Buckling analysis of functionally graded carbon nanotubes reinforced composite (FG-CNTRC) plate.

Thesis Submitted to

National Institute of Technology, Rourkela

for the award of the degree
of

Master of Technology

In Mechanical Engineering with Specialization

"Machine Design and Analysis"

by

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June 2014

Dedicated to my parents and guide

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA

CERTIFICATE

This is to certify that the work in this thesis entitled "**Buckling analysis of functionally graded carbon nanotubes reinforced composite (FG-CNTRC) plate**" by **Mr. Md. Abdul Hussain** (212ME1275) has been carried out under my supervision for award of the degree of **Master of Technology** in Mechanical Engineering with **Machine Design and Analysis** specialization during session 2012 - 2014 in the Department of Mechanical Engineering, National Institute of Technology, Rourkela.

To the best of my knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.

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ACKNOWLEDGEMENT

My first thanks are to the almighty God, without whose blessings, I wouldn't have been writing this “acknowledgments". I am extremely fortunate to be involved in an exciting and challenging research project work on “**Buckling analysis of functionally graded carbon nanotubes reinforced composite (FG-CNTRC) plate**”. It has enriched my life, giving me an opportunity to work in a new environment of ANSYS. This project increased my thinking and understanding capability as I started the project from scratch.

I would like to express my greatest gratitude to my supervisor **Prof. S. K. Panda**, for his excellent guidance, valuable suggestions and endless support. He has not only been a wonderful supervisor but also an honest person. I consider myself extremely lucky to be able to work under guidance of such a dynamic personality. He is one of such genuine person for whom my words will not be enough to express.

I would like to express my sincere thanks to **Vishesh R. Kar, Vijay K. Singh, P.V. Katariya**, **Ayushman Dehingia** and all my classmates for their precious suggestions and encouragement to perform the project work. I am very much thankful to them for giving their valuable time for me.

Finally, I express my sincere gratitude to my parents for their constant encouragement and support at all phases of my life.

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ABSTRACT

In this work, buckling responses of functionally graded single-walled carbon nanotubes (SWCNT) reinforced composite plates with temperature dependent material properties are investigated. The effective material properties of the composite plates are obtained using simple rule of mixture by introducing the CNT efficiency parameter under different thermal environment. In the present analysis, a suitable finite element model of the SWCNT reinforced composite plate is developed using ANSYS parametric design language code in ANSYS environment using Block-Lancoz's method. An eight noded serendipity shell element (SHELL281) has been used for the discretisation of the developed simulation model from the ANSYS library. The buckling responses of the SWCNT composite plate have been obtained and verified with those of the available published results. The non-dimensional critical buckling load parameters under uniaxial compression, biaxial compression and biaxial compression and tension have been obtained by varying different parameters like, CNT volume fraction, temperature, thickness ratio and support conditions. Finally, the detailed parametric study has been carried out to reveal the influence of different design parameters on the buckling responses through the simulation study.

Keywords- Buckling, CNT, FGM, volume fraction, FEM

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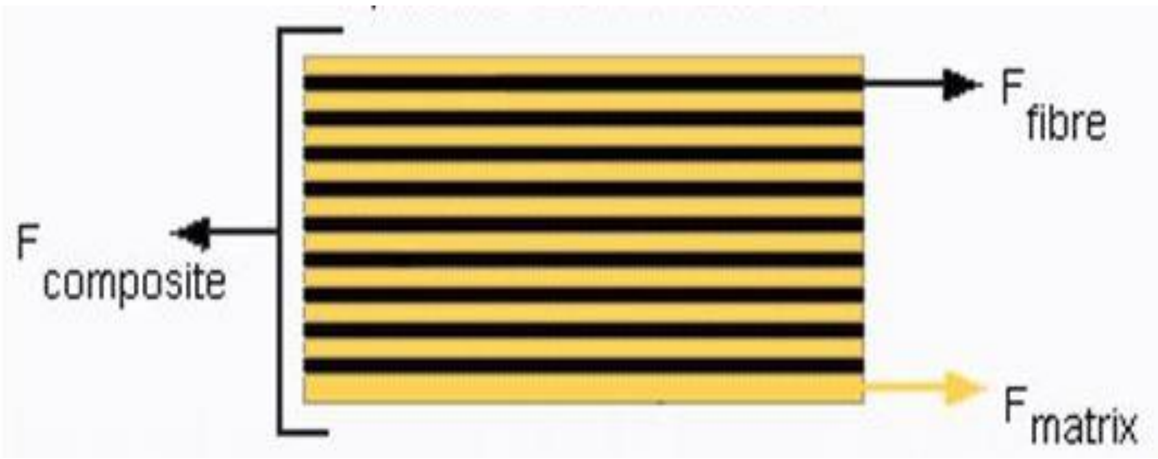
CHAPTER 1

INTRODUCTION

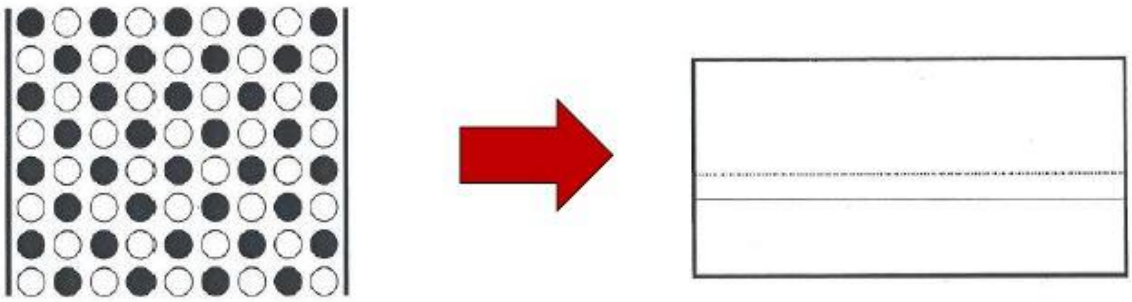
1.1 Overview

Composite materials are defined as combination of two or more materials on a microscopic scale. They are continuously use because of its better properties like stiffness, strength, low weight, corrosion resistance, thermal properties, fatigue life and wear resistance. Composites have two constituent elements namely, fiber and matrix. The fibers are used in modern composites because of its high specific mechanical properties compared to those of traditional bulk materials. Carbon and graphite are the common fiber materials used by many weight sensitive industries since last few decades. Matrix acts as a bonding element which protects fiber from external break or damage. The main function of matrix is to distribute and transfer the load to the fibers or reinforcements. Metal, ceramic and polymer are the commonly used material for matrix phase. Transformation of load depends on the bonding interface between the reinforcement and matrix. Bonding depends on the types of reinforcement and matrix and the fabrication technique.

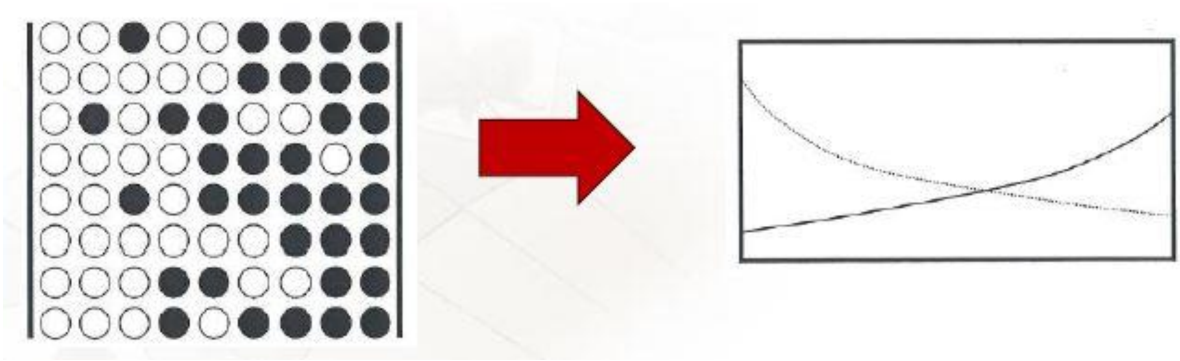
Functionally graded material (FGM) is a new kind of advanced composite material in which the constituents are gradually changed with respect to the spatial coordinate over the volume, resultant in consistent change in the properties of the material in Fig. 1. The overall properties of functionally graded material are exclusive and dissimilar from any of the individual materials that form it. Now-a-days wide range of FGM application are using in engineering field and FGM is predictable to rise as the cost of material fabrication and processing processes are reduce by improving these processes. In this study, an overview of fabrication processes area of application. Thus, material properties depend on the spatial position in the structure. The materials can be designed for specific function and applications. The properties that may be designed/controlled for desired functionality include chemical, mechanical, thermal, and electrical properties. Provide ability to control deformation, dynamic response, wear, corrosion, etc. and ability to design for different complex environments, provide ability to remove stress concentrations.



(a} Volume fraction of fibers



(b) Uniformly distributed material with properties variations



(c) Functionally graded material with properties variation

Fig. 1. Volume fraction of fiber and functionally graded material [42]

Buckling is characterized by instability of a structural member subjected to high compressive and/or tensile load. Buckling means the bending due to axial load or the effect in perpendicular direction to the cause.

Carbon nanotubes (CNTs) play very important role in engineering field. It is cylindrical macromolecules consisting of carbon atoms arranged in a periodic hexagonal structure and were invented by Sumio Iijima in 1991. CNT is continuously used in new field of research for the perfect analysis of nano size structure. CNT is used extensively as reinforcing materials at nano scale for developing new nanocomposites, because of its excellent mechanical, thermal and electrical properties**.** CNTs in polymer matrices can potentially enhance the stiffness and strength of composites significantly when compared to those reinforced with conventional carbon fibers. However, retaining these outstanding properties at macro scale poses a considerable challenge. It is well known that the CNTs have large Young’s modulus, yield strength, flexibility and conductivity properties. In addition to the above, they have strengths 20 times that of high strength steel alloys, half denser than aluminium and current carrying capacity is 10000 times that of the copper.

1.2 Types of CNTs

CNTs can be categorized as single walled carbon nanotube (SWNT) and multi walled carbon nanotube (MWNT). SWNTs are nanometer-diameter cylinders made up of a single rolled up graphene sheet to form a tube and MWCN consisting of multiple rolled up graphene sheet to form a tube in Fig. 2.

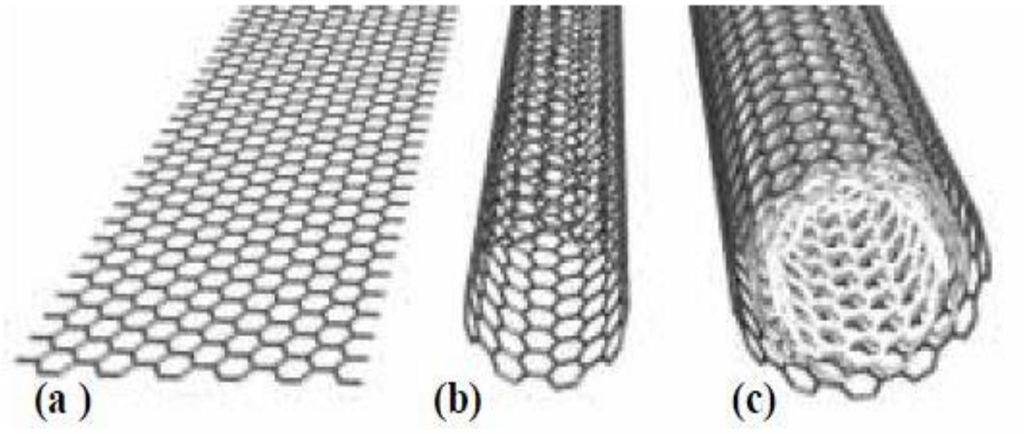
1.3 CNTs geometry

CNT have three unique geometrical arrangements of carbon atoms. These flavours can be categorized by how graphene sheet is wrapped into a tube form. Because of physical and mechanical properties of CNTs depending on its atomic arrangement, they are armchair, chiral, and zig-zag as shown in Fig. 3.

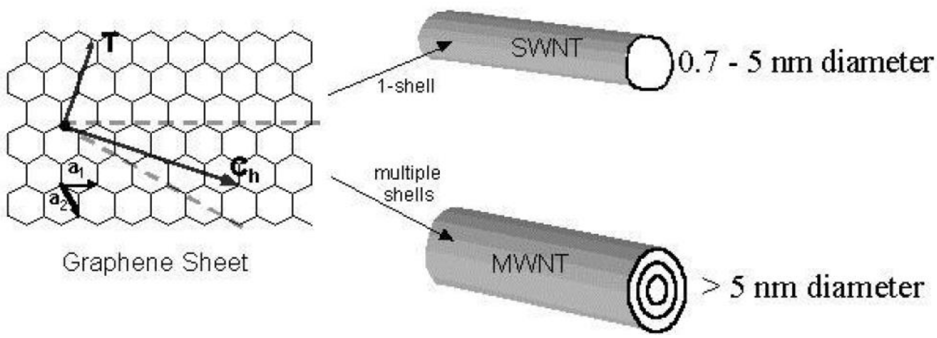
1.4 Applications of CNTs

CNTs have very usual mechanical, chemical, thermal, electronic and optical properties. Carbon nanotubes are promising to revolutionize in different fields such a nanotechnology and material science. CNTs have wide variety of unexplored potential applications in numerous technological fields such as automobile, aerospace, medicine, energy, or chemical industry, in which CNTs may be used as templates, actuators, gas absorbents, composite reinforcements, probes, catalyst supports, chemical sensors, nano reactors, nano pipes etc.

The key of using CNT based FGM is that one can obtain these properties as per the requirement just by varying the distribution and composition of CNT. That's how one can get directional properties and can control other parameters. Another advantage stated above is the stress concentration free material because the cross-section shows there are no layers inside the material and instead there is a continuous gradation of materials from top to bottom. So, there is no stress concentration and delamination of layers.

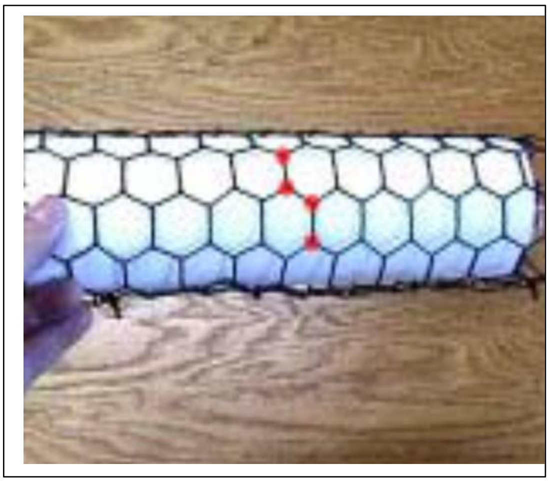


(a) A cut-out part of a graphene sheet. (b) A single walled CNTs. (c) A multi-walled CNTs

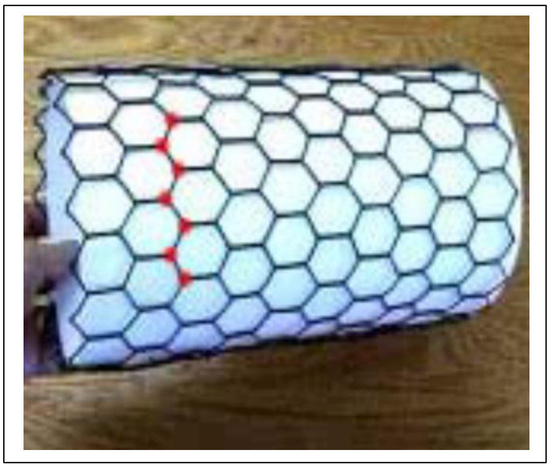


(d) Graphene sheets rolled into SWCNT and MWCNT

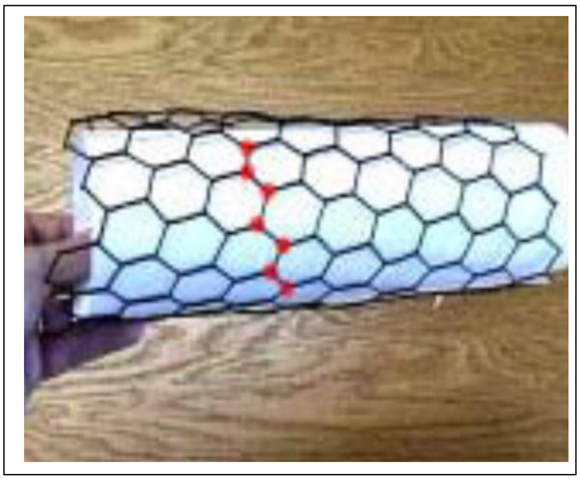
Fig. 2. Types of CNT [40]



(a) Arrangement of carbon atom for armchair



(b) Arrangement of carbon atom for zig-zag



(c) Arrangement of carbon atoms for chiral

Fig. 3. Arrangement of carbon nanotubes for armchair, zig-zag and chiral [31]

1.5 Motivation of the Present Work

The carbon nanotubes based composite plate provides excessive motivation to the engineering field because of its excellent mechanical, physical and thermal properties. CNT provides efficient size, shape, structure, strength to weight ratio, stiffness to weight ratio, better wear resistance and fatigue, good elevated temperature properties and CNT based composites having ability to fabricate directional mechanical properties and providing ability to control the deformation, dynamic response of the system, wear and corrosion of parts etc. In the recent few years, use of composite structures has increased a lot. Especially in aerospace/ aeronautical engineering which forced the engineers for its analysis. These structural components are subjected to various types of combined loading and exposed to elevated thermal environment during their service, which may lead to change in the shape of the geometry of structure. The changes in panel geometry and the interaction with loading condition affect the buckling responses greatly. The main aim of this present work is to increase the buckling load and control the instability of a structure.

1.6 Aim and Scope of the Present Thesis

The aim of this thesis is to develop a mathematical model for functionally graded single walled carbon nanotubes based composite plate under various load and environment temperature using the parametric language in ANSYS 13.0 environment and then evaluate its buckling effects subjected to compressive and tensile load alternately on its adjacent edges, based on the finite element method. A suitable finite element model is proposed and applied for the discretisation of the composite plate model. It also aims to obtain the effect of three types of FG-CNTRC (UD, FG-X and FG-V) and other geometrical parameters such as CNT volume fraction, thickness ratio, environment temperature, boundary conditions, uniaxial compression, biaxial compression and biaxial compression and tension on the buckling responses of the FG-CNTs based composite plate.

Chapter 2

LITERATURE SURVEY

Introduction

It focuses on study of developing a new nanocomposite material using carbon nanotubes because of its excellent mechanical properties, buckling, vibration, bending, analysis of SWCNTS and MWCNTs reinforced composite. It was found that the technique which was used for calculating the effective material properties of composite and method is used to find the mechanical behaviours.

Lei et al. [1] presented the buckling analysis of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) plates under various in-plane mechanical loads using first order shear deformation theory (FSDT) and calculate effective mechanical properties of nano composite using rule of mixture or Eshelby-Mori-Tanaka approach, optimised the variation in the buckling strength on composite plate with volume fraction, aspect ratio, loading conditions, width-to-thickness ratio and environment temperature. Han and Elliott [2] employed molecular dynamics (MD) and energy minimization simulation methods to examine the elastic properties of the CNT composites materials. Mehrabadi et al. [3] studied the mechanical buckling behaviour of a FG-CNTRCs rectangular plate using FSDT mid-plane kinematics. The authors utilized MD, Eshelby-Mori-Tanaka approach and the extended rule of mixture to evaluate the effect material properties of SWCNT. Zhu et al. [4] presented the vibration and bending analyses of FG-CNTRCs using finite element method based on FSDT. Shen and Zhang [5] examined the thermal buckling and postbuckling behaviour of FG-CNTRC using a micromechanical model. Shen [6] investigated the nonlinear bending of simply-supported FG-CNTRCs under thermo-mechanical loading using higher order shear deformation theory (HSDT) with von Karman nonlinearity. Alibeigloo and Liew [7] employed three dimensional theory of elasticity to obtain the bending responses of simply supported FG-CNTRCs rectangular plate subjected to thermo-mechanical loads. Fazzolari et al. [8] presented the buckling response of composite plate assemblies using HSDT and dynamic stiffness method. Ansari [9] studied the buckling behaviour of single-walled silicon carbide nanotubes using density functional theory. Neves et al. [10] investigated stability behaviour of isotropic and functionally graded sandwich plates in the framework of HSDT by using a messless technique. Murmu and Pradhan [11] examined the buckling behaviour of

SWCNTs embedded in elastic medium using Eringen's nonlocal elasticity theory and the Timoshenko beam theory. Popov et al. [12] evaluated the elastic properties of triangular close-packed crystal lattices of SWCNTs using analytical expressions based on a force-constant lattice dynamical model. Yas and Samadi [13] analysed the free vibrations and the buckling behaviour of FG-SWCNT resting on an elastic foundation using Timoshenko beam theory. Ayatollahi et al. [14] estimated the nonlinear mechanical properties of the zigzag and armchair SWNTs under axial, bending and torsional loading conditions using finite element based molecular mechanics steps. Chen and Liu [15] obtained effective mechanical properties of carbon nanotube based composite using a square representative volume element (RVE) based on continuum mechanics. Odegard et al. [16] developed a constitutive model for polymer composite systems reinforced with SWCNTs. Shen and Xiang [17] investigated the postbuckling of SWCNTs reinforced nanocomposite cylindrical shells under thermo-mechanical loading. The model has been developed based on HSDT shell theories with a von Karman type of nonlinearity kinematics. Thai [18] employed a nonlocal shear deformation beam theory to investigate the buckling, bending, and vibration of nanobeams. Guo et al. [19] employed an atomic scale finite element method (FEM) to analyse bending and buckling behaviour of SWCNTs. Zhang et al. [20] studied the buckling responses of CNTs using FEM. Mohammadimehr et al. [21] presented the buckling behaviour of double-walled carbon nanotubes embedded in an elastic medium under axial compression using non-local elasticity theory. Sears et al. [22] presented buckling of MWNTs and SWNTs, correspondingly under the axial compressive loads have been studied by MDs, and results compared with those from the analysis of equivalent continuum structures using the finite element method and Euler buckling theory. Vodenitcharova and Zhang [23] presented buckling and bending analysis of nano composite beam reinforced by SWCNTs, analysed the matrix deformation using Airy stress function method. Also it has been found that adding quantity of CNTs reinforced in matrix increased load carrying capacity of structure. Sun and Liew [24] studied a bending buckling behaviour test of SWCNTs using higher order gradient continuum and mesh free method. It also studied about various types of CNTs and the buckling mechanism. Yan et al. [25] investigated the buckling test of SWCNTs using a moving Kriging interpolation and the higher order Cauchy–Born rule to predict the mechanical response of SWCNTs. Giannopoulos et al. [26] studied the calculation for shear and Young's modulus of SWCNTs with the development of finite element formulation implemented for the computation of mechanical elastic response of zigzag and armchair SWCNT for an extensive range of value for nanotubes radius. Shima [27] presented a nonlinear mechanical bending and buckling response of CNTs based composite and studied the behaviour of CNTs under different load condition as compression, bending, tension torsion, and their combination. Lei et al. [28] investigated the vibration analysis of FG-SWCNT, using the element-free kp-Ritz method. SWCNT was reinforced into a matrix with various types of distribution. The material properties of FG-CNTRCs were assumed to be graded through the thickness direction according to several linear distributions of the volume fraction of carbon nanotubes and FSDT was used for governing equation. Rangel et al. [29] presented an analytical procedure to find out the elastic properties of SWCNTs of armchair type using finite element approach for mechanical modelling of a SWCNTs and it was found that mechanical properties of CNTs was outstanding. Simsek [30] presented forced vibration analysis of simply supported SWCNTs under the action of a moving harmonic load based on nonlocal elasticity theory. Grace [31] studied different types of CNTs like SWCNTs and MWCNTs and geometrical arrangement of carbon atom as armchair, chiral and zig-zag because of physical and mechanical properties of CNTs depending on its atomic arrangement. Lu, X., and Hu, Z., [32] studied computational simulation for predicting the mechanical properties of carbon nanotubes. It have been adopted as a powerful tool relative to the experimental difficulty. Based on molecular mechanics, an improved 3D finite element model for armchair, zigzag and chiral SWCNTs has been developed. Yu et al. [33] investigated the properties of carbon nanotubes based composite by precursor infiltration and pyrolysis process (PIP). The fiber and matrix interface coating has been arranged through chemical vapormdeposition (CVD) process using methyltrichlorosilane (MTS). An effect of the CNTs on mechanical and thermal properties of the composite has been estimated by three-point single edge notched beam test, bending test, and laser flash method. Yeetsorn [34] studied about the carbon nanotubes as an advanced composite material in the form of CNTs like armchair and zigzag. They found out CNTs have excellent mechanical, thermal, and electrical properties. CNT was developed through different technique like laser ablation, arc discharge and chemical vapour deposition. Formica et al. [35] studied of the vibrational property of CNTRC by using an equivalent continuum model based on the Eshelby–Mori–Tanaka method. Odegard et al. [36] discussed representative volume element (RVE) based on continuum mechanism for developing structural properties relationship of nano structure material. Volcov et al. [37] discussed effect of bending and buckling analysis of carbon CNTs on thermal conductivity of carbon nanotubes materials was studied in mesoscopic and atomistic simulations. Ma et al. [38] presented the dispersion, surface, interfacial properties of carbon nanotubes, and the mechanical properties of the CNTs based composite affected by CNTs functionalization were studied. Liu and Chen [39] discussed about CNTs having very high strength, resilience and stiffness and one of the best reinforcement material for the growth of a new nanocomposite. In this work the effective mechanical properties of CNTs based composites were estimated using a three dimensional nanoscale RVE based on continuum mechanism and using the FEM. The effective Young's moduli in the axial direction of the RVE have been found through extended rule of mixtures. Kreupl et al. [40] discussed about inter connected application of carbon nanotubes.

Based on the above literature, it is clear that many attempts have been made to study the mechanical buckling behaviour of FG-CNTRC but the studies with temperature dependent material properties were very rare. Hence, the authors' aim is to analyse the mechanical buckling of uniformly distributed (UD) and FG-CNTRC with temperature dependent material properties. A simulation model is developed using ANSYS parametric design language (APDL) in ANSYS environment.

Chapter 3

GENERAL MATHEMATICAL FORMULATION

3.1 ANSYS element SHELL 281 formulation for buckling

The element SHELL 281 is working for the buckling analysis. It is 8 noded linear shell elements with 6 degrees of freedom on every node. Shown in the Fig. 4 is selected from the element library of ANSYS 13.0 element library. Those are three translations along x, y, z direction and three rotations about x, y, z axis. It is fine suitable for linear, large rotation or large strain nonlinear applications, x, y, z axis per node. It's working on FSDT. The elements formulation is considered on true stress and logarithmic strain measures. Fig. (4) Shows the knowledge about the SHELL 281 element. The details of the element can be seen in reference [41].

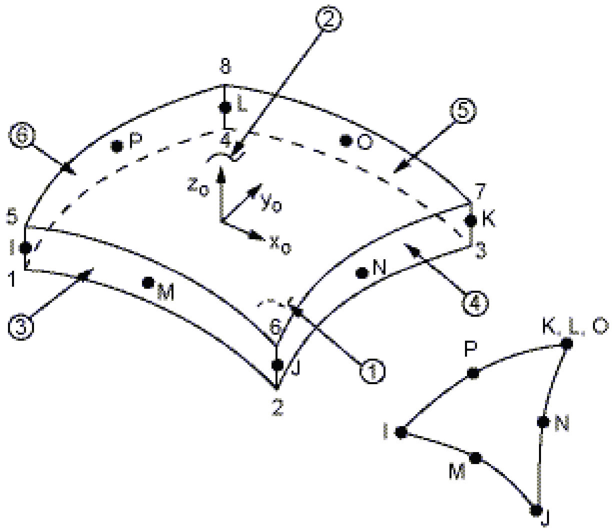


Fig. 4. shell 281 element description [41]

x = Element x-axis if element orientation is providing.

x_0 = Element x-axis if element orientation is not providing.

This is well known that mid plane kinematics of carbon nanotubes based composite has been taken as the FSDT using the inbuilt steps in ANSYS and conceded as follows:

$$
\begin{aligned}
u(x,y,z) &= u_o(x,y) + z\psi_x(x,y) \\
v(x,y,z) &= v_o(x,y) + z\psi_y(x,y) \\
w(x,y,z) &= w_o(x,y) + z\psi_z(x,y)
\end{aligned}
\tag{3.1}
$$

where, *u*, *v* and *w* represent the displacement u_0, v_0 and w_0 are mid plane displacement in x, y, z axes respectively and ψ_x, ψ_y are the rotations of the normal to the mid plane about *x* and *y* axes respectively and ψ_z is the higher order terms in Taylor's series expansion [3.1].

The displacements *u*, *v* and *w* can be expressed in terms of shape functions (N_i) as:

$$\delta = \sum_{i=1}^{j} N_i \delta_i \tag{3.2}$$

where

$$\delta_i = \left(u_{oi}\; v_{oi}\; w_{oi}\; \psi_{xi}\; \psi_{yi}\; \psi_{zi}\right)^T$$

In Eqn. (3.3) the shape functions for eight noded shell elements (j=8) are represented in natural (η- ξ) coordinates, and explanations of the elements are certain as:

$$
\begin{aligned}
N_1 &= \frac{1}{4}(1-\xi)(1-\eta)(-\xi-\eta-1), & N_5 &= \frac{1}{2}(1-\xi^2)(1-\eta), \\
N_2 &= \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1), & N_6 &= \frac{1}{2}(1+\xi)(1-\eta^2), \\
N_3 &= \frac{1}{4}(1+\xi)(1+\eta)(1+\xi-\eta), & N_7 &= \frac{1}{2}(1-\xi^2)(1+\eta), \\
N_4 &= \frac{1}{4}(1-\xi)(1+\eta)(-\xi+\eta-1), & N_8 &= \frac{1}{4}(1-\xi)(1-\eta^2),
\end{aligned}
\tag{3.3}
$$

Strains are obtained by derivation of displacements as:

$$\{\varepsilon\} = \{u,_x \quad v,_y \quad w,_z \quad u,_y + v,_x \quad v,_z + w,_y \quad w,_x + u,_z\} \tag{3.4}$$

where, $\{\varepsilon\} = \{\varepsilon_x,\ \varepsilon_y,\ \varepsilon_z,\ \gamma_{xy},\ \gamma_{yz},\ \gamma_{zx}\}^T$ is the strain matrix containing normal and shear strain components of the mid-plane in in-plane and out of plane direction.

The strain components are rearranged by using the following steps by in plane and out of plane sets.

The in-plane strain vector:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{3.5}$$

The transverse strain vector:

$$\begin{Bmatrix} \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{zo} \\ \gamma_{yzo} \\ \gamma_{xzo} \end{Bmatrix} + z \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} \tag{3.6}$$

where, the deformation components are described as:

$$\begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial x} \\ \dfrac{\partial v_o}{\partial y} \\ \dfrac{\partial u_o}{\partial y} + \dfrac{\partial v_o}{\partial x} \end{Bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \psi_x}{\partial x} \\ \dfrac{\partial \psi_y}{\partial y} \\ \dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x} \end{Bmatrix} \tag{3.7}$$

$$\begin{Bmatrix} \varepsilon_{zo} \\ \gamma_{yzo} \\ \gamma_{xzo} \end{Bmatrix} = \begin{Bmatrix} \psi_z \\ \dfrac{\partial w_o}{\partial y} + \psi_y \\ \dfrac{\partial w_o}{\partial x} + \psi_x \end{Bmatrix} \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \dfrac{\partial \psi_z}{\partial y} \\ \dfrac{\partial \psi_z}{\partial x} \end{Bmatrix} \tag{3.8}$$

The strain vector expression in term of nodal displacement vector is given as:

$$\{\varepsilon\} = [B]\{\delta\} \tag{3.9}$$

where, [B] is the strain displacement matrix containing interpolation function and derivative operator and { δ } is the nodal displacement vector.

The generalized stress-strain relationship with respect to its reference plane is expressed as:

$$\{\sigma\} = [D]\{\varepsilon\} \tag{3.10}$$

where, { σ } and {ε} is the linear stress and linear strain vector correspondingly and [D] is the rigidity matrix.

The element stiffness matrix [K] is integrated by using Gauss-quadrature integration over the domain to obtain the global stiffness and mass matrices and this can be expressed as:

$$[K] = \int_{-1}^{+1}\int_{-1}^{+1} [B]^T [D] [B]|J| d\zeta d\eta \tag{3.11}$$

where, $|J|$ is the determinant of the Jacobian matrix. The jacobian is used to map the domain from natural coordinate to the general coordinate.

The nodals displacement can be presented in the term of their shape functions and their consistent nodal values as follows,

$$u^o = \sum_{i=1}^{8} N_i u_i^o, v^o = \sum_{i=1}^{8} N_i v_i^o, w^o = \sum_{i=1}^{8} N_i w_i^o,$$
$$\psi_x = \sum_{i=1}^{8} N_i \psi_{xi}^o, \psi_y = \sum_{i=1}^{8} N_i \psi_{yi}^o, \psi_z = \sum_{i=1}^{8} N_i \psi_{zi}^o \tag{3.12}$$

The above equation can be rewritten above equation in *i*th nodal displacement as follows:

$$\left\{\delta_i^*\right\} = \left\{u_i^o \quad v_i^o \quad w_i^o \quad \psi_{xi}{}^o \quad \psi_{yi}{}^o \quad \psi_{wi}{}^o\right\} \tag{3.13}$$

$$\left\{\delta_i^*\right\} = \left[N_i\right]\left\{\delta_i\right\} \tag{3.14}$$

where $$\left[N_i\right] = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & 0 & N_i \end{bmatrix} \tag{3.15}$$

Strain energy expression obtained by substituting the value of nodal displacement in expression, is given by:

$$\left\{\varepsilon_i\right\} = \left[B_i\right]\left\{\delta_i\right\} \tag{3.16}$$

$$U = \frac{1}{2}\int [B_i]^T \left\{\delta_i\right\}^T [D][B_i]\left\{\delta_i\right\} dA - \left\{F\right\}_{mi} \tag{3.17}$$

where, $[B_i]$ is the strain displacement relation matrix and $\{F\}_m$ is the mechanical force, respectively.

The final expression of the equation obtained by minimizing the total potential energy (TPE) as follows:

$$\partial\Pi = 0$$

where, Π is the total potential energy.

$$[K]\{\delta\} = \{F\}_m \tag{3.18}$$

where, [K] is the global mass and linear stiffness matrix.

The eigenvalue type of buckling equation can be expressed as in the following steps by dropping force terms and conceded to

$$\{[K] + \lambda[K_g]\}\{\delta\} = 0 \tag{3.19}$$

where, $[K_g]$ and λ are the geometric stiffness matrix and critical mechanical load at which the structure buckling start.

3.2 Calculate for effective material properties of FG-CNTRC Plate.

A square FG-CNTRC plates of thickness *h*, length *a*, and width *b* is taken in the present work. The effective material properties of the nanocomposites are mixture of CNT and matrix can be finding according to the rule of mixture [1].

$$E_{11} = \eta_1 V_{CNTs} E_{11}^{CNTs} + V_m E^m \tag{3.20}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CNTs}}{E_{22}^{CNTs}} + \frac{V_m}{E_{12}^{CNTs}} + \frac{V_m}{E^m} \tag{3.21}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CNTs}}{G_{12}^{CNTs}} + \frac{V_m}{G^m} \tag{3.22}$$

where, E_{11}^{CNTs}, E_{22}^{CNTs} and G_{12}^{CNTs} are the elastic constants of SWCNT and E^m, G^m are characterize the elastic properties of the matrix. η_1, η_2 and η_3 are the CNT effective parameters and it can be calculated by matching the effective material properties of FG-CNTRC found from the rule of mixture and molecular dynamic simulation. V_{CNTs} and V_m are the volume fraction of the carbon nanotube and volume fraction of matrix. E_{11} and E_{22} are the effective Young's modulus of carbon nanotubes reinforced composite plates in the principal material coordinates, G_{12}, G_{13} and G_{23} are the shear modulus, ν_{12} and ν_{21} are Poisson's ratios and α_{11} and α_{22} are thermal expansion coefficients.

A uniform and two types of functionally graded distributions of the CNTs along the thickness direction of the nanocomposite plates,

$$\begin{aligned}
&V_{CNTs} = V^*_{CNTs} && (UD\,CNTRC)\\
&V_{CNTs}(z) = 2\left(1+\frac{2z}{h}\right)V^*_{CNTs} && (FG-V\,CNTRC)\\
&V_{CNTs}(z) = 2\left(\frac{2|z|}{h}\right)V^*_{CNTs} && (FG-X\,CNTRC)
\end{aligned} \tag{3.23}$$

where

$$V^*_{CNTs} = \frac{W_{CNTs}}{W_{CNTs} + (\rho^{CNTs}/\rho^m) - (\rho^{CNTs}/\rho^m)W_{CNTs}} \tag{3.24}$$

where, W_{CNTs} is the mass fraction of the CNT in the composite plate, and ρ^{CNTs} , ρ^m are the densities of the carbon nanotubes and matrix, respectively.

Relation between the CNT and matrix volume fractions is shown as

$$V_{CNTs} + V_m = 1 \tag{3.25}$$

$$\nu_{12} = (\mathrm{V}^*_{CNTs}\,\nu_{12}^{CNTs} + V_m \nu^m) \tag{3.26}$$

$$\rho = V_{CNTs}\rho^{CNTs} + V_m \rho^m \tag{3.27}$$

$$\alpha_{11} = V_{CNTs}\alpha_{11}^{CNTs} + V_m \alpha^m \tag{3.28}$$

$$\alpha_{22} = (1+\nu_{12}^{CNTs})V_{CNTs}\alpha_{22}^{CNTs} + (1+\nu^m)\,\mathrm{V_m}\,\alpha^m - \nu_{12}\alpha_{11} \tag{3.29}$$

where, ν_{12}^{CNTs} and ν^m are Poisson's ratio of CNT and matrix, respectively and, α_{11}^{CNTs}, α_{22}^{CNTs} and α^m are the thermal expansion coefficients of the CNT and matrix. Similarly, ν_{12} is constant over the thickness of the FG-CNTRC plate in Fig. 5

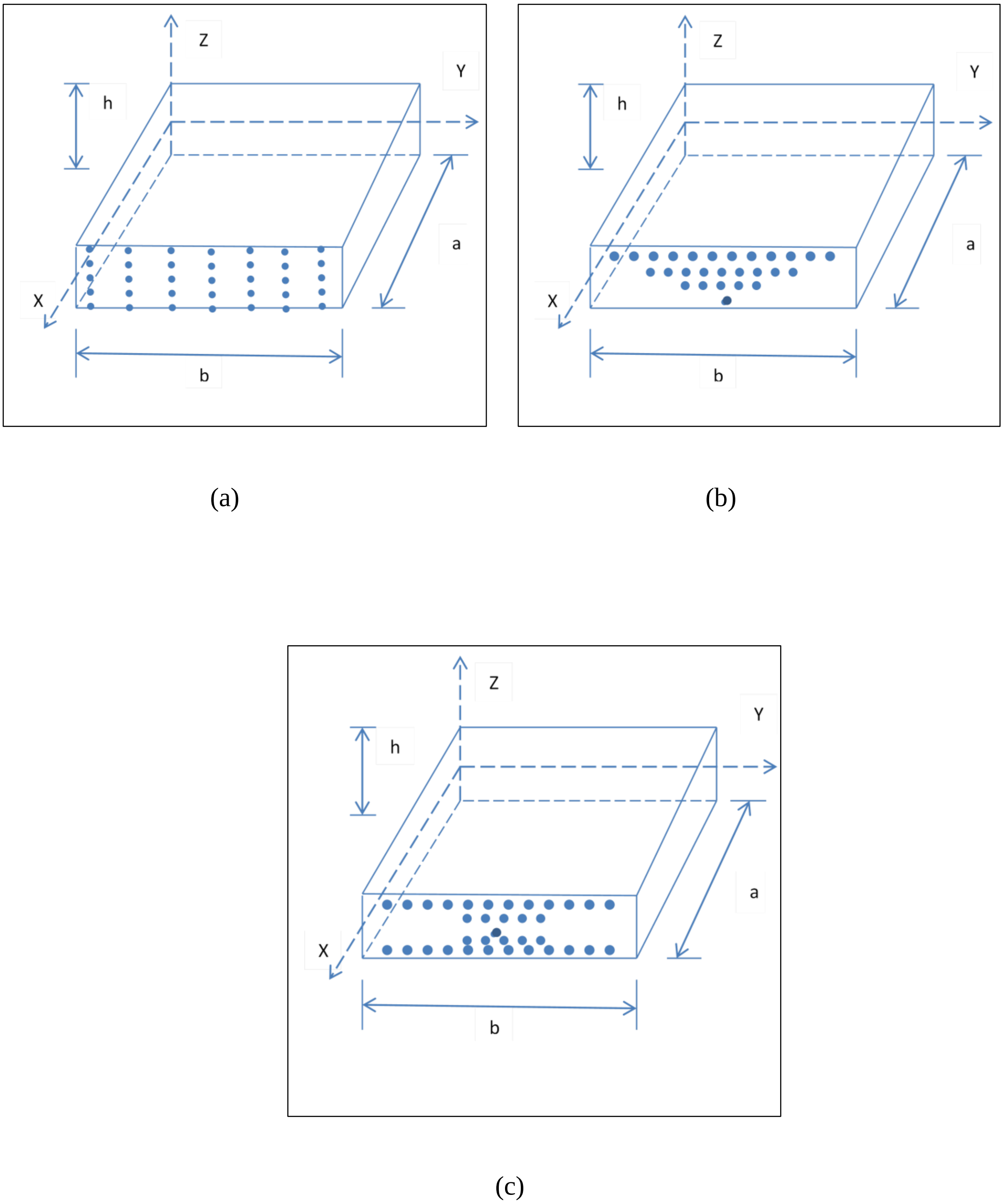


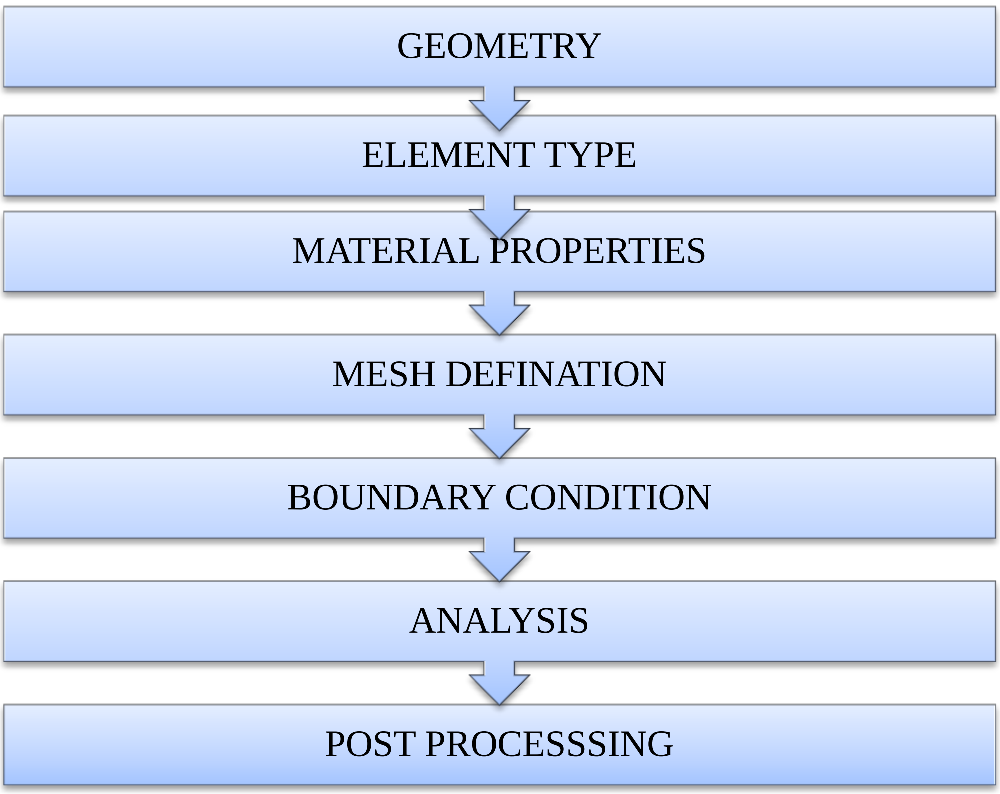
Fig. 5. Model of the FG-CNTRCs plates. (a) UD CNTRC plate (b) FG-V CNTRC plate and (c) FG-X CNTRC plate.

3.3 ANSYS modelling of FG-CNTC composites

The Finite Element Analysis (FEA) is a numerical technique for solving problems of mathematical physics and engineering. It is useful for problems with complicated geometries, material properties and loadings where analytical solutions cannot be found. Some important applications of FEA are aerospace/mechanical/civil engineering, structural/stress analysis, heat transfer, fluid flow, electromagnetic Fields and biomechanics

The finite element simulation has been prepared by commercial FEA package ANSYS 13. It is well known that, ANSYS software solve for the combined effects of multiple forces, accurately modelling combined behaviour resulting from "metaphysics interaction". The buckling responses obtained of FG-CNTRC using the effective material properties through the ANSYS parameter design language (APDL) code. ANSYS is used to make the modelling and calculate the effective material properties of the FG-CNTRC.

3.4 A layout of modelling procedure in ANSYS



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Chapter 4

NUMERICAL RESULT AND DISCUSSION

4.1 Material and geometrical parameters

In this section, the mechanical buckling behaviour of FG-CNTRC plates is examined. The material properties of matrix material are ν^m = 0.34, ρ^m = 1.15 g/cm^3 and E^m = 2.1GPa at environment temperature (300^{o}K). In this present study, the armchair (10, 10) type SWCNT is considered. The materials properties of the SWCNT are listed in Table 1 which is evaluated based on MD simulation and taken from reference [6]. In order to obtain the effective material properties of the CNTRC plate, CNT efficiency parameters η_j are given in Table 2.

Table 1: Temperature dependent materials properties of (10, 10) SWCNT (R = 0.68 nm, L = 9.26 nm, h = 0.067 nm, $\nu_{12}^{CNTs} = 0.175$)[6]

Temperature (^{o}K)	$E_{11}^{CNTs}(TPa)$	$E_{22}^{CNTs}(TPa)$	$G_{12}^{CNTs}(TPa)$
300	5.6466	7.0800	1.9445
500	5.5308	6.9348	1.9643
700	5.4744	6.8641	1.9644

Table 2: CNT efficiency parameters for different volume fractions [6].

V^*_{CNTs}	η_1	η_2
0.11	0.149	0.934
0.14	0.150	0.941
0.17	0.149	1.381

A square plate with dimension thickness '*h*', length '*a*' and width '*b*' is considered throughout the analysis. Three different types of support condition namely, simply-supported (S), Clamped (C) and Free (F) are considered individually and/or in combination to avoid rigid motion and to reduce the unknown field variables. The following support conditions are as follows:

SSSS: All the edges are simply-supported

$v_0 = w_0 = \psi_y = \psi_z = 0$ at x=0,a

$u_0 = w_0 = \psi_x = \psi_z = 0$ at y=0,b

SCSC: Two opposite edges are simply-supported and two are clamped.

$v_0 = w_0 = \psi_y = \psi_z = 0$ at x=0,a

$u_0 = v_0 = w_0 = \psi_x = \psi_y = \psi_z = 0$ at y=0,b

SSSF: Three edges are simply-supported and one is free.

$v_0 = w_0 = \psi_y = \psi_z = 0$ at x=0,a

$u_0 = w_0 = \psi_x = \psi_z = 0$ at y=0

The results are obtained for different load condition such as uniaxial compression (γ_1=-1, γ_2=0), biaxial compression (γ_1=-1, γ_2=-1) and biaxial compression (γ_1=-1, γ_2=1) as shown in Fig (6).

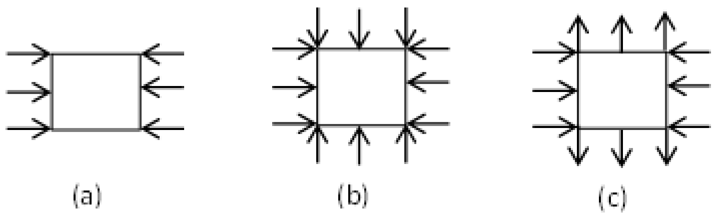

Fig. 6. (a) Uniaxial compression (γ_1=-1, γ_2=0), (b) Biaxial compression (γ_1=-1, γ_2=-1) and (c) Biaxial compression (γ_1=-1, γ_2=1)

4.2 Convergence and validation

In order to show the effectiveness of the present model, the convergence and validation study is performed for UD-CNTRC plate. Fig. 7 and 8 show the buckling load parameter $\left(\overline{N} = N_{cr}b^2/E_m h^3\right)$ of a simply supported UD FG-CNTRC (V^*_{CNT} =0.11) plate (*b/h*=10) under uniaxial compression (γ_1=-1, γ_2=0) and biaxial compression (γ_1=-1, γ_2=-1), respectively for different mesh size. It is clear from the figures that the present responses are converging well at a (21×21) mesh and showing good agreement with the published literature [1].

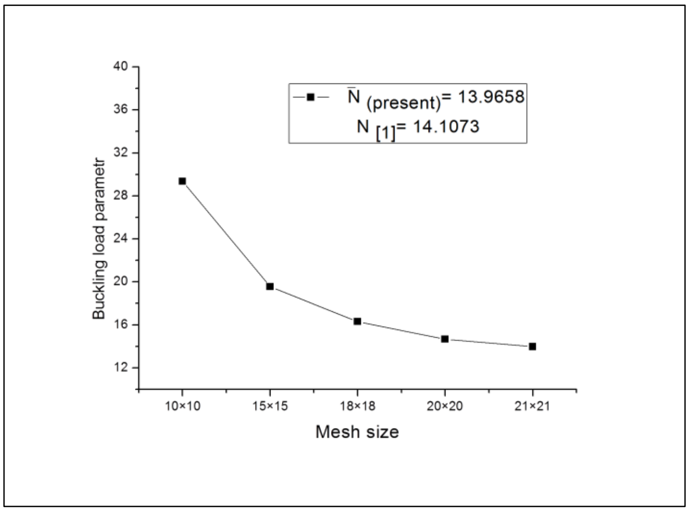

Fig. 7. Variation of the buckling load parameter of simply-supported UD CNTRC plates under uniaxial compression (γ_1=-1, γ_2=0) for different mesh size.

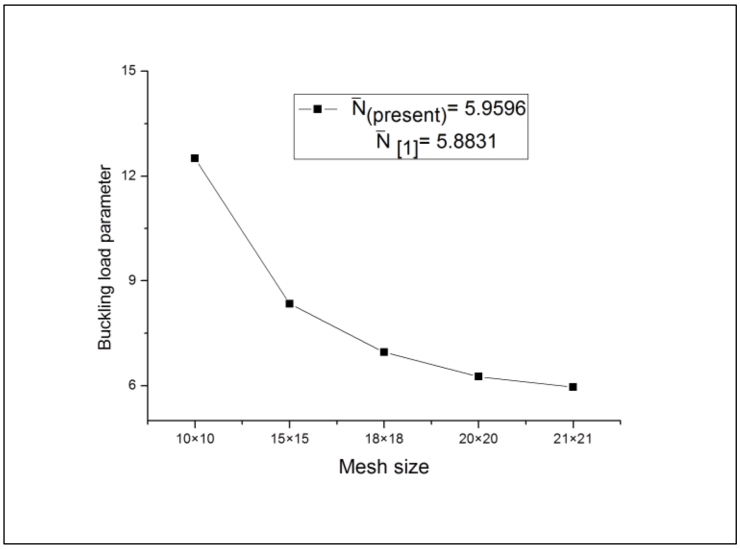

Fig. 8. Variation of the buckling load parameter of simply-supported UD CNTRC plates under biaxial compression (γ_1=-1, γ_2=-1) for different mesh size.

4.3 Numerical illustrations

In order to show the robustness of present model, the effects of different parameters like CNT volume fractions, loading conditions, thickness ratios and boundary conditions on the non-dimensional buckling load parameters of FG-CNTRC plate.

Table 3-5 exhibit the buckling behaviour of FG-CNTRC plates (b/h=10, T=300^{o}K) for three different CNT volume fractions (V^*_{CNT} =0.11, 0.14 and 0.17) and three different support conditions (SSSS, SCSC and SSSF) under uniaxial compression (γ_1=-1, γ_2=0), biaxial compression (γ_1=-1, γ_2=-1) and biaxial compression and tension (γ_1=-1, γ_2=1), respectively. It is observed from the stated tables that the buckling load parameter increases with the increase in CNT volume fractions and number of support constraints. It is interesting to note that, buckling load parameter is found maximum in FG-X type CNTRC plate whereas minimum in case of FG-V type CNTRC plate. It is also noted that, plate under biaxial compression and tension is having maximum buckling load parameter whereas plate under biaxial compression exhibits minimum buckling load parameter.

Table 6-8 exhibit the buckling behaviour of FG-CNTRC plates (b/h=10, V^*_{CNT} =0.11) for three different temperature field (300, 500 and 700^{o}K) and three different support conditions (SSSS, SCSC and SSSF) under uniaxial compression (γ_1=-1, γ_2=0), biaxial compression (γ_1=-1, γ_2=-1) and biaxial compression and tension (γ_1=-1, γ_2=1), respectively. It is observed that the buckling load parameter decreases with the increase in temperature values and increases with increase in number of support constraints. Again, the buckling load parameter is found maximum in FG-X type CNTRC plate whereas minimum in case of FG-V type CNTRC plate. It is also noted that, plate under biaxial compression and tension is having maximum buckling load parameter whereas plate under biaxial compression exhibits minimum buckling load parameter.

Table 3: The buckling load parameter $\left(\overline{N} = N_{cr}b^2/E_m h^3\right)$ of a support condition FG-CNTRC (b/h=10) plate under uniaxial compression (γ_1=-1, γ_2=0) is presented.

V^*_{CNT}	Types of FG	Boundary conditions		
		SSSS	SCSC	SSSF
0.11	UD	13.9658	17.5666	13.1676
	FG-V	12.9345	17.2654	11.2299
	FG-X	16.5819	21.7333	14.6376
0.14	UD	14.8509	18.4542	14.1816
	FG-V	13.7902	18.7452	12.10618
	FG-X	18.1138	22.5259	16.6576
0.17	UD	22.0602	27.8331	20.7519
	FG-V	20.5347	28.3023	17.7842
	FG-X	24.5714	31.4976	22.2354

Table 4: The buckling load parameter $\left(\overline{N} = N_{cr}b^2/E_m h^3\right)$ of a support condition FG-CNTRC (b/h=10) plate under biaxial compression (γ_1=-1, γ_2=-1) is presented.

V^*_{CNT}	Types of FG	Boundary conditions		
		SSSS	SCSC	SSSF
0.11	UD	5.5996	6.7102	3.4223
	FG-V	5.1241	6.2543	3.2542
	FG-X	6.0266	8.3209	3.7947
0.14	UD	6.2688	7.0642	4.258
	FG-V	6.2142	6.9841	3.9674
	FG-X	7.2431	8.6269	4.0769
0.17	UD	9.4307	10.6321	6.3928
	FG-V	8.2437	9.3452	5.9654
	FG-X	10.0122	12.3745	5.9281

Table 5: The buckling load parameter $\left(\overline{N} = N_{cr}b^2 / E_m h^3\right)$ of a support condition FG-CNTRC (b/h=10) plate under biaxial compression and tension (γ_1=-1, γ_2=1) is presented.

V^*_{CNT}	Types of FG	Boundary conditions		
		SSSS	SCSC	SSSF
0.11	UD	27.0357	29.9595	24.9119
	FG-V	26.2431	28.4235	23.9373
	FG-X	29.8785	30.2261	25.0421
0.14	UD	28.2952	31.2881	25.6238
	FG-V	27.0983	29.0123	25.0121
	FG-X	30.8928	31.6499	27.8857
0.17	UD	42.8285	47.5047	39.5928
	FG-V	41.5469	44.8712	38.4253
	FG-X	44.4833	48.3552	38.4547

Table 6: The buckling load parameter $\left(\overline{N} = N_{cr}b^2 / E_m h^3\right)$ of a support condition FG-CNTRC (V^*_{CNT} =0.11) plate under uniaxial compression (γ_1=-1, γ_2=0) with temperature differences is presented.

Temperature	Types of FG	Boundary conditions		
		SSSS	SCSC	SSSC
300^oK	UD	13.9658	17.5666	13.1676
	FG-V	12.9345	17.2654	11.2299
	FG-X	16.5819	21.7333	14.6376
500^oK	UD	8.123	17.548	7.8673
	FG-V	7.5092	10.0245	6.68761
	FG-X	10.9954	12.7326	9.6981
700^oK	UD	1.4815	1.9312	1.4744
	FG-V	1.3981	2.0014	1.2821
	FG-X	2.8645	4.6988	2.2461

Table 7: The buckling load parameter $\left(\overline{N} = N_{cr}b^2 / E_m h^3\right)$ of a support condition FG-CNTRC (V^*_{CNT} =0.11) plate under biaxial compression (γ_1=-1, γ_2=-1) with temperature differences is presented.

Temperature	Types of FG	Boundary conditions		
		SSSS	SCSC	SSSC
300°K	UD	5.5996	6.7102	3.4223
	FG-V	5.1241	6.2543	3.2542
	FG-X	6.0266	8.3209	3.7947
500°K	UD	3.3709	3.7897	2.2919
	FG-V	3.0132	3.3249	2.1234
	FG-X	3.5259	5.5202	2.0893
700°K	UD	0.6573	0.7375	0.2171
	FG-V	0.6267	0.7199	0.2012
	FG-X	0.7538	1.0351	0.4543

Table 8: The buckling load parameter $\left(\overline{N} = N_{cr}b^2 / E_m h^3\right)$ of a support condition FG-CNTRC (V^*_{CNT} =0.11) plate under biaxial compression and tension (γ_1=-1, γ_2=1) with temperature differences is presented.

Temperature	Types of FG	Boundary conditions		
		SSSS	SCSC	SSSC
300°K	UD	27.0357	29.9595	24.9119
	FG-V	26.2431	28.4235	23.9373
	FG-X	29.8785	30.2261	25.0421
500°K	UD	15.0714	16.6104	13.2423
	FG-V	14.6431	15.6342	13.0012
	FG-X	17.8757	19.5528	15.6195
700°K	UD	2.8857	2.9707	2.2879
	FG-V	2.5243	2.6543	2.3671
	FG-X	3.5583	3.7259	3.2219

Fig. 9-11 exhibit the effect of buckling behaviour of FG-CNTRC plates (b/h=10, V^*_{CNT} =0.14) for three different environment temperature field (300, 500 and 700^{o}K) and three different support conditions (SSSS, SCSC and SSSF) under uniaxial compression, biaxial compression and biaxial compression and tension, respectively. It is observed that the buckling load parameter decreases with the increase in temperature values and increases with increase in number of support constraints. It is found that the buckling load parameter is found maximum in FG-X type CNTRC plate whereas minimum in case of FG-V type CNTRC plate. It is also noted that, plate under biaxial compression and tension is having maximum buckling load parameter whereas plate under biaxial compression exhibits minimum buckling load parameter.

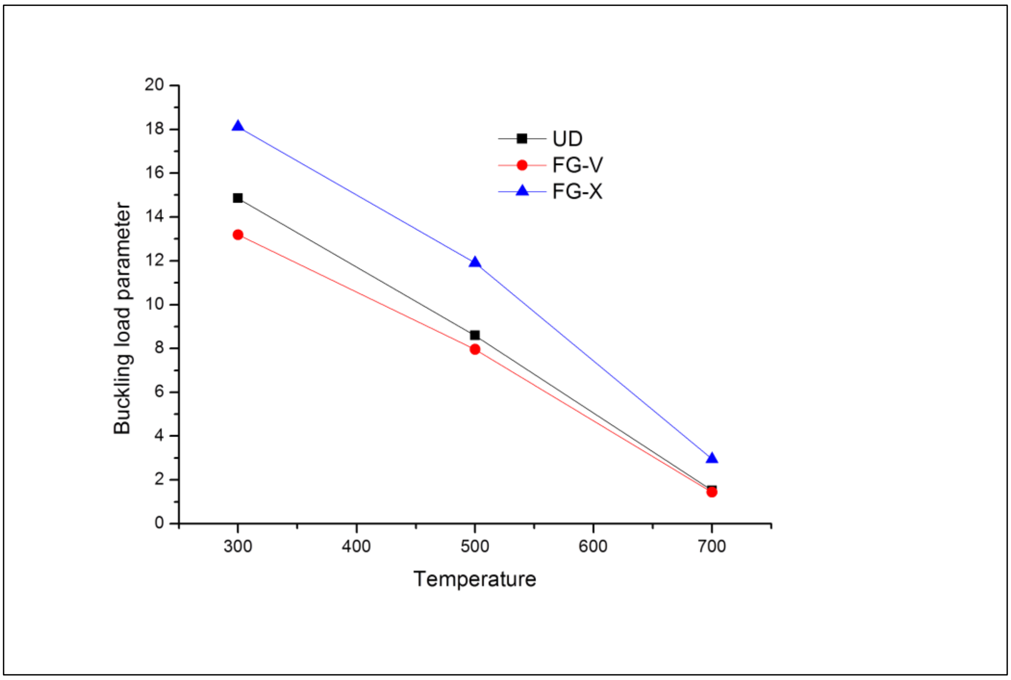

Fig. 9. Effect on the buckling load parameter of SSSS boundary condition three different types of CNTRC plate verses environment temperature under uniaxial compression.

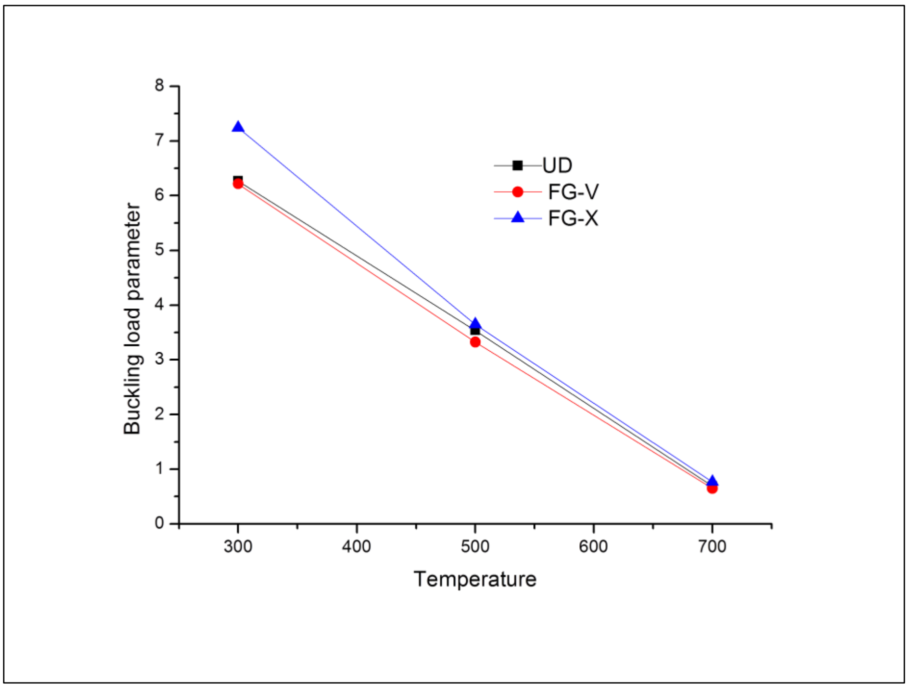

Fig. 10. Effect on the buckling load parameter of SSSS boundary condition three different types of CNTRC plate verses environment temperature under biaxial compression.

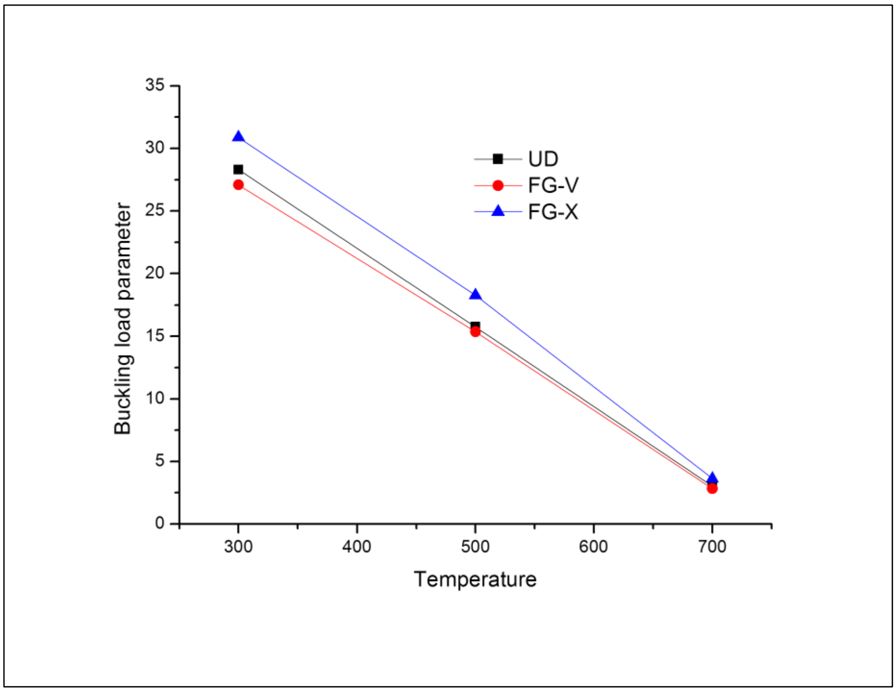

Fig. 11. Effect on the buckling load parameter of SSSS boundary condition three various types of CNTRC plates verses environment temperature under biaxial compression and tension.

Chapter 5

CONCLUSION

In this study, the buckling behaviour of SWCNT composite plate has been investigated. The effective material properties of FG-CNTRC are evaluated by using rule of mixture for different distribution type (UD, FG-X and FG-V). Finite element solutions are obtained in ANSYS 13.0 environment by using Block-Lancoz's method. The present model is validated through the comparison with those available in the literature. Some new numerical experimentation for different volume fractions, boundary conditions, temperature and loading conditions are illustrated. Based on the parametric study on the buckling behaviour of SWCNT composite plate, some points are concluded:

- The buckling load parameter increases with the increase in CNT volume fractions.
- As the number of support constraints increase, the buckling load parameter increases.
- The increment in temperature field results decrease in the buckling load parameters because as the temperature value increases the stiffness of the composite plate reduces.
- It is also found that, buckling load parameter is found maximum in FG-X type CNTRC plate whereas minimum in case of FG-V type CNTRC plate.
- The effect of loading conditions on buckling behaviour of SWCNT composite plate is found critical i.e., plate under biaxial compression and tension is having maximum buckling load parameter whereas plate under biaxial compression exhibits minimum buckling load parameter.

Future work

- Buckling analysis of functionally graded multi-walled carbon nanotubes plates can be performed.
- The effective material properties of CNT based composite material can be evaluated through different material modal such as Mori-Tanaka approach, molecular dynamics simulation, etc.
- An experimental study can be performed on CNT based composite plates for buckling analysis.

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