ANALYTICAL APPROACH TO THE DETERMINATION OF DYNAMIC CHARACTERISTICS OF A BEAM

A THESIS SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

Bachelor of Technology

in

Mechanical Engineering

By

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Department of Mechanical Engineering

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Under the Guidance of

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CERTIFICATE

This is to certify that the thesis entitled, "Analytical approach to the determination of dynamic characteristics of a cracked beam" submitted by Mr.Sunil Kumar Kanhar & Mr. Sudipta senapati in partial fulfillments for the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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Sunil kumar kanhar)

CONTENTS

Abstract

A wide spectrum of investigations devoted to the determination of natural frequencies and mode shapes of beams with an open crack are presented in the literature. However, as is well known, an open crack is a fairly crude model of a fatigue crack. The study of the dynamic characteristics of a beam with a closing crack is the main aim of the present paper. The analytical approach which enables one to determine the effect of crack parameters (crack magnitude and location) on different dynamic characteristics of a cantilever Bernoulli}Euler beam with a closing edge transverse crack is performed. Natural frequencies, mode shapes and distortion of time functions describing wave shapes of displacement, acceleration and strain of different cross-sections of a beam are considered as dynamic characteristics to be investigated. The general solution of the problem is derived from the synthesis of particular solutions obtained for the crack-free beam and for the beam with an open crack. The possibility of origination of several modes of vibrations during crack opening is taken into account as well as the peculiarity of strain distribution in the vicinity of a crack. It is shown that analytically predicted relationships between the dynamic characteristics of a cracked beam and crack parameters are well-founded. The analytical approach makes it possible to solve the inverse problem of damage diagnostics with sufficient accuracy for practical purposes.

Chapter 1

GENERAL INTROIDUCTION

Introduction

Dynamic characteristics of a damaged and undamaged body are, as a rule, different. This difference is caused by a change in stiffness and can be used for the detection of damage and for the determination of its parameters (crack magnitude and location). Many mechanical structures in real service conditions are subjected to combined or separate effects of the dynamic load, temperature and corrosive medium, with a consequent growth of fatigue cracks, corrosive cracking and other types of damage. The immediate visual detection of damage is difficult or impossible in many cases and the use of local non-destructive methods of damage detection requires time and "financial expense and frequently is inefficient.

In this connection, the use of vibration methods of damage diagnostics is promising. These methods are based on the relationships between the vibration characteristics (natural frequencies and mode shapes) or peculiarities of a non-linear vibration system behavior (for example, non-linear distortions of the displacement wave in different cross-sections of a beam, the amplitudes of sub-resonance and super-resonance vibrations, the anti-resonance frequencies, etc.) and damage parameters. It is important to note that the essential non-linearity of vibrations of a body with a fatigue crack is due to the change of stiffness at the instant of crack opening and closing and is the main difficulty in the solution of such class problems. The analytical Investigation of vibrations of damaged structures is a complicated Problem. This problem may be simplified if a structure can be represented in the form of a beam with corresponding boundary and loading conditions. This class of structures can include bridges, offshore platforms, pipelines, masts of electricity transmission, TV towers, aircraft wings, blades and rotors of turbine engines, propellers of helicopters and many others.

In earlier works, the solution of the problem of the bending vibrations of a cantilever beam with a closing crack during the "first cycle of vibration was described. It was shown that at the instant of crack opening, the so-called concomitant mode shapes differ from the initially given mode shape. This approach to the solution of the problem can be extended not only over the "first but also over the subsequent cycles.

Therefore, the aim of the study is to develop the algorithm of consecutive (cycle-by-cycle) calculation of cracked beam mode shapes amplitudes, to investigate the regularities of concomitant mode shapes origination, and to study the level of non-linear distortions of the displacement, acceleration and strain waves.

DYNAMIC CHARACTERISTICS OF A BEAM WITH AN OPEN CRACK

Let us consider a cantilever beam of constant rectangular cross-section with a mass on the end. It is well known that the free bending vibrations of such a beam with the damping effect neglected are described by the differential equation.

$$\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 y(x,t)}{\partial t^2} = 0. \tag{1}$$

where E and o are Young's modulus and density of the beam material, respectively, $I=bh^3/12$ and $A=bh$ are the moment of inertia and area of the cross-section, respectively, and b and h are the width and height of cross-section respectively.

The general solution of equation (1) can be presented in a following form:

$$Y(x,t)=\sum_{I=1}^{\infty} w_i(x)(P_i \sin \omega_i t + R_i \cos \omega_i t) \tag{2}$$

Where $w_i(x)$ and ω_i are the mode shapes and natural angular velocities, respectively, and i is the number of the mode shape. The mode shapes of the beam are described by the expression.

$$w_i(x)=A_iS(k_ix) + B_iT(k_ix) + C_iU(k_ix) + D_iV(k_ix), \tag{3}$$

Where

$$K_i=(\omega_i^2\rho A/EI)^{1/4}$$

$S(k_ix)=(\cosh k_ix+\cos k_ix)/2,$

$T(k_ix)=(\sinh k_ix+\sin k_ix)/2,$

$U(k_ix)=(\cosh k_ix - \cos k_ix)/2,$

$V(k_ix)=(\sinh k_ix - \sin k_ix)/2$

Where S, T, U,V are the Krylov functions.

The coeficients A_i , B_i, C_i and D_i in expression (3) are determined from the boundary Conditions

1) $w_i(0) = 0,$
2) $\theta_i(0) = \partial w_i(0)/\partial x = 0$ (4)
3) $M_i(L) = EI\partial^2 w_i(L)/\partial x^2 = I_m \omega_i^2 \partial w_i(L)/\partial x$

4) $Q_i(L) = EI\partial^3 w_i(L)/\partial x^3 = -I_m \omega_i^2 w_i(L)$

Where h is the angle of rotation of the cross-section, M is the bending moment, Q is the transverse force, ¸ is the length of the beam, m_L is the mass on the end, and I_m is the moment of inertia of the mass.

The characteristic equation in this case assumes the form

$$[S(k_iL) - qT(k_iL)][S(k_iL) + gV(k_iL)] - [T(k_iL) - qU(k_iL)][V(k_iL) + gU(k_iL)] = 0, \quad (5)$$

where $q = I_m k_i^3/\rho A$,
$g = m_L k_i/\rho A$.

Taking coefficient C_i to be C_i"$M(0)/EIk_i^2$ one can obtain

$$w_i(x) = M(0)/EIk_i^2\{U(k_ix) - [V(k_iL)+gU(k_iL)/S(k_iL)+gV(k_iL)]V(k_ix)\} \quad (6)$$

The coefficients P_i and R_i in equation (2) are determined by the formulae

$$P_i = w_iF_1 \sin\omega_i t_1 + F_2 \cos \omega_i t_1 /\omega_i[\int m w_i^2(x)\,dx + m_L w_i^2(L) + I_m\theta_i^2(L)]$$

$$R_i = w_iF_1 \sin\omega_i t_1 - F_2 \cos \omega_i t_1 /\omega_i[\int m w_i^2(x)\,dx + m_L w_i^2(L) + I_m\theta_i^2(L)]$$

where $m=\rho A$ is the beam mass per unit length,

$$F_1 = \int m\, y_1(x)\, w_i(x)dx + m_L\, y_1(l)\, w_i(L) + I_m\, \theta_1(L)\, \theta_i(L),$$

$$F_2 = \int m\, v_1(x)\, w_i(x)dx + m_L\, v_1(L)\, w_i(L) + I_m\, [\partial\theta(L,t)/\partial t] = \theta_i(L), \text{ at } t = t_1$$

Taking into consideration the fact that $S(0)=1$, $T(0) = U(0) = V(0) = 0$ and the first two boundary conditions, it can be shown that A_i2"B_i2"0. Residuary boundary and compatibility conditions determine the set of equations.

The solution of characteristic equation enables one to calculate the natural frequencies of the beam with an open crack:

$$\omega_i = k_i^2\sqrt{(EI/\rho A)} \quad (7)$$

When solving the set of equations by the Gauss method the coefficient C_{i3} is taken to be the same as in the case of the crack-free beam and in doing so $M_{i3}(0)=M(0)$. It is also assumed that on the boundaries of the section $j=2$, the cross-sectional moment of inertia is equal to I.

Chapter 2

ANALYTICAL WORK

ANALYTICAL WORK

Let us consider a cantilever beam of constant rectangular cross section with a mass on the end. It is well known that the free bending vibration of such beam with the damping effect neglected by the differential equation

$$\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 y(x,t)}{\partial t^2} = 0.$$

Where

Mass m= 20.4 gram

Density of ball ρ =7.78×10^-6 kg/mm³

Modulus of elasticity E= 206800 N/mm²

Length of cantilever beam L= 500mm

Cross sectional area A= 29×4 =116.10^-4m²

Base b= 29mm

Height h= 4mm

Moment of inertia I=bh³/12 =24×4³=1856×10^-8mm^4

As described before the mode shape of the cantilever beam can be expressed by the expression

$$w_i(x) = A_i S(k_i x) + B_i T(k_i x) + C_i U(k_i x) + D_i V(k_i x),$$

where

$S(k_i x) = (\cosh k_i x + \cos k_i x)/2,$

$T(k_i x) = (\sinh k_i x + \sin k_i x)/2,$

$U(k_i x) = (\cosh k_i x - \cos k_i x)/2,$

$V(k_i x) = (\sinh k_i x - \sin k_i x)/2$

Where S,t,U,V are the Krylov's function

1) Then considering the 1st boundary condition

i.e. $w_i(0) = 0$

$S(k_ix) = 1$

$T(k_ix) = 0$

$U(k_ix) = 0$

$V(k_ix) = 0$

i.e $0 = A_i \times 1 + 0 + 0 + 0$

or $A_i = 0$ (a)

2) Now considering 2nd boundary condition

i.e. $\theta_i(0) = \partial w_i(0)/\partial x = 0$

$\partial S/\partial x = -k_i(\sinh k_ix + \sin k_ix)/2 = 0$

$\partial T/\partial x = k_i(\cosh k_ix + \cos k_ix)/2 = k_i$

$\partial U/\partial x = -k_i(\sinh k_ix - \sin k_ix)/2 = 0$

$\partial U/\partial x = k_i(\cosh k_ix - \cos k_ix)/2 = 0$

i.e $\theta_i(0) = 0 + B_i \times k_i + 0 + 0$

or $B_i = 0$ (b)

3) Now considering 3rd boundary condition

i.e. $M_i(L) = EI\partial^2 w_i(L)/\partial x^2 = I_m \omega_i^2\, \partial w_i(L)/\partial x$
takind 2nd derivative of all kryloves function

$\partial^2 S/\partial x^2 = -k_i^2(\cosh k_ix + \cos k_ix)/2$

$\partial^2 T/\partial x^2 = -k_i^2(\sinh k_ix + \sin k_ix)/2$

$\partial^2 U/\partial x^2 = -k_i^2(\cosh k_ix - \cos k_ix)/2$

$\partial^2 U/\partial x^2 = -k_i^2(\sinh k_ix - \sin k_ix)/2$

As $A_i = B_i = 0$

$M_i(L)/EI = -k_i^2[C_i(\cosh k_ix - \cos k_ix)/2\ ] + k_i^2[D_i(\sinh k_ix - \sin k_ix)/2]$ (c)

4) Now considering the 4th boundary condition

i.e. $Q_i(L) = EI\partial^3 w_i(L)/\partial x^3 = -I_m \omega_i^2 w_i(L)$
taking the third derivative of all the krylov's function

$\partial^3 S/\partial x^3 = k_i^3(\sinh k_ix + \sin k_ix)/2$

$\partial^3 T/\partial x^3 = -\, ki^3(\cosh kix + \cos kix)/2$

$\partial^3 U/\partial x^3 = ki^3(\sinh kix - \sin kix)/2$

$\partial^3 V/\partial x^3 = -ki^3(\cosh kix - \cos kix)/2$

As $Ai = Bi = 0$

i.e $Qi(L)/EI = 0+0+Ci[\, ki^3(\sinh kix - \sin kix)/2] - Di[ki^3(\cosh kix + \cos kix)/2]$

or $2Qi(L)/EI\, ki^3 = Ci(\sinh kix - \sin kix) - Di(\cosh kix + \cos kix)$ (d)

taking equation (c) and (d) and by multiplication of coefficient of Di and adding

$$Ci[(\cosh kix - \cos kix)^2 + (\sinh kix - \sin kix)^2] = 2Qi(L)/EI\, ki^3 \times (\sinh kix - \sin kix) - 2Mi(L)/EI\, ki^2 \times (\cosh kix - \cos kix) \quad (e)$$

AS Qi(L) is the transverse force at the end at the end of the end of the cantilever hence the bending moment will be

$M(L) = Qi(L).L^3/3EI$

Hence equation (e) becomes

$$Ci[(\cosh kix - \cos kix)^2 + (\sinh kix - \sin kix)^2] = 2Qi(L)/EI\, ki^3 \times (\sinh kix - \sin kix)$$
.
$$-2Qi(L).L^3/3E^2I^2\, ki^2(\cosh kix - \cos kix)$$

$$Ci = 2Qi(L)/EI\, ki^2[(\sinh kix - \sin kix)/ki - L^3(\cosh kix - \cos kix)/3EI] / (\cosh kix - \cos kix)^2 + (\sinh kix - \sin kix)^2$$

(A)

Similarly by cross multiplication by coefficient of Ci and substracting

$$Di[(\sinh kix - \sin kix)^2 + (\cosh kix - \cos kix)^2] = 2Mi(L)/EI\, ki^2 \times (\sinh kix - \sin kix) - 2Qi(L)/EI\, ki^3 \times (\cosh kix - \cos kix)$$

$$\text{or } Di = -2Qi(L)/EI\, ki^2 \times [L^3(\sinh kix - \sin kix)/3EI + (\cosh kix - \cos kix)/ki] / +(\sinh kix - \sin kix)^2 + (\cosh kix - \cos kix)^2$$

$$\text{or } Di = -2Qi(L)/EI\, ki^2 \times [(\cosh kix - \cos kix)/ki + L^3(\sinh kix - \sin kix)/3EI] \quad (B)$$

From all these boundary conditions and their subsequences we can get a 4×4 matrix as follow

By simplifying the equation we will get

$-ki/2\times 2Qi(L)/EI\, ki^2\times[(\cosh kix-\cos kix)(\sinh kix-\sin kix)/ki-L^3(\cosh kix-\cos kix)^2/3EI]/$

$(\cosh kix-\cos kix)^2+(\sinh kix-\sin kix)^2$

$=2Qi(0).L^3/3E^2I^2$

Or $L^3[(\cosh kix-\cos kix)^2-(\sinh kix-\sin kix)^2]/3EI=L^3/3EI$

Or $(\sinh kix-\sin kix)^2=-(\sinh kix-\sin kix)^2$

Or $ki=0$ *(C)*

For $ki=0$ *R.H.S = L.H.S*

$Ki=\pi$ *R.H.S = L.H.S*

Hence $Ki=n\pi$, *where* $n=1,2,3,4,5$……….

Since $Ki^4=\omega i^2\rho A/EI$

Where Ki= Stiffness coefficient

ωi=natural frequency

$\omega i^2=Ki^2\sqrt{(EI/\rho A)}$

now taking all the given values and calculated values

for $Ki=\pi$

$\omega i=\pi^2\sqrt{(20.68\times 1856\times 10^{-8}/7.78\times 10^{-12}\times 116\times 10^{-4})}$

or $\omega i=\pi^2\sqrt{42.53\times 10^8}$

or $\omega i=643.64$ *KHz*

for $Ki=2\pi$

$\omega i=4\pi^2\sqrt{42.53\times{}^{\wedge}8}$

$\omega i=2571.97$ *KHz*

for $Ki=3\pi$

$\omega i = 9\pi^2\sqrt{42.53\times^8}$

$\omega i = 5792.81$ KHz

for $Ki = 4\pi$

$\omega i = 16\pi^2\sqrt{42.53\times^8}$

$\omega i = 10298.3$ KHz

and so on

Chapter 3

RESULTS AND DISCUSSION

3.1 RESULTS

3.2 DISCUSSION

3.1 RESULT

From the above calculations we got different frequencies for different valuies of *Ki*

$\omega(0)=0$

$\omega(\pi)= 643.64\ KHz$

$\omega(2\pi)= 2571.97\ KHz$

$\omega(3\pi)= 5792.81\ KHz$

$\omega(4\pi)= 10298.3\ KHz$

DISCUSSION

CRITERION FOR APPLICABILITY OF THE THEORY

The theory presented above is valid if the crack at the corresponding half-cycles is either permanently open or closed. However, when concomitant modes of vibration arise this requirement is not always fulfilled.

ESTIMATION OF VALIDITY OF THE ANALYTICAL APPROACH

The estimation of the validity of the analytical approach was carried out based on the comparison of the results of calculations with the results of laboratory tests of the specimens with fatigue and open cracks. The geometrical characteristics of the specimens are shown in Table 1.

Chapter 5

CONCLUSIONS

CONCLUSION

An analytical approach that enables investigation of dynamic characteristics of a beam with a closing (fatigue) crack is developed. It is shown that in the process of the crack opening the origination of associated mode shapes differing from the initially given mode shape takes place. In the case of the initially given first mode shape (s=1), the amplitudes of higher mode shapes are relatively small. In the case of the initially given second or more higher mode shape, the amplitudes of associated mode shapes under certain conditions can be comparable with the amplitude of the initially given mode shape. The predicted values of natural frequencies and mode shapes for the specimens with a fatigue crack are close to those obtained experimentally, as well as the results of calculation and experimental estimation of distortion of strain and acceleration wave shapes. The verification of the analytical approach with a considerable amount of experimental data and with the results of other author's calculations showed that the analytical approach enables one to obtain well-founded relationships between different dynamic characteristics and crack parameters and to solve the inverse problem of damage diagnostics with sufficient accuracy for practical purposes.

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