FINITE ELEMENT ANALYSIS OF MULTIFARIOUS MACHINE COMPONENTS

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Bachelor of Technology

In

Mechanical Engineering

By

Mitrbhanu Mahapatra

&

Satya Prakash Biswal

Department of Mechanical Engineering

National Institute of Technology

Rourkela

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Under the Guidance of

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National Institute of Technology

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CERTIFICATE

This is to certify that the thesis entitled, "Finite element analysis of multifarious machine components" submitted by Sri Mitrabhanu Mahapatra and Sri Satya Prakash Biswal in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date

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We would like to articulate our deep gratitude to our project guide Prof.N.Kavi who has always been our motivation and source of inspiration and constant guidance for carrying out the project.

It is my pleasure to refer Microsoft word 2003 of which the compilation of this report would have been impossible.

An assemblage of this nature could never have been attempted without reference to and inspiration from the works of others whose details are mentioned in reference section. We acknowledge our indebtedness to all of them.

Last but not the least to all of our friends who were patiently extended all sorts of help for accomplishing this undertaking.

Date

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Contents

Abstract

Finite element analysis is a sophisticated technology based on the principle of discretization and numerical approximation to solve scientific and engineering problems. In this methodology any structure under consideration is discretized into small geometric shapes and the material properties are analyzed over these small elements. This method scores over the general strength of material methods in the way that in this technique complex beam elements with differential cross sectional geometry can be analyzed quite easily.

In this work we apply the method of finite element analysis to one dimensional beam elements. The elements that have been analyzed range from simple beams with concentric loading to beams having non uniformly varying loads. Towards the end we have taken machine components on elastic supports and beams subjected to combined bending and torsion and axial loading. Finally we have applied a new FEA method to analyze the effect of crack in a beam. Throughout this work we have restricted ourselves to calculation of deflection and slope at each nodes of the beam. We have compared our results with the results obtained by strength of material method and enlisted them in a tabular form.

For this work we have used the standard C program for solution of simultaneous equation by Gauss elimination to solve the reduced matrix equations. To calculate the global matrix we developed our own program in C.

CHAPTER 1

INTRODUCTION

Historical Background

Advantages and disadvantages

Application

Finite element analysis is based on the principle of discretization and numerical approximation to solve scientific and engineering problems. In this method, a complex region defining a continuum is discretized into simple geometric shapes called the finite elements. The material properties and the governing relationships are considered over these elements and are expressed in terms of unknown elements at the corners. An assembly process duly considering the loading and constraints results in a set of equations. Solution of these equations gives the approximate behaviour of the continuum. The application of this method ranges from deformation and stress analysis of automotives, air crafts, buildings, bridge structures to field analysis of other flow problems. With the advent of new computer technologies and CAD systems complex problems can be modeled with relative ease. Several alternative configurations can be tested on a computer before the first prototype is built. All these above suggests that we need to keep pace with these developments by understanding the basic theory, modeling techniques and computational aspects of finite element analysis.

HISTORICAL BACKGROUND:-

The term finite element was first coined and used by Clough in 1960. Basic idea of Finite Element Method originated from advances in the air craft structural analysis. In early 1960s engineers used this method for approximate solutions of problems in stress analysis, fluid flow, heat transfer and other areas. A book by Argyris in 1955 on energy theorems and matrix methods laid a foundation for further development in finite element analysis was published by Zienkiwiz and Chung in 1967. In late 1960s and 1970s finite element analysis was applied to non-linear problems and layer deformations.

Today the advent of mainframe computational techniques and powerful microcomputers has made this method within the practical applicability of industries and engineers.

ADVANTAGES:-

The main advantages of this method are that physical problems which are so far intractable and complex for any close bound solutions can be easily solved by this method.

The other advantages are:-

1. This method can be applied efficiently to irregular geometries.

2. It can take care of any types of boundary.
3. Material anisotropy and inhomogenity can be treated without much difficulty.
4. any types of loading can be handled.

DISADVANTAGES:-

1. There are many types of problems in which other methods of analysis may prove more efficient.
2. Cost involved in this method of analysis is high.
3. For vibration and stability problems cost involved in finite element analysis may be prohibitive.
4. Stress values may vary by 25% from fine mesh analysis to average mesh analysis.
5. There are other trouble spots such aspect ratio which may affect the final results.

APPLICATION:-

1.Application to Boeing 747 air crafts:-

The finite element analysis of 747 wing body region, requires a total of over 7000 unknowns. It is common practice to divide the structure with a number of sub-structures and each of these is analyzed by finite element method.

2.Application to nuclear reactor vessel:-

A nuclear reactors vessel analyzed by finite element method provides the number of unknowns range upwards of 20,000 and it is common for the analysis to extend to the treatment of inelastic phenomena.

3.Application to Bio-Mechanics Problem:-

The femur bone is idealized as consisting of a number of finite elements and analysis is performed for various loadings of prosthesis process.

4.Application for reinforced concrete beams:-

Finite element analysis applied to RC beams by Ngo and Scordelis portays the finite element representation and the analytically described crack trajectories.

5.**Other applications**

It can also be applied to various engine components like connecting rod, crank shaft etc .

CHAPTER 2

A BRIEF THEORY OF FINITE ELEMNET ANALYSIS METHODOLOGY

Steps to solve any problem in FEA

Derivation of element stiffness matrix

Steps to solve any problem in FEA

The following steps may be taken to solve any problem by the finite element analysis.

1. At First the problem has to be defined clearly and every loading conditions, boundary conditions and end conditions should be clearly noted down.
2. Then the structure whose analysis has to be carried out is to be divided into a number of elements and nodes. If there are “N” elements then the number of nodes be “N+1”.
3. Then the free body diagram of each of the elements should be drawn and every force and loading conditions should be shown on it.
4. Then the stiffness matrix for each of the element is calculated.
5. The next step is to calculate the global stiffness matrix by combining all the stiffness matrices of the elements. If there are “n” nodes then the order of the global stiffness matrix is 2n X 2n.
6. To the global stiffness matrix, boundary conditions are applied and the reduced matrix is obtained.
7. Then the equation are solved to find out the deflection and slopes at the nodes.
8. By back calculation procedure, the loads and moments on each element can be calculated.

DERIVATION OF ELEMENT STIFFNESS MATRIX:-

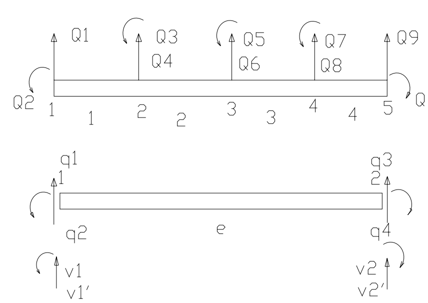

figure no:-2.1

e	1	2
1	1	2
2	2	3
3	3	4
4	4	5

Table 2.1

The beam is divided into elements as shown in the figure. Each node has two degree of freedom. Typically, the degrees of freedom of node i are Q_{2i-1} and Q_{2i} . Q_{2i-1} is the transverse displacement and Q2i is the slope or rotation. The vector

$$Q = \{Q_1 , Q_2 \dots Q_{10}\}^T \qquad \dots\dots\dots\dots\dots\dots\dots\dots 2.1$$

Represents the global displacement vector. For a single element the local degrees of freedom are

$$q = [q_1, q_2, q_3, q_4]^T \qquad \dots\dots\dots\dots\dots\dots\dots\dots 2.2$$

The local global correspondence is easy to see from the table shown below. q is same as $[v_1, v_1^1, v_2, v_2^1]^T$. the shape functions for interpolating v on an element are defined in terms of ξ on -1 to +1. Since nodal values and nodal shapes are involved we define Hermite shapes functions which satisfy nodal value and slope continuity requirements.

Each of the shape function is of cubic order and is represented by;

$H_i = a_i + b_i\, \xi + c_i\, \xi^2 + d_i\, \xi^3 \qquad i = 1,2,3,4.$

Conditions given in the following table has to be satisfied:-

H_1	H_1'	H_2	H_2'	H_3	H_3	H_4	H_4'
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0

Table 2.2

The coefficients a_i, b_i, c_i and d_i can be calculated importing the above conditions.

$H_1 = \frac{1}{4}(1-\xi)^2 (2+\xi)^2 (2 + \xi) = \frac{1}{4}(2-3\xi + \xi^3)$

$H_2 = \frac{1}{4}(1-\xi)^2 (\xi +1) = \frac{1}{4}(1-\xi - \xi^2 + \xi^3)$

$H_3 = \frac{1}{4}(1+\xi)^2 (2-\xi) = \frac{1}{4}(2+3\xi - \xi^3)$

$H_4 = = \frac{1}{4}(1+\xi)^2 (\xi - 1) = \frac{1}{4}(-1- \xi + \xi^2 + \xi^3)$ …………… ………… 2.3

Hermite shape functions can be used to write v in the form

$v(\xi) = H_1v_1 + H_2 (dv/d\xi)_1 + H_3v_2 + H_4 (dv/d\xi)_2$ ……………..……..... 2.4

The coordinates transform by the relationship

$x = 1\text{-}\xi/2\ x_1 + 1\text{+}\xi/2\ x_2$

$\Rightarrow x = x_1 + x_2/2 + x_2 - x_1/2\xi$ ……………………………………………… 2.5

Since $le = x_2 - x_1$ is the length of the element, we have $dx = le/2\ d\xi$ ..……. 2.6

$dv/d\xi = le/2\ dv/dx$ ………………………………………………. 2.7

noting that dv/dx evaluated at nodes 1 and 2 is q_2 and q_4 , we have

$v(\xi) = H_1q_1 + le/2\ H_2q_3 + le/2\ H_4q_4$ …………………………………….. 2.8

This may be generalised as $v = Hq$ ……………………………………… .2.9

Where $H = [H_1\ le/2H_2, H_3, le/2\ H_4]$ ……………………………………..2.10

In the total potential energy of the system, we consider the integrals as summation over the integrals over the elements. The element strain energy is given by

$$V_e = ½\ EI \int_e (d^2v/dx^2)^2\ dx \quad \ldots\ldots\ldots\ldots \quad 2.11$$

From equation (2.7) $dv/dx = 2/le\ dv/d\xi$ and $d^2v/dx^2\ 4/le^2\ d^2v/d\xi^2$ ……….. 2.12

Then substituting $v = Hq$ we obtain

$(d^2v/dx^2)^2 = q^T\ 16/le^4\ (d^2H/d\xi^2)^T\ (d^2H/d\xi^2)q$ ……………………………... 2.13

$(d^2H/d\xi^{2)} = [3/2\xi, -1+3/2\xi\ le/2, -3/2\xi, 1+3/2\xi\ le/2, le/2]$ …………………. 2.14

On substituting $dx = (le/2)d\xi$ we have

$$V_e = ½\ q^T\ 8EI/le^3 \int_{-1}^{+1} \begin{bmatrix} \frac{9}{4}\xi^2 & \frac{3}{8}\xi[-1+3\xi]le & \frac{-9}{4}\xi^2 & \frac{3}{8}\xi(1+3\xi)le \\ & \left(\frac{-1+3\xi}{4}\right)^2 le^2 & \frac{-3}{8}\xi(-1+3\xi)le & \frac{-1+9\xi^2}{16}le^2 \\ & Symmetric & \frac{9}{4}\xi^2 & \frac{-3}{8}\xi(1+3\xi)le \\ & & & \left(\frac{1+3\xi}{4}\right)^2 le^2 \end{bmatrix}$$

We have $\int_{-1}^{+1} \xi^2 d\xi = \frac{2}{3}$

$$\int_{-1}^{+} \xi^2 d\xi = 0$$

$$\int_{-1}^{+} d\xi = 2$$

Equating 2.15 can be written as $V_e = ½\, q^T k^e q$ ……………………2.16

$$k^e = \frac{EI}{le^3}\begin{bmatrix} 12 & 6le & -12 & 6le \\ 6le & 4le^2 & -6le & 2le^2 \\ -12 & -6le & 12 & -6le \\ 6le & 2le^2 & -6le & 4le^2 \end{bmatrix} \qquad \text{…………………………2.17}$$

k^e = is symmetric

k_e is the element stiffness matrix.

Boundary Consideration:-

Using the bending moment and shear force equations.

$M = EI\, d^2v/ dx^2$ $\qquad$ $v = dM/dx$ $\qquad$ and $v = Hq$.

Denoting the element and equilibrium loading as R_1, R_2, …. R_9

We have,

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{EI}{le^3}\begin{bmatrix} 12 & 6le & -12 & 6le \\ 6le & 4le^2 & -6le & 2le^2 \\ -12 & -6le & 12 & -6le \\ 6le & 2le^2 & -6le & 4le^2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_1' \\ v_2 \\ v_2' \end{bmatrix} \qquad \text{……………………………….2.18}$$

Then the boundary conditions may be applied to the equation 2.18 to determine the deflections slopes and forces at each of the nodes.

CHAPTER 3

APPLICATION OF FEA TECHNIQUES TO ONE DIMENSIONAL BEAM PROBLEMS

SOLUTION OF ONE DIMENSIONAL BEAM PROBLEMS WITH DIFFERENT TYPES OF LOADING

Case – 1:

Simply supported beam with a single concentrated load.

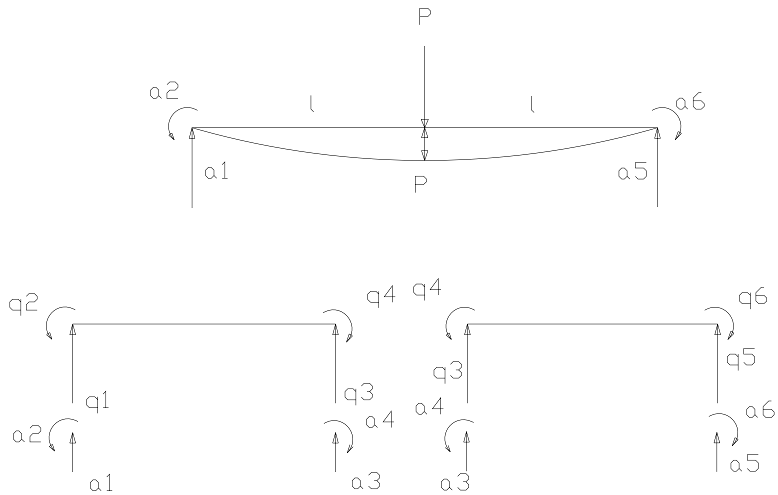

figure no:-3.1

We consider a beam of unit length 2l subjected to a concentrated load p acting at the middle. We tend to find out the slopes and deflections at each of the nodes and those of at the centre where the load is acting. For this purpose as defined in the Chapter 2 the following steps are followed.

Step 1:

The first is defining the problem. The diagram of the beam is done with all loading conditions shown clearly and distinctly at each nodes i.e. at the end points and at the centre.

Step 2 :

Next the element is divided into two distinct elements each of length l

Step 3:

Then the individual free body diagrams are drawn and the loading conditions are shown along with the respective degrees of freedom.

Step 4 :

Next the element stiffness matrix is determined as per the procedure stated in chapter 2.

For element (1) the element stiffness matrix is

[F] = [k] [a]……………………………………………………………3.1

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad \ldots\ldots\ldots\ldots 3.2$$

For second element the element stiffness matrix is

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \quad \ldots\ldots\ldots\ldots 3.3$$

Step 5 :

Next the global stiffness matrix is calculated by combining the two element stiffness matrices.

Global Matrix:-

$$[k]=\frac{EI}{l^2}\begin{bmatrix} 12 & 6l & -12 & 6l & 0 & 0 \\ 6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\ -12 & -6l & 12 & -6l & 0 & 0 \\ 6l & 2l^2 & -6l & 4l^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \frac{EI}{l^3}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 6l & -12 & 6l \\ 0 & 0 & 6l & 4l^2 & -6l & 2l^2 \\ 0 & 0 & -12 & -6l & 12 & -6l \\ 0 & 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$\Rightarrow [k]=\frac{EI}{l^2}\begin{bmatrix} 12 & 6l & -12 & 6l & 0 & 0 \\ 6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\ -12 & -6l & 24 & 0 & -12 & 6l \\ 6l & 2l^2 & 0 & 8l^2 & -6l & 2l^2 \\ 0 & 0 & -12 & -6l & 12 & -6l \\ 0 & 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots 3.4$$

Step 6:

We apply the boundary conditions to the global matrix to obtain the reduced matrix. In this case the boundary conditions are

1. Deflections at each of the end points are zero i.e. $a_1 = 0$, $a_5 = 0$
2. All the loads shown in the matrix are zero except the central load $q_3 = p = 4500$ N

Then applying the above boundary conditions we get the reduced matrix as follows.

$\therefore \quad [F] = [k][a]$……………………………………………………3.5

$$= \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l & 0 & 0 \\ 6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\ -12 & -6l & 24 & 0 & -12 & 6l \\ 6l & 2l^2 & 0 & 8l^2 & -6l & 2l^2 \\ 0 & 0 & -12 & -6l & 12 & -6l \\ 0 & 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \text{……………………..3.6}$$

Now put the boundary conditions

$a_1 = 0$, $a_5 = 0$

So the 1st and 5th rows and columns are made zero.

$$\begin{bmatrix} q_2 \\ q_3 \\ q_4 \\ q_6 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 4l^2 & -6l & 2l^2 & 0 \\ -6l & 24 & 0 & 6l \\ 2l & 0 & 8l^2 & 2l^2 \\ 0 & 6l & 2l^2 & 4l^2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ a_6 \end{bmatrix} \text{………………………...3.7}$$

$\therefore q_2 = (4l^2a_2 - 6la_3 + 2l^2a_4 + 0)EI/l^3$…………………………………...3.8

$q_3 = (-6la_2 + 24a_3 + 0 + 6la_6)\ EI/l^3$……………………………………3.9

$q_4 = (2l^2\ a_2 + 0 + 8l^2.a_4 + 2l^2.a_6)\ EI/l^3$………………………………….3.10

$q_6 = (0 + 6l.a_3 + 2l^2.a_4 + 4l^2.a_6)\ EI/l^3$…………………………………….3.11

Now using second boundary condition

$q_2 = 0$, $q_4 = 0$, $q_6 = 0$, $q_3 = p$ and $l = 1$m , $p = 4500$ml

$4a_2 - 6a_3 + 2a_4 = 0$…………………………………………………….3.12

$-6a_2 + 24a_3 + 6a_6 = 4500/EI$……………………………………………..3.13

$2a_2 + 8a_4 + 2a_6 = 0$…………………………………………………….3.14

$6a_3 + 2a_4 + 4a_6 = 0$……………………………………………………3.15

We solve the above four simultaneous equations by Gaussian elimination technique and the results obtained are

$a_2 = -1125/EI$ m.

$a_3 = -750/EI$ m.

$a_4 = 0$

$a_6 = 1125/EI$ m

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is

Conclusion:

The error might be due to the small number of elements taken in the problems. If the number of elements were increased instead of two then the result would have been better.

Case -2

Eccentric loading of beams.

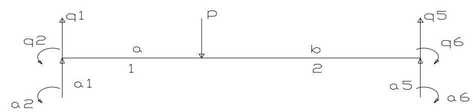

figure no:-3.2

In this problem we considered the eccentric loading of the beam. The beam was loaded in such a manner that instead of being at the centre the load was offset.

For solving this problem the following steps were followed.

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each nodes i.e at the end points and n the beam.

Step 2 :

Next the element was divided into two distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4 :

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

For element –1 the element stiffness matrix:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{EI}{a^3} \begin{bmatrix} 12 & 6a & -12 & 6a \\ 6a & 4a^2 & -6a & 2a^2 \\ -12 & -6a & 12 & -6a \\ 6a & 2a^2 & -6a & 4a^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.16$$

For element –2 the element stiffness matrix

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \frac{EI}{b^3}\begin{bmatrix} 12 & 6b & -12 & 6b \\ 6b & 4b^2 & -6b & 2b^2 \\ -12 & -6b & 12 & -6b \\ 6b & 2b^2 & -6b & 4b^2 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \quad \text{.................................3.17}$$

Step 5 :

Next the global stiffness matrix is calculated by combining the two element stiffness matrices.

Global Matrix:

$$[k] = \begin{bmatrix} 12 & 6a & -12 & 6a & 0 & 0 \\ 6a & 4a^2 & -6b & 2a^2 & 0 & 0 \\ -12 & -6a & 24 & 6(b-a) & -12 & 6b \\ 6a & 2a^2 & 6(b-a) & 4(a^2+b^2) & -6b & 2b^2 \\ 0 & 0 & -12 & -6b & 12 & -6b \\ 0 & 0 & 6b & 2b^2 & -6b & 4b^2 \end{bmatrix} \quad \text{................3.18}$$

Putting a =1, b=2;

$$[G] = \begin{bmatrix} 12 & 6 & -12 & 6 & 0 & 0 \\ 6 & 4 & -6 & 2 & 0 & 0 \\ -12 & -6 & 13.5 & -4.5 & -1.5 & -1.5 \\ 6 & 2 & -4.5 & 6 & -1.5 & 1 \\ 0 & 0 & -1.5 & -1.5 & -1.5 & -1.5 \\ 0 & 0 & 1.5 & 1 & 1 & 2 \end{bmatrix} \quad \text{....................3.19}$$

Step 6:

We applied the boundary conditions to the global matrix to obtain the reduced matrix. In this case the boundary conditions are

1. Deflections at each of the end points are zero i.e. $a_1 = 0$, $a_5 = 0$

2. All the loads shown in the matrix are zero except the central load

Then applying the above boundary conditions we get the reduced matrix as follows.

Reduced matrix:

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \mathrm{EI} \begin{bmatrix} 4 & -6 & 2 & 0 \\ -6 & 13.5 & -4.5 & 1.5 \\ -2 & -4.5 & 6 & 1 \\ 0 & 1.5 & 1 & 2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ a_6 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.20$$

By solving the above matrix in Gaussian elimination technique the deflection and slopes were found out to be.

a_2 = 55.5556/EI

a_3 = 44.4444/EI

a_4 = 22.2222/EI

a_6 = -44.4444/EI

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is

Conclusion:

The error might be due to the small number of elements taken in the problems. If the number of elements were increased instead of two then the result would have been better.

Case -3

Uniformly distributed load over cantilever

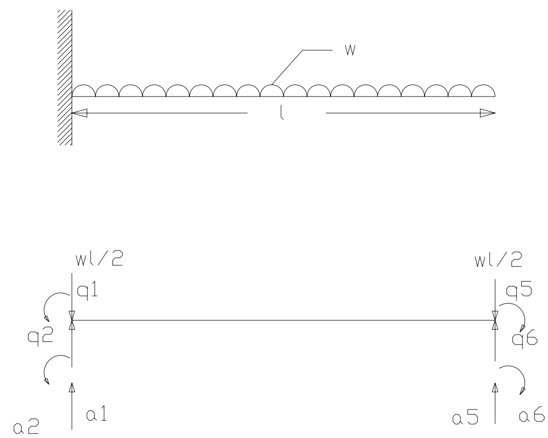

Figure no:-3.3

The present problem that is taken up is that of a cantilever beam with uniformly distributed load over it. The aim was to find the deflection and slope at each of the ends.

The following steps were followed to solve the problem

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each nodes i.e. at the end points and n the beam.

Step 2 :

Next the element was divided into two distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4 :

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

The element stiffness matrix for one element

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad \ldots\ldots\ldots 3.21$$

Step 5 :

Next the global stiffness matrix. Because there was only one element the global matrix and the element stiffness matrix were one and the same.

Step 6:

We applied the boundary conditions to the global matrix to obtain the reduced matrix

$a_1 = 0$, $a_2 = 0$

$$\begin{bmatrix} q_3 \\ q_4 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & -6l \\ 6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix}$$

$$q_3 = \frac{EI}{l^3}\left(-12a_3 - 6la_4\right)$$

$$q_4 = \frac{EI}{l^3}\left(6la_3 + 4l^2 a_4\right)$$

$$12a_3 - 6la_4 = l^3 q_3 / EI$$

$$6la_3 + 4l^3 a_4 = 0 \qquad given \;\; l = 1$$

$$12a_3 - 6a_4 = \frac{q_3}{EI}$$

$$6a_3 + 4a_4 = 0$$

$$\therefore a_3 = \frac{w}{6EI}$$

$$a_4 = -\frac{w}{4EI}$$

$$q_1 - \frac{wl}{2} = EI\left(12a_1 + 6a_2 - 12a_3 + 6a_4\right)$$

$$\therefore q_1 = -3w \quad \ldots\ldots\ldots 3.22$$

$$q_2 = -w$$

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is 33.33

Conclusion:

The large error might be due to the small number of elements taken in the problems. If the number of elements were increased instead of two then the result would have been better.

Case 4
Rechecking the result of cantilever beam.

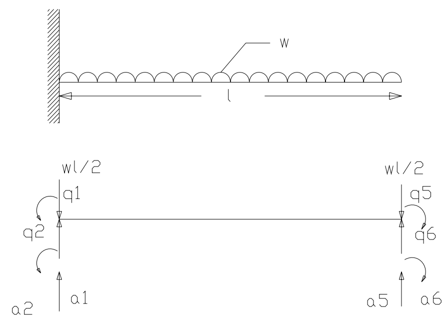

figure no:-3.5

In this case 10 elements (each element = 1m) were taken instead of just one as in the previous case. Similar force distribution were done for the rest of the elements.
Elements matrix for each element:-

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad \text{………………………………………..3.23}$$

Similar element matrices were drawn for each of the element. Then we calculated the global matrix by the computer program that was given in the previous chapter. The matrix was a 22x22 matrix

Next we derived the reduced matrix by putting the first row and column and second row and column and second row and column zero (the boundary conditions i.e. deflection and slope at
the fixed end were taken to be zero). So the resultant reduced matrix will be 20 x 20.

Due to large complexity of the problem computer program was used to calculate the global stiffness matrix.

The reduced matrix was solved by simple Gaussian elimination technique using the standard C program available for this.

Solving the equation:-

$$
\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ . \\ . \\ . \\ . \\ . \\ . \\ q_{20} \end{bmatrix}
=
\frac{EI}{l^3}
\begin{bmatrix}
12.00 & 6.00 & -12.00 & 6.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
6.00 & 4.00 & -6.00 & 2.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
-12.00 & -6.00 & 24.00 & 0.00 & -12.00 & 6.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
6.00 & 2.00 & 0.00 & 8.00 & -6.00 & 2.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & -12.00 & -6.00 & 24.00 & 0.00 & -12.00 & 6.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 6.00 & 2.00 & 0.00 & 8.00 & -6.00 & 2.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & -12.00 & -6.00 & 24.00 & 0.00 & -12.00 & 6.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 6.00 & 2.00 & 0.00 & 8.00 & -6.00 & 2.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -12.00 & -6.00 & 24.00 & 0.00 & -12.00 & 6.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 6.00 & 2.00 & 0.00 & 8.00 & -6.00 & 2.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -12.00 & -6.00 & 24.00 & 0.00 & -12.00 & 6.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 6.00 & 2.00 & 0.00 & 8.00 & -6.00 & 2.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -12.00 & -6.00 & 24.00 & 0.00 & -12.00 & 6.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 6.00 & 2.00 & 0.00 & 8.00 & -6.00 & 2.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -12.00 & -6.00 & 24.00 & 0.00 & -12.00 & 6.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 6.00 & 2.00 & 0.00 & 8.00 & -6.00 & 2.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -12.00 & -6.00 & 24.00 & 0.00 & -12.00 & 6.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 6.00 & 2.00 & 0.00 & 8.00 & -6.00 & 2.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -12.00 & -6.00 & 12.00 & -6.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 6.00 & 2.00 & -6.00 & 4.00
\end{bmatrix}
\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ . \\ . \\ . \\ . \\ . \\ . \\ a_{20} \end{bmatrix}
$$

Applying the boundary conditions the reduced matrix was obtained as follows.

The reduced matrix is

24.00, 0.00,-12.00, 6.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 8.00,-6.00, 2.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

-12.00,-6.00,24.00, 0.00,-12.00, 6.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

6.00, 2.00, 0.00, 8.00,-6.00, 2.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00,-12.00,-6.00,24.00, 0.00,-12.00, 6.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 6.00, 2.00, 0.00, 8.00,-6.00, 2.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00,-12.00,-6.00,24.00, 0.00,-12.00, 6.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 6.00, 2.00, 0.00, 8.00,-6.00, 2.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00,-12.00,-6.00,24.00, 0.00,-12.00, 6.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 6.00, 2.00, 0.00, 8.00,-6.00, 2.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,-12.00,-6.00,24.00, 0.00,-12.00, 6.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 6.00, 2.00, 0.00, 8.00,-6.00, 2.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,-12.00,-6.00,24.00, 0.00,-12.00, 6.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 6.00, 2.00, 0.00, 8.00,-6.00, 2.00, 0.00, 0.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,-12.00,-6.00,24.00, 0.00,-12.00, 6.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 6.00, 2.00, 0.00, 8.00,-6.00, 2.00, 0.00, 0.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,-12.00,-6.00,24.00, 0.00,-12.00, 6.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 6.00, 2.00, 0.00, 8.00,-6.00, 2.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,-12.00,-6.00,12.00,-6.00,

0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 6.00, 2.00,-6.00, 4.00, The above 20 simultaneous equations were solved by the Gaussian elimination method and the following results were obtained.

a_3 = -23.4167 w/EI

a_4 = -43.25 w/EI

a_5 = -87.3 w/EI

a_6 = -81.5 w/EI

a_7 = -183.75 w/EI

a_8 = -109.75 w/EI

a_9 = -304.66 w/EI

a_{10} = -131.0 w/EI

a_{11} = -443.75w/EI

a_{12} = -146.25 w/EI

a_{13}= -595.5w/EI

a_{14}= -156.5w/EI

a_{15}= -755.4161w/EI

a_{16}= -162.71w/EI

a_{17}= -920.0 w/EI

a_{18}= -166. w/EI

a_{19} = -1086.75 w/EI

a_{20} = -167.75w/EI

a_{21} = -1754.167w/EI

a_{22} = -167.5w/EI

Conclusion:-

In the previous case when we considered only element the error was 33.33%. But when we considered ten elements we calculated the error to be 0.33%. So it is clear that if the number of elements were increased the correctness of answer had increased.

Case 5

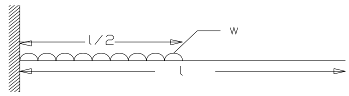

figure no:-3.6

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each nodes i.e. at the end points and n the beam.

Step 2:

Next the element was divided into two distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4 :

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

Element-1:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = EI \begin{bmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad \ldots\ldots 3.24$$

For second element:

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = EI \begin{bmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \quad \ldots\ldots 3.25$$

Step 5 :

Next the global stiffness matrix is calculated by combining the two element stiffness matrices

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = EI \begin{bmatrix} 12 & 6 & -12 & 6 & 0 & 0 \\ 6 & 4 & -6 & 2 & 0 & 0 \\ -12 & -6 & 24 & 0 & -12 & 6 \\ 6 & 2 & 0 & 8 & -6 & 2 \\ 0 & 0 & -12 & -6 & 12 & -6 \\ 0 & 0 & 6 & 2 & -6 & 4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \text{.................3.26}$$

Step 6:

We applied the boundary conditions to the global matrix to obtain the reduced matrix

Putting boundary conditions:

Putting $a_1 = 0$, $a_2 = 0$

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = EI \begin{bmatrix} 24 & 0 & -12 & 6 \\ 0 & 8 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \text{.......................................3.27}$$

The simultaneous equations were solved by Gaussian elimination technique.

$-w/2 = EI\,(24a_3 - 12a_5 + 6a_6)$……………………………………………………..3.28

$0 = EI\,(80a_4 - 6a_5 + 2a_6)$………………………………………………………3.29

$0 = EI\,(-12a_3 - 6a_4 + 12a_5 - 6a_6)$…………………………………………………..3.30

$0 = EI\,(6a_3 + 2a_4 - 6a_5 + 4a_6)$………………………………………………...3.31

$a_3 = -0.16667$ w/EI.

$a_4 = -0.2500$ w/EI.

$a_5 = -0.4167$ w/EI.

$a_6 = -0.2500$ w/EI.

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is

Case – 6

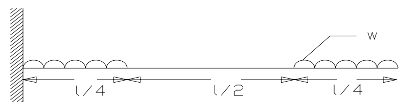

Figure no:-3.7

This is a peculiar problem common in machine components and civil structures. To solve this problem we adopt the similar procedure as enumerated above.

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each nodes i.e. at the end points and n the beam.

Step 2 :

Next the element was divided into two distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4:

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

For element -1:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = EI \begin{bmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.32$$

For element – 2

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = EI \begin{bmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.33$$

For element – 3

$$\begin{bmatrix} q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix} = EI \begin{bmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix} \begin{bmatrix} a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.34$$

For element – 4

$$\begin{bmatrix} q_7 \\ q_8 \\ q_9 \\ q_{10} \end{bmatrix} = EI \begin{bmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix} \begin{bmatrix} a_7 \\ a_8 \\ a_9 \\ a_{10} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.35$$

Step 5:

Next the global stiffness matrix is calculated by combining the two element stiffness matrices

The global form:-

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \end{bmatrix} = EI \begin{bmatrix} 12 & 6 & -12 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 4 & -6 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -12 & -6 & 24 & 0 & -12 & 6 & 0 & 0 & 0 & 0 \\ 6 & 2 & 0 & 8 & -6 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -12 & -6 & 24 & 0 & -12 & 6 & 0 & 0 \\ 0 & 0 & 6 & 2 & 0 & 8 & -6 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -12 & -6 & 24 & 0 & -12 & 0 \\ 0 & 0 & 0 & 0 & -6 & 2 & 0 & 8 & -6 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -12 & 6 & 12 & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 2 & -6 & 2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \end{bmatrix} \quad \ldots\ldots\ldots 3.36$$

Putting the conditions a_1, $a_2 = 0$ $a_9 = a_{10} = 0$

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix} = EI \begin{bmatrix} 24 & 0 & -12 & 6 & 0 & 0 \\ 0 & 8 & -6 & 2 & 0 & 0 \\ 0 & 0 & 24 & 0 & -12 & 6 \\ 0 & 0 & 0 & 8 & -6 & 2 \\ 0 & 0 & -12 & -6 & 24 & 0 \\ 0 & 0 & -6 & 2 & 0 & s \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \quad \ldots\ldots 3.37$$

The above matrix equations were solved by Gaussian elimination technique.

$q_3 = (24a_3 - 12a_5 - 16a_6)$ EI. Where $q_3 = -w/2$

$q_4 = (8a_4 - 6a_5 + 2a_6)$ EI. $q_4 = 0$

$q_5 = (24a_5 - 12a_7 + 6a_8)$ EI $q_5 = p = w$

$q_6 =. (8a_6 - 6a_7 + 2a_8)$ EI $q_6 = 0$

$q_7 = (-12a_5 - 6a_6 + -24a_7)$ EI. $q_7 = -w/2$

$q_8 = (-6a_5 + 2a_6 + 8a_8)$ EI. $q_8 = 0$

Solving the above equation

$a_3 = -0.0833$ w/EI

$a_4 = --0.0938$ w/EI

$a_5 = -0.12529$ w/EI

$a_6 = -0.000$ w/EI

$a_7 = -0.0833$ w/EI

$a_8 = -0.0937$ w/EI

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is

Case -7

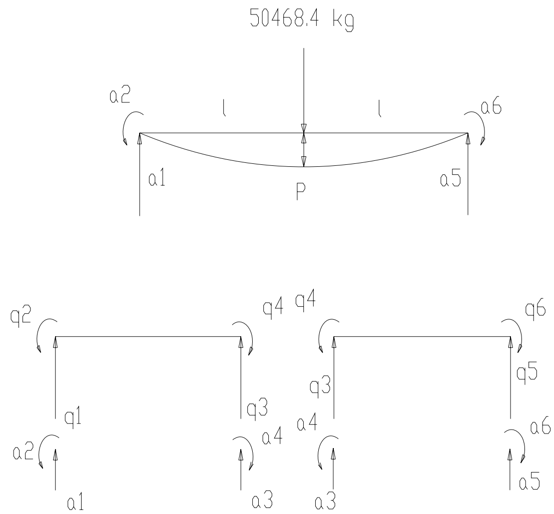

Figure no:-3.8

The next case that was taken up was the designing of the beams of the trolley structure. The trolley structure considered was a standard EOT crane with known values of loads on the beams at particular positions. We calculated the deflections and slopes on ach of the beams.

The following steps were followed to solve the problem

The following steps were followed to solve the problem

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each nodes i.e. at the end points and n the beam.

Step 2 :

Next the element was divided into two distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4 :

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

The element stiffness matrix for element -1

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad \ldots\ldots 3.38$$

For element – 2 the element stiffness matrix

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \quad \ldots\ldots 3.39$$

Step 5 :

Next the global stiffness matrix is calculated by combining the two element stiffness matrices.

$$\therefore \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l & 0 & 0 \\ 6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\ -12 & -6l & 24 & 0 & -12 & -6l \\ 6l & 2l^2 & 0 & 8l^2 & -6l & 2l^2 \\ 0 & 0 & -12 & -6l & 12 & -6l^2 \\ 0 & 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \quad \ldots\ldots 3.40$$

Step 6:

We applied the boundary conditions to the global matrix to obtain the reduced matrix

Putting boundary conditions: - a_1 and $a_5 = 0$

$$\therefore \begin{bmatrix} q_2 \\ q_3 \\ q_4 \\ q_6 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 4l^2 & -6l & 2l^2 & 0 \\ -6l & 24 & 0 & 6l \\ 2l^2 & 0 & 8l^2 & 2l^2 \\ 0 & 6l & 2l^2 & 4l^2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ a_6 \end{bmatrix} \quad \ldots\ldots 3.41$$

$q_2 = (4l^2 a_2 - 6la_3 + 2l^2 a_4)\, EI/l^3$……………………………………………………..3.42

$q_3 = (-6la_2 + 24a_3 + 6la_6)\, EI/l^3$……………………………………………………..3.43

$q_4 = (2l^2a_2 + 8l^2a_4 + 2l^2\ a_6)\ EI/l^3$……………………………………………………………3.44

$q_6 = (6la_3 + 2l^2a_4 + 4l^2\ a_6)\ EI/l^3$…………………………………………………………..3.45

$l = 1$

$q_2 = (4a_2 - 6a_3 + 2a_4)\ EI$…………………………………………………………………..3.46

$q_3 = (-6a_2 + 24a_3 + 6a_6)\ EI$……………………………………………………………….3.47

$q_4 = (2a_2 + 8a_4 + 2\ a_6)\ EI$…………………………………………………………………3.48

$q_6 = (6a_3 + 2a_4 + 4a_6)\ EI$ ………………………………………………………………….3.49

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is

Conclusion:

The large error might be due to the small number of elements taken in the problems. If the number of elements were increased instead of two then the result would have been better.

Case -8

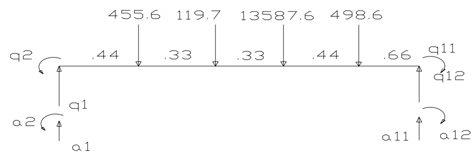

Figure no:-3.9

The following steps were followed to solve the problem

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each nodes i.e. at the end points and n the beam.

Step 2 :

Next the element was divided into two distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4:

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

For element -1 the stiffness matrix

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{EI}{.44^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.50$$

For element – 2 the stiffness matrix

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \frac{EI}{.33^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \qquad \ldots\ldots 3.51$$

For element –3 the stiffness matrix

$$\begin{bmatrix} q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix} = \frac{EI}{.33^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \qquad \ldots\ldots 3.52$$

For element – 4 the stiffness matrix

$$\begin{bmatrix} q_7 \\ q_8 \\ q_9 \\ q_{10} \end{bmatrix} = \frac{EI}{.44^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_7 \\ a_8 \\ a_9 \\ a_{10} \end{bmatrix} \qquad \ldots\ldots 3.53$$

For element – 5 the stiffness matrix

$$\begin{bmatrix} q_9 \\ q_{10} \\ q_{11} \\ q_{12} \end{bmatrix} = \frac{EI}{.66^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_9 \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix} \qquad \ldots\ldots 3.54$$

Step 5:

Next the global stiffness matrix is calculated by combining the two element stiffness matrices. Then boundary conditions were applied to it and the reduced matrix was obtained. Then the reduced matrix was solved by the Gaussian elimination technique and the values of deflection and slope were found out.

$$
\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ q_{11} \\ q_{12} \end{bmatrix}
= EI
\begin{bmatrix}
140.87, 30.99, -140.87, 30.99, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, \\
30.99, 9.09, -30.99, 4.55, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, \\
-140.87, -30.9, 474.7, 24, 1, -333.9, 55, 1, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, \\
30.99, 4.55, 24.1, 21.2, -55.1, 6.06, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, \\
0.00, 0.00, -333.92, -55.1, 667.8, 0.00, -333.9, 55, 1, 0.00, 0.00, 0.00, 0.00, \\
0.00, 0.00, 55.10, 6.06, 0.0, 24.2, -55.10, 6.06, 0.00, 0.00, 0.00, 0.00, \\
0.0, 0.0, 0.0, 0.0, -333.92, -55.1, 474.7, -24.1, -140.8, 30.9, 0.00, 0.00, \\
0.00, 0.00, 0.00, 0.00, 55.10, 6.06, -24.1, 21.2, -30.99, 4.5, 0.00, 0.00, \\
0.00, 0.00, 0.00, 0.00, 0.00, 0.00, -140.8, -30.9, 182.6, -17.22, -41.74, 13.77, \\
0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 30.99, 4.55, -17.22, 15.15, -13.77, 3.03, \\
0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, -41.7, -13.7, 41.7, 13.7, \\
0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 13.77, 3.03, -13.77, 6.06,
\end{bmatrix}
\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix}
$$

Step 6:

We applied the boundary conditions to the global matrix to obtain the reduced matrix

$$
\begin{bmatrix} q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ q_{12} \end{bmatrix} = EI
\begin{bmatrix}
9.09, -30.99, 4.55, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, \\
-30.99, 474.79, 24.10, -333.92, 55.10, 0.00, 0.00, 0.00, 0.00, 0.00, \\
4.55, 24.10, 21.21, -55.10, 6.06, 0.00, 0.00, 0.00, 0.00, 0.00, \\
0.00, -333.92, -55.10, 667.84, 0.00, -333.92, 55.10, 0.00, 0.00, 0.00, \\
0.00, 55.10, 6.06, 0.00, 24.24, -55.10, 6.06, 0.00, 0.00, 0.00, \\
0.00, 0.00, 0.00, -333.92, -55.10, 474.79, 24.10, -140.87, 30.99, 0.00, \\
0.00, 0.00, 0.00, 55.10, 6.06, -24.10, 21.21, -30.99, 4.55, 0.00, \\
0.00, 0.00, 0.00, 0.00, -140.8, -30.99, 182.61, -17.22, 13.77, \\
0.00, 0.00, 0.00, 0.00, 0.00, 30.99, 4.55, -17.22, 15.15, 3.03, \\
0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, -41.74, -13.77, -13.77, \\
0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 13.77, 3.03, 6.06,
\end{bmatrix}
\begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{12} \end{bmatrix}
$$

The equations were formed and were solved using Gaussian elimination technique.

$$\frac{q_2}{w} = 9.1a_2 - 30.9a_3$$

$$\frac{q_3}{EI} = -30.9a_2 + 477.7a_3 + 24.19a_4 - 333.1a_5 + 55.1a_6$$

$$\frac{q_4}{EI} = 4.5a_2 + 24.19a_3 + 21.2a_4 - 55.1a_5 + 6.1a_6$$

$$\frac{q_5}{EI} = 0.a_2 - 334.9a_3 - 55.1a_4 + 677.8a_5 + 0..a_6 - 333.4a_7 + 55.1a_8$$

$$\frac{q_6}{EI} = 0.a_2 + 55.1a_3 + 6.1a_4 + 0.a_5 + 24.24..a_6 - 55.197a_7 + 6.1a_8$$

$$\frac{q_7}{EI} = 0.a_2 + 0.a_3 - 0.a_4 + 337.9a_5 - 55.1.a_6 + 474.7a_7 + 24.19a_8 + 474.7a_9 + 24.a_{10}$$

$$\frac{q_8}{EI} = 0.a_2 + 0.a_3 + 55.a_5 + 6.1.a_6 + 24.19a_7 + 21.29.1a_8 + 21.12a_9 + 21.22.a_{10}$$

$$\frac{q_9}{EI} = 0.a_2 + 0.a_3 + 0.a_4 + 0.a_5 + 0.a_6 + 140.8a_7 - 30.9a_8 + 182.150_9 + 13.7.a_{10}$$

$$\frac{q_{10}}{EI} = 0.a_2 + 0.a_3 + 0.a_4 + 0.a_5 + 0..a_6 - 30.9a_7 - 9.5a_8 + 4.5a_9 + 15.2.a_{10} + 3.05a_{11}$$

$$\frac{q_{12}}{EI} = 0.a_2 + 0.a_3 + 0.a_4 + 0.a_5 + 0..a_6 + 0.a_7 - 0.a_8 + 13.7a_9 + 3.05.a_{10} + 6.1a_{11}$$

Solving we get

a_2 = -143.64 w/EI

a_3 = --42.3 w/EI

a_4 = -5.3 w/EI

a_6 = -29.67 w/EI

a_5 = -47.56 w/EI

a_7 -13.53 w/EI

a_8 = -195.55 w/EI

a_9 = -5.95w/EI

a_{10} = --72.53 w/EI

a_{12} = -22.89w/EI

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is

Conclusion:

The large error might be due to the small number of elements taken in the problems. If the number of elements were increased instead of two then the result would have been better.

Case -9

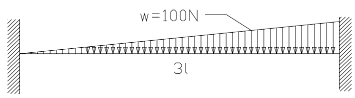

figure no:-3.10

So far the problems considered were of uniformly varying loads but in actual cases in a machine component or in any structure non uniformly varying loads pop up more prominently. In the present case the example of a non uniformly varying load is taken up. More precisely the problem that is considered is that of a beam having the triangular loads upon the whole length of it. We tend to find out the deflection and slope at each nodes of the beam.

The following steps were followed to solve the problem

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each nodes i.e. at the end points and n the beam.

Step 2 :

Next the element was divided into two distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4 :

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

The element stiffness matrix for one element is

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad \text{...3.55}$$

for the second element the element stiffness matrix is

$$\begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \quad \text{..3.56}$$

Step 5 :

Next the global stiffness matrix is calculated by combining the two element stiffness matrices.

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = EI \begin{bmatrix} 1.50, 1.50, -1.50, 1.50, 0.00, 0.00, \\ 1.50, 2.00, -1.50, 1.00, 0.00, 0.00, \\ -1.50, -1.50, 13.50, 4.50, -12.00, 6.00, \\ 1.50, 1.00, 4.50, 6.00, -6.00, 2.00, \\ 0.00, 0.00, -12.00, -6.00, 12.00, -6.00, \\ 0.00, 0.00, 6.00, 2.00, -6.00, 4.00, \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \quad \text{........................3.57}$$

Step 6:

We applied the boundary conditions to the global matrix to obtain the reduced matrix. The reduced matrix was solved by the Gaussian elimination technique and the deflections and the slopes were calculated.

$a_3 = 5.9 \times 10^{-7}$m

$a_4 = -3.9 \times 10^{-7}$m

$q_2 = -25.5$ N-m.

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is 15%

Conclusion:

The error might be reduced by taking large number of elements and analyzing the same with the steps enumerated above.

Case 10

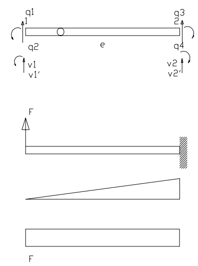

figure no:-3.11

Finite element for transversely cracked slender beams subjected to transverse loads

In this case , the effect of a crack in the beam is analyzed. The crack is replaced by a rotational linear spring connecting the uncracked pairs of the structure that are modeled as plastic elements.

The crack is specified by distance l_1 , depth d and the rotational spring stiffness k_r

The upward forces applied are F_{n1} and F_{n2} ; translations are Y_{n1} and Y_{n2}; counterclockwise moments are M_{n1}, M_{n2}; and rotations Φ_{r1} and Φ_{r2} (are taken positive). If we remove one support then it becomes a cantilever problem.

The transverse displacement of the free end is now obtained by integrating the diagram for the structure with applied force F_{n1}

$$Y_{n1}(F_{n1}) = F_{n1}\, l^3 / 3EI + F_{n1}\, l^2/ k_r + F_{n1}\, l/GA_s \ldots\ldots\ldots\ldots 3.58$$

The rotation is obtained by integrating the diagram with the same applied loading F_{n1}

$$\Phi r_1 (F_{n1}) = 0.5\, F_{n1}\, l*(-1.0)*L/EI + F_{n1}\, l_1 (-1)/ k_r \ldots\ldots\ldots\ldots 3.59$$

Now this problem is repeated and the displacement and rotation due to moment M_{n1}

$$Y_{n1}(M_{n1}) = 0.5(-M_{n1})\, l^2/ EI + (-M_{n1})*l_1 / k_r \ldots\ldots 3.60$$

Similarly for rotation we calculate

$$\Phi r_1 (F_{n1}) = 0.5\, M_{n1}\, l*(-1.0)*L/EI + M_{n1}\, l_1\, (-1)/ k_r \ldots\ldots 3.61$$

so total displacement is

$$Y_{n1} = Y_{n1}(F_{n1}) + Y_{n1}(M_{n1}) \ldots\ldots 3.62$$

$$= F_{n1}\, l^3 / 3EI + F_{n1}\, l^2/ k_r + F_{n1}\, l/GA_s - 0.5(-M_{n1})\, l^2/ EI - (-M_{n1})*l_1 / k_r$$

Similarly, total rotation is given by

$$\Phi r_1 = \Phi r_1(F_{n1}) + \Phi r_1(M_{n1}) \ldots\ldots 3.63$$

$$= -F_{n1}\, l^2 /2EI - F_{n1}\, l/ k_r + M_{n1}\, L/EI + M_{n1} / k_r$$

But we know at the free end ,

$$F_{n2} = - F_{n1}$$

And $M_{n2}= F_{n1}\, l- M_{n1}$

If the forces are applied at the right end , then we find,

$$Y_{n2}(F_{n2}) = F_{n2}\, l^3 / 3EI + F_{n2}\, (l-l_1)^2/ k_r - F_{n2}\, l/GA_s \ldots\ldots 3.64$$

And $\Phi r_2(F_{n2})= 0.5* F_{n2}\, l^2 / 2EI+ F_{n2}\, (l-l_1) / k_r \ldots\ldots 3.65$

Due to bending M_{n2}

$$Y_{n2}(M_{n2}) = 0.5(-M_{n2})\, l^2/ EI + M_{n2}*(l-l_1) / k_r \ldots\ldots 3.66$$

$$\Phi r_2 (F_{n2}) = M_{n2}\, l*(1.0)*L/EI + M_{n2}/ k_r \ldots\ldots 3.67$$

So the total displacement of the model is therefore the sum of both the partial displacements

$$Y_{n2} = Y_{n2}(F_{n2}) + Y_{n2}(M_{n2}) \ldots\ldots 3.68$$

$$= F_{n2}\, l^3 / 3EI + F_{n2}\, (l-l_1)^2/ k_r - F_{n2}\, l/GA_s +0.5(-M_{n2})\, l^2/ EI+ M_{n2}*(l-l_1) / k_r$$

and $\Phi r_2 = \Phi r_2(F_{n2}) + \Phi r_2(M_{n2}) \ldots\ldots 3.69$

$$= 0.5* F_{n2}\, l^2 / 2EI+ F_{n2}\, (l-l_1) / k_r + M_{n2}\, l*(1.0)*L/EI + M_{n2}/ k_r$$

finally the combined stiffness matrix can be written as incorporating the above terms as follows.

$$K_{cc}= 1/(k_r\, l^3(1+\beta)+EI(12\, l_1^2-12l\, l_1+l^2(4+ \beta))l$$

$$\begin{vmatrix} 12EI(EI+ k_r l) & 6EI(k_r l^2 +2EI\, l_1) & -12EI(EI+ k_r l) & 6EI(k_r l^2+ 2EI(l-l_1) \\ 6EI(k_r l^2 +2EI\, l_1) & EI(12EI\, l_1^2+ k_r l^3(4+ \beta) & -6EI(k_r l^2 +2EI\, l_1) & EI(12EI(l-l_1)l_1- k_r\, l^3(\beta-2) \\ -12EI(EI+ k_r l) & -6EI(k_r l^2 +2EI\, l_1) & 12EI(EI+ k_r l) & -6EI(k_r l^2+ 2EI(l-l_1) \\ 6EI(k_r l^2+ 2EI(l-l_1) & EI(12EI(l-l_1)l_1- k_r\, l^3(\beta-2) & 6EI(k_r l^2+ 2EI(l-l_1) & EI(12EI\, l_1^2+ k_r l^3(4+ \beta) \end{vmatrix}$$

Where $\beta=12EI/(Gl^2A_s)$

If k_r is infinite then the equation becomes a simple un cracked beam. We apply this stiffness matrix to solve a problem of cracked beam and compare the results with the results obtained by various other techniques.

Case 11

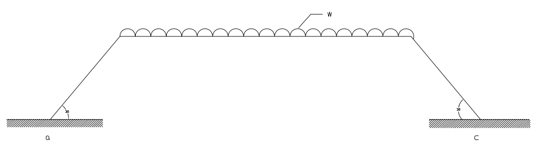

figure no:-3.12

A bent beam with uniformly distributed load

In this case such a situation is analyzed which is commonplace in the structural problems. We considered a bent beam with three arms. On the top arm there was uniformly distributed load. The lower two arms were inclined to the horizontal at 30 degree each. We intend to find out deflections and slopes at each of the points.

This particular problem was analyzed taking into consideration both bending and torsion. The assumption in this problem was the value of elasticity modulus was twice that of the rigidity modulus.

The steps to solve these problems were slightly different from the above problems. The following steps were followed.

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each node i.e. at the end points and n the beam.

Step 2 :

Next the element was divided into two distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4 :

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

Step 5:

As the problem considered was that of a bent beam the element stiffness matrix obtained was in local co-ordinates. So we converted those elements in the matrix into the global coordinates by multiplying the transformation matrix.

The element stiffness matrix is given as

K_e=

$$\begin{bmatrix} 12EI/L^3 & 0 & 6EI/L^2 & -12EI/L^3 & 0 & -6EI/L^2 \\ 0 & GJ/L & 0 & 0 & -GJ/L & 0 \\ 6EI/L^2 & 0 & 4EI/L & -6EI/L^2 & 0 & 2EI/L \\ -12EI/L^3 & 0 & -6EI/L^2 & 12EI/L^2 & 0 & -6EI/L \\ 0 & -GJ/L & 0 & 0 & GJ/L & 0 \\ 6EI/L^2 & 0 & 2EI/L & -6EI/L^2 & 0 & 4EI/L \end{bmatrix} \qquad \text{………...3.70}$$

As this is a bent beam structure in order to transform the local co-ordinates to global co-ordinate we need to multiply with it the transformation matrix. The transformation matrix is given as

$$T=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & \sqrt{3}/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{3}/2 & 1/2 \\ 0 & 0 & 0 & 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \qquad \text{…………………………….3.71}$$

The stiffness matrix in global co-ordinates can be obtained as multiplying matrix K_e and the matrix T.The stiffness matrix in global co-ordinates can be calculated for 3 elements. The global stiffness matrix is

$$\begin{bmatrix} 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4/\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 48 & 0 & 12 & -24 & 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.87 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12 & 0 & 14.92 & -12 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -24 & 0 & -12 & 48 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1.87 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12 & 0 & 4 & -12 & 0 & 14.92 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 24 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.86 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14.92 \end{bmatrix}$$

Step 6:

We applied the boundary conditions to the global matrix to obtain the reduced matrix. The reduced matrix was solved by the Gaussian elimination technique and the deflections and the slopes were calculated.

$$\begin{bmatrix} V_2 \\ M_2 \\ V_3 \\ M_3 \end{bmatrix} = EI \begin{bmatrix} 48 & 12 & -24 & 12 \\ 12 & 14.82 & -12 & 4 \\ -24 & -12 & 48 & 0 \\ 12 & 4 & -12 & 14.92 \end{bmatrix} \begin{bmatrix} \delta_2 \\ \theta_2 \\ \delta_3 \\ \theta_3 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.72$$

After solving we get

δ_2=0.4167/EI m δ_3=0.4167/EI m

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is 0.2%

Conclusion:

As the error is very less we can safely say that results obtained by FEA technique is nearly congruent to the results obtained by strength of material techniques.

Case 12

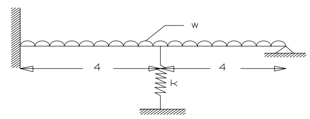

Figure no:-3.13

Beam supported with elastic foundation

In real time machine components beams supported on an elastic foundation is a common practice. The examples include railway tracks and various machine components. The problem we take up is a simple beam supported by a spring of spring constant $K=24EI/L^3$ and EI=400 units. We calculated the deflection and slope at the spring support.

As usual we follow the standard procedure to solve the FEA problems as enumerated in the Chapter 2

The following steps were followed.

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each nodes i.e. at the end points and n the beam.

Step 2 :

Next the element was divided into two distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4:

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

The element stiffness matrix for each element is same as that of the standard element stiffness matrix

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots 3.73$$

In the same line the element stiffness matrix for the second element can be drawn.

Step 5 :

Next the global stiffness matrix is calculated by combining the two element stiffness matrices.

Global stiffness matrix

$$G=EI \begin{bmatrix} 12 & 24 & -12 & 24 & 0 & 0 \\ 24 & 64 & -24 & 32 & 0 & 0 \\ -12 & -24 & 24 & 0 & -12 & 24 \\ 24 & 32 & 0 & 128 & -24 & 32 \\ 0 & 0 & -12 & -24 & 12 & -24 \\ 0 & 0 & 24 & 32 & -24 & 64 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots 3.74$$

Writing in the compact format with all the forces and moments and deflections and slopes we draw the global stiffness matrix as follows

$$\begin{bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \\ V_3 \\ M_3 \end{bmatrix} = EI \begin{bmatrix} 12 & 24 & -12 & 24 & 0 & 0 \\ 24 & 64 & -24 & 32 & 0 & 0 \\ -12 & -24 & 24 & 0 & -12 & 24 \\ 24 & 32 & 0 & 128 & -24 & 32 \\ 0 & 0 & -12 & -24 & 12 & -24 \\ 0 & 0 & 24 & 32 & -24 & 64 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \theta_1 \\ \delta_2 \\ \theta_2 \\ \delta_3 \\ \theta_3 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots 3.75$$

Step 6:

We applied the boundary conditions to the global matrix to obtain the reduced matrix. The reduced matrix was solved by the Gaussian elimination technique and the deflections and the slopes were calculated.

The reduced matrix becomes

$$\begin{bmatrix} V_2 \\ M_2 \\ M_3 \end{bmatrix} = EI \begin{bmatrix} 24 & 0 & 24 \\ 0 & 128 & 32 \\ 24 & 32 & 64 \end{bmatrix} \begin{bmatrix} \delta_2 \\ \theta_2 \\ \theta_3 \end{bmatrix} \quad \ldots\ldots 3.76$$

$$0.075 - 150\delta_2/400 = 24\delta_2 + 24\theta_3 \quad \ldots\ldots 3.77$$

$$24.375\delta_2 + 24\theta_3 = 0.075 \quad \ldots\ldots 3.78$$

$$128\theta_2 + 32\theta_3 = 0 \quad \ldots\ldots 3.79$$

$$24\delta_2 + 32\theta_2 + 64\theta_3 = 0 \quad \ldots\ldots 3.80$$

The above equations were solved by Gaussian elimination technique.

Solving these equations we get

δ_2=5.62x10^{-3} m

θ_2=6.02x10^{-4} m

θ_3=-2.40x10^{-3} m

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is 3%. This error is in the permissible range.

Conclusion:

This present problem is different from the other problems in the way that in this case we have provided an elastic support to the beam. So a spring force comes into picture along with all the external loadings.

Case:-**13**

Finite Element Analysis applied to combined bending and torsion.

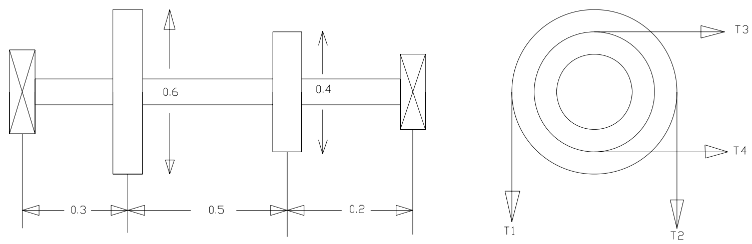

Figure no:-3.14

Lastly we considered a real machine shaft subjected to bending and torsion both by longitudinal and transverse loadings.

The shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of a belt having maximum tension of 2.25 KN. Another pulley 400mm diameter is placed 200 mm to the left of the right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180 degrees and the co-efficient of friction is 0.24. We intend to calculate the deflection and slopes at each nodes of the shaft.

The beam is subjected to the following loads

i) Loading in vertical direction

ii) Loading in horizontal direction

iii)Torsion

Let T_1 be the tension in the tight side of the belt on the pulley C and that is equal to 2250N and T_2 be the tension in the slack side of the belt on the pulley C.

We know that

$$2.3 \log (T_1/ T_2) = \mu. \theta \ldots\ldots 3.81$$

$$=0.24\pi = 0.754$$

$\log (T_1/ T_2) = 0.3278$……………………………………………………….3.82

Therefore T_2= 1058N.

Vertical load acting on the shaft at C:

$$W_c= T_1+ T_2 = 2250+1058=3308N \ldots\ldots 3.83$$

And vertical load on the shaft at D=0.

We know the torque acting on the pulley C,

$$T=(T_1- T_2)R_c=(2250-1058)0.3=357.6N \ldots\ldots 3.85$$

Let T_3 be the tension in the tight side of belt on pulley D and T_4 be the tension in the slack side of the belt on the pulley D. Since the torque on both the pulleys are same,

$(T_3 - T_4)R_d$ =T=357.6N……………………………………………………….3.86

So $T_3 - T_4$ = 1788N………………………………………………………….3.87

But we know that

$$T_1/ T_2 = T_3/ T_4=2.127 \ldots\ldots 3.88$$

So T_3=2.127 T_4

So T_3=3376N and T_4=1588N

So horizontal load acting on the the shaft at D:

W_d= $(T_3+ T_4)$=3376+1588=4964N………………………………………….3.89

And the horizontal load acting on the shaft at D is zero.

The beam is subjected to the following loads

i) Loading in vertical direction

ii) Loading in horizontal direction

iii)Torsion

Next we follow the general steps for solving any finite element analysis problem enumerated in the chapter 2

Step 1:

The first was defining the problem. The diagram of the beam was done with all loading conditions shown clearly and distinctly at each nodes i.e. at the end points and n the beam.

Step 2:

Next the element was divided into three distinct elements each of length a and b

Step 3:

Then the individual free body diagrams were drawn and the loading conditions were shown along with the respective degrees of freedom.

Step 4:

Next the element stiffness matrices were determined as per the procedure stated in chapter 2.

We considered 3 elements

The element matrix for 1 element is given as

$$
\begin{bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \\ T_1 \\ T_2 \\ H_1 \\ M^*_1 \\ H_2 \\ M^*_2 \end{bmatrix}
=
\begin{bmatrix}
12EI/L^3 & -6EI/L^3 & -12EI/L^3 & -6EI/L^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
6EI/L^2 & 4EI/L & 6EI/L^2 & -12EI/L^3 & 0 & 0 & 0 & 0 & 0 & 0 \\
-12EI/L^3 & 6EI/L^2 & 12EI/L^3 & -6EI/L^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
-6EI/L^2 & -12EI/L^3 & -6EI/L^3 & 4EI/L & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & GJ/L & -GJ/L & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -GJ/L & GJ/L & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 12EI/L^3 & 6EI/L^3 & -12EI/L^3 & -6EI/L^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 6EI/L^2 & 4EI/L & 6EI/L^2 & -12EI/L^3 \\
0 & 0 & 0 & 0 & 0 & 0 & -12EI/L^3 & 6EI/L^2 & 12EI/L^3 & -6EI/L^2 \\
0 & 0 & 0 & 0 & 0 & 0 & -6EI/L^2 & -12EI/L^3 & -6EI/L^3 & 4EI/L
\end{bmatrix}
\begin{bmatrix} \delta_1 \\ \theta_1 \\ \delta_2 \\ \theta_2 \\ \alpha_1 \\ \alpha_2 \\ \delta_1^* \\ \theta_1^* \\ \delta_2^* \\ \theta_2^* \end{bmatrix}
$$

Step 5:

Next the global stiffness matrix is calculated by combining the three element stiffness matrices.

$$
\begin{bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \\ V_3 \\ M_3 \\ V_4 \\ M_4 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ H_1 \\ M_1 \\ H_2 \\ M_2 \\ H_3 \\ M_3 \\ H_4 \\ M_4 \end{bmatrix}
= EI
\begin{bmatrix}
444.4,66.6,-444.4,66.6,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
66.6,13.7,-66.6,6.67,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
-444.4,-66.6,550.6,-42.6,-96,-24,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
66.6,6.67,-42.6,31.3,-34,4,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,-96,-24,1596,126,-1500,150,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,24,4,126,28,-150,10,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,-1500,-150,1500,150,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,150,10,-1500,20,0,0,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,2.76,-2.6,0,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,-2.76,4.43,-1.66,0,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,-1.66,5.8,-4.14,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,-4.14,4.14,0,0,0,0,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,444.44,66.67,-444.44,66.67,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,66.6,13.7,-66.6,6.67,0,0,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,-444.4,-66.6,550.6,-42.6,-96,-24,0,0 \\
0,0,0,0,0,0,0,0,0,0,0,0,66.6,6.67,-42.6,31.3,-34,4,0, \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,-96,-24,1596,126,-1500,150 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,24,4,126,28,-150,10 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1500,-150,1500,150 \\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,150,10,-1500,20
\end{bmatrix}
\begin{bmatrix} \delta_1 \\ \theta_1 \\ \delta_2 \\ \theta_2 \\ \delta_3 \\ \theta_3 \\ \delta_4 \\ \theta_4 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \delta^*{}_1 \\ \theta_1^* \\ \delta_2^* \\ \theta_2^* \\ \delta_3^* \\ \theta_3 * \\ \delta_4^* \\ \theta_4^* \end{bmatrix}
$$

Step 6:

We applied the boundary conditions to the global matrix to obtain the reduced matrix. The reduced matrix was solved by the Gaussian elimination technique and the deflections and the slopes were calculated.

δ_2 =7.15/EI

θ_2 =24.33/EI

δ_3 =2.25/EI

θ_3 =-19.17/EI

α_2 =116.59/EI

α_3 =95.02/EI

δ_2^* =5.95/EI

θ_2^*=140.36/EI

δ_3^*=-42.55/EI

θ_3^*=556.22/EI

Step 7:

In the last step the results are compared with that of the standard results obtained by strength of material techniques and the percentage of error is determined. The percentage of error is 8.67% .This error is in the permissible range.

.

CHAPTER 3

ERROR TABLE

ERROR TABLE

S.L. NO.	PROBLEM	ERROR
1.	CASE 1	0.00%
2.	CASE 2	0.00%
3.	CASE 3	7.73%
4.	CASE 4	33.3%
5.	CASE 5	0.32%
6.	CASE 6	7.73%
7.	CASE 7	0.00%
8.	CASE 8	0.00%
9.	CASE 9	11.23%
10.	CASE 10	n.a.
11.	CASE 11	0.2%
12.	CASE 12	3.86%
13.	CASE 13	8.67%

CHAPTER 4

CONCLUSION

Conclusion

The problems that have been considered in the present work are real time machine components that have been solved bystrength of material methods. We apply the finite element techniques to the above problems and calculate the deflections and slopes at each of the points. The results obtained in our method differs slightly from the results obtained from the strength of material techniques. The errors may be due to some problems in the computational techniques. Besides we have taken upto 20 elements at a max for the problems but if the number of elements are increased then the correctness of results will also increase. This is evident in the cantilever with uniformly distributed load over it problem. When the number of elements were only one the error was 33.33%, but when the number of elements were increased to 20 the error reduced to only .32%. The problem that is a real time shaft problem subjected bending and torsion due to both transeverse loads and longitudinal loads.

Lastly, a new finite element technique method has been developed to analyse the cracked beams. In this case the crack is idealised as a rotational spring and the spring constant is calculated.

Computer programs using C language have been developed to calculate the global stiffness matrix and solve the simultaneous equations by Gaussian elemination technique.

The beauty of this method lies in the fact that it is computationally easy to carry out the complex mathematical equations because of computr programs. Besides MATLAB packages and COSMOS may be used to solve the problems. Next, beams having varying cross sections like stepped beams or tapered beams which is difficult to analyse by strength of material techniques can be analysed quite easily and efficiently by this FEA methods.

So, finite element analysis provides a new horizon to solve structural and machine components. This method has to be utilised by the industries and designing firms to incoporate efficiency and accuracy to the designing aspects.

APPENDIX

Programme for finding global matrix:

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<math.h>
#define ELE 10
#define GM  2*(ELE+1)
FILE *OUTFILE;
main()
{
Float  ke[ELE][4][4],A[ELE][4][GM],AT[ELE][GM][4],KG[ELE][GM][GM],len[ELE],
        RES[ELE][GM][4],KGRES[GM][GM],xi[ELE];
 int i,j,k,l;
 OUTFILE=fopen("outputcant.txt","w");
     if(OUTFILE==NULL)
       {
               fprintf(stderr,"error opening file");
               exit(0);
       }

     for(l=0;l<ELE;l++)
      for(i=0;i<4;i++)
       for(j=0;j<GM;j++)
       {
        if(i+l*2==j)
        A[l][i][j]=AT[l][j][i]=1;
       else
         A[l][i][j]=AT[l][j][i]=0;
      }
      for(l=0;l<ELE;l++)
      for(i=0;i<GM;i++)
     {
      for(j=0;j<GM;j++)
      {
         KG[l][i][j]=0;
         KGRES[i][j]=0;
      }
        for(k=0;k<4;k++)
        {
         RES[l][i][k]=0;
        }
      }
```

```
 for(i=0;i<ELE;i++)
{
       printf("enter the length len[%d],i[%d]",i+1,i+1);
      scanf("%f%f",&len[i],&xi[i]);
}
    for(i=0;i<ELE;i++)
         {
          ke[i][0][0]=ke[i][2][2]=12*xi[i]/(len[i]*len[i]*len[i]);
          ke[i][0][1]=ke[i][1][0]=ke[i][0][3]=ke[i][3][0]=6*xi[i]/(len[i]*len[i]);
          ke[i][1][2]=ke[i][2][1]=ke[i][2][3]=ke[i][3][2]=-6*xi[i]/(len[i]*len[i]);
          ke[i][1][1]=ke[i][3][3]=4*xi[i]/len[i];
          ke[i][0][2]=ke[i][2][0]=-12*xi[i]/(len[i]*len[i]*len[i]);
          ke[i][1][3]=ke[i][3][1]=2*xi[i]/len[i];
         }
    for(i=0;i<ELE;i++)
        for(j=0;j<GM;j++)
       for(k=0;k<4;k++)
        for(l=0;l<4;l++)
          RES[i][j][k]=RES[i][j][k]+AT[i][j][l]*ke[i][l][k];


   for(i=0;i<ELE;i++)
     for(j=0;j<GM;j++)
       for(k=0;k<GM;k++)
        for(l=0;l<4;l++)
          KG[i][j][k]=KG[i][j][k]+RES[i][j][l]*A[i][l][k];
     for(j=0;j<GM;j++)
        for(k=0;k<GM;k++)
             for(i=0;i<ELE;i++)
               KGRES[j][k]=KGRES[j][k]+KG[i][j][k];
     for(j=0;j<GM;j++)
        {
         for(k=0;k<GM;k++)
          fprintf(OUTFILE,"%5.2f,", KGRES[j][k]);
        fprintf(OUTFILE,"\n\n");
        }
     getch();
    return 0;
}
```

PROGRAMME FOR SIMULTENEOUS EQUATION SOLUTION USING GAUSS ELIMINATION METHOD:

```
#include <stdio.h>
#include <math.h>
#include<conio.h>
#include<stdlib.h>
#define NMAX 4

void elgs (double a[NMAX][NMAX],int n,int indx[4])

{
 int i, j, k, itmp;
 double c1, pi, pi1, pj;
 double c[NMAX];

 if (n > NMAX)
 {
  printf("The matrix dimension is too large.\n");
  exit(0);
 }

 for (i = 0; i < n; ++i)
 {
  indx[i] = i;
 }

 for (i = 0; i < n; ++i)
 {
  c1 = 0;
  for (j = 0; j < n; ++j)
  {
   if (fabs(a[i][j]) > c1) c1 = fabs(a[i][j]);
  }
  c[i] = c1;
 }



 for (j = 0; j < n-1; ++j)
 {
  pi1 = 0;
```

```
for (i = j; i < n; ++i)
    {
      pi = fabs(a[indx[i]][j])/c[indx[i]];
      if (pi > pi1)
      {
        pi1 = pi;
        k = i;
      }
    }

    itmp = indx[j];
    indx[j] = indx[k];
    indx[k] = itmp;
    for (i = j+1; i < n; ++i)
    {
      pj = a[indx[i]][j]/a[indx[j]][j];
      a[indx[i]][j] = pj;
      for (k = j+1; k < n; ++k)
      {
        a[indx[i]][k] = a[indx[i]][k]-pj*a[indx[j]][k];
      }
    }
  }
}
void legs (double a[NMAX][NMAX],int n,double b[4],double x[4],int indx[4])


{
  int i,j;

  elgs (a,n,indx);



 for(i = 0; i < n-1; ++i)
  {
    for(j = i+1; j < n; ++j)
    {
      b[indx[j]] = b[indx[j]]-a[indx[j]][i]*b[indx[i]];
    }
  }

  x[n-1] = b[indx[n-1]]/a[indx[n-1]][n-1];
  for (i = n-2; i>=0; i--)
```



```
{

    x[i] = b[indx[i]];
```

```
    for (j = i+1; j < n; ++j)
    {
      x[i] = x[i]-a[indx[i]][j]*x[j];
    }
    x[i] = x[i]/a[indx[i]][i];
  }
}



main()


{
  double x[4], b[4]={0.5,0,0,0};
  double a[4][4]={{24,0,-12,6},{0,8,-6,2},{-12,-6,12,-6},{6,2,-6,4}};
  int i, n=4, indx[4];


  legs (a,n,b,x,indx);

  for (i=0; i<n; i++)
  {
    printf("%10.4f\n", x[i]);
  }
  getch();
}
```

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