VIBRATIONAL ANALYSIS OF A SIMPLY SUPPORTED BEAM WITH CRACK

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CERTIFICATE

This is to certify that work in this project report entitled, “**Vibrational Analysis Of Simply Supported Cracked Beam Using Perturbation Method”** by **Mr Shibani Shankar Nayak** has been carried out under my supervision and guidance in partial fulfillment of the requirements for the award of **Bachelor Of Technology In Mechanical Engineering** during session 2009-2013 in the Department of Mechanical Engineering, National Institute of Technology, Rourkela.

To the best of my knowledge, this work has not been submitted to any other university/ institute for award of any Degree or Diploma.

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ABSTRACT

Vibrational analysis of simply supported beam was investigated using an analytical method, called the perturbation method. The beam was modelled as a single degree of freedom system for simplicity and application of the physics laws into this mathematical model. The stiffness function was modelled as a time varying function due to the breathing characteristics of the crack. The governing equation of motion was found using the d Alembert's principle along with all the lumped parameters. Solving this equation lead to another equation called Mathieu equation. This Mathieu equation was solved using lindstedt method and the solution of which led to the solution of the governing equation of motion. This method is called as the perturbation method, by which the displacement equation was found analytically and theoretically. The damping ratio of the cracked beam was also found as a function of geometrical dimensions, mechanical properties and crack parameters.

NOMENCLATURE

L=length of beam

a= crack depth

w=width of beam

h= height of beam

Lo=crack distance from left end

Kc=stiffness for fully closed crack

C=flexibility

Ko =stiffness for fully open crack

ωb =breathing frequency

m=equivalent mass of the beam

z=displacement

δ, ε = non dimensional parameters of Mathieu equation

ζ =damping ratio

Ut = strain energy

Js=strain energy density

K=stress intensity factor

F= dimension less function of the ratio of crack depth to beam thickness

INRODUCTION

Vibration analysis is an effective technique used to detect fatigue cracks in structural members. The members form an important part of building or mechanical systems. Fatigue cracks in it due to repeated loading causes the structure failure. So the cracks must be detected to avoid such calamities. In the vibration analysis, many researchers have confined only to the linear model. They assumed that the crack only remains open, it does not get closed. They failed to show the non-linear effects of crack closure .thus the assumption they made lacked accuracy in the results of the dynamic behaviour. The crack detection by considering that the crack only remains open would underestimate the crack severity if the crack was propagating due to fatigue loading conditions.

The simply supported cracked beam was modelled as a single degree of freedom system. The laws of physics cannot be directly applied onto the real systems due to complexity. So assumptions are made to convert the physical model into the required mathematical description which is known as the mathematical model. Thus this mathematical model is used to analyse the motion of a system by approximation of its dynamic behaviour. Solving the governing equation of motion of the mathematical model gives the approximated dynamic behaviour of the real system.

Here in this paper, vibrational analysis of a cracked simply supported beam has been investigated by an analytical approach. The crack has been considered to be a breathing one. The beam was modelled as a single degree of freedom system and the time varying stiffness was modelled using a periodic function. All the lumped parameters were found and the governing equation of motion was derived by d' Alembert's principle .since free vibrations the response of the cracked beam depends on the initial conditions and it is a time decaying one. So by suitable variable change and mathematical operations the governing equation of motion gets in the form of an equation, called the Mathieu equation. The parameters of the Mathieu's equation were investigated accurately and the equation of motion was solved using an analytical method, called perturbation method. The analytical expressions obtained by solving Mathieu's equation showed the non-linear behaviour of the cracked beam. The dynamic response was in the form of multiplication of an exponential function and a non-linear oscillatory function. The exponential function represents the decay rate due to system damping and the oscillatory function is the solution of the Mathieu equation. The damping ratio was also determined using the constants of the Mathieu equation. Thus the displacement function can be plotted and several other parameters can be plotted with respect to the crack parameters.

LITERATURE SURVEY

[1] S. M. CHENG, X. J. WU AND W. WALLACE have developed a simple non-linear fatigue crack model. The dynamic behaviour of a cracked beam vibrating at its first mode was canvassed using this fatigue crack model. The secerned features of the dynamic response due to the fatigue crack were keyed out by investigating in both time and frequency domains.

[2] E. Douka and L.J. Hadjileontiadis have investigated the dynamic behaviour of a cantilever beam with a breathing crack both theoretically and experimentally. Empirical mode decomposition and Hilbert transform were used and the instantaneous frequency was obtained. It was seen that the instantaneous frequency oscillation revealed the breathing behaviour. The crack size was detectable with the variation trend of the frequency.

[3] T. G. CHONDROS, A. D. DIMAROGONAS, J. YAO have developed a continuous cracked beam vibration hypothesis for canvassing the lateral vibration of Euler Bernoulli beams with single edge open cracks. The differential equation and the boundary conditions of the cracked beam were developed as a one dimensional continuum by the Hu Washizu Barr variational formulation. The crack was modelled as a continuous flexibility using the displacement field in the vicinity of the crack, found with fracture mechanics methods. It had a good agreement with the experimental results.

[4] T. G. CHONDROS, A. D. DIMAROGONAS, J. YAO have employed the continuous cracked beam vibration hypothesis for the prognostication of changes in transverse vibration of a simply supported beam having breathing behaviour. The equation of motion and the boundary conditions were employed as a one-dimensional continuum. The changes in frequencies for a breathing crack were smaller than that induced by open cracks.

[5] S. Loutridis, E. Douka, L.J. Hadjileontiadis have developed a new method reliant on instantaneous frequency and empirical mode decomposition for the espial of cracks. Theoretical and experimental investigations were done on a cantilever beam with opening and closing crack due to a harmonic excitation for presenting the dynamic response. It was seen that the instantaneous frequency oscillation revealed the breathing phenomenon. The change of the instantaneous frequency increased with the crack depth and hence can be used for approximation of crack size. This time–frequency approach was better compared to Fourier analysis and is more effective for finding cracks.

[6]A.Ariaei, S.Ziaei-Rad, M.Ghayour have presented both analytical and calculation method to determine the dynamic behaviour of the un-damped Euler–Bernoulli breathing cracked beams under a point moving mass using discrete element technique and the finite element method. It was observed that the presence of crack led to higher deflections and change in beam response was seen. The largest deflection in the beam for a particular speed takes long to build up. The influence of crack and load depends on speed, crack size, time, crack location and the level of moving mass.

[7]Orhan Sadettin has canvassed the free and forced vibration of a cracked cantilever beam for detecting crack in it. He considered both Single- and two-edge cracks. The forced vibration analysis results described better changes in crack depth and location compared to the free vibration one where the change was very less.

[8]Chasalevris and Papadopoulos have canvassed the dynamic response of a cracked beam having two transverse surface cracks. Depth, position and relative angle was taken as the crack parameters. A local compliance matrix of degree of freedom equals to 2, bending in vertical and horizontal planes was used to model the rotating transverse crack in the shaft and was determined based on the formulas of the stress intensity factors and the strain energy density.

[9] Mousa Rezaee and Reza Hassannejad have described a new approach to free vibration analysis of a cracked cantilever beam with breathing characteristics. The stiffness changes was modelled as a nonlinear amplitude-dependent function and the assumptions being , during first half of a cycle, the frequencies and mode shapes of the beam changed continuously with time. Experiment revealed that the local stiffness at the crack location varied continuously between the two end values representing to the fully closed and the fully open cracks. Mechanical energy balance method was used to develop the dynamic response of the cracked beam at every time instant. The outcome revealed that for a particular crack depth, moving the crack location towards the fixed end of the beam, there was more reduction in the fundamental frequency. Also for a particular crack location, the fundamental frequency reduces and the nonlinearity of the system increased with the crack depth.

[10] A.S. Sekhar has examined different multiple-cracks, their Effects, identification methods in vibration structures such as beams, rotors, pipes, etc. It brings out the state of research on multiple cracks effects and their recognition.

[11] Nahvi and Jabbari have developed an analytical and experimental way of vibrational analysis in cantilever beams. An experimental setup was designed in which a cracked cantilever beam was excited by a hammer and the response was produced using an accelerometer attached to the beam. The crack was assumed to be open i.e. it doesn't get closed during vibration for simplicity. Shapes of the normalized frequency were plotted in terms of the normalized crack depth and location for determination of crack.

[12] M. KISA AND J. BRANDON developed a finite element scheme for computing the Eigen system for a cracked beam for different degrees of closure. Modelling of the cracked structures was done by combining the finite element analysis method, the linear elastic fracture mechanics theory and the component mode synthesis method.

[13] Yang et al. have produced an energy-based numerical model to examine the Effect of cracks on structural dynamic characteristics during beam vibration with the crack being open. Strain energy and the equivalent bending stiffness over the entire beam length were computed.

[14] G.E. Carr, M.D. Chapetti, UNMdP have examined the influence of a surface fatigue crack on vibration behaviour of tee-welded plates. Detection threshold using natural frequencies shift were determined. The effects of naturally propagated fatigue cracks on the oscillation frequencies was canvassed and compared to two and three-dimensional numerical modelling outcomes. Using the changes of the first mode natural frequencies experimental detection threshold was found. The power density spectrums of the acceleration signals were analysed. Importance of crack shape seen for the detection threshold of the method. It was seen that the minute size semi-circular fatigue cracks does not perceptibly modifies the beam natural frequency but generated super harmonics of the vibration modes that induced stresses at the samples surface concentrated by the crack. A detection threshold using this spectral analysis was lower than that of frequency shifts use. The outcomes indicated the power of the experimental technique for crack detection from a size of 0.5% of the cross section area, at 54% of the fatigue lifespan, compared to the 33% in previous methods.

[15] Papadopoulos et al. have presented a method which detects the depth and location of a transverse crack in rotating cracked shafts. A local compliance matrix of different degrees of freedom was used to modelled the transverse crack in a shaft of circular cross section, based on mathematical expressions of stress intensity factors and the associated the strain energy density.

[16] Jyoti K. Sinha has presented the observations made on the high order coherences on the numerically simulated experiment of a cantilever beam with and without cracks. The validness of the HOC in the crack detection even for the noisy response vibration data has also been brought out. Higher Order Coherences are the tools to identify the relationship between the different harmonic components in a signal. The vibration of a structure with crack produces several harmonics of the exciting frequency due to its breathing behaviour which is a non-linear phenomenon.

[17] Dharmaraju et al. have considered an Euler–Bernoulli beam in the finite element analysis method. The transverse surface crack was taken to remain open during vibration. The crack in the beam was modelled by a local compliance matrix having four degrees of freedom and this matrix contains diagonal and off-diagonal terms. A harmonic force of known amplitude and frequency was used for excitation of the beam. Many numerical examples were presented based on the method.

[18] N. Dharmaraju, R. Tiwari, S. Talukdar have proposed a general identification algorithm to approximate crack flexibility coefficients and the crack depth according to the force-response data. The algorithm used the standard dynamic reduction scheme to remove some degrees of freedom of the system. Elimination of the rotational degree of freedoms at crack element nodes was done by novel hybrid reduction scheme accordance to the physical assumption of the present problem.

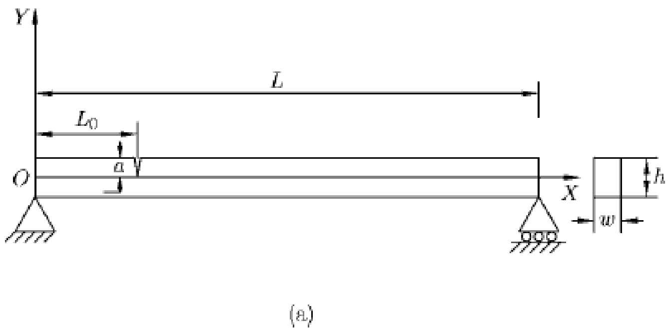
[19] Yoona Han-Ik et al. have investigated the effects of two open cracks on the dynamic response of simply supported beam, analytically and experimentally. The simply supported beam was modelled by the Euler-Bernoulli beam theory. The equation of motion was derived using the Hamilton's principle and numerical method analysis was done.

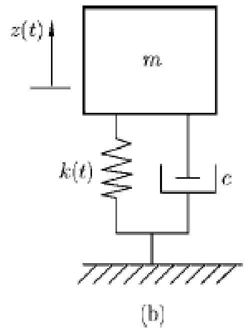
[20] Animesh Chatterjee has developed a nonlinear dynamic model of a cracked structure using higher order frequency response functions. He also approximated the bilinear restoring force by a polynomial series. A new procedure was suggested by which the severity of cracks can be approximated through the first and second harmonic amplitudes values.

[21] Ruotolo et al. have investigated forced response of a cantilever beam with a crack that either fully opens or closes, for the determination of crack depth and its location. A harmonic sine force was given at the free end of the beam. Vibration amplitude of the free end of the beam was taken into consideration. It was seen that with the variation of depth and location of the crack, the amplitude of vibration changed.

MODELLING

Modelling is that section of solution of an engineering problem whose objective is to develop a mathematical description. The laws of physics cannot be directly applied onto the real systems due to complexity. So assumptions are made to convert the physical model into the required mathematical description which is known as the mathematical model. Thus this mathematical model is used to analyse the motion of a system by approximation of its dynamic behaviour. Solving the governing equation of motion of the mathematical model gives the approximated dynamic behaviour of the real system.





Simply supported beam

SDOF system

Here in this paper, the simply supported beam was modelled as a single degree of freedom system to present the non-linear dynamic behaviour of the cracked beam. The lumped parameters constitutes the mass, spring and damper. The mass is concentrated at the centre of the beam. All the lumped parameters of the system are determined. The stiffness was calculated as a function of time and also the flexibility changes were determined. With all these required parameters the equation of motion was generated using d' Alembert's principle.

FLEXIBILITY CHANGE DUE TO CRACK

An additional rotation exists when bending moment is applied. Additional rotation is proportional to the bending moment. So according to Castiglione's theorem,

$$\theta = \frac{\partial U_T}{\partial M_b}$$

The strain energy is given by

$$U_T = \int_0^a \frac{\partial U_T}{\partial a} \mathrm{d}a = w \int_0^a J_s \mathrm{d}h$$

Strain energy density is given by

$$J_s = \frac{1-\nu^2}{E} K^2$$

The stress intensity factor considering plane stress, for the cracked beam on application of bending moment is given by

$$K = \sigma_b F_I(\alpha) \sqrt{\pi a}$$

σb = bending stress,

$$\sigma_b = \frac{6M_b}{wh^2}$$

Where the dimensionless function of crack depth and beam thickness is given by

$$F_I(\alpha) = 1.12 - 1.4\alpha + 7.33\alpha^2 - 13.1\alpha^3 + 14\alpha^4$$

So on solving for the strain energy, we get

$$U_T = \frac{36\pi(1-\nu^2)}{E}\frac{M_b^2}{wh^2}g(\alpha)$$

Where,

$$\begin{aligned} g(\alpha) = {} & 19.6\alpha^{10} - 40.7556\alpha^9 + 47.1063\alpha^8 - 33.0351\alpha^7 + 20.2948\alpha^6 \\ & -9.9736\alpha^5 + 4.5948\alpha^4 - 1.04533\alpha^3 + 0.6272\alpha^2 \end{aligned}$$

So the flexibility changes due to crack is

$$\Delta C = \frac{\partial^2 U_T}{\partial P^2} = \frac{18\pi L_0^2\left(1-\nu^2\right)}{Ewh^2}g(\alpha)$$

Governing equation of motion

A simply supported beam with uniform crack is considered here. It has length L and the crack depth is a located at a distance of Lo from the left end of beam. The width of the beam is w and the height is h. The assumption taken was that, the beam vibrates at its first mode so that it can be modelled as a single degree of freedom system for simplicity. The crack was considered to be a breathing one i.e. opening and closing during vibration. Thus the stiffness is a time varying function due to this breathing behaviour and the dynamic response shows non-linear characteristics.

First mode shape of the beam is assumed to be:

$$\phi(x) = \sin\left(\frac{\pi x}{L}\right)$$

When the crack is fully closed, the stiffness is,

$$k_c = \frac{1}{C} = \int_0^L EI\phi''^2(x)\mathrm{d}x = \frac{\pi^4 EI}{2L^3}$$

When the crack is fully open, stiffness is given by

$$k_o = 1/C_o$$

And the beam flexibility for fully open crack can be calculated as,

$$C_o = C + \Delta C$$

The stiffness varies with the opening and closing of crack, so the stiffness was modelled as a time varying function and is given by

$$k(t) = k_o + k_{\Delta c}\left[1 + \cos(\omega_b t)\right]$$

Equivalent stiffness changes

$$k_{\Delta c} = \frac{1}{2}(k_c - k_o)$$

For fully closed crack:

$$(k(t) = k_c)$$

We have,

$$\omega_b t \ = \ 2n\pi, \ n \ = \ 1, 2, ...$$

For fully open crack:

$$(k(t) = k_o)$$

We have,

$$(2n-1)\pi, \ n = 1, 2, ...$$

Crack breathing frequency can be approximated as,

$$\omega_b = \frac{2\omega_c\omega_o}{\omega_c + \omega_o}$$

The equivalent mass of the beam can be calculated as,

$$m = \int_0^L m(x)\phi^2(x)\mathrm{d}x = 0.5\bar{m}L$$

Applying d Alembert’s principle, we have the equation of motion as,

$$m\ddot{z} + c\dot{z} + \{k_o + k_{\Delta c}\left[1 + \cos(\omega_b t)\right]\} z = 0$$

ANALYTICAL APPROACH FOR SOLVING THE GOVERNING EQUATION OF MOTION

The governing equation of motion shows that the stiffness function is a time varying one due to the breathing behaviour of the uniform crack. As we know that in case of free vibrations, the response of the cracked beam will depend on the initial conditions and it will be a time decaying function. Therefore the free vibration response can be written as a function of the initial condition, a damping function and a non-linear oscillating function.

$$z(t) = A\mathrm{e}^{-[c/(2m)]t}y(t)$$

Substituting the above displacement equation and its higher derivatives in the governing equation of motion, we get

$$m\left(-\frac{c}{2m}\right)^2 \mathrm{e}^{-[c/(2m)]t}y(t) - \frac{mc}{2m}\mathrm{e}^{-[c/(2m)]t}\dot{y}(t) - \frac{mc}{2m}\mathrm{e}^{-[c/(2m)]t}\dot{y}(t) + m\mathrm{e}^{-[c/(2m)]t}\ddot{y}(t)$$
$$+\left\{\frac{c^2}{2m}\mathrm{e}^{-[c/(2m)]t}y(t) + c\mathrm{e}^{-[c/(2m)]t}\dot{y}(t)\right\} + \{k_o + k_{\Delta c}\left[1+\cos(\omega_b t)\right]\}\,\mathrm{e}^{-[c/(2m)]t}y(t) = 0$$

.... (1)

Dividing the above equation (1) by $\mathrm{e}^{-[c/(2m)]t}$ and rearranging it, we get the form

$$\ddot{y} + \left[\frac{k_o + k_{\Delta c}}{m} - \frac{c^2}{4m^2} + \frac{k_{\Delta c}}{m}\cos(\omega_b t)\right] y = 0$$

.... (2)

Then applying the following variable change,

$$t = \frac{2\tau}{\omega_b}$$

We have,

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\omega_b}{2}\frac{\mathrm{d}y}{\mathrm{d}\tau}, \quad \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = \frac{\omega_b^2}{4}\frac{\mathrm{d}^2y}{\mathrm{d}\tau^2}$$

Substituting the above expressions in equation (2) we get

$$\frac{\mathrm{d}^2y}{\mathrm{d}\tau^2} + [\delta + 2\varepsilon\cos(2\tau)]\,y(\tau) = 0$$ .... (3)

This above equation is called as the Mathieu equation

The non-dimensional parameters being given by,

$$\varepsilon = \frac{2k_{\Delta c}}{m\omega_b}$$

$$\delta = \frac{4(k_o + k_{\Delta c})}{m\omega_b^2} - \left(\frac{c}{m\omega_b}\right)^2$$

Thus the Mathieu equation shall have periodic solution when ε can be expressed in terms of δ

$$\delta = q^2 + \sum_{i=1}^{\infty} d_i \varepsilon^i$$ .... (4)

Replacing,

$$\omega_b = \frac{2\omega_c\omega_o}{\omega_c + \omega_o}$$

And

$$\omega_o = \sqrt{k_o/m},\ \omega_c = \sqrt{k_c/m} \text{ and } c = 2\zeta m\omega_b$$

We have,

$$\delta = \frac{4(k_o + k_c)}{\frac{2m\omega_o\omega_c}{\omega_o + \omega_c}} - \left(\frac{2\zeta m\omega_b}{m\omega_b}\right)^2 = \frac{4(k_o + k_c)}{\frac{4k_o k_c}{k_o + k_c + 2\sqrt{k_o k_c}}} - 4\zeta^2$$

Replacing $k_o = 1/C_o \quad k_c = 1/C \qquad c = 2\zeta m\omega_b$

We get,

$$\varepsilon = \frac{2\frac{\Delta C}{C} + \left(\frac{\Delta C}{C}\right)^2 + 2\frac{\Delta C}{C}\sqrt{1 + \frac{\Delta C}{C}}}{4\left(1 + \frac{\Delta C}{C}\right)} = \frac{\frac{\Delta C}{C}}{4\left(1 + \frac{\Delta C}{C}\right)} + \frac{\Delta C}{4C} + \frac{\frac{\Delta C}{C}}{2\sqrt{1 + \frac{\Delta C}{C}}}$$

$$\delta = \frac{8 + \frac{2\Delta C^2}{C^2} + \frac{8\Delta C}{C} + 4\left(2 + \frac{\Delta C}{C}\right)\sqrt{1 + \frac{\Delta C}{C}}}{4\left(1 + \frac{\Delta C}{C}\right)} - 4\zeta^2$$

The periodic solution of the Mathieu equation was solved using lindstedts method. The initial solution assumption was,

$$y(\tau) = \sum_{i=0}^{\infty} y_i(\tau)\varepsilon^i \quad \text{......... (5)}$$

Substituting equation (4) and (5) in the Mathieu equation, we get

$$\sum_{i=0}^{\infty} \ddot{y}_i(\tau)\varepsilon^i + \left(4 + \sum_{i=1}^{\infty} d_i\varepsilon^i\right) \sum_{i=0}^{\infty} y_i(\tau)\varepsilon^i + 2\sum_{i=0}^{\infty} \cos(2\tau)y_i(\tau)\varepsilon^{i+1} = 0$$

Rearranging it, we have

$$\sum_{i=0}^{\infty} \varepsilon^i \left[\ddot{y}_i(\tau) + 4y_i(\tau) + \sum_{j=1}^{i} d_j y_{i-j}(\tau) + 2\cos(2\tau)y_{i-1}(\tau)\right] = 0$$

Infinite set of differential equations were obtained by making the coefficient of ε equal to zero.

$$\begin{aligned}
&\ddot{y}_0(\tau) + 4y_0(\tau) = 0 \\
&\ddot{y}_1(\tau) + 4y_1(\tau) = -2\cos(2\tau)y_0(\tau) - d_1 y_0(\tau) \\
&\ddot{y}_2(\tau) + 4y_2(\tau) = -2\cos(2\tau)y_1(\tau) - d_1 y_1(\tau) - d_2 y_0(\tau)
\end{aligned}$$

And so on.

These set of differential equations were solved recursively. Some conditions were imposed on the coefficients of d (i), to obtain periodic solutions.

The zero order approximation of the first differential equation is given by,

$$y_0(\tau) = A_1 \cos(2\tau) + B_1 \sin(2\tau)$$

Applying initial conditions,

$$y(0) = 1 \text{ and } \dot{y}(0) = 0,$$

We get,

$$A_1 = 1 \text{ and } B_1 = 0.$$

Hence $y_0(\tau) = \cos(2\tau)$

So the second differential equation becomes

$$\ddot{y}_1(\tau) + 4y_1(\tau) = -2\cos^2(2\tau) - d_1 \cos(2\tau)$$

The periodicity condition for the solution of the second differential equation is satisfied when d1=0, so the solution of the second equation is

$$y_1(\tau) = -\frac{1}{4} + \frac{\cos(4\tau)}{12}$$

So in a similar manner the other differential equations were solved and is given by,

$$y_2(\tau) = \left(\frac{5}{48} - \frac{d_2}{4}\right)\tau\sin(2\tau) + \frac{\cos(6\tau)}{384}$$

$$y_3(\tau) = \frac{5}{192} + \frac{43}{13824}\cos(4\tau) + \frac{1}{23040}\cos(8\tau), \quad d_3 = 0$$

$$y_4(\tau) = \frac{29}{221184}\cos(6\tau) + \frac{1}{2211840}\cos(10\tau), \quad d_4 = -\frac{763}{13824}$$

And so on

Thus we have the periodic solution of the Mathieu equation as

$$y(\tau) = \cos(2\tau) + \varepsilon\left[-\frac{1}{4} + \frac{\cos(4\tau)}{12}\right] + \varepsilon^2\left[\frac{\cos(6\tau)}{384}\right] + \varepsilon^3\left[\frac{5}{192} + \frac{43}{13824}\cos(4\tau) + \frac{1}{23040}\cos(8\tau)\right]$$
$$+\varepsilon^4\left[\frac{29}{221184}\cos(6\tau) + \frac{1}{2211840}\cos(10\tau)\right] + \cdots$$

And

$$\delta = 4 + \frac{5}{12}\varepsilon^2 - \frac{763}{13824}\varepsilon^4$$

Now with suitable variable change and substituting the above equation solution of Mathieu equation in the approximated initial solution of the equation of motion.

We get our required dynamic response of the cracked beam as:

$$z(t) = \mathrm{e}^{-[c/(2m)]t} \left\{ \cos(\omega_b t) + \varepsilon \left[-\frac{1}{4} + \frac{\cos(2\omega_b t)}{12} \right] + \varepsilon^2 \left[\frac{\cos(3\omega_b t)}{384} \right] + \varepsilon^3 \left[\frac{5}{192} + \frac{43}{13824} \cos(2\omega_b t) \right.\right.$$
$$\left.\left. + \frac{1}{23040} \cos(4\omega_b t) \right] + \varepsilon^4 \left[\frac{29}{221184} \cos(3\omega_b t) + \frac{1}{2211840} \cos(5\omega_b t) \right] \right\}$$

The damping ratio was also determined in terms of geometrical dimensions, mechanical characteristics and the crack parameters.

$$\zeta = \sqrt{\frac{4 + \frac{\Delta C^2}{C^2} + 4\frac{\Delta C}{C} + 2\left(2 + \frac{\Delta C}{C}\right)\sqrt{1 + \frac{\Delta C}{C}}}{8\left(1 + \frac{\Delta C}{C}\right)} - \left(1 + \frac{5}{48}\varepsilon^2 - \frac{763}{55296}\varepsilon^4\right)}$$

CONCLUSION

- In this paper the mathematical model of the simply supported beam was derived using Euler-Bernoulli beam theory.
- The equation for flexibility changes of the simply supported beam due to crack was derived.
- The equation of motion for the breathing cracked beam was derived.
- Equation of motion was analytically converted to Mathieu equation for finding out the stability region of the cracked simply supported beam
- Mathieu equation was solved using the lindstedt method to obtain the periodic solution of the cracked beam.
- Finally equation for the damping ratio of the cracked beam in terms of dimension and mechanical properties and the crack parameters was derived.

FUTURE WORK

- Stiffness versus time can be plotted for a particular crack location ratio and different crack depths.
- Displacement versus time can be plotted for different crack parameters.
- Damping ratio versus crack location ratio can be plotted for different crack depth ratio.
- Displacement response by this method can be compared with that of the numerical methods response.
- The region of crack parameters for having a valid analytical solution can also be produced.
- Frequency ratio versus crack depth ratio can be plotted and comparison can be done with the experimental results and also with the proposed analytical results for an open crack model.
- Stability study of the simply supported breathing cracked beam can be done.

References

[1] Cheng,S.M., Wu,X.J. and Wallace,W., Vibrational response of a beam with a breathing crack. Journal of Sound and Vibration, 1999, 225(1): 201-208.

[2] Ariaei,A., Ziaei-Rad,S. and Ghayour,M., Vibration analysis of beams with open and breathing cracks subjected to moving masses. Journal of Sound and Vibration, 2009, 326: 709-724.

[3] Douka,E. and Hadjileontiadis,J.L., Time-frequency analysis of the free vibration response of a beam with a breathing crack. NDT&E International, 2005, 38: 3-10.

[4] Loutridis,S., Douka,E. and Hadjileontiadis,J.L., Forced vibration behaviour and crack detection of cracked beams using instantaneous frequency. NDT&E International, 2005, 38: 411-419.

[5] Zhang,W. and Testa,R.B., Closure effects on fatigue crack detection. Journal of Engineering Mechanics, 1999, 125: 1125-1132.

[6] Bovsunovsky,A.P. and Surace,C., Considerations regarding super harmonic vibrations of a cracked beam and the variation in the damping caused by the presence of the crack. Journal of Sound and Vibration, 2005, 288: 865-886.

[7] Curadelli,R.O., Riera,J.D., Ambrosini,D.andAmani,M.G., Damage detection by means of structural damping identification. Engineering Structures, 2008, 30: 3497-3504.

[8] Dimarogonas,A.D., Vibration of cracked structures: a state of the art review. Engineering Fracture Me-chanics,1996, 55: 831-857.

[9] Chondros T.G. and Dimarogonas A.D., A continuous cracked beam vibration theory. Journal of Sound and Vibration, 1998, 215(1): 17-34.

[10] Dimarogonas A.D. and Paipetis S.A., Analytical Methods in Rotor Dynamics. London: Elsevier Applied Science, 1986.

[11] Meirovitch L., Analytical Methods in Vibrations. The Macmillan Company, 1967.

[12] Clough,R.W. and Penzien,J., Dynamics of Structures. New York: McGraw-Hill, Inc, 1975.

[13] Chondros,T.G., Dimarogonas, A.D. and Yao,J., Vibration of a beam with a breathing crack. Journal of Sound and Vibration, 2001, 239(1): 57-67.

[14] Meirovitch,L., Elements of Vibration Analysis. New York: McGraw-Hill, 1986.

[15] Chondros,T.G., The continuous crack flexibility method for crack identification. Fatigue & Fracture of Engineering Materials & Structures, 2001, 24: 643-650.

[16] Loya,J.A. and Ferna'ndez-Sa'ez,L.R.J., Natural frequencies for bending vibrations of Timoshenko cracked beams. Journal of Sound and Vibration, 2006, 290: 640-653.

[17] Faverjon,B. and Sinou,J., Identification of an open crack in a beam using an a posteriori error estimator of the frequency response functions with noisy measurements. European Journal of Mechanics A/Solids, 2009, 28: 75-85.

[18] Lam,H.F., Ng,C.T. and Veidt,M., Experimental characterization of multiple cracks in a cantilever beam utilizing transient vibration data following a probabilistic approach. Journal of Sound and Vibration, 2007, 305: 34-49.

[19] Ke,L.L., Yang,J., Kitipornchai,S. and Xiang,Y., Flexural vibration and elastic buckling of a cracked Timoshenko beam made of functionally graded materials. Mechanics of Advanced Materials and Structures, 2009, 16: 488-502.

[20] Friswell,M.I. and Penny,J.E.T., A simple nonlinear model of a cracked beam. In: Proceedings 10th international modal analysis conference, San Diego, CA, 1992: 516-521.

[21] Kisa,M. and Brandon,J., The effects of closure of cracks on the dynamics of a cracked cantilever beam. Journal of Sound and Vibration, 2000, 238(1): 1-18.

[22] Abraham,O.N.L. and Brandon,J.A., Modeling of the opening and closure of a crack. Journal of Vibration and Acoustics, 1995, 117: 370-377.

[23] Cheng,S.M., Wu,X.J. and Wallace,W., Vibrational response of a beam with a breathing crack. Journal of sound and vibration, 1999, 225(1):201-208