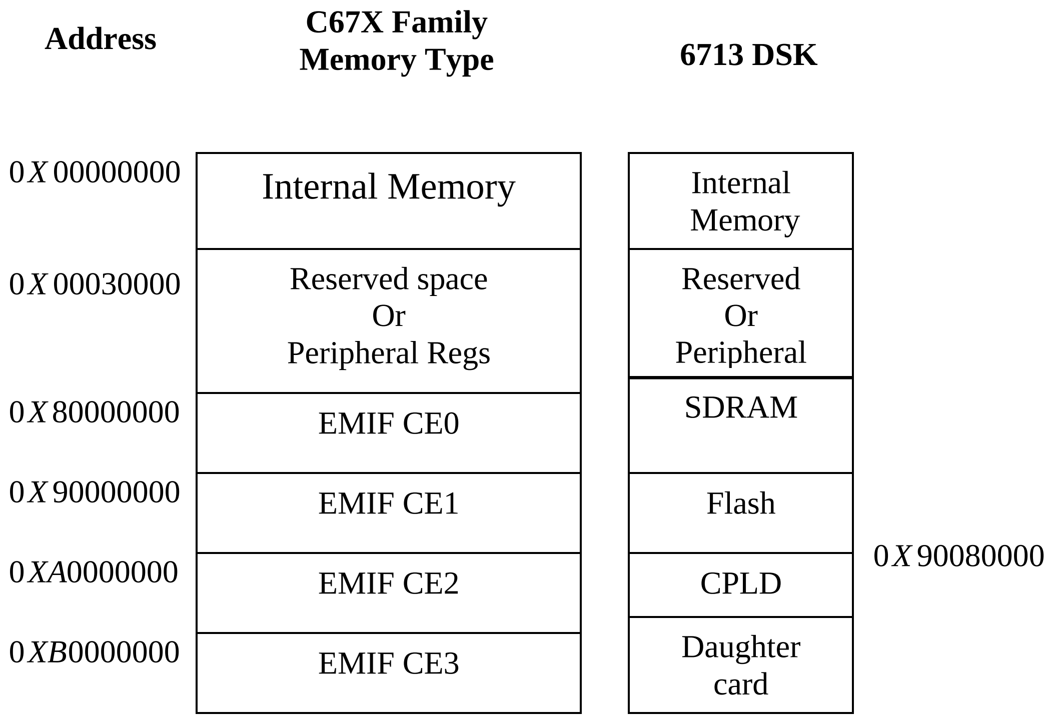


Figure 6: Memory Map for the TMS320C6713 DSP Starter KIT

The user of the processor can choose between the two orders little and big endian. That means, for the little endian mode (the default mode), the least significant byte is stored at the lowest address, literally, little end first. In contrast to this, in the big endian mode, the most significant byte is stored at the lowest address, meaning, big end first.

The TMS320C6713 also makes use of an Enhanced Direct Memory Access (EDMA) Controller. This technique enables the acquisition of audio data directly from the stereo audio codec (AIC 23). The EDMA can also be combined with the two Multichannel Bidirectional Serial Ports (McBSP)

Finally, the TMS320C6713 is provided with an optimized C/ C++ compiler for the processor's architecture. Especially the Multiply-Accumulate (MAC) command is supported and the uses of this multiply and add combination (also known as sum of products) is recommended.

3.2 The TI TMS320C6713 DSP Board

The target hardware used for the analysis of frequency recognition algorithms is the TMS320C6713 DSP Starter Kit, developed by Spectrum Digital and Texas Instrument respectively, is referred as DSP board in the following. The reason why not a Personal Computer (PC) (which nowadays has enough computational power to compete with a DSP core), was not chosen for the analysis on frequency recognition algorithms is because the aim was to target an embedded solution. The Signal Processing Laboratory was provided with two of these DSP boards by Texas Instruments. Due to financial restrictions, one further requirement of the project was that the implementation of the frequency recognition algorithms should be done on this board.

3.2.1 Chip Support Library (CSL):

To handle interrupts, scheduling tasks and their priorities and to manage memory, Texas Instruments has developed a DSP/BIOS real time operating system. It operates independently from the application. All the settings for the issues can be set up by a graphical configuration manager. When compiling, the settings are then applied by the Chip Support Library (CSL).The software interrupt (processBufferSwi) is defined or the software interrupt service routine (ISR) which contains the frequency recognition algorithm.

3.2.2 The Code Composer Studio (CCS):

The Code Composer Studio Code (CCS) is the front end of the TMS320C6713 DSP starter KIT and has a lot of advantageous properties.

The most obvious feature is the data visualization. The CCS offers the chance to observe the data which are currently present in the internal buffers. In the animation mode, even the change over time is recognizable,. i.e. a change in frequency would change the diagrams, too.

Another, very important issue is the simple data import to and export capabilities from these internal buffers. The latter scenario is very useful for debugging purposes, i.e. for verifying results and frequency recognition algorithms either in a simulation environment like MatLab or with an alternative ANSI C compiler.

This approach has been used in the course of the project and has been proved to be the most effective way.
Furthermore, the CCS has the convenient feature of a simplified file I/O which enables to trace results by using injections. Injections are soft breakpoint which halt the CPU far a short moment to perform a file I/O. Like break and animation points, they can just be used in a debug/release environment.

3.3 Usage of the Timer:

In order to use the 32-bit timer, two steps must be completed. At first, the timer has to be configured and second a calibration has to be done which measures the starting and the stopping of the timer itself.

There are three timer registers to be initialized: the control, the period and the counter register. The timer control register is used to determine the timer's mode. The timer period register stores the maximum value the timer counts to. Once this value is reached, a timer overflow occurs and the time measurements become corrupt. The maximum value, a timer can maximal count is 0xFFFFFFFF. To circumvent a timer overflow, either the overflows have to be counted or the number is downscaled through a division by constant, e.g. 1000. This constant has to be bared in mind for the correction of the exported data. The timer counter register stores the current value of the timer. To init the timer, the constant 0x00000000 is to be taken.
The second step, the calibration of the timer in conjunction with the time measurement is illustrated by

```
Open timer;
Configure timer;
Set timer to zero;
Calibrate timer;
Start timer;
Stop timer;
Calibration cycles = Stop – Start;
Start timer;
Perform algorithm;
Stop timer;
Elapsed cycles = Stop – Start – Calibration cycles;
Close timer;
```

Figure 7: Pseudo code for applying time measurements

After having opened and having configured the timer as described above, the timer needs to be calibrated because the measuring the time itself takes some time, too. This calibration is the difference between two successive timer events, i.e. the starting and the stopping of the timer. Then, the difference of both is estimated, measured in cycles.

The actual measurement of the time works as follows. The timer is started, and then the algorithm is performed. After having finished the calculation, the timer stops again and the elapsed cycles are calculated out of difference of the start time, the stop time and he calibration.

4. Implementation of the Frequency Recognition Algorithms

4.1 Methodology:

For the implementation of the algorithms, the real time characteristic is of less importance in the first place. The main purpose is to get them working properly and to verify their results in terms of correctness. In order to achieve this, several tools are very useful.
As a first approach, MatLab and Simulink are consulted for a first implementation of the algorithms. Their ease of use and their undisputable ability to monitor results graphically very quickly are of assistance to get proof of the correctness of the algorithms. The second step is a direct implementation of the algorithms in ANSI C using Microsoft Visual Studio C++ 6.0. Since ANSI C is the programming language which is used for developing applications for the Texas Instruments (TI) TMS320C6713 Digital Signal Processor (DSP) core in the Code Composer Studio (CCS), a fast implementation close to the final application is possible. This step also ensures simultaneous debugging by taking a working algorithm as a reference. This is a very effective way to verify intermediate steps. Since the Code Compose Studio offers the opportunity to export complete buffer content's into text files, MatLab is used for verifying and displaying the results.
As input sources, several options can be considered. Cleary defined unique sinusoids generated by WaveLab and played by the soundcard are valid input signals. Signals from a signal generator are preferable though because they are more reliable signal sources. At a later stage instrument samples playing musical notes are taken as input source too
To summarize, for the implementation of the algorithms onto the target hardware, MatLab and both developer studios (Microsoft Visual Studio C++ 6.0 and Code Composer Studio go hand in hand) and complement each other. This methology accelerates the developing process crucially and, what is also very important, it helps to verify the results to make sure that they are correct.

4.2 Principle of the Frequency Recognition Algorithms

The process which all frequency recognition algorithms undergo can be described as follows. At first, the sampled and quantized input signal $x(n)$ is transformed into the frequency range, in order to obtain real $\Re\{X(n)\}$ and imaginary part $\Im\{\mathrm{X(n)}\}$ of its spectrum. The calculation of magnitude $Mag(X(n))$ is initial for a peak detection mechanism, which finds the spectral component of the signal with the maximum power. The index $k_{\max}$ of this spectral component is finally evaluated with respect to the number of samples N and the sampling rate $\mathrm{f_s}$. Since the frequency is now given as a numerical value, it is ideal for a post processing MIDI conversion.

4.2.1 The Discrete Fourier Transform (DFT):

4.2.1.1 Simulation in Matlab:

The Discrete Fourier Transform is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n].e^{-j.\frac{2\pi}{N}.k.n} \qquad k = 0,1,....N-1 \qquad (20)$$

and directly implemented in MatLab (appendix D). The result is shown in figure 8 depicting the original input signal, its spectrum together with the spectrum's real and imaginary part of a 1 kHz sinusoidal input at a sampling rate of $f_s = 44100Hz$.
The signal is deliberately maximal non-coherent sampled as with a leakage factor of 0.5 as explained in section, in order to obtain a maximum leakage effect. It is noticeable that even under this worse case condition the peak detection mechanism will succeed in terms of finding the maximum energetic spectral component which underlines the decision not to apply extra windowing.

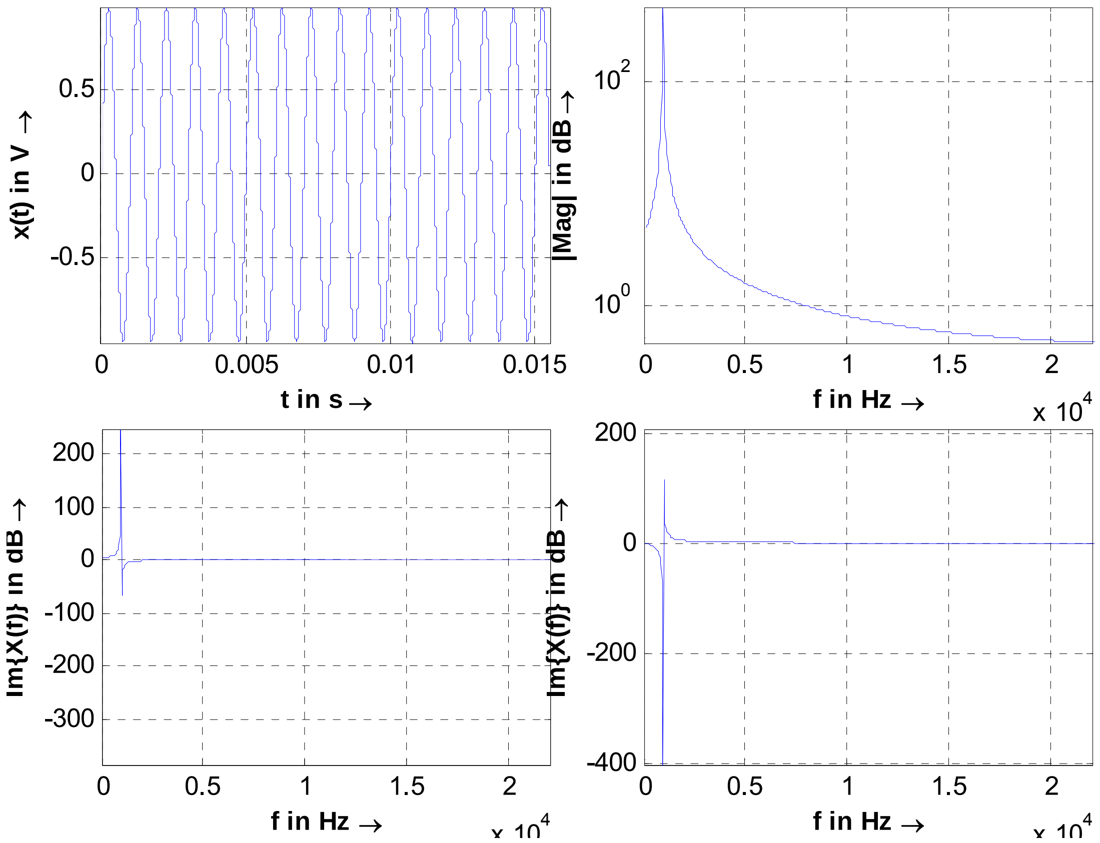


Figure 8: Frequency recognition in principle

4.2.1.2 Implementation of the DFT in C:

A direct implementation of the DFT in C as in the case of the MatLab simulation in the previous section is possible, however with slight differences. As MatLab supports the scientific notation of complex numbers but since this convenient implementation in C is not available, the twiddle factors have to be rewritten by using the Euler equation:

$$W_N^k = e^{-j.\frac{2\pi}{N}.k} = \cos(\frac{2\pi}{N}.k) - j\sin(\frac{2\pi}{N}.k) \tag{21}$$

This preliminary study is very important for the implementation of the DFT on the TMS320C671 because the real and the imaginary part of the spectrum can be regarded separately. Then the MAC (Multiply-ACumulate) command can be applied as explained. Since the architecture and the compilers of the C6000 series are specialized for this product summing up command, an implementation

of the DFT in the way as described above is promising in terms of a fast calculation speed.
What is more, as a consequence of the Nyquist-Shannon Theorem, only half the calculated spectrum is relevant for an evaluation, more precisely, the frequencies range of 0Hz until half the sampling rate $f_s / 2$. Therefore, only a scan of half the spectrum is necessary, a fact that results in an economization of computation time.

4.2.2 The Fast Fourier Transform (FFT):

4.2.2.1 Simulation in MatLab:

Since the Fast Fourier Transform (FFT) is a very efficient algorithm to apply the Discrete Fourier Transform (DFT) by exploiting the redundancy of calculating the twiddle factors, the approach to implement this algorithm implies previous knowledge and understanding of the DFT.
For the simulation of the FFT, it reads wave files which contain signals to be analyzed and outputs both its spectrum graphically and stores the input signal numerically in a user defined output file. The intention is to take this numerical data as a known reference for the implementation on the TI TMS320C6713. Another derivative of this simulation has been used, but instead of referring to wave files as input, actual buffer contents of the TI TMSC6713 were taken as the input for the MatLab file. This method ensures a direct comparison of the obtained spectra estimated both in MatLab and with the TI TMS320C6713 and is very helpful for the development of the algorithm.

4.2.2.2 Implementation of the FFT in C:

Since the implementation of the FFTW was not successfully applied, the assembly coded library has to be taken for implementing the FFT. For this, previous knowledge of the FFT as such is pre-requisite.

For a buffer size of N ≥ 32, where N has to be of the power of two, a N-point FFT can be achieved. The API function DSPF_sp_cfftr2_dit() expects a pointer to an input buffer, a pointer to an array holding the pre compiled twiddle factors and the number of elements N as an integer as input parameters.
Since the input data has to be complex, i.e. consisting of interleaved real and imaginary parts, the buffer containing this data has to have the length of 2 · N. After the execution of this function, this buffer holds the complex result. The number of twiddle factors is N/2 as a result of having taken advantage from their redundant calculation. Per definition of the API, the twiddle factors have to be reordered by bit reversal. Finally, after the calculation of the FFT, the output data has to be reordered by bit reversal. Both twiddle factor pre calculation and bit reversal can be found in the TI FFT support files.
Since there was no need to implement the FFT explicitly in another development environment rather than the Code Composer Studio (CCS), the DFT has been taken as a reference for the development of the FFT. The approach is, to export the input buffer of the algorithm in the CSS and to re-import them into the alternative development environment. Then, a debugging close to real conditions is possible.

4.2.3 The Non-Uniform Discrete Fourier Transform (NDFT):

The structure of the Non-Uniform Discrete Fourier is similar to the DFT and its complexity is N^2, as well. The only difference is the fact that the NDFT does not refer to an equidistant sampled frequency range indexed with k but to an arbitrarily frequency vector f_k as described. It is expected that the assessing to this vector stored in an array causes an additional delay that makes the NDFT a bit slower than the DFT.
The main issue of the NDFT and the key for its advantage in comparison to the DFT is the arbitrarily chosen frequency vector whose accuracy is increased, if it is ellconditioned. For frequencies in the lower range (i.e. 27.5Hz, 29.14Hz, …), a finer fragmentation of the frequency vector is required, because the spacing

between two neighboured notes is smaller than for adjoining frequencies in the higher range (i.e. … 3951.07Hz, 4186.01Hz).

The main problem is the fact that there are 88 MIDI notes, a number which differs from the buffer size N in every case. If the buffer size N was equal to 88, each MIDI note would correspond to a buffer index, but this is, as mentioned before, not possible. For a buffer size N less than 88, the accuracy would not be sufficient enough due to the fact that some of the MIDI notes are simply not assigned to one buffer index. The most interesting case is, if the buffer size is greater than the number of MIDI notes.

The spacings between each frequency point between two MIDI notes are equidistant though, however dissimilar between each MIDI note pair due to the fact that the MIDI notes themselves are not equidistant arranged to each other. Taking equation (1) with as a $f(n_{MIDI}) = 440.2^{\frac{n_{MIDI}-69}{12}} Hz$ basis, the spacings are calculated in dependence on two neighbored MIDI notes and the buffer size N.

$$\Delta_{n_{MIDI}}(N) = \frac{f(n_{MIDI}+1) - f(n_{MIDI})}{\frac{N}{88}} \qquad for\, 21 \le n_{MIDI} \le 108 \qquad (22)$$

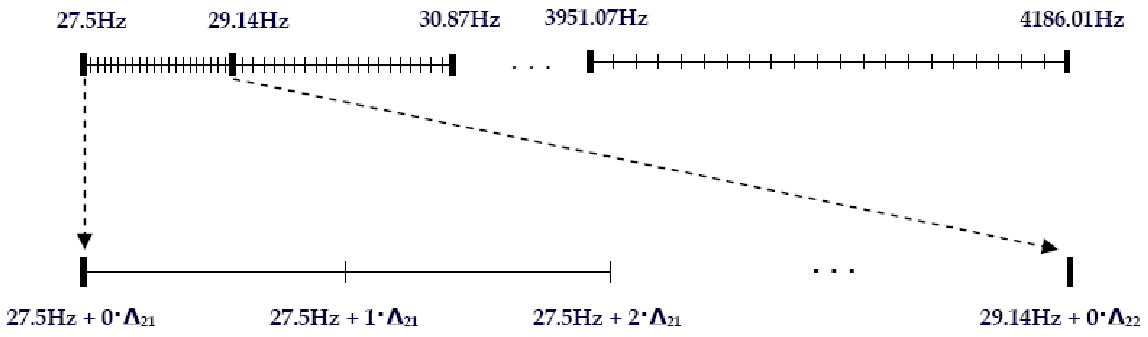


Figure 9: Arbitrarily chosen frequencies for the NDFT

The final structure of the vector holding the arbitrarily chosen frequencies and the corresponding MIDI notes is shown in figure 9 in principle. Since this vector is dependent on the buffer size N, it has to be recalculated for each buffer size N.

4.2.4 The Goertzel Algorithm:

Before coding the Goertzel algorithm up, it is quite useful to have a closer look at the Goertzel IIR filter coefficients as they are listed in table 3 . Since the filter coefficients b_0 = 1 and a_2 = -1 remain constant throughout the whole calculation and hence do not have to be determined at each stage, the filter coefficients b_1 and a_1 have to be calculated only for each Goertzel filter. It is useful to separate the complex denotation of filter coefficient b_1 into a real and an imaginary part using the Euler equation as shown in equations (41) and (42).

$$b_1 = -e^{-j.\frac{2\pi}{N}.k} = -e^{-j.2\pi.\frac{f_k}{f_s}} \qquad k = 0,1,.....88 \qquad (23)$$

$$b_1 = -\cos(2\pi.\frac{f_k}{f_s}) + j.\sin(2\pi.\frac{f_k}{f_s}) = \Re(b_1) + j.\Im(b_1) \qquad k = 0,1,.....88 \qquad (24)$$

For the sake of completeness, filter coefficient a_1 of the recursive section is given as

$$a_1 = 2\cos(\frac{2\pi}{N}.k) \qquad (25)$$

This consideration results in a set of 3 · 88 = 264 pre-calculated filter coefficients to be set up in a look-up table as shown in principle in table 3.

Table 3: Frequencies and filter coefficients for 88 MIDI notes

MIDI note		1	2	…	88
Frequencies		**27.5Hz**	**29.14Hz**	**…**	**4186.01Hz**
Filter coefficients	$\Re(b_1)$	0.999992	0.999991	…	0.827489
	$\Im(b_1)$	0.003917	0.004149	…	0.561482
	a_1	1.999985	1.999983	…	1.654978

For the buffer size N, the signal is filtered with respect to the pre-calculated filter coefficients. Since only the real and imaginary part and their magnitude are of interest only, a calculation of these intermediate values takes place after having filtered the data. Different from DTMF systems using 8 frequencies of interest, the maximum number of the frequencies to be regarded is 88. For reason of saving computation time, the pre- calculated filter coefficients are read from a two dimensional array goertzel_coeff for each loop pass. It has been shown that the speedup of the method of pre calculation the IIR filter coefficients results in a considerable factor of 10^2 .

The actual Goertzel value is obtained with the application of the Goertzel IIR filter by taking the filter coefficients into account. Again, as considered in section 3.4, the estimation of the maximum power happens promptly without and is continuously updated if the maximum is excelled instead of a down streaming scanning of a whole temporary buffer containing all successively calculated Goertzel values.

4.3 Comparison of schedulable Buffer Sizes:

A noteworthy fact is the actual memory requirement for a buffer size N for each of the algorithms. This also demonstrates the reason why a maximum buffer size of just 8192 can be applied.

To obtain N data elements, the ping and the pong buffer have to be set up, each of a size of $2.N$. Due to the codec only applying a stereo input, twice the number of samples as to be scheduled. In total, for the DFT and the Goertzel algorithm, the actual buffer equirements are 4 · N.

In particular in the case of the FFT, for the generation of the input buffer for the API function, another additional buffer is needed. This buffer has the size of $2.N$, because the input data has to be complex with interleaved real and imaginary part. The twiddle factors need an extra buffer of the size $N/2$. In total, 5.5 times more data elements have to be provided than originally scheduled for a problem size N.

The NDFT has, due to its complexity, similar basic memory requirements of $(4.N)$ as the DFT. Additionally, there is a need of N data elements for the pre-calculated arbitrarily chosen frequency vector.

Table 11 lists the actual buffer size requirements for a problem size N and states which factor has to be taken into consideration for an implementation of a problem size N for each algorithm.

5. Measurements

In order to obtain distinct signals consisting of single sinusoids only, a common signal generator is used. For monitoring reasons its signals are simultaneously displayed with an oscilloscope. Once the signal has been led into the line input of the Texas Instruments (TI) TMS320C6713 Digital Signal Processor (DSP) board it is subject to sampling and quantization before it can be analyzed by one of the to be investigated by one of the to be investigated frequency recognition algorithms. The development software, naming the Texas Instruments' Code Composer Studio is installed on the PC and is used for both coding the algorithms and for storing the measurement results in a file.

5.1 Analysis of Input Signals:

Measurements without any preceding estimation or even without a basic analysis are useless. In other words, starting a measurement without any kind of expectancy is neither engineer-like nor scientific. Therefore, this section analyses the input signals' spectra versus time, i.e. the spectral behavior during the duration of the samples. It does not take into account the spectral resolution. It

makes assumptions of the algorithms' frequency recognition capability exclusively based on an analysis of the input signals' spectra and is therefore a hypothesis. By doing this, a necessary preliminary evaluation of the frequency recognition algorithms is possible.
It is presumed that the first category of signals, which consists of a simple sinusoid, are stable over their whole duration, meaning, a simple maximum power search over the spectrum will yield the fundamental frequency. That is why, for this type of signal, frequency recognition is expected to be without complications.
For the second category, signals containing complex waveforms, samples of notes played by musical instruments are used as an input for the frequency recognition algorithms. The expectation is that not every sample holds the maximum power on the fundamental frequency because the signal can no longer be considered to be a simple sinusoid. Moreover, for some of the notes some of the harmonics actually carry more power. Additionally, this behaviour changes over time for some of the regarded notes, which might make frequency recognition more difficult rather than with simple sinusoids. These assumptions have to be confirmed both by the following spectral examination and verified by the measurements itself.

5.1.1 Sinusoidal Inputs:

The bases for the evaluation of the frequency recognition algorithms are the metrics latency, speedup and accuracy as discussed earlier. Latency and speedup can be deduced from the number of cycles an algorithm needs to estimate a frequency and the accuracy depends directly on the algorithmic estimation capability from which the absolute deviation can be calculated.
To find out the best frequency recognition algorithm, at least 10 test series for each buffer size for $32 \leq N \leq 8192$ have been applied to each algorithm. The reasons for these boundaries are as follows. The lower bound $32 \leq N$ is a limitation of the implementation of the FFT API routine (DSDF_sp_cfftr2_dit) from Texas Instruments (TI), whereas the upper bound is due to internal memory

constraints. Furthermore, for a buffer size of N > 8192, there would be unreasonably high computational costs for the calculation of direct frequency recognition algorithms such as the DFT and the NDFT, whose costs are N_2. Concluding, the common boundaries for the buffer size, $32 \leq N \leq 8192$, have been chosen for all the measurements to be able to compare all the algorithms and should be sufficient to fulfill the task.

For each algorithm test series, three cases were investigated: the note A4 with 440Hz (being the note in the centre of the MIDI scale), and two extreme cases: the highest frequency with 4186.01Hz (note C8) and the lowest one of 27.5Hz (note A0) of the target frequency range. The reason for these choices is to determine the ability to analyze the algorithms' capability to handle the highest frequency, 4186.01Hz and the highest spectral resolution demand at 27.5Hz,

However, there are a number of harmonics along with the fundamental frequency which should not occur. When we are considering the input from the function generator. The reason for this is the fact that the signal generator does not produce a pure sinusoid exclusively but has a small amount of harmonic distortion and should be ignored in this case. Additionally, there are reflections due to a mismatched connection from the signal generator to the audio jackinput of the DSP board. But here we are not considering the function generator but taking the signal directly from the system. Since the maximum power can be found at 440Hz, 4186.01Hz and 27.5Hz respectively, these undesired, less powerful frequencies do not carry weight in terms of frequency recognition. This is a good prerequisite for a solid frequency recognition capability regardless of the spectral resolution in terms of accuracy and a reasonable buffer size N.

5.1.2 Inputs of Musical Instruments:

For signals containing simple sinusoids generated by a signal generator, it is anticipated that the frequency recognition algorithms should work perfectly if maximum power detection is used. Whether they can be applied to signals containing complex waveforms is subject to a discussion in this section.

To test the frequency detection capabilities of each algorithm, four different instruments (piano, violin, trumpet and flute) playing three different notes have been taken as an input. Because not every note of the frequency range of interest can be subject to an analysis and a subsequent measurement, because of the range of each instrument, a representative selection has to be made. Thus, all the selected notes are played by each instrument to ascertain comparability.
When applying instrumental inputs, the frequency recognition capability is initially of interest only. The buffer size N is dependent on the demand for the accuracy for the notes C6, C5 and C4 respectively. Equation (44) denotes the coherence between the sampling rate f_s and the demanded accuracy depending on the MIDI note.

$$N = \frac{f_s}{accuracy(n_{MIDI})} \tag{26}$$

Table 13 lists summarizing the constraints of the notes which are subject to further measurements. They are valid each for piano, violin, trumpet and flute.

Table 4: Frequencies and filter coefficients for 88 MIDI notes

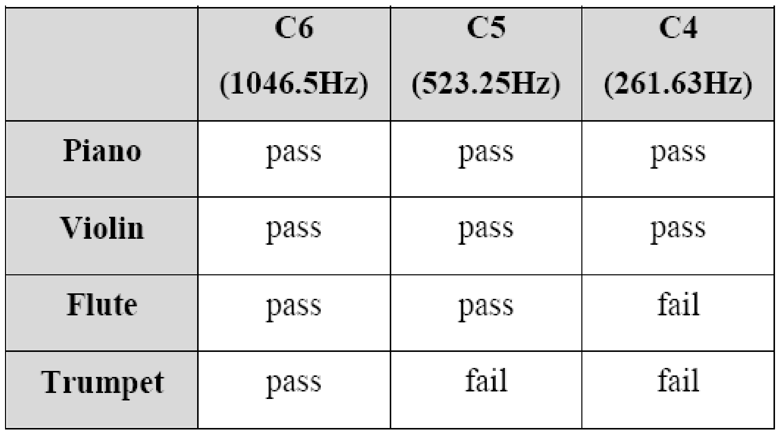
Instrument note	Frequency [Hz]	Demanded accuracy [Hz]	Lower Bound [Hz]	Upper Bound [Hz]	Buffer size N
C6	1046.5	29.37	1017.13	1075.87	$N = \frac{44100Hz}{29.37Hz} \approx 1501.53 \approx 2048$
C5	523.25	14.68	508.57	537.93	$N = \frac{44100Hz}{14.68Hz} \approx 3004.09 \approx 4096$
C4	261.63	7.34	254.29	268.97	$N = \frac{44100Hz}{7.34Hz} \approx 6008.17 \approx 8192$

5.2 Prediction of the Algorithms' Frequency Recognition Capability:

Taking simple sinusoids (440Hz, 4186.01Hz and 27.5Hz) all frequency recognition algorithms have an unambiguous input because the spectral distribution of the signal's power remains constant throughout the whole time window and spectrum's maximum peak can be found at the fundamental frequency. That is the reason why the chances of failure are very low using only a simple power detection technique. Measurements on the algorithms with these sinusoidal inputs and the subsequent analysis with the evaluation criteria will give certainty whether this prediction is right.

For instruments playing musical notes, it is important to find out whether the maximum power can actually be found on the fundamental frequency or on one of its harmonics. In the latter case, investigations have shown that the lower the frequency is the more harmonics exist. For all analyzed musical notes (1046.5Hz, 523.23Hz and 261.63Hz), string-based instruments (piano and violin), the spectral behavior over time is constant and therefore ideal for frequency recognition. It can also be seen, that the relative power distribution of the spectrum of these string based instruments remains stable over time, i.e. the maximum power can always be found on the fundamental frequency of the note being played. However, the spectrum over time of woodwind and brass instruments like the flute and the trumpet respectively is not well behaved for each of the analyzed notes. The trumpet does not have the maximum power on the fundamental frequency for MIDI note C5. Furthermore, for MIDI note C4, both the trumpet and the flute will fail according to the preliminary spectral analysis because the maximum power is not held by the fundamental frequency each. Table 5 lists the probable frequency recognition capabilities of the investigated algorithms with respect to the analyzed instrumental notes.

Table 5: *Probable frequency recognition capability of instruments*



	C6 (1046.5Hz)	C5 (523.25Hz)	C4 (261.63Hz)
Piano	pass	pass	pass
Violin	pass	pass	pass
Flute	pass	pass	fail
Trumpet	pass	fail	fail

5.3 Evaluation Criteria

In order to find the optimum frequency recognition algorithm, there is a need for a precedent definition of fixed quantities and performance metrics. The purpose is a reduction of the number of variables to a reasonable minimum which is then subject to a further analysis. Finally, this preliminary investigation leads to the conclusion that the evaluation criteria, in particular the performance metrics, are dependent on the buffer size N only.

5.3.1 Sampling Rate:

The sampling rate is fixed to $f_s = 44100Hz$ in order to take into account high frequency harmonics, since the analysis of frequency recognition algorithms is subject to a further extension to the whole audible range of 0Hz – 22000Hz. Therefore the sampling rate has to be greater than twice the maximum frequency that can appear in the expected signal in order to avoid aliasing according to the Nyquist-Shannon theorem as explained in section 5.4.1.1. Consequently, this major constraint of having a sampling rate of $f_s = 44100Hz$ is mandatory for the Generic Musical Instrument System (GMIS), despite of having a large buffer size N and thus having an increase of complexity, because the higher the sampling rate, the more sampling points are acquired.

5.3.2 Spectral Resolution:

A fine spectral resolution R is prerequisite for a solid accuracy and is given by the ratio of the sampling rate f_s and the actual buffer size N with

$$R(f_s, N) = \frac{f_s}{N} \tag{27}$$

In general, one can say, the higher the buffer size N the better is the spectral resolution R at a given sampling rate f_s . Due to the fact that an increase of the buffer size N simultaneously causes an increase of computational costs and memory requirements, a trade-off has to be found between the spectral resolution, R, and the buffer size, N.

When analyzing musical notes, the maximum spectral resolution which has to be provided, is $R_{\max} = 0.82Hz$ in order to be able to recognize all frequencies of the whole target frequency range. The reason for this high resolution is the smallest half of the MIDI channels, i.e. between MIDI note #21 and #22 with 27.5Hz and 29.14Hz respectively.

5.3.4 Time to settle:

The system's time to settle t_{settle} is the time which is needed to make sure that the buffer completely holds the signal's samples. This quantity is relevant and subject to further investigations for unbuffered systems only, but because internal buffers are applied, t_{settle} is constant the time required for filling up a buffer of the size N. Let the sampling frequency be $f_s = 44100Hz$ and assuming there is a buffer of the size N = 44100, it takes one second to acquire all 441000 samples. Therefore, the time to settle t_{settle} is defined as

$$t_{settle} = \frac{N}{f_s} \tag{28}$$

As the sampling rate f_s is fixed to 44100Hz and the buffer size N does not change during the operating mode, the time to settle t_{settle} is constant for each buffer size N, too.

5.3.5 Computational Costs of Investigated Algorithms:

The complexities of the to be investigated algorithms are listed for the buffer size N

Table 6: Fourier transform-based algorithmic complexity

Algorithm	Complexity
Discrete Fourier Transform (DFT)	N^2
Fast Fourier Transform (DFT)	$N \log_2(N)$
Goertzel algorithm	$88.N$
Non-Uniform Discrete Fourier Transform (DFT)	N^2

The comparison of the complexities of the Goertzel algorithm and the FFT shows, that at a buffer size of N > 288, the FFT's algorithmic complexity will be greater than the one of the Goertzel algorithm, but the buffer size required for this problem is unreasonable high and furthermore not feasible without massive accessing external memory. This, however, would mean additional latencies and would be a different kind of problem which cannot be taken into consideration at this stage.

Theoretically, the number of cycles corresponds directly to the algorithmic complexity, but there is an additional amount of cycles for pre and post processing of audio data to be regarded, for instance the generation of interleaved complex data, the calculation of magnitude, the spectrum's peak detection and the final evaluation of the most powerful spectral frequency component given as an index. Thus, the overall number of cycles has to be measured and taken into account when analyzing the metrics.

5.4 Performance Metrics

For the evaluation of a system's real time characteristics, it is necessary to take the algorithmic properties into account, i.e. their latency, their speedup and the algorithms' accuracy. These metrics are subject to be discussed in this section.

It is expected that according to the fixed computational costs of each algorithm the number of cycles needed for the calculation of each algorithm, will follow the same law. Consequently, the tendencies of speedup and latency will remain equal with each measurement and independent from the input, but due to the fact that the number of cycles are deduced from the computational cost and are therefore theoretical, they have to be experimentally proven. Therefore and with respect to the fact that pre and post processing of the data is not included to the theoretical considerations, there is an extra need for measuring these metrics.

5.4.1 Latency:

The latency is the time a signal needs to get from its source to its destination after processing. In the case of frequency recognition as applied in this project, latency is the difference in time between playing the note and detecting its frequency or in other words how long an algorithm needs to output the numerical value of the estimated frequency. Therefore, for the number of cycles an algorithm takes, the latency is defined as

$$lt = \frac{cycles}{f_{CPU}} \tag{29}$$

with a given central processing unit (CPU) frequency of the TI TMS320C6713 of $f_{CPU} = 225MHz$. The average in latency $lt_{average}$ is the average temporal resolution of a human ear, i.e. the time that can pass by before the listener realizes a delay in playing a note and hearing the actual sound. It has been shown that on average the latency of the human ear is $lt_{average} = 50ms$. Consequently, in order to be taken seriously into consideration for frequency recognition in real time, the algorithms have to terminate their calculations within this time limit, preferably less than that because frequency recognition will probably not be the only task for the GMIS.

5.4.2 Speedup:

An important metric for the evaluation of is the speedup according to Amdahl's Law. The speedup is the ratio of the number of cycles of an algorithm before and after its improvement and is described as

$$sp = \frac{cycles_{DFT}}{cycles} \qquad (30)$$

The speedup has to be greater than 1 if the algorithm is to be considered superior to any of the other investigated algorithms. Due to the fact that the Discrete Fourier Transform (DFT) is the slowest Fourier transform-based algorithm, it is taken as a baseline for the speedup for all the other algorithms.

5.4.3 Accuracy:

A measure for the accuracy is the absolute deviation. The absolute deviation of a value in a set of values is the absolute difference between this value and a nominal. This nominal can either be a mean of the set of these values or a threshold value, in this case the expected frequency f_{ep}. The value from which the absolute difference is taken is the estimated frequency f_{es}. Being a measure for accuracy, absolute deviation is defined as

$$ad = \left| f_{ep} - f_{es} \right| \qquad (31)$$

Due to the fact that the tolerance is the half the difference between two MIDI channels, in the worse case between MIDI note #21 and #22, the minimum acceptable error equals the maximum spectral resolution required for accurate frequency recognition, $i.e. ad_{\min} = R_{\max} = .82Hz$.

6. Results

The first category, input signals consisting of a simple sinusoid, is used to measure latency and speedup in the first step in order to prove the theoretical complexity discussed earlier and second to show the functionality of the investigated frequency recognition algorithms, i.e. to prove their frequency recognition capability and to investigate on their accuracy as derived earlier.
The second category, instrumental inputs playing musical notes, is taken to judge on the algorithms' frequency recognition capability for input signals containing complex waveforms such as in the case of musical notes. By doing this, it will be demonstrated that a simple frequency recognition algorithm with simple peak power detection as performed in the first set of measurements, will not suffice for a musical system as proposed with the GMIS.

6.1 Frequency Recognition Algorithms analyzing simple Sinusoids

6.1.1 Latency:

The behavior of the algorithmic latency depends on their individual complexity as described earlier. For the regarded frequencies 27.5Hz, 440Hz and 4189Hz, the latency is equal as it can be seen in figures 10 - 12 which underlines the preliminary considerations earlier. Therefore, just an analysis of the results concerning the latency's tendencies for each algorithm is made, not for each frequency in particular.
Figures 10 - 12 show the four algorithms' latency versus the buffer size N for different inputs containing simple sinusoids of 440Hz, 4186.01Hz and 27.5Hz respectively. The algorithms with the maximum latency for the whole range of N, with $32 \leq N \leq 8192$, are the DFT and the NDFT with a narrow difference to the advantage of the DFT as expected. This slight difference is due to the fact that

the NDFT has to read the vector with the arbitrarily chosen frequencies whereas the DFT simply uses the equidistant spectral index k for the computation of the spectrum. According to their equal structure, the tendency of their latencies is equal, too. The Goertzel algorithm has a smaller latency than the DFT and NDFT but is, as expected, still slower than the FFT.

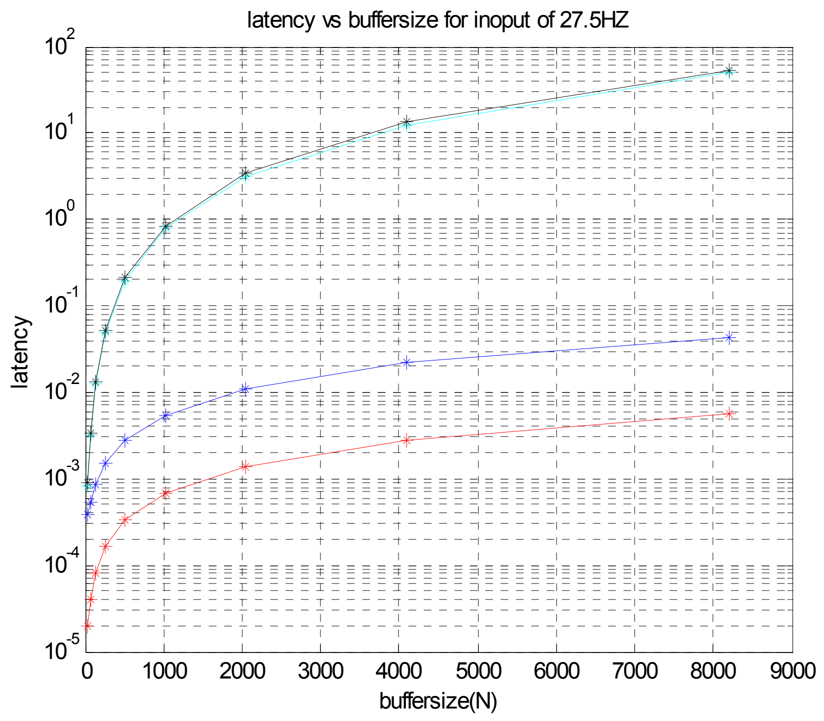


Figure 10: Latency vs. Buffer size for a 27.5Hz input

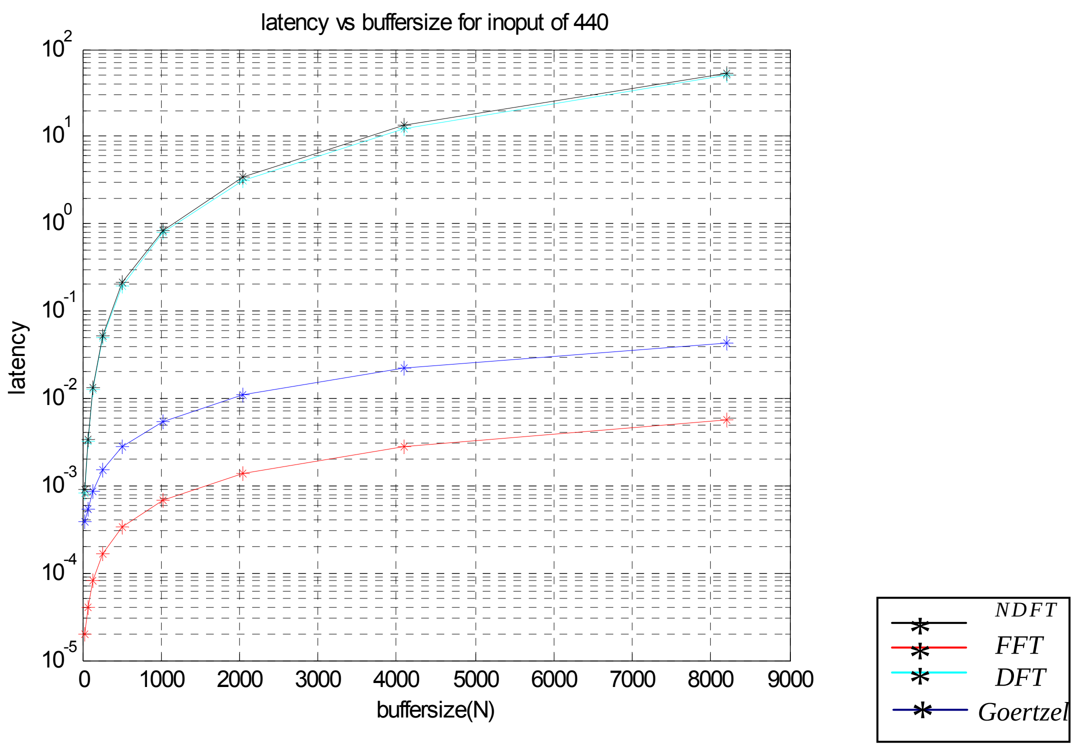


Figure 11: Latency vs. Buffer size for a 440Hz input

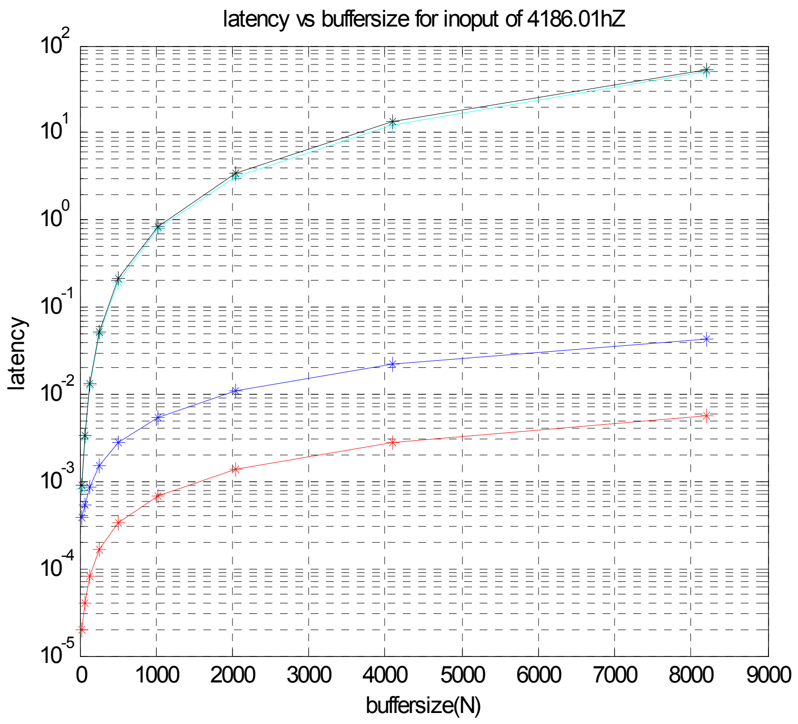


Figure 12: Latency vs. Buffer size for a 440Hz input

6.1.2 Speedup:

Figures 13- 15 show the speedup with respect to the buffer size N for 440Hz, 4186.01Hz and 27.5Hz. Again as expected, since the speedup is also directly linked with the number of cycles, the three graphs are fairly equal for each input frequency.

Taking the DFT as a baseline, just the NDFT is slightly slower which is due to implemental reasons, i.e. the reading of the vector of the arbitrary chosen frequencies takes more time than just simply referring to a loop index.

The Goertzel algorithm with a complexity of 88·N is dramatically faster than the DFT and the NDFT which computational costs are N^2, but is still more slowly than the FFT with a complexity of $N \log_2(N)$, again as expected.

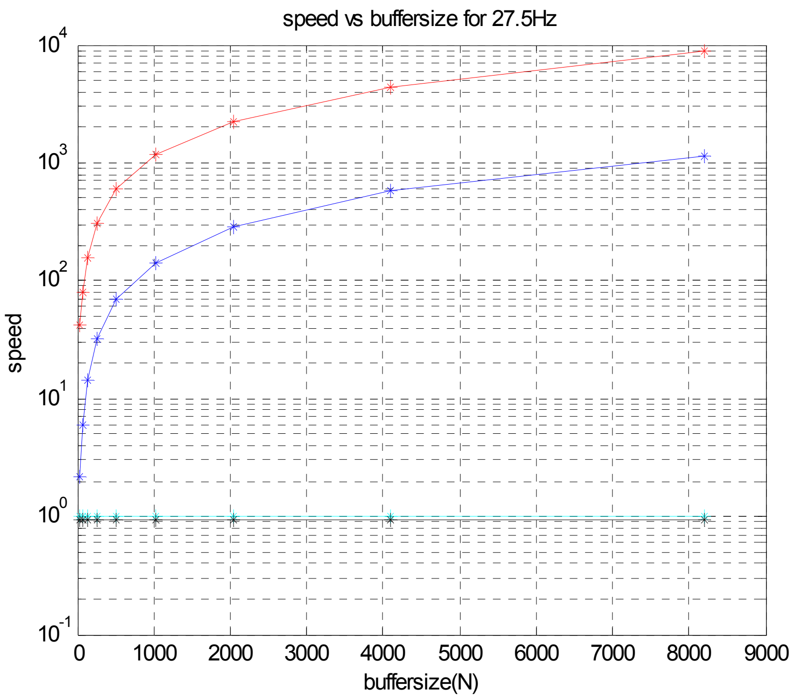


Figure 13: Speedup vs. Buffer size for a 27.5Hz input

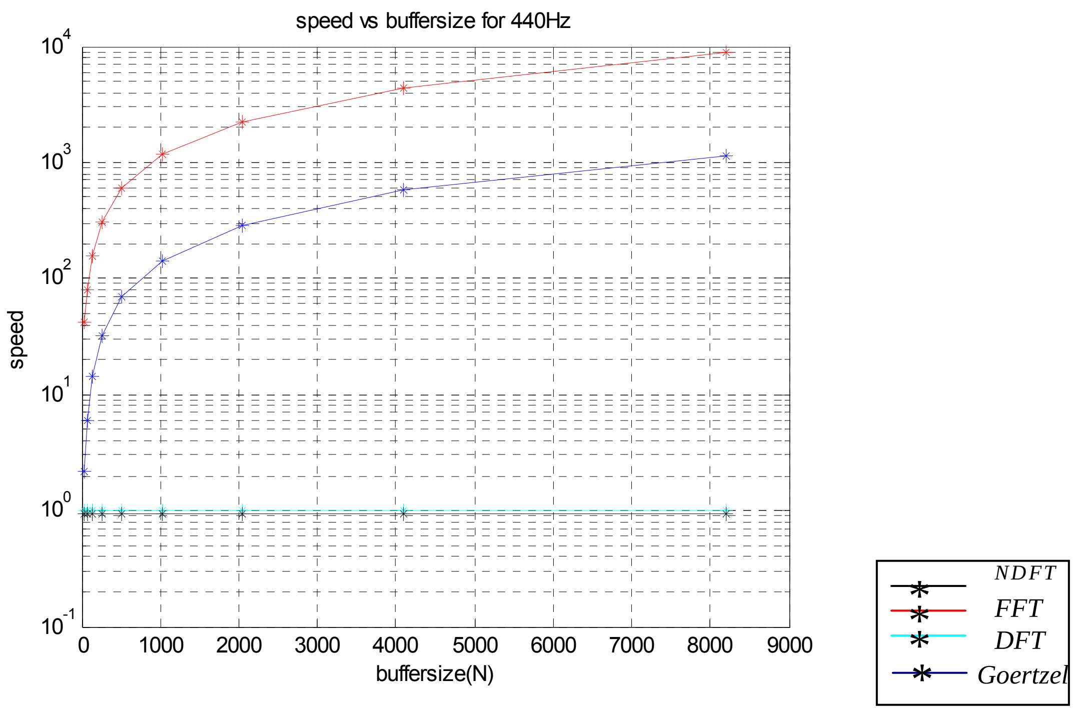


Figure 14: Speedup vs. Buffer size for a 440Hz input

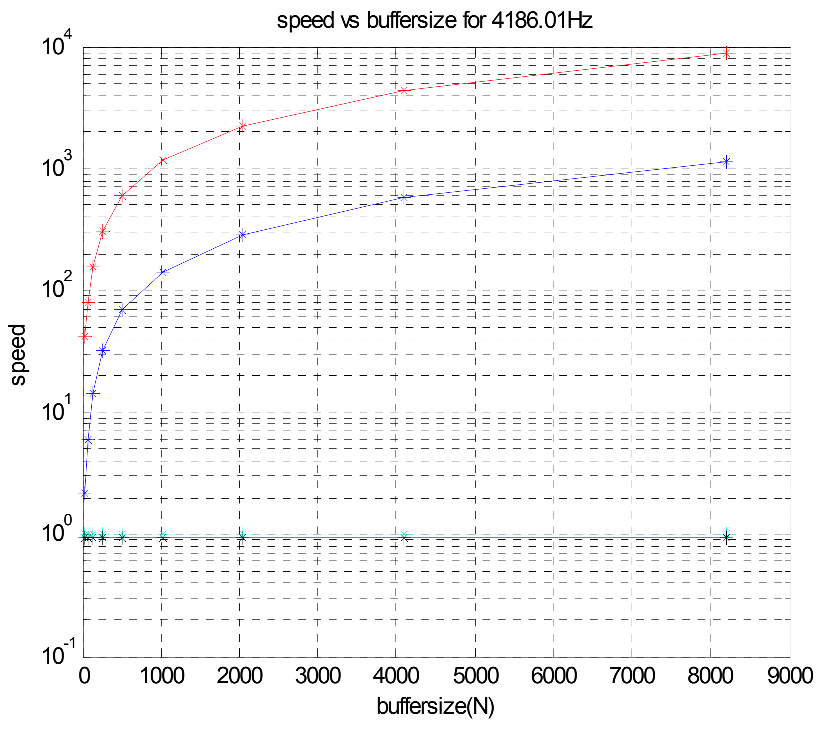


Figure 15: Speedup vs. Buffer size for a 4186.01Hz input

6.1.3 Accuracy:

Why can the fastest algorithm, the FFT, not simply be taken for frequency recognition? The answer is that, the less time an algorithm needs to fulfill its purpose the better it is, but it still has to be accurate enough to meet the requirements. Thus, the accuracy of each algorithm's frequency recognition capability has to be analyzed, too, because a fast but inaccurate algorithm is of no use. As defined, a measure for accuracy is the absolute deviation of the estimated frequency from the expected frequency, i.e. the absolute difference between these both frequencies.

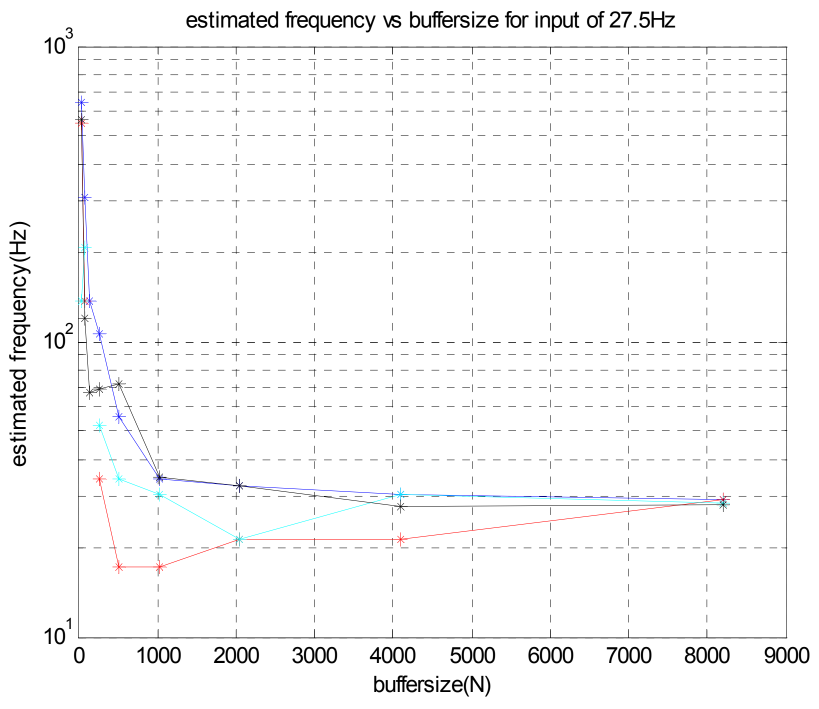


Figure 16: Estimated frequency vs. Buffer size for a 27.5Hz input

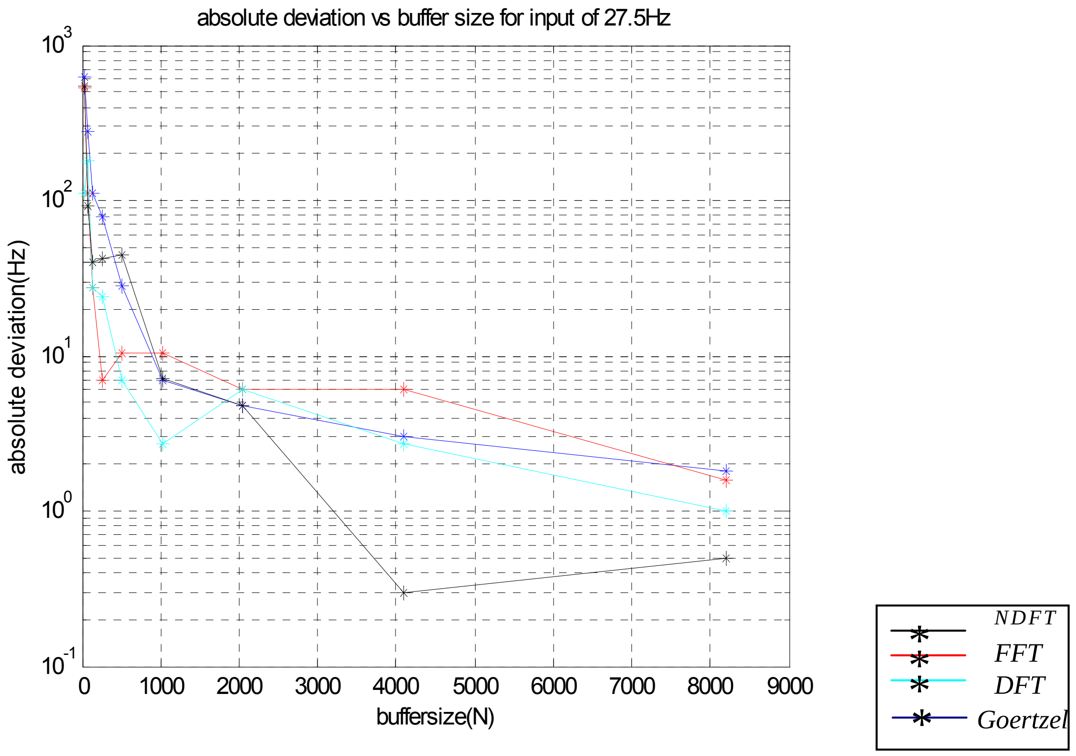


Figure 17: Absolute deviation vs. Buffer size for a 27.5Hz input

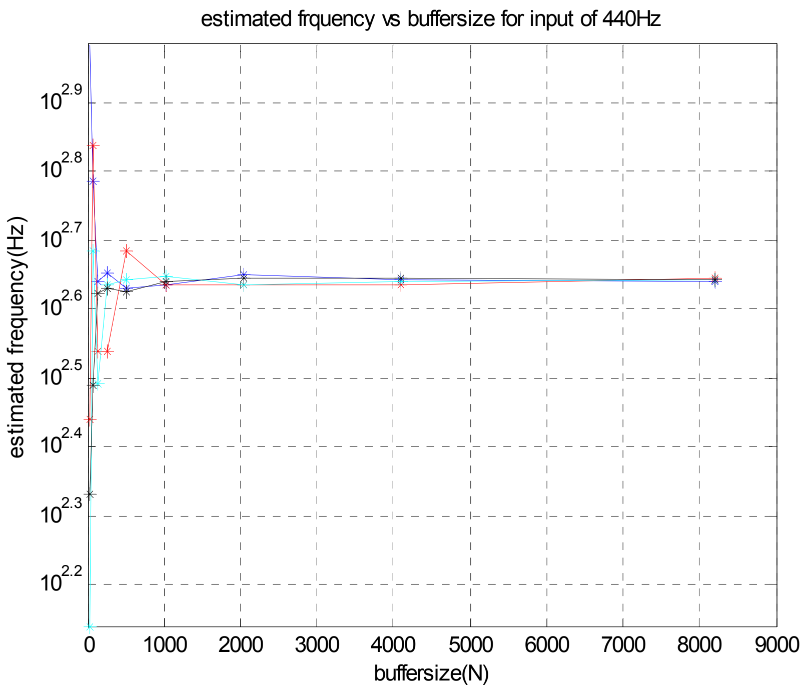


Figure 18: Estimated frequency vs. Buffer size for a 440Hz input

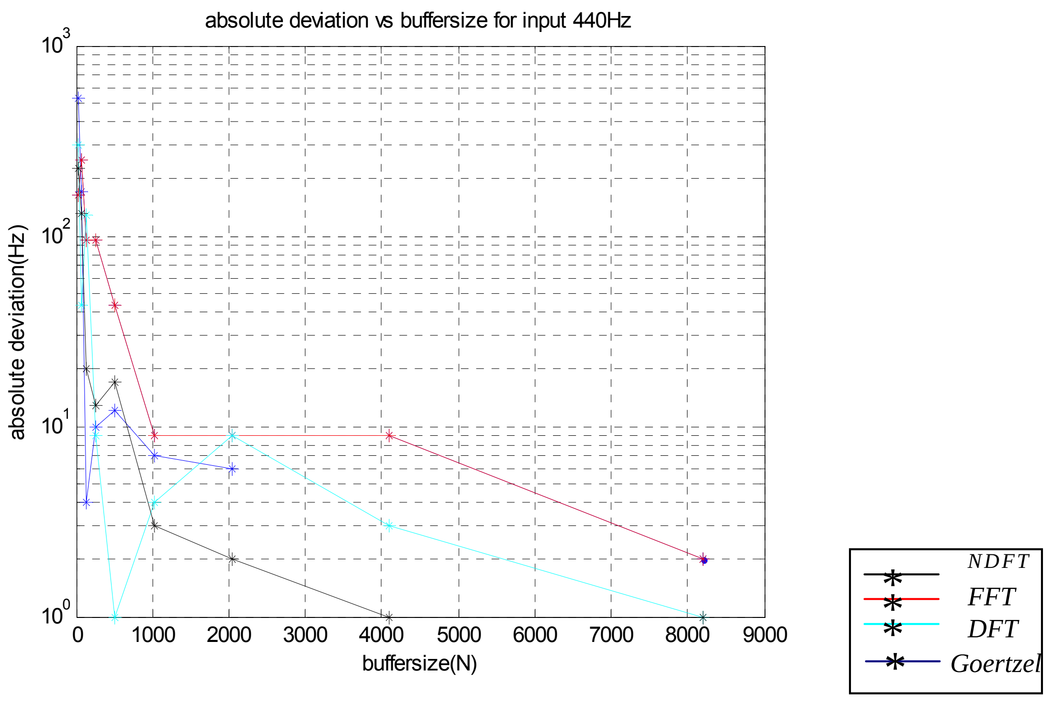


Figure 19: Absolute deviation vs. Buffer size for a 440Hz input

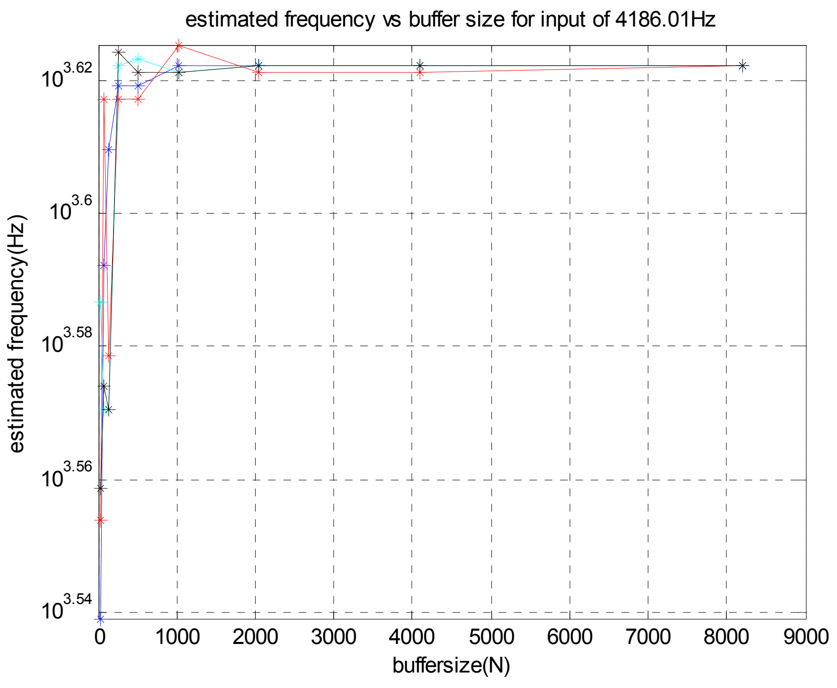


Figure 20: Estimated frequency vs. Buffer size for a 4186.01Hz input

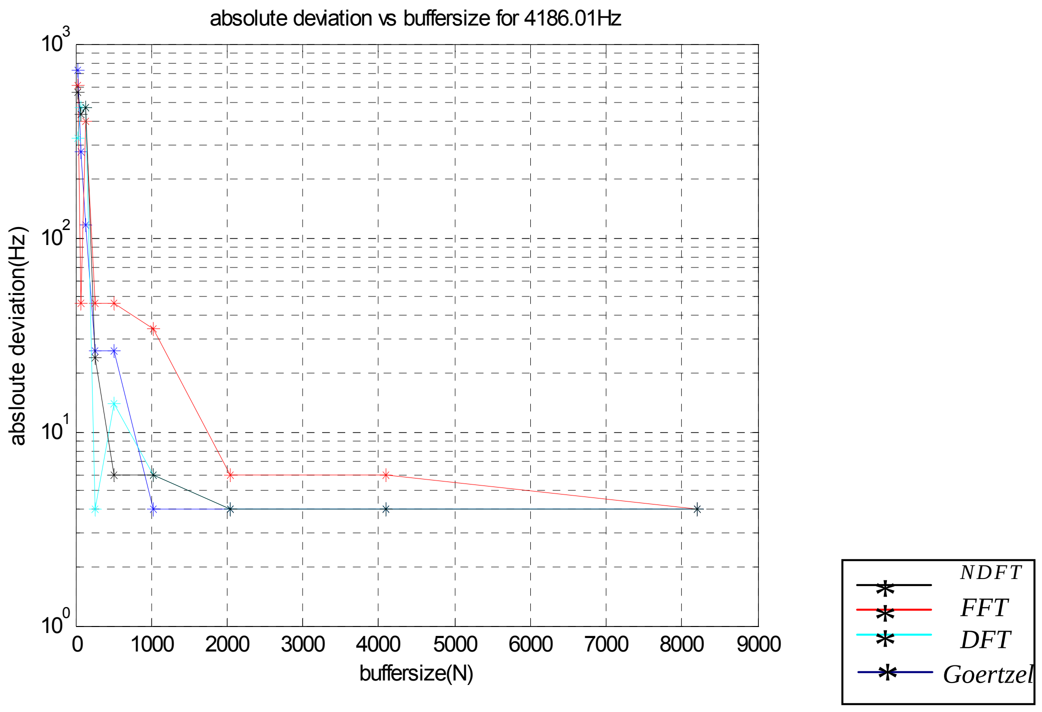


Figure 21: Absolute deviation vs. Buffer size for a 4186.01Hz input

6.1.4 Comparison of the Investigated Metrics:

From the results presented, it can be seen that the DFT and the NDFT yield the highest latency due to their equal structure. The NDFT is slightly slower, the speedup with respect to the DFT is 0.933 on average which is due to the additional reading of the vector with the arbitrarily chosen frequencies. The Goertzel algorithm is considerably faster than the DFT and NDFT but definitively slower than the FFT. Its tendency of latency and speedup is due to its algorithmic complexity of 88·N. The fastest frequency recognition algorithm is clearly the FFT with a complexity of $N \log_2(N)$.

The DFT is far too slow and inaccurate and is therefore as expected not to be taken into account for frequency recognition at all. The NDFT has an adequate accuracy at a buffer size of N = 4096, but is too slow for the average latency of the human ear. The most interesting case is the frequency input of 27.5Hz, where an accuracy of 0.82Hz is demanded. It has been clearly shown that the

FFT, the DFT and the Goertzel algorithm fail in terms of accuracy. For these two algorithms, a buffer size of N > 8192 is necessary to meet the constraints. The NDFT, however, is accurate enough for the demanded spectral resolution of 0.82Hz but is far too slow and beyond the maximum allowed latency of 50ms.

6.2 Frequency Recognition Algorithms analyzing Musical Notes

The question is whether the frequency recognition algorithms are able to cope with instrumental inputs. As analyzed, it has been analytically shown that the spectrum for a single note over time can join the maximum power on one of their harmonics rather than on the fundamental frequency, depending on the instrument's character.

6.2.1 Note C6 (1046.5Hz) for Piano, Violin, Flute and Trumpet:

Figure 22 - 23 depict the estimated frequencies over time for an 1046.5Hz input played by the two string based instruments piano and violin respectively. Since the estimated frequency is within the boundaries throughout the whole sample's duration, all investigated frequency recognition algorithms meet the restrictions for all applied instrumental inputs.

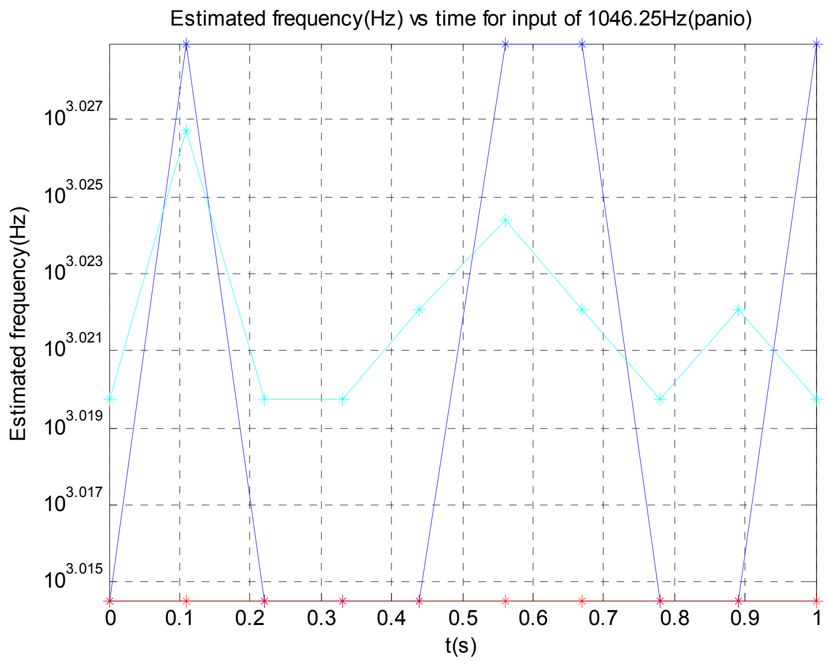


Figure 22: Estimated frequency vs. time for a 1046.5Hz input (piano)

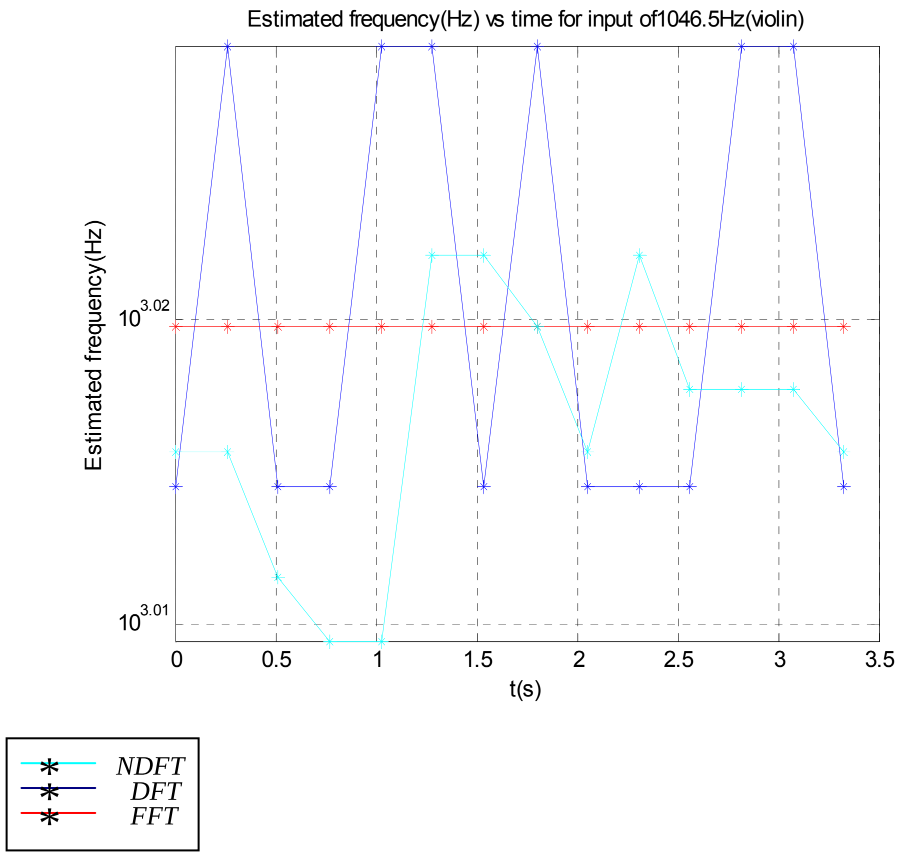


Figure 23: Estimated frequency vs. time for a 1046.5Hz input (violin)

The Goertzel algorithm is the most accurate algorithm and estimates 1046.5Hz constantly during the whole sample's duration. The FFT is the frequency recognition algorithm with the nearest close accuracy with an estimated frequency of 1033.99Hz for every moment. The DFT and the NDFT meet the constraints in terms of accuracy, too.

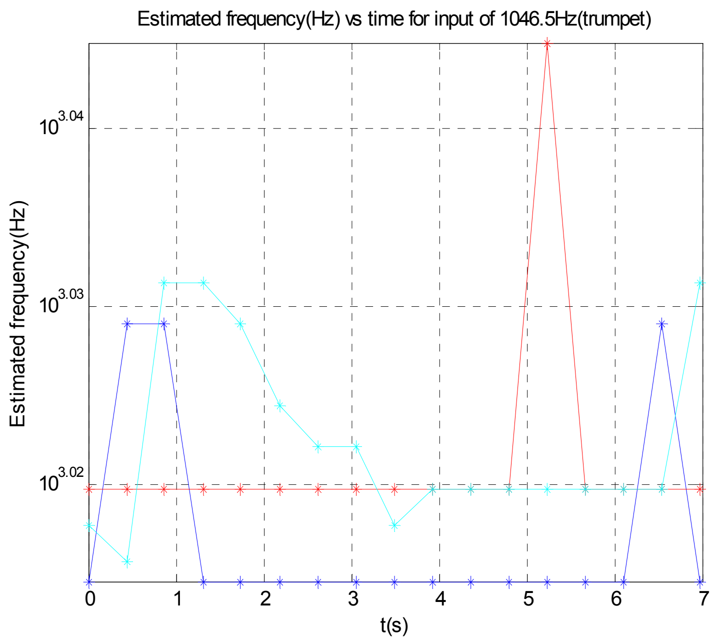


Figure 24: Estimated frequency vs. time for a 1046.5Hz input (trumpet)

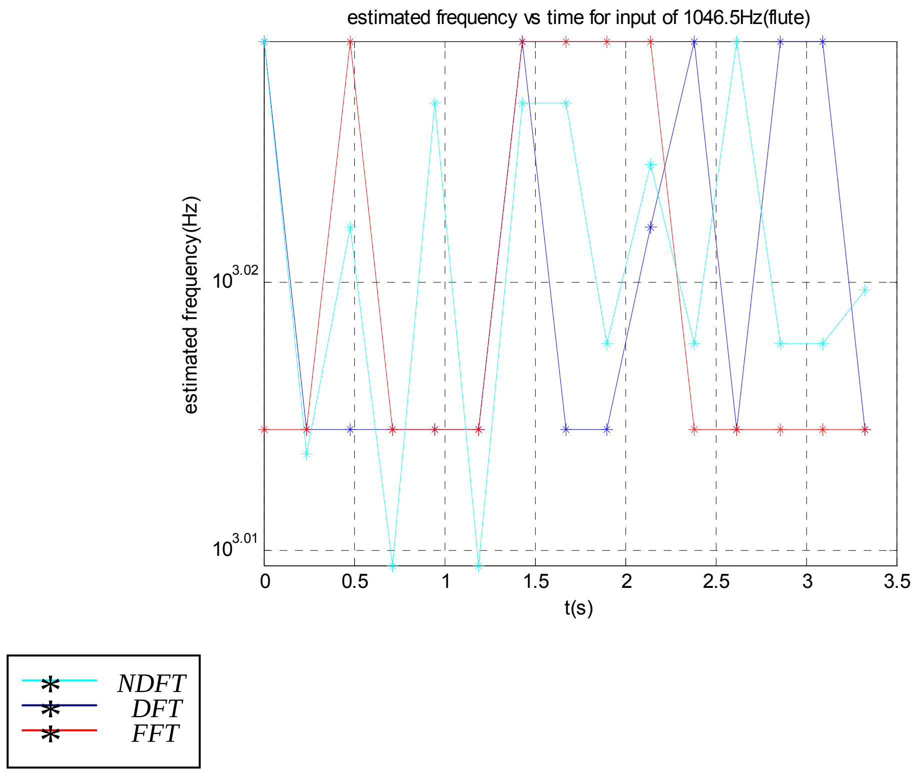


Figure 25: Estimated frequency vs. time for a 1046.5Hz input (flute)

Both the DFT and the NDFT show a similar behaviour like for the string. Both algorithms are accurate enough for meeting the restrictions but they are fluctuating throughout the whole sample's duration, whereas the DFT appears more stable than the NDFT. Again, the FFT supplies accurate frequency estimation within the limits, but is subject to variation for a violin. The only algorithm, which is unreliable in terms of frequency recognition for woodwind and brass instruments, is the Goertzel algorithm.

6.2.2 Note C5 (523.25Hz) for Piano, Violin, Flute and Trumpet:

For a string based instrumental input of 523.25Hz all algorithms succeed in estimating the played note, as shown is figures 26 and 27. Again, the Goertzel algorithm is the most accurate and hits exactly the expected frequency of 523.25Hz. The FFT is similar to the previous section the next close frequency recognition algorithm and estimates constantly 517.0Hz over the whole samples' duration and hence meets the given constraints. As expected, the DFT and the NDFT fluctuate but are is still in between the upper and lower bounds of 508.57Hz and 537.93Hz respectively.

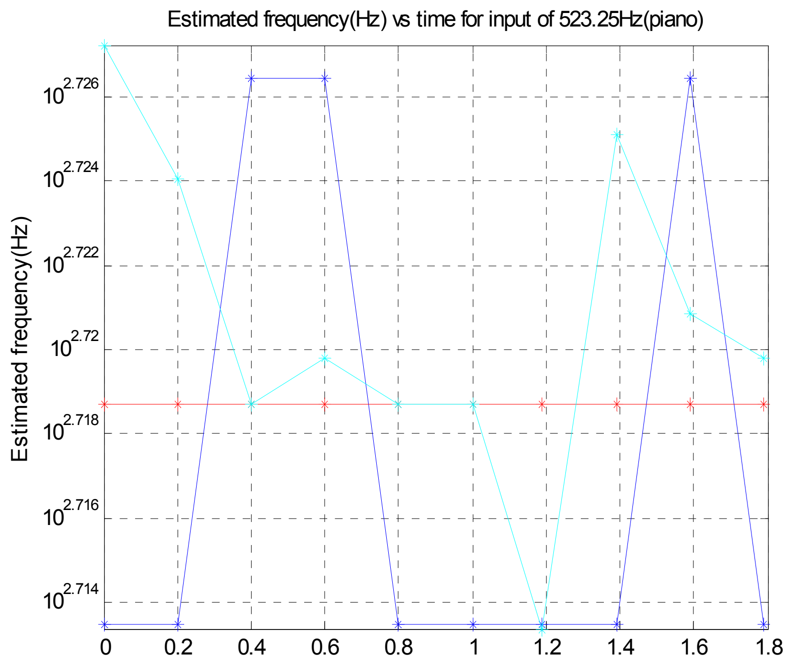
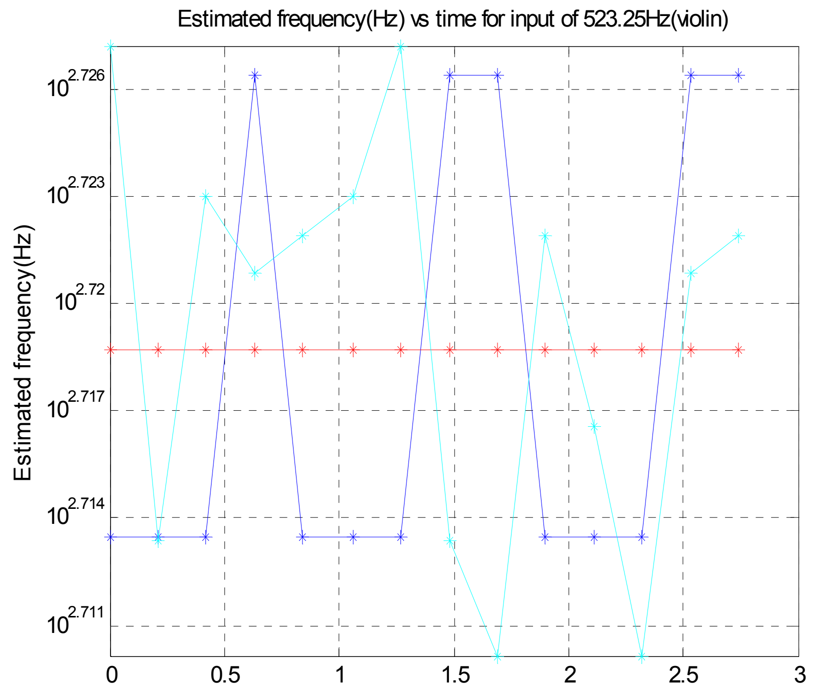


Figure 26: Estimated frequency vs. time for a 523.25Hz input (piano)



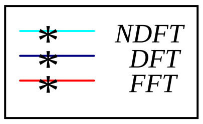


Figure 27: Estimated frequency vs. time for a 523.25Hz input (violin)

Regarding the measurement results for the flute and the trumpet, a more specific evaluation is required. Figure 28 shows the estimation capabilities of the four algorithms for a flute playing MIDI note C5. The DFT, the FFT and the NDFT are vacillating but are within the desired absolute tolerance of 14.68Hz, meaning that the estimated frequency does not cross the upper and lower limits of 508.57Hz and 537.93Hz respectively at any time. In specific, the FFT is the least fluctuating algorithm. In contrast to these three algorithms, the Goertzel algorithm fails in terms of accuracy.

For all algorithms, the estimated frequency wobbles between these two harmonics. Consequently, for a trumpet playing MIDI note C5 it is impossible to estimate the expected frequency of 523.25Hz regardless what algorithm is being applied.

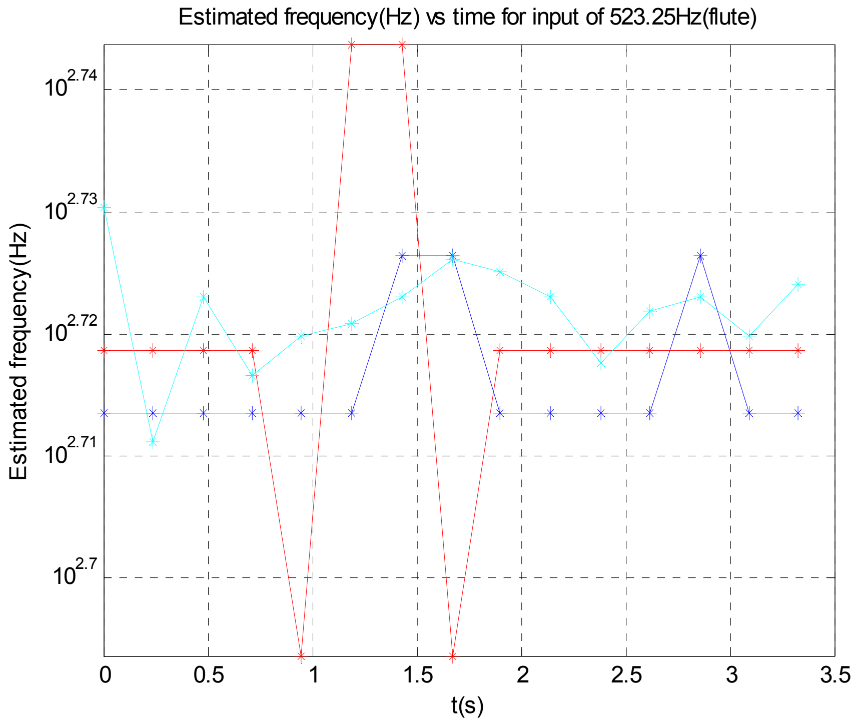


Figure 28: Estimated frequency vs. time for a 523.25Hz input (flute)

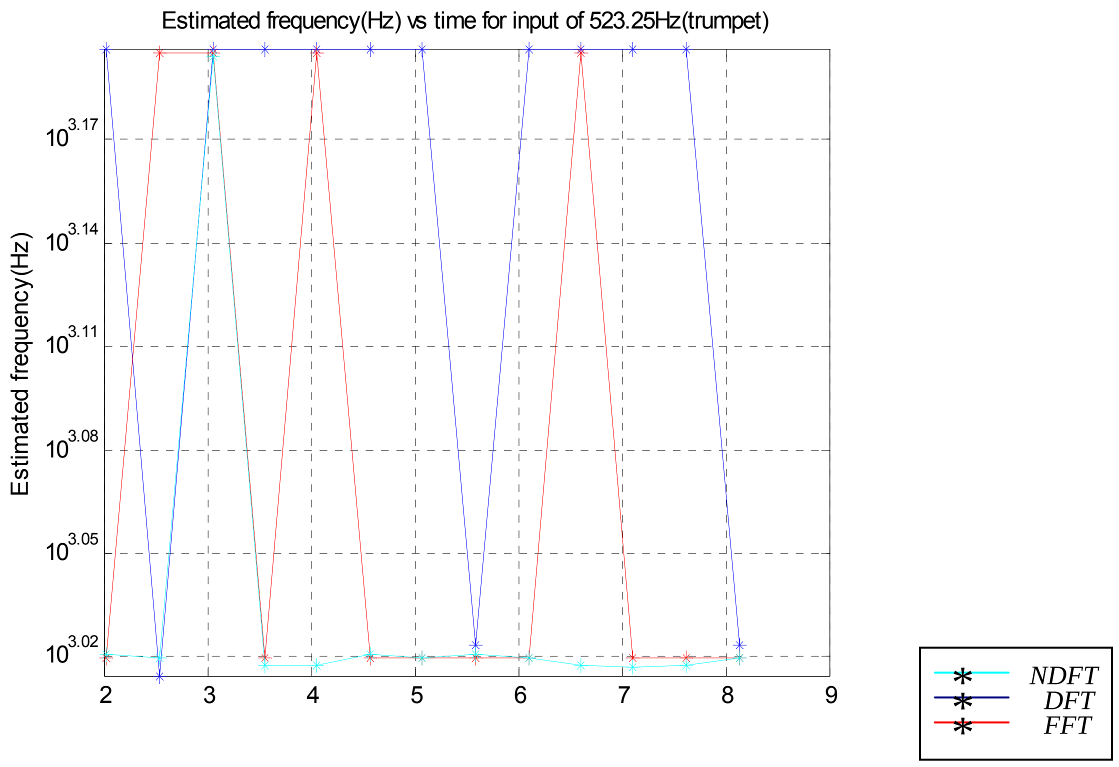


Figure 29: Estimated frequency vs. time for a 523.25Hz input (trumpet)

6.2.3 Note C4 (261.63Hz) for Piano, Violin, Flute and Trumpet:

For a string based input of a piano and a violin playing note C4 (261.63Hz) the DFT, the FFT and the NDFT meet the limits given by the absolute tolerance of 7.34Hz. The DFT varies within the compulsory boundaries of 254.29Hz and 268.97Hz respectively, and so does the NDFT but more modest than the DFT. The FFT is not fluctuating at all and stays constant at an estimated frequency of 258.5Hz. However, the Goertzel algorithm does not fulfill the required constraints at each moment and is therefore not reliable for recognizing the input's frequency for these two instruments.

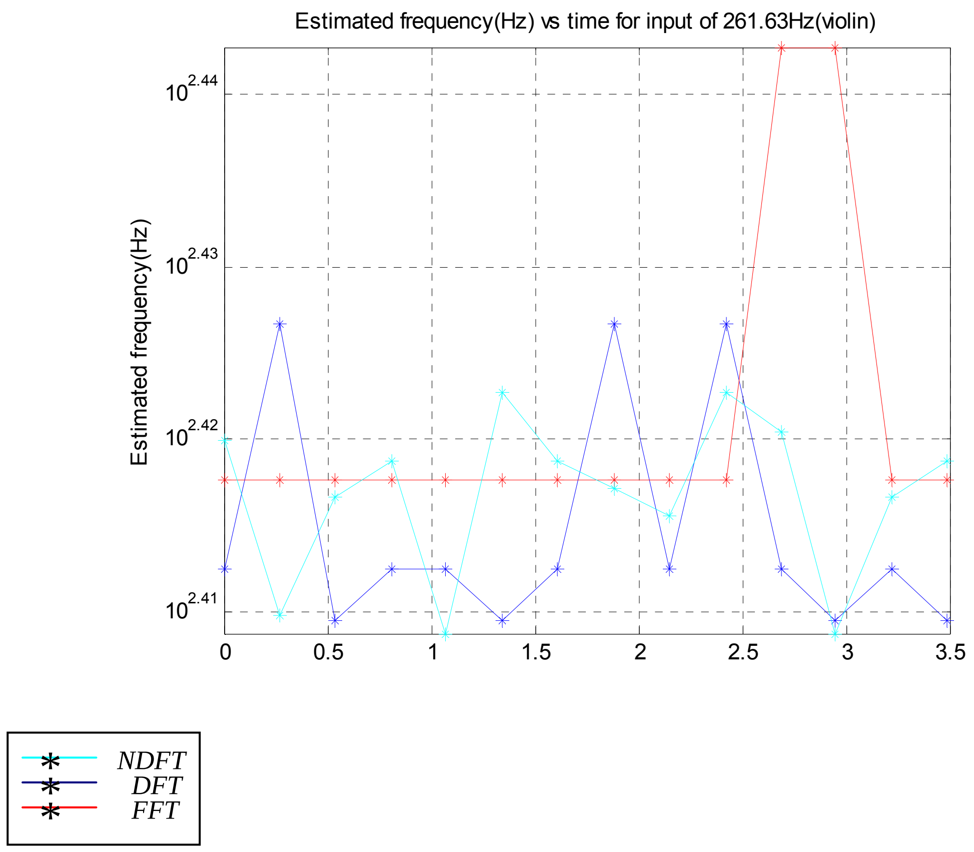


Figure 30: Estimated frequency vs. time for a 261.63Hz input (violin)

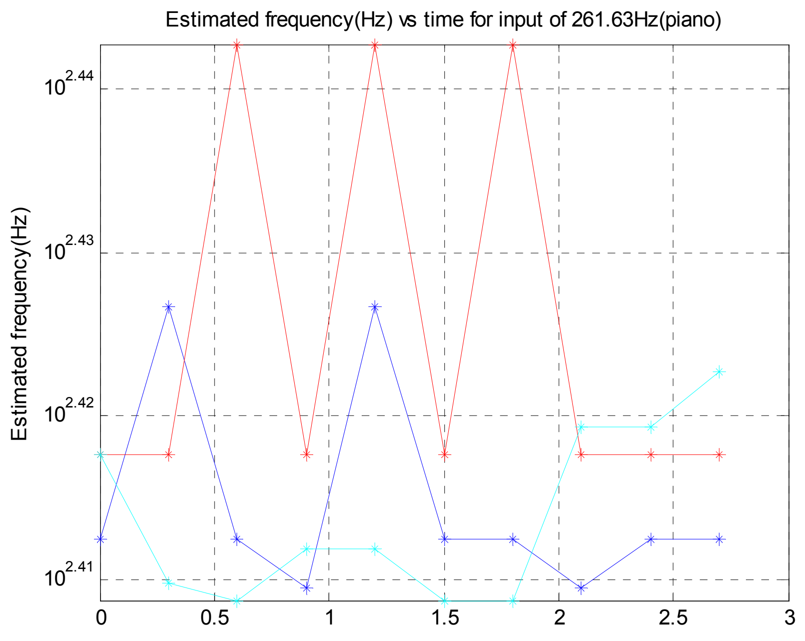


Figure 31: Estimated frequency vs. time for a 261.63Hz input (piano)

All the algorithms estimate these both overtones at 1046.5Hz and 1569.98Hz respectively because of the reasons stated above. Hence and to anticipate at this stage, a simple application of the frequency recognition algorithms is not sufficient when playing note C6 with a trumpet.

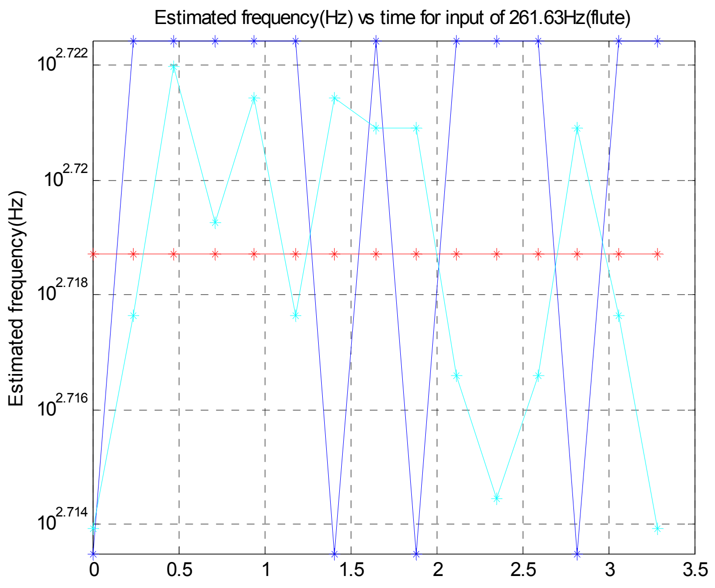
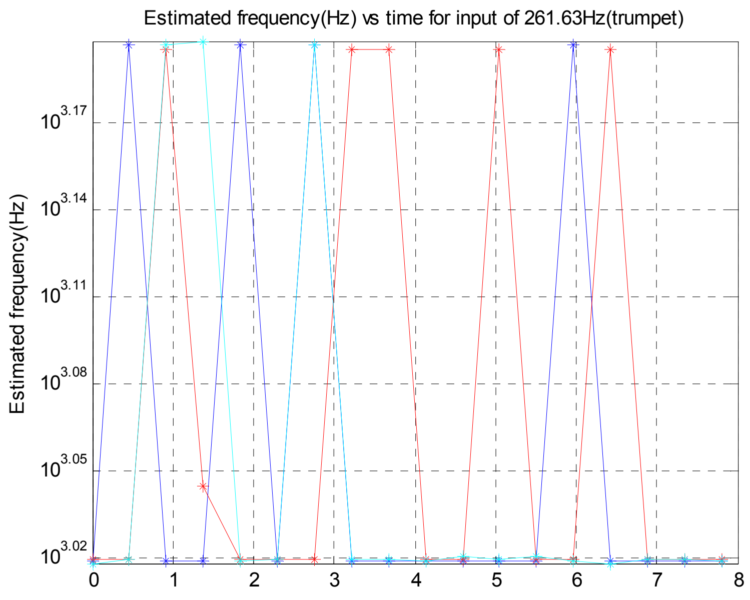


Figure 32: Estimated frequency vs. time for a 261.63Hz input (flute)



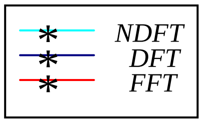


Figure 33: Estimated frequency vs. time for a 261.63Hz input (trumpet)

6.2.4 Comparison of Frequency Recognition Capability for Musical Notes:

The DFT, the FFT and the NDFT are able to estimate the expected frequency within the given constraints for note C6 for each instrument; for note C5 for all instruments except the trumpet and for C4 for string based instruments only, i.e. the piano and the violin.

Also, as expected, all algorithms fail in terms of estimating the fundamental frequency for the trumpet playing note C5 and even for both flute and trumpet producing note C4 due to the fact that the maximum power for the notes played by these instruments is joined at their harmonics as shown. In particular, note C5 (523.25Hz) played by a trumpet, holds its maximum power on its second harmonic 1046.5Hz as analytical shown and experimental verified. In specific, for note C4 (261.63Hz) played by a flute, the maximum power can be found at the second harmonic at 523.25Hz, the reason why the peak detection mechanism of all algorithms estimates this frequency. For a trumpet playing note C4, it has been theoretically and experimentally shown that it is even more difficult to find

the correct expected frequency because the maximum power is present at both the fourth and sixth harmonic of 1046.5Hz and 1569.98Hz respectively.
The exception is the Goertzel algorithm. It identifies the correct frequency of the piano and violin for the notes C6 and C5 on the one hand, but does not confirm the expected results of estimating flute and trumpet playing note C6, the flute producing C5 and the piano and the violin playing note C4 on the other hand.
Regarding the algorithms' frequency recognition capability only, it has been uniquely shown in section 6.1, that frequency recognition algorithms estimate frequencies of signals containing a simple sinusoid in most of the measured cases without complications, whereas the frequency recognition capability of each algorithm varies due to each individual nature. However, when analyzing instruments playing musical notes, it has to be taken into account that the lower the frequency is, the higher is the number of harmonics. As long as the maximum power is at the fundamental frequency, the peak detection algorithm estimates the correct frequency as it is the case with the string based instruments piano and violin. If the maximum spectral power is on one of the harmonics as been shown for note C5 played by the trumpet and for note C4 produced by trumpet and flute, a valid frequency estimation as implemented with the four algorithms, is not possible. Consequently, it can be clearly stated that an exclusive application of frequency recognition algorithms is insufficient for definite frequency recognition of musical notes.

7. Conclusions and Future Work

This dissertation details the theory behind a novel Generic Musical Instrument System (GMIS) and provides an analysis of the possibilities to adapt the advantageous techniques of Dual-Tone Multi-Frequency (DTMF) systems to musical systems such as the GMIS. Four different Fourier transform-based frequency recognition algorithms have been analyzed: the Discrete Fourier Transform (DFT) as a baseline for the evaluation of all algorithms, the Fast Fourier Transform (FFT) which is used in most of the engineering applications, the Goertzel algorithm and the Non-Uniform Discrete Fourier Transform (NDFT) which are both successfully applied to DTMF systems.

For input signals containing simple sinusoids, the DFT and the NDFT are far too slow due to their complexity of N^2. In contrast to them, the FFT is, as expected, clearly the fastest Fourier transform-based algorithm, followed by the Goertzel algorithm. Both algorithms are fast enough to undercut the average latency of a human ear of 50ms.

In specific, the DFT, the FFT and the NDFT estimate the correct frequencies for all instruments playing note C6 and C5 except for a trumpet playing note C5. For note C4, only the sting-based instruments succeed. This behaviour is due to the fact that the maximum spectral power is held by the fundamental frequency. It has been worked out, that the Goertzel algorithm estimates the correct frequency of note C6 played by the piano and the violin, but it fails for all other analyzed musical notes and does not estimate the frequencies as predicted. That is why, this algorithm cannot be considered as a candidate for frequency recognition of complex waveforms as it is required for musical systems such as the GMIS.

It has been shown that the frequency recognition capability of the investigated Fourier transform-based algorithms is not satisfactorily enough according to the constraints of the GMIS. Thus, frequency recognition algorithms using spectral power estimation could be subject to continuative research. Two representatives of this algorithm category are the Normalized Direct Frequency Estimation

Technique (NDFET) based on the Least Mean square (LMS) algorithm and the Multiple Signal Classification (MUSIC) algorithm is a power spectral estimation method.

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