FISH SCHOOL SEARCH: AN INTERVAL REPRESENTATION

A Thesis

Submitted in Partial Fulfilment of the

Requirements for the Award of Degree of

MASTER OF SCIENCE

in **Mathematics**

by

Subhashree Jena

under the supervision of

Prof. S. Chakraverty

DEPARTMENT OF MATHEMATICS

National Institute of Technology, Rourkela

Odisha - 769008

MAY, 2013

National Institute of Technology Rourkela

Declaration

I hereby certify that the work which is being presented in the thesis entitled **"Fish School Search: An Interval Representation"** in partial fulfilment of the requirements for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology, Rourkela is an authentic record of my own work carried out under the supervision of Dr. S. Chakraverty.

The matter embodied in this thesis has not been submitted by me for the award of any other degree.

Date: May, 2013 **(Subhashree Jena)**

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Dr. S. Chakraverty

Professor, Department of Mathematics,

National Institute of Technology Rourkela.

Acknowledgement

First and foremost, I thank the *almighty* for enabling me to do this project.

I express my profound sense of gratitude to my supervisor **Prof. S. Chakraverty** for his invaluable support, guidance and encouragement for the successful completion of this project. His vision and spirit will always inspire and enlighten me.

My sincere thanks to **Prof. G. K. Panda**, Head, Department of Mathematics, NIT Rourkela, the project coordinator Dr. R. S. Tungala and all the faculties of Department of Mathematics, NIT Rourkela for their guidance and technical support in due time.

I am pleased to acknowledge my sincere thanks to **my parents, Ph.D. scholars and friends** for their great inspiration and help during the course of project.

Last, but certainly not the least, I would like to thank **Prof. B. de Lima Neto Fernando** and **Mr. Anthony Lins**, Brazil for their valuable suggestions regarding the FSS procedure and programming constructs (through emails).

(Subhashree Jena)

Abstract

Fish species compared to animals show complicated behavior mostly to increase their survivability. One may understand the phenomenon by two different ways viz., for mutual protection and for synergic achievements of other collective tasks.

As per the literature, there exist some studies related to the above collective goals for finding food by considering the data as a crisp or exact form. But in actual practice the positions of fish at each instant of time may not be obtained in crisp. But those should be taken in uncertain form.

Here this uncertainty has been taken in terms of interval. Hence in the thesis, a new form of fish school search has been proposed. Accordingly the interval computation has been implemented to obtain the fish position and hence the optimization process goes in a new direction.

Table of Contents

List of Figures

List of Tables

List of Abbreviations

SPA	**Search Problems and Algorithms**
PBA	**Population Based Algorithms**
GA	**Genetic Algorithm**
AIS	**Artificial Immune System**
ACO	**Ant Colony Optimization**
PSO	**Particle Swarm Optimization**
FSS	**Fish School Search**
EA	**Evolutionary Algorithm**

Introduction

There are some fish species which spent their entire lives in aquariums as a result their individual freedom in terms of swimming ability reduces and competetion level increases in the regions of scarce food. Purpose of living in a school is to increase mutual survivability [4].

Fish school search is an engineering approach to describe the natural behavior of the fish school through a computer model. In this model a group of fish search food where their positions play an important role to optimize the search process. Fish school search is a nature-inspired searching technique which is [4,5]

1. able to handle the high dimensionalities of search spaces and
2. a population based approach affected by the collective emerging behavior to increase mutual survivability.

The search process in FSS is based on a population of limited memory individuals. Each fish represents a possible solution. FSS is driven out by the success of some individual members of the population. The fishes contain only their innate memory (i.e. their weights) that help to keep a log of best positions visited, their velocities and other competitive global variables. The bary-center of the whole school guides expansion and contraction of the school invoking exploration and exploitation when necessary [3,4].

Development of the FSS technique is based on the following categories of behaviors:[12]

1. **Feeding** is inspired by natural instinct of individuals to find food in order to grow strong. Food is a metaphor to obtain the candidate solution in the search process. Weight of the fish increases or decreases depending on the region it swims in.

2. **Swimming** aims at mimicking the coordinated and the collective movement produced by all the fish. It is driven by feeding needs.

Another major feature of FSS is the idea of evaluation through a combination of some collective swimming i.e., operators that select among different modes of operation during the search process on the basis of instantaneous results.

FSS is composed of operators that can be grouped in the following categories feeding, swimming and breeding. These operators together afford computational features [12] such as:

a. high-dimensional search abilities

b. on-the-swim selection between exploration and exploitation and

c. self-adaptable guidance towards sought solutions.

1.2 Background:

1.2.1 Search problems and Algorithms(SPA): [4,5]

There are several approaches for searching but unfortunately no general optimal search strategy exists. Although custom made algorithms have valuable option for specific problems, a more generalized automatic search engine would be great for tackling problems of high dimensionality.

Search problems are highly varying. For example, they can be classified into two groups with regard to the structure of their search space viz., **structured** or **unstructured**. For the structured case, there are many traditional techniques that are quite efficient. FSS may be a valuable option for searching in high dimensional and unstructured spaces.

1.3 **Population-based Algorithms (PBA):** [4]

Many nature inspired algorithms such as genetic algorithms (GA), artificial immune system (AIS), ant colony optimization (ACO) are based on the concept of population. In all these approaches the computing discrimination power and memorization ability of past experiences are distributed among the individuals of population in varying degrees.

Real world problems are quite often complex in nature and most of the time they are hard to compute since they are associated with the large dimensionality of the search space and the high cardinality of solutions. Therefore searching parameters or candidate solutions is costly and sometimes unfeasible by single-track computation.

Distributed representation and computation provide parallization features in the search algorithms. The obvious trade off is the cost of control (i.e. communication among the individuals) which is opposed to the lower costs associated with the centralized control.

1.3.1 **Particle Swarm Optimization (PSO):** [10]

PSO is an intelligent computational technique proposed by **Kennedy and Eberhart** in 1995. This technique is inspired by the social behavior of bird flocks and is used for the optimization of non-linear functions. The idea behind PSO is to create particles that simulate the movements of birds to achieve a specific goal within the search space. The entire swarm uses a specific communication mechanism and the candidate solution emerge by flocking behavior around more successful individuals with the notion of adjustable speed according to the degree of success achieved.

Bratton and Kennedy defined a standard for comparison of different PSO procedures

[11]. It produced good results for search problems with high-dimensionality. However, the PSO technique struggles in some multimodal problems.

1.3.2 **Genetic Algorithm (GA):**[13]

In the field of Computer Science and Artificial Intelligence, the genetic algorithm is a search heuristic that mimics the process of natural evolution. In GA, a population of candidate solutions (called individuals) to an optimization problem is evolved towards better solutions. Each candidate solution has a set of properties which can be altered. Traditionally, solutions are represented in binary strings of 0s and 1s.

The evolution usually starts from a population of randomly generated individuals and is an iterative process, with the population in each iteration called as a *generation*. In each generation, the fitness of every individual in the population is evaluated. The fitness is the value of the objective function in the optimization problem. More fit individuals are stochastically selected from the current population and each individual's genome is modified to form a new generation. The new generation of candidate solutions is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations have been produced or a satisfactory fitness level has been reached for the population. A typical genetic algorithm requires :

1. a genetic representation of the solution domain and
2. a fitness function to evaluate the solution domain.

The procedure of genetic algorithm is described in Figure 1.1 in the form of a data flow diagram.

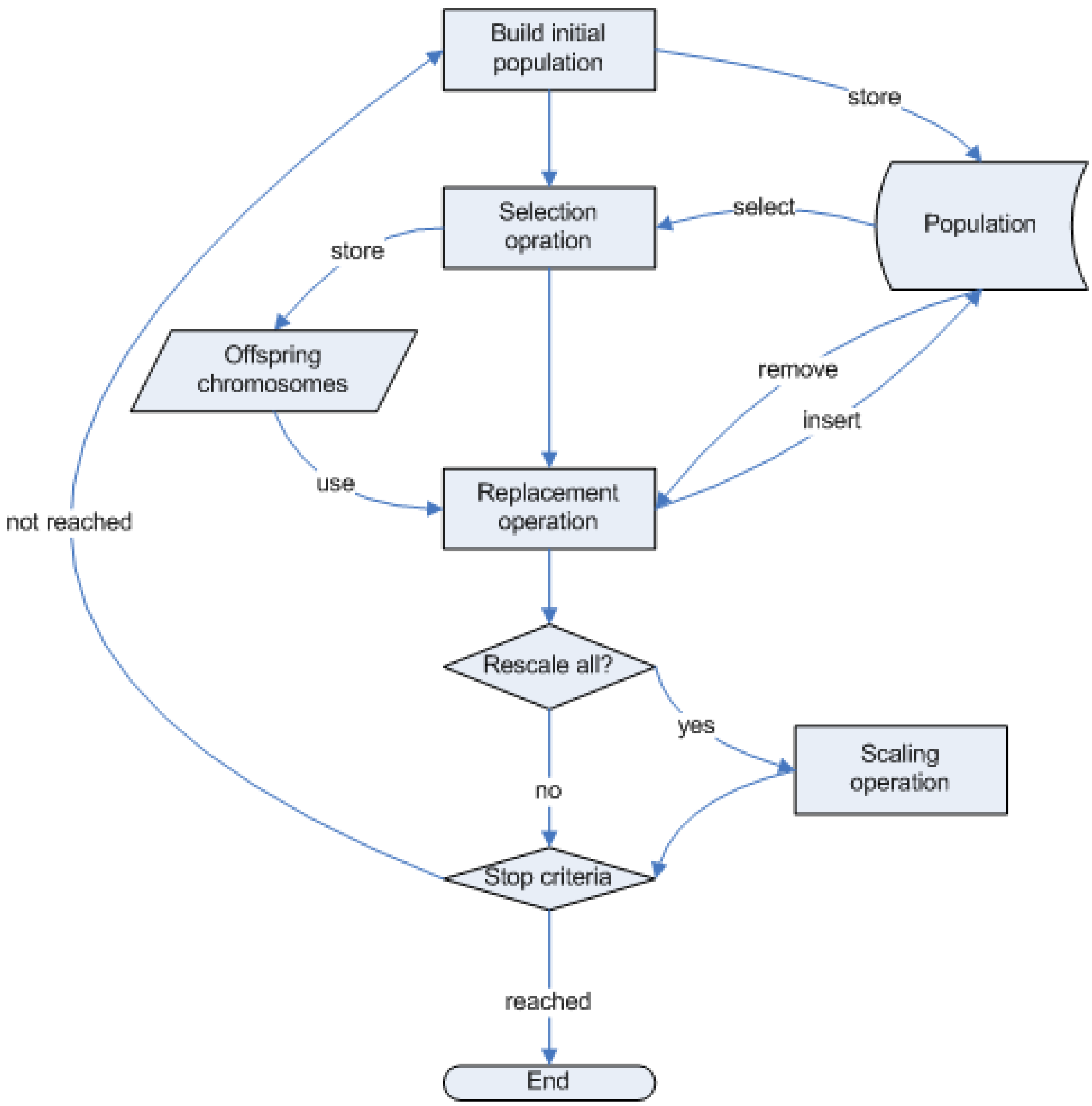

Fig. 1.1 Flowchart of GA [13]

1.3.3 **Ant Colony Optimization (ACO):** [9]

Naturally ants wander randomly (initially). After finding food ants return to their colony laying down pheromone trails. A short path gets marched over more frequently and thus the pheromone density becomes higher on those paths than the longer ones. Pheromone evaporation also has the advantage of avoiding the convergence to a locally optimal solution. If there were no evaporation at all the paths chosen by the first ants would end to be excessively attractive to the following ones. In that case the exploration of the solution space would be constrained. Thus when one ant finds a good (i.e. short) path from the colony to a food source, other ants follow that path and positive feedback eventually leads all the ants following a single path.

The ant colony optimization algorithm is a probabilistic technique for solving computational problems reduced to find good paths through graphs. It was initially proposed by Marco Dorigo in 1992. The first algorithm aimed to search an optimal path in a graph based on the behaviour of ants. It is now highly diversified to solve a wider class of numerical problems. The idea of the ant colony algorithm is to mimic this behaviour with "simulated ants" walking around the graph representing the problem to solve. In the following figure1.2 how the ant colony search for food is shown.

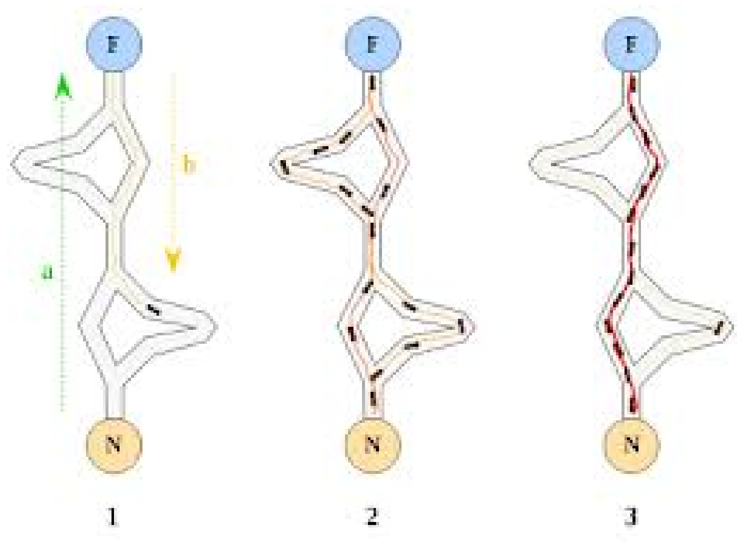

Fig. 1.2 Procedure of ant colony in search of food [13]

1.4 **Literature Survey:**

Tian and Sannomiya [1] proposed an aggregated model for studying the behavior of a fish school with many individuals. In this model the motion of a fish school is described by the motion of the center of gravity of the school and four representative individuals which are located at the boundary of the search space.

Janecek and Tan [2] investigated and compared different weight update strategies for the recently developed Fish School Search (FSS) algorithm. For the first time they introduced a new dilation multiplier as well as different weight update steps in which the individual and volitive step parameters decrease non-linearly. It speeds up the convergence of the procedure.

Filho et.al [3] introduced a approach for searching in high-dimensional spaces based on the behaviors of fish schools. This search process is highly benefited from the collective emerging behavior of the school. [3] is then extended in a book of Springer publication [4]. Filho et.al [5] again investigated certain update strategies to provide exploitation behavior in FSS procedure and speed up the process of convergence of the process.

Amintoosi et. al [6] developed the fish school clustering algorithm based on the fish school behaviors and extended the classical flock model of Reynolds with leader fishes and follower fishes. They also applied it to section the students in an institute and their time-tabling problem.

In real life measurements can not be exact always because of the instrumental errors. Also in case of programming constructs, we obtain the approximate solution but not the exact. Hence the efficiency and exactness of the procedures is measured based on certain error bound. In traditional FSS all the positions are considered as crisp values and the search process continues. To overcome the uncertainty in the crisp values the interval computation procedure is introduced thereby improving the process.

Traditional FSS Procedure

There are some fish species which spent their entire lives in aquariums that reduces individual freedom in terms of swimming ability and increases competetion level. Purpose of living in a school is to increase mutual survivability viz., **Mutual protection** by reducing the chances of being chased and caught by the predators and **Doing collective tasks** means achieving collective goals of finding food. [3,5,6]

The behavior of the fish school is due to learning or genetic responce and the fish aggregation helps to overcome the drawbacks. The main characteristics of fish school search can be categorized into two types and they are:[2,3,4,5]

Feeding: It is inspired by natural instinct of fish to find food in order to grow strong and able to breed. In the search process food is considered as a metaphor for the evaluation of candidate solutions. Depending on the regions the fish swims in, the indivisual may lose or gain weight.

Swiming: It is the most observable and elaborated behaviour in the search process since it is the only remarkable and coordinated collective movement of the school. The swimming operator depends upon the feeding needs.

Based on the behaviors of the fish school some operators are defined as follows:

a. Feeding operator

b. Swimming operator

3.1 **Feeding operator:** [3]

The fishes are attracted towards the food scattered in various locations in different concentrations. As a result a fish can grow or shrink in weight, depending on its success or failure in obtaining the food. The fish's weight variation is proportional to the normalized difference between the evaluation of fitness function of current and previous fish position with respect to food concentration at these spots.

$$w_i(t+1) = w_i + \frac{f[x_i(t+1)] - f[x_i(t)]}{max|f[x_i(t+1)] - f[x_i(t)]|} \tag{1}$$

where $w_i(t)$ represents weight of fish i,

$x_i(t)$ represents position of fish i,

$f[x_i(t)]$ evaluates the fitness function of fish i at position $x_i(t)$,

Initially for each fish i, $1 <= w_i = \frac{W_{scale}}{2} = W_{scale}$.

3.2 **Swimming operator** [3]

Here swimming is considered to be an elaborate form of reaction for living since swimming is related to all important individual and collective behaviors such as searching food, feeding, escaping from predators and also moving into livable regions. In FSS swimming pattern of fish school is the result of a combination of three different movements i.e.

a. Individual movement

b. Collective instinctive movement

c. Collective volitive movement

3.2.1 **Individual movement:** [3]

It occurs for each fish in every cycle. To determine the displacement an individual movement parameter $step_{ind}$ is estimated which decreases linearly.

$$step_{ind}(t+1) = step_{ind}(t) - \frac{step_{indinitial} - step_{indfinal}}{totalno.ofiterations} \tag{2}$$

3.2.1 **Collective instinctive movement:** [3]

A weighted average of individual fish movements based on instantaneous success of all fishes is carried out in this phase i.e.

$$x_i(t+1) = x_i(t) + \frac{\sum_{i=1}^{N} \Delta x_{indi} f[x_i(t+1)] - f[x_i(t)]}{\sum_{i=1}^{N} f[x_i(t+1)] - f[x_i(t)]} \tag{3}$$

3.2.1 **Collective volitive movement:** [3]

It is the final positional adjustment of all fishes in the school. A parameter $step_{vol}$ is defined which will be inwards or outwards with respect to the school's bary-center. The bary-center is given by

$$Bari(t) = \frac{\sum_{i=1}^{N} x_i(t) * w_i(t)}{\sum_{i=1}^{N} w_i(t)} \tag{4}$$

If the overall weight of the school increases, it represents success of swimming. Consequently which makes the radius of contract towards the bary-center and the corresponding positions will be

$$x_i(t+1) = x_i(t) - step_{vol} * rand * [x_i(t) - Bari(t)] \tag{5}$$

If the overall weight of the school decreases, it represents success of swimming. Consequently which makes the radius of contract towards the bary-center and the corresponding positions will be

$$x_i(t+1) = x_i(t) + step_{vol} * rand * [x_i(t) - Bari(t)] \tag{6}$$

Numerical Evaluation through Traditional FSS

EXAMPLE 1:

The selected example [4] considers a small school with three fishes is set to find the minimum of the sphere function in two dimensions i.e. $\sum_{i=1}^{n}(x_i)^2$
with the given parameters as:

Feasible space	$= [-10,10]$	Initial $step_{vol}$	$= 0.1$
No. of iterations	$= 20$	Final $step_{vol}$	$= 0.01$
W_{scale}	$= 10$		
Initial $step_{ind}$	$= 1$		
final $step_{ind}$	$= 0.1$		

The initial positions,weights and corresponding fitness values of fishes are selected in a random manner and given in table-3.1.

Table.3.1 Initial conditions of fishes in the example

Fish	Weight	Position	Fitness
fish-1	5	(9,7)	130
fish-2	5	(5,6)	71
fish-3	5	(8,4)	80

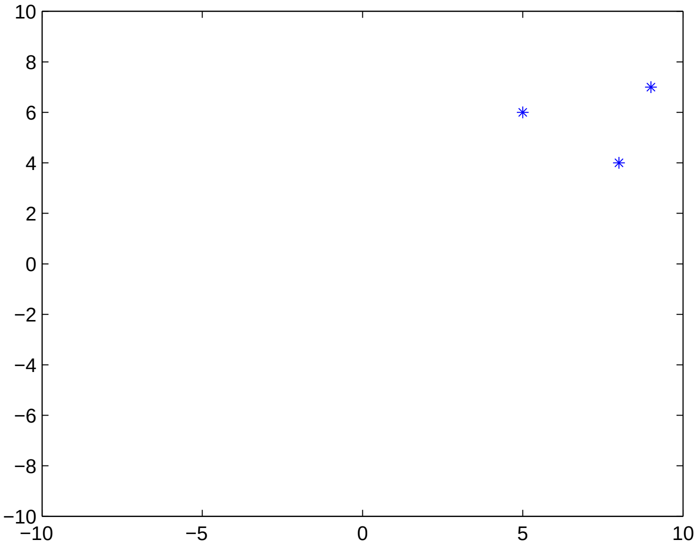

Fig. 3.1 Plot of school (initial position)

After 1st Iteration:

Table.3.2 Result after 1st iteration

Fish	Weight	Position	Fitness
fish-1	5	(9.4197,8.0037)	152.7906
fish-2	4	(4.4580,5.4994)	50.1174
fish-3	4.7068	(7.7208,5.0751)	50.1174

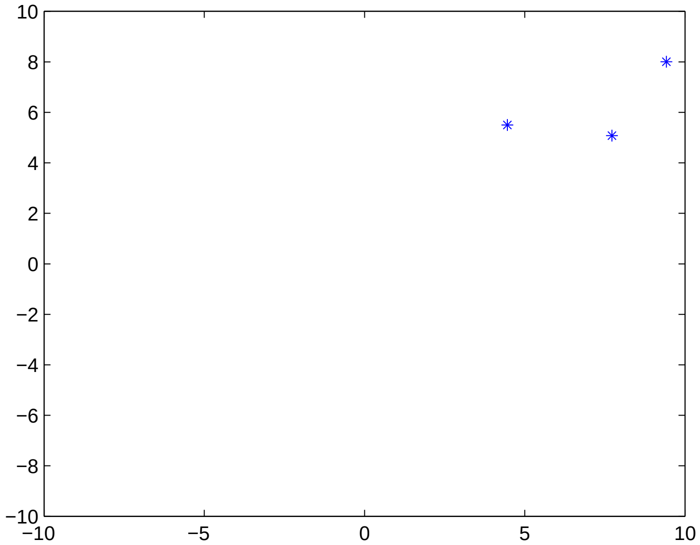

Fig. 3.2 Plot of school first iteration

After 5th Iteration:

Table.3.3 Result after 5th iteration

Fish	Weight	Position	Fitness
fish-1	2	(10.9318,10.2190)	223.9339
fish-2	2.9850	(4.9885,5.7812)	58.3072
fish-3	3.7996	(7.4647,2.2294)	50.1174

Here the position of fish-1 is exceeding the search space boundary of the given problem. Hence this position is neglected and randomly generated position assigned to fish-1 is $x_1 = (2.3226, 3.6785)$.

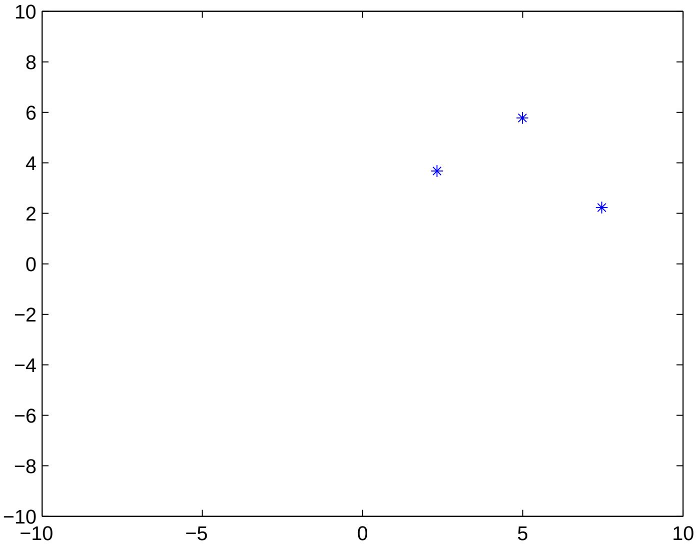

Fig. 3.3 Plot of school after 5th iteration

After 10th Iteration:

Table.3.4 Result after 10th iteration

Fish	Weight	Position	Fitness
fish-1	3.9500	(1.4623,4.7566)	24.7641
fish-2	4	(5.5713,7.0285)	80.4386
fish-3	2.5968	(8.7160 7.3983)	130.7045

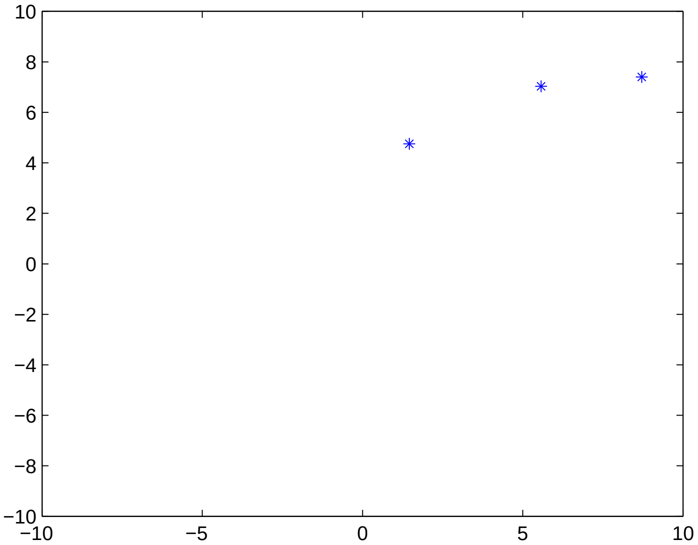

Fig. 3.4 Plot of school after 10th iteration

After 20th Iteration:

Table.3.5 Result after 20th iteration

Fish	Weight	Position	Fitness
fish-1	1.7526	(3.5088, 1.3662)	14.1782
fish-2	3.5009	(3.7662, -0.9629)	15.1115
fish-3	5.0233	(4.8780, 6.0525)	60.4276

Interval Computation of FSS

Interval data representation is very useful to study group of objects described by quantitative variables. Describing a group of objects on each variable by an interval of values rather than by a mean value, allows to reflect the variability that underlies the observed measurement. Many data analysis techniques have been extended to treat such new data description. A question frequently asked is ”Are the results obtained with intervals different than those obtained with means?” It is very difficult to answer this question because the data tables are different.

5.1 Interval Arithmetic:

Definition: [7]

The interval arithmetic is an extension of ordinary arithmetic . We shall denote A^I by a closed interval of the form ,

$$A^I = [\underline{a}, \bar{a}] = a|\underline{a} \leq a \leq \bar{a}, \underline{a}, \bar{a} \in \mathbb{R}$$

and define the center and radius of A^I respectively as follows ,

$$\text{center}: \; a^c = \tfrac{1}{2}(\underline{a} + \bar{a}) \; ,$$
$$\text{radius} \; \triangle a = \tfrac{1}{2}(\bar{a} - \underline{a})$$

thus,

$$A^I = [a^c - \triangle a, a^c + \triangle a] = a^c + \triangle a[-1, 1]$$

The right side of the above equation is so called center-radius representation of the interval. The absolute value is defined by the following equation and can be written in terms of center and radius

$$\begin{aligned}|A^I| &\triangleq max(|\underline{a}|, |\bar{a}|) \\ &= |a^c + \triangle a|\end{aligned}$$

In an interval $[\underline{a}, \bar{a}]$, we denote $\underline{a}$ as R and $\bar{a}$ as L.

Let $A^I and B^I$ be two intervals and * be one of the binary operators (+,-,*,/). The interval arithmetic of two intervals is a set defined as

$$A^I * B^I = a * b | a \;\in\; A^I, b \in B^I$$

i.e.

Let $[\underline{x}, \bar{x}]$ and $[\underline{y}, \bar{y}]$ be two elements then the following arithmetic are well known [8]

(i) $[\underline{x}, \bar{x}] + [\underline{y}, \bar{y}] = [\underline{x} + \underline{y}, \bar{x} + \bar{y}]$

(ii) $[\underline{x}, \bar{x}] - [\underline{y}, \bar{y}] = [\underline{x} - \bar{y}, \bar{x} - \underline{y}]$

(iii) $[\underline{x}, \bar{x}] * [\underline{y}, \bar{y}] = [min\ (\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}), max\ (\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y})\ ]$

(iv) $[\underline{x}, \bar{x}]/[\underline{y}, \bar{y}] = [min\ (\underline{x}/\underline{y}, \underline{x}/\bar{y}, \bar{x}/\underline{y}, \bar{x}/\bar{y}), max\ (\underline{x}/\underline{y}, \underline{x}/\bar{y}, \bar{x}/\underline{y}, \bar{x}/\bar{y})\ ]$

5.2 Interval FSS Pseudo-code:

The following procedure is adopted in accordance with the traditional method, for the interval computation of the fish school search algorithm.

Algorithm:

Initiate randomly all fishes;

Section the problem into LR and RL parts;

For each section do

while (stop criterion is not reached) do

For each fish in school do

Individual movement;

Evaluate fitness function+feeding operator

End

For each fish in school do

Collective instinctive movement

End

For each fish in school do

Collective volitive movement;

Evaluate fitness function;

End

Update $step_{ind}$ *parameter;*

Update $step_{vol}$ *parameter;*

End while

End for

The FSS procedure consists of a group of individuals and the procedure starts with random initialization of fish positions in the search space. First we consider the RL part of the given data and the FSS procedure is carried out. Next considering the LR part evaluation procedure is driven out. Then we combine the results of both the sections to obtain the desired output.

Numerical Evaluation through Interval FSS

The selected example considers a small school with three fishes is set to find the minimum of the sphere function in two dimensions i.e. $\sum_{i=1}^{n}(x_i)^2$ with the given parameters as:

Feasible space	= [-10,10]	Initial $step_{vol}$	= 0.1
No. of iterations	= 20	Final $step_{vol}$	= 0.01
W_{scale}	= 10		
Initial $step_{ind}$	= 1		
final $step_{ind}$	= 0.1		

The initial positions,weights and corresponding fitness values of fishes are selected in a random manner and given in table-5.1.1 and table 5.1.2

Table.5.1.1 Initial LR conditions of fishes in the example

Fish	Weight	Position	Fitness
fish-1	5	(8.5,7.5)	128.50
fish-2	5	(4.5,6.5)	62.50
fish-3	5	(7.5,4.5)	76.50

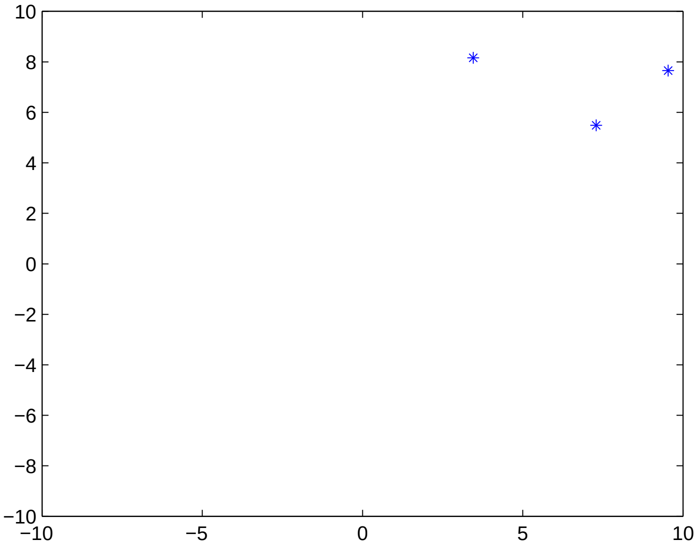

Fig. 5.1.2 LR Plot of school (initial position)

Table.5.1.2 Initial RL conditions of fish in the example

Fish	Weight	Position	Fitness
fish-1	5	(9.5,6.5)	132.5
fish-2	5	(5.5,5.5)	60.5
fish-3	5	(8.5,3.5)	84.5

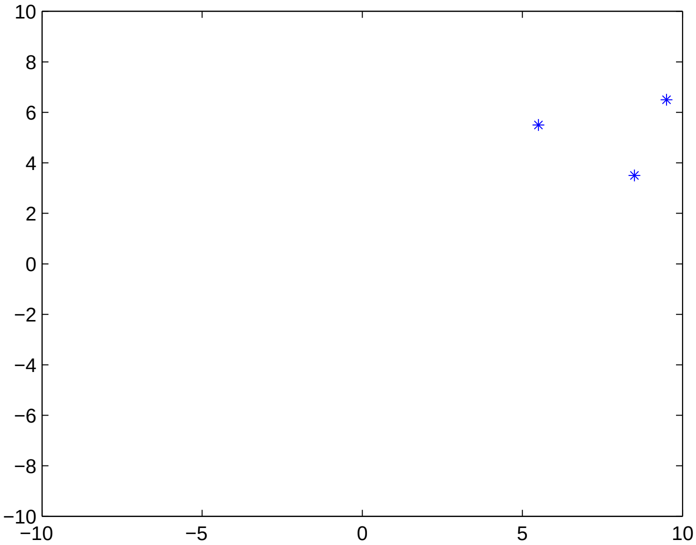

Fig. 5.1.2 RL Plot of school (initial position)

After 1st Iteration:

Table.5.2.1 LR result after 1st iteration

Fish	Weight	Position	Fitness
fish-1	4.4157	(6.1412,9.3340)	124.8385
fish-2	5.2922	(3.2167,7.3549)	64.8835
fish-3	5.3885	(6.0242, 5.8429)	70.4308

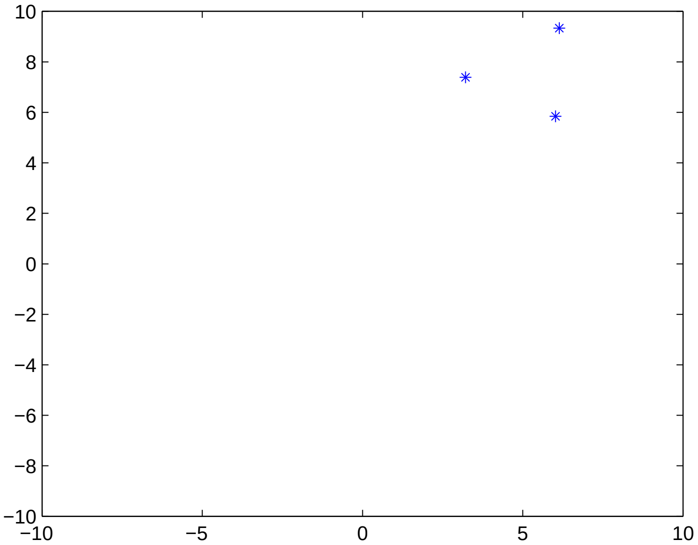

Fig. 5.2.1 LR Plot of school after 1st iteration

Table.5.2.2 RL Result after 1st iteration

Fish	Weight	Position	Fitness
fish-1	4.3147	(1.2770,3.7959)	
fish-2	5.4058	(5.1973,5.3033)	55.1363
fish-3	4.0022	(6.9503, 5.1112)	74.4314

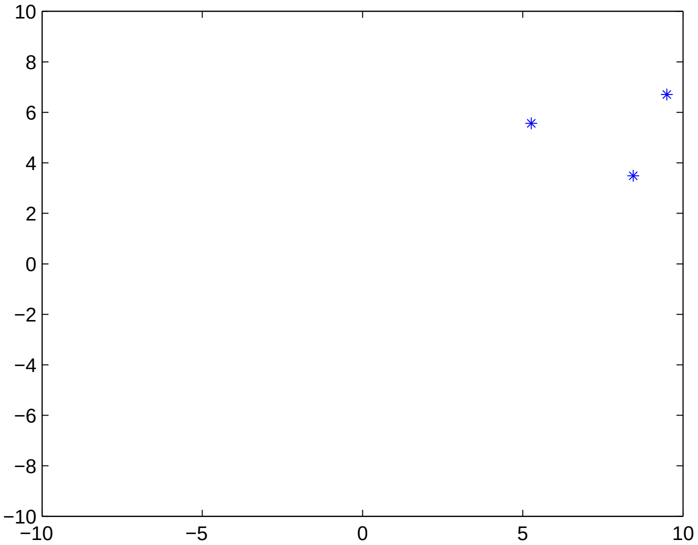

Fig. 5.2.2 RL Plot of school after 1st iteration

After 5th Iteration:

Table.5.3.1 LR result after 5th iteration:

Fish	Weight	Position	Fitness
fish-1	1.7448	(3.1903,6.6019)	53.7632
fish-2	4.1021	(2.3480,5.1300)	31.8303
fish-3	4.3885	(2.6482,4.1028)	23.8456

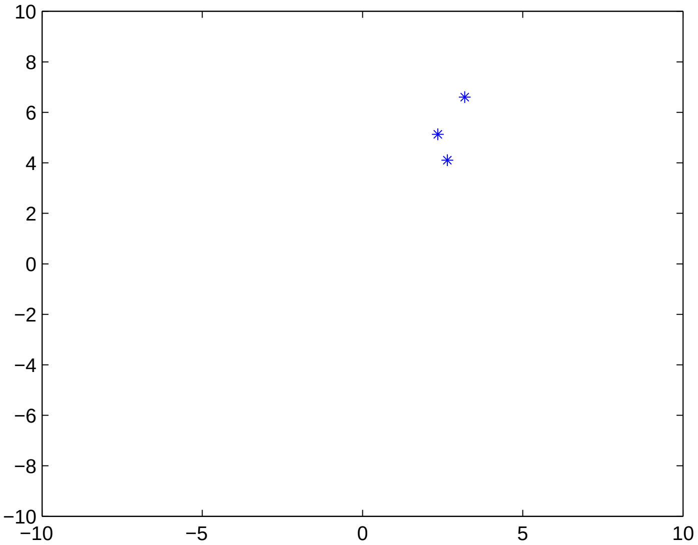

Fig. 5.3.1 LR Plot of school after 5th iteration

Table.5.3.2 RL result after 5th iteration:

Fish	Weight	Position	Fitness
fish-1	2.0075	(7.2140, 3.9063)	67.3012
fish-2	4.2932	(3.8236, 3.3695)	25.9736
fish-3	2.3237	(4.7893, 2.7748)	30.6373

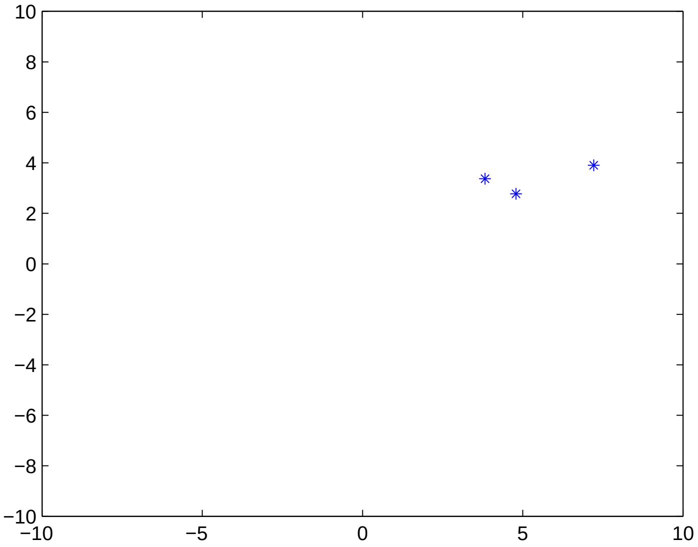

Fig. 5.3.2 RL Plot of school after 5th iteration

After 10th Iteration:

Table.5.4.1 LR Result after 10th iteration

Fish	Weight	Position	Fitness
fish-1	2.2552	(1.3608,2.9562)	10.5910
fish-2	2.4816	(2.9933,1.8066)	12.2238
fish-3	2.8500	(1.5531,0.6969)	2.8977

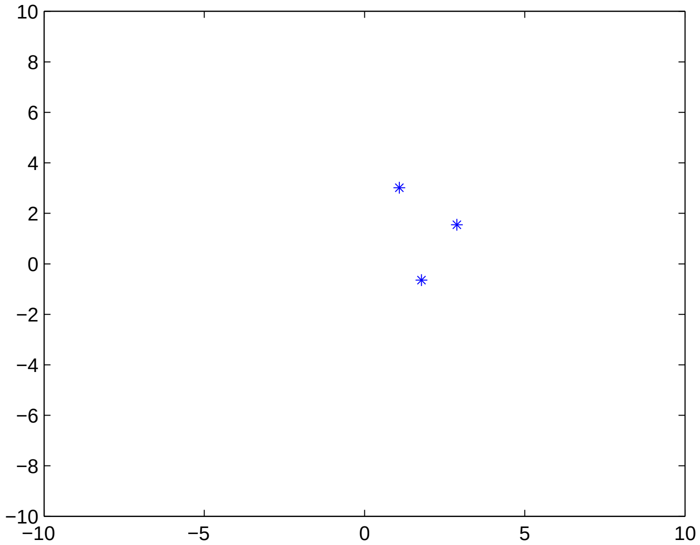

Fig. 5.4.1 LR Plot of school after 10th iteration

Table.5.4.2 RL Result after 10th iteration

Fish	Weight	Position	Fitness
fish-1	3.5595	(3.4092,4.3919)	30.9113
fish-2	2.2348	(0.7708, 3.6639)	14.0184
fish-3	5	(2.0824, 2.7120)	11.6916

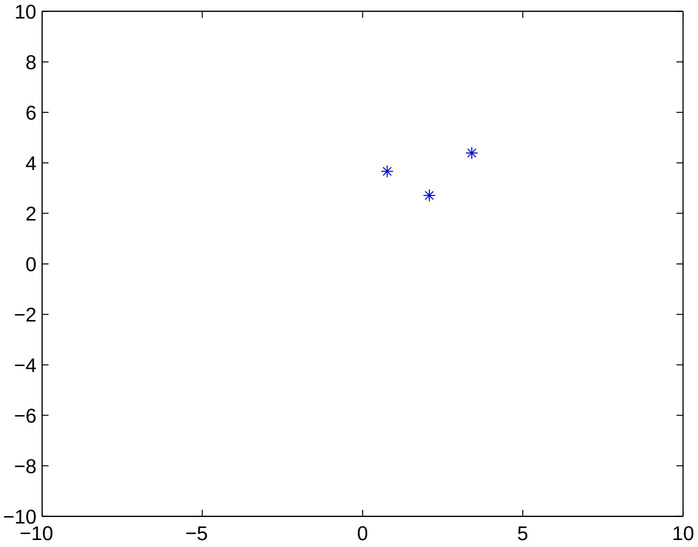

Fig. 5.4.2 RL Plot of school after 10th iteration

Conclusion and Future work

General ideas and principles embedded in FSS are described. This novel search algorithm is quite promising as a search tool dealing with high dimensional problems. Examples are illustrated for FSS in crisp case and interval case. In interval form, the positions are considered as intervals. Results of the examples are shown in tables for both crisp and interval FSS, which shows that it gives a bound for the measurement errors and helps in obtaining correct position. Here the Breeding operator is implemented.

The process can be improved and utilized in a large range or section of problems for finding a optimal solution. Bary centre gravity can be implemented for future purposes.

Bibliography

1. Tian Y. J. and Sannomiya N. (1997) **Simulation and analysis of the behavior of a fish school with many individuals using an aggregated model**. In: International Journal of systems Science (28-4), pp. 357-364.

2. Janecek Andreas and Tan Ying (2011) **Feeding the Fish Weight Update Strategies for the Fish School Search Algorithm**. In: ICSI 2011, Part-II. LNCS 6729 pp. 553-562.

3. Filho C. J. A. Bastos , Fernando, Lima P., Anthony J. C. C. Lins, Antnio I. S. Nascimento and Marilia P. Lima (2008) **A Novel Search Algorithm based on Fish School Behavior**. In: 2008 IEEE International Conference on Systems, Man and Cybernetics (SMC 2008).

4. Filho C.J.A. Bastos, Fernando , Anthony J. C. C. Lins, Marilia P. Lima (2009) **Fish School Search**. In: Springer, SCI 193, R. Chiong (Ed.), pp. 261-277.

5. Filho C.J.A. Bastos, Fernando , Lins Anthony, Jeanes Andrew (2010) **Fish School Search Some update strategies**. In: Springer, SCI.

6. Mahmood Amintoosi, Mahmoud Fathy, Naser Mozayani, Adel T. Rahmani (2007) **A Fish School Clustering Algorithm: Applied to Student Sectioning Problem**. In: Proceeding of 2007 International Conference on Life System Modeling and Simulation (LSMS).

7. Moore R.E. (1979) **Methods and Applications of Interval Analysis**. In: SIAM studies in Applied Mathematics, Philadelphia.

8. Cloud, Michael J., Moore R. E. (2009) **Introduction to Interval Analysis**. In: Philadelphia Society for Industrial and Applied Mathematics, ISBN 0-89871-669-1.

9. Dorigo M. and Gambardella L. (1997) **Ant Colony Systems: A Cooperative Learning Approach to the Traveling Salesman Problem**. In: IEEE Transactions on Evolutionary Computation, 1(1)53-66.

10. Kennedy J. and Eberhart R. (1995) **Particle Swarm optimization**. In: Proceedings of IEEE International Conference on Neural Networks, Piscataway.

11. Bratton D. and Kennedy J. (2007) **Defining a Standard for Particle Swarm Optimization**. In: Proceedings of the 2007 IEEE Swarm Intelligence Symposium (SIS 2007), Hawaii, USA (2007).

12. *http://www.fbln.pro.br/fss/versions.htm*

13. *en.wikipedia.org/wiki*