VIBRATION ANALYSIS OF VISCOELASTIC SANDWICH BEAM USING FINITE ELEMENT METHOD

A thesis submitted in the partial fulfilment

of the requirements for the degree of

Master of Technology

in

Mechanical Engineering

(Specialization: Machine Design and Analysis)

Submitted by

TATAPUDI NAVEEN KUMAR

(Roll No: 212ME1269)

DEPARTMENT OF MECHANICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA-769008, INDIA.

MAY-2014

VIBRATION ANALYSIS OF VISCOELASTIC SANDWICH BEAM USING FINITE ELEMENT METHOD

A thesis submitted in the partial fulfilment

of the requirements for the degree of

Master of Technology

in

Mechanical Engineering

(Specialization: Machine Design and Analysis)

Submitted by

TATAPUDI NAVEEN KUMAR

(Roll No: 212ME1269)

Under the guidance of

Prof. R.K.BEHERA

DEPARTMENT OF MECHANICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA-769008, INDIA

MAY-2014

NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA

CERTIFICATE

This is to certify that the thesis entitled, "**VIBRATION ANALYSIS OF VISCOELASTIC SANDWICH BEAM USING FINITE ELEMENT METHOD**" submitted by Mr.**TATAPUDI NAVEEN KUMAR (212ME1269)** in partial fulfilment of the requirements for the award of **Master Of Technology** degree in **Mechanical Engineering** with specialization in **Machine Design and Analysis** at the National Institute of Technology, Rourkela (India) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date:

Dr. R.K.BEHERA
Assoc. Professor
Department of Mechanical Engineering
National Institute of Technology
Rourkela-769008.

ACKNOWLEDGEMENT

It is my pleasure to have this opportunity to express my heartfelt and deep regards to those who have been helped me in the way of part of my life journey as student.

In particular, I would like to express my gratitude and regards to my thesis guide **Prof.R.K.BEHERA** for his valuable guidance, encouragement, and kind cooperation in the successful completion of work.

I also would like to express my sincere thanks to **Prof. K.P.Maity** Head of the department, Mechanical Engineering for providing valuable departmental facilities.

I also thankful to my parents and my sister for their encouragement and support for the successful completion of my project work.

I also thankful to research scholars Pavankishore, Ramu, Alok Ranjan Biswal, Jakeer Hussain,Varalakshmi of Mechanical engineering department for their support.

Finally, I would like to thank my fellow post graduate students and the people who have been involved directly or indirectly in my endeavour.

TATAPUDI NAVEEN KUMAR

Roll No: 212ME1269

Dept.Of Mechanical Engg.

Contents

ABSTRACT

In this research work vibration analysis of a viscoelastic sandwich beam has been studied. A finite element model has been developed for the three layer viscoelastic sandwich beam. The sandwich beam is modelled using linear displacement field at face layer and non-linear displacement field at core layer. The equation of motion for the viscoelastic sandwich beam is derived by using the Hamilton's principle. Different specimens have been modelled by varying the core layers and face layers and studied under the fixed-fixed and cantilever boundary conditions for modal analysis. The Natural frequencies are obtained for various models using different core thickness and boundary conditions. The results obtained are compared with the earlier existing and experimental results.

Key words: Sandwich beam, Viscoelastic, Damping, Finite Element Method

List of Figures

List of Tables

Nomenclature

A	: cross section area
E	: young's Modulus
E_c	: young's modulus of core layer
G	: shear Modulus
G_c	: complex shear modulus of core
h_1	: thickness of the top layer
h_2	: thickness of the core layer
h_3	: thickness of the bottom layer
K	: stiffness matrix
K^*	: complex stiffness matrix
M	: mass matrix
L	: beam length
N	: shape function
P	: transverse load
Q	: external load
t	: time
T	: kinetic energy
U	: strain energy
u	: longitudinal displacement
w	: transverse displacement
α	: shear deformation
β	: transverse normal deformation in core
u_1	: longitudinal displacement at the centroid of the top layer
u_2	: longitudinal displacement at the centroid of the core layer
u_3	: longitudinal displacement at the centroid of the bottom layer
w_1	: transverse displacement at the centroid of the top layer
w_2	: transverse displacement at the centroid of the core layer
w_3	: transverse displacement at the centroid of the bottom layer
υ	: Poisson's ratio
ρ	: Mass/unit length
τ	: shear force
ω	: forcing frequency
x	: Cartesian coordinate
y	: Cartesian coordinate
z	: Cartesian coordinate
r	:$2(1+\upsilon)$
q	: nodal displacement
q_e	:element nodal displacement

Chapter: 1

Introduction

Vibration mainly influences the life of engineering structures and their performance and invariably, damping in structures influences its behavior. Many types of damping mechanisms have been developed over time to control the undesired vibration of structures. Basically damping refers to the extraction of mechanical energy from a vibrating system, mainly by converting the mechanical energy into heat energy by means of some dissipation mechanism. Mostly all materials exhibit some amount of internal structural damping.Most of the time it is not substantially effective to minimize the vibration around resonant frequencies. Hence, by bringing these materials in contact with the highly damped and dynamically stiffed material it is possible to control the vibration.

Passive damping treatment is one of the ways to control the vibration and noise in structures. The structure borne and airborne noise and vibration are frequent in most systems. The common passive control methods that include the use of mufflers, absorbers, barriers, mufflers, silencers, etc., are for airborne noise. For the systems with constant excitation frequency, modification of mass or system's stiffness reduces the unwanted vibrations as these parameters alter the resonant frequencies.

Viscoelastic materials are one such that they are capable of storing strain energy when they are deformed; these types of materials exhibit the material characteristics of both viscous fluid and elastic solid. Viscoelastic damping property was exhibited by the large variety of polymeric materials ranging from synthetic/natural rubbers to various thermoset/thermostat materials used in different industries. Here polymers display rheological behavior intermediate between a simple fluid and crystalline solids, due to having tangled molecules and large molecular order. This type of viscoelastic materials offers a wide range of possibilities for developing a desire damping level provided by the designer to completely comprehend their mechanical behavior. In viscoelastic material the mechanical energy is released through normal deformation and cyclic shear.

There are mainly three methods of treatment of viscoelastic material viz., unconstrained layer or free layer treatment, constrained layer and partially constrained layer treatment. Depending upon the functional requirements in obtaining efficient properties of all layers sandwich structures utilizes the constrained layer treatment. In this constrained layer damping treatment, the viscoelastic material was sandwiched between the surface of structure and thin facings of elastic metallic materials.

Normally Sandwich construction includes a relative thick core of low density material, sandwiched between the bottom and top face sheets (face layers) of relatively thin in size. The schematic diagram of a sandwich beam is shown in Figure 1.1.

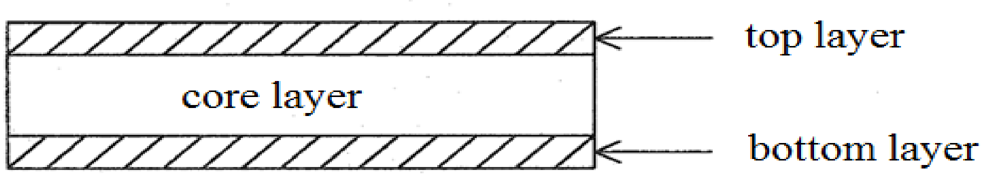

Figure 1.1: Sandwich beam model

1.2.1 Material properties of Sandwich Structure

The material choice in sandwich structures depends upon the need of employment such as high strength, high temperature resistivity, surface finish etc. In recent times the number of available cores has increased enormously due to the introduction of more competitive cellular plastics. Combining options of face sheet materials with different core materials give the new ideas to be integrated with a wide range of applications.

It is the obligation of the designer to have reliable information about the strength and stiffness of the materials used in the design for efficient analysis and design of sandwich structures. The best practice is to devote to tests for obtaining adequate material properties. The ample number of material choices may appear as an additional complexity, but is really one the main features of using sandwich structures. The materials best suited for a particular application may be utilized and some drawbacks can be overcome by geometrical sizing. The elementary objective of the designer is to achieve an efficient design that will utilize each material component to perform the function with good efficiency.

It is the need of the designer to have reliable information about the stiffness and strength of the materials used in the design for efficient design and analysis of sandwich structures. The suitable practice is to devote to test for obtaining adequate material properties. The enormous number of material choices may appear as additional complexity, but is really one the main features of using the sandwich structures. The beat suited materials for a particular application may be used and some drawbacks can be overcome by geometrical correction. The basic objective of the designer is to make a good design that will utilize the each material component to perform the function with good efficiency.

1.2.2 Core Material

The function of the cores is to give support for the thin skin layers so that they do not deform outwardly or inwardly, and to keep them in relative position to each other. The main requirements of the core are generally the shear and compressive modulus and strength. The main objective of any designer in choice of core material is that it would not fail under the any applied load and there should not be any deformation of core in thickness wise, thus requiring a high modulus of elasticity perpendicular to face layers. The core layer is exposed to shear so that global deformations and core shear stresses are developed by the shear strains in the core. The thickness of core and core material are two main parameters that decide the most of the properties of the sandwich structure.

The core layer consists of some typical features as given below,

- Lower density
- Damping of vibration and noise
- Shear strength and shear modulus
- Stiffness perpendicular to the top and bottom faces
- Thermal insulation

1.2.3 Face Material

The bottom and top layers of conventional sandwich structure are called as face layers or face sheets (as layers are in sheet form). From any structural materials that are available in the form of thin sheets can be used as a face material. The top and bottom layers face materials carry the compressive and tensile stresses in the sandwich. The flexural rigidity is often very small and it can be ignored. Fiber glass-reinforced plastics are common and acceptable to choose as face materials.

The face layer consists of some typical features as given below,

- High impact resistance
- High compressive and tensile strength
- Wear resistance
- Resistance to different conditions (chemical, heat, etc.)
- High stiffness giving high flexural rigidity
- Good surface finish

Various types of materials used as face materials are as follows:

Metals and alloys: Metals and their alloys possess all most all required properties of face materials. Conventional materials and their alloys such as steel, stainless steel and aluminum are often used as face material.

Composites: Most composites offer properties similar to or even higher than those of metals, they have been substantially used in construction of sandwich structures. Particularly fibre reinforced composites are suitable for sandwich structures even though the stiffness is often lower in magnitude. Thus with a light core, the composites produce high rigidity. Even wood also can used as face material in sandwich structures.

1.3 Design Considerations

A sandwich structure is designed to make sure that it is capable of taking structural loads throughout its design life. In addition, it should maintain its structural integrity in the in-service environments. The structure should satisfy the following criteria:

- The face sheets should have sufficient stiffness to withstand the tensile, compressive, and shear stresses produced by applied loads.
- The core should have sufficient stiffness to withstand the shear stresses produced by applied loads.
- The core should have sufficient shear modulus to prevent overall buckling of the sandwich structure under loads.
- Stiffness of the core and compressive strength of the face sheets should be sufficient to prevent the wrinkling of the face sheets under applied loads.
- The core cells should be small enough to prevent inter-cell buckling of the face sheets under design loads.
- The core shall have sufficient compressive strength to prevent crushing due to applied loads acting normal to the face sheets or by compressive stresses produced by flexure.
- The sandwich structure should have sufficient flexural and shear rigidities to prevent excessive deflections under applied loads.
- Sandwich materials (face sheet, core and adhesive) should maintain the structural integrity during in-service environments.

1.4 Application Areas of Sandwich Structures

In damped structures for effective vibration damping

Aerospace field

Building Construction

Naval ships

Rail Industry

Automotive Industry

1.5 Present Consideration

In the present study the constrained layer damping treatment has been used for the study of vibration behavior in sandwich beams. The viscoelastic material has been bonded between the top and bottom elastic layers to form the sandwich beam model.

Top layer: Elastic material (Steel, Aluminum)

Core layer: viscoelastic material (Rubber, Neoprene)

Bottom layer: Elastic material (Steel, Aluminum)

1.6 Objectives of the Present Research Work

The main objective is to model the viscoelastic sandwich beam for the modal analysis using the Finite Element Method in face layer displacement fields. The face and core layers are varied to model the different configuration of the sandwich beams and these modeled sandwich beams are investigated for natural frequencies using FEA and Experiment for various boundary conditions. The damping effect on the sandwich beams has to be studied by increasing the core layer thickness. Finally, harmonic analysis has to be made for the specimens modeled.

Chapter: 2

Literature Review

In the early, Kerwin [1] presented damping effective of the constrained viscoelsatic layers and mentioned that the damping effect depends on the wavelength of bending waves, thicknesses and elastic modulli and formulated the complex shear modulus for the damping layer and he predicted that the heat dissipation takes place through the shearing phenomenon. For a number of constraining layers damping factors have determined experimentally by neglecting the boundary condition.

Ditoranto [2] has derived auxiliary equation for the effect of viscoelastic layers. The use of this equation with the ordinary bending equation formed for homogeneous beams for solving static and dynamic bending problems. They formed the six orders, complex, homogeneous differential equation of the viscoelastic layered finite length beam and determined the natural frequencies and loss factors for the freely vibrating beam.

Mead [3] they extended the Ditoranto work by decoupling the sixth order equation and modeling the sixth order equation of motion in terms of transverse displacements in a three layered sandwich beam with viscoelastic core for the forced vibration analysis and complexity in mode will only exist when the beam was excited by the damped normal loads which are proportional to the transverse inertia loading on the beam.

Bai and Sun [4] Effects of viscoelastic adhesive layer on the structural damping and dynamic response of the structure are studied by a newly developed sandwich beam theory. They formed the new non linear displacement field in the core layer for achieving the accurate kinematics of the flexible viscoelastic. The properties of viscoelastic are assumed in a complex modulus with the function of frequency for a given temperature. They studied the effect of adhesive layer on the damping and obtained the storage modulus and loss factors for the simply supported beam under harmonic loading and also obtained the results for driving point impedance for all the set of frequencies which are nearly matched with the available data reported by Lu and Douglas.

Banerjee et al.[5] has discussed a dynamic stiffness model for unequal thickness of the three layered sandwich beam and used for investigating free vibration characteristics by using Timoshenko beam theory they modeled their layers and have developed a dynamic stiffness matrix by relating amplitudes of harmonic varying loads. The accuracy of their theory was confirmed with the earlier literature and experiment.

Lu and Douglas [6] the motive of their work is to compare the analytical and experimental forced vibration response of the three layered damped laminate in a format of mechanical impedance to verify the analytical information given by the Mead and Markus. They used the analytical model given in reference of Mead and Markus to get the mechanical impedance at the mid span with a sinusoidal transverse force for the damped laminated beam with free-free boundary condition.

Mace [7] modeled the viscoelastic sandwich beams by using the finite element model, in the layer wise displacement field for studying the dynamic behavior. The model developed is applicable duly to the very thin core layer of viscoelastic sandwich beam and the model which he made was in 3D model approach it is very difficult and costly for the implementation and it also generates the difficulties in the mesh for the analysis.

Barber et al. [8] modeled the finite element finite element model for the 3 layer viscoelastic sandwich beam considering the non-linear displacement field in the viscoelastic layer. They used the approximation that the viscoelastic core layers variable are to be expressed in the top and bottom layer displacement field and predicted the displacements at the resonant driving frequencies and compared the test data available with the literature.

Chen et al. [9] used the Euler Bernoulli beam theory for deriving the equation of motion for the system. The resonant frequencies and loss factors of the cantilever beam are analyzed by applying the mass at the free end of the constrained layer. They defined that the variation in the resonance frequency and loss factor are mainly depend on the physical properties and geometry of the constraining layer.

Yazhuk [10] developed the sandwich beam using piezoelectric layers and coupled with the electromechanical forced vibration. They investigated the effect of passive layer on the beam in nonlinear behavior. They obtained the comparison between the calculated transient responses of the beam using the full model with the approximate model.

Sakihama et al. [11] developed a method for analyzing the free vibration of the sandwich beam with both elastic and viscoelastic are using the arbitrary conditions. They obtained the characteristic equation for free vibration using the Green function which defines from the discrete solution of governing differential equation. By using this characteristic equation the behavior of sandwich beam can be easily analyzed with the trial and error approach for free vibration.

Won et al. [12] detected the problem with the mead and markus two sets of differential equation of motion for the three layered constrained sandwich beam, to resolve that they taken the symmetric straight damped and constrained three layered sandwich structures are derived using the virtual kinetic energy and strain energy are mentioned in terms of axial displacement and the transverse shear strain of a viscoelastic core layer. They compared and validated their model using the NASTRON 3D-solid element.

Khalili [13] in their paper they used the finite element formulation and the dynamic stiffness method for performing the vibration analysis of a three layered sandwich beam consists of sprung mass. Some numerical examples are used for discussing the finite element formulation and dynamic stiffness matrix.

Banerjee et al. [14] developed a 3-layered sandwich beam using the dynamic stiffness theory for calculating the free vibration characteristics. They considered the top and bottom layers to behave as a Rayleigh beams, while the core layer as a Timoshenko beam for the harmonic analysis the equations they developed are found in exact with analytical form, they discussed the natural frequencies and mode shapes of various problems.

Amirani et al. [15] discussed the free vibration analysis of a sandwich beam with the FGM as a core layer. They constrained the Galerkin method and formulation for two dimensional elastic plastic problems. Finally, they obtained the first ten natural frequencies using the finite element analysis.

Fei Lin and Mohan [16] they presented a modeling technique for multi layered viscoelastic laminated beams to obtain the vibroacoustics. In their model they provide the non-linear behavior for their core layer using the Biot damping. The FEA method has been used for the vibration analysis of the multilayered sandwich beam.

Mohammadi et al. [17] in their paper, sandwich cylindrical structures are investigated for vibration analysis and damping characteristics for various boundary conditions. They used the electroheological fluids to cover the untreated portions of the unconstrained viscoelastic material. The results showed that the sandwich treated partially with the electroheologic fluids provides better damping performance than fully treated for some boundary condition.

Grewel et al. [18] have modeled a sandwich beam using the linear and nonlinear displacement at its core layer by using the finite element method. Parametric studies were carried to find out the effect of core layer thickness on the natural frequencies and the loss factor for the sandwich beam structure and they considered the partial treatment of the structure to obtain the more damping for the fixed free and fixed -fixed boundary conditions.

Yadav [19] discussed about the vibration damping in the four layered sandwich beam. He used the method of equilibrium forces and beam theory for deriving the equation of motion for the vibration analysis. They conducted the analysis with the mass and rubber spring mounted on a sandwich beam structure for the simply supported boundary conditions.

Daya [20] modeled the viscoelastic sandwich beam for the non linear vibrations using the elementary theory. Galerkin analysis was used for coupling the harmonic balance and discussed about the effects of temperature and the boundary conditions on the vibration response.

Barbosa et.al [21] focused on the passive damping systems as viscoelastic materials in the laminated places. Golla Hugles Method (GHM) has been used in characterizing the viscoelastic materials and GHM based finite element model has been presented and validated with the various numerical and classic formulation comparisons.

Jacques et al. [22] modeled a zigzag model using FEA to describe the displacement field for non linear vibrations of sandwich beams and investigated the influence of amplitude on the damping properties of sandwich beams. The behavior of viscoelastic has handled by using the hereditary integrals with complex modulus.

Bekuit et al. [23] for the dynamic and static analysis they considered the quasi-two-dimensional finite element formulation. The model is of three layers and consists of the both longitudinal and transverse displacement field. These formulations were independent of flexibility of the core layer.

Mohammadi and Sedaghati [24] studied the semi-analytical finite element method to know the damping characteristics of the thick and thin core viscoelastic beams. They developed an efficient algorithm to solve the eigenvalue problem due to the frequency dependent properties in viscoelastic material. Effect of imperfect bonding with in the layers has also investigated.

Chapter: 3

Finite Element Method

3.1 Fundamental Concept of FEM

The main rule that involved in finite element method is "DEVIDE and ANALYZE". The greatest unique feature which separates finite element method from other methods is "It divides the entire complex geometry into simple and small parts, called 'finite elements". These finite elements are the building blocks of the finite element analysis. Based on the type of analysis going to be performed, these elements divided into several types. Division of the domain into elements is called 'mesh'. The forces and moments are transferred from one element to next element are represented by degrees of freedom (DOFs) at coordinate locations which are called as 'nodes'. Approximate solutions of these finite elements give rise to the solution of the given geometry which is also an approximate solution.

The approximate solution becomes exact when

1. The geometry is divided into numerous or infinite elements.

2.Each element of geometry must define with a complete set of polynomials (infinite terms).

3.2 General steps of the Finite Element Method

The following general steps discussed below are for structural analysis case.

1. Discretization and choosing element types:

This step includes division of geometry into an equivalent set of finite elements with associated nodes and selecting the best suitable element which resembles the actual physical behavior of the given system to be analyzed. Engineer needs to focus in the matters of selecting the number elements, variation in size and type of elements. For getting best results it is advisable to choose as small elements as possible. One of the major tasks of the engineer is the selection of the appropriate element for a particular problem.

2. Select a Primary variable function:

This step involves selecting a primary variable (displacement) function within each element. The function is defined within the element using the nodal values of the element. Polynomial functions are generally used because they are easy to work within finite element formulation. In case of two dimensional elements, the primary variable function is function of the coordinates in its plane. The functions are expressed in terms of the nodal unknowns.

3. Define relations:

The relations among stresses, strains and displacements are essential for obtaining the equations for each finite element. In the case of one –dimensional deformation, say, in the x direction, we have strain ε_x related to displacement u by

$$\varepsilon_x = \frac{du}{dx}$$

for small strain cases. The definition of material behavior is also important in obtaining acceptable results.

4. Extraction of the element stiffness Matrix and Equations:

The element stiffness matrix and equations are deriving by using the following methods.

Direct Equilibrium Method

According to this method, the stiffness matrix and element equations relating nodal forces to nodal displacements are obtained using force equilibrium conditions for a basic element, along with force or deformation relationships. This method can easily adaptable to line or one-dimensional elements.

Work or Energy Methods

For the extraction of the stiffness and equations for two and three-dimensional elements, the application of work or energy methods are very friendly. The principle of minimum potential energy, the principle of virtual work methods used for derivation of element equations.

Generally, for elastic materials the principle of minimum potential energy is suitable whereas the principle of virtual work can adopt for any other material behavior.

Methods of Weighted Residuals

The methods of weighted residuals are useful for developing the element equations particularly popular is Galerkin's method. These methods yield the same results as the energy methods wherever the energy methods are applicable. They are especially useful when a functional such as potential energy is not readily applicable.

5. Assembling the Element equations and Apply boundary conditions:

In this step, the equilibrium equations of nodes that are obtained in previous step are combined into the global nodal equilibrium equations. One more direct method of superposition, whose basis is nodal force equilibrium, can be used to obtain the global equations.

The global equation can be written in matrix form as

$$\{F\} = [K]\{\delta\}$$

Where $\{F\}$ represents the Force vector, $[K]$is the total stiffness matrix $\{\delta\}$ is the vector of generalised displacements.

At this stage, the global stiffness matrix $[K]$is a singular matrix because its determinant is equal to zero. To remove this we need to call upon certain boundary conditions in order to avoid to the movement of the structure as rigid body.

6. Solve for the primary unknowns:

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{1n} \\ K_{21} & K_{22} & K_{2n} \\ K_{n1} & K_{n2} & K_{nn} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_n \end{Bmatrix}$$

These general equations can be solved for the primary unknowns by using an elimination method or an iterative method. The primary variables are different for various problems. In case of the structural problem the primary unknown is displacement.

7. Solve for secondary unknowns:

In this step secondary unknowns are determined by using the displacement equations which are already obtained from previous step. Commonly strain and stress, shear force and moments are secondary unknowns for structural problem, are determined by using mathematical techniques.

8. Interpret the results:

The results obtained in previous step need to analyze for use in the design or analysis process. For better design and to avoid the failure of the structure it is important to determine the locations in the structure where large deformations and stresses are occur. Postprocessor computer programs help the user to interpret the results by displaying them in graphical form.

3.3 Applications of the Finite Element Method

The finite element method is an effective tool to analyze both structural and non-structural problems. Even in some biomechanical engineering problems, includes analyses of human spine, hip joints, skull, hip joints, heart and eye etc.

Structural areas:

1. stress analysis, including truss and frame analysis, and stress concentration problems commonly associated with holes, fillets, or other changes in geometry in a body

2. Buckling analysis

3. Vibration analysis

Non-structural problems

1. Heat transfer

2. Fluid flow

3. Distribution of electric or magnetic potential

3.4 Advantages of the Finite Element Method

Some of the advantages of the Finite element method that includes the ability to

1. Easy modeling of irregularly shaped bodies

2. Handle general load conditions without difficulty

3. Model bodies composed of different materials

4. Handle unlimited numbers and kinds of boundary conditions

5. Includes dynamic effects

6. Handle nonlinear behavior with large deformations and nonlinear materials

3.5 Limitations of the Finite Element Method

In spite of many advantages some drawbacks of finite element method are as follows

1. Stress values depend on the size of mesh fine to average

2. In some cases the approximations used may not provide accurate results

3. For vibration and stability problems the cost of analysis by FEA is prohibitive

Software packages for FEM

Below list provides some of the commercially available software packages of FEM

ABACUS, ANSYS, COSMOS/M, LS-DYNA, NASTRAN

Chapter: 4

Sandwich Beam Model

The sandwich beam model described here based on the following assumptions

1. Top and bottom layers are considered as ordinary beams with axial and bending resistance.
2. The core layer carries negligible longitudinal stress, but takes the non linear displacement fields in x and z directions.
3. All the three layers are assumed to be perfectly bonded and there is no slippage between the layers.
4. Transverse displacements of top and bottom layers equal transverse displacement of core at interfaces.

The sandwich beam considered here consists of three layers with viscoelastic material as a core layer, the top and bottom layers are isotropic and linear elastic material with thickness h_1 and h_3. The viscoelastic core layer has a thickness of h_2 under harmonic loading exhibits complex modulus in the form of $Ec = E^{'}(1 + i\eta)$ where η is the loss factor. The model of the viscoelastic sandwich beam has shown in the figure given below

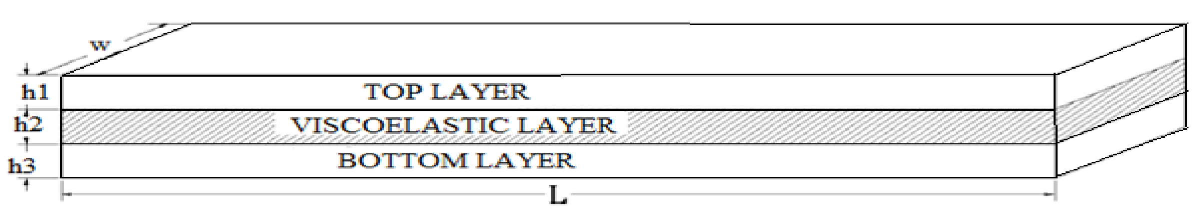

Figure 4.1 Sandwich Beam model

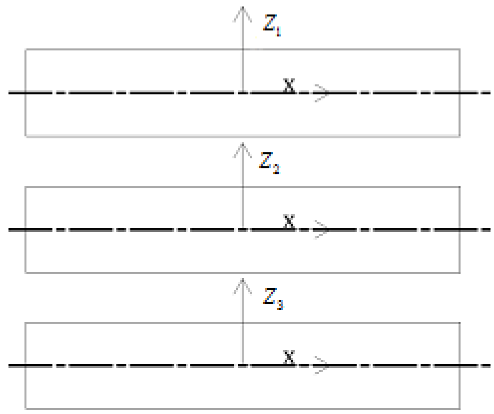

Figure 4.2 Local coordinates of sandwich beam model

4.2 Displacement Field

The assumed displacement fields in the sandwich beam are as follows,

The longitudinal and transverse displacement fields for the top layer are given by:

$$u_1(x, z_1, t) = u_1^0(x,t) - z_1 \frac{\partial w_1^0}{\partial x}(x,t) \tag{4.1}$$

$$w_1(x, z_1, t) = w_1^0(x,t) \tag{4.2}$$

Where

u_1 = longitudinal displacement in the top layer for any (x, z) location

u_1^0 = longitudinal displacement at the centroid of the top layer

z_1 = distance from centroid of top layer in transverse direction

w_1 = transverse displacement in the top layer for any (x, z) location

w_1^0 = transverse displacement at the centroid of the top layer

Similarly, for the bottom layer the displacements are considered as

$$u_3(x, z_3, t) = u_3^0(x,t) - z_3 \frac{\partial w_3^0}{\partial x}(x,t) \tag{4.3}$$

$$w_3(x, z_3, t) = w_3^0(x,t) \tag{4.4}$$

Where

u_3 = longitudinal displacement in the bottom layer for any (x, z) location

u_3^0 = longitudinal displacement at the centroid of the bottom layer

z_3 = distance from centroid of the bottom layer in transverse direction

w_3 = transverse displacement in the bottom layer for any (x, z) location

w_3^0 = transverse displacement at the centroid of the bottom layer

Displacements field in the viscoelastic core layer varies nonlinearly in both x and z directions. By taking an elastic analysis, Bai and Sun [4] assumed that the longitudinal and transverse displacement of the core is

$$u_2(x, z_2, t) = u_2^0(x,t) + z_2\left(e\alpha(x,t) - \frac{\partial w_2^0(x,t)}{\partial x}\right) - \frac{{z_2}^2}{2}\frac{\partial \beta(x,t)}{\partial x} + \frac{{z_2}^3}{6}\frac{\partial^2 \alpha(x,t)}{\partial x^2} \tag{4.5}$$

$$w_2(x, z_2, t) = w_2^0(x,t) + z_2\beta(x,t) - \frac{{z_2}^2}{2}\frac{\partial \alpha(x,t)}{\partial x} \tag{4.6}$$

Where

u_2 = longitudinal displacement in the core layer for any (x, z) location

u_2^0 = longitudinal displacement at the centroid of the core layer

z_2 = distance from centroid of core layer in transverse direction

α =shear deformation in core

β =transverse normal deformation in core

w_2 =transverse displacement in core layer for any (x, z) location

w_2^0 = transverse displacement at the centroid of the core layer

e = 2 (1+ϑ_c)

ϑ_c = Poisson's ratio of viscoelastic core layer

To describe the displacement field for the core layer, four generalized degrees of freedom u_2^0 , w_2^0 , α and β are required. Non linear displacement field of a viscoelastic core layer allows the transverse displacement of top constrained layer and bottom layer to remain independent of each other. This leads to transversal extension and compression of the core layer. As assumed in the assumptions that there is the perfect bond between the three layers and to obtain the continuity between all the three layers the following relations are used:

At top interface,

$$u_2(x, h_2/2, t) = u_1(x, -h_1/2, t) \tag{4.7}$$

$$w_2(x, h_2/2, t) = w_1(x, -h_1/2, t) \tag{4.8}$$

At bottom interface,

$$u_2(x, h_2/2, t) = u_3(x, -h_3/2, t) \tag{4.9}$$

$$w_2(x, h_2/2, t) = w_3(x, -h_3/2, t) \tag{4.10}$$

Now substituting the displacement fields of the viscoelastic core layer from Eq. (4.5) and Eq. (4.6) into Eqs. (4.7), (4.8), (4.9), and (4.10), out of four degrees of freedom it is possible to eliminate the three degrees of freedom in core layer, specifically through the following relations

$$u_2^0 = \frac{1}{2}\left[u_1^0 + u_3^0 - \left(\frac{h_1}{2} + \frac{h_2}{4}\right)\frac{dw_1^0}{dx} - \left(\frac{h_3}{2} + \frac{h_2}{4}\right)\frac{dw_3^0}{dx}\right] \tag{4.11}$$

$$w_2^0 = \frac{w_1^0 + w_3^0}{2} + \frac{{h_2}^2}{8}\frac{\partial \alpha}{\partial x} \tag{4.12}$$

$$\beta = \frac{w_1^0 - w_3^0}{h_2} \tag{4.13}$$

The remaining degree of freedom for the core layer, α (x, t), cannot be eliminated easily, but it is related to the top and bottom face layers by the following partial differential equation:

$$\frac{\partial^2 \alpha(x,t)}{\partial x^2} - \frac{12e}{h_2^2}\alpha = \frac{12}{h_2^3}\left(u_3^0 - u_1^0 + \frac{(h_1 + h_2)}{2}\frac{dw_1^0}{dx} + \frac{(h_3 + h_2)}{2}\frac{dw_3^0}{dx}\right) \tag{4.14}$$

The generalized degrees of freedom to describe the displacement fields in vector form for both core and elastic layers as follows

$$q(x,t) = [u_1^0, \theta_1, w_1^0, u_3^0, \theta_3, w_3^0]^T \tag{4.15}$$

It should be noted that from Eq (4.14), the term α cannot be directly eliminated as it related to the top and bottom face layers variables through the partial differential equation and the above partial differential equation can be modified into the ordinary differential equation for steady state harmonic response independent of variable time t as follows:

$$\frac{\partial^2\alpha(x)}{\partial x^2}-\frac{12e}{h_2^2}\alpha=\frac{12}{h_2^3}\left(u_3^0-u_1^0+\frac{(h_1+h_2)}{2}\frac{\partial w_1^0}{\partial x}+\frac{(h_3+h_2)}{2}\frac{\partial w_3^0}{\partial x}\right) \tag{4.16}$$

The solution of the equation has been solved by neglecting the higher order terms by Baber et.al (1998) and final approximate solution of the above equation can be written as

$$\alpha(x)\approx-\frac{1}{eh_2}\left[u_3^0-u_1^0-\frac{(h_1+h_2)}{2}\frac{dw_1^0}{dx}-\frac{(h_3+h_2)}{2}\frac{dw_3^0}{dx}\right]. \tag{4.17}$$

4.3 Strain Displacement Relations

It can be easy to formulate the strains developed in all the three layers by using the assumed displacement fields. By following the assumed Euler-Bernoulli bending of top and bottom layers, the only relevant strains in these layers are longitudinal strains. They are

Longitudinal strain in top layer is taken as

$$\varepsilon_1^{xx}=\frac{\partial u_1^0}{\partial x}-z_1\frac{\partial^2 w_1^0}{\partial x^2} \tag{4.18}$$

Longitudinal strain in bottom layer is taken as

$$\varepsilon_3^{xx}=\frac{\partial u_3^0}{\partial x}-z_3\frac{\partial^2 w_3^0}{\partial x^2} \tag{4.19}$$

The relevant strains in the core layer are vertical normal strain

$$\varepsilon_2^{zz}=\beta(x,t)-z_2\frac{\partial\alpha(x,t)}{\partial x} \tag{4.20}$$

and shear strain in the core layer is

$$\gamma_2^{xz} = e\alpha(x,t) \tag{4.21}$$

4.4 Strain Energy

The total strain energy consists of sum of several different contributions from the top, bottom, and core layers and given by the following equation.

$$U = U_1 + U_2 + U_3 + U_4 + V \tag{4.22}$$

Where

$$U_1 = \frac{1}{2}\int_0^L E_1 h_1 \left[\left(\frac{\partial u_1^0}{\partial x} \right)^2 + \frac{h_1^2}{12} \left(\frac{\partial^2 w_1^0}{\partial x^2} \right)^2 \right] dx, \tag{4.23}$$

$$U_2 = \frac{1}{2}\int_0^L E_3 h_3 \left[\left(\frac{\partial u_3^0}{\partial x} \right)^2 + \frac{h_3^2}{12} \left(\frac{\partial^2 w_3^0}{\partial x^2} \right)^2 \right] dx, \tag{4.24}$$

$$U_3 = \frac{1}{2}\int_0^L G_2 h_2 (e\alpha)^2 dx, \tag{4.25}$$

$$U_4 = \frac{1}{2}\int_0^L E_2 h_2 \left[\beta^2 + \frac{h_2^{\,2}}{12} \left(\frac{\partial \alpha}{\partial x} \right)^2 \right] dx, \tag{4.26}$$

$$V = -\int_0^L q w_1 dx \tag{4.27}$$

4.5 Kinetic Energy

The kinetic energy for all the three layers can be written as

$$T = T_1 + T_2 + T_3 \tag{4.28}$$

Where

$$T_1 = \frac{1}{2}\int_0^L \left\{ \rho_1 h_1 \left[\left(\frac{\partial u_1^0}{\partial t} \right)^2 + \left(\frac{\partial w_1^0}{\partial t} \right)^2 + \frac{h_1^2}{12} \left(\frac{\partial^2 w_1^0}{\partial x \partial t} \right)^2 \right] \right\} dx \tag{4.29}$$

$$T_2 = \frac{1}{2}\int_0^L \left\{ \rho_3 h_3 \left[\left(\frac{\partial u_3^0}{\partial t} \right)^2 + \left(\frac{\partial w_3^0}{\partial t} \right)^2 + \frac{h_3^2}{12} \left(\frac{\partial^2 w_3^0}{\partial x \partial t} \right)^2 \right] \right\} dx \tag{4.30}$$

$$T_3 = \frac{1}{2}\int_0^L \int_{\frac{-h}{2}}^{\frac{h}{2}} \rho_2 \left[\left(\frac{\partial u_2^0}{\partial t} \right)^2 + \left(\frac{\partial w_2^0}{\partial t} \right)^2 \right] dz dx \tag{4.31}$$

4.6 Finite Element Formulation

As discussed in above section the beam modeled over here consists of three layers with two nodes at each node there are six degrees of freedom as shown in the figure given below:

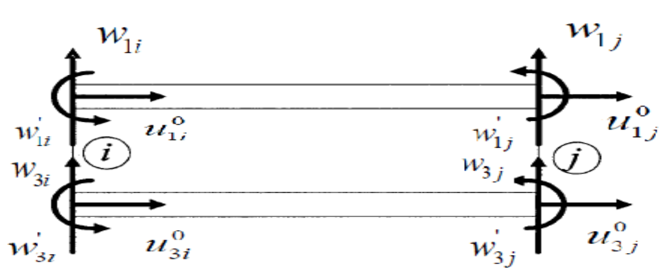

Figure 4.3: Two-node sandwich beam element with displacement field

The response of the sandwich beam has been expressed in face layers only, so the face plates are assumed to behave as Euler-Bernoulli beams (with rotary inertia added). The general shape function for the beam analysis can be used to determine the displacement of the face layers. That is

$$\left.\begin{aligned} u_k^0(x) &= N_{1a}(x)u_{ki}^0 + N_{2a}(x)u_{kj}^0 \\ w_k^0(x) &= N_{1f}(x)w_{ki}^0 + N_{2f}(x)w_{ki}^{0^1} + N_{3f}(x)w_{kj}^0 + N_{4f}(x)w_{kj}^{0^1} \end{aligned}\right\} \quad k= 1, 3 \tag{4.32}$$

where i ,j denotes the two nodes of the beam element as shown in the above figure and

the shape functions are given as follows.

$$\begin{aligned} N_{1a}(x) &= 1-\frac{x}{L}, \\ N_{2a}(x) &= \frac{x}{L}, \\ N_{1f}(x) &= 1-3\left(\frac{x}{L}\right)^2+2\left(\frac{x}{L}\right)^3, \\ N_{2f}(x) &= L\left[\frac{x}{L}-2\left(\frac{x}{L}\right)^2+\left(\frac{x}{L}\right)^3\right], \\ N_{3f}(x) &= 3\left(\frac{x}{L}\right)^2-2\left(\frac{x}{L}\right)^3, \\ N_{4f}(x) &= L\left[-\left(\frac{x}{L}\right)^2+\left(\frac{x}{L}\right)^3\right] \end{aligned} \tag{4.33}$$

The generalized degrees of freedom q are related to the element nodal degrees of freedom q_e related to shape functions as follows

$$q = Nq_e \tag{4.34}$$

$$q_e = (u_{1i}^0, w_{1i}^0, w_{1i}^{0^1}, u_{3i}^0, w_{3i}^0, w_{3i}^{0^1}, u_{1j}^0, w_{1j}^0, w_{1j}^{0^1}, u_{3j}^0, w_{3j}^0, w_{3j}^{0^1})^T \tag{4.35}$$

Substituting the equations no's (4.32) and (4.33) into Eqs no's (4.11), (4.12), (4.13), (4. 17) the stiffness and mass matrix for the beam element can be obtained as follows

The resulting element stiffness matrix may be written in the form of

$$K = K_1 + K_2 + K_3 + K_4 \tag{4.36}$$

In the above equation the stiffness matrix consists of K_1 which was contributed by the strain energies of top and bottom layers U_1 and U_2 and K_2 has been contributed by shearing strain energy of the core appearing in U_3. K_3 and K_4 has been contributed by the two terms appearing in the U_4.

The resulting element mass matrix may be written in the form of

$$M = M_1 + M_2 + M_3 \tag{4.37}$$

In the above equations the mass matrix consists of M_1 contributed by kinetic energy of the face layers T_1 and T_2 and M_2 and M_3 contributed by the two terms in kinetic energy T_3 of the core layer.

4.7 Equation of Motion

The equation of motion for the mentioned element has been solved by using the Hamilton's principle

$$\int_{t_1}^{t_2} \left(\delta \left(T - U \right) + \delta w \right) dt = 0 \tag{4.38}$$

Now, substituting the expressions for strain energy, kinetic energy and work done given equations Eq (4.22), Eq (4.28) in to Eq (4.38) and integrating over time, we get the Eq (4.38) as follows

$$[M]\{\ddot{q}\}+[K(\omega)]q-\{F(t)\}=0 \tag{4.39}$$

Where [M], [K], {F} are the mass element matrix, stiffness matrix and force vector for the given element respectively. As mentioned earlier the strain energy of the core layer is the function of excitation frequency and it is in complex quantity. Thus the stiffness matrix obtained here was formed with all the strain energies so it is also the function of frequency and complex matrix. So the above equation can now be written as

$$[M]\{\ddot{q}\}+[K^{*}(\omega)]q-\{F(t)\}=0 \tag{4.40}$$

Where $K^{*}(\omega)=K^{'}(\omega)+iK^{"}(\omega)$

If the harmonic load has been applied, then

$$\{F(t)\}=\{F_o\}e^{i\omega t} \tag{4.41}$$

where ω is the forcing frequency .Both the amplitude and mode of vibration are depending on the exciting frequency.

It should be assumed that the response due to the applied harmonic load will also be in harmonic and at the same frequency, then

$$\begin{aligned} q &= \{Q\}e^{i\omega t} \\ \ddot{q} &= -\omega^2\{Q\}e^{i\omega t} \end{aligned} \tag{4.42}$$

Now, substituting the above equations in to the Eq (4.32) then the equation of motion becomes

$$\left[K^*(\omega)-\omega^2 M\right]Q = F_o \tag{4.43}$$

Here K^* and M are matrices and Q, F_o are the vectors.

The above equation can be easily solved for every frequency after applying certain boundary conditions by using the inverse of matrix inside the brackets to the other side. Here we are not interested for every excited frequency and when the exciting frequency gets close to the natural frequency, the response of the system elevated as inertial forces become prominent along external exciting forces. Natural frequency of the system can be obtained easily by equating the exciting force amplitude in Eq (4.36) to zero as

$$\left[K^*(\omega)-\omega^2 M\right]Q = 0 \ . \tag{4.44}$$

Chapter: 5

Experimentation

5.1 Description of the Experimental Setup

The experimental setup consists of 1) power amplifier 2) Function Generator 3) Vibration Generator 4) Accelerometer and 5) Oscilloscope.

The function of each instruments are described below:

1) Power amplifier: Power amplifier was used for the subsequent amplification of the signal generated by the function generator and give input to the vibration exciter for vibrating of the specimen.

2) Function Generator: Function generator is used to generate the sine function of the required frequency and it is given as input to the power amplifier for giving the excitation in the vibration generator.

3) Vibration Generator: Vibration Generator gets the signal from the amplifier through the function generator and vibrates at the given range of frequency.

4) Accelerometer: Accelerometer measures the response of the specimen which is vibrated by the vibration generator and gives the response to the oscilloscope.

5) Oscilloscope: Oscilloscope was used to observe the response of vibration pickups of the accelerometer from the vibrated specimen. It gives the signals in the form of graph between the amplitude and time.

The schematic diagram of the experimental setup is as shown in the Figure 5.1 and photograph with setup in Figure 5.2.

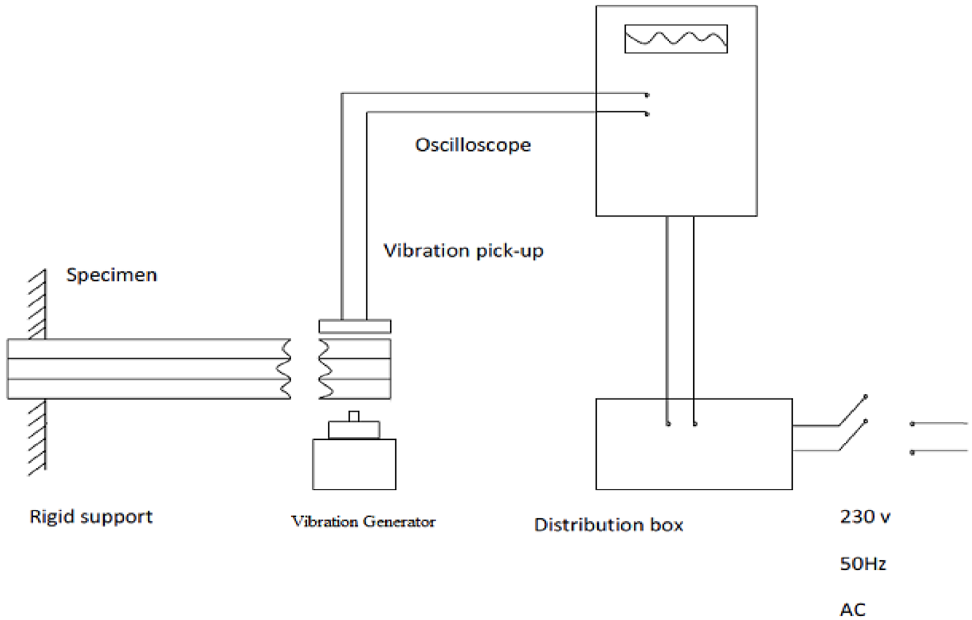

Figure 5.1: The schematic diagram of the experimental setup

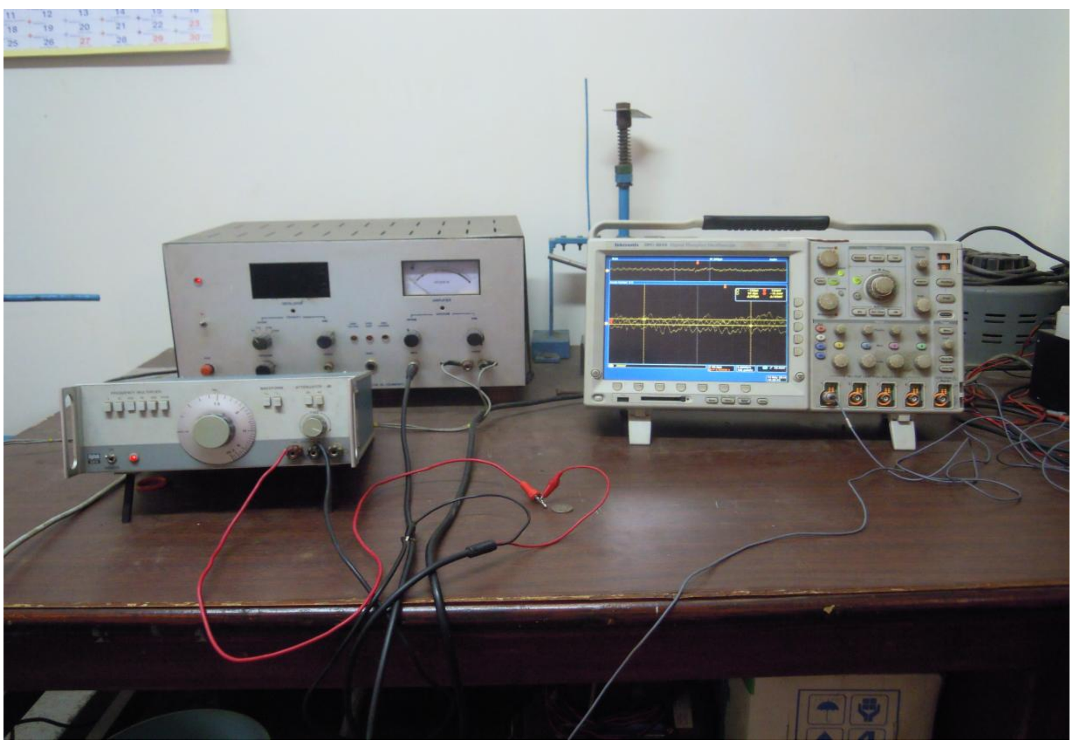

Figure 5.2: Photograph of the experimental setup

Figure 5.3: Beam is arranged on frame for fixed-fixed and cantilever conditions

Figure 5.4: Specimens prepared for the Experimental Investigation

5.2 Preparation of Sandwich Beam Specimens

Sandwich beams are made with the aluminum and steel sheets as the face layers and the core layers as rubber and neoprene. In preparing the sandwich beam specimens the face layers are made free from grease, dirt etc by cleaning their surface with acetone and carbon tetrachloride. The adhesive used for bonding the layers was commercially available Araldite. After application of thin layer and equal amount of adhesive on surfaces of all layers, the specimens were allowed to settle down for 24 hr's for perfect bonding under the load and proper care was taken to avoid the slippage between the layers by providing the positioning guides at all the edges of the specimen. The details of physical and geometrical properties of specimen are given as: the thickness of top and bottom layers 1 mm, core layer thickness as 5 mm, the length and width of beam are taken as 500 mm and 25 mm respectively.

The material properties of sandwich beam considered here are given in Table 5.1.

Table 5.1: Material properties of sandwich beam for face and core layers

Type of material	Young's Modulus E (GPA)	Shear Modulus G (GPA)	Density ρ in Kg/m^3	Poisson's Ratio υ
Aluminum	71	27.3	2766	0.33
Steel	210	80	7850	0.3
Rubber	0.00154	0.005	950	0.45
Neoprene	0.0008154	0.000273	960	0.49

Four different types of sandwich beam specimens were made for experimental investigation which consists of

Specimen1: Aluminum – Rubber- Aluminum

Specimen2: Aluminum-Neoprene- Aluminum

Specimen3: Steel- Rubber- Steel

Specimen4: Steel-Neoprene-Steel

The natural frequencies for all the specimens were determined experimentally for the cantilever and fixed-fixed boundary conditions.

5.3 Procedure for Conducting the Experiment:

1) The testing specimen was arranged in the frame for obtaining the necessary boundary conditions.
2) The accelerometer was placed on the testing specimen at a position where the vibration measurement was to be taken.
3) Now, the excitation was given to the loaded specimen by using the vibration generator which was driven by the power amplifier connected to the function generator.
4) Responses of the excited beam were measured by the accelerometer which gives the input signal to the oscilloscope.
5) From oscilloscope one can get the response of the beam in the wave form i.e., sine wave given from the function generator between the amplitude and time for the exciting frequency of the specimen.

6) With the help of these response readings the corresponding peak amplitudes were noted for different exciting frequencies. The graphs were plotted between the peak amplitudes of vibration response and the exciting frequencies which gives the natural frequency of the tested specimen.

Chapter: 6

Results and Discussion

6.1 Numerical Results

This section represents the validation of the result for the present developed finite element model for the viscoelastic sandwich beam .Total twenty number elements are used for validating the present model with the model already been developed in reference[5].

The sandwich beam model has been validated with results of the Banerjee et al. [5]. They developed a dynamic stiffness method for the three layered sandwich beam and discussed about the natural frequencies and mode shapes for the two different types of cases. In case 1, they considered the core layer as the rubber and in case 2, core layer as lead. For face layers they considered steel for both the cases. The geometric properties of the case 1 are given as length of the beam is 0.5 m with rectangular cross section. The bottom and top layer are made up of steel with thickness 15 and 10 mm respectively, the core layer is of rubber material with thickness 20 mm, and width is 40 mm for all the layers. The properties used for the steel and rubber are as follows

For steel E_s =210 Gpa, G_s =80 Gpa, ρ_s =7850 kg/m^3 and

For rubber E_r =1.5 Mpa, G_r =0.5 MPa, ρ_r =950 kg/m^3.

For case 2, it is similar to the first one except that only the core layer of rubber is replaced with the lead with material properties as:

For lead E_1 =16 Gpa, G_1 =5.5 Gpa, ρ_1 =11,100 kg/m^3

The first four natural frequencies of the two cases with cantilever boundary conditions are as shown in the Table 6.1 together with results obtained by using the theory of Banerjee et al. [2007].

Table 6.1: Natural frequencies of a three layered sandwich beam with cantilever boundary conditions.

Mode No	Natural Frequency (rad/sec)			
	case 1		case 2	
	Reference [5]	Present Result	Reference [5]	Present Result
1	291.50	291.94	776.4	776.79
2	1684.48	1685.03	3841.1	3841.62
3	4623.98	4624.17	8753.1	8752.8
4	8945.18	8945.07	11459.2	11468.02

The natural frequencies obtained here are in good agreement with results of the Banerjee [5]. Here small deviations between the previous and current results are noticed. In the previous result, reference [5] they have used Timoshenko beam theory for the core layer. But in present theory for face layer Euler-Bernoulli beam theory has been taken.

6.2 Experimental Results

After the validation of the present developed model, an experiment has been conducted for further validation of the present theory. The prepared specimens which are discussed in the earlier section [5.2] are discussed here for modal analysis with the Fixed-Fixed and cantilever boundary conditions. The first three natural frequencies of the specimen which are experimented are compared here with theoretical results. The natural frequencies obtained through experiment are given in Tables 6.2 through 6.5.

Table 6.2: Natural frequencies of Specimen 1 (Aluminum-Rubber-Aluminum)

Mode No	Cantilever beam			Fixed- Fixed beam		
	Experiment (Hz)	Theoretical (Hz)	% Error	Experiment (Hz)	Theoretical (Hz)	% Error
1	21.64	18.36	15.16	25.64	22.73	11.34
2	38.42	44.20	15.04	58.20	53.59	7.92
3	91.88	89.275	2.83	97.85	103.37	5.64

Table 6.3: Natural frequencies of Specimen 2 (Aluminum-Neoprene-Aluminum)

Mode No	Cantilever beam			Fixed- Fixed beam		
	Experiment (Hz)	Theoretical (Hz)	% Error	Experiment (Hz)	Theoretical (Hz)	%Error
1	14.02	15.08	7.56	22.10	20.605	6.76
2	47.60	45.24	8.79	62.35	55.68	10.69
3	89.74	94.482	5.28	117.32	109.79	6.14

Table 6.4: Natural frequencies of Specimen 3 (Steel-Rubber-Steel)

Mode No	Cantilever beam			Fixed- Fixed beam		
	Experiment (Hz)	Theoretical (Hz)	% Error	Experiment (Hz)	Theoretical (Hz)	%Error
1	22.36	20.54	8.14	28.10	25.88	7.90
2	60.21	55.169	8.372	71.63	67.31	6.03
3	102.25	113.77	11.26	119.23	132.03	10.74

Table 6.5: Natural frequencies of Specimen 4 (Steel-Neoprene-Steel)

Mode No	Cantilever beam			Fixed- Fixed beam		
	Experiment (Hz)	Theoretical (Hz)	% Error	Experiment (Hz)	Theoretical (Hz)	%Error
1	22.63	18.91	16.43	25.26	24.27	3.92
2	59.10	54.29	8.13	74.54	66.58	10.67
3	121.65	113.25	6.90	120.67	131.58	9.04

From the tabular results one can infer that the values obtained using experiment and theoretical are in good agreement with acceptable errors.

6.3 Modal Analysis using FEA:

In this section the thickness of the damping layer has varied to study the damping effect on the sandwich beam for the specimens which are modeled in the previous chapter. The thickness of top and bottom layers are taken as 1 mm and 1.5 mm respectively and core layer varied as 5mm, 10 mm, 15 mm, and 20 mm. The length and width of the beam are 500mm and 40mm respectively. The material properties considered here are similar to that taken in the previous section. The natural frequencies obtained here for fixed-fixed and cantilever boundary conditions are given in Tables 6.0 through 6.13.

Fixed-Fixed boundary condition:

Table 6.6: Natural frequency of Specimen1 (Aluminum-Rubber-Aluminum) using FEA

Mode No	Natural Frequency (ω) in Hz			
	h_2= 5 mm	h_2= 10 mm	h_2= 15 mm	h_2= 20 mm
1	97.73	72.81	62.08	55.75
2	142.89	124.85	116.76	111.49
3	236.40	221.73	213.39	206.93
4	372.25	356.96	346.21	337.04
5	546.22	527.84	513.20	500.18

Table 6.7 Natural Frequency of Specimen2 (Aluminum-Neoprene- Aluminum) using FEA

Mode No	Natural Frequency (ω) in Hz			
	h_2= 5 mm	h_2= 10 mm	h_2= 15 mm	h_2= 20 mm
1	45.43	41.90	40.06	38.72
2	113.47	109.55	106.51	503.82
3	219.51	213.20	207.67	202.60
4	361.37	351.42	342.40	334.06
5	538.51	523.80	510.30	497.80

Table 6.8: Natural frequency of specimen3 (Steel-Rubber-Steel) using FEA

Mode No	Natural Frequency (ω) in Hz			
	h_2= 5 mm	h_2= 10 mm	h_2= 15 mm	h_2= 20 mm
1	68.17	56.07	51.22	48.47
2	127.85	120.88	117.78	115.70
3	233.60	228.16	224.92	222.28
4	379.15	373.42	369.13	365.29
5	562.39	555.39	549.48	543.95

Table 6.9: Natural Frequency of Specimen4 (Steel-Neoprene-Steel) using FEA

Mode No	Natural Frequency (ω) in Hz			
	h_2= 5 mm	h_2= 10 mm	h_2= 15 mm	h_2= 20 mm
1	43.89	42.58	41.87	41.33
2	116.64	115.16	113.93	112.79
3	227.51	225.07	222.82	220.66
4	375.29	371.42	367.73	364.16
5	559.66	553.93	548.40	543.04

Cantilever boundary condition:

Table 6.10: Natural Frequency of specimen1 (Aluminum-Rubber-Aluminum) using FEA

Mode No	Natural Frequency (ω) in Hz			
	h_2= 5 mm	h_2= 10 mm	h_2= 15 mm	h_2= 20 mm
1	89.12	61.54	49.14	41.66
2	97.76	72.71	61.91	55.55
3	143.17	125.02	116.87	111.57
4	236.48	221.67	213.25	206.73
5	372.07	356.63	345.75	336.45

Table 6.11: Natural frequency of Specimen2 (Aluminum-Neoprene- Aluminum) using FEA

Mode No	Natural Frequency (ω) in Hz			
	h_2= 5 mm	h_2= 10 mm	h_2= 15 mm	h_2= 20 mm
1	21.65	15.53	12.84	11.25
2	44.90	41.34	39.50	38.16
3	113.47	109.53	106.46	103.75
4	219.33	212.99	207.41	202.29
5	360.99	360.99	341.84	333.38

Table 6.12: Natural frequency of Specimen3 (Steel-Rubber-Steel) using FEA

Mode No	Natural Frequency (ω) in Hz			
	h_2= 5 mm	h_2= 10 mm	h_2= 15 mm	h_2= 20 mm
1	54.119	38.17	31.10	26.88
2	67.93	55.70	50.80	48.02
3	127.96	120.93	117.81	117.71
4	233.52	228.03	224.76	222.08
5	378.86	373.07	368.73	364.82

Table 6.13: Natural Frequency of Specimen4 (Steel-Neoprene-Steel) using FEA

Mode No	Natural Frequency (ω) in Hz			
	h_2= 5 mm	h_2= 10 mm	h_2= 15 mm	h_2= 20 mm
1	14.13	10.92	9.58	8.82
2	43.28	41.95	41.24	40.70
3	116.63	115.14	113.90	112.76
4	227.33	224.88	222.61	220.42
5	374.92	371.02	361.90	359.66

From the above results one can clearly see that with increase in the thickness of the core layer there is a decrease in natural frequency for each mode. This means the damping effect of the sandwich has been increased. The results obtained clearly shows that the beams modeled with Neoprene as a core layer has more damping effect as compared to the rubber for the fixed-fixed and cantilever boundary conditions.

6.4 Harmonic Analysis:

In this section harmonic analysis has been performed for the prepared specimens using FEA. For harmonic analysis the width of the beam considered as 50mm and length as 500 mm and rest of geometrical properties are taken as same in modal analysis and load of 1N is applied at the centre of the beam. The amplitude and frequency response plots have been plotted for fixed-fixed and cantilever boundary conditions.

For fixed-fixed boundary condition the frequency response is as follows:

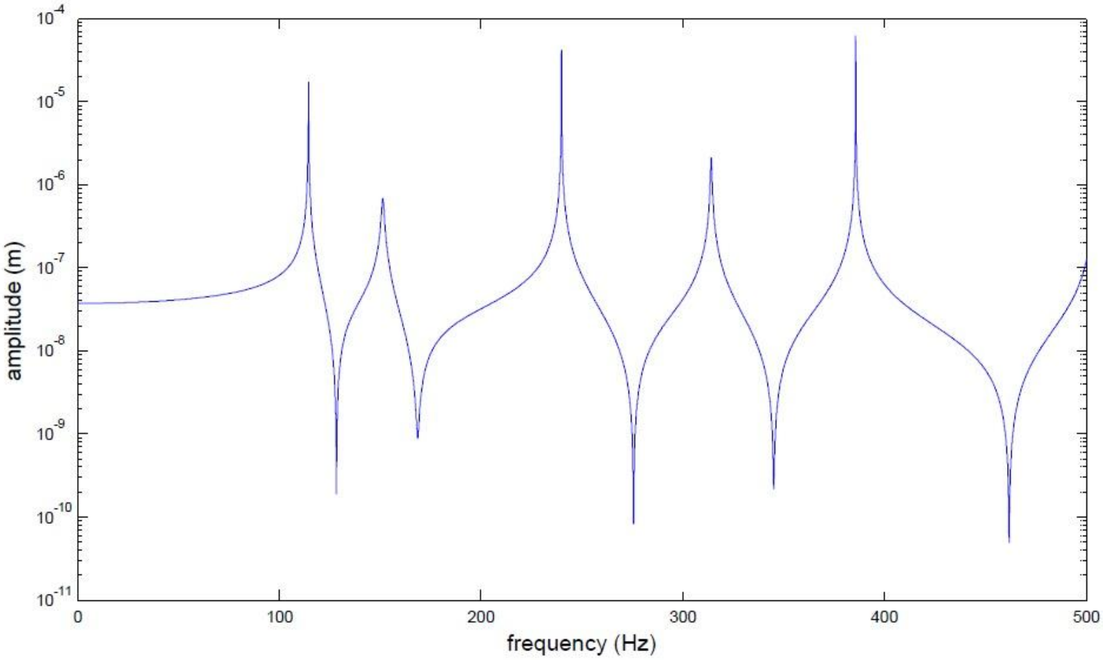

Figure 6.1: Frequency response of Aluminum-Rubber-Aluminum specimen

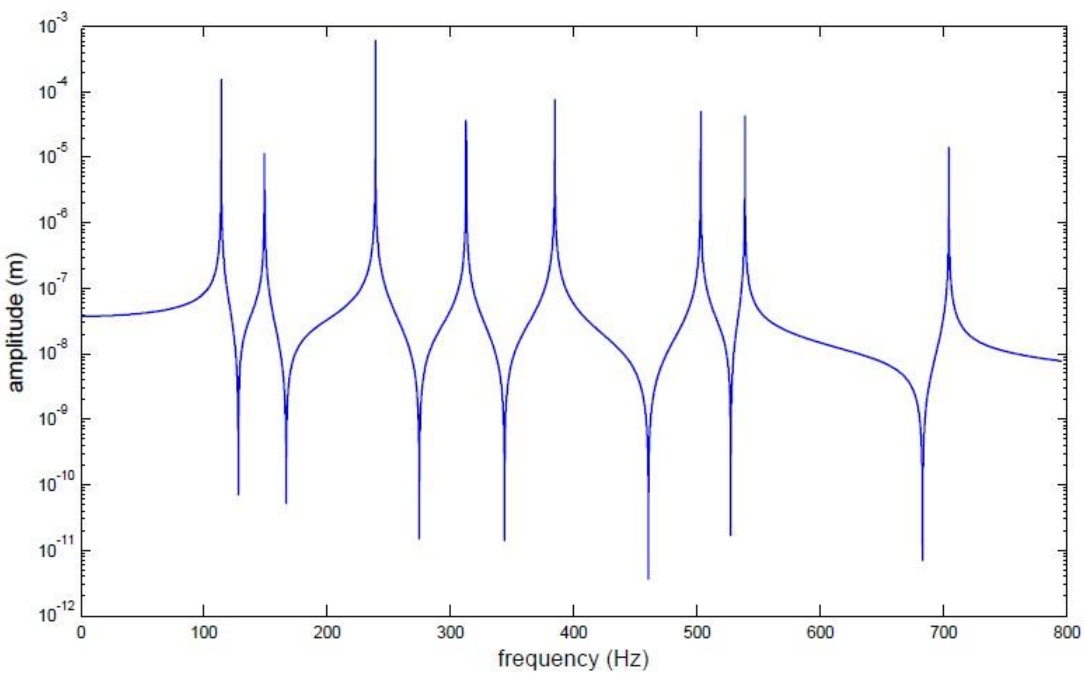

Figure 6.2: Frequency response of Aluminum-Neoprene-Aluminum specimen

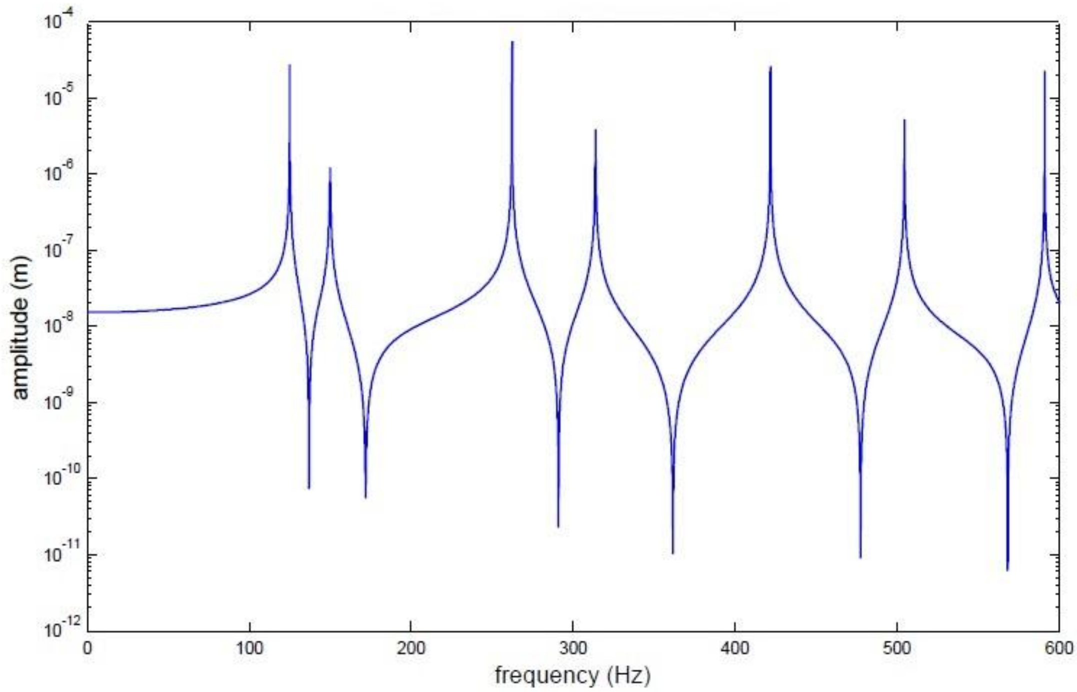

Figure 6.3: Frequency response of Steel-Rubber-Steel specimen

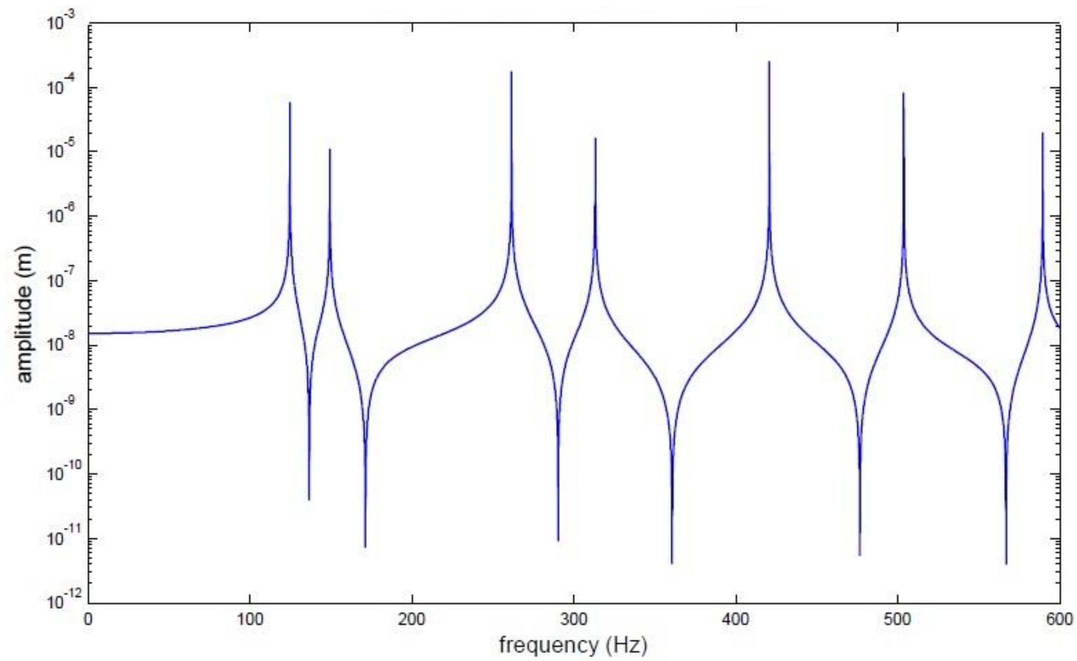

Figure 6.4: Frequency response of Steel-Neoprene-Steel specimen

For Cantilever boundary condition the frequency response is as follows

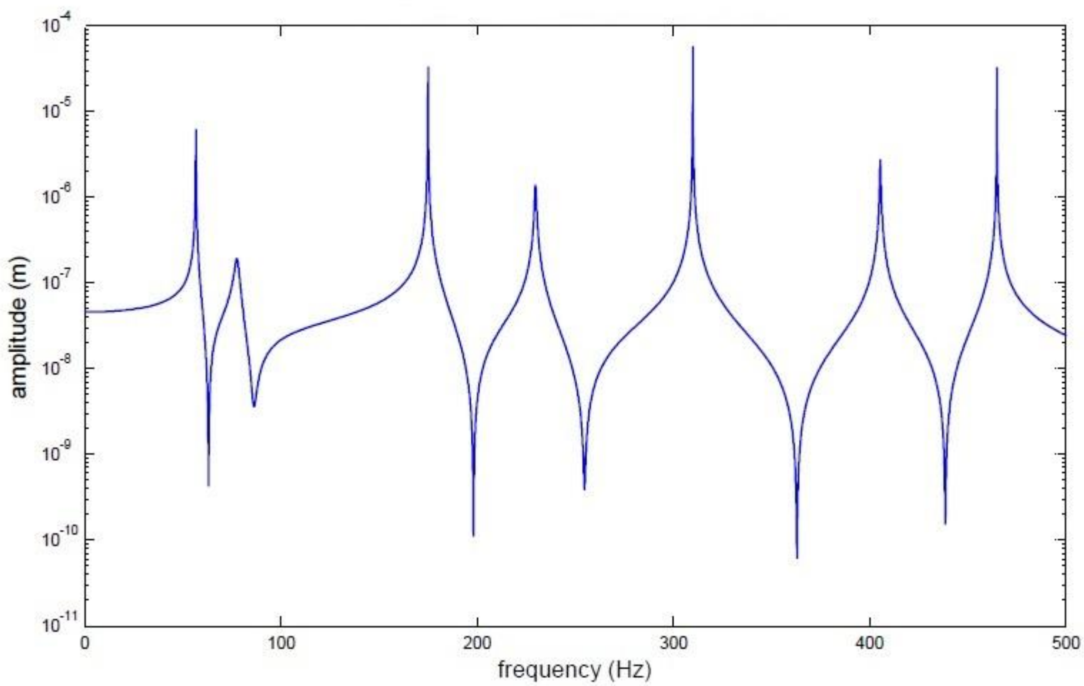

Figure 6.5: Frequency response of Aluminum-Rubber-Aluminum specimen

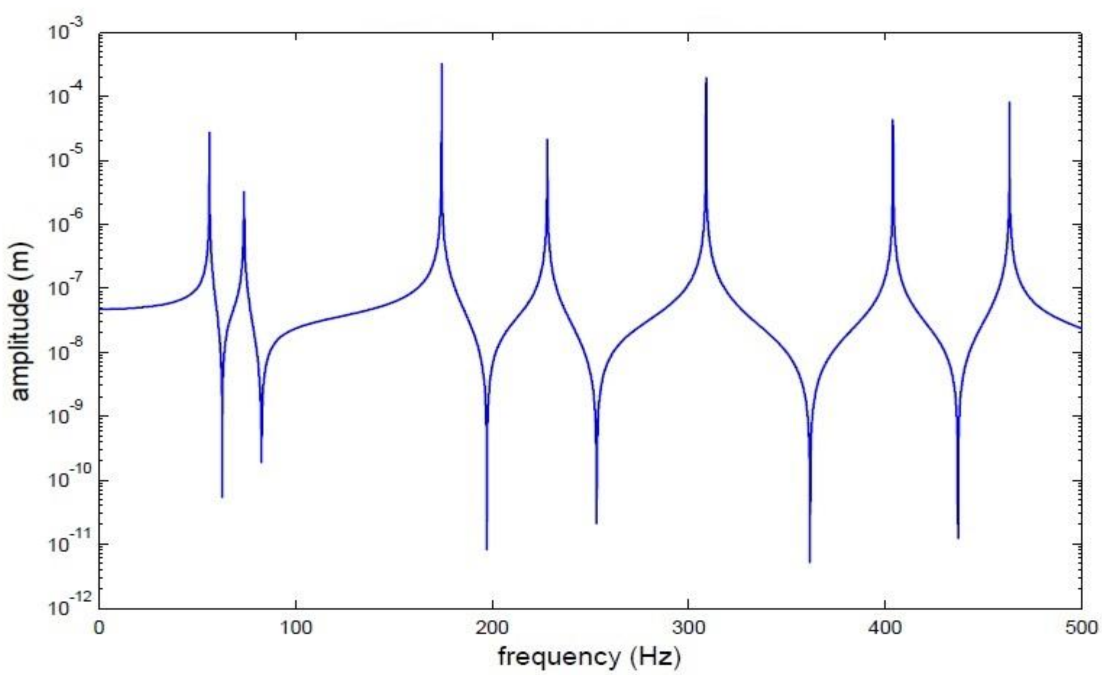

Figure 6.6: Frequency response of Aluminum-Neoprene-Aluminum specimen

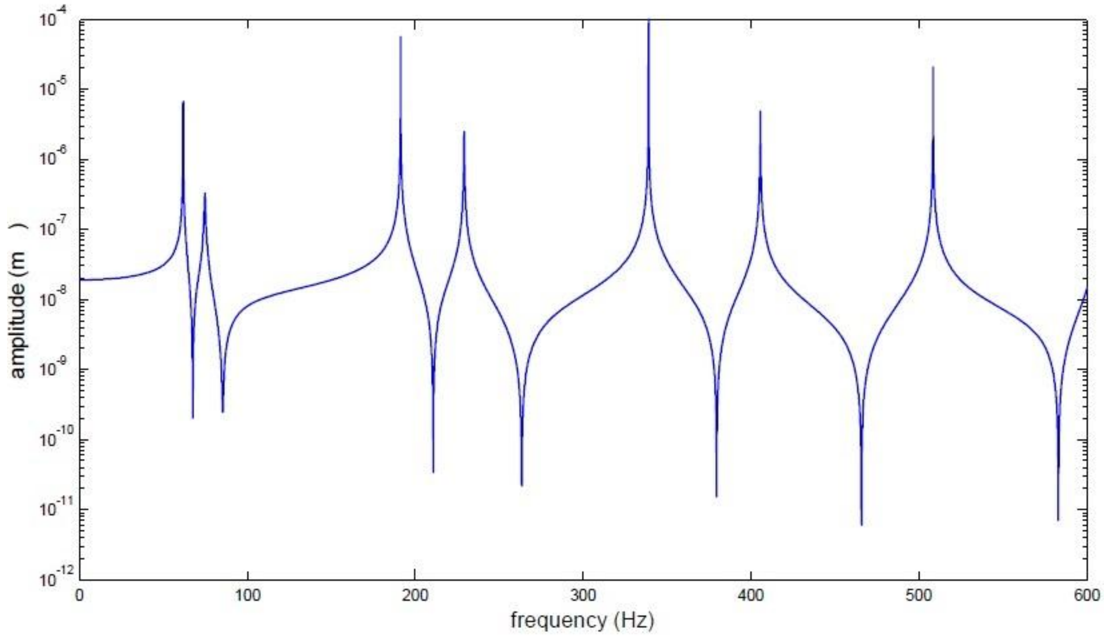

Figure 6.7: Frequency response of Steel-Rubber-Steel specimen

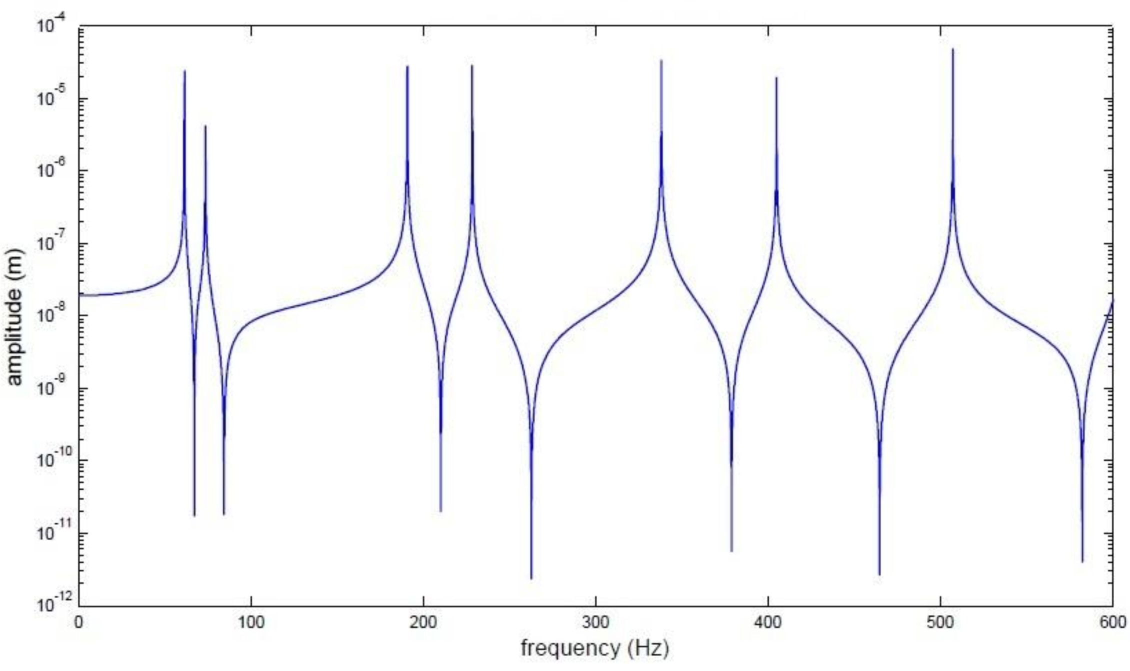

Figure 6.8: Frequency response of Steel-Neoprene-Steel specimen

These are harmonic response of the beams for fixed-fixed and cantilever boundary conditions for the specimens considered in the modal analysis and experimental analysis with varying the core and face layers. From the frequency response curves the sandwich beam with core layer as rubber provides less amplitude when compared to the neoprene under load conditions.

Chapter: 7

Conclusion

The viscoelastic sandwich beam has been successfully modeled using finite element method. The developed model have been validated with the earlier theory, experimental verification has also been done for the different types of sandwich beams modeled. The sandwich beams modeled here with varying of face and core layers. The sandwich beams modeled here are carried out for modal analysis using finite element method by varying the core thickness to study the damping effect on the beams for the fixed-fixed and cantilever boundary conditions. The results obtained from the modal analysis clearly shows that with increase in the thickness of the core layer there is a decrease in the natural frequency for the same mode. From the results one can infer that damping characteristics for neoprene viscoelastic material has significant effect when compared with the rubber viscoelastic material. Finally the frequency responses of the modeled sandwich beams have been plotted for the fixed-fixed and cantilever boundary conditions. Results show that the viscoelastic constrained layer damping treatment has a great significance in controlling the vibration of structures like beams, plates, etc.

7.1 Scope for future work

- The developed model can be extended to study the forced vibration of the sandwich beams for various boundary conditions.
- The thermal effects in the viscoelastic layer can be included for vibration analysis of sandwich beam.
- The face layers and core layers of the sandwich beam can be replaced with different materials like FGM and Composite materials for the vibration analysis and dynamic study.
- Vibration analysis can be done for the multiple boundary conditions for the multiple layers of the viscoelastic sandwich beams.

References

1. Kerwin EM. Damping of flexural waves by a constrained viscoelastic layer. J Acoust So Am 1959; 31: 952–962.

2. Di Taranto RA. Theory of vibratory bending for elastic and viscoelasic layered finite length beams. ASME J.ApplMech 1965; 32: 881–886.

3. Mead DJ and Markus S. The forced vibration of a three-layer damped sandwich beam with arbitrary boundary conditions. J Sound Vib 1969; 10: 163–175.

4. Bai JM and Sun CT. The effect of viscoelastic adhesive layers on structural damping of sandwich beams. Mech Struct Mach 1995; 23: 1–16.

5. J.R.Banerjee.Free vibration of a three-layered sandwich beam using the dynamic stiffness method and experiment. International journal of solids and structures 2007;44:7543-7563

6. Lu and Douglas. On the forced vibrations of three layer damped sandwich beams. J sound vib 1974; 32:513.516.

7. Mace M. Damping of beam vibrations by means of a thin constrained viscoelastic layer: evaluation of new theory. J Sound Vib 1994; 172: 577–591.

8. Baber TT, Maddox RA and Orozco CE. A finite element model for harmonically excited viscoelastic sandwich beams. Comput Struct 1998; 66: 105–113.

9. Chen Q and Chan YW. Integral finite element method for dynamical analysis of elastic and viscoelastic composite structures. Comput Struct 2000; 74: 51–64.

10. Y.A. Zhuk , I.A.Guz ,and C.M.Sands. Monoharmonic approximation in the vibration analysis of a sandwich beam containing piezoelectric layers under mechanical or electrical loading. . J Sound Vib 330 (2011) 4211–4232.

11. T Sakiyama et.al .Free vibration analysis of sandwich Beam with elastic or viscoelastic core by applying the discrete green function. Journal of Sound and Vibration(1996) 191(2),189-206.

12. S.G. Won et al. Three-layered damped beam element for forced vibration analysis of symmetric sandwich structures with a viscoelastic core. Finite Elements in Analysis and Design 68 (2013) 39–51.

13. S.M.R. Khalili Free vibration analysis of sandwich beam carrying sprung masses International Journal of Mechanical Sciences 52 (2010) 1620–1633 .

14. J.R. Banerjee and A.J. Sobey Dynamic stiffness formulation and free vibration analysis of a three-layered sandwich beam International Journal of Solids and Structures 42(2005) 2181–2197.

15. M. Chehel Amirani Free vibration analysis of sandwich beam with FG core using the element free Galerkin method Composite Structures 90 (2009) 373–379.

16. Fei Lin and Mohan D. Rao Vibroacoustical Analysis of Multiple-Layered Structures with Viscoelastic Damping Cores: Hindawi Publishing Corporation ISRN Mechanical Engineering Volume 2013, Article ID 645232, 13 pages.

17. Farough Mohammadi and Ramin Sedaghati. Vibration analysis and design optimization of sandwich cylindrical panels fully and partially treated with electrorheological fluid materials. Journal of Intelligent Material Systemsand Structures 23(15) 1679–1697.

18. Jasrobin Singh Grewal, Ramin Sedaghati and Ebrahim Esmailzadeh. Vibration analysis and design optimization of sandwich beams with constrained viscoelastic core layer. Journal of Sandwich Structures and Materials 15(2) 203–228.

19. Binod P. Yadav, Vibration damping using four-layer sandwich. Journal of Sound and Vibration. 317 (2008) 576–590.

20. E.M. Daya, L. Azrar, M. Potier-Ferry. An amplitude equation for the non-linear vibration of viscoelastically damped sandwich beams. Journal of Sound and Vibration 271 (2004) 789–813.

21. F.S. Barbosa_, M.C.R. Farage. A finite element model for sandwich viscoelastic beams: Experimental and numerical assessment. Journal of Sound and Vibration 317 (2008) 91–111.

22. N. Jacques , E.M.Daya , M.Potier-Ferry. Nonlinear vibration of viscoelastic sandwich beams by the harmonic balance and finite element methods. Journal of Sound and Vibration 329 (2010) 4251–4265.

23. J.-J.R.B. Bekuit, D.C.D. Oguamanam, O. Damisa Aquasi-2D finite element formulation for the analysis of sandwich beams. Finite Elements in Analysis and Design 43 (2007) 1099 – 1107

24.Farough Mohammadi, Ramin Sedaghati. Vibration analysis and design optimization of viscoelastic sandwich cylindrical shell. Journal of Sound and Vibration 331 (2012) 2729–2752.