BEHAVIOUR OF LAMINATED COMPOSITE TWISTED PLATES SUBJECTED TO IN-PLANE LOADING

Submitted by

VIKAS KACHHAP, 211CE2024

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Prof. A. V. ASHA

Department of Civil Engineering

National Institute of Technology Rourkela

Orissa – 769008

DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

ODISHA, INDIA

CERTIFICATE

This is to certify that the thesis entitled "**BEHAVIOUR OF LAMINATED COMPOSITE TWISTED PLATES SUBJECTED TO IN-PLANE LOADING**", submitted by **MR. VIKAS KACHHAP** bearing **Roll no. 211CE2024** in partial fulfillment of the requirements for the award of ***Master of Technology*** in the Department of Civil Engineering, National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university/institute for the award of any Degree or Diploma.

Place: Rourkela

Date:

Prof. A. V. ASHA

Civil Engineering Department

National Institute of Technology, Rourkela

DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA 769008

A C K N O W L E D G E M E N T

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Place: Rourkela

Date**:**

VIKAS KACHHAP

M. Tech., Roll No: 211CE2024

Specialization: Structural Engineering

Department of Civil Engineering

National Institute of Technology, Rourkela

ABSTRACT

The twisted panel has various applications in turbine blades, compressor blades, fan blades and particularly in gas turbines. Many of these plates are subjected to in-plane load due to fluid or aerodynamic pressures.Hence it is necessary to study their behaviour under differnt types of loads. In these days,composite materials are increasingly used as load bearing structural components in aerospace and naval structures, automobiles, pressure vessels, turbine blades and many other engineering applications because of their high specific strength and stiffness.

The analysis is carried out using ANSYS software. An eight-node isoparametric quadratic element is considered in the present analysis with five degrees of freedom per node.In ANSYS, the shell 281 element with five degrees of freedom per node is used. An eight by eight mesh is found to give good accuracy. The vibration and stability behaviour of composite laminated twisted plate under various types of non-uniform in-plane loading is studied. The effect of number of layers, changing angle of twist, width to thickness ratio, aspect ratio, etc on the vibration and buckling loads are presented.

It is observed that for increasing angles of twist of laminated composite plate with different in-plane load conditions, the vibration and buckling both decreases. Also as the number of layers increases, the vibration and buckling parameters of the laminated twisted plate are both observed to increases.

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Nomenclature

a ,b	panel dimension of the twisted panel
a/ b	aspect ratio of the twisted plate
$A_{ij}, B_{ij}, D_{ij} and\ S_{ij}$	Extensional, bending-stretching coupling, Bendingand transverse shear stiffnesses
b/h	width to thickness ratio of the twisted plate
[B]	Strain-displacement matrix for the element
[D]	stress-strain matrix
$[D_p]$	stress-strain matrix for plane stress
dx, dy	element length in x and y-direction
dV	volume of the element
E_{11}, E_{22}	Modulii of elasticity in longitudinal and Transverse directions
G_{12}, G_{13}, G_{23}	Shearmodulii
J	Jacobian
k	shear correction factor

$[K_e]$	global elastic stiffness matrix
$[k_E]$	element bending stiffness matrix with Shear deformation of the panel
$[K_g]$	global geometric stiffness matrix
$[K_P]$	plane stiffness matrix
k_x, k_y, k_{xy}	Bending strains
[M]	Global consistent mass matrix
$[m_e]$	element consistent mass matrix
M_x, M_y, M_{xy}	Moment resultants of the twisted panel
n	number of layers of the laminated panel
[N]	Shape function matrix
N_i	Shape functions
N_{cr}	Critical load
N_X, N_Y, N_{XY}	In-plane stress resultants of the twisted plate
$N_X^0,\ N_y^0$	External loading in the X and Y directionsrespectively
[P]	Mass density parameters
q	Vector of degrees of freedom
Q_x, Q_y	Shearing force

R_X, R_Y, R_{XY}	Radii of curvature of shell in x and y directions and radius of twist
T	transformation matrix
u, v, w	displacement components in the x, y, z directions at any point
u_0, v_0, w_0	Displacement components in the x, y, z directions at the midsurface
U_0	Strain energy due to initial in-plane stresses
U_1	Strain energy associated with bending withtransverse shear
U_2	Work done by the initial in-plane stresses and the nonlinear strain
V	kinetic energy of the twisted panel
w	out of plane displacement
X, Y, Z	global coordinate axis system
γ	shear strains
$\xi_x, \xi_x, \gamma_{xy}$	Strains at a point
$\varepsilon_{xnl}, \varepsilon_{xnl}, \gamma_{xnl}$	Non-linear strain components
θ_x, θ_y	rotations of the midsurface normal about

	the x- and y- axes respectively
ξ, η	local natural coordinates of the element
(ρ)	mass density of k^{th} layer from mid-plane
ρ	mass density of the material
σ_x^0, σ_y^0 and σ_{xy}^0	in-plane stresses due to external load
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	Shear stresses in xy, xz and yz planes
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	Shear stresses in xy, xz and yz planes Respectively
ω	Frequencies of vibration
$\overline{\omega}$	Non-dimensional frequency parameter
Φ	angle of twist of the twisted panel
$[\,]^{-1}$	Inverse of the matrix
$[\,]^T$	Transpose of the matrix
$\frac{\partial}{\partial x}, \frac{\partial}{\partial x}$	Partial derivatives with respect to x and

CHAPTER 1

1.1: INTRODUCTION

The twisted cantilever panels have significant applications in wide chord turbine blades, compressor blades, fan blades, aircraft or marine propellers, helicopter blades, and particularly in gas turbines. Today twisted plates are key structural units in the research field. Because of the use of twisted plates in turbo-machinery, aeronautical and aerospace industries etc, it is required to understand the deformation and vibration characteristics of the rotating blades. The twisted plates are also subjected to different types of loads due to fluid pressure or transverse loads. The present study is mainly focused on behavior, both vibration and stability, of laminated composite twisted plates under in- plane loading.

1.2: IMPORTANCE OF THE PRESENT STUDY

The blades are often subjected to axial periodic forces due to axial components of aerodynamic or hydrodynamic forces acting on the blades. Composite materials are being increasingly used in turbo-machinery blades because of their specific strength and stiffness and these can be tailored through the variation of fiber orientation and stacking sequence to obtain an efficient design.

Hence it is required to have an understanding of the behaviour of the laminated composite twisted plates under loads.

1.3: OUTLINE OF THE PRESENT WORK

The present study mainly deals with the behaviour of homogeneous laminated composite twisted cantilever panels. Under the influence of different end loads applied axially. The influence of various parameters like angle of twist, side to thickness ratios, number of layers, lamination sequence, and ply orientation, degree of orthotropy on the vibration and stability behaviour of twisted panels are examined.

The governing equations for the laminated composite doubly curved twisted panels/shells subjected to in-plane loading are developed. The governing differential equations have been developed using the first order shear deformation theory (FSDT).

This thesis contains five chapters. In this chapter, a brief introduction of the importance of this study has been outlined.

In chapter 2, a detailed review of the literature pertinent to the previous works done in this field has been listed. The aim and scope of the present study is also outlined in this chapter.

In chapter 3, a description of the theory and formulation of the problem and the finite element procedure used to analyse the vibration and buckling of laminated composite twisted cantilever panels subjected to in-plane loads is explained in detail. The study is done using ANSYS software. The computer program used to implement the formulation is briefly described.

In chapter 4, the results and discussions obtained in the study have been presented in detail. The effects of various parameters like twist angle, lamination sequence, ply orientation, degree of orthotropy, aspect ratio, width to thickness ratio and in-plane load parameters on the vibration and buckling behavior has been presented. The studies have been done for cross-ply laminated composite twisted cantilever panels.

Finally, in chapter 5, the conclusions drawn from the above studies are stated. There is also a brief note on the scope for further study in this field.

CHAPTER 2

2.1: LITERATURE REVIEW

The vast use of turbomachinery blades lead to significant amount of research over the years. Due to its wide range of application in the practical field, it is important to understand the nature of deformation, vibration and stability behaviour of cantilever twisted plates.

Crispino and Benson [4] studied the stability of thin, rectangular, orthotropic plates which were in a state of tension and twist. Results were presented, in a compact non-dimensional form, for a range of material, geometric and loading parameters.

The effect of thermal gradient and tangency coefficient on the stability of a pretwisted, tapered, rotating cantilever with a tip mass and subjected to a concentrated partial follower force at the free end was investigated by Kar and Neogy [8]. The non-self adjoint boundary value problem was formulated with the aid of a conservation law using Euler-Bernoulli theory. The associated adjoint boundary value problem was introduced and an opposite variationalprinciple was derived. Approximate values of critical load were calculated on the basis of this variational principle and the influence of different parameters on the stability of the system was studied.

A parametric study was presented by Nemeth [15] of the buckling behavior of infinitely long symmetrically laminated anisotropic plates subjected to combined loads. The loads considered in this report are uniform axial compression, pure in-plane bending, transverse tension and compression, and shear. Results are presented that were obtained by using a special purpose nondimensional analysis that is well suited for parametric studies of clamped and simply supported plates. An important finding of the present study is that the effects of flexural anisotropy on the buckling resistance of a plate can be significantly more important for plates subjected to combined loads than for plates subjected to single-component loads.

Vibration of cross-ply laminated composite plates subjected to initial in-plane stresses was studied by Matsunaga [14]. Natural frequencies, modal displacements and stresses of cross-ply laminated composite plates subjected to initial in-plane stresses were analyzed for the effects of higher-order deformations and rotatory inertia.

Wang et al [23] studied the plastic buckling of rectangular plates subjected to intermediate and end in-plane loads. The plate had two opposite simply supported edges that were parallel to the load direction while the other remaining edges were any combination of free, simply supported or clamped conditions. Both the incremental theory of plasticity and the deformation theory of plasticity were considered in bounding the plastic behaviour of the plate.

Static, free vibration and buckling analysis of anisotropic thick laminated composite plates on distributed and point elastic supports using a 3-D layer-wise FEM was done by Karami [20]. A three-dimensional elasticity based layer-wise finite element method (FEM) was used to study the static, free vibration and buckling responses of general laminated thick composite plates. Various mixed boundary conditions and free edge conditions were conveniently and accurately implemented. Elastic line and point supports were also successfully incorporated for thick plates.

Shukla et al [21] estimated the critical/ buckling loads of laminated composite rectangular plates under in-plane uniaxial and biaxial loadings. The formulation was based on the first-order shear deformation theory and von-Karman-type non-linearity. An incremental iterative approach was used for estimating the critical load. Various combinations of support conditions were considered. The effects of aspect ratio, lamination scheme, number of layers and material properties on the critical loads were studied.

Free vibration of laminated composite plates subjected to in-plane stresses using trapezoidal p-element was studied by Leung et al [11]. A new trapezoidal p-element was applied to solve the free vibration problem of polygonal laminated composite plates subjected to in-plane stresses with various boundary conditions. The element stiffness and mass matrixes were analytically integrated in closed form.

Buckling of symmetrical cross-ply composite rectangular plates under a linearly varying in-plane load was studied by HongzhiZhong and Chao Gu [21]. It was developed based on the first-order shear deformation theory for moderately thick laminated plates. Buckling loads of cross-ply rectangular plates with various aspect ratio were obtained and the effects of load intensity variation and layup configuration on the buckling load were investigated. The results were verified using the computer code ABAQUS.

Natural frequencies of laminated composite plates using third order shear deformation theory were carried out by Aagaah et al [1]. The natural frequencies of square laminated composite plates for different supports at edges were presented. Laminated plates were supposed to be either angle-ply or cross-ply.

Buckling analysis of symmetrically laminated composite plates by the extended Kantorovich method was conducted by Ungbhakorn and Singhatanadgid [22].They investigated the buckling problem of rectangular laminated composite plates with various edge supports. The principle of minimum total potential energy with a separable displacement function was utilized to derive a set of ordinary differential equations. The buckling load and mode were determined from iterative calculations of the governing equations using the initial trial function which can be selected arbitrarily. The accuracy of this method was confirmed with the available Le´vy and Rayleigh–Ritz solutions.

Failure analysis of laminated structures by FEM based on nonlinear constitutive relationship was found by Huang [6]. The influence of different load increments, mesh scales, and stiffness discount schemes on the finite element (FE) analyzed results was studied. Strength envelopes and stress–strain or load–deflection curves of several laminates subjected to in-plane as well as bending loads were obtained.

Three-dimensional dynamic analysis of laminated composite plates subjected to moving load was done byMalekzadeh et al [13]. For accurately determining the dynamic response of cross-ply laminated thick plates subjected to moving load, a solution procedure based on the three-dimensional (3D) elasticity theory was developed. Plates with simply supported edges and subjected to point moving load were considered. The layer wise theory was used to discretize the equations of motion and the related boundary conditions through the thickness of the plates. Then, the modal analysis in conjunction with the differential quadrature method was employed for the in-plane and the temporal discretization of the resulting system of differential equations, respectively.

Spectral element model for axially loaded bending–shear–torsion coupled composite Timoshenko beams was studied by Lee and Jang [10]. In this paper, the spectral element model was developed for an axially loaded bending–shear–torsion coupled composite laminated beam which was represented by the Timoshenko beam model based on the first-order shear deformation theory.

Buckling and vibration analysis of laminated composite plate/shell structures via smoothed quadrilateral flat shell element with in-plane rotations was studied by Nguyen-Van et al [16].This paper presented the buckling and free vibration analysis of composite plate/shell structures of various shapes, modulus ratios, span-to-thickness ratios, boundary conditions and lay-up sequences via a novel smoothed quadrilateral flat element. The element was developed by incorporating a strain smoothing technique into a flat shell approach.

Bending and vibration responses of laminated composite plates using an edge-based smoothing technique were investigated by Cui et al [3]. The analysis was carried out using a novel triangular composite plate element based on an edge-based smoothing technique. The present formulation was based on the first-order shear deformation theory, and the discrete shear gap (DSG) method was employed to mitigate the shear locking.

In-plane free vibrations of single-layer and symmetrically laminated rectangular composite plates was carried out by Dozio [5]. This work presented accurate upper-bound solutions for free in-plane vibrations of single-layer and symmetrically laminated rectangular composite plates with a combination of clamped and free boundary conditions. In-plane natural frequencies and mode shapes were calculated by the Ritz method with a simple, stable and computationally efficient set of trigonometric functions.

The FEM analysis of laminated composite plates with rectangular hole and various elastic moduli under transverse loads was done by Ozben and Arslan [17].The aim of this work was to predict the expansion of the plastic zone and residual stresses in layers of fiber-reinforced, thermoplastic laminated composite plates with rectangular hole. The effects of the material properties (constant modulus ratio, E1= E2) on residual stresses and expansion of the plastic zone were studied. The elastic and elasto-plastic stresses were analyzed using the finite element method (FEM) by a developed computer program.

Inelastic buckling behavior of stocky plates under interactive shear and in-plane bending was carried out by Alinia et al [2]. Plate girder web panels and infill plates in steel shear wall systems are two typical structural elements that are commonly subjected to interactive shear and in-plane bending. In general, flat plates may buckle before, concurrent with, or after the material's nonlinearity limit point.

Expansion of plastic zone and residual stresses in the thermoplastic-matrix laminated plates ([0/h]2) with a rectangular hole subjected to transverse uniformly distributed load expansion was carried out by Tamer et al[18]. The work focused on the understanding of elastic stress, residual stress and plastic zone growth in layers of stainless steel woven fiber-reinforced thermoplastic matrix composite laminated plates with rectangular hole by using the finite element method (FEM) and first-order shear deformation theory for small deformations.

The deformation of in-plane loaded unsymmetrically laminated composite plates was studied by Majeed[12]. This study focused on the response of flat unsymmetric laminates to an in-plane compressive loading that for symmetric laminates were of sufficient magnitude to cause bifurcation, buckling, post buckling, and secondary buckling behavior. In particular, the purpose of this study was to investigate whether or not the concept of bifurcation buckling is applicable to unsymmetric laminates.

The dynamic failure analysis of laminated composite plates was conducted by Jam and Nia[7]. A developed finite element analysis investigation into the failure behavior of laminated composite plates subjected to impulsive loads was undertaken using ANSYS. The study presented the effects of pulse duration and pulse shapes on the predicted critical static and dynamic failure modes as well as free vibration, for several layer configurations. The effect of parameters like size of plates, boundary conditions and fiber orientation angles was also studied.

Stability of laminated composite pretwisted cantilever panels was studied by Sahuand Asha[19].This study deals with the stability analysis of angle-ply laminated composite twisted panels using the finite element method. Here, an eight-nodedisoparametric quadratic shell element is used to develop the finite element procedure. To investigate the vibration andstability behavior of twisted panels, the effect of various geometrical parameters like angle of twist, aspect ratio, lamination parameters, shallowness ratio, etc. are studied.

Composite plates with two concentric layups under compression was analysed by Kassapoglou[9]. A new concept for designing composite panels with improved performance under compression was presented. In this concept, the panel consisted of two different concentric layups. A Rayleigh–Ritz-based approach to model such rectangular panels under compression was presented. The buckling load and the in-plane stresses everywhere in the plate were determined using an energy minimization approach.

2.2: METHODOLOGY

This project develops FEA models of a laminated composite twisted plate under an in plane load. The first step is to develop an ANSYS model of a laminated composite plate. The model will be subjected to vibration and buckling with in-plane loads and compared with the previous results with in-plane loading. Then a laminated composite twisted plate is studied for its characteristics with in-plane loads. Vibration characteristics will be analyzed and validated with the calculations using MATLAB**.**

2.3: OBJECTIVE OF THE PRESENT STUDY

The present study mainly aims to analyse the laminated composite twisted plates under the in plane loading conditions. Literature shows a lot of work has been done on the vibration of laminated composite twisted plates. A lot of literature is also available for forced vibration of untwisted plates. There is very little work done in the area of behavior of laminated composite twisted plates subjected to various loading conditions. The present study aims to understand the vibration and buckling behavior of twisted plates subjected to various loading conditions.

CHAPTER 3

FORMULATION

3.1: Governing Differential Equations

The differential equations of motion are obtained by taking a differential element of the twisted panel, as shown in figure 1. This figure shows an element with internal forces like membrane forces N_x ,N_y andN_{xy}, shearing forces(Q_x andQ_y) and the moment resultants (M_x, $M_y\ and\ M_{xy}$).

The governing differential equations of equilibrium for a shear deformable doubly curved pretwisted panel subjected to external in-plane loading can be expressed as (Chandrashekhara , Sahu and Dutta)[19]

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial y} + \frac{Q_x}{R_x} + \frac{Q_y}{R_{xy}} = P_1\frac{\partial^2 u}{\partial t^2} + P_2\frac{\partial^2 \theta_x}{\partial t^2}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial x} + \frac{Q_y}{R_y} + \frac{Q_x}{R_{xy}} = P_1\frac{\partial^2 v}{\partial t^2} + P_2\frac{\partial^2 \theta_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_x}{R_y} - 2\frac{N_{xy}}{R_{xy}} + N_x^0\frac{\partial^2 w}{\partial x^2} + N_y^0\frac{\partial^2 w}{\partial y^2} = P_1\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = P_1\frac{\partial^2 \theta_x}{\partial t^2} + P_2\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = P_1\frac{\partial^2 \theta_y}{\partial t^2} + P_2\frac{\partial^2 v}{\partial t^2} \qquad (3.1)$$

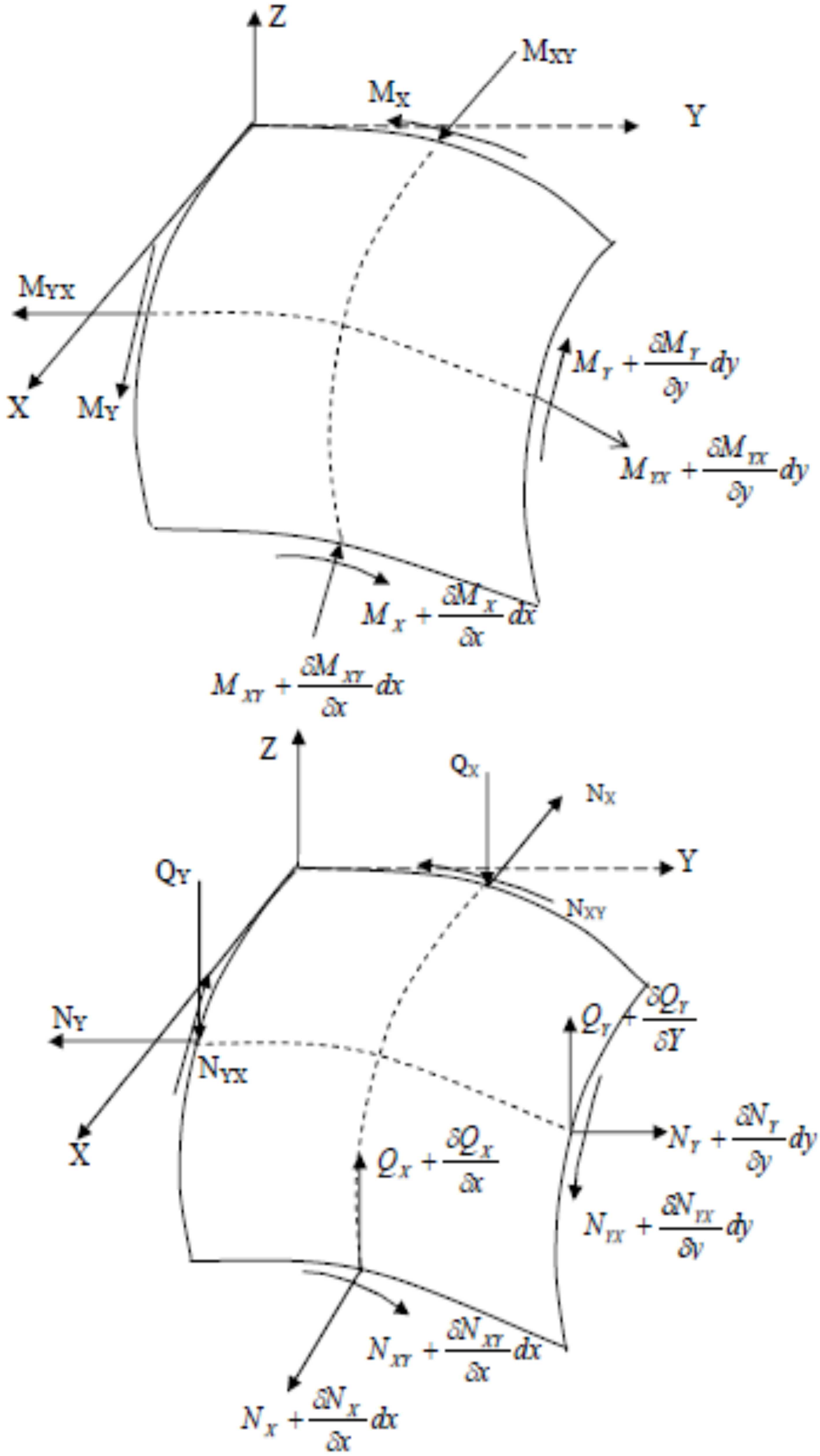

Figure 3.1: element of a twisted shell panel

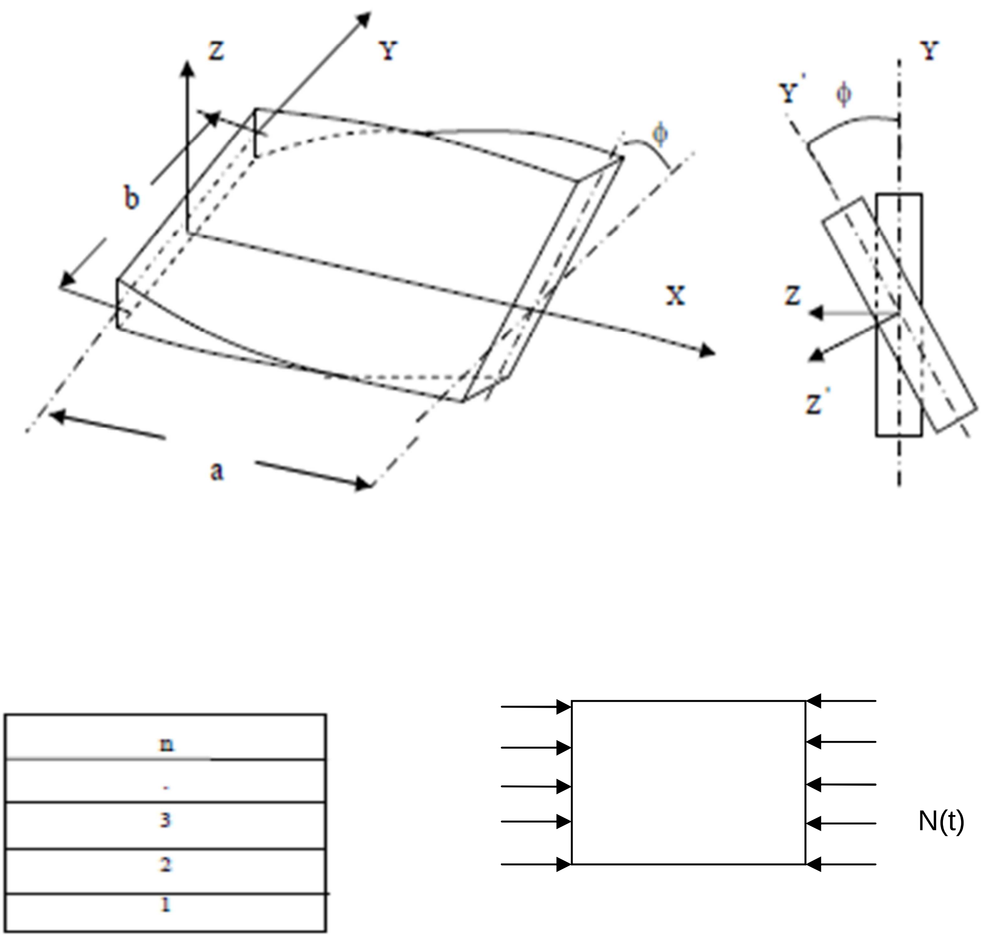

Figure 3.2: laminated composite twisted panel with in-plane loads

Here n = number of layers of panel

∅= angle of twist

a and b are length and width of the panel

Also N_x^0 and N_x^0 are the external loading in the X and Y direction respectively. The constants R_x, R_y and R_{xy} are the radii of curvature in the x and y directions and the radius of twist.

$$(P_1, P_2, P_3) = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} (\rho)_k (1, z, z^2) dz \qquad (3.2)$$

where n= number of layers of the laminated composite twisted curved panel and $(\rho)_k$ = mass density of k_{th} layer from the mid-plane.

3.2: Energy equations

The laminated composite doubly curved twisted panel is subjected to initial in-plane edge loads $N_X^0,\ N_y^0\ and\ N_{xy}^0$. Here there are in-plane stresses of $\sigma_x^0,\ \sigma_y^0$ and σ_{xy}^0 and it is a plane stress problem. The total stresses at any layer are the sum of the initial stresses plus the stresses due to bending and shear deformation. The strain energy due to initial in-plane stresses is written as

$$U_0 = \frac{1}{2}\int\int\{\varepsilon^0\}^r\{\sigma^0\}dA \tag{3.3}$$

where

$$\{\varepsilon^0\}^r = \left[\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right]^r = \left[\frac{\partial u^0}{\partial x}, \frac{\partial v^0}{\partial y}, \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x}\right] \tag{3.4}$$

And the stresses are

$$\{\sigma^0\} = [D_P]\{\varepsilon^0\} \tag{3.5}$$

The strain can be expressed in term of initial in-plane deformationu^0, v^0as

$$\{\varepsilon^0\} = [B_P]\{q^0\} \tag{3.6}$$

Substituting the value of stress and strain in the equation and we get

$$U_0 = \frac{1}{2}\int\int\{q^0\}^T[B_P]^T[B_P]\{q^0\}dA \tag{3.7}$$

The strain energy is expressed as

$$U_0 = \frac{1}{2}\{q^0\}^T[K_P]\{q^0\} \tag{3.8}$$

Where

$$[K_P] = \int\int[B_P]^T[D_P]\,[B_P]dA$$

Considering the prestressed state as the initial state, the strain energy stored due to

bending and shear deformation in the presence of initial stresses and neglecting higher order terms

$$U_0 = U_1 + U_2 \tag{3.9}$$

where

U_1= Strain energy accomplished with bending with transverse shear and

U_2= Work done by the initial in-plane stresses and the nonlinear strain

$$U_1 = \frac{1}{2} \int \int \int [\{\varepsilon_l\}^T [D] \{\varepsilon_l\}] \, dV \tag{3.10}$$

Where the strain can be expressed in term of deformations

$$\{\varepsilon_l\} = [B]\{q^0\} \tag{3.11}$$

And

$$U_2 = \frac{1}{2} \int \int \int [\{\sigma^0\}^T \{\varepsilon_{nl}\}] \, dV \tag{3.12}$$

The integration method is performed through the thickness of the twisted panel and thus the generalized force and moment resultants can directly be related to the strain components through the laminate stiffness. The kinetic energy V of the curved panel can be derived as

$$V = \int \int \left[\frac{h}{2} \left\{ \frac{\partial \bar{u}^2}{\partial t} + \frac{\partial \bar{v}^2}{\partial t} + \frac{\partial \bar{w}^2}{\partial t} \right\} + \frac{h^3}{12} \left\{ \frac{\partial \theta_x^2}{\partial t} + \frac{\partial \theta_y^2}{\partial t} \right\} \right] dxdy \tag{3.13}$$

The energies now can be written in matrix form

$$U_0 = \frac{1}{2} \{q\}^T [K_P] \{q\}$$

$$U_1 = \frac{1}{2} \{q\}^T [K_e] \{q\} \tag{3.14}$$

$$U_2 = \frac{1}{2} \{q\}^T [K_g] \{q\}$$

$$V = \frac{1}{2}\{\dot{q}\}^T[M]\{\dot{q}\}$$

Where

$[K_P]$= plane stiffness matrix of the twisted panel

$[K_P]$= Bending stiffness matrix with shear deformation of the panel

$[K_g]$= geometric stiffness or stress stiffness matrix of the twisted panel

$[M]$= Consistent mass matrix of the twisted panel

3.3: Finite element formulation

For complex geometrical and boundary conditions, analytical method are not so easily adaptable, so numerical methods like finite element method have been used. The finite element formulation is developed here by for the structural analysis of composite twisted shell panels using first order shear deformation theory..

The plate is made up with bonded layers, where each lamina is considered to be homogenous and orthotropic and made of unidirectional fiber-reinforced material. The orthotropic axes of symmetry in each lamina are oriented at an arbitrary angle to plate axes. An eight–nodedisoparametric quadratic shell element is used in the present analysis with five degree of freedom u,v,w, θ_x and $\theta_y$per node as shown in figure 3. The isoparametric element shall be oriented in the natural coordinate system and transferred to the cartesian coordinate system by using the Jacobian matrix. In the analysis of thin shell, where the element is assumed to have mid-surface nodes, the shape function of the element of the element is derived using the interpolation polynomial as follows

$$u(\xi,\eta)=\alpha_1+\alpha_2\,\xi+\alpha_3\,\eta+\alpha_4\,\xi^2+\alpha_5\xi\eta+\alpha_6\,\eta^2+\alpha_7\,\xi^2\eta+\alpha_8\,\xi\eta^2 \qquad (3.15)$$

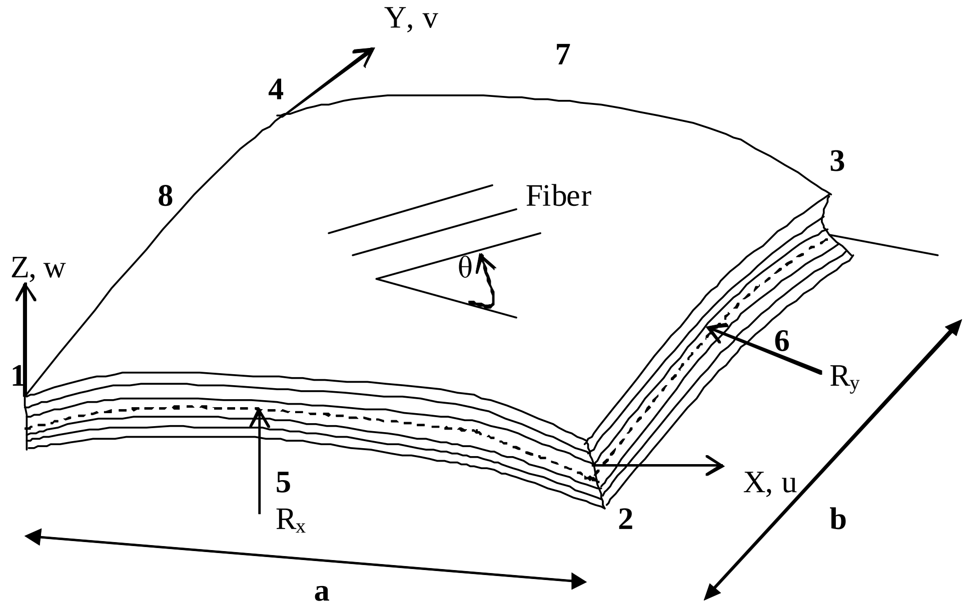

Figure 3.3: Isoparametric quadratic shell element

The element and displacement field are expressed by the shape function N_i

The shape function N_i *are defined as*

N_i=(1+ξξ_i)(1+ηη_i) (ξξ_i+ηη_i-1)/4 i=1 to 4

$N_i = (1-\xi^2)$ (1+ηη_i)/2 i= 5,7 (3.16)

N_i=(1+ξξ_i) (1- η^2)/2 i = 6,8

Where ξ and η are the local natural coordinates of the element and ξ_i *and* η_i are the values at ith node.

The derivatives of the shape function are N_i with respect to x and y are expressed in term of their derivatives with respect to ξ and η by the following relationship.

$$\begin{bmatrix} N_{i,x} \\ N_{i,y} \end{bmatrix} = [J]^{-1} \begin{bmatrix} N_{i,\xi} \\ N_{i,\eta} \end{bmatrix} \quad (3.17)$$

Where

$$[J]=\begin{bmatrix} X_{I,\xi} & Y_{I,\xi} \\ X_{i,\eta} & Y_{i,\eta} \end{bmatrix} \quad (3.18)$$

is the Jacobian matrix . The shell with the initial stresses undergoes small lateral deformations. First order shear deformation theory is used and the displacement field assumes that the mid – plane normal remains straight before and after deformation, but not necessarily normal after deformation, so that

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\,\theta_y(x,y) \\ v(x,y,z) &= u_0(x,y) + z\,\theta_x(x,y) \quad (3.19) \\ w(x,y,z) &= w_0(x,y) \end{aligned}$$

Where u,v,w and u_0, v_0, w_0 are displacement in the x,y,z directions at any point and at the midsurface respectively θx *and* θ_y and are the rotations of the midsurface normal about the x and y axes respectively . Also

$$\begin{aligned} & x = \sum N_i x_i, \qquad y = \sum N_i y_i \\ & u_0 = \sum N_i u_i \; v_0 = \sum N_i v_i \; w_0 = \sum N_i w_i \quad (3.20) \\ & \theta_x = \sum N_i \theta_{xi} \; \theta_y = \sum N_i \theta_{yi} \end{aligned}$$

Strain Displacement Relations

Green-Lagrange’s strain displacement relations are used throughout the structural analysis. The linear part of the strain is used to derive the elastic stiffness matrix and the nonlinear part of the strain is used to derive the geometric stiffness matrix. The linear strain displacement relations for a twisted shell element are:

$$\xi_{xl} = \frac{\partial u}{\partial x} + \frac{w}{R_x} + zk_x$$

$$\xi_{yl} = \frac{\partial v}{\partial y} + \frac{w}{R_y} + zk_y$$

$$\gamma_{xyl} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} + zk_{xy} \quad (3.21)$$

$$\gamma_{xzl} = \frac{\partial w}{\partial x} + \theta_x - \frac{u}{R_x} - \frac{v}{R_{xy}}$$

$$\gamma_{yzl} = \frac{\partial w}{\partial y} + \theta_y - \frac{v}{R_y} - \frac{u}{R_{xy}}$$

Where the bending strains are expressed as

$$k_x = \frac{\partial \theta_x}{\partial x}, \ k_y = \frac{\partial \theta_y}{\partial y}$$

$$k_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \quad (3.22)$$

The linear strains can be expresses interm of displacements as:

$$\{\varepsilon\} = [B]\{d_e\} \quad (3.23)$$

Where

$$\{d_e\} = \{u_1 v_1 w_1 \theta_{x1} \theta_{y1} \ldots\ldots\ldots\ldots\ldots\ldots. u_8 v_8 w_8 \theta_{x8} \theta_{y8}\} \quad (3.24)$$

$$[B] = [[B_1], [B_2].. \ \ldots\ldots\ldots\ldots\ldots\ldots. [B_8]] \quad (3.25)$$

$$[B_i] = \begin{bmatrix} N_{i,}x & 0 & \frac{N_i}{R_x} & 0 & 0 \\ 0 & N_{i,}y & \frac{N_i}{R_y} & 0 & 0 \\ N_{i,}y & N_{i,}x & 2\frac{N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,}x & 0 \\ 0 & 0 & 0 & 0 & N_{i,}y \\ 0 & 0 & 0 & N_{i,}y & N_{i,}x \\ 0 & 0 & N_{i,}x & N_i & 0 \\ 0 & 0 & N_{i,}y & 0 & N_i \end{bmatrix} \quad (3.26)$$

Constitutive Relations

The basic composite twisted curved panel is considered to be composed of composite material laminates (typically thin layers). The material of each lamina consists of parallel, continuous fibers (e.g. graphite,boron,glass) of one material embedded in a matrix material(e.g. epoxy resin). Each layer may be regarded on a macroscopic scale as being homogeneous and orthotropic. The laminated fiber reinforced shell is assumed to consist of a number of thin laminates as shown in figure 4. The principle material axes are indicated by 1 and 2 and moduli of elasticity of a lamina along these directions are E_{11} and E_{22} respectively. For the plane stress state,$\sigma_{0=0}$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \quad (3.27)$$

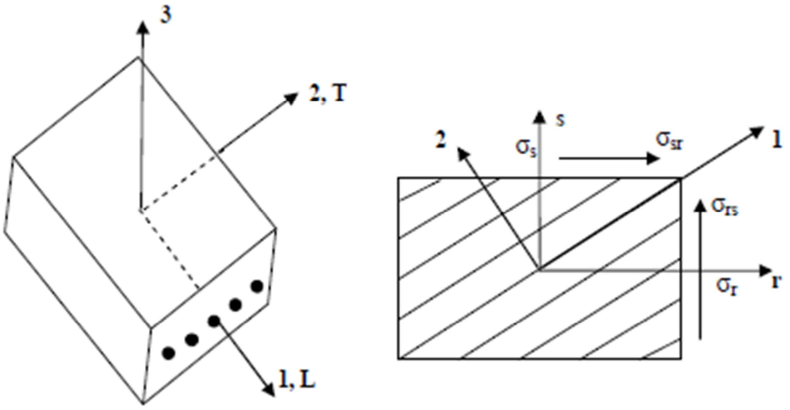

Figure 3.4: Laminated shell element showing principal axes and laminate directions

Where

$$Q_{11} = \frac{E_{11}}{(1-v_{12}v_{21})}$$

$Q_{12}= \frac{E_{11}v_{21}}{(1-v_{12}v_{21})}$

$Q_{21}= \frac{E_{22}}{(1-v_{12}v_{21})}$

$Q_{22}= \frac{E_{22}}{(1-v_{12}v_{21})}$(3.28)

$Q_{66}= G_{12}$

$Q_{44}=$ kG_{13}

$Q_{55}= kG_{23}$

The on –axis elastic constant matrix corresponding to the fiber direction is given by

$$[Q_{ij}] = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \tag{3.29}$$

If the major and minor Poisson's ratio are ν_{12} and ν_{21},then using reciprocal relation one obtains the following well known expression

$$\frac{\vartheta_{12}}{E_{11}} = \frac{\vartheta_{21}}{E_{22}} \tag{3.30}$$

Standard coordinate transformation is required to obtain the elastic constant matrix for any arbitrary principle axes with which the material principal axes makes an angle θ. Thus the off-axis elastic contant matrix is obtained from the on-axis elastic constant matrix as

$$[\bar{Q}_{ij}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \tag{3.31}$$

$$[\bar{Q}_{ij}] = [T]^T][Q_{ij}][T] \tag{3.32}$$

Where T is the transformation matrix. After transformation the elastic stiffness coefficient are.

$$\bar{Q}_{11} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4)$$

$$\bar{Q}_{22} = Q_{11}n^4 + 2(Q_{12} + Q_{66})m^2n^2 + Q_{22}m^4 \tag{3.33}$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3m$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 + Q_{66}(n^4 + m^4)$$

The elastic constant matrix corresponding to transverse shear deformation is

$$\bar{Q}_{44} = G_{13}m^2 + G_{23}n^2$$

$$\bar{Q}_{45} = (G_{13} - G_{23})mn \tag{3.34}$$

$$\bar{Q}_{55} = G_{13}n^2 + G_{23}m^2$$

Where m = cosθ and n = sinθ

The stress strain relations are

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{3.35}$$

The forces and moment resultants are obtained by integration through the thickness h for stresses as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_Y \\ M_{xY} \\ Q_x \\ Q_Y \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_x \\ \tau_{xy} \\ \sigma_x z \\ \sigma_{yz} \\ \tau_{xy} z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{3.36}$$

Where σ_x, σ_y are the normal stresses along X and Y direction τ_{xy},τ_{xz} and τ_{yz} are shear stresses in xy , xz and yz planes respectively.

Considering only in-plane deformation, the constitutive relation for the initial plane stress analysis is

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{31} & A_{32} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3.37}$$

The extensional stiffness for an isotropic material with matrial properties E and v are

$$[D_P] = \begin{bmatrix} \frac{Eh}{1-v^2} & \frac{Eh}{1-v^2} & 0 \\ \frac{vEh}{1-v^2} & \frac{Eh}{1-v^2} & 0 \\ 0 & 0 & \frac{Eh}{2(1+v)} \end{bmatrix} \tag{3.38}$$

The constitutive relationships for bending transverse shear of a doubly curved shell becomes

$$\begin{Bmatrix} N_x \\ N_P \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{112} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{3.39}$$

This can also be stated as

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix} \tag{3.40}$$

Or $\{F\} = [D]\{\varepsilon\}$ (3.41)

Where A_{ij},B_{ij},D_{ij} and S_{ij} are the extensional , bending-stretching coupling, bending and transverse shear stiffness. They may be defined as:

$A_{ij} = \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})$

$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^2 - z_{k-1}^2)$

$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^3 - z_{k-1}^3)\,; i,j = 1,2,6$ (3.42)

$S_{ij} = k \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})\,; i,j = 4,5$

And k is the transverse shear correction factor. The accurate prediction for anisotropic laminates depends on a number of laminate properties and is also problem dependent. A shear correction factor of5/6 is used in the present formulation or all numerical computations.

Derivation of element matrices

The element matrices in natural coordinate system are deriver as:

1. Element plane elastic stiffness matrix

$[k_P] = \int_{-1}^{1} \int_{-1}^{1} [B_P]^T [D_P][B_P]|J| d\xi d\eta$ (3.42)

2. Element elastic stiffness matrix

$[k_e] = \int_{-1}^{1} \int_{-1}^{1} [B]^T [D][B]|J| d\xi d\eta$ (3.43)

3. Generalized element mass matrix matrix or consistent mass matrix

$[m_e] = \int_{-1}^{1} \int_{-1}^{1} [N]^T [P][N]|J| d\xi d\eta$ (3.44)

Where the shape function matrix

$$[N] = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \text{ i=1, 2…..8} \tag{3.45}$$

$$[P] = \begin{bmatrix} P_1 & 0 & 0 & P_2 & 0 \\ 0 & P_1 & 0 & 0 & P_2 \\ 0 & 0 & P_1 & 0 & 0 \\ P_2 & 0 & 0 & P_3 & 0 \\ 0 & P_2 & 0 & 0 & P_3 \end{bmatrix} \tag{3.46}$$

And

$$(P_1, P_2, P_3) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_1} (\rho)_k (1, z, z^2) dz \tag{3.47}$$

Where [B],[D],[N] arethe strain-displacement matrix stress-strain and shape function matrix and $|J|$is the Jacobian determinant . [P] involves mass density parameters as explained earlier.

Geometric stiffness matrix

The element geometric stiffness matrix for the twisted shell is derived using the non-linear in-plane Green's strains with curvature component using the procedure explained by Cook, Malkus and Plesha[1989]. The geometric stiffness matrix is a function of in-plane stress distribution in the element due to applied edge loading. Plane stress analysis is carried out using the finite element technique to determine the stresses and these are used to formulate the geometric stiffness matrices.

$$U_2 = \int_V [\sigma^0]^T \{\varepsilon_{nl}\} dV \tag{3.48}$$

The non-linear strain components are sa follow:

$$\varepsilon_{xnl} = \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right] \qquad (3.49)$$

$$\varepsilon_{ynl} = \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right]$$

$$\gamma_{xnl} = \frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)\left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right) + z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right]$$

Using the non-linear strains, the strain energy can be written as

$$U_2 = \int_A \frac{h}{2}\left[\sigma_x^0\left\{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)^2\right\} + \sigma_y^0\left\{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \right.\right.$$

$$\left.\left.\left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)^2\right\} + 2\tau_{xy}^0\left\{\left(\frac{\partial u}{\partial y}\frac{\partial u}{\partial y}\right) + \left(\frac{\partial v}{\partial y}\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)\left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)\right\}\right] dxdy +$$

$$\int_A \frac{h^2}{24}\left[\sigma_x^0\left\{\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2 + \right\} + \sigma_y^0\left\{\left(\frac{\partial \theta_y}{\partial y}\right)^2 + \left(\frac{\partial \theta_x}{\partial y}\right)^2\right\} + 2\tau_{xy}^0\left\{\left(\frac{\partial \theta_y}{\partial x}\frac{\partial \theta_x}{\partial y}\right) + \right.\right.$$

$$\left.\left.\left(\frac{\partial \theta_x}{\partial x}\frac{\partial \theta_y}{\partial y}\right)\right\}\right] dxdy \qquad (3.50)$$

This can also be written as

$$U_2 = \frac{1}{2}\int_v [f]^r[S][f]dV \qquad (3.51)$$

Where

$$\{\mathrm{f}\} = \left[\frac{\partial \mathrm{u}}{\partial \mathrm{x}}, \frac{\partial \mathrm{u}}{\partial \mathrm{y}}, \frac{\partial \mathrm{v}}{\partial \mathrm{x}}, \frac{\partial \mathrm{v}}{\partial \mathrm{y}}, \left(\frac{\partial \mathrm{w}}{\partial \mathrm{x}} - \frac{\mathrm{u}}{\mathrm{R_x}}\right), \left(\frac{\partial \mathrm{w}}{\partial \mathrm{y}} - \frac{\mathrm{v}}{\mathrm{R_y}}\right), \frac{\partial \theta_\mathrm{x}}{\partial \mathrm{x}}, \frac{\partial \theta_\mathrm{x}}{\partial \mathrm{y}}, \frac{\partial \theta_\mathrm{y}}{\partial \mathrm{x}}, \frac{\partial \theta_\mathrm{y}}{\partial \mathrm{y}}\right]^\mathrm{r}$$

(3.52)

And

$$[S] = \begin{bmatrix} [s] & 0 & 0 & 0 & 0 \\ 0 & [s] & 0 & 0 & 0 \\ 0 & 0 & [s] & 0 & 0 \\ 0 & 0 & 0 & [s] & 0 \\ 0 & 0 & 0 & 0 & [s] \end{bmatrix} \tag{3.53}$$

Where

$$[s] = \begin{bmatrix} \sigma_x^0 & \tau_{xy}^0 \\ \tau_{xy}^0 & \sigma_y^0 \end{bmatrix} = \frac{1}{h} \begin{bmatrix} N_x^0 & N_{xy}^0 \\ N_{xy}^0 & N_y^0 \end{bmatrix} \tag{3.54}$$

The in-plane stress resultantsN_x^0, $N_x^0$and N_x^0 at each Gauss point are obtained separately by plane stress analysis and the geometric stiffness matrix is formed for these stress resultants

$$\{f\} = [G]\{q_e\} \tag{3.55}$$

Where

$$\{q_e\} = [u \;\; v \;\; w \;\; \theta_x \theta_x]^T \tag{3.56}$$

The strain energy becomes

$$U_2 = \frac{1}{2}[q]^r [G]^T [S][G]\{q\}dV = \frac{1}{2}\{q_e\}^T [K_e]_e [q_e] \tag{3.57}$$

Where the element geometric stiffness matrix

$$[K_e]_e = \int_{-1}^{1}\int_{-1}^{1} [G]^T [S][G]|J| d\xi d\eta \tag{3.58}$$

$$[G] = \begin{bmatrix} N_{i,}x & 0 & 0 & 0 & 0 \\ N_{i,}y & 0 & 0 & 0 & 0 \\ 0 & N_{i,}x & 0 & 0 & 0 \\ 0 & N_{i,}y & 0 & 0 & 0 \\ 0 & 0 & N_{i,}x & 0 & 0 \\ 0 & 0 & N_{i,}y & 0 & 0 \\ 0 & 0 & 0 & N_{i,}x & 0 \\ 0 & 0 & 0 & N_{i,}y & 0 \\ 0 & 0 & 0 & 0 & N_{i,}x \\ 0 & 0 & 0 & 0 & N_{i,}y \end{bmatrix} \tag{3.59}$$

3.4: ANSYS methodology

Introduction

In ANSYS problems include: static/dynamic structure analysis (both linear and non-linear), heat transfer and fluid problems, as well as acoustic and electro-magnetic problems. There are three stages:

Preprocessing

It is defining the problem

1. Define key points/lines/area/volumes
2. Define element type and material/geometric properties
3. Mesh lines/area/volumes as required

Solution

Assigning loads, constraints and solving

Here we specify the loads (point or pressure), constraints (translational and rotational) and finally solve the resulting set of equations.

Post-processing

Further processing and viewing the results in this stage one may wish to see.

1. List of nodal displacement

2. Element forces and moments

3. Deflection plots

4. Stress contour diagram

The present problem has been solved using ANSYS software. The twisted plate was first solved without loading in order to validate the methodology and the results compared to previous results for free vibration and buckling. Also the methodology was tested for a laminated composite plate with different types of edge loading and results compared to a result from a previous paper. The results matched closely in most cases. Then the software was run for studying the twisted plate with different types of in-plane loads.

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1: Introduction

In this chapter, the results of the vibration and buckling analysis of laminated composite twisted cantilever panels under different types of in-plane loads is presented. The evaluation is done using ANSYS software.

As explained in the previous chapter, the eight node isoparametric quadratic shell element is used to develop the finite element procedure. Shell element eight node 281is used with 8 by eight mesh.The first order shear deformation theory is used to model the twisted panels considering the effects of transverse shear deformation and rotary inertia. The vibration and stability characteristics of laminated composite pre-twisted cantilever panels subjected to in plane loads are studied.

4.2: Convergence study

The convergence study is first done for the three lowest non dimensional frequencies of free vibration of the square laminated composite twisted cantilever plates with an angle of twist of 20° for different mesh divisions and is shown in Table 4.1. Based on this study, an 8 x 8 mesh was chosen for solving the problem.

Table 4.1: Convergence of non-dimensional fundamental frequencies of two-layer laminated composite twisted plates.$[0^0/90^0]$

a/b =1, b/h = 250, $\Phi = 20^0$

a = b = 500mm, h = 2mm, ρ= 1580kg/m^3

E_{11} = 141.0GPa, E_{22} = 9.23GPa, ν_{12} = 0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa.

Mesh divison	Angle of twist	Non-dimensional frequency		
		1st frequency	2nd frequency	3rd frequency
4 x 4		21.56	22.3974	23.449
6 x 6	$\mathbf{20^0}$	22.7614	23.2433	24.1643
8 x 8		21.56	22.3974	23.449

4.3: Comparison with previous studies

In this study, the accuracy and efficiency of the solution are established through comparison with previous studies. The lowest non-dimensional frequency of a square laminated composite pre-twisted cantilever plate obtained by ANSYS is compared with those obtained by Sahu and Asha[2008] in Table 4.2. The results have also been obtained using the finite element formulation described in previous chapter using MATLAB.

Non-dimensional frequency $\varpi = \omega a^2 \sqrt{\dfrac{\rho}{E_{11} h^2}}$

Non-dimensional buckling load $\lambda = \dfrac{N_x b^2}{E_{22} h^3}$

Table 4.2: Comparison of non-dimensional frequency parameter for cross-ply plates with different ply lay-ups with previous study

a/b =1, b/h=250

Angle of twist	Non-dimensional frequency parameter	Sahu&Asha[2008]	Present study(ANSYS)	Present study[FEM]
0^0	$0^0/90^0$	0.4829	0.4827	0.4828
	$0^0/90^0/0^0/90^0$	0.6872	0.6831	0.6871
	$0^0/90^0/90^0/0^0$	0.9565	0.9562	0.9563
	$0^0/90^0/0^0/90^0/0^0/90^0/0^0/90^0$	0.7294	0.7291	0.7293
10^0	$0^0/90^0$	0.4800	0.4862	0.4799
	$0^0/90^0/0^0/90^0$	0.6831	0.6759	0.6830
	$0^0/90^0/90^0/0^0$	0.9508	0.9447	0.9505
	$0^0/90^0/0^0/90^0/0^0/90^0/0^0/90^0$	0.7251	0.7147	0.7249
20^0	$0^0/90^0$	0.4708	0.5015	0.4707
	$0^0/90^0/0^0/90^0$	0.6700	0.6728	0.6899
	$0^0/90^0/90^0/0^0$	0.9326	0.9295	0.9323
	$0^0/90^0/0^0/90^0/0^0/90^0/0^0/90^0$	0.7112	0.7112	0.7110
30^0	$0^0/90^0$	0.4540	0.4710	0.4539
	$0^0/90^0/0^0/90^0$	0.6461	0.6426	0.6459
	$0^0/90^0/90^0/0^0$	0.8993	0.8312	0.8990
	$0^0/90^0/0^0/90^0/0^0/90^0/0^0/90^0$	0.6858	0.6783	0.6856

In Table 4.3, the results of buckling of laminated composite twisted plates with different angles of twist and lamination done using ANSYS software are compared with previous results.

Table 4.3: Comparison of non-dimensional buckling load with angle of twist for cross-ply plates with different ply lay-ups

Angle of twist	Non-dimensional buckling load	Sahu&Asha[2008]	Present study (ANSYS)	Present study[FEM]
0^0	$0^0/90^0$	0.7106	0.7104	0.7097
	$0^0/90^0/0^0/90^0$	1.4432	1.4391	1.4383
	$0^0/90^0/90^0/0^0$	2.7891	2.7878	2.7861
	$0^0/90^0/0^0/90^0/0^0/90^0/0^0/90^0$	1.6254	1.6213	1.6256
10^0	$0^0/90^0$	0.6473	0.6947	0.6921
	$0^0/90^0/0^0/90^0$	1.4078	1.4072	1.3987
	$0^0/90^0/90^0/0^0$	2.7273	2.7260	2.7237
	$0^0/90^0/0^0/90^0/0^0/90^0/0^0/90^0$	1.5860	1.5853	1.5854
20^0	$0^0/90^0$	0.6473	0.6471	0.6433
	$0^0/90^0/0^0/90^0$	1.3114	1.3108	1.3112
	$0^0/90^0/90^0/0^0$	2.5405	2.5392	2.5336
	$0^0/90^0/0^0/90^0/0^0/90^0/0^0/90^0$	1.4774	1.4767	1.4771
30^0	$0^0/90^0$	0.5689	0.5687	0.5642
	$0^0/90^0/0^0/90^0$	1.1526	1.1520	1.1498
	$0^0/90^0/90^0/0^0$	2.2329	2.2317	2.2321
	$0^0/90^0/0^0/90^0/0^0/90^0/0^0/90^0$	1.2985	1.2978	1.2975

As seen from Table 4.2 and Table 4.3,the results are in good agreement.

Laminated composite plate with load

The non-dimensional buckling load factors for simply supported symmetric cross-ply square plates subjected to various linearly varying loads is compared with the results presented by Zhong and Gu[2007] and ABAQUS results(Zhong and Gu, 2007). The compressive force is expressed as

$$N_x = N_o \left(1 - \eta \frac{y}{b}\right)$$

Here η parameter determines the linear variation of the in-plane load. η =0 for example represents uniform compression.

Non-dimensional critical load is $k = \frac{N_o b^2}{E_T h^3} = \frac{\lambda_{cr} b^2}{E_T h^3}$

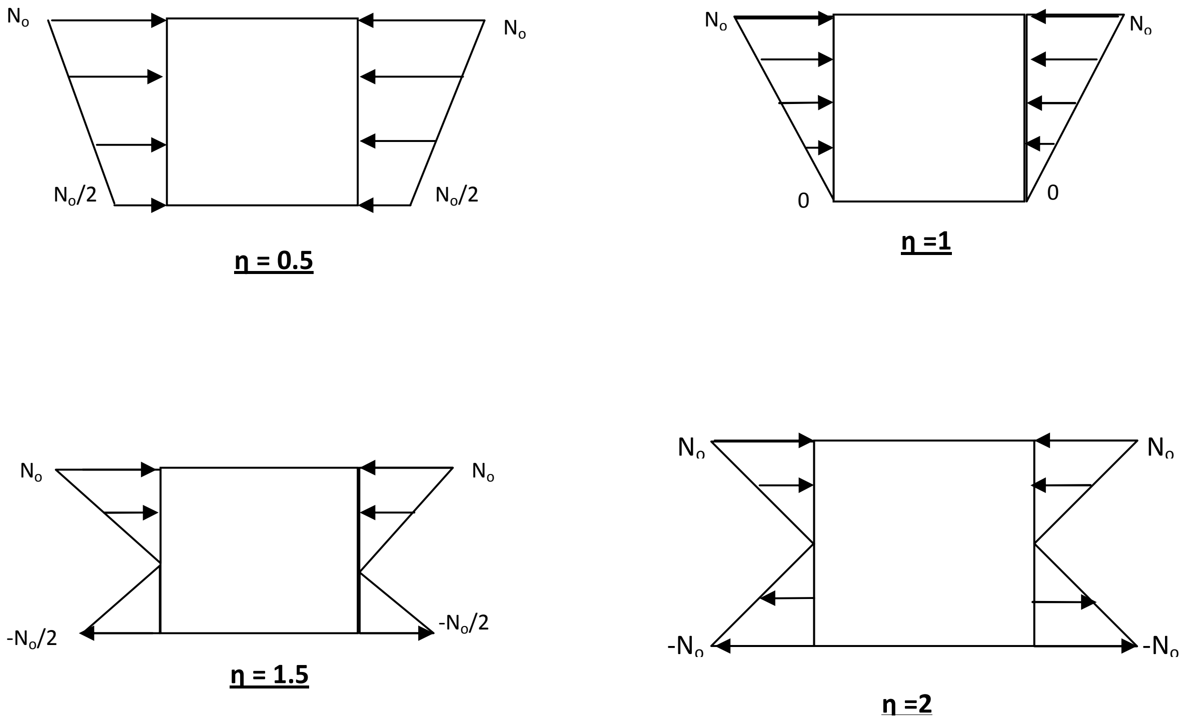

Figure 4.1: in-plane loads applied

Table 4.4: Comparison of non-dimensional buckling load factors for symmetric cross-ply square plates [$0^0/90^0/0^0$] subjected to various linearly varying loads

$\frac{E_L}{E_T}$ =40, $\frac{G_{LT}}{E_T} = \frac{G_{LZ}}{E_T}$=0.6, $\frac{G_{ZT}}{E_T}$=0.5, υ_{LT}=0.25

η	**h/b**	**Zhong and Gu [2007]**	**ABAQUS[Zhong and Gu][2007]**	**Present [ANSYS]**
0.5	0.01	47.267	47.267	47.404
	0.05	41.075	41.849	40.6
	0.1	29.432	29.121	29.12
	0.15	20.364	20.037	19.847
1.0	0.01	64.982	64.847	65.54
	0.05	56.705	56.392	56.145
	0.1	40.999	40.599	40.3
	0.15	21.131	21.559	27.54
2.0	0.01	129.785	129.450	166.67
	0.05	114.837	114.296	103.83
	0.1	47.872	49.955	51.81
	0.15	21.277	22.214	23.334

4.4: Numerical results

After validation, the studies are extended to investigation of the vibration and buckling characteristics of laminated composite pre-twisted cantilever panels under the action of various in-plane loads as described in Figure 4.1. Numerical results are presented to show the effect of various geometrical parameters like angle of twist, aspect ratio, thickness ratio, number of layers, etc. on the vibration and stability characteristics of laminated composite twisted cantilever panels under in-plane loads. Only cross-ply laminated twisted plates have been studied.

The geometrical and material properties of the twisted panels are

a = b= 500mm, h = 2mm, ρ = 1580kg/m^3

E_{11} = 141.0GPa, E_{22} = 9.23GPa, ν_{12} = 0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa.

Boundary conditions

The clamped boundary condition of the laminated composite twisted cross-ply panel using the first order shear deformation theory is

u = v = w=θ_x=θ_y= 0 at the left edge.

Vibration studies

The frequencies of vibration of twisted cantilever plates for different cross-ply and for varying angles of twist and subjected to different in-plane loads are shown in Table 4.5. The laminated composite material properties are as mentioned earlier. Two, four and eight layer anti symmetric lay-ups as well as a four layer symmetric cross-ply lay-up was taken for the study.

The compressive force is expressed as

$$N_x = N_o\left(1 - \eta\frac{y}{b}\right)$$

Here η parameter determines the linear variation of the in-plane load. η = 0 for example represents uniform compression (figure 4.1)[24]

It is found that as the angle of twist increases for a particular cross-ply orientation, the frequency increases for a particular load pattern. This is true for both the symmetric as well as anti-symmetric cross-ply stacking sequences. And for load (η=0.5, 1) the frequencies obtained are quite similar.

Table 4.5: Variation in frequency (Hz) with changing angle of twist for symmetric cross-ply square plates [$0^0/90^0$] subjected to various linearly varying loads

a/b=1, b/h = 250, a = b= 0.5m, h= 0.002m

Angle of twist	frequency in Hz	Free vibration(rad/sec)	η=0.5	η = 1	η =2
$\mathbf{0^0}$	1st frequency	36.50	5.772	5.772	6.899
$\mathbf{10^0}$	1st frequency	36.28	6.043	6.122	6.101
$\mathbf{20^0}$	1st frequency	35.58	6.003	6.003	5.957
$\mathbf{30^0}$	1st frequency	34.315	5.633	5.633	5.694

Table 4.6 shows the variation of the frequency in hertz for a particular in-plane load given by η = 0.5 with changing angle of twist for a two layer cross-ply twisted plate. The frequencies are

observed to increase as the angle of twist increases upto 10° and then it decreases as the angle of twist increases.

Table 4.6: Variation in frequency in Hz with changing angle of twist for symmetric cross-ply square plates [$0^0/90^0$] subjected to linearly varying load(η = 0.5)

b/h = 250, h =0.002 m

frequency in Hz	**Angle of twist**		
	10^0	20^0	30^0
1st frequency	6.0426	6.003	5.633
2nd frequency	36.876	35.737	32.284
3rd frequency	85.915	101.52	97.75

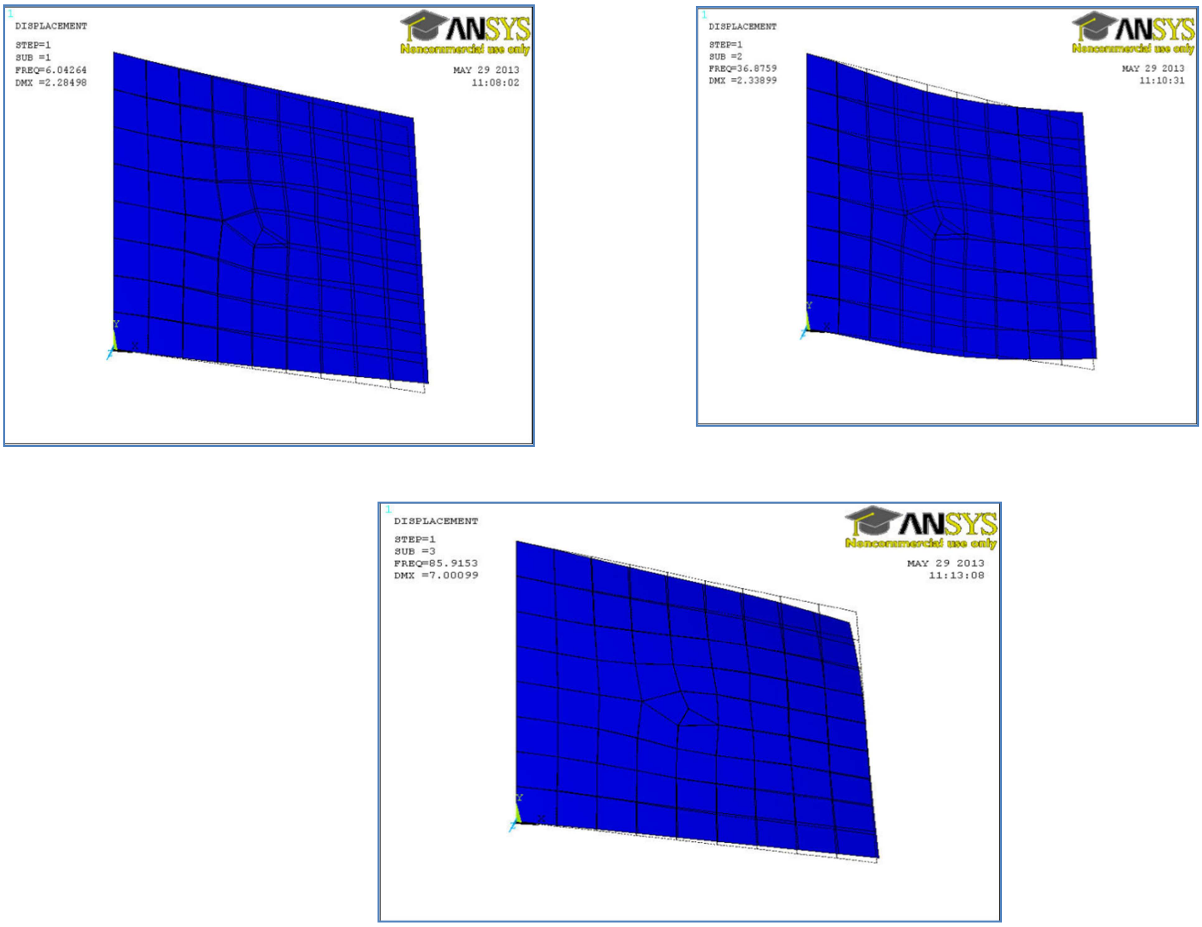

Fig. 4.2deformed shape of twisted plate in first three modes for η = 0.5, two-layer plate [$0^0/90^0$], Φ =10^0

In Table 4.7, the aspect ratio is varied for a two-layer cross-ply twisted plate with angle of twist Φ = 20° and the effect of an in-plane load with η = 0.5 is studied.

Table 4.7: Variation in frequency in Hz with changing aspect ratio for symmetric cross-ply square plates [$0^0/90^0$] subjected to linearly varying load(η = 0.5)

b/h = 250, h =0.002m

		Aspect ratio(a/b)	
		1	**2**
Frequency in Hz	**1st frequency**	6.003	1.599
	2nd frequency	35.737	9.692
	3rd frequency	101.52	27.16

Here it is observed that the frequency decreases as the aspect ratio increases.

The following table investigates the behavior of the twisted plate under load as the number of layers changes. The load is kept same.

Table 4.8: Variation of frequency in Hz with number of layers for cross-ply square plates subjected to linearly varying loads

a/b=1,η=0.5

h=0.002m, Φ=20^0

		Lamination sequence		
		$\mathbf{0^0/90^0}$	$\mathbf{0^0/90^0/0^0}$	$\mathbf{0^0/90^0/0^0/90^0}$
Frequency in Hz	**1st frequency**	6.003	8.065	8.055
	2nd frequency	35.737	36.767	48.31
	3rd frequency	101.52	113.09	139.54

As can be observed, the frequency in hertz generally increases as the number of layers increases.

Table 4.9 studies the variation in frequency parameter as the b/h ratio changes.

Table 4.9: Variation of frequency in Hz with b/h ratio for cross-ply square plates [$0^0/90^0$] subjected to a linearly varying load

a/b =1,a = b= 0.5m, η= 0.5, Φ =20^0

		b/h ratio		
		100	**250**	**300**
Frequency in Hz	**1st frequency**	15.10	6.003	5.00
	2nd frequency	90.07	35.737	29.78
	3rd frequency	173.98	101.52	84.84

Here too the frequency is observed to decrease as the b/h ratio increases.

Buckling studies

The buckling behaviour of twisted plates subjected to various in-plane loads is next studied. In the first study, keeping the number of layers same, the loads are varied, i.e., η is changed and the behavior is investigated. This is shown in Table 4.10.

Table 4.10: Variation in buckling load in N/m with changing angle of twist and load for a two-layer cross-ply twisted square plate [0^0/90°] subjected to linearly varying loads

a/b =1, b/h = 250, a =b = 0.5m, h= 0.002m

Angle of twist	**Buckling load in N/m**	**uniform compression**	**η =0.5**	**η = 1**	**η =2**
$\mathbf{0^0}$	1st mode	209.88	276.94	386.26	905.8
$\mathbf{10^0}$	1st mode	204.99	299.53	444.29	2777.3
$\mathbf{20^0}$	1st mode	190.95	288.61	430.7	3788.5
$\mathbf{30^0}$	1st mode	167.83	270.4	406.45	4035.8

The behavior with changing aspect ratio is studied next. Table 4.11 shows that as aspect ratio increases, the buckling load decreases.

Table 4.11: Variation of buckling load with aspect ratio for a two-layer cross-ply square plates [$0^0/90^0$] subjected to a linearly varying load

$\eta = 0.5$, h = 0.002m, $\Phi = 20^0$

		Aspect ratio(a/b)	
		1	**2**
Buckling load in N/m	**1^{st} mode**	288.61	87.32
	2^{nd} mode	2501.06	721.92
	3^{rd} mode	6908.60	1974.30

Here it is observed that the buckling load decreases as the aspect ratio increases

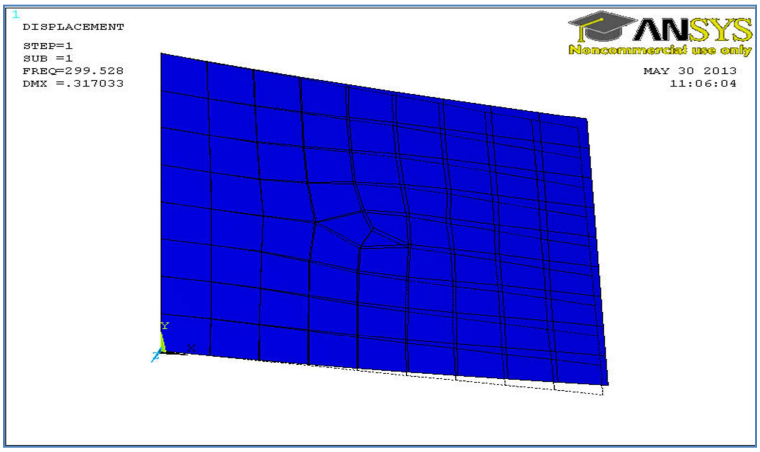

Fig. 4.3 First mode of buckling for $\eta = 0.5$, two-layer plate [$0^0/90^0$], $\Phi = 10^0$

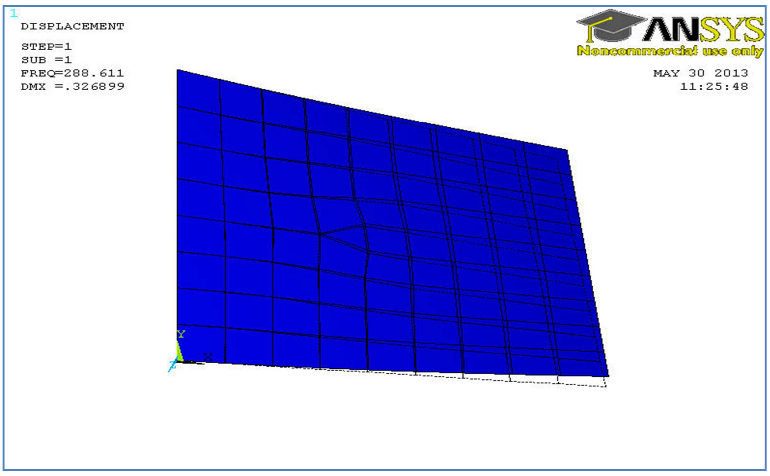

Fig. 4.4 First mode of buckling for $\eta = 0.5$, two-layer plate [$0^0/90^0$], $\Phi = 20^0$

The effect of changing lamination sequence is presented in Table 4.12.

Table 4.12: Variation of buckling load with number of layers for cross-ply square plates subjected to linearly varying loads

a/b =1,η = 0.5, h = 0.002m, Φ =20^0

		Lamination sequence		
		$0^0/90^0$	$0^0/90^0/0^0$	$0^0/90^0/0^0/90^0$
Buckling load in N/m	**1st mode**	288.61	374.54	542.01
	2nd mode	2501.06	3042.80	4661.80
	3rd mode	6908.60	9623.50	12809

The buckling load increases with increase in the number of layers as can be observed from Table 4.12. Next the effect of b/h ratio on the buckling load is studied keeping the load pattern same.

Table 4.13: Variation of buckling load with b/h ratio for a two-layer cross-ply square plate [$0^0/90^0$] subjected to linearly varying load

a/b =1, a = b= 0.5m, η=0.5, Φ =20^0

		b/h ratio		
		100	250	300
Buckling load in N/m	**1st mode**	4681.40	288.61	172.80
	2nd mode	38448.00	2501.06	1457.10
	3rd mode	84919.00	6908.60	4204.40

As the b/h ratio increases, the buckling load decreases.

CHAPTER 5

CONCLUSION

The behavior of vibration and buckling of laminated composite twisted plate subjected to various types of in-plane loading was studied. The works are done in ANSYS and MATLAB. Here effects of geometrical parameters like angle of twist, aspect ratio, number of layers and change in thickness of laminated composite plate on the vibration and buckling parameters has been analysed.

Based on the study, it is observed that for increasing angles of twist of laminated composite plate with different in-plane load conditions, the vibration and buckling both decreases.

Also as the number of layers increases, the vibration and buckling parameters of the laminated twisted plate are both observed to increase.

Asthe b/h ratio of laminated composite twisted plate increases, the vibration and buckling parameters are decrease.

Scope of further study:

The study may be extended to anisotropic laminated composite twisted plates since here only cross-ply lamination was considered.

The effects of harmonic loading may also be studied. Also the effect of lateral loads is another field for further study since fluid pressure may also act normally.

The study may also be extended to functionally graded materials since these are being increasingly used nowadays.

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