Lean Metric Evaluation in Fuzzy Environment

Thesis submitted in partial fulfillment of the requirements for the Degree of

Bachelor of Technology (B. Tech.)

In

Mechanical Engineering

By

MALAY KUMAR MOHANTA

Roll No. 109ME0387

Under the Guidance of

Prof. SAURAV DATTA

NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

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<u>Certificate of Approval</u>

This is to certify that the thesis entitled **Lean Metric Evaluation in Fuzzy Environment** submitted by ***Malay Kumar Mohanta*** has been carried out under my supervision in partial fulfillment of the requirements for the Degree of ***Bachelor of Technology*** in ***Mechanical Engineering*** at **National Institute of Technology**, **NIT Rourkela**, and this work has not been submitted elsewhere before for any other academic degree/diploma.

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Malay Kumar Mohanta

Abstract

This research aims to assess the leanness of an organizational supply chain using fuzzy based Multi-Criteria Decision Making (MCDM) approach. During this research, a leanness measurement module incorporated with fuzzy logic has been designed. After computing organizational overall leanness index, the barriers (ill-performing areas) towards successful lean implementation have been identified. The approach presented in this research is expected to be used fruitfully as a test kit for periodically monitoring and evaluating organizational supply chain leanness and related aspects.

Keywords: Leanness, Multi-Criteria Decision Making, Fuzzy Logic

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1. Introduction and State of Art

Lean manufacturing, **lean production** or **lean enterprise** often simply, **Lean**, is a production practice that considers the expenditure of resources for any goal other than the creation of value for the end consumer to be wasteful, and thus a target for reduction. Working from the perspective of the consumer or service, 'value' is defined as any process or action that a customer would be willing to pay for.

Essentially, lean is focused on *preserving value with less work*. Lean manufacturing is a management philosophy derived mostly from the *Toyota Production System* (TPS) (hence the term Toyotism is also prevalent) and identified as 'Lean' only in the 1990s. TPS is renowned for its focus on reduction of the original Toyota *seven wastes* to improve overall customer value, but there are changing focus area on how this is best achieved. The steady growth of *Toyota*, became the world's largest automaker company from a small company, has focused attention on how it has succeeded.

Lean manufacturing is a variation on the theme of efficiency based on optimizing flow; it is a present-day instance of the recurring theme in human history toward increasing efficiency, reducing waste, and using empirical methods to decide what matters, rather than accepting pre-existing ideas. As such, it is a chapter in the superior narrative that also includes such ideas as the folk wisdom of thrift, motion study and time, Taylorism, the Efficiency Movement, and Fordism. Lean manufacturing is seen as a more refined version of earlier efficiency efforts, building upon the work of earlier leaders such as Taylor or Ford, and learning from their faults.

Lean Manufacturing is an operational approach oriented toward achieving the shortest possible cycle time by eliminating waste. It is derived from the *Toyota Production System* and its key thrust is to increase the value-added work by eliminating waste and reducing incidental work. In today's competitive global marketplace the concept of lean manufacturing has gained vital consciousness to all manufacturing sectors, their supply chains and hence a logical measurement index system is indeed required in implementing leanness in practice. Such leanness estimation can help the enterprises to assess their existing leanness level; can compare different industries who are adapting this lean concept.

The present work is aimed to exhibit an efficient fuzzy-based leanness assessment system using generalized fuzzy numbers set. The concept of properties of fuzzy numbers is to be explored here to identify ill-performing areas towards lean achievement.

The methodology to be described here may prove fruitful while applying for a particular industry, in India, as a case study. Apart from estimating overall lean performance metric, the model can be extended to identify ill-performing areas towards lean achievement.

Zanjirchi et al. (2010) developed a methodology for measuring leanness degree in manufacturing companies using fuzzy logic. The evaluation methods were based on human perceptions; made this kind of measuring unreliable. Considering the shortage, this research developed an approach based on linguistic variables and fuzzy numbers for measuring organizational leanness, and lastly use the method for measuring a manufacturing organization`s leanness. The method developed was usable simply by practitioners and made more precise approximate for leanness and then better improvement path for them.

Singh et al. (2010) discussed the concept of leanness and to provide an efficient measurement method for measuring leanness. Measurement method was based on the judgment and evaluation given by leanness measurement team (LMT) on various leanness parameters such as supplier's issues, investment importance, and various waste addressed by lean and customers' issues.

Further fuzzy set theory was introduced to remove the bias of human judgment and finally defuzzification is done and results were presented in the form of leanness index.

Vinodh et al. (2011) presented a study in which fuzzy association rules mining approach was used for leanness evaluation of an Indian modular switches manufacturing organization. The experiences gained as a result of the conduct of the study indicated that leanness evaluation could be performed by the decision makers without any constraints.

Saurin et al. (2011) introduced a framework for assessing the use of lean production (LP) practices in manufacturing cells (MCs). The development of the framework included four stages: (a) defining LP practices applicable to MC, based on criteria such as the inclusion of practices that workers could observe, interact with and use on a daily basis; (b) defining attributes for each practice, emphasising the dimensions which were typical of their implementation in LP environments; (c) defining a set of evidence and sources of evidence for assessing the existence of each attribute – the sources of evidence included direct observations, analysis of documents, interviews and a feedback meeting to validate the assessment results with company representatives; (d) drawing up a model of the relationships among the LP practices, based on a survey with LP experts. This model supported the identification of improvement opportunities in MC performance based on the analysis of their interfaces. A case study of an MC from an automobile parts supplier was also presented to illustrate the application of the framework.

Vinodh & Balaji (2011) reported a study which was carried out to assess the leanness level of a manufacturing organization. During this research study, a leanness measurement model was designed. Then the leanness index was computed. Since the manual computation was time consuming and error-prone, a computerized decision support system has been developed. This decision support system was designated as FLBLA-DSS (decision support system for fuzzy logic based leanness assessment). FLBLA-DSS computed the fuzzy leanness index, Euclidean distance and identified the weaker areas which need improvement. The developed DSS was test implemented in an Indian modular switches manufacturing organization.

Vinodh et al. (2011) reported a research carried out to assess the leanness of an organization using multi-grade fuzzy approach. During this research, a leanness measurement model incorporated with multi-grade fuzzy approach was designed. This is followed by the substitution of the data gathered from a manufacturing organization. After the computation of leanness index, the areas for leanness improvement were identified. The approach contributed in this project could be used as a test kit for periodically evaluating an organization's leanness.

Eswaramoorthi et al. (2011) conducted a survey to identify the status of lean practices in the machine tool manufacturing, which was one of the constituents of automobile value chain. A questionnaire tool applicable to machine tool environment was designed and validated. The data recorded through the survey across the core machine tool manufacturers were analyzed, and the results were presented. The results showed that the status of lean implementation in the machine tool sector is still in infant stage. The reasons for low priority towards lean practices among the industries were identified, and suitable measures were suggested to address the problems. This would further assist the machine tool industries to gauge their level of leanness and would serve as a foundation for future research.

Behrouzi and Wong (2011) identified the underlying lean supply chain performance components and related measures with a special focus on small and medium enterprises in Iran's automotive industry. An initial list of supply chain performance measures was compiled and then modified by six experts in order to extract the appropriate lean supply chain performance measures. Following this, a questionnaire was designed and sent to 580 supply chain practitioners working at small and medium enterprises in Iran's automotive industry in order to score the measures. Principal component analysis was applied to identify and group the lean supply chain performance components and connected measures. From the initial list, a total of 28 performance measures were chosen by the experts and considered essential to monitor the leanness of a supply chain. Four underlying constituents were identified (quality, cost, flexibility, and delivery and reliability) and the 28 related measures were subsequently grouped under these performance components. By identifying and validating a multi-dimensional list of lean supply chain performance components and related measures for small and medium enterprises, the research can be useful for practitioners or academics that are going to evaluate the leanness of a supply chain.

Vinodh & Vimal (2012) presented the 30 criteria based leanness assessment methodology using fuzzy logic. Fuzzy logic was used to overcome the disadvantages with scoring method such as impreciseness and vagueness. During this research, a conceptual model for leanness assessment was designed. Then the fuzzy leanness index which indicated the leanness level of the organization and fuzzy performance importance index which helped in identifying the obstacles for leanness was computed. The results indicated that the model was capable of effectively assessing leanness and had practical relevance.

Literature depicts that considerable volume of work has been carried out towards leanness assessment in manufacturing, organizational supply chain. Pioneer researchers proposed exploration of fuzzy theory to tackle subjective decision-making information. Lean barriers (obstacles to achieve leanness) have been identified too. The theory of ranking fuzzy numbers using 'maximizing and minimizing set' has been adapted to identify lean barriers. However, this approach invites mathematical complexity and tedious computation. In this context the theory of ranking and comparing fuzzy numbers reported by [Thorani et al., 2012] has been proposed for this purpose. The study presents a fuzzy based leanness appraisement module followed by identification of lean barriers by exploring theories of generalized fuzzy numbers, the concept of crisp equivalent, fuzzy operational rules to facilitate managerial decision-making.

2. Fuzzy Preliminaries

To deal with vagueness in human thought, Zadeh (1965) first introduced the fuzzy set theory, which has the capability to represent/manipulate data and information possessing based on nonstatistical uncertainties. Moreover fuzzy logic has been designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision inherent to decision making problems. Some basic definitions of fuzzy number, fuzzy set and linguistic variables are reviewed from Zadeh (1975), Buckley (1985), Negi (1989), Kaufmann and Gupta (1991).The basic definitions and notations below will be used throughout this paper until otherwise stated.

2.1 Definitions of fuzzy sets:

Definition 1. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval $[0,1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ **(Kaufmann and Gupta, 1991)**.

Definition 2. A fuzzy set $\tilde{A}$ in a universe of discourse X is convex if and only if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right) \quad (1)$$

For all x_1, x_2 in X and all $\lambda \in [0,1]$, where min denotes the minimum operator (Klir and Yuan, 1995).

Definition 3. The height of a fuzzy set is the largest membership grade attained by any element in that set. A fuzzy set $\tilde{A}$ in the universe of discourse X is called normalized when the height of $\tilde{A}$ is equal to 1 (Klir and Yuan, 1995).

2.2 Definitions of fuzzy numbers:

Definition 1. A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal. Fig. 1 shows a fuzzy number $\tilde{n}$ in the universe of discourse X that conforms to this definition (Kaufmann and Gupta, 1991).

Definition 2. The α -cut of fuzzy number $\tilde{n}$ is defined as:

$$\tilde{n}^{\alpha} = \{x_i : \mu_{\tilde{n}}(x_i) \geq \alpha, x_i \in X\}, \qquad (2)$$

Here, $\alpha \in [0,1]$

The symbol $\tilde{n}^{\alpha}$ represents a non-empty bounded interval contained in X , which can be denoted by $\tilde{n}^{\alpha} = [n_l^{\alpha}, n_u^{\alpha}]$, n_l^{α} and n_u^{α} are the lower and upper bounds of the closed interval, respectively (Kaufmann and Gupta, 1991; Zimmermann, 1991). For a fuzzy number $\tilde{n}$, if $n_l^{\alpha} > 0$ and $n_u^{\alpha} \leq 1$ for all $\alpha \in [0,1]$, then $\tilde{n}$ is called a standardized (normalized) positive fuzzy number (Negi, 1989).

Definition 3. Suppose, a positive triangular fuzzy number (PTFN) is $\tilde{A}$ and that can be defined as (a,b,c) shown in Fig. 2. The membership function $\mu_{\tilde{n}}(x)$ is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & if\ a \leq x \leq b, \\ (c-x)/(c-b), & if\ b \leq x \leq c, \\ 0, & otherwise, \end{cases} \qquad (3)$$

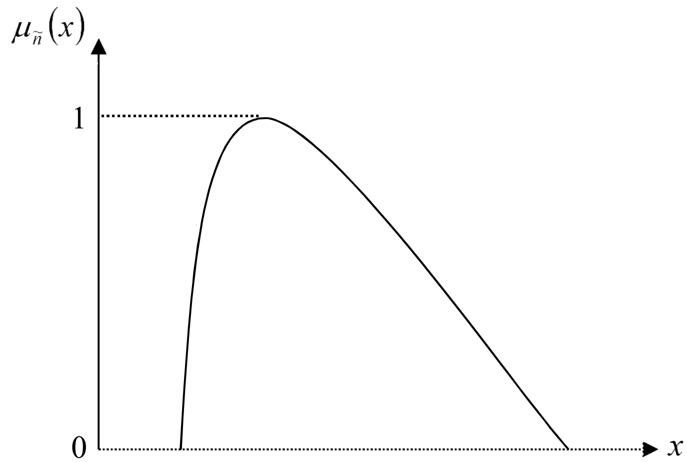

Fig. 1. A fuzzy number $\tilde{n}$

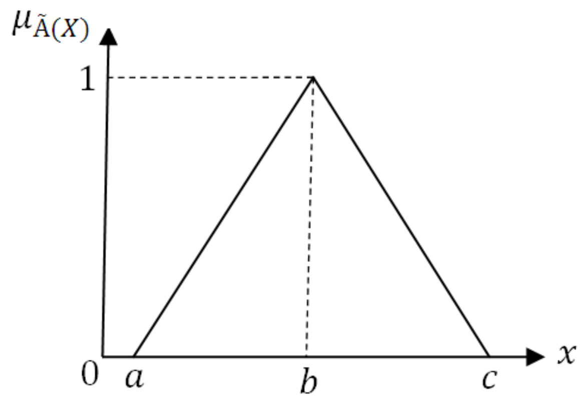

Fig. 2. A triangular fuzzy number $\tilde{A}$

Based on extension principle, the fuzzy sum $\oplus$ and fuzzy subtraction Θ of any two triangular fuzzy numbers are also triangular fuzzy numbers; but the multiplication $\otimes$ of any two triangular fuzzy numbers is only approximate triangular fuzzy number (Zadeh, 1975). Let's have a two positive triangular fuzzy numbers, such as $\tilde{A}_1 = (a_{1,}\ b_1, c_1)$, and $\tilde{A}_2 = (a_2, b_2, c_2)$, and a positive real number $r = (r, r, r)$, some algebraic operations can be expressed as follows:

$$\widetilde{A}_1 \oplus \widetilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \quad (4)$$

$$\widetilde{A}_1 \Theta \widetilde{A}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2), (5)\ \widetilde{A}_1 \otimes \widetilde{A}_2 = (a_1 a_2, b_1 b_2, c_1 c_2), \quad (6)$$

$$r \otimes \widetilde{A}_1 = (ra_1, rb_1, rc_1), \quad (7)$$

$$\widetilde{A}_1 \varnothing \widetilde{A}_2 = (a_1/c_2, b_1/b_2, c_1/a_2), \quad (8)$$

The operations of $\vee$ (max) and $\wedge$ (min) are defined as:

$$\widetilde{A}_1(\vee)\widetilde{A}_2 = (a_1 \vee a_2, b_1 \vee b_2, c_1 \vee c_2), \quad (9)$$

$$\widetilde{A}_1(\wedge)\widetilde{A}_2 = (a_1 \wedge a_2, b_1 \wedge b_2, c_1 \wedge c_2), \quad (10)$$

Here, $r > 0$, and $a_1, b_1, c_1 > 0$,

Also the crisp value of triangular fuzzy number set $\widetilde{A}_1$ can be determined by defuzzification which locates the Best Non-fuzzy Performance (BNP) value. Thus, the BNP values of fuzzy number are calculated by using the center of area (COA) method as follows: (Moeinzadeh and Hajfathaliha, 2010)

$$\text{BNP}_i = \frac{[(c-a)+(b-a)]}{3} + a, \qquad \forall_i, \quad (11)$$

Definition 4. A matrix $\widetilde{\mathbf{D}}$ is called a fuzzy matrix if at least one element is a fuzzy number (Buckley, 1985).

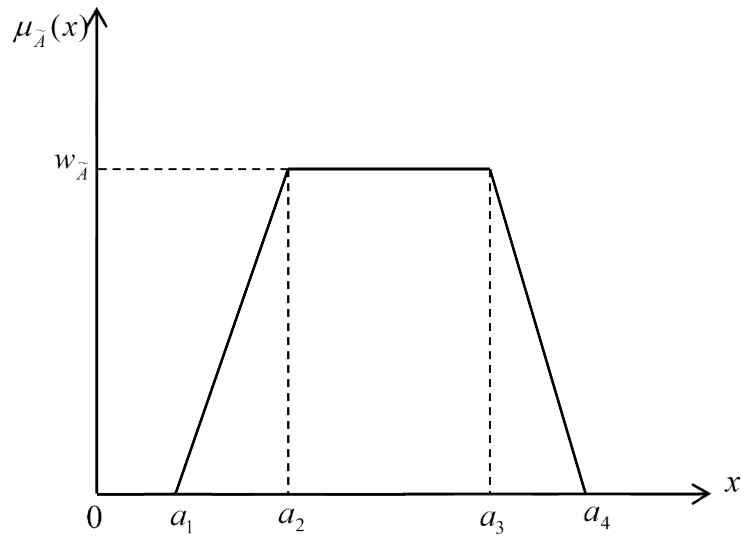

Fig. 3 Trapezoidal fuzzy number $\widetilde{A}$

2.3 Linguistic variable:

Definition 1. A linguistic variable is the variable whose values are not expressed in numbers but words or sentences in a natural or artificial language (Zadeh, 1975). The concept of a linguistic variable is very useful in dealing with situations, which are complex or not well-defined to be reasonably described in conventional quantitative expressions (Zimmermann, 1991). For example, 'weight' is a linguistic variable whose values are 'very low', 'low', 'medium', 'high', 'very high', etc. Fuzzy set can also represent these linguistic values.

2.4 The concept of generalized trapezoidal fuzzy numbers

By the definition given by (Chen, 1985), a generalized trapezoidal fuzzy number can be defined as $\widetilde{A} = (a_1, a_2, a_3, a_4; w_{\widetilde{A}})$, as shown in Fig. 3.

and the membership function $\mu_{\widetilde{A}}(x): R \to [0,1]$ is defined as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}\times w_{\tilde{A}}, & x\in(a_1,a_2)\\ w_{\tilde{A}}, & x\in(a_2,a_3)\\ \dfrac{x-a_4}{a_3-a_4}\times w_{\tilde{A}}, & x\in(a_3,a_4)\\ 0, & x\in(-\infty,a_1)\cup(a_4,\infty)\end{cases} \quad (12)$$

Here, $a_1 \le a_2 \le a_3 \le a_4$ and $w_{\tilde{A}} \in [0,1]$

The elements of the generalized trapezoidal fuzzy numbers $x \in R$ are real numbers, and its membership function $\mu_{\tilde{A}}(x)$ is the regularly and continuous convex function, it shows that the membership degree to the fuzzy sets. If $-1 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$, then $\tilde{A}$ is called the normalized trapezoidal fuzzy number. Especially, if $w_{\tilde{A}} = 1$, then $\tilde{A}$ is called trapezoidal fuzzy number (a_1, a_2, a_3, a_4); if $a_1 < a_2 = a_3 < a_4$, then $\tilde{A}$ is reduced to a triangular fuzzy number. If $a_1 = a_2 = a_3 = a_4$, then $\tilde{A}$ is reduced to a real number.

Suppose that $\tilde{a} = (a_1, a_2, a_3, a_4; w_{\tilde{a}})$ and $\tilde{b} = (b_1, b_2, b_3, b_4; w_{\tilde{b}})$ are two generalized trapezoidal fuzzy numbers, then the operational rules of the generalized trapezoidal fuzzy numbers $\tilde{a}$ and $\tilde{b}$ are shown as follows (Chen and Chen, 2009):

$$\tilde{a} \oplus \tilde{b} = (a_1, a_2, a_3, a_4; w_{\tilde{a}}) \oplus (b_1, b_2, b_3, b_4; w_{\tilde{b}}) =$$

$$(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(w_{\tilde{a}}, w_{\tilde{b}})) \quad (13)$$

$$\tilde{a} - \tilde{b} = (a_1, a_2, a_3, a_4; w_{\tilde{a}}) - (b_1, b_2, b_3, b_4; w_{\tilde{b}}) =$$

$$(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1; \min(w_{\tilde{a}}, w_{\tilde{b}})) \quad (14)$$

$$\tilde{a} \otimes \tilde{b} = (a_1, a_2, a_3, a_4; w_{\tilde{a}}) \otimes (b_1, b_2, b_3, b_4; w_{\tilde{b}}) =$$

$$(a, b, c, d; \min(w_{\tilde{a}}, w_{\tilde{b}})) \quad (15)$$

Here,

$$a = \min\left(a_1 \times b_1, a_1 \times b_4, a_4 \times b_1, a_4 \times b_4\right)$$

$$b = \min\left(a_2 \times b_2, a_2 \times b_3, a_3 \times b_2, a_3 \times b_3\right)$$

$$c = \max\left(a_2 \times b_2, a_2 \times b_3, a_3 \times b_2, a_3 \times b_3\right)$$

$$d = \max\left(a_1 \times b_1, a_1 \times b_4, a_4 \times b_1, a_4 \times b_4\right)$$

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ are real numbers, then

$$\tilde{a} \otimes \tilde{b} = \left(a1 \times b1, a2 \times b2, a3 \times b3, a4 \times b4; \min\left(w_{\tilde{a}}, w_{\tilde{b}}\right)\right)$$

$$\tilde{a} / \tilde{b} = \left(a_1, a_2, a_3, a_4; w_{\tilde{a}}\right) \Big/ \left(b_1, b_2, b_3, b_4; w_{\tilde{b}}\right)$$

$$= \left(a_1 / b_4, a_2 / b_3, a_3 / b_2, a_4 / b_1; \min\left(w_{\tilde{a}}, w_{\tilde{b}}\right)\right) \tag{16}$$

Chen and Chen (2003) proposed the concept of COG point of generalized trapezoidal fuzzy numbers, and suppose that the COG point of fuzzy number

$\tilde{a} = \left(a_1, a_2, a_3, a_4; w_{\tilde{a}}\right)$ is $\left(x_{\tilde{a}}, y_{\tilde{a}}\right)$, then:

$$y_{\tilde{a}} = \begin{cases} \dfrac{w_{\tilde{a}} \times \left(\dfrac{a_3 - a_2}{a_4 - a_1} + 2\right)}{6}, & \text{if } a_1 \neq a_4 \\ \dfrac{w_{\tilde{a}}}{2}, & \text{if } a_1 = a_4 \end{cases} \tag{17}$$

$$x_{\tilde{a}} = \frac{y_{\tilde{a}} \times \left(a_2 + a_3\right) + \left(a_1 + a_4\right) \times \left(w_{\tilde{a}} - y_{\tilde{a}}\right)}{2 \times w_{\tilde{a}}} \tag{18}$$

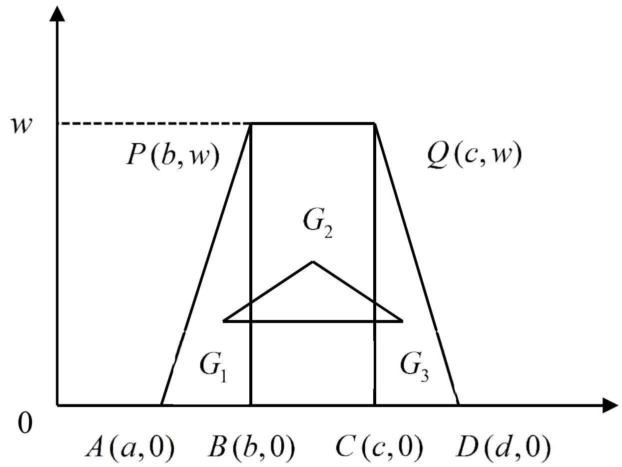

Fig. 4. Trapezoidal Fuzzy Number [Thorani et al. (2012)]

2.5 Ranking of Generalized Trapezoidal Fuzzy Numbers **[**Thorani et al. (2012)]

The centroid of a trapezoid is considered as the balancing point of the trapezoid (Fig. 4). Divide the trapezoid into three plane figures. These three plane figures are a triangle (APB), a rectangle (BPQC), and a triangle (CQD), respectively. Let the centroids of the three plane figures be G_1, G_2, and G_3 respectively. The In center of these Centroids G_1, G_2 and G_3 is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point are balancing points of each individual plane figure, and the In center of these Centroid points is a much more balancing point for a generalized trapezoidal fuzzy number. Hence, this point would be a better reference point than the Centroid point of the trapezoid.

Consider a generalized trapezoidal fuzzy number $\tilde{A} = (a,b,c,d;w)$, (Fig. 4). The Centroids of the three plane figures are $G_1 = \left(\frac{a+2b}{3}, \frac{w}{3}\right)$, $G_2 = \left(\frac{b+c}{2}, \frac{w}{2}\right)$ and $G_3 = \left(\frac{2c+d}{3}, \frac{w}{3}\right)$ respectively.

Equation of the line $\overline{G_1G_3}$ is $y=\frac{w}{3}$ and G_2 does not lie on the line $\overline{G_1G_3}$. Therefore, G_1G_2 and G_3 are non-collinear and they form a triangle.

We define the Incentre $I_{\tilde{A}}(\bar{x}_0,\bar{y}_0)$ of the triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number $\tilde{A}=(a,b,c,d;w)$ as

$$I_{\tilde{A}}(\bar{x}_0,\bar{y}_0)=\left(\frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma},\frac{\alpha\left(\frac{w}{3}\right)+\beta\left(\frac{w}{2}\right)+\gamma\left(\frac{w}{3}\right)}{\alpha+\beta+\gamma}\right) \tag{19}$$

Here

$$\alpha=\frac{\sqrt{(c-3b+2d)^2+w^2}}{6}$$

$$\beta=\frac{\sqrt{(2c+d-a-2b)^2}}{3}$$

$$\gamma=\frac{\sqrt{(3c-2a-b)^2+w^2}}{6}$$

As a special case, for triangular fuzzy number $\tilde{A}=(a,b,c,d;w)$, i.e. $c=b$ the incentre of Centroids is given by

$$I_{\tilde{A}}(\bar{x}_0,\bar{y}_0)=\left(\frac{x\left(\frac{a+2b}{3}\right)+yb+z\left(\frac{2b+d}{3}\right)}{x+y+z},\frac{x\left(\frac{w}{3}\right)+y\left(\frac{w}{2}\right)+z\left(\frac{w}{3}\right)}{x+y+z}\right) \tag{20}$$

Here

$$x = \frac{\sqrt{(2d-2b)^2 + w^2}}{6}$$

$$y = \frac{\sqrt{(d-a)^2}}{3}$$

$$z = \frac{\sqrt{(2b-2a)^2 + w^2}}{6}$$

The ranking function of the generalized trapezoidal fuzzy number $\tilde{A} = (a,b,c,d;w)$, which maps the set of all fuzzy numbers to a set of real numbers is defined as,

$$R(\tilde{A}) = x_0 \times y_0 = \left(\frac{x\left(\frac{a+2b}{3}\right) + yb + z\left(\frac{2b+d}{3}\right)}{x+y+z} \times \frac{x\left(\frac{w}{3}\right) + y\left(\frac{w}{2}\right) + z\left(\frac{w}{3}\right)}{x+y+z} \right) \tag{21}$$

This is the Area between the in center of the centroids $I_{\tilde{A}}(\bar{x}_0, \bar{y}_0)$ as defined in Eq. (19) and the original point.

The Mode (m) of the generalized trapezoidal fuzzy number $\tilde{A} = (a,b,c,d;w)$, is defined as:

$$m = \frac{1}{2}\int_0^w (b+c)dx = \frac{w}{2}(b+c) \tag{22}$$

The Spread(s) of the universal trapezoidal fuzzy number $\tilde{A} = (a,b,c,d;w)$, is defined as:

$$s = \int_0^w (d-a)dx = w(d-a) \tag{23}$$

The left spread (ls) of the generalized trapezoidal fuzzy number $\tilde{A} = (a,b,c,d;w)$, is defined as:

$$ls = \int_0^w (b-a)dx = w(b-a) \tag{24}$$

The right spread (rs) of the generalized trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; w)$, is defined as:

$$rs = \int_0^w (d - c)dx = w(d - c) \tag{25}$$

Using the above definitions we now define the ranking procedure of two generalized trapezoidal fuzzy numbers.

Let $\widetilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\widetilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers.

The working procedure to compare $\widetilde{A}$ and $\widetilde{B}$ is as follows:

Step 1: Find $R(\widetilde{A})$ and $R(\widetilde{B})$

Case (i) If $R(\widetilde{A}) > R(\widetilde{B})$ then $\widetilde{A} > \widetilde{B}$

Case (ii)If $R(\widetilde{A}) < R(\widetilde{B})$ then $\widetilde{A} < \widetilde{B}$

Case (iii) If $R(\widetilde{A}) = R(\widetilde{B})$ comparison is not possible, then go to ***step 2***.

Step 2: Find $m(\widetilde{A})$ and $m(\widetilde{B})$

Case (i) If $m(\widetilde{A}) > m(\widetilde{B})$ then $\widetilde{A} > \widetilde{B}$

Case (ii)If $m(\widetilde{A}) < m(\widetilde{B})$ then $\widetilde{A} < \widetilde{B}$

Case (iii) If $m(\widetilde{A}) = m(\widetilde{B})$ comparison is not possible, then go to ***step 3***.

Step 3: Find $s(\widetilde{A})$ and $s(\widetilde{B})$

Case (i) If $s(\widetilde{A}) > s(\widetilde{B})$ then $\widetilde{A} < \widetilde{B}$

Case (ii)If $s(\widetilde{A}) < s(\widetilde{B})$ then $\widetilde{A} > \widetilde{B}$

Case (iii) If $s(\widetilde{A}) = s(\widetilde{B})$ comparison is not possible, then go to ***step 4***.

Step 4: Find $ls\left(\widetilde{A}\right)$ and $ls\left(\widetilde{B}\right)$

Case (i) If $ls\left(\widetilde{A}\right) > ls\left(\widetilde{B}\right)$ then $\widetilde{A} > \widetilde{B}$

Case (ii)If $ls\left(\widetilde{A}\right) < ls\left(\widetilde{B}\right)$ then $\widetilde{A} < \widetilde{B}$

Case (iii) If $ls\left(\widetilde{A}\right) = ls\left(\widetilde{B}\right)$ comparison is not possible, then go to ***step 5***.

Step 5: Examine w_1 and w_2

Case (i) If $w_1 > w_2$ then $\widetilde{A} > \widetilde{B}$

Case (ii) If $w_1 < w_2$ then $\widetilde{A} < \widetilde{B}$

Case (iii) If $w_1 = w_2$ then $\widetilde{A} \approx \widetilde{B}$

3. Proposed Appraisement Module

A fuzzy based performance appraisement module in lean organization proposed in this paper has been present below. General hierarchy criteria (GHC) for evaluating overall organizational leanness extent, adapted in this paper has been shown in Table 1 [Zanjirchi et al., 2010]. It consists of two-level index system; which aims at achieving the target to evaluate overall appraisement index. 1st level lists out a number of lean capabilities/ enablers; 2nd level comprises of various lean attributes. Procedural steps for leanness evaluation have been presented as follows:

1. Selection of linguistic variables towards assigning priority weights (of individual lean capabilities as well as attributes) and appropriateness rating (performance extent) corresponding to each 2nd level lean attributes.

2. Collection of expert opinion from a selected decision-making group (subjective judgment) in order to express the priority weight as well as appropriate rating against each of the evaluation indices.

3. Representing decision-makers' linguistic judgments using appropriate fuzzy numbers set.

4. Use of fuzzy operational rules towards estimating aggregated weight as well as aggregated rating (pulled opinion of the decision-makers) for each of the selection criterion.

5. Calculation of computed performance rating of 1st level lean capabilities and finally overall lean performance index called Fuzzy Performance Index (FPI).

Appropriateness rating for each of the 1st level capability U_i (rating of i_{th} lean capability) has been computed as follows:

$$U_i = \frac{\sum U_{ij} \otimes w_{ij}}{\sum w_{ij}} \tag{26}$$

In this expression (Eq. 26) U_{ij} is denoted as the aggregated fuzzy appropriateness rating against j_{th} lean attribute (at 2nd level) which is under i_{th} main criterion in the 1st level. w_{ij} is the aggregated fuzzy weight against j_{th} lean attribute (at 2nd level) which is under i_{th} main criterion in 1st level. The *Fuzzy Performance Index (FPI)* has been computed as:

$$U(FPI) = \frac{\sum U_i \otimes w_i}{\sum w_i} \tag{27}$$

In this expression (Eq. 27) U_i is denoted as the computed fuzzy appropriateness rating (obtained using Eq. 26) against i_{th} lean capability at 1st level. w_i is the aggregated fuzzy priority weight against i_{th} lean capability in 1st level.

6. Investigation for identifying ill-performing areas those seek for future improvement.

4. Numerical Illustrations

The proposed appraisement module has been implemented in a famous service sector at eastern part of India. The module encompasses of various lean capabilities as well as lean attributes. An evaluation team has been deployed to assign priority weights (importance extent) against different lean capabilities/ attributes considered in the proposed appraisement model. A questionnaire has been formed and circulated among the decision-makers (experts) to provide the required detail. Collected data has been explored to investigate application feasibility of the proposed appraisement platform. After critical investigation and scrutiny each decision-maker has been instructed to explore the linguistic scale (Table 2) towards assignment of priority weight and appropriateness rating against each evaluation indices. Appropriateness rating for 2nd level lean attributes has been furnished in Table 3. Tables 4-5 provide subjective judgment of the evaluation team members expressed through linguistic terms in relation to weight assignment against various lean capabilities as well as attributes, respectively. These linguistic expressions (human judgment) have been converted into appropriate generalized trapezoidal fuzzy numbers as presented in Table 2. The method of *simple average* has been used to obtain aggregated priority weights and aggregated ratings of 2nd level lean attributes (Tables 6). Computed fuzzy performance ratings (obtained by using Eqs. 27) and aggregated fuzzy priority weight for 1st level lean capabilities and tabulated in Table 7. Finally, Eq. 28 has been used to obtain overall FPI which becomes: (0.25, 0.65, 1.56).

The concept of '*Ranking of fuzzy numbers*' **[**Thorani et al. (2012)] has been adapted here to indentify ill-performing areas of lean implementation. 2nd level lean attributes have been ranked based on their individual *Fuzzy Performance Impotance Index* (FPII) [Lin et al., 2006]. It has been computed as follows:

$$FPII_j = [1 - w_{ij}] \otimes U_{ij} \tag{28}$$

Here $FPII_j$ is denoted as the *Fuzzy Performance Importance Index* of j_{th} lean attribute; whose aggregated performance rating is U_{ij} and aggregated priority weight w_{ij}. The equivalent crisp measure corresponding to $R(FPII_{Individual})$ has been computed; thus, lean criterions have been ranked accordingly (Table 8).

5. Conclusions

Lean paradigm has become an important avenue in recent days. Many organizations around the world have been attempting to implement lean concepts. The leanness metric is an important indicator in lean performance measure. Aforesaid study aimed to develop a quantitative analysis framework and a simulation methodology to evaluate the efficacy of lean practices by exploring the concept of fuzzy numbers. The procedural hierarchy presented here could help the industries to assess their existing lean performance extent, to compare and to identify week-performing areas towards lean implementation successfully.

The contribution of this research has been furnished below.

1. Development of fuzzy-based integrated leanness appraisement module. Industries/ enterprises can utilize this appraisement module as a test kit to assess and improve leanness degree.
2. Estimation of overall lean index; identification of lean barriers.
3. Based on estimated overall lean index; different lean industries can be ranked accordingly.

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Table 1: Organizational Leanness Appraisement Module [Zanjirchi et al., 2010]

Goal	Focus area	1st level indices	2nd level indices
Lean Production	Supply Management/ Supplier Related	Supplier Feedback, C_1	We frequently are in close contact with our supplier, C_{11}
			Our suppliers seldom visit our plants, C_{12}
			We seldom visit our supplier's plants, C_{13}
			We give our suppliers feedback on quality and delivery performance, C_{14}
			We strive to establish long-term relationship with our suppliers, C_{15}
		JIT Delivery by Suppliers, C_2	Suppliers are directly involved in the new product development process, C_{21}
			Our key suppliers deliver to plant on JIT basis, C_{22}
			We have a formal supplier certification program, C_{23}
		Supplier Development, C_3	Our suppliers are contractually committed to annual cost reductions, C_{31}
			Our key suppliers are located in close proximity to our plants, C_{32}
			We have corporate level communication on important issues with key suppliers, C_{33}
			We take active steps to reduce the number of suppliers in each category, C_{34}
			Our key suppliers manage our inventory, C_{35}
			We evaluate suppliers on the basis of total cost and not per unit price, C_{36}
	Customer Relation/involvement Customer Related	Customer Relation/involvement, C_4	We frequently are in close contact with our customers, C_{41}
			Our customers seldom visit our plants, C_{42}
			Our customers give us feedback on quality and delivery performance, C_{43}
			Our customers are actively involved in current and future product offerings, C_{44}
			Our customers are directly involved in current and future product offerings, C_{45}
			Our customers frequently share current and future demand information with

			marketing department, C_{46}
			We regularly conduct customer satisfaction surveys, C_{47}
	Internally Related	Pull, C_5	Production is 'pulled' by the shipment of finished goods, C_{51}
			Production at stations is 'pulled' by the current demand of the next station, C_{52}
			We use a 'pull' production system, C_{53}
			We use kanban, squares, or containers of signals for production control, C_{54}
		Continuous Flow, C_6	Products are classified into groups with similar processing requirements, C_{61}
			Products are classified into groups with similar routing requirements, C_{62}
			Equipment is grouped to produce a continuous flow of families of products, C_{63}
			Families of products determine our factory layout, C_{64}
			Pace of production is directly linked with the rate of customer demand, C_{65}
		Setup time reduction, C_7	Our employees practice setups to reduce the time required, C_{71}
			We are working to lower set up times in our plant, C_{72}
			We have low set up times of equipment in our plant, C_{73}
			Long production cycle times prevent responding quickly to customer requests, C_{74}
			Long supply lead times prevent responding quickly to customer requests, C_{75}
		Statistical Process Control, C_8	Large number of equipment/processes on shop floor are currently under SPC, C_{81}
			Extensive use of statistical techniques to reduce process variance, C_{82}
			Charts showing defect rates are used as tools on the shop-floor, C_{83}
			We use fishbone diagram to identify causes of quality problems, C_{84}
			We conduct process capability studies before product launch, C_{85}
		Employee	Shop-floor employees are key to problem solving teams, C_{91}

		Involvement, C_9	Shop-floor employees drive suggestion program, C_{92}
			Shop-floor employees lead product/process improvement efforts, C_{93}
			Shop-floor employees undergo cross functional training, C_{94}
		Total Productive/Preventive Maintenance Employee, C_{10}	We dedicate a portion of everyday to planned equipment maintenance, $C_{10,1}$
			We maintain all our equipment regularly, $C_{10,2}$
			We maintain excellent records of all equipment maintenance, $C_{10,3}$
			We post equipment maintenance records on shop-floor for active sharing with employees, $C_{10,4}$

Table 2: Seven-member linguistic terms and their corresponding fuzzy numbers

Linguistic terms for weight assignment	fuzzy numbers	Linguistic terms for ratings	fuzzy numbers
Very low, VL	(0, 0.05, 0.15)	Worst, W	(0, 0.05, 0.15)
Low, L	(0.1, 0.2, 0.3)	Very Poor, VP	(0.1, 0.2, 0.3)
Fairly low, FL	(0.2, 0.35, 0.5)	Poor, P	(0.2, 0.35, 0.5)
Medium, M	(0.3, 0.5, 0.7)	Fair, F	(0.3, 0.5, 0.7)
Fairly High, FH	(0.5, 0.65, 0.8)	Good, G	(0.5, 0.65, 0.8)
High, H	(0.7, 0.8, 0.9)	Very Good, VG	(0.7, 0.8, 0.9)
Very High, VH	(0.85, 0.95, 1.0)	Excellent, E	(0.85, 0.95, 1.0)

Table 3: Appropriateness rating (in linguistic scale) of 2nd level indices assigned by DMs

2nd level indices	Appropriateness rating (in linguistic scale) of 2nd level indices assigned by DMs				
	DM1	DM2	DM3	DM4	DM5
C_{11}	E	VG	G	VG	VG
C_{12}	VG	VG	VG	G	G
C_{13}	E	E	VG	E	E
C_{14}	VG	E	E	E	E
C_{15}	F	G	F	F	G
C_{21}	P	F	F	F	F
C_{22}	P	F	P	F	P
C_{23}	F	F	F	F	F
C_{31}	E	G	G	VG	VG
C_{32}	VG	G	VG	G	G
C_{33}	E	E	G	E	E
C_{34}	VG	E	E	E	E
C_{35}	F	G	F	F	G
C_{36}	P	F	F	F	F
C_{41}	P	F	F	F	P
C_{42}	F	F	F	F	F
C_{43}	E	VG	G	VG	VG
C_{44}	VG	VG	G	G	G
C_{45}	E	E	VG	E	E

C_{46}	VG	E	E	E	E
C_{47}	F	G	F	F	G
C_{51}	P	F	P	F	F
C_{52}	P	F	P	F	P
C_{53}	F	F	F	F	F
C_{54}	E	VG	G	VG	VG
C_{61}	VG	VG	G	G	G
C_{62}	E	E	G	E	E
C_{63}	VG	E	E	E	E
C_{64}	F	G	F	F	G
C_{65}	P	P	F	F	F
C_{71}	P	F	P	F	P
C_{72}	F	F	F	F	F
C_{73}	E	VG	G	VG	VG
C_{74}	VG	VG	G	G	G
C_{75}	E	E	G	G	E
C_{81}	VG	E	E	E	E
C_{82}	G	G	F	F	G
C_{83}	P	F	F	F	F
C_{84}	F	F	P	F	P
C_{85}	F	F	F	F	F
C_{91}	E	VG	G	VG	VG
C_{92}	VG	VG	VG	VG	G

C_{93}	E	E	VG	VG	E
C_{94}	VG	E	E	E	E
$C_{10,1}$	F	G	F	F	G
$C_{10,2}$	P	F	F	F	F
$C_{10,3}$	P	F	P	F	P
$C_{10,4}$	G	F	G	F	G

Table 4: Priority Weight (in linguistic scale) of 2nd level indices assigned by DMs

2nd level indices	Priority Weight (in linguistic scale) of 2nd level indices assigned by DMs				
	DM1	DM2	DM3	DM4	DM5
C_{11}	VH	H	H	H	H
C_{12}	VH	VH	H	H	VH
C_{13}	M	FH	H	M	FH
C_{14}	H	FH	FH	FH	H
C_{15}	M	M	FH	M	M
C_{21}	VH	H	FH	VH	VH
C_{22}	VH	VH	H	H	H
C_{23}	VH	H	H	H	VH
C_{31}	M	FH	H	M	FH
C_{32}	H	FH	FH	FH	H
C_{33}	M	M	FH	M	M

C_{34}	VH	H	FH	VH	VH
C_{35}	VH	H	VH	H	H
C_{36}	VH	VH	H	H	VH
C_{41}	M	FH	H	M	FH
C_{42}	H	FH	FH	FH	H
C_{43}	M	M	FH	M	M
C_{44}	VH	H	FH	VH	VH
C_{45}	VH	H	H	H	H
C_{46}	VH	H	H	H	VH
C_{47}	M	H	H	M	FH
C_{51}	H	FH	FH	FH	H
C_{52}	M	M	FH	M	M
C_{53}	VH	H	H	VH	VH
C_{54}	VH	H	H	H	H
C_{61}	VH	VH	H	H	VH
C_{62}	M	FH	H	M	FH
C_{63}	H	FH	H	FH	H
C_{64}	M	M	FH	M	M
C_{65}	VH	H	FH	VH	VH
C_{71}	H	H	H	H	H
C_{72}	VH	VH	H	H	VH
C_{73}	M	FH	H	M	FH
C_{74}	H	FH	FH	FH	H

C_{75}	M	M	FH	M	M
C_{81}	VH	H	FH	VH	VH
C_{82}	H	H	H	H	H
C_{83}	VH	VH	H	H	VH
C_{84}	M	FH	H	M	FH
C_{85}	H	FH	H	FH	H
C_{91}	M	M	FH	M	M
C_{92}	VH	H	H	VH	VH
C_{93}	VH	H	H	H	H
C_{94}	VH	VH	H	H	VH
$C_{10,1}$	M	FH	H	M	FH
$C_{10,2}$	H	FH	FH	FH	H
$C_{10,3}$	M	M	H	M	M
$C_{10,4}$	VH	H	H	VH	VH

Table 5: Priority Weight (in linguistic scale) of 1st level indices assigned by DMs

2nd level indices	Priority Weight (in linguistic scale) of 1st level indices assigned by DMs				
	DM1	DM2	DM3	DM4	DM5
C_1	M	M	FH	M	M
C_2	VH	H	FH	VH	VH
C_3	H	H	H	H	H
C_4	VH	H	H	H	VH
C_5	M	FH	H	M	FH
C_6	H	FH	FH	FH	H
C_7	M	M	FH	M	M
C_8	VH	H	FH	H	VH
C_9	H	H	H	H	H
C_{10}	VH	VH	H	H	VH

Table 6: Aggregated Fuzzy Priority Weight and Aggregated Fuzzy Rating of 2nd level indices

2nd level indices	Aggregated Fuzzy Priority Weight, w_{ij}	Aggregated Fuzzy Rating, U_{ij}
C_{11}	(0.73,0.83,0.92)	(0.69,0.80,0.90)
C_{12}	(0.79,0.89,0.96)	(0.62,0.74,0.86)
C_{13}	(0.46,0.62,0.78)	(0.82,0.92,0.98)
C_{14}	(0.28,0.47,0.78)	(0.82,0.92,0.98)
C_{15}	(0.34,0.53,0.72)	(0.38,0.56,0.74)
C_{21}	(0.75,0.86,0.94)	(0.28,0.47,0.78)
C_{22}	(0.76,0.86,0.94)	(0.24,0.41,0.58)
C_{23}	(0.76,0.86,0.94)	(0.30,0.50,0.70)
C_{31}	(0.46,0.62,0.78)	(0.65,0.77,0.88)
C_{32}	(0.28,0.47,0.78)	(0.58,0.71,0.84)
C_{33}	(0.34,0.53,0.72)	(0.78,0.89,0.96)
C_{34}	(0.75,0.86,0.94)	(0.82,0.92,0.98)
C_{35}	(0.76,0.86,0.94)	(0.38,0.56,0.74)
C_{36}	(0.79,0.89,0.96)	(0.28,0.47,0.78)
C_{41}	(0.46,0.62,0.78)	(0.26,0.44,0.62)
C_{42}	(0.28,0.47,0.78)	(0.30,0.50,0.70)
C_{43}	(0.34,0.53,0.72)	(0.69,0.80,0.90)
C_{44}	(0.75,0.86,0.94)	(0.58,0.71,0.84)
C_{45}	(0.73,0.83,0.92)	(0.82,0.92,0.98)
C_{46}	(0.76,0.86,0.94)	(0.82,0.92,0.98)

C_{47}	(0.50,0.65,0.80)	(0.38,0.56,0.74)
C_{51}	(0.28,0.47,0.78)	(0.26,0.44,0.62)
C_{52}	(0.34,0.53,0.72)	(0.24,0.41,0.58)
C_{53}	(0.79,0.89,0.96)	(0.30,0.50,0.70)
C_{54}	(0.73,0.83,0.92)	(0.69,0.80,0.90)
C_{61}	(0.79,0.89,0.96)	(0.58,0.71,0.84)
C_{62}	(0.46,0.62,0.78)	(0.78,0.89,0.96)
C_{63}	(0.69,0.80,0.90)	(0.82,0.92,0.98)
C_{64}	(0.34,0.53,0.72)	(0.38,0.56,0.74)
C_{65}	(0.75,0.86,0.94)	(0.26,0.44,0.62)
C_{71}	(0.70,0.80,0.90)	(0.24,0.41,0.58)
C_{72}	(0.79,0.89,0.96)	(0.30,0.50,0.70)
C_{73}	(0.46,0.62,0.78)	(0.69,0.80,0.90)
C_{74}	(0.28,0.47,0.78)	(0.58,0.71,0.84)
C_{75}	(0.34,0.53,0.72)	(0.71,0.83,0.92)
C_{81}	(0.75,0.86,0.94)	(0.82,0.92,0.98)
C_{82}	(0.70,0.80,0.90)	(0.42,0.59,0.76)
C_{83}	(0.79,0.89,0.96)	(0.28,0.47,0.78)
C_{84}	(0.46,0.62,0.78)	(0.26,0.44,0.62)
C_{85}	(0.72,0.74,0.86)	(0.30,0.50,0.70)
C_{91}	(0.34,0.53,0.72)	(0.69,0.80,0.90)
C_{92}	(0.79,0.89,0.96)	(0.66,0.77,0.88)
C_{93}	(0.73,0.83,0.92)	()0.79,0.89,0.96

C_{94}	(0.79,0.89,0.96)	(0.82,0.92,0.98)
$C_{10,1}$	(0.46,0.62,0.78)	(0.38,0.56,0.74)
$C_{10,2}$	(0.28,0.47,0.78)	(0.28,0.47,0.78)
$C_{10,3}$	(0.38,0.56,0.74)	(0.24,0.41,0.58)
$C_{10,4}$	(0.79,0.89,0.96)	(0.42,0.59,0.76)

Table 7: Aggregated Fuzzy Priority Weight and Computed Fuzzy Rating of 1st level indices

2nd level indices	Aggregated Fuzzy Priority Weight, w_i	Computed Fuzzy Rating, U_i
C_1	(0.34,0.53,0.72)	(0.43,0.79,1.38)
C_2	(0.75,0.86,0.94)	(0.23,0.46,0.81)
C_3	(0.70,0.80,0.90)	(0.37,0.71,1.24)
C_4	(0.76,0.86,0.94)	(0.40,0.72,1.26)
C_5	(0.34,0.53,0.72)	(0.28,0.56,1.09)
C_6	(0.28,0.47,0.66)	(0.39,0.70,1.19)
C_7	(0.34,0.53,0.72)	(0.28,0.62,1.21)
C_8	(0.72,0.83,0.92)	(0.43,0.59,0.97)
C_9	(0.70,0.80,0.90)	(0.56,0.85,1.25)
C_{10}	(0.79,0.89,0.96)	(0.22,0.52,1.14)

Table 8: Ranking order of 2^{nd} level indices

2^{nd} level indices	FPII	Crisp Value	Ranking Order
C_{11}	(0.186,0.136,0.072)	0.030	26
C_{12}	(0.130,0.081,0.034)	0.018	36
C_{13}	(0.443,0.350,0.216)	0.079	8
C_{14}	(0.590,0.488,0.216)	0.114	1
C_{15}	(0.251,0.263,0.207)	0.056	12
C_{21}	(0.070,0.066,0.040)	0.014	41
C_{22}	(0.058,0.057,0.035)	0.012	44
C_{23}	(0.072,0.070,0.042)	0.015	40
C_{31}	(0.351,0.293,0.194)	0.065	11
C_{32}	(0.418,0.376,0.185)	0.082	6
C_{33}	(0.515,0.418,0.269)	0.095	2
C_{34}	(0.205,0.129,0.056)	0.029	28
C_{35}	(0.091,0.078,0.044)	0.017	37
C_{36}	(0.059,0.052,0.026)	0.011	47
C_{41}	(0.140,0.167,0.136)	0.035	22
C_{42}	(0.216,0.265,0.154)	0.053	14
C_{43}	(0.455,0.376,0.252)	0.085	4
C_{44}	(0.145,0.099,0.050)	0.022	34
C_{45}	(0.221,0.156,0.078)	0.035	23

C_{46}	(0.197,0.129,0.059)	0.029	29
C_{47}	(0.190,0.196,0.148)	0.042	17
C_{51}	(0.187,0.233,0.136)	0.047	16
C_{52}	(0.158,0.193,0.162)	0.040	20
C_{53}	(0.063,0.055,0.028)	0.012	45
C_{54}	(0.186,0.136,0.072)	0.030	27
C_{61}	(0.122,0.078,0.034)	0.017	38
C_{62}	(0.421,0.338,0.211)	0.076	9
C_{63}	(0.254,0.184,0.098)	0.041	20
C_{64}	(0.251,0.263,0.207)	0.056	13
C_{65}	(0.065,0.062,0.037)	0.013	43
C_{71}	(0.072,0.070,0.042)	0.017	39
C_{72}	(0.063,0.055,0.028)	0.012	46
C_{73}	(0.373,0.304,0.198)	0.068	10
C_{74}	(0.418,0.376,0.185)	0.082	7
C_{75}	(0.469,0.390,0.258)	0.088	3
C_{81}	(0.205,0.129,0.056)	0.029	30
C_{82}	(0.126,0.118,0.076)	0.025	32
C_{83}	(0.059,0.052,0.026)	0.011	48
C_{84}	(0.140,0.167,0.136)	0.035	24
C_{85}	(0.084,0.130,0.098)	0.026	31
C_{91}	(0.455,0.376,0.252)	0.085	5
C_{92}	(0.139,0.085,0.035)	0.019	35

C_{93}	(0.213,0.151,0.077)	0.034	25
C_{94}	(0.172,0.101,0.039)	0.023	33
$C_{10,1}$	(0.205,0.129,0.056)	0.045	17
$C_{10,2}$	(0.202,0.249,0.145)	0.050	15
$C_{10,3}$	(0.149,0.180,0.151)	0.038	21
$C_{10,4}$	(0.088,0.065,0.030)	0.014	42