STUDY ON PARAMETRIC BEHAVIOUR OF SINGLE CELL BOX GIRDER UNDER DIFFERENT RADIUS OF CURVATURE

A thesis
Submitted by

Laxmi Priya Gouda
(210CE2025)

In partial fulfillment of the requirements for the award of the degree of

Master of Technology
In
Civil Engineering
(Structural Engineering)

Department of Civil Engineering
National Institute of Technology Rourkela
Odisha -769008, India
May 2013
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Under The Guidance of
Dr. Robin Davis P., Prof. Dept. of Civil Engineering &
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May 2013
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Laxmi Priya Gouda

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ABSTRACT

Bangalore metropolis, the silicon valley of India, has experienced phenomenal growth in population in the last two decades. So, to meet the traffic demands, Metro Rail Transport started. Bangalore Metro Rail Corporation; is constructing some phase of Metro Rail to be of elevated one. There are different structural elements for a typical box girder bridge. The present study focus on the parametric study of single cell box girder bridges curved in plan.

For the purpose of the parametric study, five box girder bridge models with constant span length and varying curvature. In order to validate the finite element modelling method, an example of box girder bridge is selected from literature to conduct a validation study. The example box girder is modelled and analysed in SAP 2000 and the responses are found to be fairly matching with the results reported in literature.

For the purpose of the parametric study, the five box girder bridges are modelled in SAP2000. The span length, cross-section and material property remains unchanged. The only parameter that changes is the radius of curvature. The cross section of the superstructure of the box girder bridge consists of single cell box. The curvature of the bridges varies only in horizontal direction. All the models are subjected to self weight and moving load of IRC class A tracked vehicle. A static analysis for dead load and moving load, and a modal analysis are performed. The longitudinal stress at top and bottom of cross sections, bending moment, torsion, deflection and fundamental frequency are recorded. The responses of a box girder bridge curved in plan are compared with that of a straight bridge. The ratio of responses is expressed in terms of a parameter. From the responses it is found that; the parameters like torsion, bending moment, and deflection is increasing as curvature of the bridges increase.
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<td>L</td>
<td>Length span</td>
</tr>
<tr>
<td>H</td>
<td>Depth box girder</td>
</tr>
<tr>
<td>$b_{tf}$</td>
<td>Width top flange</td>
</tr>
<tr>
<td>$t_{tf}$</td>
<td>Thickness top flange</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Width of web</td>
</tr>
<tr>
<td>$b_{bf}$</td>
<td>Width bottom flange</td>
</tr>
<tr>
<td>$t_{bf}$</td>
<td>Thickness bottom flange</td>
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<td>$b_{boxts}$</td>
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<td>L_{cant}</td>
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</tr>
<tr>
<td>H_{box}</td>
<td>Depth webs</td>
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<tr>
<td>$g$</td>
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<tr>
<td>R</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>Parameter curved/ Parameter straight</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Density of concrete</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Area of concrete</td>
</tr>
<tr>
<td>$Z_{cb}$</td>
<td>Distance from bottom to centroidal axis</td>
</tr>
<tr>
<td>$Z_{ct}$</td>
<td>Distance from top to centroidal axis</td>
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<tr>
<td>$I_c$</td>
<td>Second moment of area of the concrete section</td>
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<tr>
<td>$W_b$</td>
<td>Section modulus bottom</td>
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<tr>
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<td>Section modulus top</td>
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<tr>
<td>$u$</td>
<td>Perimeter concrete box girder</td>
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<tr>
<td>$E$</td>
<td>Young's modulus</td>
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<tr>
<td>$G$</td>
<td>Shear modulus</td>
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<tr>
<td>$\nu$</td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td>$A$</td>
<td>Coefficient of thermal expansion</td>
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<td>$f_{c'}$</td>
<td>Specific compressive strength of concrete</td>
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CHAPTER 1

INTRODUCTION

1.1 GENERAL

Bangalore metropolis, the silicon valley of India, has experienced phenomenal growth in population in the last two decades. To meet the traffic demands, Metro Rail Transport has been started. Some part of the metro rail is elevated one, known as viaduct. The viaduct has trapezoidal box girders of single cell. There are different structural elements for a typical box girder bridge.

Box girders, have gained wide acceptance in freeway and bridge systems due to their structural efficiency, better stability, serviceability, economy of construction and pleasing aesthetics. Analysis and design of box-girder bridges are very complex because of its three dimensional behaviours consisting of torsion, distortion and bending in longitudinal and transverse directions. A typical box girder bridge constructed in Bangalore Metro Rail Project is shown in Figure.1.1.

![Viaduct for metro rail.](image)

Figure 1.1. Viaduct for metro rail.
Box girders can be classified in so many ways according to their method of construction, use, and shapes.

Box girders can be constructed as single cell, double cell or multicell. It may be monolithically constructed with the deck, called closed box girder or the deck can be separately constructed afterwards called open box girder. Or box girders may be rectangular, trapezoidal and circular.

![Box Girder Diagram](image-url)

Figure 1.2 Different types of box girder bridges, Gupta et. al (2010)

A box girder is particularly well suited for use in curved bridge systems due to its high torsional rigidity. High torsional rigidity enables box girders to effectively resist
the torsional deformations encountered in curved thin-walled beams. There are three box girder configurations commonly used in practice. Box girder webs can be vertical or inclined, which reduces the width of the bottom flange.

In bridges with light curvature, the curvature effects on bending, shear and torsional shear stresses may be ignored if they are within acceptable range. Treating horizontally curved bridges as straight ones with certain limitations is one of the methods to simplify the analysis and design procedure. But, now a days higher level investigations are possible due to the high capacity computational systems available. It is required to examine these bridges using finite element analysis with different radius of curvatures configurations (i.e. closed box girders).

1.2. OBJECTIVES

The objectives of the present study are:

1. Literature review of the analytical methods, previous experimental and theoretical research work, and general behaviour of curved box girder bridges.

2. To Study the behaviour of curved box girders compared a straight bridge.

1.4 SCOPE

The present work is about the study of the behaviour of trapezoidal box girder bridges. Present study is limited to constant span length and variable radius of curvatures. The cross section of the bridge is limited to that of a single cell trapezoidal shape. Pre-stressed bridges are not included in the scope. Super elevation is not considered in the modelling. Only Linear static analysis is considered for the bridge. Typical box girder for metro rail is considered.
1.5 METHODOLOGY

Five box girder bridge models are considered with constant span length and varying curvature. In order to validate the finite element modelling method, an example of box girder bridge is selected from literature to conduct a validation study. The example of box girder is modelled and analysed in SAP 2000 and the responses are found to be fairly matching. For the purpose of the parametric study, the five box girder bridges are modelled in SAP2000. The span length, cross-section and material property remains unchanged. The only parameter that changes is the radius of curvature in plan. The cross section of the superstructure of the box girder bridge consists of single cell box. All the models are subjected to self weight and moving load of IRC class A tracked vehicle. A static analysis for dead load and moving load, and a modal analysis are performed. The longitudinal stress at top and bottom of cross sections, bending moment, torsion, deflection and fundamental frequency are recorded. The responses of a box girder curved in plan and straight are compared. The ratio of responses is expressed in terms of a parameter.

1.6. OUTLINE OF THESIS

This thesis contains four chapters. Chapter-1 is introduction to this chapter followed by objective and scope of the study.

In chapter-2, there is in study of previously published theoretical, experimental work on Box Girders, Horizontal curved bridges.

Chapter-3 contains three parts. Part 1 presents Validation study of a Rectangular cross-section Box Girder by already published journal values in SAP2000. Second part the modelling of single cell box girders under different values of radius of
curvatures. Third the parametric study on the models, how they behave in different curvatures under same loading conditions, same material property, same boundary condition and under same span length.
CHAPTER 2
LITERATURE REVIEW

2.1. GENERAL
This chapter is about the description of various literatures on curved type bridge. This discusses; the origin of curved beam theory, development of design approach, analytical methods such as finite element and thin walled beam theory, general behaviour and torsional behaviour in closed sections, formulation on deformation equation by Tung and Fountain (1970). First, the curved beam theory was given by first Saint-Venant (1843) and later the thin-walled beam theory by Vlasov (1965) has put foundation for all research works published to till date on the analysis and design of straight and curved box-girder bridges.

2.2. ORIGIN OF CURVED BEAM THEORY
In recent past of modern bridge design, engineers were reluctant to use curved girders due to the mathematical complexities associated with the design of such systems. Curved girders are subjected to not only major axis flexural stresses but also to significant torsional stresses, even under gravitational loading alone. Deflection, cross section distortion, and large deflection effects are much more pronounced in curved girder systems. The inherent rotation characteristics of horizontally curved girders require that the diaphragms and bracing that are used in straight girder systems simply to prevent premature lateral buckling become very important (primary) load carrying components in curved systems. In the past two decades, the availability of digital computers to carry out the complex structural analysis and design of such girders,
along with advancements in fabrication and erection technology, have made horizontally curved girder superstructures a viable and cost efficient option for designers.

Design considerations are kept different in the curved I-girder bridge design compared with the curved box girder design. As, the I-girder is an "open" section and is characterized by very low torsional resistance. The twisting of the I-girder results in significant normal stresses in the flanges. The closed box girder has generally improved torsional resistance over the I-section.

St. Venant (1843) worked on curved beam theory which is 150 years ago. Since then, a number of other European and Japanese researchers have contributed towards the analysis of curved beams. These researchers include Gottfield (1932), Umanskii (1948), Dabrowski (1964, 1965, 1968), Vlasov (1961), Timoshenko (1905), Shimada and Kuranashi (1966), and others. Comprehensive presentations of the basic theory of thin walled beams including flexure, torsion, distortion, and stress distribution is provided in several texts (Nakai and Yoo 1988, Vlasov 1961, Dabrowski 1968, Kollbrunner 1969, Heins 1975).

2.3 DEVELOPMENT OF CURVED BRIDGE DESIGN APPROACH

Research prior to the mid-sixties on the behaviour of curved girders was generally limited to theoretical work on the linear elastic static behaviour of isolated curved members and based on strength of materials assumptions, namely, that the cross section does not distort, that Hooke's law applies, and that small deflection theory applies. Since the mid-sixties an emphasis in curved girder research in the United States and Japan has been placed on the practical use of curved beam theory towards the design of horizontally curved bridges. Theory has been formulated for the
horizontally curved girder in the curved bridge system including buckling behaviour, large displacement behaviour, and ultimate strength behaviour.

In 1965, U.S. Steel (Highway 1965) published an approximate procedure called "V-load Analysis" for determining moments and shears in horizontally curved open-framed highway bridges.

Heins and Jin (1984) provided V-load method for analysing curved bridge design. Previously it has been noted that the live load distribution factors used in straight bridge design do not appropriately model the distribution in curved bridges. But it has certain limitations. Accuracy of this method with regard to live load depends upon the ability of the user. This method is also given by Brockenbrough 1986).

Rapid advancement in computer technology over the past 20 years has encouraged both theoretical and analytical investigations on many aspects of the behaviour of horizontally curved girders. In addition to the general ease in producing numerical solutions to solve complex mathematical relationships, such as in using the finite difference method to provide solutions to insolvable differential equations, this technology has encouraged the development of more general use structural analysis tools, namely, the finite element method. The use of these tools for designing curved bridges has also been encouraged by the requirement that the entire curved bridge superstructure be analyzed as a system. Many software packages have been developed exclusively for the design and analysis of curved bridges, resulting in less use of the V-load method, which was used for over 75 percent of curved bridge design prior to 1973 (AISC 1986).

In 1969, a comprehensive pooled funds research project sponsored by 25 participating state highway departments was initiated under the direction of the Federal Highway Administration to study the behaviour of curved bridges and to develop design
requirements. This project, referred to as CURT (Consortium of University Research Teams), was comprised of Carnegie Mellon University, the University of Pennsylvania, the University of Rhode Island, and Syracuse University.

This work was performed throughout the 1970's and resulted in the AASHTO Guide Specifications for Horizontally Curved Highway Bridges (subsequently referred to as the Guide Specifications), which was officially adopted in 1980. The CURT project involved:

(1) Reviewing all published information on the subject of curved bridges;
(2) Conducting analytical and experimental studies to confirm or supplement this published information and assimilate information from related research programs sponsored by state highway departments;
(3) Developing simplified analysis and design methods along with supporting computer programs and design aids; and
(4) Correlating the developed analysis and design methods with analytical and experimental data.

The Guide Specifications in its original form was disjointed and difficult to use. The strength predicted by the formulation did not feel appropriate that predicted by the formulation for straight girders as the radius of the curved girder approaches infinity.

The commentary was incomplete and lacked the detail.

In 1993, NCHRP Project 12-38, "Improved Design Specifications for Horizontally Curved Steel Girder Highway Bridges," was initiated.
2.4. ANALYTICAL METHODS FOR BOX GIRDER BRIDGES

2.4.1 Finite Element Method

The finite element method of analysis is generally the most powerful, versatile and accurate analytical method of all the available methods and has rapidly become a very popular technique for the computer solution of complex problems in engineering. It is very effective in the analysis of complicated structures such as that of a box girder bridge with complex geometry, material properties and support conditions and subjected to a variety of loading conditions. Canadian Highway Bridge Design Code has recommended the finite element method for all type of bridges.

A large number of elements have been developed for use in the finite element technique that includes one-dimensional beam-type elements, two dimensional plate or shell elements or even three-dimensional solid elements. Since the structure is composed of several finite elements interconnected at nodal points, the individual element stiffness matrix, which approximates the behaviour in the continuum, is assembled based on assumed displacement or stress patterns. Then, the nodal displacements and hence the internal stresses in the finite element are obtained by the overall equilibrium equations. By using adequate mesh refinement, results obtained from finite element model usually satisfy compatibility and equilibrium

Zienkeiwicz (1977), Sisodiya,et.al (1970) presented finite element analyses for single box girder skew bridges that were curved in plan. The bridge that could be analyzed by this method may be of varying width, curved in any shape, not just a circular shape and with any support conditions. They used rectangular elements for the webs and parallelogram or triangular elements for top and bottom flanges. This approximation would require a large number of elements to achieve a satisfactory solution. Such an approach is impractical, especially for highly curved box bridges.
Chapman, et al. (1971) conducted a finite element analysis on steel and concrete box girder bridges with different cross section shapes to investigate the effect of intermediate diaphragms on the warping and distortional stresses. They showed that curved steel boxes even with symmetrical load components gave rise to distortional stresses, and showed that the use of sloping webs resulted in an increase in distortional stresses.

Lim, et al. (1971) developed an element that has a beam-like-in-plane displacement field which is trapezoidal in shape, and hence, can be used to analyze right, skew, or curved box-girder bridges with constant depth and width.

Fam and Turkstra (1975) developed a finite element scheme for static and free vibration analysis of box girders with orthogonal boundaries and arbitrary combination of straight and horizontally curved sections. Four-node plate bending annular elements with two straight radial boundaries, for the top and bottom flanges, and conical elements for the inclined web members were used. The importance of warping and distortional stresses in single-cell curved bridges was established in relation to the longitudinal normal bending stresses, using the finite element method.

Dezi (1985) examined the influence of some parameters including transverse and longitudinal locations of external loads, span-to-radius ratio, width-to-depth of the cell, and number of cross diaphragms on the deformation of the cross section in curved single-cell box beams over those in straight single-cell box beams.

Galuta and Cheung (1995) developed a hybrid analytical solution that combines the boundary element method with the finite-element method to analyze box-girder bridges. The finite-element method was used to model the webs and bottom flange of the bridge, while the boundary element method was employed to model the deck. The bending moments and vertical deflection were found to be in good agreement when compared with the finite strip solution.

2.4.2 Thin Walled Beam Theory Method

Saint-Venant (1843) established the curved beam theory for the case of a solid curved bar loaded in a direction normal to the plane of curvature. In general, curved beam theory cannot be applied to curved box girders bridges, because it cannot account for warping, distortion, and bending deformations of the individual wall elements of the box. Curved beam theory can only provide the designer with an accurate distribution of the resultant bending moments, torque, and shear at any section of a curved beam if the axial, torsional and bending rigidities of the section are accurately known. The thin-walled beam theory was established by Vlasov (1965) for axisymmetric sections, and then extended by Dabrowski (1968) for asymmetric section who derived the fundamental equations that account for warping deformations caused by the gradient of normal stresses in individual box element.

The theory assumes non-distortional cross section and, hence, does not account for all warping or bending stresses. The predication of shear lag or the response of deck slabs to local wheel load cannot be obtained using the theory.

Oleinik and Heins (1975), and Heins and Oleinik (1976) analyzed the curved box girders in two parts. In the first part of the analysis, the box sections were assumed to retain their shape under the load. The load-deformation response of such a curved box
that considers bending, torsion and warping deformations was developed by Vlasov. Vlasov’s differential equations were solved using a finite difference approach to calculate the normal bending and normal warping stresses. In the second part of the analysis, the effect of cross sectional deformations was considered. These cross sectional deformations were calculated using a differential equation developed by Dabrowski. This equation was also solved using the finite difference approach and the normal stresses that resulting from cross-sectional deformations were calculated. The effects of both parts were summed to give the total normal stress distribution.


Mavaddat and Mirza (1989) utilized Maisel’s formulations to develop computer programs to analyze straight concrete box beams with one, two, or three cells and side cantilevers over a simple span or two spans with symmetric midspan loadings.

The structure was idealized as a beam, and the normal and shear stresses were calculated using the simple bending theory and Saint- Venant’s (1843) theory of torsion. Then, the secondary stresses arising from torsional and distortional warping and shear lag were calculated.

Fu and Hsu (1995) generated a new finite element based on Vlasov’s theory of curved thin walled beams. The horizontally curved thin walled beam element stiffness was developed directly in the cylindrical coordinate system.

2.5. GENERAL BEHAVIOUR OF SINGLE CELL BOX GIRDER

A general loading on a box girder, for single cell box, has components which bend, twist, and deform the cross section. Thin walled closed section girders are so stiff and strong in torsion that the designer might assume, after computations based on the
elemental torsional theory. In this theory torsional component of loading has negligible influence on box girder response. If the torsional component of the loading is applied as shears on the plate elements that are in proportion to St. Venant torsion shear flows, the section is twisted without deformation of the cross section. The resulting longitudinal warping stresses are small, and no transverse flexural distortion stresses are induced. However, if the torsional loading is applied, there are also forces acting on the plate elements, which tend to deform the cross section. Movements of the plate elements of the cross section cause distortion stresses in the transverse direction and warping stresses in the longitudinal direction.

Horizontally curved bridges will undergo bending and associated shear stresses as well as torsional stresses because of the horizontal curvature even if they are only subjected to their own gravitational load. Figure 2.1 shows the general behaviour of an open box section under gravity load showing separate effect.

Figure 2.1 General behaviour of an open box section under gravity load showing separate effect, (Zakia Begum, 2010)
2.6. TORSION IN CURVED CLOSED SECTION GIRDERS

There are two types of torsion that act on cross-sections; one is Saint-Venant torsion and other is Warping torsion. Saint-Venant torsion is a result of shear flow around the cross-section, while the other one is warping is due to bending deformation in the cross-section. In closed sections, warping torsion is neglected (Kolbrunner and Basler, 1969).
2.7. **DEFORMATION EQUATION FOR CURVED GIRDER BY TUNG AND FOUNTAIN (1970)**

It is difficult to calculate exact moments and stresses for curved girders. The analysis requires sophisticated computer analysis programs. One approximate method is to analyse that the girder is straight.

Tung and Fountain (1970) demonstrated that this approximation is acceptable for girders that have subtended angle per span is up to 40\(^\circ\). The bending moment of the girder can be determined by neglecting the curvature and using traditional beam theory for straight girders. But, for torsion the same assumption cannot be adopted.

The parameters such as Moment, Torque, Vertical Deflection and Rotation of the Curved Box Girder can be calculated using Tung and Fountain's equation.

A single span girder with radial support is employed in the derivation to show the behaviour due to curvature, as shown in Figure 2.2.
For curved girder under self-weight or gravity of concrete deck (distributed vertical load), the girder’s deformation has an interaction of bending and twisting. Tung and Fountain (1970) described the deformation of curved girder with some basic differential equations, shown as Equation 2.1 ~ Equation 2.3. For an infinitesimal segment at subtended angle $\beta$, the force equilibrium equation is shown as

$$\frac{dv}{R \, d\beta} = \frac{dv}{dx} = -w$$  \hspace{1cm} (2.1)

$$\frac{dM}{R \, d\beta} = \frac{dM}{dx} = \frac{T}{R} + N$$  \hspace{1cm} (2.2)

$$\frac{dT}{R \, d\beta} = \frac{dT}{dx} = \frac{M}{R} - t$$  \hspace{1cm} (2.3)

Assumption: $\frac{T}{R}$ term is small and its effect can be neglect for moment.

Then Moment can be approximated as straight girder

$$M = \frac{wx}{2} (L - x)$$  \hspace{1cm} (2.4)

Torque: for cases only has vertical load $w$, $t = 0$

$$\frac{d^2T}{dx^2} = \frac{1}{R} \frac{d(M/dx)}{dx} = \frac{1}{R} \frac{dV}{dx} = -\frac{w}{R}$$  \hspace{1cm} (2.5)
Integrating Equation 2.5 and get the general expression of \( T(x) \)

\[
T(x) = \int \int \int - \frac{w}{R} \, dx = - \frac{w}{6R} (x^3 + ax^2 + bx + c) \tag{2.6}
\]

Boundary conditions:

\[
\begin{align*}
T\left( \frac{L}{2} \right) &= 0 \\
T''(0) &= T(L) = 0 \\
T''\left( \frac{L}{2} \right) &= 0 \tag{2.7}
\end{align*}
\]

Thus, the approximate Torque is

\[
T(x) = \frac{wx^3}{24R} \left( \frac{4x^3}{L^2} - 6 \frac{x^2}{L} + L \right) \tag{2.8}
\]

Similarly, deformation equilibrium is shown in Equation 2.9 and Equation 2.10:

\[
\frac{d\phi}{R \, dx} = \frac{d\phi}{dx} = \frac{\phi}{R} + \frac{T}{GJ} \tag{2.9}
\]

\[
\frac{d\theta}{R \, dx} = \frac{d\theta}{dx} = \frac{\phi}{R} + \frac{M}{EI} \tag{2.10}
\]

Where, \( \phi \) is rotation about the radially (about R-axis) and \( \theta \) is rotation longitudinally.

Differentiating Equation 2.9 by \( x \) and substitute Equation 2.10 and Equation 2.3, the equation will transform into Equation 2.11.

\[
\frac{d^2\phi}{dx^2} = \frac{1}{R} \frac{d\phi}{dx} + \frac{1}{GJ} \frac{dT}{dx} = - \frac{\phi}{R^2} + \frac{1}{EI} \left( \frac{dT}{dx} + t \right) + \frac{1}{GJ} \frac{dT}{dx} \tag{2.11}
\]

\( \frac{\phi}{R^2} \) Usually has small value compared with other terms in the Equation 2.11. If no distributed torque \( t \) is applied, the equation is simplified as Equation 2.12.

\[
\frac{d^2\phi}{dx^2} = \left( \frac{1}{EI} + \frac{1}{GJ} \right) \frac{dT}{dx} \tag{2.12}
\]

Thus, the longitudinal rotation of the curved girder is presented as

\[
\phi(x) = \left( \frac{1}{EI} + \frac{1}{GJ} \right) \int_0^x T(s) \, ds \tag{2.13}
\]

For curved girder with vertical load \( w \), if substituting the torque Equation 2.8 into Equation 2.13, the rotation would be
\[
\phi_w(x) = \left(1 + \frac{GJ}{EI}\right) \int_0^x T(s) ds = \frac{wL^3}{24R} \left(\frac{1}{EI} + \frac{1}{GJ}\right) \int_0^x \left(\frac{4s^3}{L^2} + \frac{6s^2}{L} + L\right) ds
\]

(2.14)

\[
\phi_w(x) = \frac{wx}{24EI} \left(1 + \frac{EI}{GJ}\right) \left(x^3 - 2Lx^2 + L^3\right)
\]

(2.15)

At mid-span, the maximum rotation is

\[
\phi_w\left(\frac{L}{2}\right) = \frac{5wL^4}{384EI} \left(1 + \frac{EI}{GJ}\right)
\]

(2.16)

The deflection of curved girder can be approximate by straight girder deflection and curved effect, such as

\[
\Delta_w(x) = \frac{wx}{24EI} \left(x^3 - 2Lx^2 + L^3\right) + \phi_w(x) d_0
\]

(2.17)

From Figure 2.2, \(d_0 = R(1 - \cos(\frac{\beta_0}{2})\), which can be substituted into Equation 2.17 to give

\[
\Delta_w(x) = \frac{wx}{24EI} \left(x^3 - 2Lx^2 + L^3\right) \left[1 + \left(1 + \frac{EI}{GJ}\right)(1 - \cos(\frac{\beta_0}{2}))\right]
\]

(2.18)

\[
\Delta_w\left(\frac{L}{2}\right) = \frac{5wL^4}{384EI} \left[1 + \left(1 + \frac{EI}{GJ}\right)(1 - \cos(\frac{\beta_0}{2}))\right]
\]

(2.19)

These equations can be used to calculate the parameters like torsion, deflection of a curved bridge under dead load case.

**2.8 SUMMARY**

The literature review presented above shows that there are a number of published work on study of box girder bridges under various radius of curvatures. Also, tells about birth of curved bridge to modern day popularity of curved bridges. Due to rapid advancement of computers; it became easy to analyse the complex model of curved bridges and the finite element method for the analysis of curved models made it
popular. The thin walled beam theory made it easier to understand about the behaviour of a beam in curvature.
CHAPTER 3

BEHAVIOUR OF SINGLE CELL BOX GIRDER BRIDGE UNDER DIFFERENT RADIUS OF CURVATURES

3.1 INTRODUCTION

There are many methods available for analyzing curved bridges, as mentioned earlier in Chapter 2. However, of all the available analysis methods, the finite element method is considered to be the most powerful, versatile and flexible method. Due to recent development in computer technology, the method has become an important part of engineering analysis and design because nowadays finite element computer programs are used practically in all branches of engineering. A complex geometry, such as that of continuous curved steel box girder bridges, can be readily modelled using the finite element technique. The method is also capable of dealing with different material properties, relationships between structural components, boundary conditions, as well as statically or dynamically applied loads. The linear and nonlinear structural response of such bridges can be predicted with good accuracy using this method. In the current research, various structural elements are modelled using finite element method.

Program SAP2000 that was utilized throughout this study for the structural modelling and analysis and finally the description of the models of the straight and curved box bridges is presented. Before conducting a parametric study, the finite element model is validated using a recent study presented by Gupta et al (2010).

3.2 VALIDATION OF THE FINITE ELEMENT MODEL

To validate the finite element model of bridge deck in SAP-2000 a numerical example reported by Gupta et al (2010) has been considered.
SAP is a commercially available, general-purpose finite element-modelling package for numerically solving a wide variety of civil engineering problems. These problems include static/dynamic analysis. The program employs the matrix displacement method of analysis based on finite element idealization.

The shell element has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes.

In the recent past Yapping Wu (2003) has given an initial value solution of the static equilibrium differential equation of thin walled box beam, considering both shear lag and shear deformation.

Figure 3.1 shows the cross section of the simply supported box beam bridge model used for the validation study. It is subjected to two equal concentrated load (P=2x800N) at the two webs of mid span.

![Cross-section of simply supported rectangular box beam](image)

Figure 3.1. Cross-section of simply supported rectangular box beam
Length of Span is considered as 800mm, Modulus of elasticity (E) as 2.842GPa and Modulus of rigidity (G) as 1.015GPa. The model is Modelled in SAP refer Figure 3.2.

The rectangular box girder is modelled with Bridge Wizard having Shell elements. The boundary condition is taken is simply supported. It is assigned with point loads along the negative Z direction. Static analysis is conducted for the model.

![Model without load](image1.png) ![Model with load](image2.png)

Figure 3.2. Single cell rectangular box girder bridge modelled in SAP 2000

The bending moment, shear force and deflection at quarter span and mid span are monitored. The comparison of the values obtained and the values reported in literature, Gupta et. al (2010) are presented in the Table 3.1.

The table shows that percentage error in the values obtained for BM, SF and deflections are very negligible. Hence the finite element model can be considered as validated. The same modelling approach is followed for further studies on modelling of straight and curved box girder bridges.
Table 3.1 Comparison of responses obtained in present study and Gupta et. al (2010)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Location</th>
<th>Present Study</th>
<th>Gupta et.al(2010)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Moment (KN-m)</td>
<td>L/4th of span</td>
<td>0.16</td>
<td>0.16</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Mid span</td>
<td>0.32</td>
<td>0.32</td>
<td>0</td>
</tr>
<tr>
<td>Shear Force (KN)</td>
<td>L/4th of span</td>
<td>0.8</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Mid span</td>
<td>0.8</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>Deflection (mm)</td>
<td>Mid-span</td>
<td>4.35</td>
<td>4.91</td>
<td>12.87</td>
</tr>
</tbody>
</table>

3.3 MODELLING OF BOX GIRDER BRIDGES FOR PARAMETRIC STUDY

All the five models are modelled in SAP2000 for the parametric study.

3.3.1. Case study of Bridge Models

A 66m strip of a viaduct of elevated system of Metro Rail of BMRC is modelled as a bridge which is horizontally curved in plan. Its slab is monolithically constructed with the concrete box girder. The Girder is a single cell Box Girder having Trapezoid in cross-section. The bridge is simply supported. The reason for simply supported, is that the restraining effect of slab is not considered here. In reality the slab will help to restrict the rotation of girder. There are five models, among which one is straight and other four are curved in plan, modelled in SAP2000; which is Structural analysis software.

3.3.2 Types of design loads

The loads that are to be considered on the superstructure of a typical box girder bridges are listed below:

A) Permanent Loads:
• Dead Loads
• Superimposed Dead Loads
• Pressures (earth, water, ice, etc.)

B) Temporary Loads:
• Vehicle Live Loads
• Earthquake Forces
• Wind Forces
• Channel Forces
• Longitudinal Forces
• Centrifugal Forces
• Impact Forces
• Construction Loads

C) Deformation and Response Loads:
• Creep
• Shrinkage
• Settlement
• Uplift
• Thermal Forces

D) Group Loading Combinations.
Although there are various kinds of loading present in a typical bridge, for the present parametric study of bridges curved in plan, the scope is limited to Dead Loads and Vehicle live loads only.

3.3.3 Loading Placement

I.R.C Class A Tracked Vehicle loading are first applied on a simply supported girder, as lane loads with a span equal to 66m. Subsequently two loading cases were
considered for each bridge prototype, I.R.C Class A Tracked Vehicle loading, and bridge dead load.

3.4 MODELLING OF BRIDGES

The finite element modelling of one straight and four curved bridges are conducted in SAP2000.

3.4.1. Curved girder bridge modelled in SAP2000

The curved Box Girder Model is made using Bridge module with shell elements of SAP2000. The Horizontal alignment from Bridge Wizard is made curved by horizontally. Four curved bridges are modelled having radius of curvatures 205m, 210m, 220m and 306m. The curved bridge with radius of curvature of 205m is denoted as 205R. Similarly, 210R represents a curved bridge with radius 210m. Similar notation is followed for all the other cases. The Box Girders has Trapezoid in cross section. The Deck section was taken as per BMRC model and having a single span, of length 66m. The boundary condition is simply supported.

Table 3.2 Material properties.

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight /unit volume</td>
<td>25000 N/m³</td>
</tr>
<tr>
<td>Young's modulus (E)</td>
<td>$32500 \times 10^6$ N/m²</td>
</tr>
<tr>
<td>Poisson's ratio (v)</td>
<td>0.15</td>
</tr>
<tr>
<td>Shear Modulus (G)</td>
<td>$1.413 \times 10^3$ N/m²</td>
</tr>
<tr>
<td>Coefficient of thermal expansion (A)</td>
<td>$1.17 \times 10^{-5}$/°C</td>
</tr>
<tr>
<td>Specific compressive strength of concrete ($f_c'$)</td>
<td>$45 \times 10^6$ N/m²</td>
</tr>
</tbody>
</table>
3.4.2 Straight girder bridge modelled in SAP2000

The straight Box Girder Model is made using Bridge Wizard Commands with shell elements of SAP2000. The Horizontal alignment from Bridge Wizard is made straight by horizontally. The Box Girders has Trapezoid in cross section. The Deck section was taken as per BMRC model and having a single span, of length 66m. The boundary condition is simply supported. The material property is same as for the curved models.
3.5. CONFIGURATION OF BOX GIRDER

3.5.1. Cross-sectional properties

The cross-sectional properties for the trapezoidal box girder like span length, width of bridge, depth of bridge, thickness of top flange, width of top flange, width of bottom flange etc. is shown in Figure 3.5 and Table 3.3 and the material properties like cross sectional area, moment of inertia, distance from bottom to centroidal axis etc. are given in Table 3.4

Figure 3.5 Details of cross-section at mid span
Table 3.3 Cross-sectional dimensions (refer Figure 3.5)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notation</th>
<th>(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length span</td>
<td>L</td>
<td>66</td>
</tr>
<tr>
<td>Depth box girder</td>
<td>H</td>
<td>2.31</td>
</tr>
<tr>
<td>Width of top flange</td>
<td>b_{tf}</td>
<td>9.6</td>
</tr>
<tr>
<td>Thickness of top flange</td>
<td>t_{tf}</td>
<td>0.381</td>
</tr>
<tr>
<td>Width web</td>
<td>b_{w}</td>
<td>0.381</td>
</tr>
<tr>
<td>Width bottom flange</td>
<td>b_{bf}</td>
<td>4</td>
</tr>
<tr>
<td>Thickness bottom flange</td>
<td>t_{bf}</td>
<td>0.381</td>
</tr>
<tr>
<td>Width box top side</td>
<td>b_{boxts}</td>
<td>5.445</td>
</tr>
<tr>
<td>Cantilever length top flange</td>
<td>L_{cant}</td>
<td>2.080</td>
</tr>
<tr>
<td>Depth webs</td>
<td>H_{box}</td>
<td>1.547</td>
</tr>
</tbody>
</table>

The Angle of webs with vertical axis can be calculated as,

\[ \text{Angle of webs with vertical axis } (\alpha_w) = \tan^{-1} \left[ \frac{(b_{boxts} - b_{tf})/2}{(H_{box} + t_{bf})} \right] = 20^\circ 40' \]

Table 3.4 Material Property of Trapezoidal Box Girder.

<table>
<thead>
<tr>
<th>Cross sectional parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional area of concrete</td>
<td>A_c</td>
<td>6.375 m²</td>
</tr>
<tr>
<td>Distance from bottom to centroidal axis</td>
<td>Z_{cb}</td>
<td>1.47 m</td>
</tr>
<tr>
<td>Distance from top to centroidal axis</td>
<td>Z_{ct}</td>
<td>0.83 m</td>
</tr>
<tr>
<td>Second moment of area of the concrete section</td>
<td>I_c</td>
<td>4.45 m⁴</td>
</tr>
<tr>
<td>Section modulus bottom</td>
<td>W_b</td>
<td>3 m³</td>
</tr>
<tr>
<td>Section modulus top</td>
<td>W_z</td>
<td>5.3 m³</td>
</tr>
<tr>
<td>Perimeter concrete box girder</td>
<td>u</td>
<td>22.66 m</td>
</tr>
</tbody>
</table>

Dead of the box girder bridge per metre is calculated as,

Dead load box girder per meter = A_c * \rho_c * g = 156.34 KN/m

where,

A_c = cross-sectional area ; refer table 3.4,
\[ \rho_c = \text{density of concrete; } 2500 \text{ Kg/m}^3. \]

\[ g = \text{acceleration due to gravity; } 9.81 \text{m/s}^2. \]

Table 3.5 Comparison of Torsion and bending moment due to self weight

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Hand calculation using Tung and Fountain (1970)</th>
<th>SAP2000</th>
<th>%error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torsion</td>
<td>887t-m</td>
<td>924t-m</td>
<td>4</td>
</tr>
<tr>
<td>Bending Moment</td>
<td>72110KNm</td>
<td>60000KN-m</td>
<td>20</td>
</tr>
</tbody>
</table>

**3.6 RESPONSE OF BOX GIRDER BRIDGE WITH CURVATURE**

Analyses of the curved and straight box girder bridge models are for dead load and moving load conducted. The responses such as torsion, bending moment, longitudinal stress, deflections are monitored in each analysis. The non-dimensional parameter (L/R) is considered to plot the variation of the maximum responses curvature of the bridges.

The effect of the ratio of span length to the radius of curvature is insignificant for straight bridges with L/R=0. However, for curved bridges the ratio increases with increase in span length for a constant radius of curvature; but, it decreases, if the bridge has constant span length and radius of curvature increases.

**3.6.1 Torsion**

Torsion for all the bridge models is considered under dead load and moving load. The variation of torsion is plotted across the span length. A non-dimensional parameter \( \alpha \), is introduced here which represents the ratio of maximum torsion for curved bridge to that of straight from moving load analysis. The variation of \( \alpha \) is considered with the non dimensional parameter (L/R) for plotting the graphs.
3.6.1.1 Torsion Due To Self Weight

The analysis is conducted for self weight for all the cases. The torsion along the span is monitored and a graph is plotted between torsion and the span length. Figure 3.6 shows the variation of torsion along the span for various models.

The straight beam (R =∞, curvature, 1/R =0 ) shows no torsion for dead loads. But as the curvature increases or (radius of curvature decreases) the torsion arises in the girder, and it varies across the span as shown in the Figure 3.6

The maximum value of the torsion in the cross section increases as the curvature increases. As the radius increases beyond 210m for the particular span considered, the box girder shows a double twist. This behaviour may not be a favourable one to the girder as the shear flow changes sign at that section.

A 33% decrease in maximum torsion is observed when the radius of curvature is increased from 205m to 210m, 210m to 220m and 220m to 306m.
3.6.1.2 Torsion due to moving load

The analysis is conducted for live load of IRC.6 class A Tracked vehicle load for all the cases. The torsion along the span is monitored and a graph is plotted between torsion and the span length. Figure 3.7 shows the variation of torsion along the span for various models.

The variation of torsion along the span length for the five models considered is shown in Figure 3.7. The straight bridge model experiences torsion due to lane loading; but, in comparison with the curved bridge models it is almost negligible. In the curved bridges as the curvature increases torsion increases in general.

When the radius of curvature increased from 205m to 210m, the maximum torsion in the cross section decreased by 20.5%. An increase in maximum torsion of 7% is observed for in the increase in radius from 210m to 220m. When the radius of curvature increased from 220m to 306m, the maximum torsion is decreased by 16%.

![Figure 3.7 Variation of Torsion across the span under IRC class A loading](image-url)
3.6.1.3 Relation between maximum torsion to the ratio of span to radius of curvature

From the relation

\[ T = \frac{G\theta I}{L} \]  

where,

- \( T \) = Torsional Moment
- \( G \) = Bulk Modulus
- \( \theta \) = Subtended angle
- \( I \) = Polar Moment of Inertia
- \( L \) = Span length

From the relation the graph from Figure 3.8 shows that as \( \theta \) increases the Torsion increases. Thus with increase in \( \theta \), \( R \) will decrease for constant span. In comparison to straight model the curved models has much higher values of torsional moments.

Figure 3.8. Variation of \( \alpha_{\text{torsion}} \) with the radius of curvature
Maximum Torsion can also be expressed in terms of L/R ratio by the following Linear Equation

\[ \alpha_{\text{torsion}} = -1308.0 \frac{L}{R} + 792.7 \]  \hspace{1cm} 3.2

where,

\[ \alpha_{\text{torsion}} = \frac{\text{max. torsion, curved}}{\text{max. torsion, straight}} \]

L/R = span to radius of curvature

3.6.2 Deflection of the box girder

The deflection is recorded both along transverse direction of the trapezoidal box girder and also along the longitudinal direction of box girder.

3.6.2.1 Mid span vertical deflection along the width of box girder

The vertical deflection of the girder is considered along the transverse direction of the single cell box girder. The variation of vertical displacement across the traverse direction is shown in Figure 3.9. It shows that the mid span vertical displacement is increasing as per the radius of curvature is increasing.

When the radius of curvature is increased from 205m to 210m, the maximum mid span deflection along the transverse direction is increased by 14%. For the radius of curvature increased from 210m to 220m, the maximum deflection increased by almost 12.5%. Also for the radius of curvature is increased from 220m to 306m, the maximum deflection is increases by 11.11%.
3.6.2.2 Variation of mid span vertical deflection along the width of box girder compared with a dimensionless value (L/R)

The ratio of maximum deflection along the horizontal width of curved box girder to the maximum deflection of straight is taken as \( \alpha \), which is a dimensionless value is plotted with span to the radius of curvature.

Figure.3.9 Mid-span vertical displacement along the width of box girder

Figure.3.10 \( \alpha \) deflection along transverse direction to span to radius of curvature
The Figure 3.10 shows that, $\alpha_{\text{deflection}}$ is increasing as the L/R ratio decreases; it means that mid span vertical deflection along the transverse direction increases with increase in radius of curvature.

$\alpha_{\text{deflection}}$ can also be expressed approximately using a linear equation in terms of L/R.

$$\alpha = -4.479\frac{L}{R} + 3.304$$  

where,

$\alpha =$mid span deflection along the width of box girder

$L/R =$span to radius of curvature

3.6.2.3. Maximum deflection along the length of box girder

The deflection parameter is recorded for all the bridge models along the length of box girder. The $\alpha_{\text{deflection}}$ is a non dimensional parameter plotted along with L/R ratio and it is shown in Figure 3.11

When the radius of curvature increased from 205m to 210m, the maximum mid span deflection along the span is decreased by 7.7%. As the radius of curvature is increased to 220m, the maximum deflection is decreased by almost 16.67%. Also, when the radius of curvature is increased from 220m to 306m, the maximum deflection is found to be decreased by 75%, which behaves more like a straight bridge.

The dimensionless ratio $\alpha_{\text{maximum deflection}}$(deflection of curved to deflection of straight girder) decreases with decrease in span to radius of curvature. The deflection along the span of bridge increases with increase in curvature.
The Equation for maximum deflection along the length of box girder can be approximately found out by

\[ \alpha = 38.08L/R - 7.217 \]

where,

- \( \alpha \) = maximum deflection along the length of girder
- \( L/R \) = span to radius of curvature

![DEFLECTION (MAX)](image)

**Figure 3.11** \( \alpha \) to span to radius of curvature

### 3.6.3 Bending Moment

Bending moment parameter is recorded along the span of the box girder for all the models under their self weight and IRC Class A Tracked Vehicle Load. A static analysis and moving load analysis is carried out for the five models. Also a non dimensional value \( \alpha \) (bending moment), which is the ratio between maximum bending moment in the curved girder to the straight girder, is compared with span to radius of curvature ratio, which is a non dimensional value.
3.6.3.1 Bending moment due to self-weight

Bending moment for the all the curved models are compared with the bending moment of the straight model, and it is plotted along the span of the bridge, refer Figure 3.12.

It shows the Bending moment diagram of five different radius of curvature models under self weight. As, it can be seen that, the 205R model shows maximum bending moment than that of other bridge models The bending moment decreases by almost 29% as the radius of curvature decreases from 205m to 210m, 210m to 220m, and 220m to 306m.

Figure 3.12 Bending Moment along the span of girder in their self weight

3.6.3.2 Bending moment due to moving load

The models are loaded with rail load on one lane and the rail IRC class A tracked vehicle on the lane. A moving load analysis is done in SAP2000. The results have been plotted and compared, as shown in Figure 3.13.
Figure 3.13, shows the Bending moment values under IRC Class A Tracked Vehicle loading. The 205 radius of curvature model shows higher bending moment values than that of straight and all other curved models shows almost close value that with straight one i.e. horizontal radius of curvature more for 205 and for other model has less significance on bending moment values. The bending moment decreases by almost 18.75% as the radius of curvature decreases from 205m to 210m, 210m to 220m, and 220m to 306m.

From the relation $\frac{M}{I} = \frac{E}{R}$ 3.5

Bending moment is inversely proportional to Radius of curvature.

Where, $M$ = Bending moment, $I$ = Moment of Inertia, $E$ = Modulus of Elasticity, $R$ = Radius of curvature.

The Figure 3.13 shows that; there is decrease in bending moments as the radius of curvature increased.

![Graph showing bending moment values](image)

Figure 3.13 Bending moment along the span of box girder under vehicular live load
3.6.3.3 $\alpha_{\text{bending moment}}$ (bending moment of curved girder to the straight girder) with L/R ratio

The graph is plotted between $\alpha$, (The ratio between maximum bending moment of curved to the bending maximum bending moment of curved model) to the ratio of span to the radius of curvature and it is shown in Figure.3.14

![Graph showing relationship between bending moment and L/R ratio](image)

**Figure.3.14 $\alpha_{\text{bending moment}}$ to the span to radius of curvature**

From the Figure 3.14, it is found that, there is increase in $\alpha_{\text{bending moment}}$ as L/R ratio increases. As the curvature increases the deflection increases.

The bending moment can be approximately found out by using the equation

$$\alpha = 0.157 \frac{L}{R} + 0.965$$

where,

$$\alpha = \text{maximum bending moment (curved/straight)}$$

[3.6]
L/R = span to radius of curvature ratio

3.6.4 Longitudinal stresses due to moving load

All the models are analysed for a moving load analysis under IRC Class A Tracked Vehicle load. The longitudinal stress parameter is recorded across the span length for left overhanging portion, centre, right side overhanging portion of the box girder. Also, $\alpha_{\text{longitudinal stress}}$ for top and bottom face of box girder is plotted for span to radius of curvature ratio.

3.6.4.1 Longitudinal Stress (Top and Bottom) - Left side overhanging potion of box girder

The longitudinal stress at the top and bottom of the left overhanging potion of girder is found out from moving load analysis.

The bottom of left side of girder refer Figure 3.15, shows that, the maximum stress increases for 205 radius of curvature bridge model (205R) to 210 radius of curvature (210R) bridge model, and then it decreases for models 220R and 306R. But, it is observed that all the curved models have stresses more than straight model.

For bottom face of left overhang part of girder, the longitudinal stress is increased by 12% as the radius of curvature is increased from 205m to 210m and it is increased by 17.5% as radius of curvature is increased from 210m to 220m radius of curvature, 5.2% when the radius of curvature is increased from 220m to 306m. For top face, the longitudinal stress for all the curved models increases about 6% from straight model.
Figure 3.15 Longitudinal stress; top and bottom on left side of girder

### 3.6.4.2 Longitudinal Stress - Top and Bottom – Centre box part of cross section

The longitudinal stress at centre part of box girder at its top and bottom of girder along the span is considered for all the models and plotted as shown in figure 3.16.

A longitudinal stress increase of 12%, 25% and 1% are observed between the models 205R to 210R, 210R to 220R and 220R to 306R. For top face the longitudinal stress for all the curved models increases fairly 6% from straight model.

Figure 3.16 Longitudinal stresses at centre of girder (top and bottom)
3.6.4.3 Longitudinal Stress - Top and Bottom - Right side over hang portion

The longitudinal stress at right side over hanging part of box girder at its top and bottom of girder along the span is taken for all the models and plotted as shown in figure 3.17.

Under IRC class A loading, for bottom face of right overhang part of girder, the longitudinal stress is increased 16% from 205R to 210R and it is decreased by 28% both from 210R to 220R and 210R to 306R. For top face, longitudinal stress increases by 2% from 205R to 210R. and a 33% from 210R to 220R and also from 210R to 306R.

Figure 3.17 Longitudinal stress for right side (over hanging portion) of girder (top and bottom)

3.6.4.4 Longitudinal stress (Top)

The ratio of maximum longitudinal stress at the top face of the curved girder to the maximum longitudinal stress at the top face of the straight girder is taken as $\alpha$ and plotted with span to radius of curvature.
The longitudinal stress on top of box girder can also be expressed approximately in terms of L/R ratio using the following Linear relation

$$\alpha = -22.78\frac{L}{R} + 13.10$$

$$\alpha = \text{longitudinal stress, top}$$

$$L/R = \text{span to radius of curvature}$$

3.6.4.5 Maximum longitudinal stress (bottom face of girder)

The ratio of maximum longitudinal stress at the top face of the curved girder to the maximum longitudinal stress at the top face of the straight girder is taken as $\alpha$ and plotted with span to radius of curvature, as shown in Figure 3.19.
The longitudinal stress for bottom of trapezoidal box girder for the curved models show that; there is a sharp decrease in stress from 205R to 220R and it again increase for 306R model.

The maximum longitudinal stress (bottom) can be approximately found by using the relation

$$\alpha = -0.082 \frac{L}{R} + 1.028$$

$\alpha = \text{longitudinal stress; bottom , (curved/straight)}$

$L/R = \text{span to radius of curvature ratio}$

**3.6.5 FREQUENCIES FOR THE BRIDGE MODELS**

A modal analysis is performed for all the five box bridge models out of which one is straight and other four are curved in plan. All the five models have same span length and same material properties.
Table 3.6 Frequencies for the bridge models

<table>
<thead>
<tr>
<th>Output case</th>
<th>Frequency (Cycle/sec) R=205</th>
<th>Frequency (Cycle/sec) R=210</th>
<th>Frequency (Cycle/sec) R=220</th>
<th>Frequency (Cycle/sec) R=306</th>
<th>Frequency (Cycle/sec) R=0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode1</td>
<td>0.493</td>
<td>0.493</td>
<td>0.494</td>
<td>0.493</td>
<td>0.492</td>
</tr>
<tr>
<td>Mode2</td>
<td>0.963</td>
<td>0.964</td>
<td>0.971</td>
<td>1.006</td>
<td>0.998</td>
</tr>
<tr>
<td>Mode3</td>
<td>3.269</td>
<td>3.260</td>
<td>3.260</td>
<td>3.239</td>
<td>3.219</td>
</tr>
<tr>
<td>Mode4</td>
<td>3.771</td>
<td>3.772</td>
<td>3.803</td>
<td>3.829</td>
<td>3.798</td>
</tr>
<tr>
<td>Mode5</td>
<td>4.512</td>
<td>4.512</td>
<td>4.516</td>
<td>4.555</td>
<td>4.549</td>
</tr>
<tr>
<td>Mode6</td>
<td>7.644</td>
<td>7.640</td>
<td>7.646</td>
<td>7.698</td>
<td>7.642</td>
</tr>
<tr>
<td>Mode7</td>
<td>8.072</td>
<td>8.065</td>
<td>8.070</td>
<td>8.110</td>
<td>7.642</td>
</tr>
</tbody>
</table>

3.6.5.1 Frequencies for the bridge models

Modal analysis is performed for all the five models. In result seven step frequencies taken. as shown in Figure 3.20.

![Figure 3.20 Various mode shapes with the different radius of curvature](image)

The Figure 3.20 shows there is no change in mode shapes for the curvatures considered in the present study. As the length and the material property is same for all
the models so, the mass for all the five models are same; therefore there is no change in mode shapes.

3.6.5.2 Fundamental frequency to L/R ratio

The ratio of fundamental frequency of curved to the fundamental frequency for straight is compared with span to radius of curvature for all the five models. As the mass remains same the $\alpha$ (fundamental frequency) remains unchanged with span to radius of curvature. So, there is no effect on fundamental frequency if curvature changes, refer figure 3.21

![Figure 3.21 Fundamental mode frequencies to the L/R ratio](image)

3.7 SUMMARY

This chapter discuss about the parametric study conducted on five box girder bridge models with constant span length and varying curvature. In order to validate the finite element modelling method, an example of box girder is selected from. The example of box girder is modelled and analysed in SAP 2000 and the responses are found to be
fairly matching. For the purpose of the parametric study, the five box girder bridges are modelled in SAP2000. The span length, cross-section and material property remains unchanged. The only parameter that changes is the radius of curvature. The cross section of the superstructure of the box girder bridge consists of single cell box. The curvature of the bridges varies only in horizontal direction. All the models are subjected to self weight and moving load of IRC class A tracked vehicle. A static analysis for dead load and moving load, and a modal analysis are performed. The longitudinal stress at top and bottom of cross sections, bending moment, torsion, deflection and fundamental frequency are recorded. The responses of a box girder bridges curved in plan and straight are compared. The ratio of responses is expressed in terms of a parameter.

Under dead load; it is recorded that, there is a 33% decrease in maximum torsion is observed when the radius of curvature is increased from 205m to 210m, 210m to 220m and 220m to 306m.

Under IRC class A loading; it shows that the as radius of curvature increased from 205m to 210m, the maximum torsion in the cross section decreased by 20.5%. An increase in maximum torsion of 7% is observed for in the increase in radius from 210m to 220m. When the radius of curvature increased from 220m to 306m, the maximum torsion is decreased by 16%.

From this it concluded that, as the curvature increases the torsion also increases.

Under IRC Class A loading, it shows that; when the radius of curvature increased from 205m to 210m, the maximum mid span deflection along the transverse direction increases by 14%. For the radius of curvature increased to 220m, the maximum
deflection increases by almost 12.5%. Also for the radius of curvature is increased from 220m to 306m, the maximum deflection is increases by 11.11%.

Under IRC Class A loading, it shows that; when the radius of curvature increased from 205m to 210m, the maximum mid span deflection along the span is decreased by 7.7%. As the radius of curvature is increased to 220m, the maximum deflection is decreased by almost 16.67%. Also, when the radius of curvature is increased from 220m to 306m, the maximum deflection is found to be decreased by 75%, which behaves more like a straight bridge.

Under dead load, the bending moment decreases by almost 29% as the radius of curvature decreases from 205m to 210m, 210m to 220m, and 220m to 306m.

Under IRC Class A loading, the bending moment decreases by almost 18.75% as the radius of curvature decreases from 205m to 210m, 210m to 220m, and 220m to 306m.

Under IRC class A loading, for bottom face of left overhang part of girder, the longitudinal stress is increased by 12% as the radius of curvature is increased from 205m to 210m and it is increased by 17.5% as radius of curvature is increased from 210m to 220m radius of curvature, 5.2% when the radius of curvature is increased from 220m to 306m. For top face, the longitudinal stress for all the curved models increases about 6% from straight model.

Under IRC class A loading, the longitudinal stress increase of 12%, 25% and 1% are observed between the models 205R to 210R, 210R to 220R and 220R to 306R. for the bottom face of central cross section. For top face the longitudinal stress for all the curved models increases fairly 6% from straight model.
Under IRC class A loading, for bottom face of right overhang part of girder, the longitudinal stress is increased 16% from 205R to 210R and it is decreased by 28% both from 210R to 220R and 210R to 306R. For top face, longitudinal stress increases by 2% from 205R to 210R, and a 33% from 210R to 220R and also from 210R to 306R.

For fundamental frequency; it shows, as the mass remains same the α (fundamental frequency) remains unchanged with span to radius of curvature. So, there is no effect on fundamental frequency if curvature changes.

Therefore, it shows that the parameters, torsion, deflection and bending moment is increasing as the curvature increases.
CHAPTER 4

CONCLUSIONS

4.1 SUMMARY

The present study focus on the parametric study of single cell box girder bridges curved in plan. For the purpose of the parametric study, five box girder bridge models with constant span length and varying curvature. In order to validate the finite element modelling method, an example of box girder bridge is selected from literature to conduct a validation study. The example box girder is modelled and analysed in SAP 2000 and the responses are found to be fairly matching. The five box girder bridges are modelled in SAP2000. The span length, cross-section and material property remains unchanged. The only parameter that changes is the radius of curvature. The cross section of the superstructure of the box girder bridge consists of single cell box. The curvature of the bridges varies only in horizontal direction. All the models are subjected to self weight and moving load of IRC class A tracked vehicle. A static analysis for dead load and moving load, and a modal analysis are performed. The longitudinal stress at top and bottom of cross sections, bending moment, torsion, deflection and fundamental frequency are recorded. These four models are analysed and have been compared with the straight model.

4.2 CONCLUSIONS

The major conclusions are listed below:

- The vertical displacement of simply supported curved box girder bridges at mid-span is related to horizontal radius of curvature. The deflection is taken along the width of the box girder. When the radius is below 210m,
displacement increases more rapidly, but, when the radius is more than 210 m, displacement curve gradually tends to level, the characteristics is the same as straight bridge.

- Under IRC Class A loading, it shows that; when the radius of curvature increased from 205m to 210m, the maximum mid span deflection along the transverse direction increases by 14%. For the radius of curvature increased to 220m, the maximum deflection increases by almost 12.5%. Also for the radius of curvature is increased from 220m to 306m, the maximum deflection is increases by 11.11%.

- The mid span vertical displacement along the width of box girder showed that the deflection on the right side of girder is more than left side of girder (centre of curvature is on right).

- Under IRC Class A loading, it shows that; when the radius of curvature increased from 205m to 210m, the maximum mid span deflection along the span is decreased by 7.7%. As the radius of curvature is increased to 220m, the maximum deflection is decreased by almost 16.67%. Also, when the radius of curvature is increased from 220m to 306m, the maximum deflection is found to be decreased by 75%, which behaves more like a straight bridge.

- As the span to radius of curvature increases the value of $\alpha$ (for all cases) increases. The range of $\alpha$ is in between 1 to 6 except for the torsion. This means that the forces or deflections in the curved bridge can be obtained by multiplying the straight bridge with the corresponding values of $\alpha$.

- Under dead load; it is recorded that, there is a 33% decrease in maximum torsion is observed when the radius of curvature is increased from 205m to 210m, 210m to 220m and 220m to 306m.
- Under IRC class A loading; it shows that as radius of curvature is increased from 205m to 210m, the maximum torsion in the cross section decreased by 20.5%. An increase in maximum torsion of 7% is observed for in the increase in radius from 210m to 220m. When the radius of curvature increased from 220m to 306m, the maximum torsion is decreased by 16%.
- For relation of torsional moment to the L/R ratio, it showed that with decrease in span to radius of curvature, the dimensionless value α for maximum torsion is increasing.
- Under dead load, the bending moment decreases by almost 29% as the radius of curvature decreases from 205m to 210m, 210m to 220m, and 220m to 306m.
- Under IRC Class A loading, the bending moment decreases by almost 18.75% as the radius of curvature decreases from 205m to 210m, 210m to 220m, and 220m to 306m.
- As the curvature increases the bending moment values also increases.
- Under IRC class A loading, for bottom face of left overhang part of girder, the longitudinal stress is increased by 12% as the radius of curvature is increased from 205m to 210m and it is increased by 17.5% as radius of curvature is increased from 210m to 220m radius of curvature, 5.2% when the radius of curvature is increased from 220m to 306m. For top face, the longitudinal stress for all the curved models increases about 6% from straight model.
- Under IRC class A loading, the longitudinal stress increase of 12%, 25% and 1% are observed between the models 205R to 210R, 210R to 220R and 220R to 306R for the bottom face of central cross section. For top face the
longitudinal stress for all the curved models increases fairly 6% from straight model.

- Under IRC class A loading, for bottom face of right overhang part of girder, the longitudinal stress is increased 16% from 205R to 210R and it is decreased by 28% both from 210R to 220R and 210R to 306R. For top face, longitudinal stress increases by 2% from 205R to 210R and by 33% from 210R to 220R and also from 210R to 306R.

- The fundamental mode is same for all the five models of bridges; as the mass and stiffness remains almost the same.
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