STATIC ANALYSIS OF CROSS - PLY LAMINATED COMPOSITE PLATE USING FINITE ELEMENT METHOD

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology in Mechanical Engineering
(Machine Design and Analysis Specialization)

By

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CERTIFICATE

This is to certify that the thesis entitled ***"static analysis of Cross-Ply laminated composite plate using Finite Element Method"*** submitted by Mr. Venkata sai Gopal .K , in partial fulfillment of the requirements for the degree of Master of Technology in Mechanical Engineering with specialization in Machine Design and Analysis during session 2005-2007 in the department of Mechanical Engineering, National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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ABSTRACT

Finite element Analysis is carried out to perform static analysis on a cross-ply laminated composite square plate based on the First order Shear Deformation Theory (FSDT). The theory accounts for constant variation of transverse shear stresses across the thickness of the laminate; and it uses a shear correction factor .The element formulated is an 8-noded iso-parametric quadratic (Serendipity) element. In this analysis, the square plate is analyzed for transverse loading viz., sinusoidal varying load and uniformly distributed load under simply supported boundary conditions. A program is written in MATLAB to obtain the finite element solutions for transverse displacements, normal stresses and transverse shear stresses. Solutions are obtained for 3, 4 and 5 layers of the laminate with cross-ply orientation for different values of side to thickness ratios. Reduced integration scheme is adopted to alleviate shear locking effects. Stresses are found at Gauss points from constitutive relations. The solutions are compared with closed form solutions of FSDT, 3D elasticity solutions and Classical Laminate Plate Theory (CLPT) solutions. It is observed that the results are in close agreement with the available solutions. The element being a C^0 continuous element, it ensures the continuity of generalized displacements only; not strains and thus stresses. The model is validated by its good convergence with the analytical results. Analysis can be done on thin as well as moderately thick plates satisfactorily by using this model.

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Chapter-1

INTRODUCTION

1.1 INTRODUCTION:

Composite materials are increasingly used in aerospace, under water, and automotive structures and space structures. The application of composite materials to engineering components has spurred a major effort to analyze structural components made from them Composite materials provide unique advantages over their metallic counterparts, but they also present complex and challenging problems to analysts and designers. To take advantage of full potential of composite materials, accurate models and design methods are required the most common structural elements are plates and shells. An accurate modeling of stress fields is of paramount importance in the design of such components. Laminated composites are one of the classifications of the composites which are used in structural elements like leaf springs, automobile drive shafts, and gears, and axles.

The Navier, Levy, and Rayleigh-Ritz developed solutions to composite beams and plate problems. However, exact analytical or variational solutions to these problems cannot be developed when complex geometries, arbitrary boundary conditions or nonlinearities ar involved. Therefore one must resort to approximate methods of analysis that are capable of solving such problems. There are several theories available to describe the kinematics of the laminates. Classical Laminate Plate Theory and First order Shear Deformation Theory are some among them.

The Finite element Method is such an approximate method and powerful numerical technique for the solution of differential and integral equations that arise in various fields of engineering and applied science. FEM is an effective method of obtaining numerical solutions to boundary value, initial value and eigen value problems.

1.2Objective of the thesis:

The objective of the present work is to determine transverse displacements, normal stresses, and transverse shear stresses of 3, 4 and 5 layered cross ply laminated composite square plate subjected to transverse loading viz., sinusoidal varying load and uniformly distributed load when it is simply supported at the edges.

In this present work, a displacement based finite element model is formulated based on First order Shear Deformation Theory. It is an iso parametric element with 8 nodes and 3 degrees of freedom at each node. The 3 degrees of freedom are; transverse displacement, rotation about x and rotation about y axes.

Chapter- 2

LITERATURE SURVEY

2.1Literature Survey:

Closed form solutions to 3D elasticity problem of laminated structures are scarce and limited in scope. Pagano developed solutions for simply supported rectangular plates with symmetric lamination undergoing cylindrical and bidirectional bending; the fiber orientation is 0° and 90°. Ren (1987) has extended the cylindrical bending solution to infinitely long cylindrical shells. Noor and Burton (1990a), (1992) have provided solutions for the bending, buckling and vibration of anti-symmetrically laminated rectangular plates, periodic in the in-plane directions. Savithri and Varadan (1992) studied plates under uniformly distributed and concentrated loads. All these approaches use Fourier expansions in the in-plane directions resulting in sets of ordinary differential equations with constant coefficients, which can be solved exactly. The unknown coefficients of the solutions are determined by boundary and interface conditions in thickness direction. J.N.Reddy and W.C.Chao studied and derived Closed form solutions and Finite Element Solutions for Laminated Anisotropic Rectangular Plates.[1]. AA.Khedier and J.N.Reddy derived Exact solutions for bending of thin and thick cross-ply laminated beams[2]. T.J.R.Hughes and T.E.Tezduyar developed Four-Node Bilinear Isoparametric Element based upon Mindlin Plate Theory [3]. The state-space concept in conjunction with the Jordan canonical form is presented to solve the governing equations for the bending of cross-ply laminated composite beams by J.N.Reddy1997[4].An elastic-plastic stress analysis was carried out on simply supported and clamped aluminum metal-matrix laminated plates. The thin plate model is also used for the study of rectangular plates with practically important mixed edge constraints by Liew, Hua & Lim and Laura & Gutierrez.A composite material model is presented to analyze progressive failure in composite structures by Raimondo Luciano,Raffaele Zinno in 2000. Song Cen, Zhen-Han Yao developed a new 4-node quadrilateral finite element for the analysis of composite plates in 2002. A new analytical method was developed by M.R.Khalili and R.K.Mittal to analyze the response of laminated composite plates subjected to static and dynamic loading.(2005).B.N.Pandya and T.Kant worked on Finite Element analysis of Laminated Composite Plates Using a Higher – Order Displacement Model [5].B.R.Somashekar,G.Pratap and C.Ramesh Babu developed a simple and efficient Four nodded, Laminated Anisotropic Plate Element[6]. Xiao-Ping Shu,Kostas P.Soldatos determined Stress distributions in angle-ply laminated plates subjected to cylindrical bending[7].

Exact solutions for rectangular bidirectional composites and sandwich plates were developed by Pagano,N.J.,[8].Reddy, J.N., Khdeir, A.A. developed Levy type solutions for symmetrically laminated rectangular plates using first order shear deformation theory[9].An exact approach to the elastic stat of stress of shear deformable antisymmetic angle ply laminated plates was developed by Khedeir A. A., [10]. He also compared shear deformable and kirchoff theories for bending buckling and vibration of antisymmetric angle ply laminated plates.[11].Srinivas and Jogarao got some results from exact analysis of thick laminates in vibration and buckling[12]. A review is made on plate bending finite elements by Hrabok, M.M. and Hrudey [13].Fraeijis de veubeke developed a conforming finite element for plate bending [14]. A triangular refined plate bending element was suggested by Bell K. [15]. Irons B.M. developed a conforming quartic triangular element for plate bending [16].Strcklin, J.a. haisler, w developed a rapidly converging triangular plate element [17]. A study was made on 3 node triangular plate bending element by Batoz, J.L and Bathe, K.J. [18].

Chapter-3

FIRST ORDER SHEAR DEFORMATION THEORY

3.1Introduction:

The use of composite materials in structural components is increasing due to their attractive properties such as high strength-to-weight ratio, ability to tailor the structural properties, etc. Plate structures find numerous applications in the aerospace, military and automotive industries. The effects of transverse shear deformation are considerable for composite structures, because of their high ratio of extensional modulus to transverse shear modulus. Most of the structural theories used to characterize the behavior of composite laminates fall into the category of equivalent single layer (ESL) theories. In these theories, the material properties of the constituent layers are combined to form a hypothetical single layer whose properties are equivalent to through the thickness integrated sum of its constituents. This category of theories has been found to be adequate in predicting global response characteristics of laminates, like maximum deflections, maximum stresses, and fundamental frequencies, or critical buckling loads

In the context of ESL theories, the simplest one is the CLT which neglects the shear contribution in the laminate thickness. However, flat structures made of fiber-reinforced composite materials are characterized by non negligible shear deformations in the thickness direction, since the longitudinal elastic modulus of the lamina can much higher than the shear and the transversal moduli; hence the use of a shear deformation laminate theory is recommended. The extension of the Reissner and Mindlin model to the case of laminated anisotropic plates, i.e. FSDT ,accounts for shear deformation along the thickness in the simplest way. It gives satisfactory results for a wide class of structural problems, even for moderately thick laminates, requiring only $C0$-continuity for the displacement field. The transverse shearing strains (stresses) are assumed to be constant along the plate thickness so that stress boundary conditions on the top and the bottom of the plate are violated; *shear correction factors* must be introduced. The determination of shear correction factors is not a trivial task, since they depend both on the lamination sequence and on the state of deformation .

Assumptions:

1) The layers are perfectly bonded
2) The material of each layer is linearly elastic and has two planes of material symmetry
3) The strains and displacements are small
4) Deflection is wholly due to bending strains only
5) Plane sections originally perpendicular to the longitudinal plane of the plate remain plane, but not necessarily perpendicular to longitudinal plane
6) The transverse shearing strains (stresses) are assumed to be constant along the plate thickness

3.2Kinematic relations:

The displacement field of the first-order theory is of the form

$$u(x, y, z, t) = u_0(x, y, t) + z\phi_x(x, y, t)$$
$$v(x, y, z, t) = v_0(x, y, t) + z\phi_y(x, y, t) \quad \text{-----------------------} (1)$$
$$w(x, y, z, t) = w_0(x, y, t)$$

Where $(u_0, v_0, w_0, \phi_x, \phi_y)$ are unknown functions, called the *generalized displacements.* (u_0, v_0, w_0), denote the displacements of a point on the plane z = 0.

$$\frac{\partial u}{\partial z} = \phi_x, \frac{\partial v}{\partial z} = \phi_y$$

Indicate that ϕ_x and ϕ_y are the rotations of the transverse normal about the y- and x- axes, respectively, owing to bending only.

The strains associated with the displacement field (1) are given by:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y}, \varepsilon_{zz} = \frac{\partial w}{\partial z}, \gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \gamma_{xz} = \frac{\partial w}{\partial x} + \phi_x, \gamma_{yz} = \frac{\partial w}{\partial y} + \phi_y \text{-------}(2)$$

Substituting the expressions for u,v,w from eq. (1) in equation (2) gives:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} + z\frac{\partial \phi_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial v_0}{\partial y} + z\frac{\partial \phi_y}{\partial y}, \quad \varepsilon_{zz} = 0$$

$$\gamma_{xy} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right), \gamma_{xz} = \frac{\partial w_0}{\partial x} + \phi_x, \gamma_{yz} = \frac{\partial w_0}{\partial y} + \phi_y \text{--------------------} (3)$$

In Matrix form the above equations are given by

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial w_0}{\partial x} + \phi_x \\ \dfrac{\partial w_0}{\partial y} + \phi_y \end{Bmatrix} + z \begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial \phi_y}{\partial y} \\ \left(\dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \right) \\ 0 \\ 0 \end{Bmatrix}$$

The above matrix is in the form:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{xx} \\ \gamma^0_{xy} \\ \gamma^0_{xz} \\ \gamma^0_{yz} \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \\ 0 \\ 0 \end{Bmatrix}$$

For *plate bending* problem, the in- plane displacements (u,v) are uncoupled from (w_0, ϕ_x, ϕ_y). Hence, the equations reduce as follows:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} zk_x \\ zk_y \\ zk_{xy} \\ \phi_x + \dfrac{\partial w_0}{\partial x} \\ \phi_y + \dfrac{\partial w_0}{\partial y} \end{Bmatrix} = \begin{Bmatrix} z\dfrac{\partial \phi_x}{\partial x} \\ z\dfrac{\partial \phi_y}{\partial y} \\ z\left(\dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \right) \\ \phi_x + \dfrac{\partial w_0}{\partial x} \\ \phi_y + \dfrac{\partial w_0}{\partial y} \end{Bmatrix} \text{------------------(4)}$$

3.3Constitutive Relations:

The Stress-Strain relations for a typical lamina k with reference to the lamina co-ordinate axes (1-2-3) are given by

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}^k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}^k$$

$$\begin{Bmatrix} \tau_{23} \\ \tau_{13} \end{Bmatrix}^k = K \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix}^k \begin{Bmatrix} \gamma_{23} \\ \gamma_{13} \end{Bmatrix}^k \text{---------------------(5)}$$

In which $(\sigma_1, \sigma_2, \tau_{12}, \tau_{23}, \tau_{13})$ are the stress and $(\varepsilon_1, \varepsilon_2, \gamma_{12}, \gamma_{23}, \gamma_{13})$ are the linear strain components referred to the lamina co-ordinate axes (1-2-3). The Q_{ij}'s are the plane stress reduced elastic constants of the k^{th} lamina and the following relations hold between these and the engineering constants.

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} \quad , \quad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \quad , \quad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} \quad ,$$

$$Q_{33} = G_{12} \ , \quad Q_{44} = G_{23} \ , \quad Q_{55} = G_{13} \ .$$

The stress-strain relations are the basis for the stiffness and stress analysis of an individual lamina subjected to forces in its own plane. The relations are therefore indispensable in the analysis of laminates. There are 4 independent material properties, E_1, E_2, G_{12}, and ν_{12} the reciprocal relation is given by

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

Stress – Strain relations for a lamina of arbitrary orientation

From elementary mechanics of materials the transformation equations for expressing stresses in 1-2 coordinate system(principal coordinate system) in terms of stresses in x-y coordinate system.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \begin{Bmatrix} \sigma_X \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \text{-------------------(6)}$$

Where θ is the angle from the x-axis to the axis 1. Following the usual transformation of Stress-Strain between the lamina and laminate coordinate systems, the Stress-Strain relations for the kth lamina in the laminate coordinates (x,y,z) are written as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{13}} \\ \overline{Q_{21}} & \overline{Q_{22}} & \overline{Q_{23}} \\ \overline{Q_{31}} & \overline{Q_{32}} & \overline{Q_{33}} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^k$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k = K \begin{bmatrix} \overline{Q_{44}} & \overline{Q_{45}} \\ \overline{Q_{54}} & \overline{Q_{55}} \end{bmatrix}^k \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^k \text{-----------------------(7)}$$

in which , $\sigma = \left(\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz}\right)^t$ and $\varepsilon = \left(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}\right)^t$ are the stress and linear strain vectors with respect to the laminate axes and $\overline{Q_{ij}}$ s are the plane stress reduced elastic constants in the plate (laminate) axes of the k[th] lamina given in **Appendix**. The superscript t denotes the transpose of a matrix. *K* refers to the Shear Correction Factor used in FSDT. Normally its value is 5/6.

Stress Resultants:

The resultant forces and moments acting on a laminate are obtained by integration of the stresses in each layer or lamina through the laminate thickness, for example,

$$N_x = \int_{-t/2}^{t/2} \sigma_x dz \qquad M_x = \int_{-t/2}^{t/2} \sigma_x z dz$$

Actually, N_x is a force per unit length (width) of the cross section of the laminate. Similarly, M_x is a moment per unit length. The entire collection of force and moment resultants for an *N*-layered laminate is shown and is defined as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k dz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k dz$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k zdz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k zdz \quad \text{-----------------(8)}$$

Where, Z_k and Z_{k-1} are defined in figure shown in appendix. $Z_0 = -t/2$.These force and moment resultants do not depend on Z after integration, but are functions of x and y, the coordinates in the plane of the laminate middle surface.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{13}} \\ \overline{Q_{21}} & \overline{Q_{22}} & \overline{Q_{23}} \\ \overline{Q_{31}} & \overline{Q_{32}} & \overline{Q_{33}} \end{bmatrix}^k \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon^0{}_x \\ \varepsilon^0{}_y \\ \gamma^0{}_{xy} \end{Bmatrix} dz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} zdz \right\}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{13}} \\ \overline{Q_{21}} & \overline{Q_{22}} & \overline{Q_{23}} \\ \overline{Q_{31}} & \overline{Q_{32}} & \overline{Q_{33}} \end{bmatrix}^k \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon^0{}_x \\ \varepsilon^0{}_y \\ \gamma^0{}_{xy} \end{Bmatrix} zdz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} z^2dz \right\}$$

Thus the above equations can be written as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon^0{}_x \\ \varepsilon^0{}_y \\ \gamma^0{}_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon^0{}_x \\ \varepsilon^0{}_y \\ \gamma^0{}_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

Where

$$A_{ij} = \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \left(Z_k - Z_{k-1}\right) \quad B_{ij} = \frac{1}{2} \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \left(Z^2{}_k - Z^2{}_{k-1}\right) \quad D_{ij} = \frac{1}{3} \sum_{k=1}^{N} \left(\overline{Q_{ij}}\right)_k \left(Z^3{}_k - Z^3{}_{k-1}\right) \quad \text{---(9)}$$

Where

A_{ij} Coefficients are called extensional stiff nesses,

B_{ij} Coefficients are called coupling stiff nesses,

D_{ij} Coefficients are called bending stiff nesses.

In this present work , as only cross ply laminates only analyzed, the Bij terms get vanished. D16=D26=0.

As there present no inplane forces, they are uncoupled from the equations. So only D matrix is used in this plate bending analysis.

<u>3.4Virtual Work Statement</u>

The variational formuations form a powerful basis for obtaining approximate solutions to real world/practical problems. The variational method uses the variational principles, such as the Principle of Virtual displacements, to determine approximate displacements as continuous functions of position in the domain. In the Classical sense, variational principle has to do with the minimization of a functional, which includes all the intrinsic features of the problem, such as the governing equations, boundary and /or initial conditions, and constraint conditions.

One of the concepts of Variational formulation is Principle of virtual work. It is the work done on a particle or a deformable body by actual forces in displacing the particle or the body through a hypothetical displacement that is consistent with the geometric constraints. The applied forces are kept constant during the virtual displacement. The Principle of virtual displacement states that the virtual work done by actual forces in moving through virtual displacements is zero if the body is in equilibrium.

The principle of virtual work states that "*a continuous body is in equilibrium if the virtual work of all forces acting on the body is zero in a virtual displacement*". The principle of virtual work is independent of any constitutive law and applies to elastic (linear and non linear) and in elastic continuum problems. The plate to be analyzed may have curved or straight boundaries as well as different boundary conditions. The principle of virtual work statement for the plate can be stated as

$$\delta W_I + \delta W_E = 0$$

Where

δW_I = Virtual work resulting from internal forces

δW_E = Virtual work resulting from external forces

The governing equations of the first order shear deformation theory are derived using the dynamic version of the principle of virtual displacements for the displacements (w, ϕ_x, ϕ_y):

$$0 = \int_0^T \left(\delta K - (\delta U + \delta V)\right) dt \text{ --------------(10)}$$

Here $\delta W_I = \delta U$ and

$\delta W_E = \delta K + \delta V$

Where δU is virtual strain energy, δV is virtual work done by applied forces and δK is virtual kinetic energy. On substituting the expressions for δU, δV and δK in the equation noA. we get

$$0 = \int_{V_e} \begin{pmatrix} \left\{ \rho z^2 \delta\phi_x \dfrac{\partial^2 \phi_x}{\partial t} + \rho z^2 \delta\phi_y \dfrac{\partial^2 \phi_y}{\partial t} + \rho \delta w \dfrac{\partial^2 w}{\partial t} \right\} \\ + \left\{ \delta\varepsilon_{xx}\sigma_{xx} + \delta\varepsilon_{yy}\sigma_{yy} + 2\delta\varepsilon_{xy}\sigma_{xy} + 2\delta\varepsilon_{xz}\sigma_{xz} + 2\delta\varepsilon_{yz}\sigma_{yz} \right\} \\ - \left\{ \displaystyle\int_{\Omega e} \delta w q \, dx dy \right\} \end{pmatrix}$$

Carrying out integration with respect to z, we get

$$\begin{aligned} 0 = \int_{\Omega e} [& \int_{-h/2}^{h/2} \rho z^2 \delta\phi_x \frac{\partial^2 \phi_x}{\partial t^2} + \int_{-h/2}^{h/2} \rho z^2 \delta\phi_y \frac{\partial^2 \phi_y}{\partial t^2} + \int_{-h/2}^{h/2} \rho \delta w \frac{\partial^2 w}{\partial t^2} \\ & + \int_{-h/2}^{h/2} \frac{\partial \delta\phi_x}{\partial x} \sigma_{xx} z + \int_{-h/2}^{h/2} \frac{\partial \delta\phi_y}{\partial y} \sigma_{yy} z + \int_{-h/2}^{h/2} \left(\frac{\partial \delta\phi_x}{\partial y} + \frac{\partial \delta\phi_y}{\partial x} \right) \sigma_{xy} z \\ & + \int_{-h/2}^{h/2} \left(\delta\phi_x + \frac{\partial \delta w}{\partial x} \right) \sigma_{xz} z + \int_{-h/2}^{h/2} \left(\delta\phi_y + \frac{\partial \delta w}{\partial y} \right) \sigma_{yz} z - \int_{-h/2}^{h/2} q \delta w] dx dy dz \end{aligned}$$

On simplification the above equation yields to

$$0=\int_{\Omega e}\left[\begin{array}{l} I_0\delta w\frac{\partial^2 w}{\partial t^2}+I_2\left(\delta\phi_x\frac{\partial^2\phi_x}{\partial t^2}+\delta\phi_y\frac{\partial^2\phi_y}{\partial t^2}\right)+M_{xx}\frac{\partial\delta\phi_x}{\partial x}+M_{yy}\frac{\partial\delta\phi_y}{\partial y} \\ +M_{xy}\left(\frac{\partial\delta\phi_x}{\partial y}+\frac{\partial\delta\phi_y}{\partial x}\right)+Q_x\left(\delta\phi_x+\frac{\partial\delta w}{\partial x}\right)+Q_y\left(\delta\phi_y+\frac{\partial\delta w}{\partial y}\right)-q\delta w \end{array}\right]dxdy$$

Where

$$I_0=\int_{-h/2}^{h/2}\rho dz\ ,\quad I_2=\int_{-h/2}^{h/2}\rho z^2 dz\ ,\quad Q_x=K\int_{-h/2}^{h/2}\sigma_{xz}dz\ ,\quad Q_y=K\int_{-h/2}^{h/2}\sigma_{yz}dz\ ,$$

$$M_{xx}=\int_{-h/2}^{h/2}\sigma_{xx}zdz\quad ,\quad M_{yy}=\int_{-h/2}^{h/2}\sigma_{yy}zdz\quad ,\quad M_{xy}=\int_{-h/2}^{h/2}\sigma_{xy}zdz$$

Where, I_0 is the mass moment of inertia term,

I_2 is the rotary inertia term and

K is the shear correction factor.

For *Static case*, the above virtual displacement equation becomes,

$$0=\int_{\Omega e}\left[M_{xx}\frac{\partial\delta\phi_x}{\partial x}+M_{yy}\frac{\partial\delta\phi_y}{\partial y}+M_{xy}\left(\frac{\partial\delta\phi_x}{\partial y}+\frac{\partial\delta\phi_y}{\partial x}\right)+Q_x\left(\delta\phi_x+\frac{\partial\delta w}{\partial x}\right)+Q_y\left(\delta\phi_y+\frac{\partial\delta w}{\partial y}\right)-q\delta w\right]dxdy$$

The Virtual Work Statement contains 3 weak forms for the 3 displacements (w, ϕ_x, ϕ_y).They are identified by collecting the terms involving δw , $\delta\phi_x$ and $\delta\ \phi_y$ separately and equating them to zero:

$$0=\int_{\Omega e}\left(Q_x\frac{\partial\delta w}{\partial x}+Q_y\frac{\partial\delta w}{\partial y}-q\delta w\right)dxdy$$

$$0=\int_{\Omega e}\left(M_{xx}\frac{\partial\delta\phi_x}{\partial x}+M_{xy}\frac{\partial\delta\phi_x}{\partial y}+Q_x\delta\phi_x\right)dxdy$$

$$0=\int_{\Omega e}\left(M_{xy}\frac{\partial\delta\phi_y}{\partial x}+M_{yy}\frac{\partial\delta\phi_y}{\partial y}+Q_y\delta\phi_y\right)dxdy$$

The governing equations of FSDT are obtained from the weak forms given above

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0$$

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0 \quad \text{------------------(11)}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = 0$$

Chapter-4

FINITE ELEMENT METHOD

4.1Introduction:

The finite Element Method is a powerful computational technique for the solution of differential and integral equations that arise in various fields of engineering and applied science. The method is a generalization of the Classical Variational (i.e., the Rayleith-Ritz) and weighted – residual (Galerkin, Least-squares etc.)methods. Since most real-world problems are defined on domains that are geometrically complex and may have different typers of boundary conditions on different portions of the boundary of the domain, it is difficult to generate approximation functions required in the traditional variational methods. The basic idea of the finite element method is to view a given domain as an assemblage of simple geometric shapes called finite elements, for which it is possible to systematically generate the approximation functions needed in the solution methods. The ability to represent domains with irregular geometries by a collection of finite elements makes the method a valuable practical tool for the solution of boundary, initial, and eigen value problems arising in various fields of engineering. The approximation functions are often constructed using ideas from interpolation theory, and hence they are also called interpolation functions. Thus, the finite element method is a piecewise application of the variational and weighted –residual methods.

Finite Element Modelling

In the FEM, the total solution domain is discretized in to N elements (sub-domains).Then, the finite element model of the problem is developed using variational method .The variational formuations form a powerful basis for obtaining approximate solutions to practical problems. The variational method uses the variational principles, such as the Principle of Virtual displacements, to determine approximate displacements as continuous functions of position in the domain. In the Classical sense, variational principle has to do with the minimization of a functional, which includes all the intrinsic features of the problem, such as the governing equations, boundary and /or initial conditions, and constraint conditions.

4.2Principle of Virtual Displacements: It states that "a deformable body is in equilibrium if the total external virtual work is equal to the total internal virtual work for every virtual displacement satisfying the kinematic boundary conditions".

$$\delta W_I = \delta W_E$$

Where

δW_I = Virtual work resulting from internal forces

δW_E = Virtual work resulting from external forces

The Principle of Virtual work for the plate can be stated as

$$\int_V \delta\varepsilon^T \sigma dv + \int_V \delta\gamma^T \tau dv = \int_V q\delta d dA \text{------------(12)}$$

The integration over the thickness reduces eq 12 as follows:

$$\int_A \left\{ \delta\varepsilon_{xx}^T \sigma_{xx} + \delta\varepsilon_{yy}^T \sigma_{yy} + \delta\gamma_{xy}^T \sigma_{xy} + \delta\gamma_{xz}^T \sigma_{xz} + \delta\gamma_{yz}^T \sigma_{yz} \right\} dA \text{-------------------(13)}$$

Where A is the cross-sectional area and V is the volume of the plate. Using the lamina constitutive relation eq 12 leads to the following form:

$$\int_A \left[\varepsilon^t \{M\} + \Phi^t \{Q\} \right] dA$$

Replacing the stress resultants by the product of rigidity matrix and strains in the strain energy expression in equation-------------, we get

$$\int_A \left\{ \delta\varepsilon_{xx}^T D_B \varepsilon_{xx} + \delta\varepsilon_{yy}^T D_B \varepsilon_{yy} + \delta\gamma_{xy}^T D_B \gamma_{xy} + \delta\gamma_{xz}^T D_s \gamma_{xz} + \delta\gamma_{yz}^T D_s \gamma_{yz} \right\} dA \text{-----------------(14)}$$

Which is the final form of the virtual work principle as it is required for finite element calculations.

4.3Stiffness matrix derivation :

In this work, an eight noded isoparametric element (Serendipity Element) is chosen to discretize the plate domain. The variation of displacement u is expressed by the polynomial in natural coordinates as:

$$u = \alpha_1 + \alpha_2 r + \alpha_3 s + \alpha_4 r^2 + \alpha_5 rs + \alpha_6 s^2 + \alpha_7 r^2 s + \alpha_8 rs^2 \text{ ----------(15)}$$

In the above polynomial, cubic terms have been omitted.

The nodal displacement vector $\{d\}$ is obtained by substituting the coordinates for the nodes as:

$$\{d\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 \\ 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{Bmatrix}$$

$$\{\alpha\} = [A]^{-1}\{d\}$$

where $[A]^{-1}$ is as given below:

$$\text{And } [A]^{-1} = \frac{1}{4}\begin{bmatrix} -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 & -2 & 0 & -2 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & -2 & 0 & -2 \\ -1 & -1 & 1 & 1 & 2 & 0 & -2 & 0 \\ -1 & 1 & 1 & -1 & 0 & -2 & 0 & 2 \end{bmatrix}$$

$$\{\delta\}^T = [1, r, s, r^2, rs, s^2, r^2 s, rs^2]$$

$$[A]^{-1} = \frac{1}{4}\begin{bmatrix} -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 & -2 & 0 & -2 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & -2 & 0 & -2 \\ -1 & -1 & 1 & 1 & 2 & 0 & -2 & 0 \\ -1 & 1 & 1 & -1 & 0 & -2 & 0 & 2 \end{bmatrix}$$

Let δ be expressed as $\{\delta\}^T = [1, r, s, r^2, rs, s^2, r^2s, rs^2]$ then, the shape functions for this element are given by

$[N]^T = \{\delta\}^T [A]^{-1}$ which is given as

$$[N] = \frac{1}{4}\begin{Bmatrix} (1-r)(1-s)(-r-s-1) \\ (1+r)(1-s)(r-s-1) \\ (1+r)(1+s)(r+s-1) \\ (1-r)(1+s)(-r+s-1) \\ 2(1+r)(1-r)(1-s) \\ 2(1+r)(1+s)(1-s) \\ 2(1+r)(1-r)(1+s) \\ 2(1-r)(1+s)(1-s) \end{Bmatrix} \text{------------------(16)}$$

The shape functions can be expressed in concise form as follows

$[N]^T = \frac{1}{4}[N_1 N_2 N_3 N_4 N_5 N_6 N_7 N_8]$

In the present work, of laminated plate bending, the transverse displacement w, the rotation about x-axis ϕ_x and the rotation about y-axis ϕ_y are considered as the only 3 degrees of freedom at each node of the element. The in- plane displacements (u, v) are neglected in this study.

The displacement vector *d* is given as

$$d = (w, \phi_x, \phi_y)$$

The generalized displacements at any point (x, y) in the element are expressed in terms of the nodal values of displacements and shape functions as given below:

$$w = \sum_{i=1}^{8} N_i w_i \qquad \phi_x = \sum_{i=1}^{8} N_i \phi_{xi} \qquad \phi_y = \sum_{i=1}^{8} N_i \phi_{yi}$$

Adopting the same shape function 'N' to define all the components of the generalized displacement vector, d, we can write

$$d = \sum_{i=1}^{N} N_i d_i \text{ -----------(17)}$$

The nodal displacements are given by

$$\{d\}^T = \left[w_1\theta_{x1}\theta_{y1}w_2\theta_{x2}\theta_{y2}w_3\theta_{x3}\theta_{y3}w_4\theta_{x4}\theta_{y4}w_5\theta_{x5}\theta_{y5}w_6\theta_{x6}\theta_{y6}w_7\theta_{x7}\theta_{y7}w_8\theta_{x8}\theta_{y8} \right]$$

In which, N is the number of nodes in the element. Now, referring to the expressions in equation (4), the bending curvatures and the shear strains can be written in terms of nodal displacements d using the matrix notation as follows:

$$\{\varepsilon\} = L_B d \qquad \{\Phi\} = L_S d$$

In which the subscripts B and S refer to bending and shear respectively and the matrices L_b And L_s attain the following form

$$L_B = \begin{bmatrix} 0 & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}$$

$$L_S = \begin{bmatrix} \frac{\partial}{\partial x} & 1 & 0 \\ \frac{\partial}{\partial y} & 0 & 1 \end{bmatrix} \text{-------(18)}$$

Knowing the generalized displacement vector, d, at all points within the element, the generalized strain vectors at any point are determined with the aid of equations (17) and (18) as follows:

$$\{\varepsilon\} = L_B d = L_B \sum_{i=1}^{N} N_i d_i = \sum_{i=1}^{N} B_{iB} d_i = B_B a$$
$$\{\Phi\} = L_S d = L_S \sum_{i=1}^{N} N_i d_i = \sum_{i=1}^{N} B_{iS} d_i = B_S a \qquad \text{----------------------------(19)}$$

In which

The B matrix for the i th node can be written as

$$B_i = \begin{bmatrix} B_{iB} \\ B_{iS} \end{bmatrix}$$

$$\left[B_{iB}\right] = [L_B][N_i] = \begin{bmatrix} 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} \\ 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \quad , \quad B_B = \sum_{i=1}^{N} B_{iB}$$

$$[B_{iS}] = [L_S][N_i] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 1 & 0 \\ \frac{\partial N_i}{\partial y} & 0 & 1 \end{bmatrix} \quad , \quad B_S = \sum_{i=1}^{N} B_{iS}$$

and $a = (d_1^T, d_2^T, d_3^T, \ldots\ldots\ldots\ldots, d_N^T)$

Substituting the above strain-displacement matrix B, in the virtual work statement derived above results in

$$\int_A \left[a^t B_B^t D_B B_B a + a^t B_s^t D_s B_s a \right] dA \qquad \text{or} \qquad \int_A (a^t K^e a) dA$$

In which K^e is the element stiffness matrix and is expressed as

$$K^e = \int_A \left[B_B^t D_B B_B + B_s^t D_s B_s \right] dA$$

Because of the symmetry of the stiffness matrix, only the blocks *Kij* lying on one side of the main diagonal are formed for simplification. The integral is evaluated numerically using the Gauss quadrature rule, in the limits of -1 to +1

$$K_{ij}^e = \int_{-1}^{1} \int_{-1}^{1} B_i^t D B_j \left| J \right| dr ds$$

$$K_{ij}^e = \sum_{a=1}^{g}\sum_{b=1}^{g} W_a W_b B_i^t D B_j \left|J\right|$$

$$W_e = a^t F_c + a^t \int_A (N_i^T q + N_i^T P) dA$$

$$P_i = \sum_{a=1}^{g}\sum_{b=1}^{g} W_a W_b \left|J\right| N_i^T \{100\}^T \left(q + P_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \right)$$

Where, *Wa* and *Wb* are the weights of the Gauss points determined using Legendre Polynomials. g's are the Gauss sampling points at which numerical integration is carried out. $\left|J\right|$ is the determinant of the Jacobian matrix [*J*].Subscripts i and j vary from one to the number of odes per element. The matrices *Bi* and *D* are given above and *Bj* is obtained by replacing *i* by *j*.

For this flexural analysis , the total external work done by the applied external loads for an element e, is given by

$$W_e = a^t F_c + a^t \int_A (N_i^T q + N_i^T P) dA$$

In which suffix, *i* , varies from one to number of nodes per element. *Fc* is the vector of concentrated nodal loads corresponding to nodal degrees-of-freedom. *q* and *P* are the uniform and sinusoidal distributed load intensities acting over an element e in the z-direction.

The integral of the above equation is evaluated numerically using the Gauss quadrature rule as follows

$P_i = \sum_{a=1}^{g}\sum_{b=1}^{g} W_a W_b \left|J\right| N_i^T \{100\}^T \left(q + P_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \right)$ in which a and b are the plate dimensions, x and y are the Gauss point coordinates and m and n ar the usual harmonic numbers.

Present work:

In the present study of Cross-Ply laminated composite plate, using finite element method, a square laminated composite plate is taken as focus of study. Let, the side of the square plate is ‘a’ unit. And the Laminate consists of ‘N’ number of laminas. The laminas have either 0 degree or 90 degree orientation with respect to the material coordinates, i.e., the lamina’s are cross-plied.

Here, the laminated square plate is considered as the domain. The domain is discretized in to sub-domains/finite elements using 8- noded isoparametric quadratic element (Serendipity Element). As the Plate is symmetric about both the axes in its plane, a quarter of the plate is considered for the study. This quarter plate model is again discretized with 2×2 mesh. So, there are 4 numbers of elements in the quarter plate. The 2×2 mesh in quarter plate model is equivalent to full plate model with 4×4 mesh.

After discretizing the domain into sub-domains, the finite element model of the problem is developed using classical variational method as explained above. Finally, the element stiffness matrix is obtained as:

$$[K^e] = \begin{bmatrix} K^{11} & K^{12} & K^{13} \\ & K^{22} & K^{23} \\ & & K^{33} \end{bmatrix}$$

Where,

$$K_{ij}^{11} = \int_A \left(A_{55} \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + A_{44} \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) dxdy$$

$$K_{ij}^{12} = \int_A \left(A_{55} \frac{\partial N_i}{\partial x} N_j \right) dxdy$$

$$K_{ij}^{13} = \int_A \left(A_{44} \frac{\partial N_i}{\partial y} N_j \right) dxdy$$

$$K_{ij}^{22} = \int_A \left(D_{11} \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + D_{33} \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + A_{55} N_i N_j \right) dxdy$$

$$K_{ij}^{23} = \int_A \left(D_{12} \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial y} + D_{33} \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial x} \right) dxdy$$

$$K_{ij}^{33} = \int_A \left(D_{33} \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + D_{22} \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + A_{44} N_i N_j \right) dxdy$$

Because of the symmetry of the stiffness matrix, only the blocks *Kij* lying on one side of the main diagonal are formed for simplification.

Subscripts i and j vary from one to the number of nodes per element. The integral is evaluated numerically using the Gauss quadrature rule, in the limits of -1 to +1.

$$K_{ij}^{e} = \int_{-1}^{1} \int_{-1}^{1} B_i^t D B_j \left| J \right| drds$$

$$K_{ij}^{e} = \sum_{a=1}^{g} \sum_{b=1}^{g} W_a W_b B_i^t D B_j \left| J \right|$$

$$\overline{\sigma_{yz}} = 0$$

$$\phi_x = 0$$

$$\phi_y = 0$$

Here, the bending stiffness and shear stiffness values are evaluated separately, to avoid shear locking of problem. The bending terms are evaluated using 3×3 order of integration i.e. at 9 sampling points and the shear terms are evaluated using 2×2 order of integration i.e., at 4 sampling points for each element. Reduced integration scheme is adopted for shear terms.

The values of sampling points and weights for each order are given as below:

For 3×3 order, First weight is 0.8888888889 and the corresponding sampling point is 0
The other weight is 0.555555555 and the corresponding sampling point is +/- 0.7745966692
For 2×2 order, the weights are 1 and sampling points are +/- 0.5773502692

Since there are 3 degrees of freedom per node of an element viz., transverse displacement w, rotation about x- axis ϕ_x and rotation about y- axis ϕ_y and the domain is discretized in to 2×2 meshes i.e., there are 4 elements present. And there are a total of 21 nodes and correspondingly, 21×3=63 degrees of freedom will be present. After getting the

Element stiffness matrix for all the elements, they are assembled to obtain the Global stiffness matrix. Therefore, there present 63 simultaneous equations in [K][d]=[Q] form.

When the assembly of the element stiffness matrices is over, the boundary conditions are applied at the boundaries of the plate. Initially the plate is simply supported on all the edges of the plate. Since, a quarter plate model is considered for the analysis, only two of the edges are simply supported and the remaining are free. The applied boundary conditions are described as below:

$$w = 0 \quad \text{at x=0 and y=0}$$

$$\phi_x = 0 \quad \text{at x=0 and y= 0}$$

$$\phi_y = 0 \quad \text{at x=0 and y=0}$$

After applying the boundary conditions, the plate is transversely loaded. The loading can be with a uniformly distributed load of magnitude 'q' units and a sinusoidal varying load on the plate surface acting individually. So, the stiffness matrix after applying boundary conditions is reduced from 63×63 matrix to 44×44 matrix. So, there are a total of 44 simultaneous equations present that are to be solved in the reduced stiffness matrix. These equations are solved to obtain the displacement vector matrix [d] like, $[d] = [K]^{-1}[Q]$.The inverse of the reduced stiffness matrix and the solution to obtain generalized displacement vector is carried out using the program written in **MATLAB**.

4.4Post computation of Stresses and Strains:

Once the generalized displacements at the nodes are determined by solving the assembled equations of the problem , the transverse displacements and slopes at any (x,y) can be calculated using eq-----------.Strains at any point (x , y, z) in a typical element 'e' can be computed from the strain-displacement relations stated above. It is to be noted here that, only displacements are continuous across the element boundaries. Strain continuity across the boundaries is not ensured as we are using a C^0 continuous element. That is, along a boundary common to two elements, the strains and hence stresses take different values on the two sides of the interface. However, strains and hence stresses are continuous within an element. The stesses can be calculated using the constitutive relations stated above.

Since the displacements in the finite element models are referred to the global coordinates (x,y,z), the stresses are computed in the global coordinates using the relations at the sampling points ; not at the nodes. The stresses can be transformed to principal material coordinates using the stress transformation relations. Similarly, strains can also be transformed to principle material coordinates.

Chapter-5

RESULTS AND DISCUSSION

5.1Results :

Numerical results are obtained for a specific problem whose data is given below:

Material: Graphite-Epoxy composite with material properties

E1=175 GPa

E2=7 GPa

G12=G13=0.5 E2=3.5 GPa

G23=0.2 E2= 1.4 GPa

ν 12=0.25

Shear correction factor K=5/6

Boundary conditions: Simply supported on all edges

Loading : a)Sinusiodal varying load and

b) Uniformly distributed load acting individually

The Non-dimensionalized displacement, and stresses results are tabulated and given below.

Side to Thickness ratio a/h	Type of solution	$\bar{w}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\sigma_{xy}}$	$\overline{\sigma_{xz}}$	$\overline{\sigma_{yz}}$
10	FEM	100.35	0.7568	0.2865	0.0487	0.7157	0.2869
	Closed form	102.19	0.7719	0.3072	0.0514	0.7548	0.3107
20	FEM	69.25	0.7138	0.218	0.0423	0.7869	0.2654
	Closed form	75.72	0.7983	0.227	0.0453	0.7697	0.2902
100	FEM	62.78	0.7895	0.1856	0.0409	0.7496	0.2687
	Closed form	66.97	0.8072	0.1925	0.0426	0.7744	0.2842
	CLPT	66.00	0.8075	0.1912	0.0425	0.7191	0.3791

Table 1: Non dimensionalized maximum deflection and stresses of simply supported cross-ply (0/90/0) square plate subjected to uniformly distributed loading

Side to Thickness ratio a/h	Type of solution	$\overline{w}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\sigma_{xy}}$	$\overline{\sigma_{xz}}$	$\overline{\sigma_{yz}}$
10	FEM	101.80	0.7493	0.4989	0.0438	0.7896	0.3406
	Closed form	102.50	0.7577	0.5006	0.0470	0.7986	0.3499
20	FEM	76.77	0.7959	0.3905	0.0408	0.8298	0.3184
	Closed form	76.94	0.8045	0.3968	0.0420	0.8305	0.3228
100	FEM	65.19	0.8394	0.3527	0.0315	0.8395	0.3092
	Closed form	68.33	0.8420	0.3558	0.0396	0.8420	0.3140
	CLPT	67.96	0.8236	0.3540	0.0395	0.6404	0.4548

Table 2: Non dimensionalized maximum deflection and stresses of simply supported cross-ply (0/90/90/0) square plate subjected to uniformly distributed loading

Side to Thickness ratio a/h	Type of solution	$\overline{w}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\sigma_{xy}}$	$\overline{\sigma_{xz}}$	$\overline{\sigma_{yz}}$
10	FEM	93.5	0.7459	0.5268	0.0394	0.6751	0.3658
	Closed form	97.27	0.7649	0.5525	0.0436	0.6901	0.4410
20	FEM	71.59	0.7952	0.4685	0.0401	0.6927	0.4089
	Closed form	75.81	0.8080	0.4844	0.0403	0.7166	0.4188
100	FEM	65.35	0.7995	0.4227	0.0351	0.7186	0.3982
	Closed form	68.74	0.8264	0.4559	0.0386	0.7267	0.4108
	CLPT	68.44	0.8272	0.4546	0.0385	----------	----------

Table 3: Non dimensionalized maximum deflection and stresses of simply supported cross-ply (0/90/0/90/0) square plate subjected to uniformly distributed loading

Side to Thickness ratio a/h	Type of solution	$\overline{w}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\sigma_{xy}}$	$\overline{\sigma_{xz}}$	$\overline{\sigma_{yz}}$
10	FEM	66.92	0.5098	0.2518	0.0250	0.4060	0.0908
	Closed form	66.93	0.5134	0.2536	0.0252	0.4089	0.0915
	3D-Elasticity	---------	0.590	0.288	0.029	0.357	0.123
20	FEM	49.21	0.5281	0.1983	0.0222	0.4176	0.0754
	Closed form	49.21	0.5318	0.1997	0.0223	0.4205	0.0759
	3D-Elasticity	----------	0.552	0.210	0.234	0.385	0.092
100	FEM	43.36	0.5346	0.1791	0.0212	0.4215	0.0699
	Closed form	43.37	0.5384	0.1804	0.0213	0.4247	0.0703
	3D-Elasticity	---------	0.539	0.181	0.0213	0.395	0.083
	CLPT	43.13	0.5387	0.1796	0.0213	0.3951	0.0823

Table 4: Non dimensionalized maximum deflection and stresses of simply supported cross-ply (0/90/0) square plate subjected to sinusoidal loading

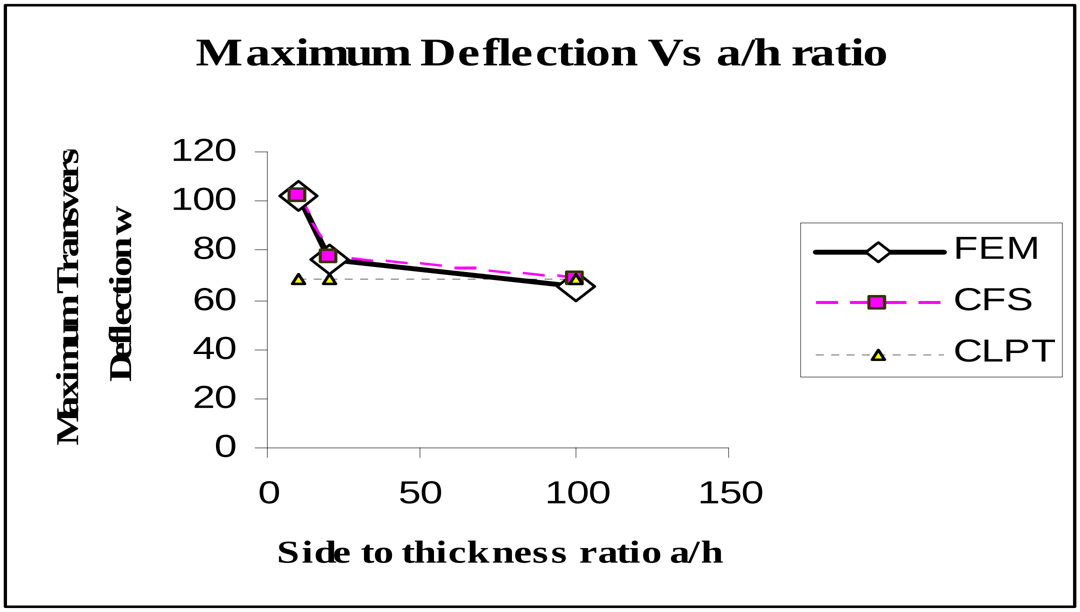
Side to Thickness ratio a/h	Type of solution	$\overline{w}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\sigma_{xy}}$	$\overline{\sigma_{xz}}$	$\overline{\sigma_{yz}}$
10	FEM	66.75	0.4906	0.3512	0.0258	0.398	0.112
	Closed form	66.27	0.4989	0.3614	0.0241	0.416	0.129
	3D-Elasticity	73.70	0.5590	0.4010	0.0276	0.301	0.196
20	FEM	49.25	0.5198	0.2906	0.0228	0.430	0.126
	Closed form	49.12	0.5273	0.2956	0.0221	0.437	0.109
	3D-Elasticity	51.28	0.5430	0.3080	0.0230	0.328	0.156
100	FEM	43.18	0.5215	0.2598	0.0208	0.448	0.118
	Closed form	43.37	0.5382	0.2704	0.0213	0.445	0.101
	3D-Elasticity	43.47	0.5390	0.2710	0.0214	0.339	0.139
	CLPT	43.13	0.5387	0.2667	0.0213	0.339	0.138

Table 5 : Non dimensionalized maximum deflection and stresses of simply supported cross-ply (0/90/90/0) square plate subjected to sinusoidal loading

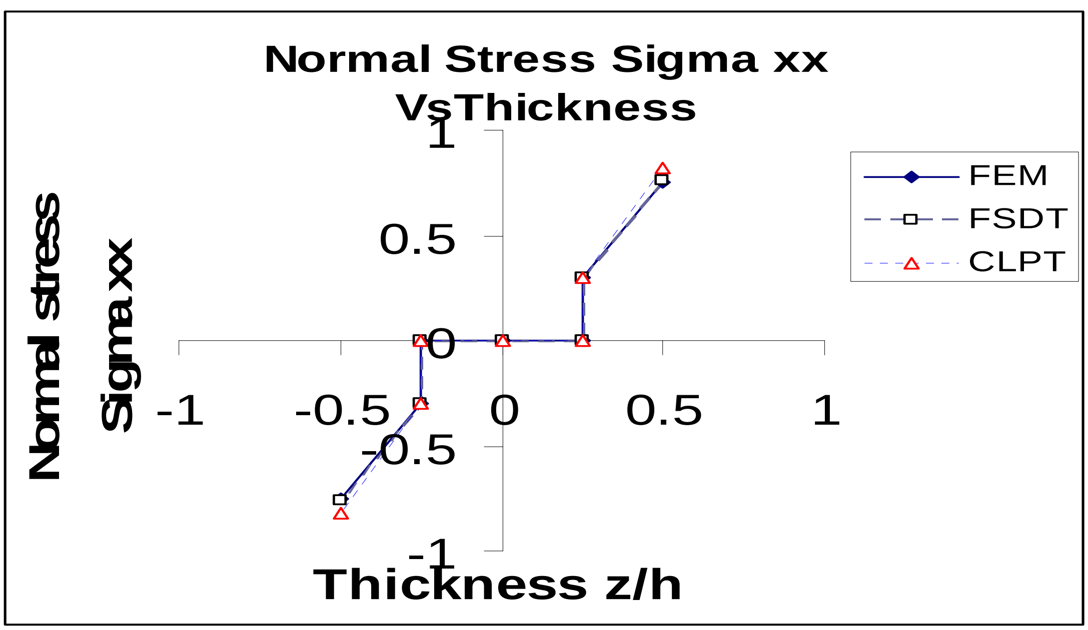
Side to Thickness ratio a/h	Type of solution	$\overline{w}$	$\overline{\sigma_{xx}}$	$\overline{\sigma_{yy}}$	$\overline{\sigma_{xy}}$	$\overline{\sigma_{xz}}$	$\overline{\sigma_{yz}}$
10	FEM	62.12	0.4986	0.4078	0.0219	0.3435	0.1984
	Closed form	62.13	0.5021	0.4107	0.0221	0.3459	0.1998
	3D-Elasticity	67.71	0.545	0.430	0.0247	0.258	0.223
20	FEM	47.96	0.5239	0.3722	0.0214	0.3592	0.1827
	Closed form	47.96	0.5276	0.3748	0.0215	0.3617	0.1840
	3D-Elasticity	49.38	0.539	0.380	0.0222	0.268	0.212
100	FEM	43.31	0.5345	0.3573	0.0211	0.3655	0.1761
	Closed form	43.32	0.532	0.3598	0.0213	0.3683	0.1774
	3D-Elasticity	43.38	0.539	0.360	0.0213	0.272	0.205
	CLPT	43.13	0.5387	0.3591	0.0213	0.2722	0.2052

Table 6 :Non dimensionalized maximum deflection and stresses of simply supported cross-ply (0/90/0/90/0) square plate subjected to sinusoidal loading

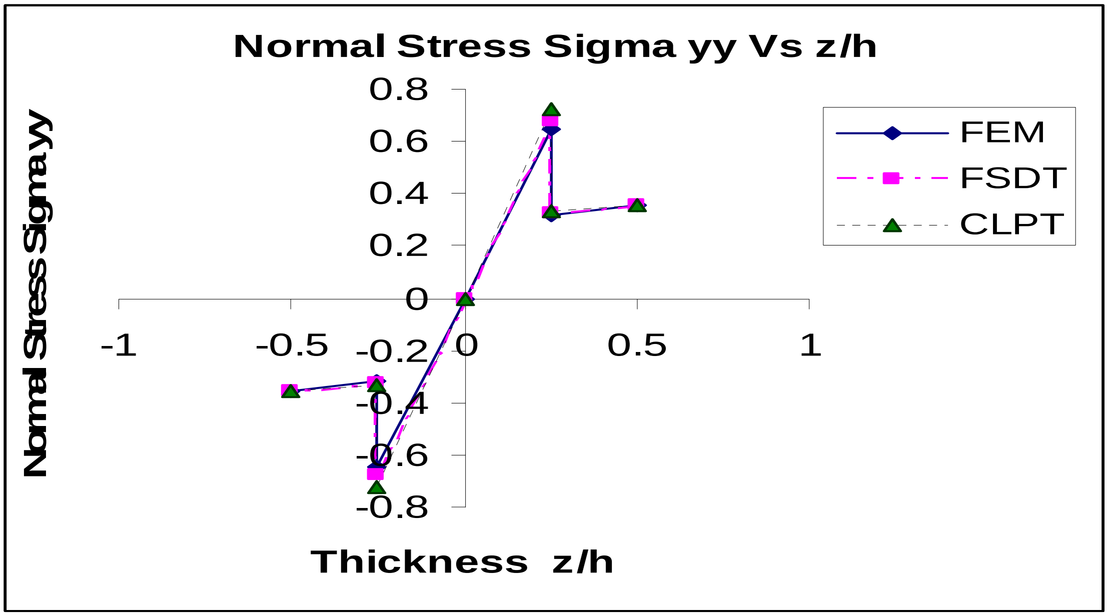
Graphs:



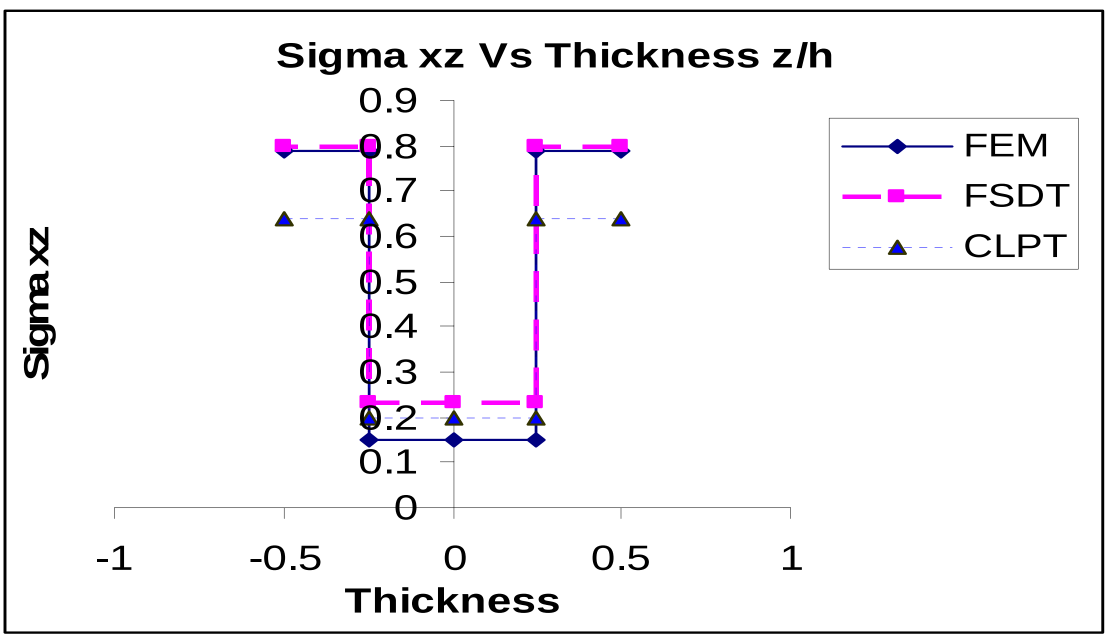
Graph 1: Non dimensionalized central transverse deflection versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed loading



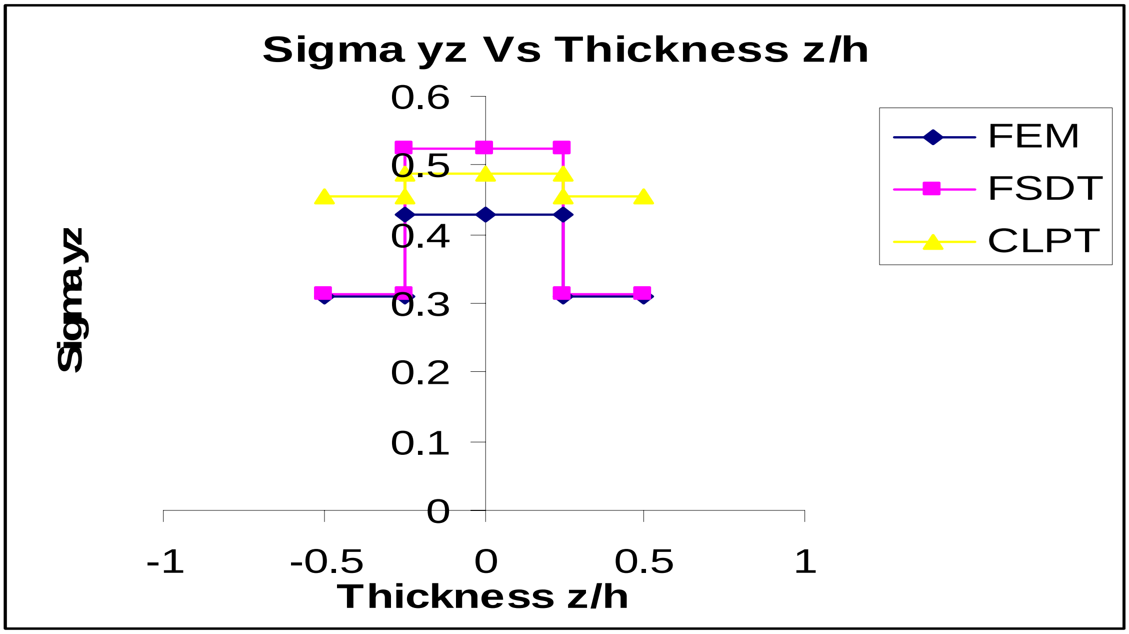
Graph 2: Non dimensionalized normal stress sigma xx versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed loading



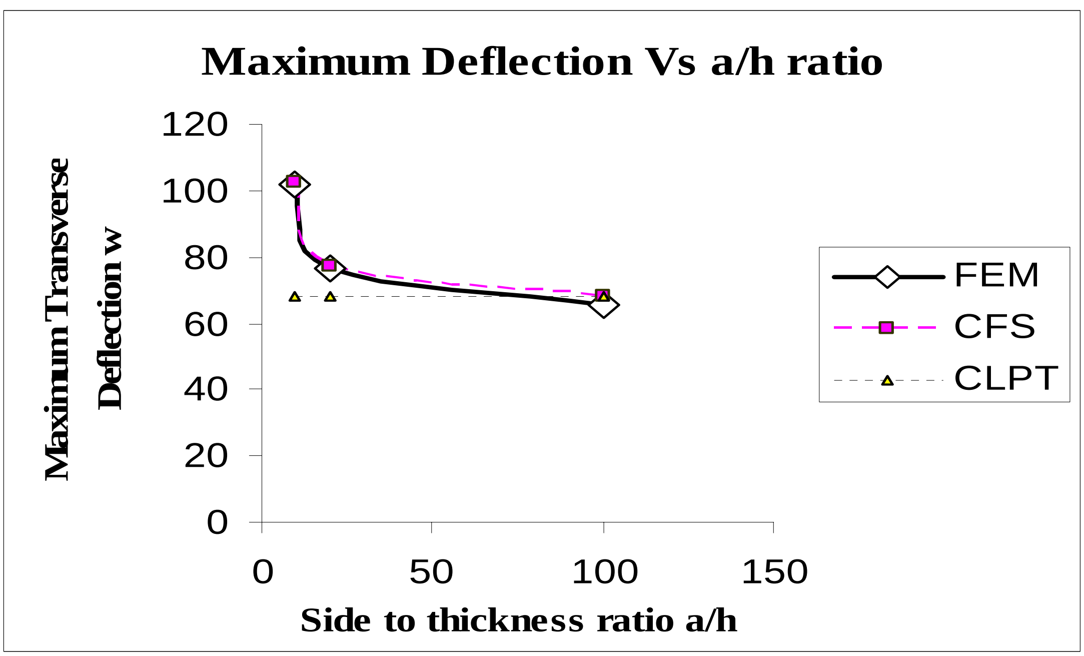
Graph 3: Non dimensionalized normal stress sigma yy versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed loading



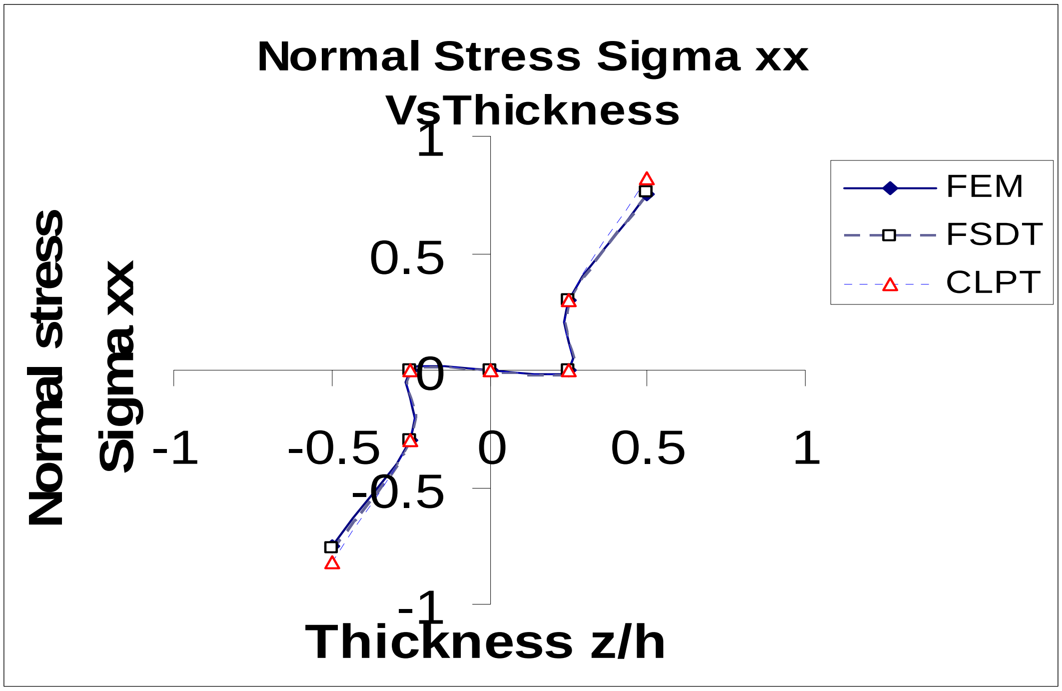
Graph 4: Non dimensionalized transverse shear stress sigma xz versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed loading



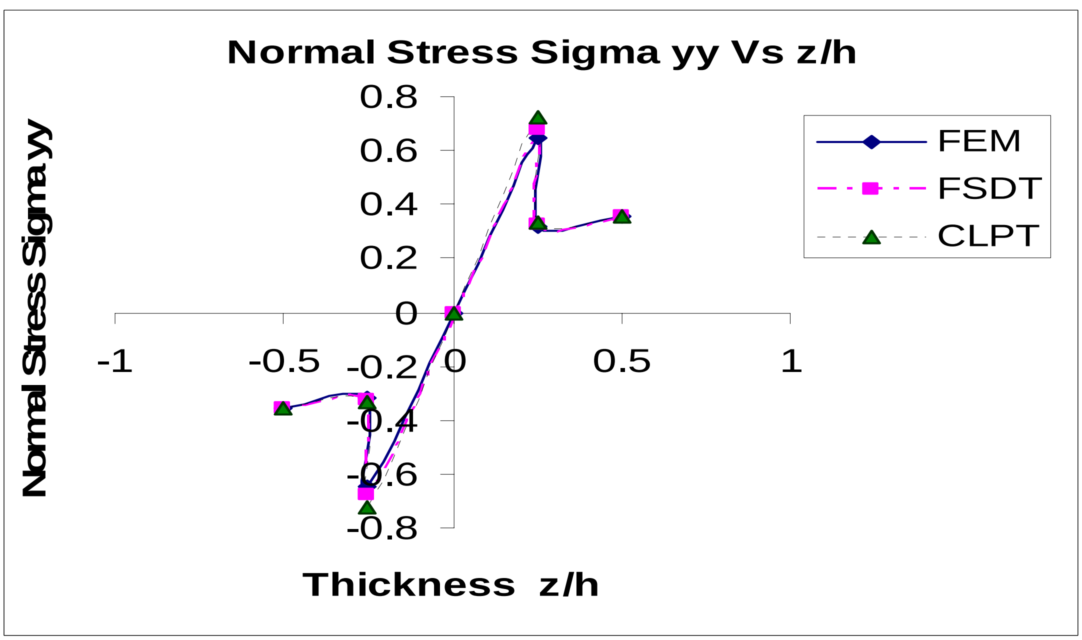
Graph 5: Non dimensionalized transverse shear stress sigma yz versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to uniformly distributed Loading



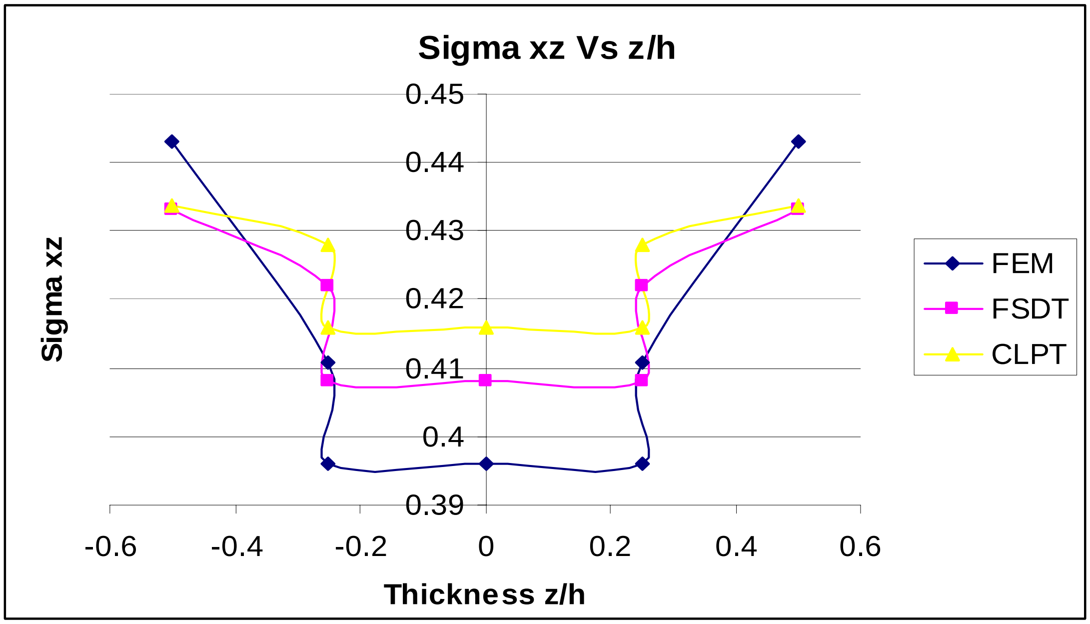
Graph 6: Non dimensionalized central transverse deflection versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to sinusoidal varying load



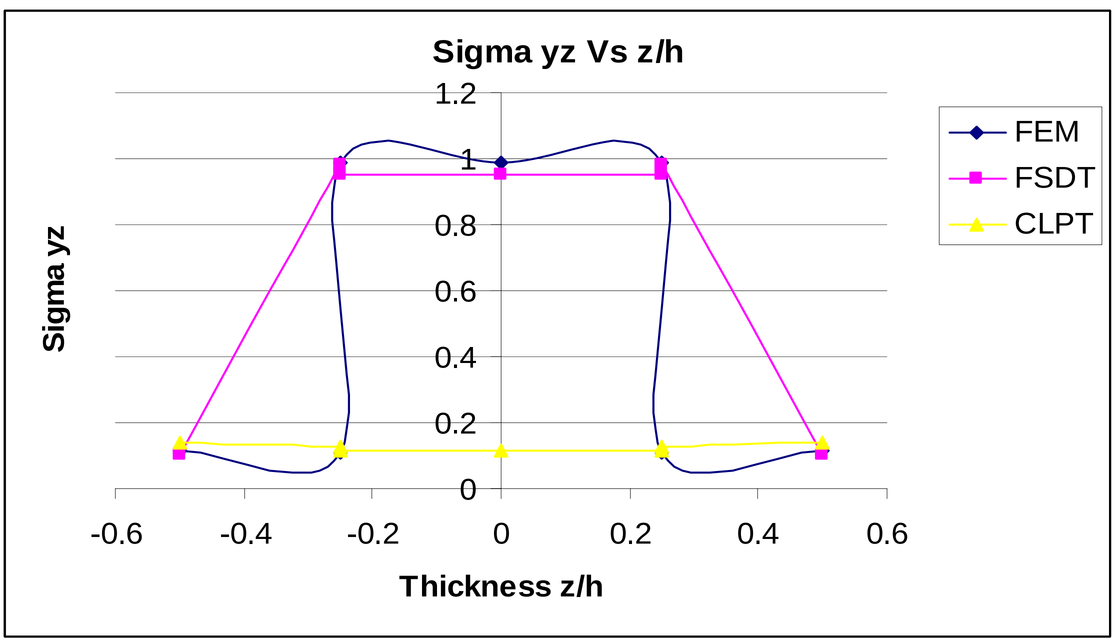
Graph 7: Non dimensionalized normal stress sigma xx versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to sinusoidal varying load



Graph 8: Non dimensionalized normal stress sigma yy versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to sinusoidal varying load



Graph 9: Non dimensionalized transverse shear stress sigma xz versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to sinusoidal varying load



Graph 10: Non dimensionalized transverse shear stress sigma yz versus side to thickness ratio for simply supported cross ply (0/90/90/0) square laminate subjected to sinusoidal varying load

5.2Discussion:

The following non-dimensional quantities are used to get the non dimensionalized stresses and deflections from the actual ones.

$$\overline{w} = w_0(0,0)\frac{E_2h^2}{a^4q_0} \qquad \overline{\sigma_{xy}} = \sigma_{xy}(a/2, a/2, -h/2)\frac{h^2}{a^2q_0}$$

$$\overline{\sigma_{xx}} = \sigma_{xx}(0,0,h/2)\frac{h^2}{a^2q_0} \qquad \overline{\sigma_{yy}} = \sigma_{yy}(0,0,h/4)\frac{h^2}{a^2q_0}$$

$$\overline{\sigma_{xz}} = \sigma_{xz}(a/2,0,k=1,4)\frac{h}{aq_0} \qquad \overline{\sigma_{yz}} = \sigma_{yz}(0,a/2,k=1,4)\frac{h}{aq_0}$$

The origin of the coordinate system is taken at the centre f the plate, -a/2<(x,y)<a/2 and –h/2<z<h/2. As mentioned earlier, the stresses are computed at the reduced Gauss points. The Gauss coordinates are mentioned like (A, A). The finite element solutions of the present study is compared with the closed form solutions obtained using FSDT and that of Classical Laminated Plate Theory (CLPT) for uniformly distributed loading case when the edges of the plate are simply supported. Similarly, the results obtained from present finite element model are compared with the closed form solutions of FSDT and CLPT as well as 3-D elasticity solutions for sinusoidal variation of loading case when all the edges of the plate are simply supported.

Comparison is made, between non dimensional quantities of transverse displacements, for different values of side to thickness ratios. Comparison is also made with respect to the lamina orientation in the laminate for different side to thickness values between the transverse displacements.

The results of non dimensional quantities of normal stresses, in-plane shear stresses and transverse shear stresses are compared, at various thickness co -ordinates for different values of side to thickness ratios. The results are obtained for different lamina orientation schemes (0 or 90 degrees i.e., cross – ply orientation with symmetry) in the laminate as well as for varying number of layers i.e., for 3 layer (0/90/0), 4 layer (0/90/90/0) and 5 layer (0/90/0/90/0) orientations.

5.3Observations:

It is observed from the results presented in the tables that, the FEM results obtained using 8-noded isoparametric Serendipity element are giving near approximations for a/h ratios <20, i.e., for thick plates. And as the ratio increases, the values are not that much satisfactory. That is, the present finite element model using First order shear deformation theory can well predict the results for a thick plate. As the plate a/h ratio is increasing i.e. when the plate is becoming thin, the results are not that much in good comparison as that for the thick plates. Reduced integration alleviated this phenomenon called shear locking to some extent.

From the results it can be observed that the Finite Element Solutions are in well agreement with the results of closed form solutions of FSDT. The displacements converge faster than stresses. This is expected because; the rate of convergence of gradients of the solution is one order less than the rate of convergence of the solution. However, the results based on the finite element solutions should not be expected to agree well with the 3-D elasticity solutions. The finite element solutions should only converge to the closed form solutions of the FSDT.

It is also observed that, the normal stresses are varying non-linearly across the thickness of the laminate. However, they are varying linearly for an individual lamina. The stresses are discontinuous across the thickness of the laminate. That means, there exist different values of stresses at the interfaces of the laminate. The stress at the bottom surface of a lamina zk, is different from that at the top surface of the adjacent lamina zk-1.This is obvious from the First order shear deformation theory. The element is a C^0 continuous element. The generalized displacements only are continuous across the thickness of the laminate. But, the strains and thus the stresses are not continuous at the boundaries.

The stress values are maximum at the top and bottom of the laminate with + ve and – ve signs respectively. The transverse shear stresses are constant throughout the thickness. It is because of the use of a constant in calculating the shear stresses, the shear correction factor. Its value varies with lamina orientation and stacking sequence. In the present study it’s value is taken as 5/6.

Since the stresses in the finite element analysis are computed at locations different from the analytical solutions, they are expected to be different.

Chapter -6

CONCLUSIONS

Conclusions

Finite element analysis of cross ply laminated composite square plate is carried out, using a 8 – noded isoparametric quadratic element to predict the transverse displacements , normal stresses and transverse shear stresses, when it is subjected to transverse loading under simply supported boundary conditions. The present model is developed based on the First order Shear Deformation Theory (FSDT). This theory uses a shear correction factor to approximate the transverse shear stresses. A computer program is written in MATLAB to get various results. The accuracy of results obtained using the present formulation is demonstrated by comparing the results with three-dimensional elasticity solution ,closed form solutions of FSDT and Classical Laminated Plate Theory. The present analysis gives accurate values for displacements and stresses compared to Classical Laminated Plate Theory. It is observed that the results are in close agreement with closed form solutions of FSDT and 3-D elasticity solutions. It is found that, the transverse shear stresses vary constantly through the thickness. This is attributed to the use of shear correction factor in the theory. But, the actual variation of the transverse shear stresses is parabolic according to 3D elasticity using equilibrium relations in predicting the same. More over, the results of stresses are calculated at Gauss points and they are expected to differ from the analytical solutions. Adoption of reduced integration scheme alleviated the shear locking effects. The present model accurately predicts the transverse displacements and various stresses for thin as well as thick laminated composite plates. As the present model is developed using a non-conforming element, the results can be further improved using a conforming element with improved mesh size thereby increased no of elements. Infact , the FEM results approach the true solutions, with the increase in the number of elements.

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Chapter-7

SCOPE FOR FUTURE WORK

Scope for future work:

Results can be expected with excellent agreement with the analytical/experimental solutions by using a conforming element with increased mesh size.

Analysis can be done for different loading conditions with various boundary conditions.

There is a need to develop a mechanics based theory to find out the optimal stacking sequence, which will significantly help the designers.

Study can be made on real life problems pertaining to stress concentration, non-linearity and complicated geometries

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APPENDIX

Composite: A combination of two or more materials on a macroscopic scale which are physically distinct.

Classification:

1) *Fiber reinforced composites* which consists of fibers in a matrix
2) *Laminated composites* which consists of layers of various materials
3) *Particulate composites* which consists of particles in a matrix

Advantages of composites:

They usually exhibit the best qualities of their constituents and often some qualities that neither constituent possesses. The properties that can be improved by forming a composite material include:

Strength	Fatigue life	stiffness	Corrosion resistance
Temperature-dependent behavior	Wear resistance	Weight	

Mechanical behavior of composite materials

Composite materials unlike *isotropic* materials are often both *inhomogeneous* and *non-isotropic*

Anisotropic body has material properties that are different in all directions at a point in the body. There are no planes of material property symmetry. Again, the properties are a function of orientation at a point in the body.

Orthotropic body has material properties that are different in three mutually perpendicular planes of material symmetry. Thus, the properties are a function of orientation at a point in the body.

Micromechanics are the study of composite material behavior where in the interaction of the constituent materials is examined on a microscopic scale.

Macro-mechanics is the study of composite material behavior wherein the material is presumed homogeneous and the effects of the constituent materials are detected only as averaged apparent properties of the composite.

<u>Basic terminology of laminated fiber-reinforced composite materials:</u>

Lamina: It is a flat (sometimes curved as in a shell) arrangement of unidirectional l fibers or woven fibers in a matrix.

Laminate: It is a stack of lamina with various orientations of principal material direction in the lamina as shown .fig.

The major purpose of lamination is to tailor the directional dependence of strength and stiffness of a material to match the loading environment of the structural element. Laminates are uniquely suited to this objective since the principal material direction of each layer can be oriented according to need.

Geometry of the N- layered laminate is as shown:

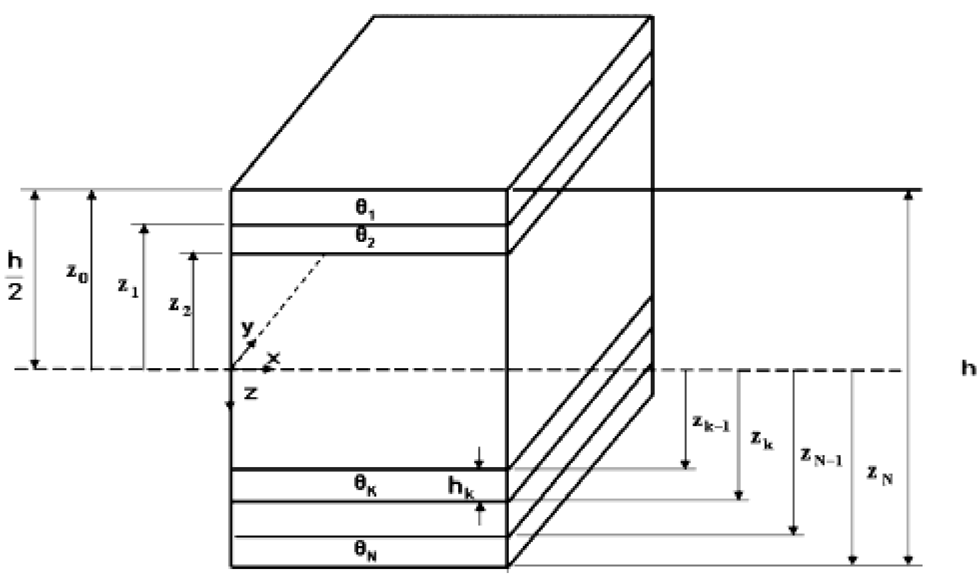

Fig. 1. Geometry of an *N*-layered laminated plate.

The transformed reduced stiffness matrix terms can be expressed as

$$\overline{Q_{11}} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{33})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta$$

$$\overline{Q_{12}} = (Q_{11} + Q_{22} - 4Q_{33})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta)$$

$$\overline{Q_{22}} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{33})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta$$

$$\overline{Q_{13}} = (Q_{11} - Q_{12} - 2Q_{33})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{33})\sin^3\theta\cos\theta$$

$$\overline{Q_{23}} = (Q_{11} - Q_{12} - 2Q_{33})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{33})\sin\theta\cos^3\theta$$

$$\overline{Q_{33}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{33})\sin^2\theta\cos^2\theta + Q_{33}(\sin^4\theta + \cos^4\theta)$$

<u>Jacobian Matrix:</u>

The derivatives of the shape functions with regard to x and y can be obtained by transformation from natural coordinate's r and s, using Jacobian matrix. The Jacobian matrix is given by

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial s} \\ \frac{\partial x}{\partial r} & \frac{\partial y}{\partial s} \end{bmatrix}$$

The transformation from (r, s) coordinates to (x, y) coordinate system is done by

$$\begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial s} \end{Bmatrix}$$

For the present problem with 8-noded Serendipity element, the transformation is as follows:

$$\begin{bmatrix} \frac{\partial N_1}{\partial x} \frac{\partial N_2}{\partial x} \frac{\partial N_3}{\partial x} \cdots\cdots\cdots \frac{\partial N_8}{\partial x} \\ \frac{\partial N_1}{\partial y} \frac{\partial N_2}{\partial y} \frac{\partial N_3}{\partial y} \cdots\cdots\cdots \frac{\partial N_8}{\partial y} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial N_1}{\partial r} \frac{\partial N_2}{\partial r} \frac{\partial N_3}{\partial r} \cdots\cdots\cdots \frac{\partial N_8}{\partial r} \\ \frac{\partial N_1}{\partial s} \frac{\partial N_2}{\partial s} \frac{\partial N_3}{\partial s} \cdots\cdots\cdots \frac{\partial N_8}{\partial s} \end{bmatrix}$$