SLIDING MODE CONTROLLER FOR 3-PHASE BOOST RECTIFIER CIRCUIT

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SLIDING MODE CONTROLLER FOR 3-PHASE BOOST RECTIFIER CIRCUIT

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by

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Dedicated

То

My loving parents and my sister



Department of Electrical Engineering

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CERTIFICATE

This is to certify that the thesis entitled "Sliding Mode Controller For 3-Phase Boost Rectifier Circuit" being sub-mitted by Mr. Sandeep Kumar Pradhan, to the National Institute of Technology, Rourkela (Deemed University) for the award of degree of Master of Technology in Electrical Engineering with specialization in "Control Automation", is a bonafide research work carried out by him in the Department of Electrical Engineering, under my supervision and guidance. I believe that this thesis fulfills a part of the requirements for the award of degree of Master of Technology. The research reports and the results embodied in this thesis have not been submitted in parts or full to any other University or Institute for the award of any other degree or diploma.

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ABSTRACT

Feedback control is the elementary mechanism through which many systems, like electrical, mechanical, or biological system maintain their equilibrium. Feedback control may be defined as the use of various signals that are determined by comparing the actual values of the system variables to their desired values, as a means of a control system.

At the present, boost type three phase rectifiers are increasingly used for industrial applications such as uninterruptible power supplies (UPS), battery chargers, motor drives. Also, such as MOSFET, IGBT are commonly used by semiconductors, operate in high frequency switching devices which are free to switch at frequencies much higher than the main frequency, allowing the controller whose dynamic response is very high. The efficient conversion of power is a problem in the modern world. The power factor, one of the most vital characteristics in the AC/DC power conversion, describes the efficiency and superiority of such process. The unity-value power factor converter does not introduce any alteration to the energy source and exploits performance of the power conversion. Many schemes and solutions that are available in the field of power factor correction (PFC) are presented in the work.

The thesis shows a power factor correction for a class of three-phase boost ac/dc power converters. Here a full-bridge boost converter has been studied for power factor control, using output sliding mode control. The sliding mode control drives the output voltage to the preferred dc level in the existence of external disturbances and internal parameter uncertainties providing a value of power factor close to unity. The controlled converters is simulated and studied for the effectiveness of the proposed control algorithms and good consistency is achieved.

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NOTATIONS AND ABBREVIATIONS

AC	Alternating Current
DC	Direct Current
EMI	Electromagnetic Interference
HOSM	Higher Order Sliding mode Algorithm
HVDC	High Voltage Direct Current
IGBT	Insulated gate Bipolar Transistor
PFC	Power Factor Control
PFP	Power Factor Pre compensators
SMC	Sliding Mode Control
SMPS	Switch Mode Power Supply

SOSM	Second Order Sliding Mode
THD	Total Harmonic Distortion
UPS	Uninterruptible Power Supply
VSR	Voltage Source Rectifier
VSCS	Variable Structure Control System
VSS	Variable Structure System
r	Internal phase Resistance
С	Output Capacitance
L	Phase Inductor
r	Internal phase Resistance
R	Load Resistance
U	Control Input
U_{g1}, U_{g2}, U_{g3}	Source Input Voltage
U_1, U_2, U_3	Control Signals
${U}_0$	Output Voltage
i_1, i_2, i_3	Input Phase Current
λ	Gain Matrix
IP_{j}	Input power taken by the j^{th} phase
I_{j}	Magnitude of phase current in j^{th} phase
E_{j}	Magnitude of phase voltage in j^{th} phase
ϕ	Relative phase shift

I_{dj}	Desired current
V_d	Desired voltage
E_{i}	Magnitude of input voltages
S	Sliding surface
$\sigma(x,t)$	Sliding Surface w.r.t. time
f^+,f^-	State velocity vector
$f_{\scriptscriptstyle N}^{ +}, f_{\scriptscriptstyle N}^{ -}$	Normal vector

Introduction

The efficient conversion of power is an important problem that is very relevant in the modern world. It is essential to have effectual energy systems because now government is focusing on global warming, dependence on foreign oil, and the focus on renewable energy. Now a days boost type three-phase rectifiers are useful in industrial applications, such as variable speed drives, power supplies, dc motor drives, front-end converters in adjustable-speed AC drives, HVDC transmission, SMPS, in process technology like welding such as variable-speed drives and ac–dc power supplies for telecommunications equipment, aerospace, military environment and so on. The design, advancement and fruitful application of single-phase, improved quality converters in domestic, commercial and industrial environment has made possible the design and development of three-phase, improved quality converters and their widespread use in different applications. To overcome the problems of harmonic distortion and low power factor in power system several converters and control schemes have been proposed in recent years.

Conventionally diode rectifiers or thyristor bridge converters were used to synthesize dc voltage from the ac voltage. A few advantages of boost topology are

- High power density
- High efficiency
- Improvement of power quality at input and output.

Hence boost type converters not only provide regulated DC output voltage but also maintain near unity power factor and low THD on current at the input. One of the main concerns in controlling the output voltage of boost converter is keeping the output voltage in a desirable value under the variations of load and parameter uncertainties.

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The challenge behind the necessity of finding the most suitable control method to overcome the main problems rising and affecting the circuit performance. These problems are nonlinearity, stability, reduction of cost, reduction of EMI. Linear controllers have been used for linear systems and for the linearized version of a nonlinear system also gives good results at an equilibrium point near which the system behavior is approximately linear. Nonlinear approaches such as

- fuzzy logic control [12]
- passivity based control [13,14]
- back-stepping technique control [15]
- Lyapunov-based control [16]
- the Linearization approximation
- the Describing function concept
- the Piecewise-linear approximation
- Sliding mode control[3-4]

From aforementioned control methods sliding mode control is used because it has described the characteristics of robustness and effectiveness by some researchers [3-4], which is easy and effective to deal with the nonlinear behavior of the boost rectifier as the system order is reduced and the non-minimum phase behavior is canceled out. One of the most intriguing aspects of sliding mode control is the discontinuous nature of the control action. The main function of each of the feedback control is to switch between the two different system structures, so that a new type of system motion called sliding mode exists in a manifold $\sigma = 0$. Two main objectives are followed in order to design the efficient ac/dc power converter.

- 1) To maintain power factor as close as possible to unity.
- 2) To achieve a stabilized DC output voltage.

Both objectives are achieved by the SMC theory that permits not only accomplishment of high performance (power factor close to unity) [5,6] of the system but also the keeping the functionality in the presence of parameter uncertainties and external disturbances.

1.2 Literature Review

In the literature survey, I have studied about modelling of 3-ph AC/DC boost converter and to get unity value of power factor different nonlinear control technique are studied, among them I came to know that sliding mode controller (SMC) is best. The following papers describe their contributions.

Jianxing Liu*et.al.***[1]**discussed a full-bridge boost power converter topology has been studied for power factor control, using out-put high order sliding mode control. The proposed controller forces the input currents tracking the desired values which can control indirectly the output voltage while keeping power factor close to one, and ensuring minimum frequency deviation and phase difference between voltage and current.

KadaHartani*et.al.*[2]proposed the space vector pulse width modulation (SVPWM) control scheme for three-phase voltage source PWM rectifier. The proposed control can stabilize the minimum of the systems storage function at the desired equilibrium point determined by unity power factor and sinusoidal current on the AC side, and constant output voltage on the DC side.

Yu Wang *et.al.*[3]introduced the topology of Three-Phase PWM Voltage Source Rectifier (VSR). Then the mathematical models in three-phase static and two-phase rotary coordinate system are built. Based on that theory and the voltage-oriented vector control's idea, a dual-channel closed-loop control strategy with current-inner-loop and voltage-outer-loop. Active power channel is aim to make DC side voltage remain steady, and reactive power channel can regulate power factor by set current q-axis component reference. In order to prove that voltage-outer-loop is necessary, the paper builds a comparative model without a voltage loop.

R. Schaeffel*et.al.*[4] explained power factor correction (PFC) for a full bridge 3-phase AC/DC boost power converter. The purpose of this paper is to design and study the effectiveness of the traditional sliding mode controllers (SMC) and second order sliding mode (SOSM) controllers

[12,13] that convert AC power into DC power at a desired voltage level in a 3-phase AC/DC boost power converter while driving the power factor to a unity level.

J.T.Boyset.al.[5]A voltage sourced reversible rectifier(VSRR)which achieves bidirectional power flow between a 3-phase AC supply and a DC bus voltage is described. The device features a pulse Width modulated voltage control strategy that confined unwanted harmonics to known frequency bands where they can be easily filtered out while operating at near unity power factor, with sinusoidal currents, and maintaining a constant DC bus voltage.

Cecati.C.al.c[6] proposed that active rectifiers include current or voltage sensors, and the number should be reduced to obtain low cost systems. Here a active filter with feed forward fuzzy logic control is offered and discussed. Here mathematical description of the system and design of fuzzy logic controller is introduced and analyzed.

Sira-Ramirez H.al.c[7] proposed to incorporate the sliding mode controller design for the regulation of DC-DC power converters of the "boost" type, the energy dissipation and passivity properties of the switch -regulated system. It is announced that for the "traditional" sliding mode controller, the integral of the stored energy is infinite while for a mixed passivity-based sliding mode controller the same index becomes finite.

H.Komurcugil.al.c[8] discussed a technique for 3-phase pulse width modulation (PWM)AC/DC voltage source converter with the control laws proposed so far is not only unstable against large-signal disturbances, but also has the problem that its stability depends on the circuit parameters such as the DC output capacitance. This paper describes a new control law based on Lyapunov's stability theory. It is shown that the converter can be stabilized globally for handling large-signal disturbances. The resulting closed-loop system not only guarantees a sufficient

stability region (independent of the circuit parameters) in the state space, but also exhibits good transient response both in the rectifying and regenerating modes.

Y.Shtssel.al.c[9] understanding the problem of power factor correction a full-bridge3-phase AC/DC converter hardware topology is designed and used for efficient power conversion with high values of power factor. The sliding mode control drives the output voltage to the desired dc level in the presence of external disturbances and internal parameter uncertainties while providing a value of power factor that is close to unity (greater than 97%).sliding mode observers are engaged to estimate variation in load and phase resistances-subject of real-time adjustment in the control law.

A.Levant[10] has introduced universal finite-time-convergent controller is developed capable to control the output of any uncertain single-input-single-output system with a known permanent relative degree r. The tracking error σ is steered to zero by means of a control dependent only on σ , $\sigma^{(r-1)}$ and continuous everywhere except the set $\sigma=\sigma^{:}=\cdots=\sigma^{(r-1)}=0$. A robust output-feedback controller version provides for the tracking accuracy proportional to the sampling noise magnitude.

1.3 Motivation

The design of an efficient power converter should be considered as a difficult problem, because of mainly two reasons. They are electrical circuit (hardware topology) selection and control algorithm design and implementation. The main goal conversion of power is not only the production of electrical signals (voltage or current) but also the consideration of the effect of power converters to the energy source to provide better quality power conversion. The power factor, which reflects the efficiency and quality of the process is one of the most vital feature in the field of ac/dc power conversion. The unity-value power factor converter does not introduce any distortion to the energy source and exploits the performance factor of the power conversion. In the area of power factor correction (PFC) many schemes and solutions are introduced. Also different hardware topologies and control methods are used in this work. In order to design the efficient ac/dc power converter two main goals should be achieved.

- Power factor should be as close as possible to unity.
- Achieve a constant DC output voltage.

Both goals are addressed by the SMC theory that allows not only the achievement of the high performance of the system (with power factor closed to unity) but also its robustness, and effective to deal with the nonlinear behavior of the boost rectifier, as the system order is reduced.

1.4 Objective of Research work

Due to variation of parameters like nonlinear component in the converter, line and load variation, electromagnetic interferences (EMI) the converter was deviated from the desired operating condition. The converter will not operate in steady state, if the parameter deviation increases. Many control methods are used to control and solve the above problem. Each control method has its own advantages and weaknesses due to which a particular control method is used to be the most suitable control method under specific conditions. A particular control method is demanded which has the best performances under any conditions. The DC portion of the output voltage should equal a desired constant output voltage value (V_d). The AC portion of the output voltage should be reduced to an acceptable range. The input phase currents should be in phase with their corresponding phase voltage. The phase currents should only have the frequency corresponding to the frequency of the respective phase voltage.

The thesis defines the cause by which a specific control method is selected, i.e. the sliding mode control (SMC), among all control methods. A detailed research analysis is done of the sliding mode control is implemented in some AC/DC converter topologies.

1.5 Organization of Thesis

The following presents the outline of the work in details.

Chapter 1: presents a review on the available literature on the modeling and control techniques of 3-phase AC/DC converter. The importance of 3-phase AC/DC converter is widely used for various applications is given. This chapter explains the various types of linear and nonlinear control techniques. Among them SMC is the best control method for efficient conversion of power, because of its robustness, capability to generate control function in high frequency switching format and discontinuous nature of control action.

Chapter 2: In this chapter modeling of three phase AC/DC converter is discussed. Here the control objectives are inductor current and the capacitor voltage. The mathematical model is in terms of these two parameters which describes the circuit behavior is given in scalar format or vector format. Again the quality of the control algorithm should be analyzed via calculating the actual value of power factor.

Chapter 3: It describes in detail structure of SMC and a concisely reviews the history of SMC. The sliding mode control technique for 3-phase boost converter is analyzed, and comparative analysis of the discussed control methods are done. A review of SMC theory like the existence condition, the reaching condition and the chattering is explained. The benefit of using sliding-mode controller is discussed. The result shows that SM controller offers better steady state and transient response than other control methods.

Chapter 4: The SMC is implemented to the 3-phase AC/DC Boost converter and the aforementioned SMC theories are proven. The results are verified with the simulation results and it includes the thesis conclusions and further research directions. Finally, the researches on and applications of the SMC for AC/DC converters are given in details.

Chapter – 2

MODELLING OF 3-PHASE AC/DC CONVERTER

2.1 Introduction

In this chapter we are interested in the performance improvement of ac-to-dc converters. To overcome the main disadvantages found in dc to dc converters namely low input-power factor and the considerable pollution presented in the line source the simple circuit ac-to-dc converters are used. They are generally behave as power factor pre compensators (PFPs), since they are mainly used to ensure a unity power factor functioning, thus seeming as a primary step in a power source. Three-phase single stage ac-dc Boost converter having six switches, used to obtain improved power factor and sinusoidal currents. The switches have bidirectional power flow, such as IGBTs or MOSFETs, Therefore additional series diode will be required. As there is variation in DC load the robustness must be a vital factor for rectifier controllers. Power electronic converters are nonlinear systems. In designing converters and control systems there are used both mathematical modeling of their behavior, software, capable of simulating various stages of work in order to verify the perfection of the projected solutions to design and control algorithms.

BOOST-TYPE three-phase rectifiers are useful in industrial applications, such as variablespeed drives and ac-dc power supplies for telecommunications apparatus. Traditional rectifying techniques adopts phase-control method or passive diode rectifier. They have some drawbacks, like slower dynamic respond, passive diode rectifier can absorb harmonic current from power grid and the DC-side energy cannot feedback. Three-Phase boost power converter techniques with

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sliding mode is used to overcome the phase-control method and passive diode rectifier's disadvantages, and has higher power factor, lower harmonic current and rapid dynamic respond. Here we design a mathematical model for 3-phase ac-dc boost converter in terms of current and voltages.

2.2 3-phase boost rectifier model

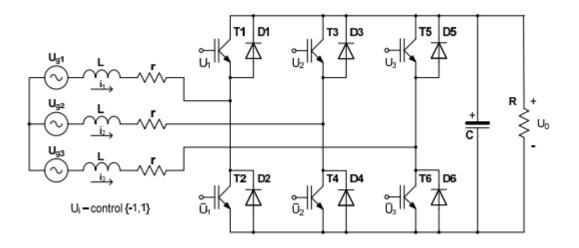


Figure 2.1 3-Phase boost AC/DC power converter

Fig. 1 shows a full boost converter circuit that is used to convert from 3-phase AC power into DC power. To design a math model, it is assumed that the AC voltage is a balanced three phase supply, switch is ideal and lossless, and filter is linear. The circuit consists of three main parts, storage elements, bi-directional switches, and RC filter. The Inductors are the energy storage elements used for higher output DC voltages that could be obtained by rectifier. The IGBTS with anti-parallel diode make the needed switches. The switches are bidirectional because when the switches are closed, they short the diode, therefore current can flow in either direction. The output is an RC filter which is used for smoothing and reducing the output variations to a dc voltage level [1, 8]. The six on-off signals of the converter are signified by

$$\boldsymbol{s} = \begin{bmatrix} \boldsymbol{s}_a & \boldsymbol{s}_b & \boldsymbol{s}_c & \boldsymbol{s}_d & \boldsymbol{s}_e & \boldsymbol{s}_f \end{bmatrix}^T$$
(2.1)

For simplicity the six on-off signals appeared as control inputs and defined as $U = \begin{bmatrix} U_1 & U_2 & U_3 \end{bmatrix}^T$. *U* represents a set of distorted control inputs (-1, 1) instead of (0,1).

The switches are arranged so that when the top switches (s_a, s_b, s_c) are open the bottom switches (s_d, s_e, s_f) are closed. Also, when the top switches are closed the bottom switches are opened. Thus, the following relation holds:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} s_a \\ s_b \\ s_c \\ s_d \\ s_e \\ s_f \end{bmatrix}$$
(2.2)

 $U_1 = +1$ means $s_a = 1$ and $s_d = 0$, corresponds to the conducting state for the upper switching element and nonconducting state for the bottom switching element. The opposite state will happen when $U_1 = -1$ i.e. $s_a = 0$ and $s_d = 1$. At any moment out of six switching elements only three conduct. The mathematical model of the boost AC/DC converter in phase coordinate frame can be obtained From Equation (2.2).

The voltage equation are given by

$$U_{g1} = L \frac{di_{1}}{dt} + ri_{1} + U_{a}$$

$$U_{g2} = L \frac{di_{2}}{dt} + ri_{2} + U_{b}$$

$$U_{g3} = L \frac{di_{3}}{dt} + ri_{3} + U_{c}$$
(2.3)

Where U_a, U_b, U_c are the input voltage to the rectifier.

The source voltage are expressed as

$$\begin{bmatrix} U_{g1} \\ U_{g2} \\ U_{g3} \end{bmatrix} = E \begin{bmatrix} \sin \theta \\ \sin(\theta - \frac{2\pi}{3}) \\ \sin(\theta + \frac{2\pi}{3}) \end{bmatrix}$$
(2.4)

The current across the capacitor C is

$$i_{c} = i_{dc} - i_{L}$$

$$\Rightarrow C \frac{dU_{0}}{dt} = (i_{1}U_{1} + i_{2}U_{2} + i_{3}U_{3}) - \frac{U_{0}}{R}$$

$$\Rightarrow \frac{dU_{0}}{dt} = \frac{1}{C}(i_{1}U_{1} + i_{2}U_{2} + i_{3}U_{3}) - \frac{U_{0}}{RC}$$
(2.5)

The input rectifier voltage can be expressed as

$$U_{a} = \left\{ U_{1} - \frac{U_{1} + U_{2} + U_{3}}{3} \right\} U_{0}$$
(2.6)

$$\Rightarrow U_{a} = \left\{ \frac{2U_{1} - U_{2} - U_{3}}{3} \right\} U_{0}$$

$$U_{b} = \left\{ U_{2} - \frac{U_{1} + U_{2} + U_{3}}{3} \right\} U_{0}$$

$$\Rightarrow U_{b} = \left\{ \frac{2U_{2} - U_{1} - U_{3}}{3} \right\} U_{0}$$
(2.7)

Likewise

$$U_{c} = \left\{ U_{3} - \frac{U_{1} + U_{2} + U_{3}}{3} \right\} U_{0}$$

$$\Rightarrow U_{c} = \left\{ \frac{2U_{3} - U_{1} - U_{2}}{3} \right\} U_{0}$$
(2.8)

By putting the value of U_a, U_b, U_c from equation (2.3) in the above (2.6, 2. 7, 2. 8) the input voltages are

$$U_{g1} = L \frac{di_{1}}{dt} + ri_{1} + \frac{(2U_{1} - U_{2} - U_{3})}{3}U_{0}$$
(2.9)

$$\Rightarrow \frac{di_{1}}{dt} = -\frac{r}{L}i_{1} + \frac{U_{g1}}{L} - \frac{U_{0}}{3L}(2U_{1} - U_{2} - U_{3})$$
(2.10)

$$U_{g2} = L \frac{di_{2}}{dt} + ri_{2} + \frac{(2U_{2} - U_{1} - U_{3})}{3}U_{0}$$
(2.10)

$$\Rightarrow \frac{di_{2}}{dt} = -\frac{r}{L}i_{2} + \frac{U_{g2}}{L} - \frac{U_{0}}{3L}(2U_{2} - U_{1} - U_{3})$$
(2.11)

$$U_{g3} = L \frac{di_{3}}{dt} + ri_{3} + \frac{(2U_{3} - U_{1} - U_{2})}{3}U_{0}$$
(2.11)

$$\Rightarrow \frac{di_{3}}{dt} = -\frac{r}{L}i_{3} + \frac{U_{g3}}{L} - \frac{U_{0}}{3L}(2U_{3} - U_{1} - U_{2})$$
(2.11)

Finally the mathematical model which describes the circuit is given in scalar format

$$\frac{di_{i}}{dt} = -\frac{r}{L}i_{i} + \frac{1}{3L}\left(2U_{g1} - U_{g2} - U_{g3}\right) - \frac{U_{0}}{6L}\left(2U_{1} - U_{2} - U_{3}\right)
\frac{di_{2}}{dt} = -\frac{r}{L}i_{2} - \frac{U_{0}}{6L}\left(2U_{g2} - U_{g1} - U_{g3}\right) + \frac{1}{3L}\left(2U_{2} - U_{1} - U_{3}\right)
\frac{di_{3}}{dt} = -\frac{r}{L}i_{3} - \frac{U_{0}}{6L}\left(2U_{g3} - U_{g1} - U_{g2}\right) + \frac{1}{3L}\left(2U_{3} - U_{1} - U_{2}\right)
\frac{dU_{0}}{dt} = -\frac{U_{0}}{RC} + \frac{1}{2C}\left(i_{1}U_{1} + i_{2}U_{2} + i_{3}U_{3}\right)$$
(2.12)

or in vector format

$$\frac{di}{dt} = -\frac{r}{L}i - \frac{U_0}{6L}\lambda U + \frac{1}{3L}\lambda U_g$$

$$\frac{dU_0}{dt} = -\frac{U_0}{RC} + \frac{1}{2C}U^T i$$
(13)

Where

- r internal phase resistance (impedance of switching elements in open state);
- R Load (output) resistance;
- L phase inductor (presumed to be same for all phases);
- C Output capacitor;

 U_0 Output voltage;

 $i = \begin{bmatrix} i_1 & i_2 & i_3 \end{bmatrix}^T$ Input phase currents;

 $U_{g} = (U_{g1}, U_{g2}, U_{g3})^{T}$ Source (input) voltages;

 $U = (U_1, U_2, U_3)^T$ Control signals well-defined by finite set of values $\{+1, -1\}$.

In equation (13) λ is gain matrix and defined as follows;

$$\lambda = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

The input voltages U_{gj} can have similar frequency but dissimilar voltage magnitudes and phase shift of 120 degrees from each other, thus the vector U_g can be denoted as:

$$U_{g} = \begin{bmatrix} E_{1} & 0 & 0\\ 0 & E_{2} & 0\\ 0 & 0 & E_{3} \end{bmatrix} \begin{vmatrix} \sin(\omega t)\\ \sin\left(\omega t - \frac{2\pi}{3}\right)\\ \sin\left(\omega t + \frac{2\pi}{3}\right) \end{vmatrix} = E * \sin$$
(14)

Where E_i is the magnitude of input voltages i^{th} phase. In order to extract the sinusoidal signal the phase voltage is divided by its magnitude.

The current and voltage that come out of an uncontrolled boost power converter is not in phase and does not have the exact harmonics. Also, it does not boost the voltage to preferred levels. The estimation and correction of power factor (PF) value is very essential in the quality analysis of the designed control law

Hence the control objectives are like this

• The dc portion of the output voltage U_0 should be equivalent to some preferred constant value V_d , on the other hand the ac component has to be attenuated to a given level.

• The input phase currents should be in phase with their respective phase voltage. The phase currents should have frequency equivalent to the frequency of the respective phase voltage.

2.3 Current Profile Design

The maximal output voltage that can be stabilized by the circuit appears to be a function of main voltage magnitudes and circuit parameters. By analogy to [2] and [6], the problem solvability state is to be conveyed in terms of the desired magnitudes of input phase currents. These magnitudes should be selected to provide the dc power balance between the input and output of the converter while preserving dc level of the output voltage in a desired state.

Let's consider the DC power which is consumed by j^{th} phase where j could be 1, 2 or 3: The input power should be denoted as $\{IP_j\}_{dc}$

$$\{IP_{j}\}_{dc} = \{i_{j}U_{gj} - ri_{j}^{2}\}_{dc}$$
$$= \{I_{j}E_{j}\sin(\omega t + \phi)^{2} - rI_{j}^{2}\sin(\omega t + \phi)^{2}\}_{dc}$$
$$= \frac{1}{2}(I_{j}E_{j} - rI_{j}^{2})$$
(15)

Where

- IP_j = The input power taken by the j^{th} phase;
 - I_j = Magnitude of phase current in j^{th} phase;
 - E_i = Magnitude of phase voltage in j^{th} phase;
 - ϕ =Relative phase shift

Since I_{dj} is the magnitude of the desired value of the input current is the matter of concern the input-side dc power which is taken from the energy source by j^{th} phase, can be rewritten as:

$$\left\{ IP_{j} \right\}_{dc} = \frac{1}{2} \left(I_{dj} E_{j} - r I_{dj}^{2} \right)$$
(16)

Furthermore, the output-side dc power is given by;

$$\left(OP\right)_{dc} = \frac{V_d^2}{R}$$

Where V_d the required dc level of output voltage and R is the load resistance.

Let's assume there is uniform contribution to the output dc power from each phase, the required magnitude for the j^{th} phase input current can be found by solving the following equation.

$$\left\{IP_{j}\right\}_{dc}=\frac{1}{3}\left(OP\right)_{dc}$$

The above has two real solutions

$$I_{dj} = \frac{E_j}{2r} \pm \sqrt{\left(\frac{E_j}{4r^2} - \frac{2}{3}\frac{V_d^2}{rR}\right)}$$

 I_{dj} is the desired current profile for each phase. V_d is the desired voltage level.

Additionally, a condition for V_d keeping I_{dj} real is the following;

$$V_d \le E_j \sqrt{\frac{3R}{8r}}$$

This corresponds to an upper limit for V_d . Between the two possible solutions one is optimal means minimal energy consumption (with "-" sign) and should be used as the desired magnitude of j^{th} the phase current.

$$I_{dj} = \frac{E_j}{2r} - \sqrt{\left(\frac{E_j}{4r^2} - \frac{2}{3}\frac{V_d^2}{rR}\right)}$$
(17)

In the same way to the phase voltages, the desired phase currents can be written as:

$$I_{d} = \begin{bmatrix} I_{1} & 0 & 0\\ 0 & I_{2} & 0\\ 0 & 0 & I_{3} \end{bmatrix} \begin{bmatrix} \sin(\omega t) \\ \sin(\omega t - \frac{2\pi}{3}) \\ \sin(\omega t + \frac{2\pi}{3}) \end{bmatrix} = I_{d} * \sin$$
(18)

The sin term is the same as in (14) due to the desired phase current curves need to be in phase with their equation.

2.4 Chapter Conclusion

In this chapter a mathematical model of 3-phase ac/dc boost converter is designed. The input current and output voltage are the two parameters which are required to be controlled. Therefore all the equations are in the terms of these two parameters. Also we have evaluated the desired amount of current and voltage which are necessary for design of sliding mode controller in the next chapter.

Chapter – 3

DESIGN OF A SLIDING MODE CONTROLLER

3.1 Introduction

In the design of any control problem there will be divergences between the actual plant and the mathematical model developed for controller design. This mismatch may be due to the deviation in system parameters or the approximation of complex plant behavior.

It must be confirmed that the resulting controller has the capability to produce the desired performance levels in practice despite of the plant/model mismatches. This aroused intense interest in the development of robust control methods, which pursues to solve this problem. The SMC methodology gives one particular approach to robust controller design.

The Variable Structure Control System (VSCS) is a class of systems where the control law is intentionally changed during the control process according to some well-defined rules that depend on the state of the system. The SMC is a particular type of the VSCS, which is characterized by a feedback control laws and switching function, of the current system behavior and produces as an output the particular feedback controller, it should be used at that instant of time. A Variable structure system (VSS) is a combination of subsystems, where each subsystem has a fixed control structure and is effective for specified regions of the system behavior. The instances at which the varying of the structure occurs are determined by the current state of the system. Sliding mode control (SMC) is a nonlinear control method that changes the dynamics of a nonlinear system by application of a high frequency switching control that forces the system to slide along the

Cross section of the system's normal behavior. The state-feedback control law can switch from one continuous structure to another based on the existent position in the state space. Therefore sliding mode control is a variable structure control method. The multiple control structures are designed so that trajectories always move toward an adjacent region with a different control structure, and so ultimate trajectory will not exist entirely within one control structure. Instead, it will slide along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consist of the boundaries is called the sliding surface.

This control method offers several advantages over the other control methods which are:

- ✤ Low sensitivity to plant parameter uncertainty
- ✤ Greatly reduced-order modeling of plant dynamics
- Finite-time convergence (due to discontinuous control law)
- Stability even for large line and load variations
- Robustness
- Good dynamic response
- Simple implementation.

3.2 Sliding Mode Control in Electrical and Mechanical Systems

As the applications of SMC is increasing day by day researchers are working to prove the efficiency of the SMC in electrical and mechanical systems. Recently many researches have done research on the applying the SMC in electrical and mechanical systems. Some of the main fields of research on the topic are shown in Figure 3.1.

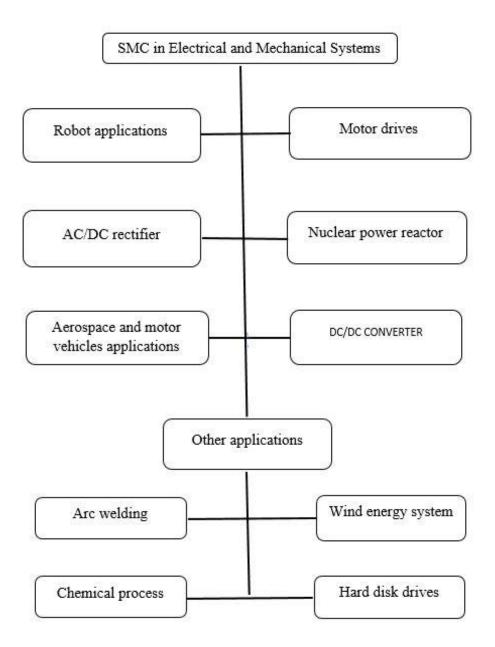


Fig. 3.1 SMC in Electrical and Mechanical system

3.3 Basic principle of Sliding Mode Control

There are two basic ideas to design of SM control

- first a sliding(hyper) surface in state space
- second is to design a control law

The control law force the system state trajectory starting from any arbitrary initial state to reach the sliding surface in predetermined time, and finally it should come to a point where the system equilibrium state exists. The existence, stability and hitting condition are the three factors for the stability of sliding mode control. SM control principle is graphically represented in Figure 1, where S = 0, represent the sliding surface and x_1 and x_2 are the voltage error variable and voltage error dynamics respectively. The sliding line divides the phase plane into two sections. Each section is specified with a switching state and when the trajectory arrives at the system equilibrium point, the system is considered as stable.

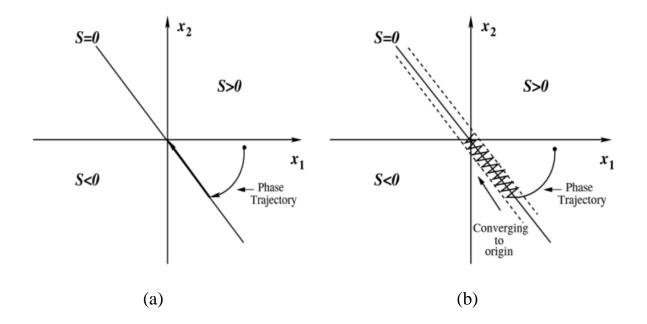


Figure 3.2 (a)Ideal Sliding mode control

(b)Actual Sliding mode control

Let us consider a system with scalar control as an example for better understanding of design procedure of SM control.

$$\dot{x} = f(x, t, u)$$

where \mathcal{X} is the column vector which signifies the state of the system, \mathcal{F} is a function vector with n dimension, \mathcal{U} is the control input that influences the system motion. The function vector f is discontinuous on the sliding surface $\sigma(x, t) = 0$ which can be represented as,

$$f(x,t,u) = \begin{cases} f^{+}(x,t,u^{+}) for \sigma(x,t) > 0\\ f^{-}(x,t,u^{-}) for \sigma(x,t) < 0 \end{cases}$$

Where $\sigma(x, t) = 0$ is the sliding surface (hyper surface). The system is in sliding mode if all the representative points move on the sliding surface.

3.4 Existence and Reachability Condition

Existence and reaching condition are two basic need for a stable SM control system. The existence condition of sliding mode requires that the phase trajectories of the two subsystems corresponding to the two different values of the vector function f must be directed toward the sliding line (surface) $\sigma(x, t) = 0$. In other words, while approaching the sliding line from the points which satisfy $\sigma(x,t) < 0$ the resultant state vector f^- must be directed toward the sliding surface, and the same happens for the points above the surface $\sigma(x, t) > 0$ for which the state velocity vector is f^+ .

The state velocity vectors (f^+, f^-) of the function f are denoted with subscript N orthogonal to the sliding surface, which is given by

$$\lim_{\sigma \to 0^+} f_N^+ < 0 \Longrightarrow \lim_{\sigma \to 0^+} \nabla \sigma(x, t) f^+ < 0$$
$$\lim_{\sigma \to 0^-} f_N^- < 0 \Longrightarrow \lim_{\sigma \to 0^-} \nabla \sigma(x, t) f^- < 0$$

Where $\nabla \sigma$ is the gradient of surface $\sigma(x, t)$. This is expressed as

$$\frac{d\sigma}{dt} = \sum_{i=1}^{n} \frac{\delta\sigma}{\delta x_{i}} \frac{dx_{i}}{dt} = \nabla\sigma(x,t)f$$

Therefore, the sliding-mode existence condition is defined as follows

$$\lim_{\sigma(x,t)\to 0} \sigma(x,t) \frac{d\sigma(x,t)}{dt} < 0$$

This condition is the sufficient condition for the system to reach the sliding surface and holds the entire state space and the region around the sliding surface.

The reachability condition means trajectory will reach the sliding surface within predetermined time interval. The scalar input \mathcal{U} at any instant depends upon the trajectory in state space at that instant. Hence, the control input for the system can be written in mathematical form as,

$$u = \begin{cases} u^+ for \sigma(x,t) > 0\\ u^- for \sigma(x,t) < 0 \end{cases}$$

Where u^+ and u^- are the switching function which belong to the region $\sigma(x,t) > 0$ and $\sigma(x,t) < 0$ respectively. Let $[p^+]$ and $[p^-]$ be the steady state points corresponding to the inputs u^+ and u^- . Then a sufficient condition for reaching the sliding surface is given by:

$$\begin{bmatrix} p^+ \end{bmatrix} \in \sigma(x,t) < 0$$
$$\begin{bmatrix} p^- \end{bmatrix} \in \sigma(x,t) > 0$$

If the steady-state point for one subsystem belongs to the region of the phase space reserved to the other subsystem, then earlier or later the system trajectory will strike the sliding (hyper) surface.

3.5 Chattering

Chattering phenomenon in SMC due to non-linearity of the system. The trajectory moves toward the sliding line and oscillates in a defined boundary. It is an oscillatory motion in the neighborhood of the sliding surface. Suitable reasons that cause chattering include non- idealities of switching devices for control realization or the existence of parasitic dynamics in series with the plant. The chattering phenomenon shown in Figure 3.2 is inevitable in the real application of the SMC.

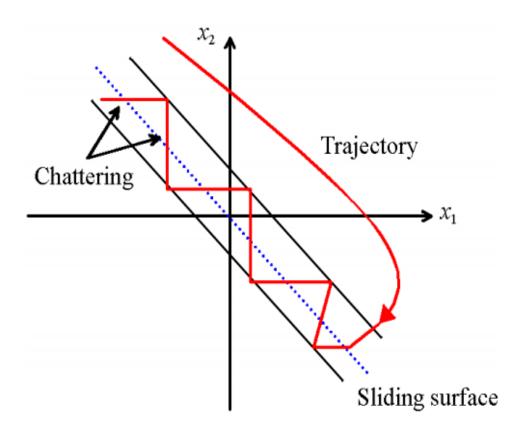


Figure 3.3 Chattering effect in SMC

To reduce the effect of chattering many solutions have been developed over the last few decades.

In particular some methods have been developed.

- •The use of a continuous approximation of the relay.
- To restrain chattering in the observer dynamics by use of an asymptotic stateobserver.
- The use of higher-order sliding mode control algorithms

(HOSM)

3.6 Sliding Mode Control for Boost Rectifier

One of the most important aspect of the sliding mode control in the VSCS is the ability to achieve response that is not dependent of the system parameters. Therefore the AC/DC converter is particularly suitable for the application of the SMC, because of controllable state that is the system is controllable if every state variable can be changed by an input signal. For measurement the output voltage and its derivative are both continuous and manageable.

There are some assumptions that we have to take when studying controlling the 3-phase AC/DC boost converter. They are as follows.

- All phase voltages have the same frequency.
- All of the phases have the same phase inductance.
- The phase current, phase voltage, output voltage, and control signals can be calculated.
- The phases equally contribute to output DC power.
- The control vector U equals to {-1,1}instead of {1,1,1} or {-1,-1,-1} since this would cause the system to lose controllability.

The design of an SMC, the input current i had to follow the reference signal i_d . However, first of all we have to discuss about the control obstacles to overcome. The control vector U looks in the system equations through the singular gain matrix β . This fact produces to a loss of controllability in the case when U= {1, 1, 1} or U= {-1, -1, -1}. At first we have to introduce three sliding variables σ_1 , σ_2 and σ_3 and combine them into the vector format $\sigma = {\sigma_1, \sigma_2, \sigma_3}^T$. Every scalar quantity represents the current tracking error in the equivalent phase. To make all the sliding variables to zero, providing convergence of all tracing errors is the responsibility of controller. The errors are

 $\sigma_j = i_j - i_{dj}$, For j = 1, 2, 3

OR

 $\sigma = i - i_d = i - I_d \sin i$, Therefore $i = \sigma + I_d \sin i$

The sliding variables σ are supposed to be driven to zero

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} i_1 - i_{d1} \\ i_2 - i_{d2} \\ \int_0^t (U_1 + U_2 + U_3) \end{bmatrix}$$

According to Kirchhoff's current law, it is not required to track all three phase currents at the similar time. Also, tracking the third current gives rise a singularity. The third condition evades the unnecessary requirement to track the third current and is used to claim control symmetry.

In the process of controller design we have to now find the derivatives of sliding variables along the system trajectory.

$$\begin{bmatrix} \dot{\sigma}_1 \\ \dot{\sigma}_2 \\ \dot{\sigma}_3 \end{bmatrix} = \begin{bmatrix} \dot{i}_1 - I_{d1}\omega\cos(\omega t) \\ \dot{i}_2 - I_{d2}\omega\cos(\omega t + \frac{2\pi}{3}) \\ U_1 + U_2 + U_3 \end{bmatrix} = \alpha + \beta U$$

Where

$$\alpha = \begin{bmatrix} \frac{1}{3L} (2E_1 - E_2 - E_3 - 3rI_{d1}) \sin(\omega t) - I_{d1} \cos(\omega t) + U_T \\ \frac{1}{3L} (2E_2 - E_1 - E_3 - 3rI_{d2}) \sin(\omega t + \frac{2\pi}{3}) - I_{d1} \cos(\omega t + \frac{2\pi}{3}) + U_T \\ 0 \end{bmatrix}$$

$$\beta = \begin{bmatrix} \frac{-2U_0}{6L} & \frac{U}{6L} & \frac{U}{6L} \\ \frac{U}{6L} & \frac{-2U_0}{6L} & \frac{U}{6L} \\ 1 & 1 & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} \qquad U_T = \frac{U_0}{6L} (U_1 + U_2 + U_3)$$

Here β is a nonsingular matrix. So U can be decoupled by the following equation.

$$\sigma^* = eta^{-1} \sigma$$

 $\dot{\sigma}^* = eta^{-1} lpha + eta^{-1} . eta . U = F + U$

Where

$$\beta^{-1} = \begin{bmatrix} \frac{-2L}{U_0} & 0 & \frac{1}{3} \\ 0 & \frac{-2L}{U_0} & \frac{1}{3} \\ \frac{2L}{U_0} & \frac{2L}{U_0} & \frac{1}{3} \end{bmatrix}$$

Then control function becomes

$$U_{j} = sign(\dot{\sigma}_{j}^{*}) \qquad j = 1, 2, 3$$

3.7 Simulation Results

The multi rate simulation of the proposed three-phase ac/dc boost power converter has been conducted with the parameters shown in (Table 3.1). The system uses a control rate that is slower than the simulation rate in order to test for the controller implementation. The load resistance changes after 1.5 seconds to test for varying loads. The frequency changes after 1.0 second to test the controller's ability to handle varying frequencies. The phase voltages have different magnitudes to test the controller's ability to handle phase voltages that are not equal.

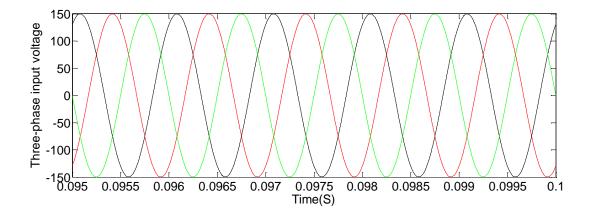


Fig 3.4 shows the relationship between 3-phase input voltages.

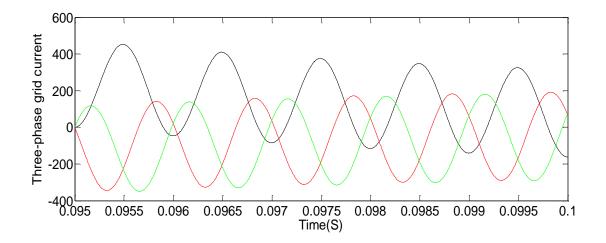


Fig 3.5 shows the relationship between 3-phase grids current

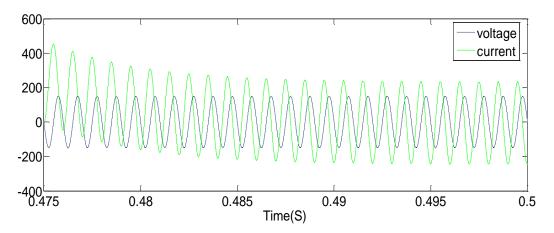


Fig 3.6 Relationship between input voltage and current

Paramet	ter used	l for a	simul	ation

Parameter	Value	Description	
Method	Euler	Integration method	
ΔT	10 ⁻⁶	Integration step size, sec	
f_1	10 ⁶	Pseudo-analog simulation rate, Hz	
f_2	10 ⁵	Control evaluation rate, Hz	
f_3	10^{4}	Pulse width modulator rate, Hz	
R	$30 \rightarrow 40$	Load resistance, Ω	
r	0.02	Parasitic phase resistance, Ω	
L	2	Phase inductor value, <i>mH</i>	
С	100	Output capacitance, μF	
$\left\{E_1, E_2, E_3\right\}$	{155,145,150}	Main voltage, V	
f	75→150	Main voltage frequency, <i>Hz</i>	
<i>U</i> ₀ (0)	1	Initial output voltage, V	
V_d	650	Desired output DC voltage, V	

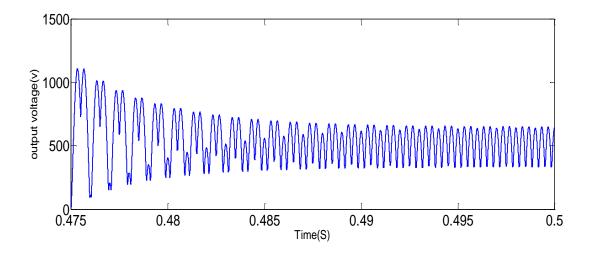


Fig 3.7 Output voltage of 3-phase boost converter with sliding mode control

3.8 Conclusion

A brief history of SMC was given with a short and the theory of SMC was given in details. A comparison of SMC researches and applications in electrical and mechanical system were specified .The theoretical study of SMC is applied to the 3-phase boost converter was given in detail and the simulation results are discussed.

Chapter-4

Conclusion

In this thesis we attempt to apply the sliding mode control concept in 3-phase boost rectifier circuit with full-bridge configuration. The mathematical formulation of power circuit under this proposed method has been done. To verify the capability of suggested method, a preliminary study of 3-phase rectifier circuit has been done using MATLAB/Simulink software. The simulation results shows a good performance of proposed method at start-up during load variations, providing a desirable output voltage. However analysis of such sliding mode controlled ac/dc converter systems needs to be investigated further in detail.

Scope of Future Work

The following are the few areas of future study which can be considered for further work.

- Apply super twisting sliding mode controller to the boost converter circuit and study the power factor and output dc voltage.
- Concept of higher order sliding mode controllers.

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