Vibration and Buckling Behaviour of Laminated Composite Plate

A Thesis submitted in partial fulfilment of the requirements for the degree of

Bachelor of Technology

In

Mechanical Engineering

By

Ravi

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National Institute of Technology, Rourkela

Odisha, 769008 India

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Under the guidance of

Prof. Subrata Kumar Panda

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Odisha, 769008 India

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DEPARTMENT OF MECHANICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA 769008

ODISHA, INDIA

CERTIFICATE

This is to certify that the thesis titled "**Vibration and Buckling Behaviour of Laminated Composite Plate**", submitted to the National Institute of Technology, Rourkela by **Ravi (Roll No. 109ME0357)** for the award of **Bachelor of Technology** in Mechanical Engineering, is a bonafide record of research work carried out by him under my supervision and guidance.

The candidates have fulfilled all the prescribed requirements.

The thesis which is based on candidate's own work, has not submitted elsewhere for a degree/diploma.

In my opinion, thesis is of standard required for the award of a **Bachelor of Technology** in Mechanical Engineering.

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ABSTRACT

Free vibration and buckling responses of laminated composite plate in the framework of first order shear deformation theory is analysed. The model has been developed in ANSYS using ANSYS parametric design language code. The model has been developed in ANSYS using ANSYS parametric design language code. In this study two shell elements (SHELL181/SHELL281) have been chosen from the ANSYS element library to discretise and obtain the elemental equations. The governing differential eigenvalue equations have been solved using Block-Lanczos algorithm. The solution predicts fundamental natural frequencies and critical buckling load of laminated composite plate. To establish the correctness of the proposed model, a convergence study has been done and the results obtained by using the model are compared with the available published literature. Effect of different parameters such as the thickness ratios, the aspect ratios, the modular ratios and the boundary conditions on the free vibration and buckling behavior of laminated composite plate is discussed.

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1.   Introduction

A structural composite is consisting of two or more phases on a microscopic scale and their mechanical performance/properties are designed to be superior to those of the constituent materials acting independently. Out of the two phases one is said fibre/reinforcement usually discontinuous, stiffer and stronger. The second one is less stiff weaker and continuous phase namely, matrix phase. The properties of a composite depend on the properties of the constituents, their geometry and the distribution of the phase. Composite system includes concrete reinforced with steel and epoxy reinforced with graphite fibres, etc. The high performance structural composite is normally continuous fibre reinforcement and it also determines the mechanical properties like stiffness and strength in the fibre direction. The matrix phase provides protection to fibre, bonding, support and local stress transfer from one fibre to another. Laminated composite structures are being increasingly used in many industries such as aerospace, marine, and automobile due to their high strength to weight ratio, high stiffness to weight ratio, low weight and resistances to electrochemical corrosion, good electrical and thermal conductivity and aesthetics.

The most popular numerical technique to solve governing differential equations today is the finite element method (FEM) and to reduce the computational cost many finite element software are also available in market for modelling and analysis of composite and advanced material structures.

Analyses of composite plate have been based on the following approaches:

(1) Equivalent single layer theories (2-D)
  (a) Classical laminated plate theory
  (b) Shear deformation laminated plate theories
(2) three dimensional elastic theories (3-D)
  (a) Traditional 3-D elasticity formulation
  (b) Layerwise theories
(3) Multiple model methods (2-D and 3-D)

The equivalent single layer (ESL) plate theories are derived from the 3-D elasticity theory by making suitable assumption concerning the kinematics of deformation or the stress state through the thickness of laminate. In the three-dimensional elasticity theory, each layer

is modelled as a 3-D solid. The simplest ESL laminated plate theory is the classical laminated plate theory (CLPT), which is an extension of the Kirchhoff theory. To overcome the shortcomings of the classical theory, first order shear deformation theory (FSDT) has been developed. The FSDT extends the kinematics of the CLPT by including a gross transverse shear deformation in its kinematics assumption.

The objective of present work to developed a finite element model to analyse the free vibration and buckling behaviour of laminated composite plate. The present model has been developed in ANSYS and solved using ANSYS parametric design language (APDL) code. Effect of different parameters such as thickness ratios, aspect ratios, modular ratios and boundary conditions on the laminated composite plate has been discussed.

2.   Literature Review

In recent years, many researchers have been studied the free vibration and buckling behaviour of laminated plate to meet new challenges in the real world. Here, a short discussion on the different behaviour and analysis steps of composite plate has been discussed to connect the purpose of the work as discussed in aforementioned chapter. Aydogdu [1] investigated laminated composite plates and using an inverse method based on a new shear deformation theory. Zhen *et al*. [2] solved free vibration analysis of laminated composite and sandwich plates using Navier's technique and the model has been developed based on higher order theory. Wang *et al*. [3] examined rectangular laminated composite plates via mesh less method using the FSDT plate model. Kant and Swaminathan [4] reported analytical solutions of free vibration behaviour of laminated composite and sandwich plates based on a higher order refined theory. Subramanian [5] analysed dynamic behaviour of laminated composite beams using higher order theories and finite element steps. Lee [6] studied the free vibration analysis of delaminated composite beams by using a layerwise theory and equation of motion are derived using Hamilton's principles. Chen *et al*. [7] studied free vibration of generally laminated beams via state space based differential quadrature using the technique of matrix theory. Ferreira *et al*. [8] examined free vibration cases of symmetric laminated composite plates by radial basis functions and the plate kinematics is considered as the FSDT. Leung *et al*. [9] analysed free vibration of laminated composite plates subjected to in-plane stresses. Thai and Kim [10] studied free vibration of laminated composite plates using two variable refined plate theories. Khdeir and Reddy [11] examined the free vibrations of laminated composite plates in the framework of second order shear deformation theory. Tseng *et al*. [12] studied the in-plane vibration of laminated curved beams with variable curvature based on the Timoshenko type curved theory. Xiang and Kang [13] examined the free vibration analysis of laminated composite plates using the meshless local collocation method. Zhang *et al*. [14] studied recent developments in finite element analysis for laminated composite plates. Koutsawa and Daya [15] investigate the static and free vibration analysis of laminated glass beam on viscoelastic supports. Dong *et al*. [16] examined the vibration analysis of a stepped laminated composite Timoshenko beam. Aydogdu *et al*. [17] studied vibration behaviour of cross ply laminated square plates with general boundary conditions by using the two dimensional shear deformation theories. Lanhe *et al*. [18] studied vibration responses of generally laminated composite plates by the moving least squares differential quadrature method. Hu *et al*. [19] examined the vibration of twisted laminated composite conical shells.

Buckling is one of the major modes of failure of laminated structures and it is necessary to predict the critical load of the structural component for the easy replacement with good load bearing capacity. Wu *et al*. [20] studied the thermo-mechanical buckling of laminated composite and sandwich plates based on the global–local higher order theory. Guo *et al*. [21] studied buckling behaviour by taking the effect of elastic and geometric stiffness matrices of stiffened laminated plates using layerwise finite element formulation. Topal and Uzman [22] reported optimization of thermal buckling load of laminated composite plates based on the FSDT and a modified feasible direction (MFD) method. Matsunaga [23] examined thermal buckling of cross-ply laminated composite and sandwich plates based on the global higher order deformation theory. Shufrin *et al*. [24] studied the buckling of symmetrically laminated rectangular plates with general boundary conditions using a semi-analytical approach. Ovesy and Assaee [25] examined effects of bend–twist coupling on postbuckling characteristics of composite plate using the finite strip approach. Shukla and Nath [26] studied thermo-mechanical postbuckling of cross ply laminated rectangular plates in the framework of the FSDT by taking von-Korman type nonlinearity in the formulation. Shukla and Nath [27] presented analytical solution of buckling and postbuckling of angle ply laminated plate under thermo-mechanical loading based on the FSDT and von-Karman type geometric nonlinearity. Shariyat [28] studied nonlinear dynamic and thermo-mechanical buckling behaviour of the imperfect laminated and sandwich cylindrical shells based on zigzag layer wise shell theory.

It is true from the above literature that many attempts have been made in past to exploit the vibration and buckling strength of laminated structures using different theory and commercial tool. The present work aims to develop an ANSYS model of laminated structure using ANSYS parametric design language (APDL) code as discussed in earlier chapter to obtain the eigenvalue solutions of vibration and buckling cases.

3.   ANSYS and its application

In modern world design process is too close to precision so, the finite element method (FEM) has got worldwide appreciation due to its trustworthiness in design and analysis. It helps to predict the responses of various products, parts, subassemblies and assemblies by simulating the real life cases as par to the experimental results. Modelling and simulation step helps to reduce time of prototyping and moderates the physical verification expenses. It also increases the innovation at a faster rate. The optimization step in FEM tool adds a new paradigm to achieve the designer quest. ANSYS is now being used in a number of different engineering fields such as power generation, transportation, medical components, electronic devices, and household appliances for its easy applicability.

The first ANSYS concept was discussed in a public forum during 1976. The designing was improved slowly from 2D-3D modeling. Initially, the modeling was confined to beam to shell and then it is extended to volume elements. In addition to that, graphics were introduced for better modeling and prediction. It is well known that, the FEM was initially involved to discretize the structure into nodes and joining them by specific rule elements are obtained and the respective responses are calculated. Today ANSYS can be used many fields such as fatigue analysis, nuclear power plant, medical applications, and to find the eigenvalues of magnetic field, etc. ANSYS is also very useful in electro-thermal analysis of switching elements of a super conductor, ion projection lithography, detuning of an HF oscillator by the mechanical vibration of an acoustic sounder. It is used to analyze the vehicle simulation and in aerospace industries as well.

Based on the above discussion on the capability of ANSYS the present work has been modeled in ANSYS by choosing two different shell elements namely SHELL181 and SHELL281 from ANSYS library. SHELL181 element is used for the vibration and SHELL281 is used in the buckling analysis of laminated structures. In the following steps property and applicability of those two elements are discussed to have more clear idea.For present work the analysis is done by choosing two different shell elements from ANSYS library.

SHELL181 is a four noded shell element and six degrees of freedom per node (three translations in x, y, z direction and rotation about x, y, z axis). It can model plastic behaviour, so the laminated composites are preferred to model because the matrix phase is not purely elastic in nature. Thin to moderately thick structures can be analyzed using the same element. The analysis of shells is easy whose thickness changes under nonlinear analysis of plates.

This shell element can also be used for analyzing layered shell structures. A pictorial presentation of SHELL181 is given in Fig. 1 [32]
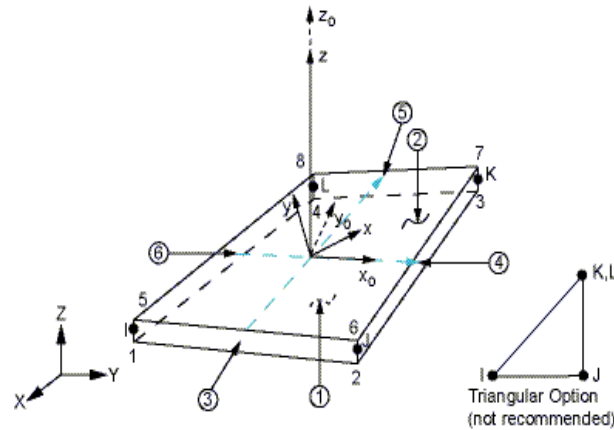

Fig. 1 SHELL181 geometry

x_o = Element x-axis if ESYS is not provided.

x = Element x-axis if ESYS is provided.

One more element SHELL281 is employed for the buckling analysis. This is an eight-node linear shell element with six degrees of freedom at each node. Those are translation in x, y, z direction and rotation about x, y, z axis. It is well-suited for linear, large rotation, and/or large strain nonlinear applications. It uses the same theory and analyses all the elements that uses SHELL181. The element formulation is based on logarithmic strain and true stress measures. Fig. 2 shows the idea regarding the SHELL281 element. The details of the element can be seen in reference [32].
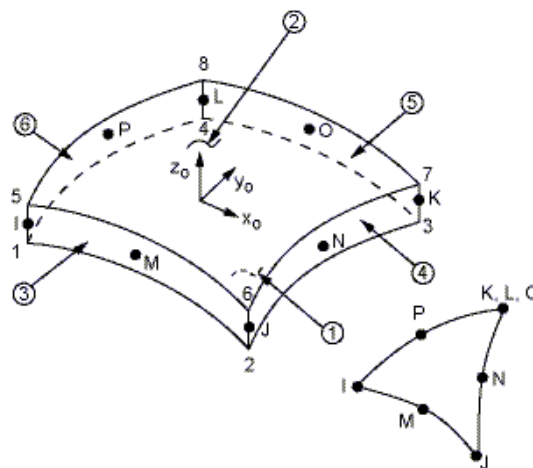

Fig. 2 SHELL281 geometry

x_o = Element x-axis if element orientation is not provided.

x = Element x-axis if element orientation is provided.

4. Mathematical formulation

It is well known that midplane kinematics of laminated composite has been considered as the FSDT using the inbuilt steps in ANSYS and conceded as follows:

$$u(x, y, z) = u_0(x, y) + z\theta_x(x, y)$$
$$v(x, y, z) = v_0(x, y) + z\theta_y(x, y)$$
$$w(x, y, z) = w_0(x, y) + z\theta_z(x, y)$$

(1)

where, u, v and w represents the displacements of any point along the (x, y, z) coordinates. u_0, v_0 are the in-plane and w_0 is the transverse displacements of the mid-plane and θ_x, θ_y are the rotations of the normal to the mid plane about y and x axes respectively and θ_z is the higher order terms in Taylor's series expansion. The geometry of two-dimensional laminated composite plates with positive set of coordinate axis is shown in Fig. 3
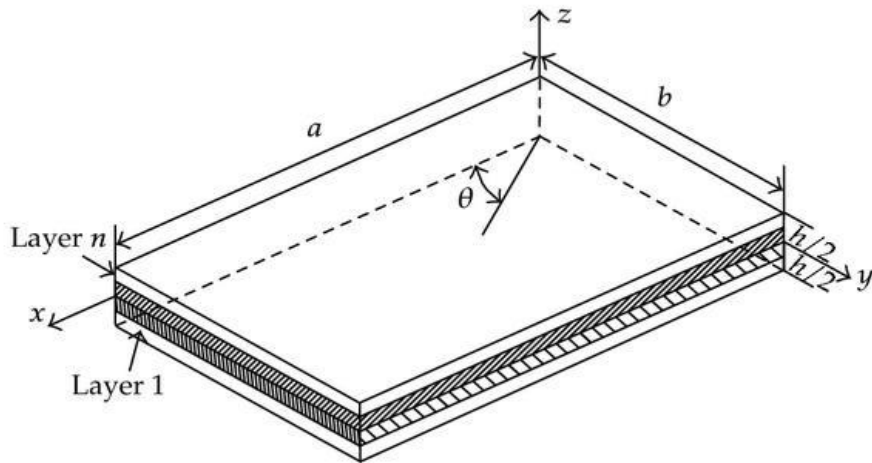

Fig. 3 Geometry of laminated composite plate with positive set of coordinate axis

where, $\delta_i = \begin{bmatrix} u_{0_i} & v_{0_i} & w_{0_i} & \phi_{x_i} & \phi_{y_i} & \phi_{z_i} \end{bmatrix}^T$. The shape functions for four noded shell element (j=4) and eight noded shell element (j=8) are represented in Eqn. (1) and (2), respectively in natural (ξ-η) coordinates, and details of the element are given as

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$
$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$
$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$
(2)

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)(-\xi-\eta-1) ; \qquad N_2 = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1) ; \qquad N_4 = \frac{1}{4}(1-\xi)(1+\eta)(-\xi+\eta-1)$$

$$N_5 = \frac{1}{2}(1-\xi^2)(1-\eta) ; \qquad N_6 = \frac{1}{2}(1+\xi)(1-\eta^2)$$

$$N_7 = \frac{1}{2}(1-\xi^2)(1+\eta) ; \qquad N_8 = \frac{1}{2}(1-\xi)(1-\eta^2)$$
(3)

Strains are obtained by derivation of displacements as:

$$\{\varepsilon\} = \left\{ u_{,x} \quad v_{,y} \quad w_{,z} \quad u_{,y}+v_{,x} \quad v_{,z}+w_{,y} \quad w_{,x}+u_{,z} \right\}^T$$
(4)

where, $\{\varepsilon\} = \left\{ \varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{xy} \quad \gamma_{yz} \quad \gamma_{xz} \right\}^T$, is the normal and shear strain components of in plane and out of plane direction.

The strain components are now rearranged in the following steps by in plane and out of plane sets.

The in-plane strain vector:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \gamma_{xy_0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$
(5)

The transverse strain vector:

$$\begin{Bmatrix} \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{z_0} \\ \gamma_{yz_0} \\ \gamma_{xz_0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix}$$
(6)

where, the deformation components are described as:

8

$$\begin{Bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \gamma_{xy_0} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} ; \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \theta_x}{\partial x} \\ \dfrac{\partial \theta_y}{\partial y} \\ \dfrac{\partial \theta_x}{\partial y} + \dfrac{\partial \theta_y}{\partial x} \end{Bmatrix} \tag{7}$$

$$\begin{Bmatrix} \varepsilon_{z_0} \\ \gamma_{yz_0} \\ \gamma_{xz_0} \end{Bmatrix} = \begin{Bmatrix} \theta_z \\ \dfrac{\partial w_0}{\partial y} + \theta_y \\ \dfrac{\partial w_0}{\partial x} + \theta_x \end{Bmatrix} ; \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \dfrac{\partial \theta_z}{\partial y} \\ \dfrac{\partial \theta_z}{\partial x} \end{Bmatrix} \tag{8}$$

The strain vector expressed in terms of nodal displacement vector:

$$\{\varepsilon\} = [B]\{\delta\} \tag{9}$$

where, $[B]$ indicates the strain displacement matrix containing interpolation functions and their derivatives and $\{\delta\}$ is the nodal displacement vector.

The generalized stress strain relation with respect to the reference plane is expressed as:

$$\{\sigma\} = [D]\{\varepsilon\} \tag{10}$$

where, $\{\sigma\}$ and $\{\varepsilon\}$ is the stress and strain vectors, respectively and $[D]$ is the rigidity matrix. The element stiffness matrix $[K]$ and mass matrix $[M]$ can be easily derived with the help of virtual work method which may be expressed as:

$$[K] = \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D][B]|J| d\xi d\eta \tag{11}$$

$$[M] = \int_{-1}^{+1} \int_{-1}^{+1} [N]^T [m][N]|J| d\xi d\eta \tag{12}$$

where, $|J|$ Is the determinant of the Jacobian matrix, $[N]$ is the shape function matrix and $[m]$ is the inertia matrix. The integration has been carried out using the Gaussian quadrature method.

The free vibration analysis is used to determined natural frequencies by given equation:

$$\left([K] - \omega_n^2 [M] \right) = 0 \tag{13}$$

The eigenvalue type of buckling equation can be expressed as in the following steps by dropping force terms and conceded to

$$\{[K] + \lambda_{cr} [K_G]\}\{\delta\} = 0 \tag{14}$$

where, $[K_G]$ is the geometric stiffness matrix and λ_{cr} is the critical mechanical load at which the structure buckles.

5.   Results and Discussion

In the present study, the free vibration and buckling behaviour of laminated composite panels has been investigated using APDL code developed in ANSYS environment. Two elements are used for the analysis as discussed earlier, a four nodded six degrees of freedom structural shell element, SHELL181 and eight-nodded six degrees of freedom element SHELL281. The numerical values for the fundamental natural frequency and buckling load are calculated using Block-Lanczos method. The non-dimensional frequency and buckling load parameters are as follows: $(\varpi) = \omega b^2 \left\{ \rho / \left(E_2 h^2 \right) \right\}^{\frac{1}{2}}$ and $N_{xx} = \lambda a^2 \big/ E_2 h^3$

The material properties for vibration and buckling behaviour are presented in Table 1 and 2 respectively.

Table 1

$E_1 = 40E_2$	$G_{12} = 0.6E_2$	$v_{12} = 0.25$
$E_2 = E_3$	$G_{23} = 0.5E_2$	$v_{23} = 0.25$
$E_2 = 1$	$G_{13} = 0.6E_2$	$v_{13} = 0.25$

Table 2

$E_1 = 20E_2$	$G_{12} = 0.6E_2$	$v_{12} = 0.25$
$E_2 = E_3$	$G_{23} = 0.5E_2$	$v_{23} = 0.49$
$E_2 = 1$	$G_{13} = 0.6E_2$	$v_{13} = 0.25$

The boundary conditions are used for the computation purpose are expressed as follows:

Simply supported (SSSS): $v=w=\theta_y=0$ at   x=0, and *a*;
                          $u=w=\theta_x=0$ at   y=0 and *b*;

Clamped (CCCC): $u= v =w=\theta_x= \theta_y=0$ at   x=0 and *a* and   y=0 and *b*;

Convergence and validation study

In this numerical analysis two different problem of square (*a/b* = 1) laminated composite angle ply $(\pm 45^0)_2$ and symmetric cross-ply $(0^0/90^0/0^0)$ plates have been analyzed for the validation of free vibration and buckling behavior, respectively. As a first step, the developed ANSYS model is validated by comparing the results with references [11] for non-dimensional fundamental natural frequency with different mesh sizes and is shown in Fig. 4. It is seen from the Fig. 4 that the frequency values showing good results with the reference [11], a (18×18) mesh size is sufficient and it has been used further investigation of vibration behavior of laminated composite plate. The buckling analysis of laminated plate has been carried out under biaxial loading. The convergence of the present developed model has been obtained and tabulated in Table 3 and a (14×14) mesh is sufficient to give the result. It can be

seen that the present results are showing good agreement in comparison to the HSDT model.

Table 3. Convergence of buckling load for cross ply $(0^0/90^0/0^0)$ simply supported laminate

Mesh size	Non dimensional buckling load
10×10	5.6231
11×11	5.1125
12×12	4.6868
13×13	4.3266
14×14	4.0178
Wu and Chen [20]	**4.963**
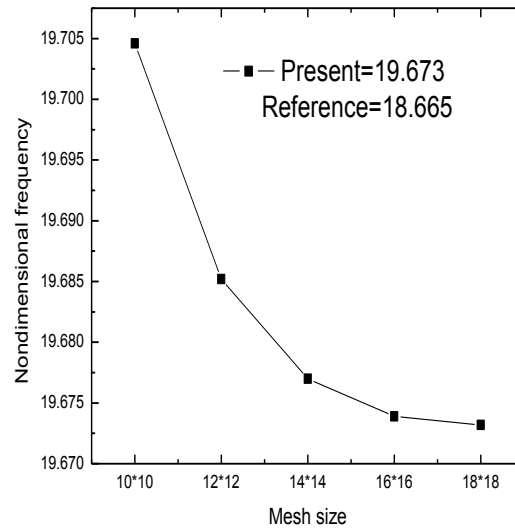
Fig. 4 Variation of nondimensional fundamental frequency of a simply supported square plate under different mesh sizes

Some new results are obtained using SHELL181 and SHELL281 elements with various thickness ratios (a/h), aspect ratios (a/b), modular ratios (E_1/E_2) and different boundary conditions and five different modes of vibration are presented. Fig. 5 and 6 shows that the frequency values (different modes) of different mode increases as thickness ratio increases for both support conditions. It is clear from Fig. 7 that the increase in aspect ratio changes the shapes of the plate and increases the values of frequency. It can be seen from Fig.8 that the frequency values of laminated plate increases with the increasing modular ratio. The laminated composite plate structure shows higher value of non-dimensional frequency under clamped support condition for different modes. The non-dimensional buckling loads

parameters for simply supported laminated plate for different thickness ratios (a/h) and the aspect ratios (a/b) under biaxial load have been computed and shown in Fig.10 and 11, respectively. It can easily be seen that the buckling load decreases with the increase in thickness ratios and increases with increase in aspect ratios.
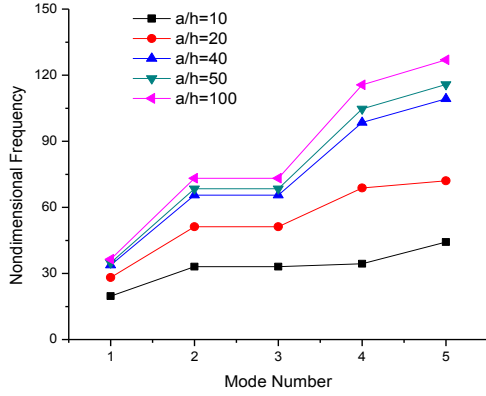

Fig. 5 Variation of nondimensional frequency of a simply supported square plate under different modes and thickness ratio
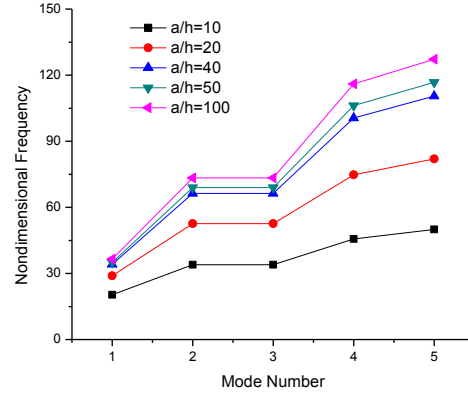

Fig. 6 Variation of nondimensional frequency of a clamped square plate under different modes and thickness ratio
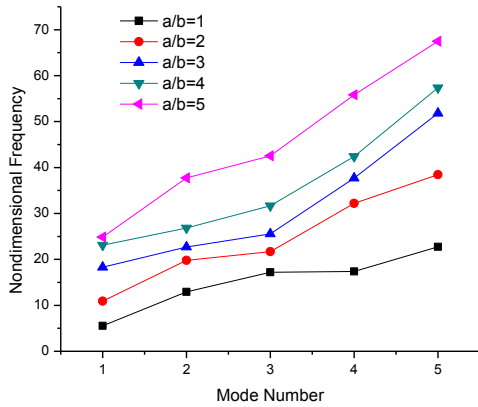

Fig. 7 Variation of nondimensional frequency of a simply supported plate under different modes and aspect ratio
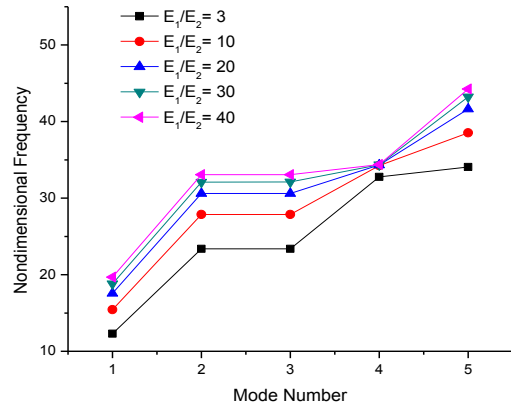

Fig. 8 Variation of nondimensional frequency of a simply supported square plate under different modes and modular ratio
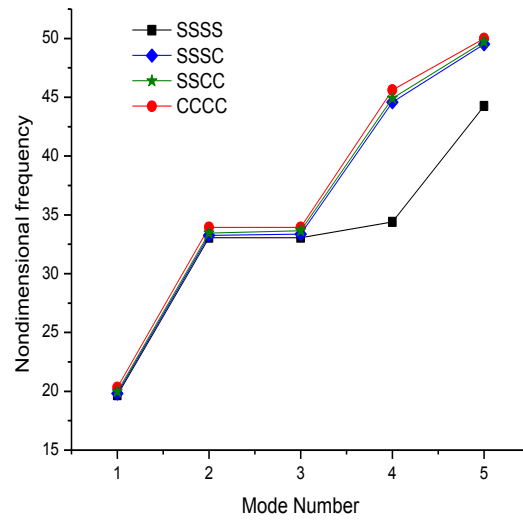
Fig. 9 Variation of nondimensional frequency of a square laminated plate under different modes and boundary conditions.
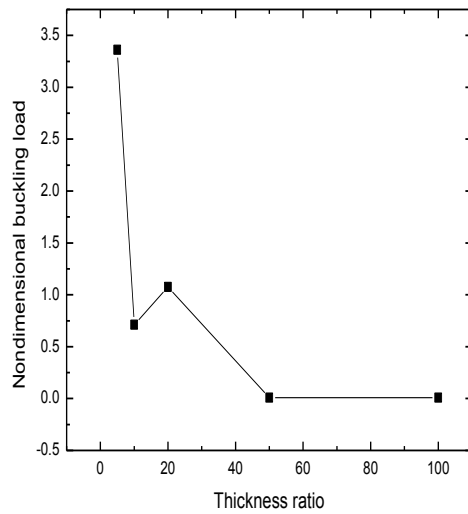

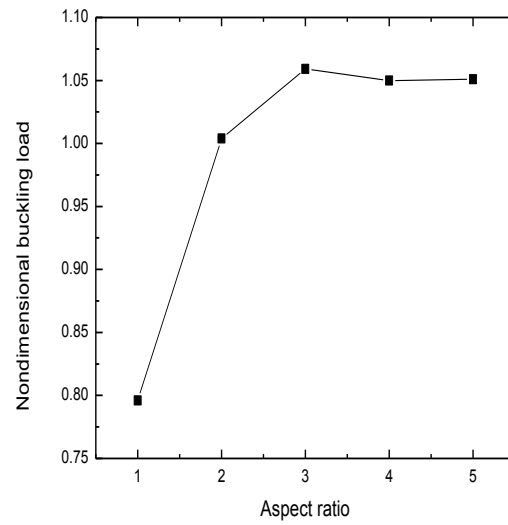Fig.10 Buckling load of a square laminated plate under different thickness ratio

Fig. 11 Buckling load of a square laminated plate under different aspect ratio

6.   Conclusions

In this present work, a finite element model is developed in ANSYS using APDL code and its comprehensive testing has been done. The ANSYS model is used to obtain the free vibration and buckling responses for laminated plates with different aspect ratios, thickness ratios and modular ratio under different support condition for different modes. The following points can be concluded from the present study are as follows:

❖ The convergence study is clearly showing that the present developed model is capable to solve different vibration and buckling problems with good accuracy and less computational cost.

❖ The non-dimensional frequency value increases with increase in the thickness ratios and aspect ratios.

❖ Similarly, the frequency of laminated plate increases with increase in modular ratio i.e., because of the fact that the degree of orthotropicity increases as the modular ratio increases.

❖ In addition to that the non-dimensional frequency of clamped plate is higher as compared to other.

❖ The buckling load is decreasing with the increase in thickness ratio and the response shows a reverse response for aspect ratio.

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