VIBRATIONAL ANALYSIS OF LAMINATED COMPOSITE TURBO MACHINERY BLADES

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VIBRATION ANALYSIS OF LAMINATED COMPOSITE TURBOMACHINERY BLADES

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in

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By

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CERTIFICATE

*This is to certify that the work in the thesis entitled "**Vibration Analysis of Laminated Composite Turbomachinery Blades**", submitted by **Sindhuja Gantayet**, bearing roll no. **110CE0409**, is a record of an authentic research work done by her under my guidance and supervision in partial fulfillment of the requirements for the award of the degree of **Bachelor of Technology** in **Civil Engineering, National Institute of Technology, Rourkela.***

To the best of my knowledge, the results embodied in this thesis have not been submitted to any other University/Institute for the award of any Degree or diploma.

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ABSTRACT

Composites or the composite materials have profound applications in various aspects of engineering, ranging from aerospace to civil, automobile to marine engineering. The present study deals with the study of twisted plates or turbo-machinery blades made of composite materials which form the fundamental element of many structural components. Blades are the key structural units in turbo machinery of the aeronautical and aerospace industries. Rotating structures having the shape of pre-twisted blades commonly occur in several types of practical engineering areas such as turbines, airplane propellers, helicopter rotors and aircraft rotary wings. Turbo machinery is widely used in industries as turbo engines, turbo generators, turbofans etc. These plates are subjected to in-plane load on account of fluid or aerodynamic pressures. The structures implementing turbo machinery blades are subjected to high dynamic loadings. It is therefore, quite significant for design, safety and life of the machinery to determine their dynamic character accurately as they are working at high speeds. This fact necessitates the use of composites as load bearing structural components in aerospace and naval structures, automobiles, pressure vessels, turbine blades and many other engineering applications because of their high specific strength and stiffness.

The present study deals with the vibration analysis of the laminated composite twisted plates subjected to free vibrations. The analysis commences with the review of the previous works done in order to gather a rough idea regarding the said matter. The composite plates of different layers with different dimension are manufactured using woven glass fiber and epoxy matrices. The study proceeds with the determination of elastic constants of the laminated composite plates used for characterization, prepared in the laboratory by performing the tensile testing of the specimen. Free vibration characteristics are studied using FFT analyzer, accelerometer using impact hammer excitation. The FRFs are studied to obtain a clear understanding of the vibration characteristics of the specimens. The modal testing of the rectangular laminated composite plates is done in the laboratory and the results obtained experimentally are compared with the results obtained from the program based on FEM and coded in MATLAB environment.

The analysis further proceeds with the study of the effects of the various parameters such as increase in the number of layers, aspect ratio, side to thickness ratios, angle of twist etc using ANSYS. An eight-node iso-parametric quadratic element is considered in the present analysis with 5 degrees of freedom per node. In ANSYS, the shell 281 element with five degrees of freedom per node is used. From the convergence studies, an eight by eight mesh is found to supply good accuracy. The vibration behaviour of composite laminated twisted plate under

various types of non-uniform in-plane loading is studied. The effect of number of layers, changing angle of twist, width to thickness ratio, aspect ratio, etc on the vibration are presented.

LIST OF SYMBOLS

Symbol	Description
‘a’, ‘b’	dimensions of the twisted panel
(a/b)	aspect ratio of the twisted plate
A_{ij},B_{ij},D_{ij},S_{ij}	Extensional, bending-stretching-coupling, Bending and transverse shear stiffness
(b/h)	width to thickness ratio of the plate
[B]	Strain displacement Matrix of the element
[D]	Stress-strain Matrix
[D_p]	Stress strain matrix for plane stress
dx,dy	length of the element in respective axes
dV	Volume of the element
E_{11} , E_{22}	Elasticity Modulus in longitudinal and Transverse direction
G_{12}, G_{13}, G_{23}	Shear moduli
J	Jacobian
k	shear correction factor
[K_e]	global elastic stiffness matrix
[k_E]	element bending stiffness matrix
[K_g]	geometric stiffness matrix
[K_p]	plane stiffness matrix
$K_{x.}$ $k_{y,}$ k_{xy}	Bending strains
[M]	Global consistent mass matrix

$[m_e]$	Element consistent mass matrix
M_x, M_y, M_{xy}	Moment resultants
$[N]$	Shape function matrix
N_i	Shape function
N_{cr}	Critical Load
N_x, N_y, N_{xy}	In-plane stress resultants
$N_x°$, $N_y°$	External loading
$[P]$	Mass Density parameters
q	Vector of degrees of freedom
R_x, R_y, R_{xy}	Radii of curvature
u, v, w	displacement components at any point
u_0, v_0, w_0	displacement components at midpoint
w	out of plane displacement
X,Y,Z	Global Coordinate System
Ξ_x, ξ_y, ξ_{xy}	strains at a point
E_{xnl}, γ_{xnl}	Non-linear strain components
‘ξ’,’η’	local nature coordinates
‘ρ’	mass density of the element
‘σ_x^0’, ‘σ_y^0’, ‘σ_{xy}^0’	in-plane stresses due to external load
‘τ_x’, ‘τ_y’, ‘τ_{xy}’	shear stresses
‘ω’	frequencies of vibration
‘Φ’	angle of twist

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1

INTRODUCTION

1.1 Introduction

Composite materials are extending the horizons of all branches of engineering as they have marked their presence in different engineering structures with the domain ranging in the field of aerospace, marine, civil, biomedical, automobiles etc. The composites undergo the process of optimization where materials are combined in such a way that their virtues such as high specific strength, excellent fatigue resistance, high hygroscopic sensitivity and high resistance, can be put to use in a better way while minimizing the extents of the effect of their deficiencies.

The term “turbomachinery” is a quite a renowned one owing to its applications in turboengines, turbogenerators, turbofans etc. Blades play a pivotal role as the structural units in the turbomachinery of aeronautical and aerospace industries. Rotating structures employing twisted cantilever panels have significant applications in wide chord turbine blades, fan blades, compressor blades, aircraft or marine propellers, helicopter rotors, aircraft rotary wings and particularly in gas turbines. Thus the above mentioned applications of the twisted plates highlight the essence of twisted cantilever panels in present research field.

1.2 Importance of the Present Study

The structures implementing turbo machinery blades are subjected to high dynamic loadings. The twisted plates are also subjected to different types of loads due to fluid pressure or transverse loads. The blades are constantly subjected to axial periodic forces as they form the part of the axial components of aerodynamic or hydrodynamic forces acting on the blades. It is therefore, quite significant for design, safety and life of the machinery to determine their dynamic character accurately as they are working at high speeds. Failure of turbine blades often occurs as a result of sustained blade vibration at or near natural frequencies, hence knowledge of these frequencies is of fundamental importance. Moreover, to ensure reliable and economic delivery of the designs of the structures, it is necessary to estimate the vibration characteristics of those structures accurately.

1.3 Outline of the Present Work

The present study deals with the vibration characteristics of the laminated composite twisted plates. The project includes the determination of tensile characteristics of the laminated composite plates developed in the laboratory followed by the examination of the modal characteristics of the plates. The work subsequently proceeds with the study of the influence of various parameters listed as aspect ratio, side to thickness ratio, angle of twist and number of layers on the vibration of the twisted panels using finite element package, ANSYS software.

This thesis is broadly divided into seven chapters. The current chapter introduces the application of composite materials and the twisted plates followed by the briefing about the importance of the present study in the research arena.

In chapter 2, a detailed review of the literature pertinent to the previous works along with a critical discussion of the investigations conducted in this field has been listed. The aim and scope of the present study is also outlined in this chapter.

In chapter 3, the entire set of equations governing the fundamentals of vibration of plates are presented in a detailed manner for better mathematical understanding of the current aspect.

In chapter 4, the procedural steps for tensile testing of the plates for characterization, modal testing of the plates and study of ANSYS have been outlined in an expanded manner.

In chapter 5, a thorough discussion regarding the results obtained by introducing variation in parametric conditions while modal testing of the laminated composite pre-twisted plate has been done.

In chapter 6, the inferences that are drawn by analyzing the numerical results obtained in the previous chapter are summarized.

In chapter 7, some of possible the areas where further study can be done have been pointed out.

2

REVIEW OF LITERATURE

2.1 Review of Literature

The widespread domain of the applications of the turbo machinery blades in the industrial and technological arenas have paved the way for a large number of research works concerning twisted plates. It is highly essential to study the vibration characteristics of such plates in order to determine its behavior under dynamic loadings that they are subjected to. The following paragraphs present the previous works that have been done to investigate the frequency variation pertaining to variation in loadings, support conditions, parameters etc.

Walker employed shell theory for the analysis of curved twisted fan blades by preparing a conforming finite shell element. The element is assumed to be a doubly curved right helicoidal shell with low aspect ratio. Element stiffness and mass formulations are based on Mindlin's theory and the effects of transverse shear and rotary inertia are included.

Free vibration characteristics of a rotating small aspect ratio pre-twisted blade are determined by Rao and Gupta using classical bending theory of thin shells. Analysis of differential geometry of the blade in curvilinear coordinates was done and strain-displacement relations were formed. Determination of the strain and kinetic energies of the rotating and vibrating blades was done followed by the establishment of the Lagrangian function. The natural frequencies and mode shapes of the blade are calculated.

Khader Naim developed a ten node triangular shell element of thirty degrees of freedom and applied to shell static analysis as well as pre-twisted and cambered fan blades. The natural frequencies of cambered and untwisted fan blades having a rectangular platform with constant thickness are calculated for blade thickness and blade different shallowness ratio, blade tip twist angle etc. Coarse mesh size was used to land up with accurate results.

Chen utilized a finite element model to analyze the vibration behavior of a pre-twisted rotating blade with a single edge crack. The study focused on the influences of the crack location and the crack size on buckling loads, natural frequencies and dynamic instability and developing a model that satisfies the geometric boundary conditions and natural boundary conditions on the blades.

Chung, Kwak and Yoo studied the effects of dimensionless parameters on the modal characteristics of the rotating blades with a concentrated mass through numerical analysis. The resulting equations for the vibration analysis are converted into a dimensionless form in which dimensionless parameters are identified.

Chou and Choi proposed Modified Differential Quadrature Method (MDQM) analysis for elastically supported turbo machinery blades with varying cross sections. The equations of motions for the coupled flexural and torsional vibrations are obtained using Hamilton’s principle. Validation of the numerical results of the elastically supported blades with or without a shroud derived by the MDQM , was done by comparing with the analytic solutions.

Sakiyama, Hu, Matsuda and Lorita devised a method for vibration of a rotating cantilever blades with pre-twist developed by the principle of virtual work and Rayleigh-Ritz method. The study includes the analysis of the deformation and stress resultants caused by the rotation, formulation of eigen frequency equation of a rotating cantilever conical shell with pre-twist using equilibrium of energy for vibration and investigation of the effects of parameters such as an angular velocity, a setting angle, a radius of a hub, a subtended angle, a twist angle and a tapered ratio of cross section on fundamental vibration.

The study of the natural frequencies of folded plate structures using finite element transfer matrix method was done by Liu and Huang. The division of the cantilever folded plate structures into a series of parallel strips was done which were further divided into sequence of elements.

Lee Sen-Yung investigated the divergence instability and vibration of a rotating Timoshenko beam with pre-cone and pitch angle using Hamilton's principle.

2.2 Methodology

The project aims to study the vibration characteristics of a laminated composite twisted plate using ANSYS software in the environment of the Finite Element Analysis. The study commences with the development of a model of the laminated twisted plate which is subjected to free-free vibrations and cantilever type boundary conditions. For better accuracy the developed model is meshed into 8×8 mesh size. The modal analysis is done with a frequency range of 0 Hz to 500 Hz. The above procedure is adopted to investigate the influence of various parameters, namely aspect ratio, side to thickness ratio, angle of twist and number of layers on vibration of the modeled twisted panel.

2.3 Critical Discussion

After a brief insight to the review of literature, it is quite clear how the study of the vibration characteristics of the twisted plates has captured the attention of a large number of researchers. As regards to methodology, there has been a subtle shift i.e. the focus took a turn from analytic methods to numerical ones with the use of finite element methods and experimental methods.

From the above review of literature, we can get an idea regarding the inherent lacunae in the earlier investigations which seek further attention. Earlier the methods adopted revolve were primarily analytical. The present day studies investigate in the light of recently developed software which is more accurate, as a result of which effects of the numerous assumptions considered have been minimized. Also the effect of wide range of parameters is also taken into consideration.

2.4 Objective and Scope of the Present Study

Vibration characteristics of rotating blades are required for efficient performance prediction and high specific strength. Complexity of configuration such as twist, curvature, non-uniform cross section, thickness etc. needs to be studied for better functioning of such blades. In the past several decades both analytical and experimental techniques have been developed to analyze their vibration behavior. The present study is aimed to analyze the dynamic properties of turbo machinery blades using ANSYS. The project intends to include the following tests and experiments to get a complete analysis regarding vibration characteristics of turbo machinery blades:

- Tensile testing for characterization
- Modal testing of turbo machinery blades using ANSYS
- Tests to study the effect of parameters such as aspect ratio, twist angle and number of layers.

3

MATHEMATICAL FORMULATION

3.1 Governing Differential Equation

The differential element of the twisted panel is used to obtain the differential equations as shown in figure 3.1. This figure displays an element having internal forces like shearing forces Q_x and Q_y, membrane forces N_x, N_y and N_{xy}, and the moment resultants M_x, M_y and M_{xy}.

The differential equations governing the equilibrium of a shear deformable pre-twisted curved panel exposed to external in-plane loading can be given by

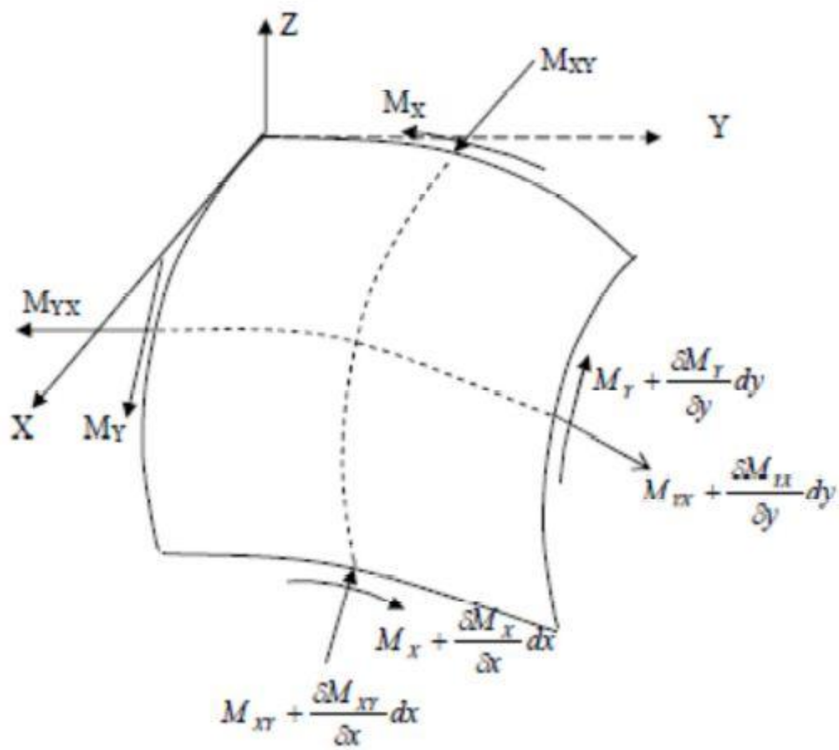
$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial y} + \frac{Q_x}{R_x} + \frac{Q_y}{R_{xy}} = P_1\frac{\partial^2 u}{\partial t^2} + P_2\frac{\partial^2 \theta_x}{\partial t^2}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial x} + \frac{Q_y}{R_y} + \frac{Q_x}{R_{xy}} = P_1\frac{\partial^2 v}{\partial t^2} + P_2\frac{\partial^2 \theta_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_x}{R_y} - 2\frac{N_{xy}}{R_{xy}} + N_x^0\frac{\partial^2 w}{\partial x^2} + N_y^0\frac{\partial^2 w}{\partial y^2} = P_1\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = P_1\frac{\partial^2 \theta_x}{\partial t^2} + P_2\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = P_1\frac{\partial^2 \theta_y}{\partial t^2} + P_2\frac{\partial^2 v}{\partial t^2} \qquad (3.1)$$



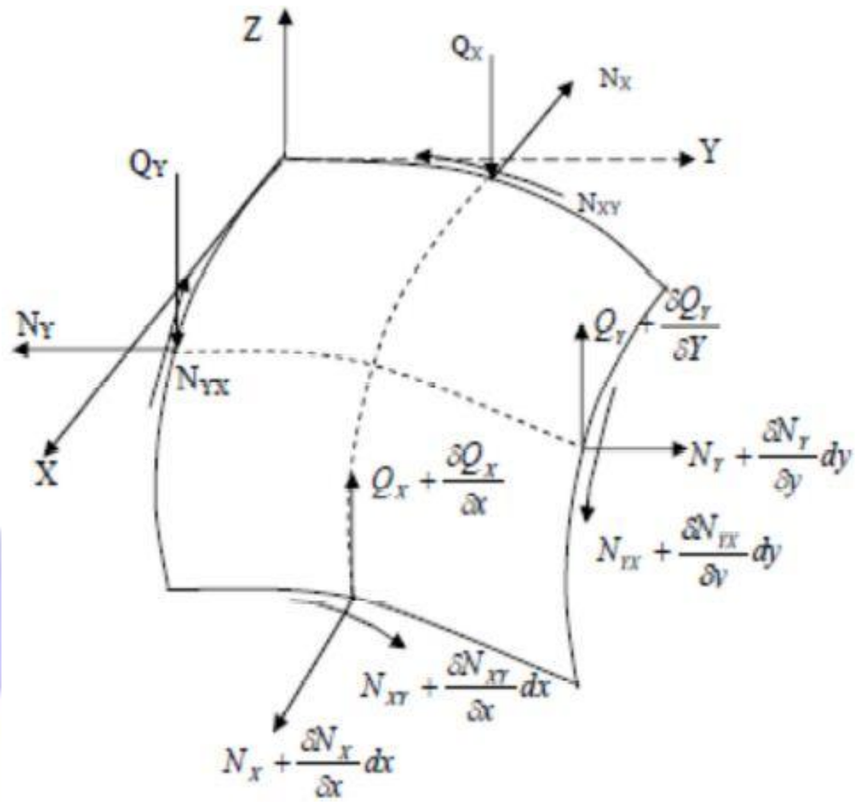
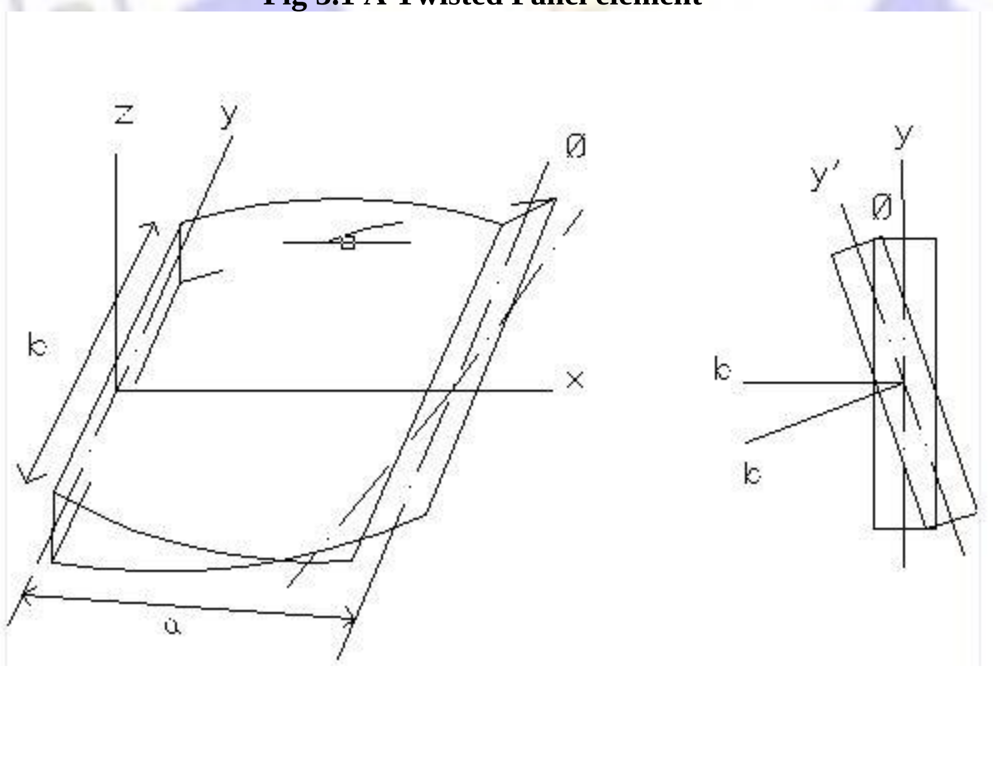


Fig 3.1 A Twisted Panel element



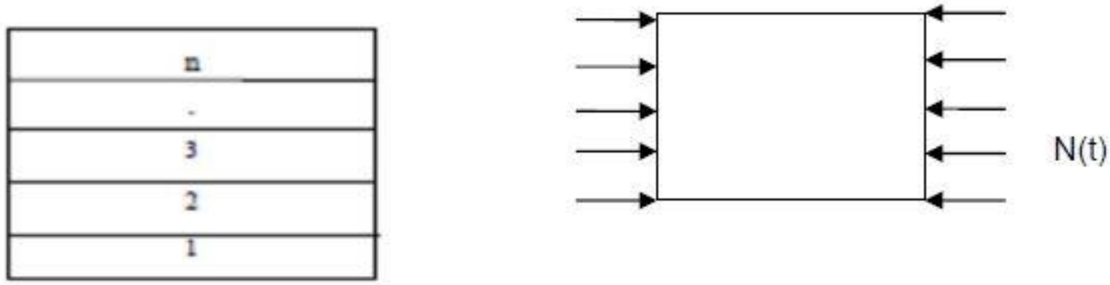


Fig 3.2 Laminated Composite Twisted plate with in plane loads

Where n is number of layers in the panel
Φ is the angle of twist
'b' and 'a' are the width and length of the plate
'N_x' and 'N_y' are the external loading and
'R_x' , 'R_y' & 'R_{xy}' are the radii of curvature (where x and y denotes X-direction and Y- direction respectively)

$$(P_1, P_2, P_3) = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} (\rho)_k (1, z, z^2) dz \quad (3.2)$$

Where n is the number of layers in the laminated composite twisted plate and (ρ_k) is the mass density of k_{th} layer from the mid plane.

3.2 Energy Equations

The in-plane stresses developed in the panel are $\sigma_x°$, $\sigma_y°$ and $\sigma_{xy}°$.
Total stresses at any layer = Initial stresses + Stresses due to bending
And shear
The strain energy arising due to in-plane stresses is:

$$U_0 = \frac{1}{2} \int \int \{\varepsilon^0\}^r \{\sigma^0\} dA \quad (3.3)$$

Here

$$\{\varepsilon^0\}^r = \left[\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right]^r = \left[\frac{\partial u^0}{\partial x}, \frac{\partial v^0}{\partial y}, \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x}\right] \quad (3.4)$$

Stresses are:

$$\{\sigma^0\} = [D_P]\{\varepsilon^0\} \quad (3.5)$$

And the strain can be given by:

$$\{\varepsilon^0\} = [B_P]\{q^0\} \quad (3.6)$$

By putting stress and strain values in the equation we obtain:

$$U_0 = \frac{1}{2} \int \int \{q^0\}^T [B_P]^T [B_P]\{q^0\} dA \quad (3.7)$$

$$[K_P] = \int\int [B_P]^T [D_P]\,[B_P] dA$$

We know that

$U_0 = U_1 + U_2$ (3.8)

$$U_1 = \frac{1}{2}\int\int\int [\{\varepsilon_l\}^T [D] \{\varepsilon_l\}]\, dV \tag{3.9}$$

$$U_2 = \frac{1}{2}\int\int\int [\{\sigma^0\}^T \{\varepsilon_{nl}\}]\, dV \tag{3.10}$$

Where

U_1 = Strain energy achieved from bending with transverse shear

U_2 = Work done by the nonlinear strain and initial in plane stresses

The kinetic energy is given by:

$$V = \int\int \left[\frac{h}{2}\left\{\frac{\partial \bar{u}^2}{\partial t} + \frac{\partial \bar{v}^2}{\partial t} + \frac{\partial \bar{w}^2}{\partial t}\right\} + \frac{h^3}{12}\left\{\frac{\partial \theta_x^2}{\partial t} + \frac{\partial \theta_y^2}{\partial t}\right\}\right] dxdy \tag{3.11}$$

So presenting in the matrix form we have

$$U_0 = \frac{1}{2}\{q\}^T [K_P] \{q\}$$

$$U_1 = \frac{1}{2}\{q\}^T [K_e] \{q\}$$

$$U_2 = \frac{1}{2}\{q\}^T [K_g] \{q\} \tag{3.12}$$

$$V = \frac{1}{2}\{\dot{q}\}^T [M] \{\dot{q}\}$$

Where

$[K_p]$= plane stiffness matrix of the twisted plate

$[K_e]$= Bending stiffness matrix along with shear deformation of the plate

$[K_g]$= geometric stiffness/stress stiffness matrix of the twisted plate

$[M]$= Consistent mass matrix of the twisted plate

3.3 Finite Element Formulation

Finite element methods have been used for complex geometrical and boundary conditions. The finite element formulation is developed using first order shear deformation theory. Each lamina of the bonded layer is assumed to be homogenous and orthotropic and made up of unidirectional fiber reinforced material. The shape function of the element is derived using the interpolation polynomial as follows:

$$u(\xi,\eta)=\alpha_1+\alpha_2\,\xi+\alpha_3\,\eta+\alpha_4\,\xi^2+\alpha_5\xi\eta+\alpha_6\,\eta^2+\alpha_7\,\xi^2\eta+\alpha_8\,\xi\eta^2 \qquad (3.13)$$

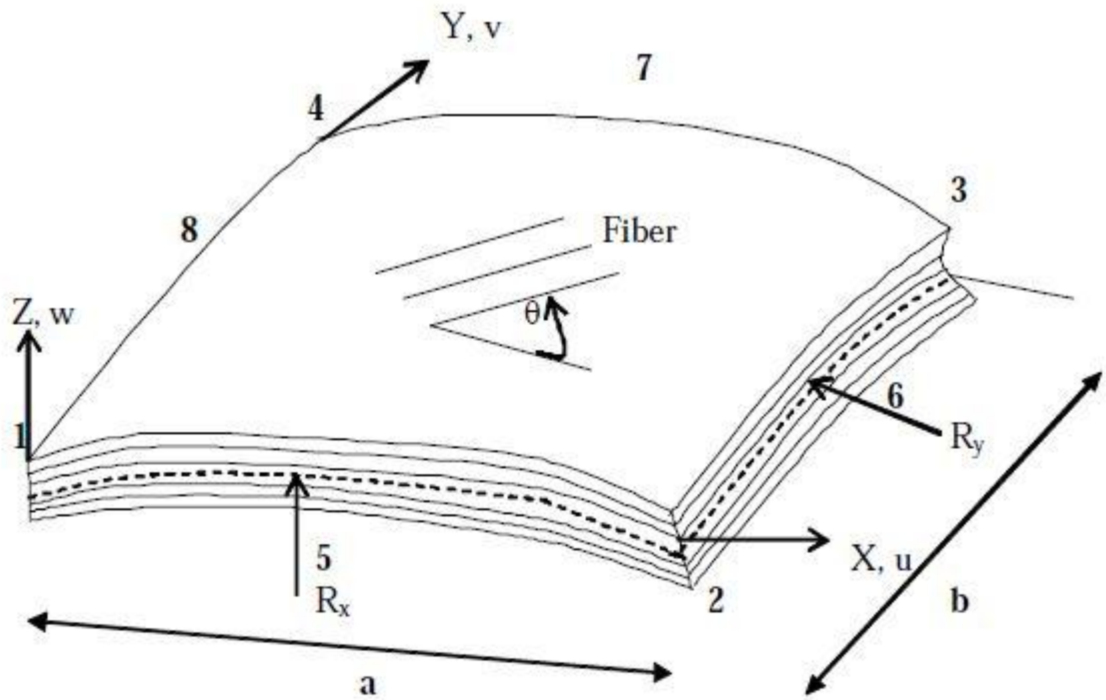


Fig 3.3 Isoparametric quadratic shell element

The shape function N_i are defined as:

$$N_i=(1+\xi\xi_i)(1+\eta\eta_i)\,(\xi\xi_i+\eta\eta_i-1)/4 \qquad i=1\text{ to }4$$

$$N_i=(1-\xi^2)\,(1+\eta\eta_i)/2 \qquad i=5,7 \qquad (3.14)$$

$$N_i=(1+\xi\xi_i)\,(1-\eta^2)/2 \qquad i=6,8$$

Where ξ and η are local coordinates.
The derivatives of the shape function are given by the following relationship:

$$\begin{bmatrix} N_{i,x} \\ N_{i,y} \end{bmatrix} = [J]^{-1}\begin{bmatrix} N_{i,\xi} \\ N_{i,\eta} \end{bmatrix} \qquad (3.15)$$

Where

$$[J]=\begin{bmatrix} X_{I,\xi} & Y_{I,\xi} \\ X_{i,\eta} & Y_{i,\eta} \end{bmatrix} \tag{3.16}$$

Is the Jacobian matrix

We have

$$u(x,y,z)= u_0(x,y)+ z\,\theta_y(x,y)$$

$$v(x,y,z)= u_0(x,y)+ z\,\theta_x(x,y) \tag{3.17}$$

$$w(x,y,z)= w_0(x,y)$$

Where u,v,w and u_0, v_0, w_0 are displacements in the x, y and z directions.

Also

$$x =\sum N_i x_i, \qquad y = \sum N_i y_i$$

$$u_0 =\sum N_i u_i\ v_0= \sum N_i v_i\ w_0 = \sum N_i w_i \tag{3.18}$$

$$\theta_x = \sum N_i \theta_{xi}\ \theta_y = \sum N_i \theta_{yi}$$

3.4 Strain-Displacement Relations

The linear strain displacement relation derived for a twisted shell element is given by:

$$\xi_{xl}= \frac{\partial u}{\partial x} + \frac{w}{R_x} + zk_x$$

$$\xi_{yl}= \frac{\partial v}{\partial y} + \frac{w}{R_y} + zk_y$$

$$\gamma_{xyl}= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} + zk_{xy} \tag{3.19}$$

$$\gamma_{xzl}= \frac{\partial w}{\partial x} + \theta_x - \frac{u}{R_x} - \frac{v}{R_{xy}}$$

$$\gamma_{yzl}= \frac{\partial w}{\partial y} + \theta_y - \frac{v}{R_y} - \frac{u}{R_{xy}}$$

And the bending strains are:

$$k_x = \frac{\partial \theta_x}{\partial x},\ k_y = \frac{\partial \theta_y}{\partial y}$$

$$k_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{3.20}$$

The linear strains are expressed as:

$$\{\varepsilon\} = [B]\{d_e\} \tag{3.21}$$

Where

$$\{d_e\} = \{u_1 v_1 w_1 \theta_{x1} \theta_{y1} \ldots\ldots\ldots\ldots\ldots\ldots u_8 v_8 w_8 \theta_{x8} \theta_{y8}\} \tag{3.22}$$

$$[B] = [[B_1], [B_2] \ldots\ \ldots\ldots\ldots\ldots\ldots\ldots [B_8]] \tag{3.23}$$

$$[B_i] = \begin{bmatrix} N_{i,}x & 0 & \frac{N_i}{R_x} & 0 & 0 \\ 0 & N_{i,}y & \frac{N_i}{R_y} & 0 & 0 \\ N_{i,}y & N_{i,}x & 2\frac{N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,}x & 0 \\ 0 & 0 & 0 & 0 & N_{i,}y \\ 0 & 0 & 0 & N_{i,}y & N_{i,}x \\ 0 & 0 & N_{i,}x & N_i & 0 \\ 0 & 0 & N_{i,}y & 0 & N_i \end{bmatrix} \tag{3.24}$$

3.5 Constitutive Relations

The composite twisted plate constituting of lamina of composite material (glass fiber) embedded in a matrix material (epoxy) is studied here. The principal axes are shown by 1 and 2 and the elastic moduli along these axes are shown by E_{11} and E_{22}.

$$
\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \qquad (3.25)
$$

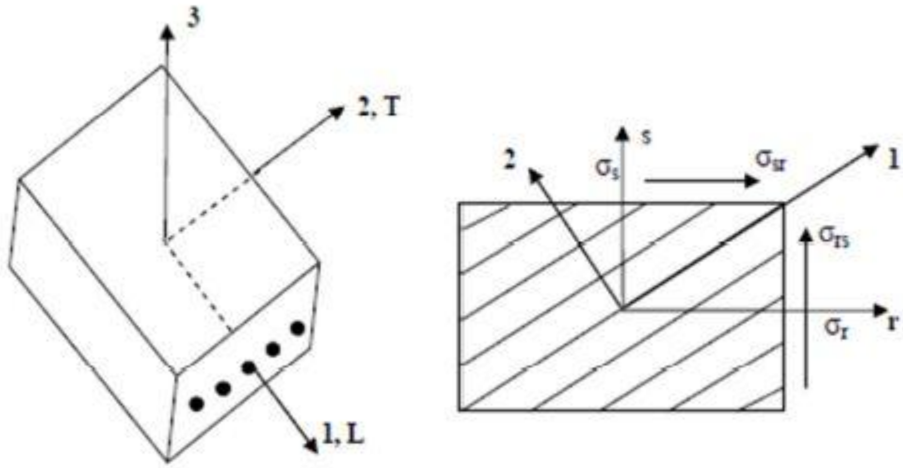


Fig 3.4 Laminated shell element with principal axes and laminate directions

Where

$$Q_{11} = \frac{E_{11}}{(1 - v_{12}v_{21})}$$

$$Q_{12}= \frac{E_{11}v_{21}}{(1-v_{12}v_{21})}$$

$$Q_{21}= \frac{E_{22}}{(1-v_{12}v_{21})}$$

$$Q_{22}= \frac{E_{22}}{(1-v_{12}v_{21})}$$

$$Q_{66}= G_{12}$$

$$Q_{44}= \mathrm{k}G_{13}$$

$$Q_{55}= kG_{23}$$

(3.26)

The on axis elastic constant matrix is:

$$[Q_{ij}] = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix}$$

(3.27)

If major and minor Poisson's ratio is ν_{12} and ν_{21}, then we have

$$\frac{\vartheta_{12}}{E_{11}} = \frac{\vartheta_{21}}{E_{22}}$$

(3.28)

The off axis elastic constant matrix is:

$$[\bar{Q}_{ij}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}$$

(3.29)

$$[\bar{Q}_{ij}] = [T]^T][Q_{ij}][T]$$

(3.30)

Where T denotes the transformation matrix.

Now, the elastic stiffness coefficients are

$$\bar{Q}_{11} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4)$$

$$\bar{Q}_{22} = Q_{11}n^4 + 2(Q_{12} + Q_{66})m^2n^2 + Q_{22}m^4$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3m$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 + Q_{66}(n^4 + m^4)$$

(3.31)

The elastic constant matrix with respect to transverse shear deformation is:

$$\bar{Q}_{44} = G_{13}m^2 + G_{23}n^2$$

$$\bar{Q}_{45} = (G_{13} - G_{23})mn$$

$$\bar{Q}_{55} = G_{13}n^2 + G_{23}m^2$$

(3.32)

here, m= cosθ, n= sinθ

The stress –strain relations:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

(3.33)

The forces and moment relations:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_Y \\ M_{xY} \\ Q_x \\ Q_Y \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_x \\ \tau_{xy} \\ \sigma_x Z \\ \sigma_{yz} \\ \tau_{xy} Z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{3.34}$$

Where σ denotes normal stresses, τ denotes shear stresses and x, y denotes direction.

The constitutive relation for the in-plane stress analysis is:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{31} & A_{32} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3.35}$$

The extensional stiffness is:

$$[D_P] = \begin{bmatrix} \frac{Eh}{1-v^2} & \frac{Eh}{1-v^2} & 0 \\ \frac{vEh}{1-v^2} & \frac{Eh}{1-v^2} & 0 \\ 0 & 0 & \frac{Eh}{2(1+v)} \end{bmatrix} \tag{3.36}$$

The constitutive relationships given for bending transverse shear:

$$\begin{Bmatrix} N_x \\ N_P \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{112} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{3.37}$$

Also

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix} \tag{3.38}$$

Or

$$\{F\} = [D]\{\varepsilon\} \tag{3.39}$$

Where

$$A_{ij} = \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^3 - z_{k-1}^3)\ ; i,j = 1,2,6$$

$$S_{ij} = k \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})\ ; i,j = 4,5 \tag{3.40}$$

Where k denotes transverse shear correction factor

3.5 Derivation of Element Matrices

1. Element plane elastic stiffness matrix

$$[k_P] = \int_{-1}^{1}\int_{-1}^{1} [B_P]^T [D_P][B_P]|J|\, d\xi\, d\eta \tag{3.41}$$

2. Element elastic stiffness matrix

$$[k_e] = \int_{-1}^{1}\int_{-1}^{1} [B]^T [D][B]|J|\, d\xi\, d\eta \tag{3.42}$$

3. Consistent mass matrix

$$[m_e] = \int_{-1}^{1}\int_{-1}^{1} [N]^T [P][N]|J|\, d\xi\, d\eta \tag{3.43}$$

Shape function matrix:

$$[N] = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \text{ i=1, 2…..8} \tag{3.44}$$

$$[P] = \begin{bmatrix} P_1 & 0 & 0 & P_2 & 0 \\ 0 & P_1 & 0 & 0 & P_2 \\ 0 & 0 & P_1 & 0 & 0 \\ P_2 & 0 & 0 & P_3 & 0 \\ 0 & P_2 & 0 & 0 & P_3 \end{bmatrix} \tag{3.45}$$

Also

$$(P_1, P_2, P_3) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_1} (\rho)_k (1, z, z^2) dz \tag{3.46}$$

Where

[B], [D] and [N] are strain-displacement, stress-strain and shape function matrices.

3.6 Geometric Stiffness Matrix

The above said matrix is arrived at using the non linear in plane Green's strain with curvature component. It is a function of in plane stress distribution in element because of applied edge loading.

As we have

$$U_2 = \int_V [\sigma^0]^T \{\varepsilon_{nl}\} dV \tag{3.47}$$

The non linear strain components are:

$$\varepsilon_{xnl} = \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right]$$

$$\varepsilon_{ynl} = \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right]$$

$$\gamma_{xnl} = \frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)\left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right) + z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right] \tag{3.48}$$

The strain energy is:

$$U_2 = \int_A \frac{h}{2}\left[\sigma_x^0\left\{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)^2\right\} + \sigma_y^0\left\{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)^2\right\} + 2\tau_{xy}^0\left\{\left(\frac{\partial u}{\partial y}\frac{\partial u}{\partial y}\right) + \left(\frac{\partial v}{\partial y}\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)\left(\frac{\partial w}{\partial y} - \frac{u}{R_y}\right)\right\}\right] dxdy + \int_A \frac{h^2}{24}\left[\sigma_x^0\left\{\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2 +\right\} + \sigma_y^0\left\{\left(\frac{\partial \theta_y}{\partial y}\right)^2 + \left(\frac{\partial \theta_x}{\partial y}\right)^2\right\} + 2\tau_{xy}^0\left\{\left(\frac{\partial \theta_y}{\partial x}\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_x}{\partial x}\frac{\partial \theta_y}{\partial y}\right)\right\}\right] dxdy \qquad (3.49)$$

It can be rewritten as

$$U_2 = \frac{1}{2}\int_v [f]^r[S][f]dV \qquad (3.50)$$

Where

$$\{f\} = \left[\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right), \left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right), \frac{\partial \theta_x}{\partial x}, \frac{\partial \theta_x}{\partial y}, \frac{\partial \theta_y}{\partial x}, \frac{\partial \theta_y}{\partial y}\right]^r \qquad (3.51)$$

And

$$[S] = \begin{bmatrix} [s] & 0 & 0 & 0 & 0 \\ 0 & [s] & 0 & 0 & 0 \\ 0 & 0 & [s] & 0 & 0 \\ 0 & 0 & 0 & [s] & 0 \\ 0 & 0 & 0 & 0 & [s] \end{bmatrix} \qquad (3.52)$$

Where

$$[s] = \begin{bmatrix} \sigma_x^0 & \tau_{xy}^0 \\ \tau_{xy}^0 & \sigma_y^0 \end{bmatrix} = \frac{1}{h}\begin{bmatrix} N_x^0 & N_{xy}^0 \\ N_{xy}^0 & N_y^0 \end{bmatrix} \qquad (3.53)$$

The geometric matrix is formed for in-plane stress resultants

$$\{f\} = [G]\{q_e\} \qquad (3.54)$$

where

$$\{q_e\} = [u\ \ v\ \ w\ \ \theta_x \theta_x]^T \qquad (3.55)$$

The strain energy is now:

$$U_2 = \frac{1}{2}[q]^r[G]^T[S][G]\{q\}dV = \frac{1}{2}\{q_e\}^T[K_e]_e[q_e] \tag{3.56}$$

And the element stiffness geometric matrix:

$$[K_e]_e = \int_{-1}^{1}\int_{-1}^{1}[G]^T[S][G]|J|d\xi d\eta \tag{3.57}$$

$$[G] = \begin{bmatrix} N_{i,}x & 0 & 0 & 0 & 0 \\ N_{i,}y & 0 & 0 & 0 & 0 \\ 0 & N_{i,}x & 0 & 0 & 0 \\ 0 & N_{i,}y & 0 & 0 & 0 \\ 0 & 0 & N_{i,}x & 0 & 0 \\ 0 & 0 & N_{i,}y & 0 & 0 \\ 0 & 0 & 0 & N_{i,}x & 0 \\ 0 & 0 & 0 & N_{i,}y & 0 \\ 0 & 0 & 0 & 0 & N_{i,}x \\ 0 & 0 & 0 & 0 & N_{i,}y \end{bmatrix} \tag{3.58}$$

3.8 Computer programming

A program is coded in FORTRAN for the comparison of the numerical results obtained by calculations with the results obtained experimentally. The flow chart of the said program is as follows:

The boundary condition assumed in the code is simply-supported where as in the actual experimental conditions in the laboratory the boundary condition is free-free. To account for that, the “shift technique” as suggested by Baffe is introduced which will modify the results by considering free-free boundary conditions.

FLOW CHART:

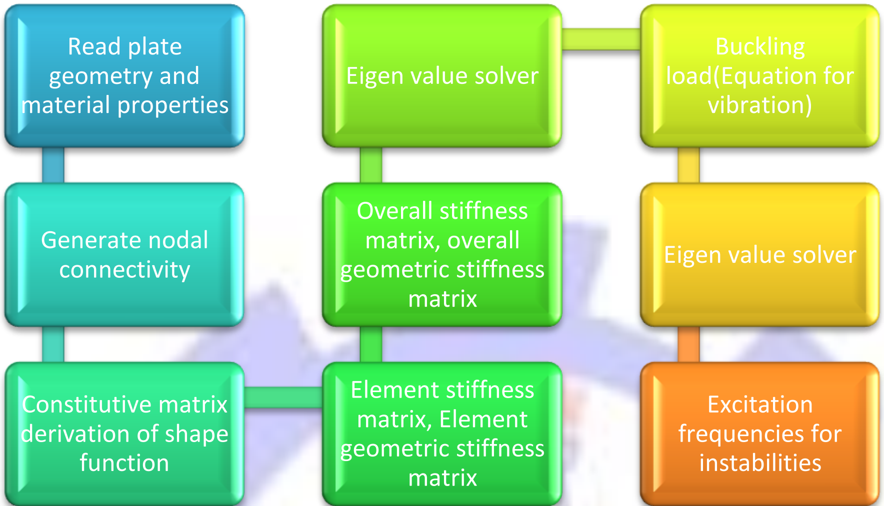


Fig 3.5 Flow chart showing coding in FORTRAN

4
EXPERIMENTAL PROGRAMME

4.1 Introduction

This section deals with the experimental works carried out for determination of material constants (tensile testing) and vibration effect (modal testing). A laminated composite plate and shell were fabricated and their material properties were determined from the tensile tests. The study can be properly divided broadly into two categories:

- Tensile testing of the specimens to determine elastic constants
- Dynamic stability of the plate and the shell

4.2 Materials

The following list shows the constituent materials used while fabrication of plate and shell:

- Glass woven fibers (reinforcement material)
- Epoxy (resin)

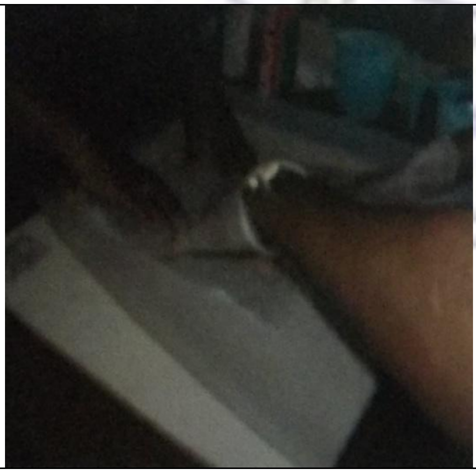
- Hardener (8-10% of the weight of epoxy) (catalyst)
- Polyvinyl Alcohol (PVA) (releasing agent)

4.3 Fabrication Procedure

The hand-layup technique was used for the casting of the FRP composite specimens. In this method, liquid resin was placed with the reinforcement (glass fiber) against finished surface of the open mould. Chemical reactions present in the resin hardened the material into a strong and light weight product. Fiber and matrix were taken as 50:50 in weight for the fabrication of the plates. A plywood rigid platform q(preferably flat) was selected. Glass fiber was cut from the roll of woven roving. On the platform, a plastic sheet which is a mould releasing sheet is placed and a thin film of polyvinyl alcohol (PVA) is applied to facilitate release. A mixture of epoxy and hardener, known as gel coat, is applied on the mould with a brush. This should be done as fast as it can be, otherwise the mixture may harden. The quantity of epoxy used should be approximately equal to the weight of the glass fiber sheets and the quantity of hardener should be approximately 8-10% of the weight of the glass fiber sheets.

The gel coat is applied to ensure a smooth external surface and for the protection of the fibers from direct exposure to the environment. The procedure proceeds with the subsequent stacking of layers of reinforcement with the application of gel coat. Any air which may have been entrapped was removed by steel roller which gives proper compaction. The process concludes with the final placement of another plastic sheet with the application of polyvinyl chloride inside the sheet. The entire setup of layers of glass woven fibers and plastic sheet is compressed by putting another flat ply board and a heavy metal rigid platform at its top.

The plates were left for a minimum of 48 hours in room temperature for curing before it is cut to the required shape. The following pictures highlight the various procedural steps:



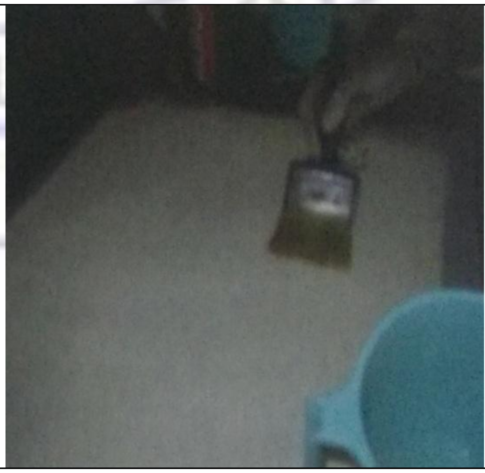


Fig.4.1(a) Application of gel coat on mould releasing sheet	Fig4.1(b) Placing woven roving fiber on gel coat
	
Fig4.1(c). Removal of air entrapment using steel roller	Fig.4.1(d) Fabricated laminated composite plate

Fig 4.1 Procedural steps in fabrication

4.4 Determination of Material Constants

After the plate is dried, it is cut to the required shape. The specimens were cut from the plate by using diamond cutter. At least four replicate sample specimens were prepared. Following this, the next step of determination of its material properties is carried out.

The material constants to be determined is

- Elastic modulus, E_{11} and E_{45}
- Shear modulus, G_{12}

where the suffixes indicate the principal material directions. For material characterization, sample of 8 layers Glass/Epoxy plates were tested by INSTRON 1195 machine for determining tensile strength and Young's modulus in different directions. From the test data E1, E2, G12 were calculated.

For the measurement of Young's modulus, the specimen is loaded in INSTRON 1195 universal testing machine, monotonically to a failure with a recommended rate. Specimens were first fixed in the upper jaw and then subsequently gripped in the movable jaw (i.e. lower jaw). To avoid slippage, 50 mm in each side is taken for gripping. The load and extension were recorded digitally using the load cell and the extensometer respectively. The engineering stress-strain curve was plotted. The initial slope of this curve provides Young's modulus. Poisson's ratio is directly obtained by the ratio of transverse to longitudinal strain by means of two strain gauges in longitudinal and transverse direction.

The shear modulus was calculated from the formula below from Jones [1975]:

$$G12 = \frac{1}{\frac{4}{E45} - \frac{1}{E1} - \frac{1}{E2} + \frac{2\nu}{E1}}$$

4.5 Description of Test Specimen

The geometrical dimensions of the woven roving Glass/Epoxy composite plate and shell, fabricated for the present experimental study have been outlined below:

Table 4.1: Properties of the woven roving G/E composite plate and shell

Property	Value
Lay-up	Woven-Roving
Number of layers	8
Length of plate	150mm
Overall Length of plate	235mm
Width of specimen	25mm
Thickness of plate	2.35mm
Thickness of shell	2.62mm
Density of plate	1600kg/m^3
Density of shell	1600kg/m^3

4.6 Vibration of Composite Plate

As already mentioned, the importance of material and dynamic properties of the twisted plates and its effect on efficient performance of the plates is of utmost importance. In order to avoid the problems posed by dynamic loading on such plates, it is necessary to determine natural frequency of the system. The natural frequencies are quite sensitive to the orthotropic properties of the composite plates and various design tailoring tools may aid in controlling the fundamental frequency.

4.6.1 Equipments

4.6.1.1 Modal Hammer

The modal hammer used in the present experiment is 2302-5. It has three interchange tips which determine the width of the input pulse. Its main purpose is to excite and measure impact forces on the specimens.

Fig 4.2. Modal Hammer

4.6.1.2 Accelerometer:

The accelerometer type used in the present experiment is type 4507. It is fixed on Plates by using bee-wax. It is highly sensitive, light weight, dimensionally small and hence can be easily fitted to wide range of test objects by a selection of mounting clips.

Fig 4.3. Accelerometer

4.6.1.3 FFT Analyzer:

The present experiment uses Bruel & Kjaer FFT analyzer 3560-B type. The system has some channels to connect the cables for analyzing both input and output signals. It is useful both in the case of free vibration and forced vibration.

Fig 4.4. FFT Analyzer

4.6.1.4 PULSE Software:

The signal of the resulting vibrations of the specimens, which were excited at selected points by means of Modal Hammer, were received to the FFT Analyzer by an accelerometer mounted on the specimen by using bee-wax and is displayed in the display unit, which is generally in the form of PC or laptop. The output gets displayed in the graphical form which includes graph of force amplitude spectrum, response amplitude spectrum, coherence and frequency response functions as shown in the following figures.

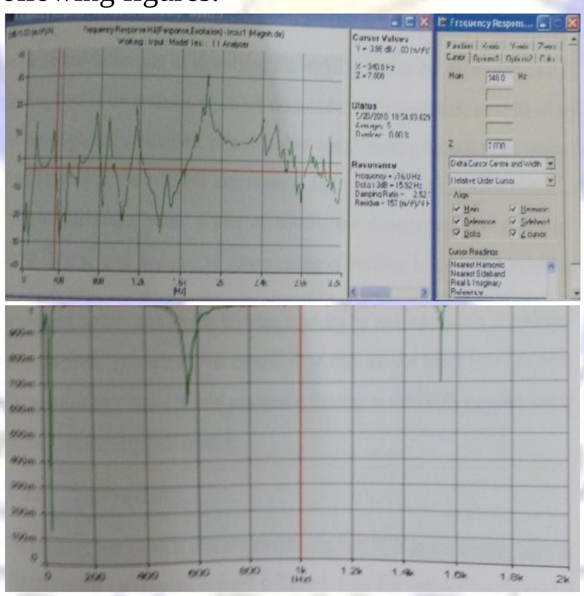


Fig 4.5. Readings of Pulse Software

4.6.2 Set up and Test Procedure

The test frame used in the system is designed for different boundary conditions i.e. free-free, four sides simply supported, cantilever and fully clamped.

The procedure began with the proper fitting of the test specimen to the iron frame followed by ensuring of connections of FFT analyzer, transducers, laptop, cables and modal hammer to the system. Impact hammer (Model 2302-5) was used to excite the plate at selected points and the resulting vibrations were recorded by means of an Accelerometer (B&K, Type 4507) held to the specimen through bees wax. At each selected point, the hammer was made to strike for five times to obtain the Frequency Response Function (FRF). The screen displayed the average value of the response. The Modal Hammer, Accelerometer, FFT Analyzer and the FRFs are described and shown in the previous section.

The strokes made were ensured to be perpendicular to the surface of the plates. To maintain the coherency of the results, the strokes should be made at approximately same points. The peaks of the FRFs provide the frequencies. Output is displayed on the analyzer screen by means of pulse software.

4.6.3 Setting up the template in Pulse lab Report

There are for important windows to set up a template as shown below:

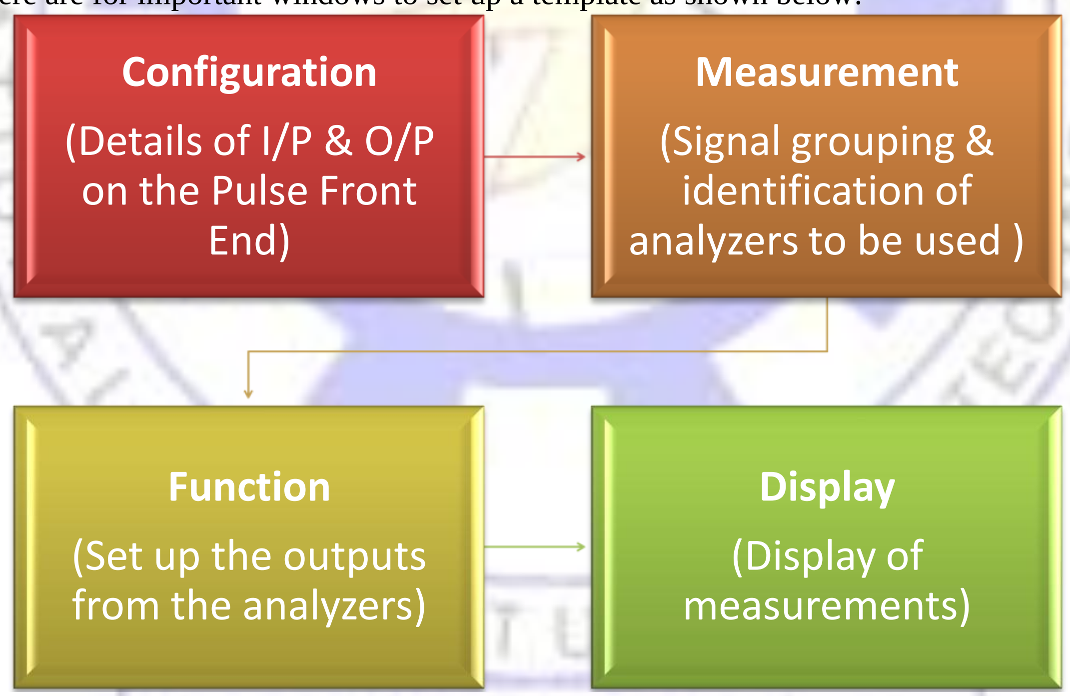


Fig 4.6 Windows showing setting up of template in pulse lab report

4.6.4 Pulse Report

The Figure 4.5 shows a typical pulse report (FRF) taken from FFT Analyzer followed by coherence curve. The different modes of vibrations are shown by different peaks of the FRF. The coherence value shown in the figure suggests that the impact is given almost at the same point in all the five strokes.

4.7 Analysis in ANSYS

ANSYS is used to study linear and non-linear static/dynamic structure analysis, acoustic and electromagnetic problems and heat transfer and fluid problems.

- Preprocessing (Defining the Problem)

  It is defined by:
  - Element type/material and geometric properties
  - Key-points/volumes/areas
  - Mesh lines/volumes/areas

- Solutions (Assignment of Loads)

  It is assigned:
  - Loads(points/pressure)
  - Constraints(translational/rotational)
    And the solution of resulting equation is done.

- Post-processing (Results viewing)

  The following results can be obtained
  - List of nodal displacement
  - Element forces and moments
  - Deflection plots
  - Stress Contour diagrams

The present analysis was done following the above procedure in ANSYS. The twisted plate was first solved without loading to validate the methodology and the results compared to previous results for free-free vibration. The methodology was verified for a laminated composite plate for different parameters and results compared to a result from a previous paper.

5
RESULTS AND DISCUSSIONS

5.1 Introduction

The composite plates are found as part of different structural elements each claiming variation in terms of geometry, loads, boundary conditions etc. Since these plates are subjected to dynamic impact during their lifetime, therefore the study of the frequencies which may mark their failure is of utmost importance. The present chapter presents the results of the tensile testing of laminated composite plates and shells, prepared in the laboratory. The results of the modal testing done using FFT Analyzer are also presented along with the corresponding values obtained from computer programming in the environment of FORTRAN. The chapter concludes with the compilation of frequencies obtained by subjecting the modeled twisted plates to free vibrations under different parametric conditions.

5.2 Results of Tensile Testing

The following table compiles the values of Young's Modulus, Poisson's ratio, density for the woven fiber laminated composite specimen. The value of the shear modulus was calculated by putting the values of the Young's modulus in the equation stated above (Eqn).

Table 5.1: Material constants of the plate

Material Property	Value
Elastic Modulus , E_1 E_2 E_{45}	10.846 GPa 10.846 GPa 7.253 Gpa
Shear Modulus, G_{12}	2.42 GPa
Poisson's Ratio, ν	0.25
Density, ρ	1600kg/m^3

5.3 Results of Vibration Analysis

<u>PLATES</u>

Table 5.2: Comparison of the experimental results with the results obtained from numerical analysis in terms of frequencies for plates

EXPERIMENTAL VALUES	*NUMERICAL VALUES*
68	55
152	126
172	160
204	160
424	282
424	321
456	342
504	380

<u>Comparison through Graph:</u>

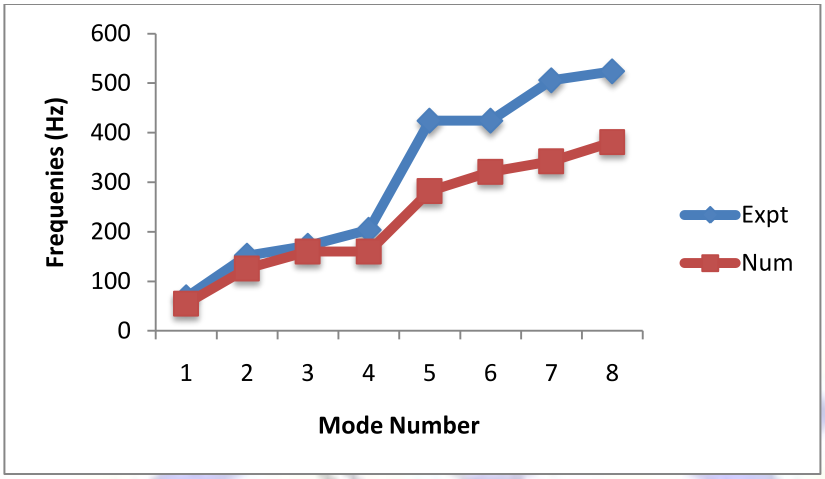


Fig 5.1. Comparison of the experimental results with the results obtained from numerical analysis in terms of frequencies for plates

<u>SHELLS</u>

Table 5.3: Comparison of the experimental results with the results obtained from numerical analysis in terms of frequencies for shell

EXPERIMENTAL VALUES	*NUMERICAL VALUES*
72	58
156	129
208	168
240	182
404	321
532	359
648	377
840	421

<u>Comparison through Graph:</u>

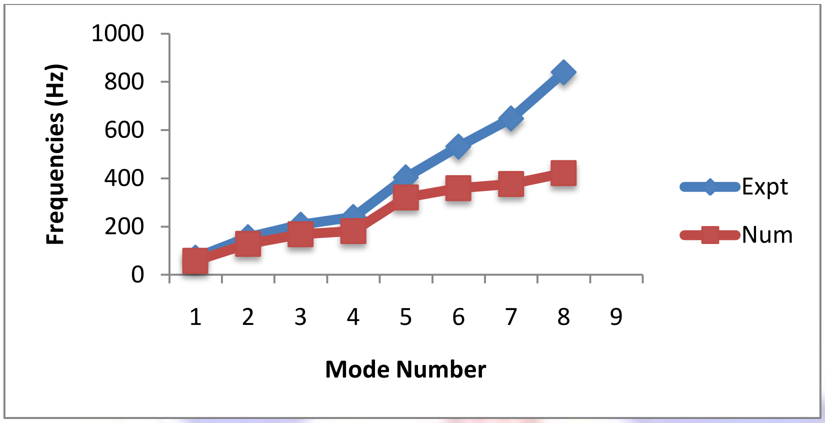


Fig 5.2. **Comparison of the experimental results with the results obtained from numerical analysis in terms of frequencies for shell**

5.4 Convergence Study

The convergence studies are conducted for the three lowest non-dimensional frequencies of a two-layer square laminated twisted plate having cantilever support and subjected to free vibrations. The angle of twist adopted for this study is 10° and the study is done for different mesh divisions.

The following table Table 5.1 shows the results obtained for different mesh divisions. For the following analysis a mesh division of 8×8 was considered.

Table 5.4: Convergence study of free vibration on non-dimensional frequencies of 2-layer laminated twisted composite cantilever plates. [0°/90°]

a/b =1, b/h = 250, Φ = 10°
a = b = 500mm, h = 2mm, ρ= 1520kg/m^3, ν_{12} = 0.313
E_{11} = 140.0GPa, E_{22} = 9.23GPa
G_{12} = 5.95GPa, G_{23}= 2.94GPa.

Mesh Division	**Angle of Twist**	**Non-Dimensional Frequency**		
		1st Frequency	**2nd Frequency**	**3rd Frequency**
4×4	**10°**	22.10	22.94	23.56
8×8		23.38	24.01	24.67
6×6		22.10	22.94	23.56

5.5 Comparison with the Previous works

The efficiency and accuracy of the solution are established with the aid of comparison with previous studies done. The non-dimensional frequency for a square laminated composite panel is solved through ANSYS and the results are compared with Sahu & Asha in Table 5.5

Non-dimensional frequency, $\Omega = \omega a^2 \sqrt{(\rho/E_{11} h^2)}$

Table 5.5: Comparison with previous study non-dimensional frequency parameter for cross-ply plates with different ply lay-ups

Angle of Twist	Non-dimensional frequency parameter	Results from previous work	Results from ANSYS
10°	0° /90°	0.4800	0.4842
	0° /90° /0° /90°	0.6831	0.6789
	0° /90° /90° /0°	0.9508	0.9467
	0° /90° /0° /90° /0° /90° /0° /90°	0.7251	0.7134
20°	0° /90°	0.4708	0.5003
	0° /90° /0° /90°	0.6700	0.6754
	0° /90 ° /90° /0°	0.9326	0.9259
	0° /90° /0° /90° 0° /90° /0° /90°	0.7112	0.7102
30°	0° /90°	0.4540	0.4614
	0° /90° /0° /90°	0.6461	0.6334
	0° /90° /90° /0°	0.8993	0.8765
	0° /90° /0° /90° /0° /90° /0° /90°	0.6858	0.6632

5.6 Numerical Results

The study further proceeds with the investigation of vibration analysis of laminated composite twisted panels under free vibration and cantilever support. Numerical studies are done to judge the variation in frequencies arising due to variation of angle of twist, number of layers, aspect ratios and width to thickness ratio. The mode shapes in each of the above cases are given for better understanding of the results. The red zone in the mode shape shows maximum displacement and the blue zone shows the minimum displacement.
The Table 5.6 lists the material properties of the specimen for which the study in ANSYS was carried out.

Table 5.6: Material Properties of the specimen

Property	*Values*
E	200 GPa
a=b=	0.5m
N	0.28
Loading Conditions	Free Vibration

P	1520kg/m^3
Type	Isotropic
Boundary Conditions	Cantilever

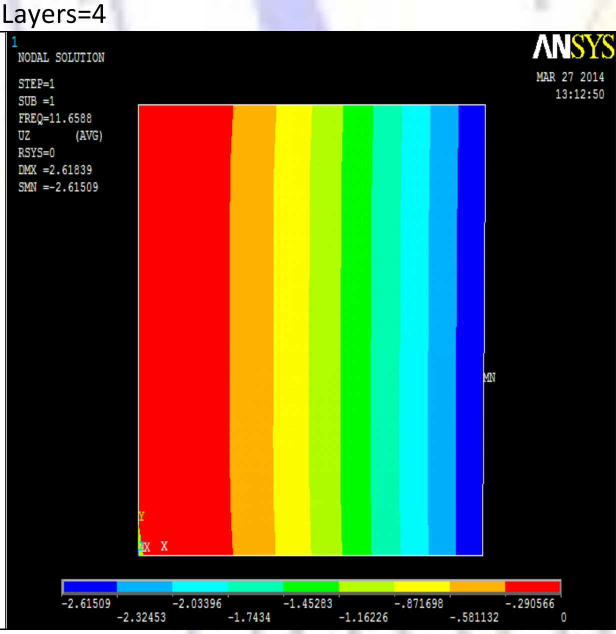
Effect of layers:

Table 5.7 : Variation in the frequencies (Hz) obtained by varying number of layers for laminated composite twisted plate $[0^\circ/90^\circ]$ subjected to free vibrations

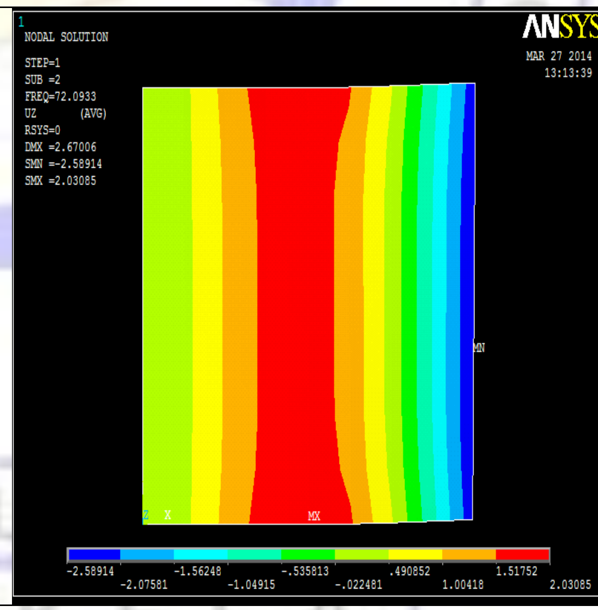
	Frequency (Hz)		
Sl.No	***Layers=4***	***Layers=6***	***Layers=8***
1	11.659	17.468	23.272
2	72.053	107.88	143.48
3	181.27	206.62	220.42

As it can be observed the frequencies of the specimen undergo an appreciable increase of about 40-50 % for the first mode of frequency range, as the number of layers increases, which decreases further to about 30% for the next mode and further reduces in the subsequent modes.

Layers=4

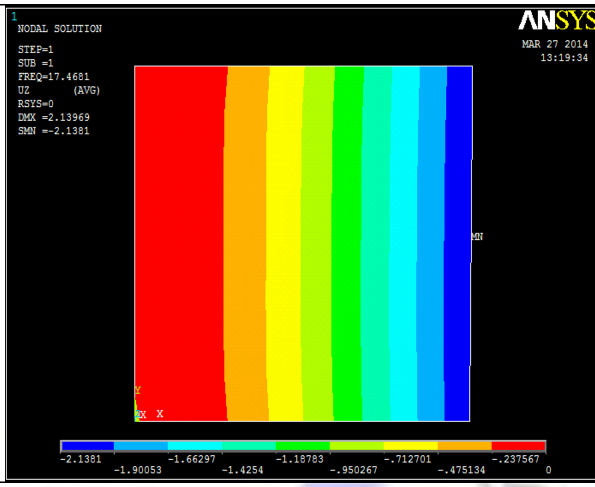


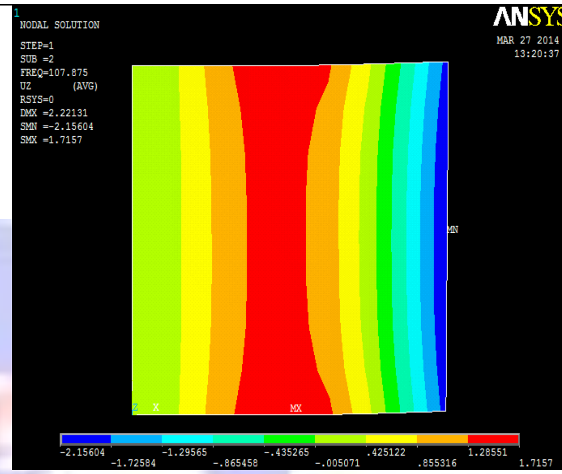
Mode 1



Mode 2

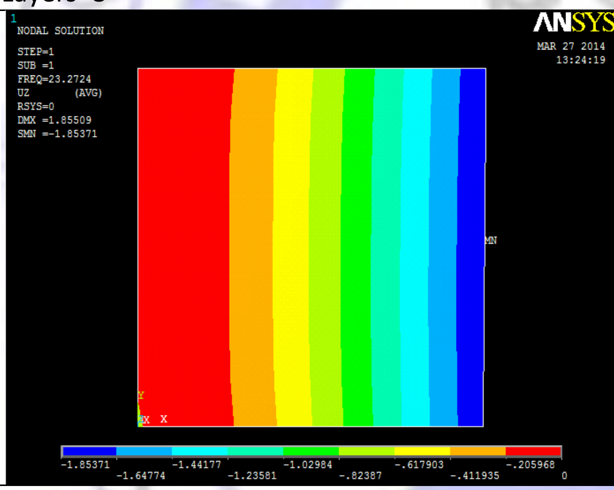
Layers=6

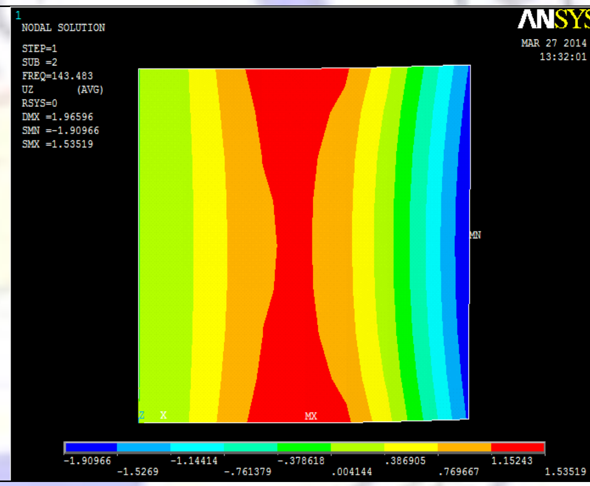




Mode 1 | Mode 2

Layers=8





Mode 1 | Mode 2

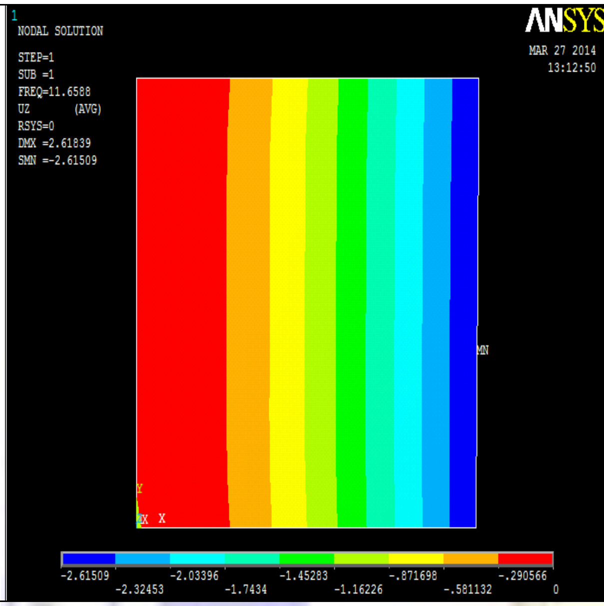
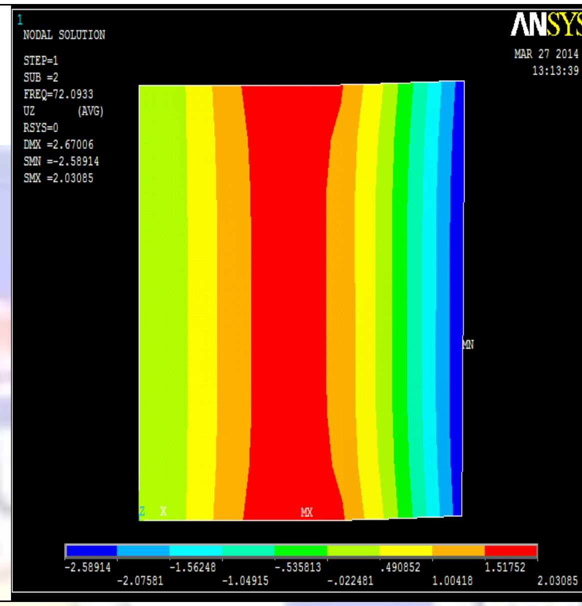
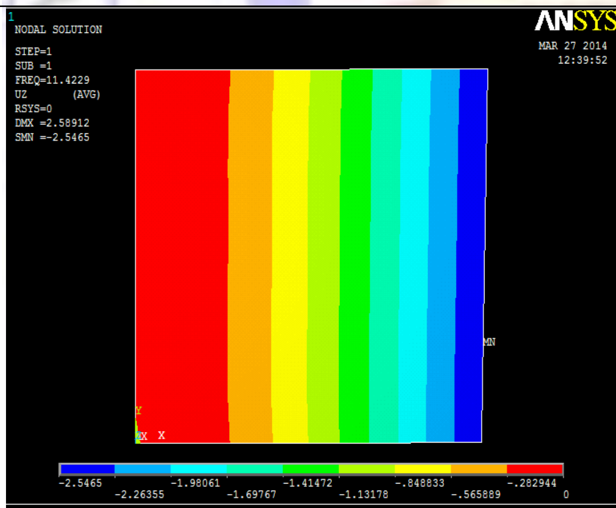
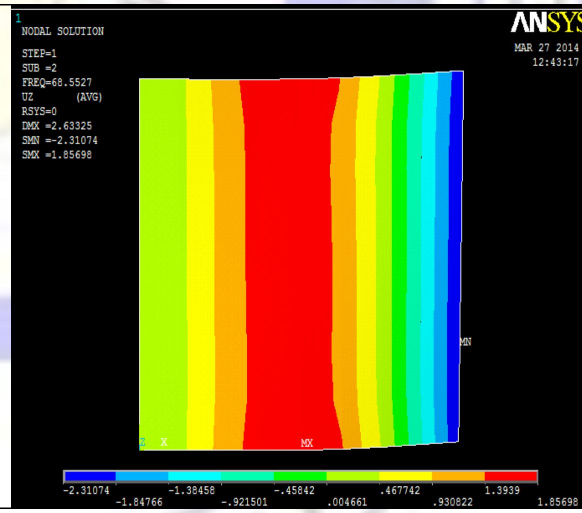
Fig 5.3 Mode shapes for twisted plate by varying the number of layers

Effect of twist angles:

Table 5.8: Variation in the frequencies (Hz) obtained by varying angles of twist for laminated composite twisted plate [0° /90°] subjected to free vibrations

	Frequency (Hz)		
Sl.No	***Angle=10°***	***Angle=20°***	***Angle=30°***
1	11.659	11.423	10.995
2	72.053	68.553	62.649
3	181.27	179.44	170.58

As it can be observed the frequencies of the specimen undergo a marginal decrease of approximately 1-2 % in the values of frequencies for corresponding modes as the angle of twist increases.

Mode 1	Mode 2
Φ=10°	
	
Φ=20°	
	
Φ = 30°	

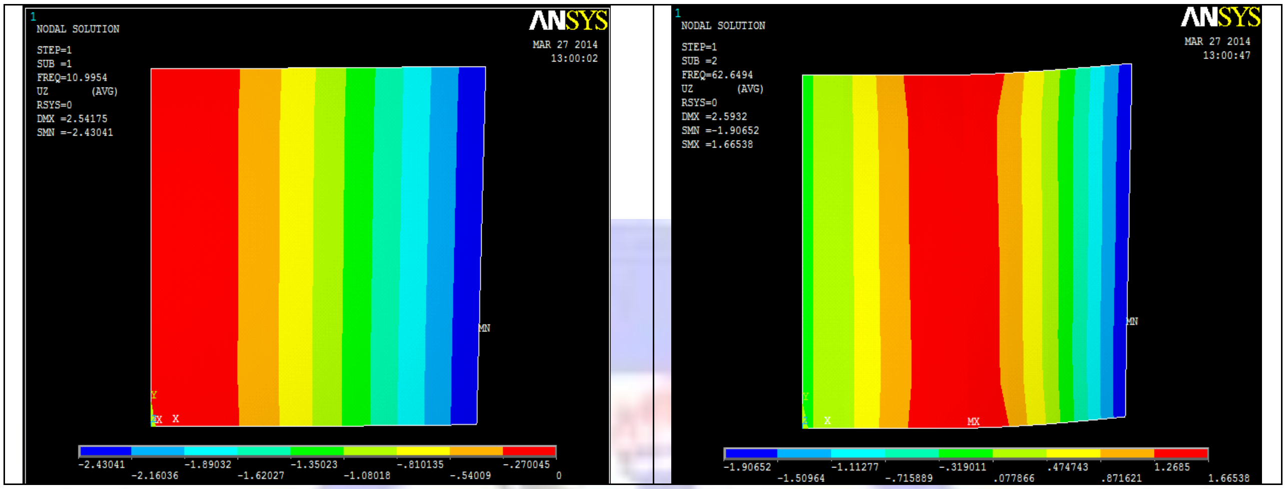


Fig 5.4 Mode shapes for twisted plate by varying angles of twist

Effect of aspect ratio:

Table 5.9: Variation in the frequencies (Hz) obtained by varying aspect ratios for laminated composite twisted plate [0° /90°] subjected to free vibrations

b/h=250, Φ = 10°

	Frequency (Hz)	
Sl.No	***a/b=1***	***a/b=2***
1	270.58	152.11
2	371.29	269.41
3	496.53	387.92

As it can be observed the frequencies of the specimen undergo appreciable decrease in the values of the frequencies as the aspect ratio increases. The decrease in the frequencies reduces as the mode increases.

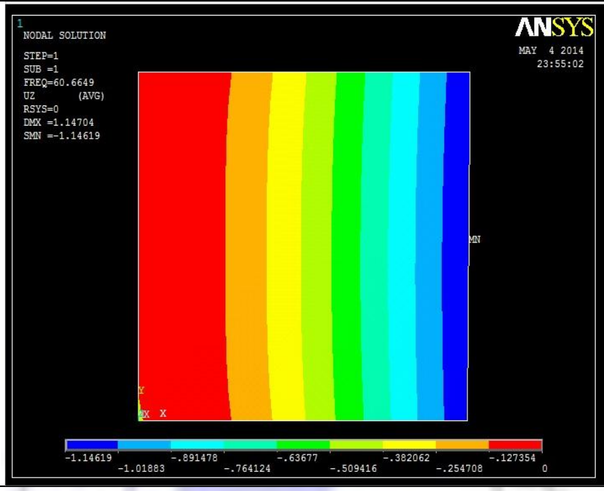
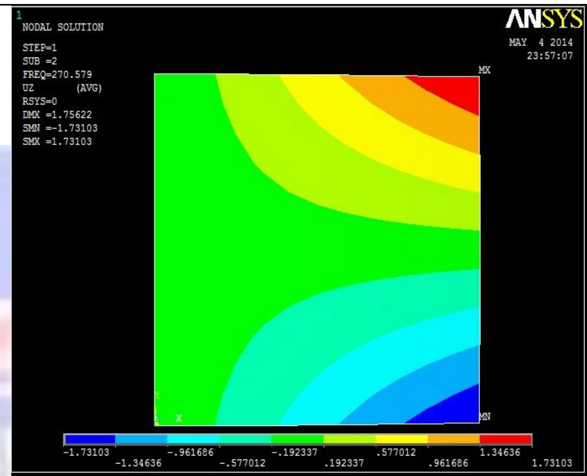
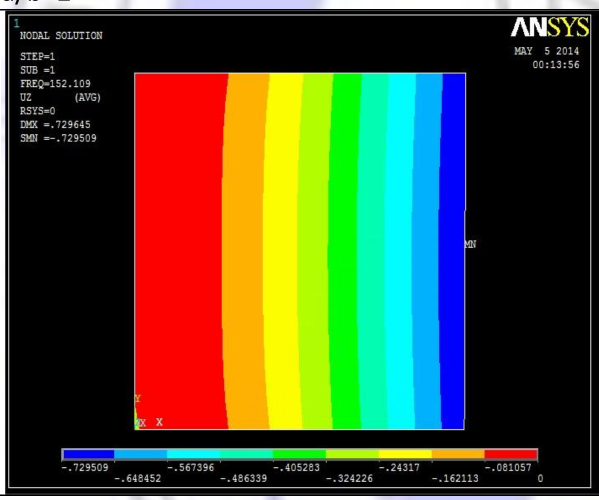
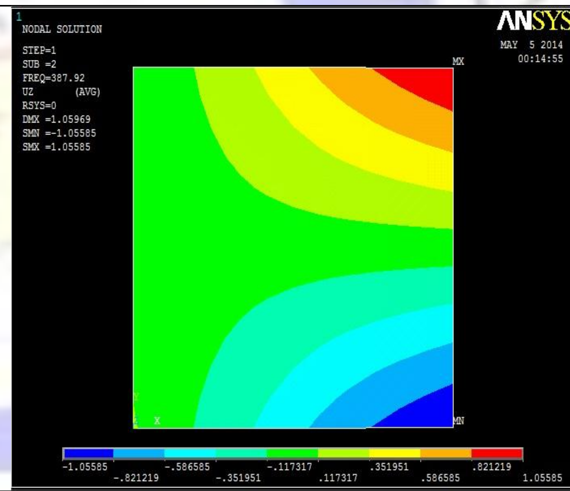
<table>
<tr><td>Mode 1</td><td>Mode2</td></tr>
<tr><td>a/b=1</td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td>a/b=2</td><td></td></tr>
<tr><td></td><td></td></tr>
</table>

Fig 5.5 Mode shapes for twisted plate by varying aspect ratio

Effect of ratio (b/h):

Table 5.10: Variation in the frequencies (Hz) obtained by varying width to thickness ratio for laminated composite twisted plate [0° /90°] subjected to free vibrations

a/b=1, Φ = 10°

	Frequency (Hz)	
Sl.No	***b/h=100***	***b/h=250***
1	151.35	60.66
2	340.75	270.58
3	430.89	371.29

As it can be observed the frequencies of the specimen undergo appreciable decrease in the values of the frequencies as the b/h ratio increases. The decrease in the frequencies reduces as the mode increases, the first mode witnesses a huge decrease of about 50-60 % decrease whereas the subsequent decrease comes down to about 20% as the mode number increases.

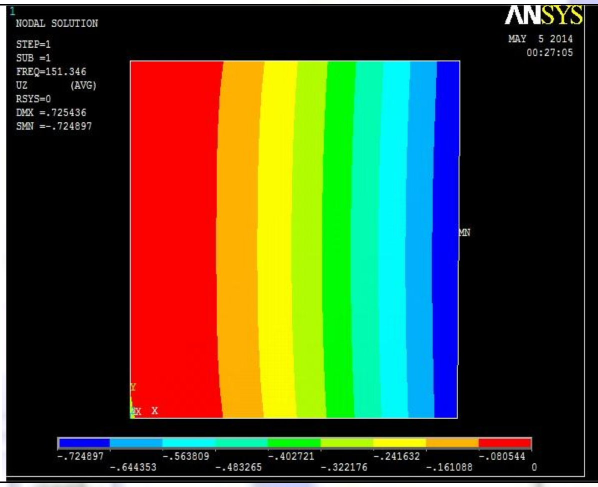
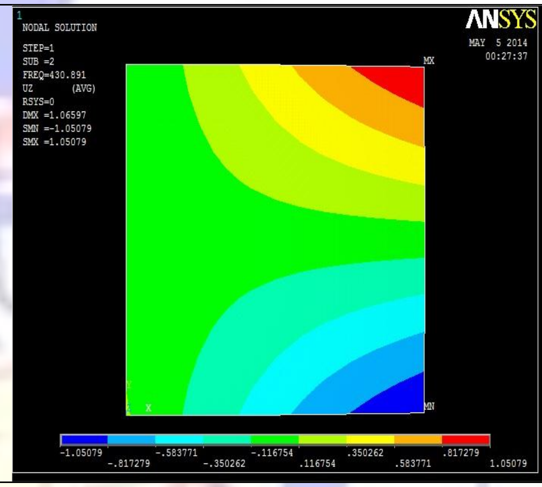
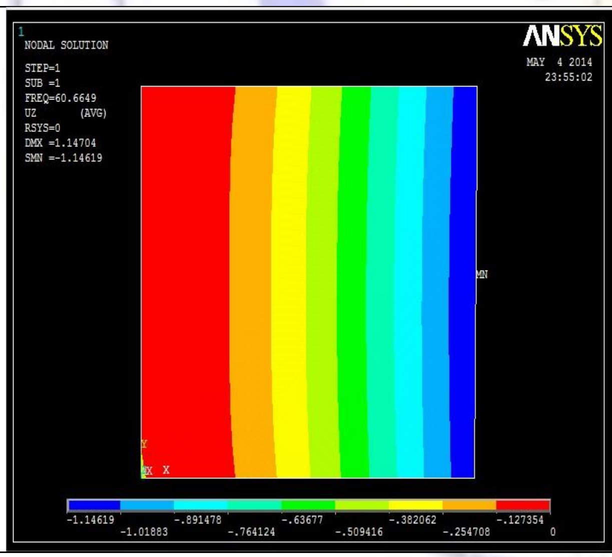
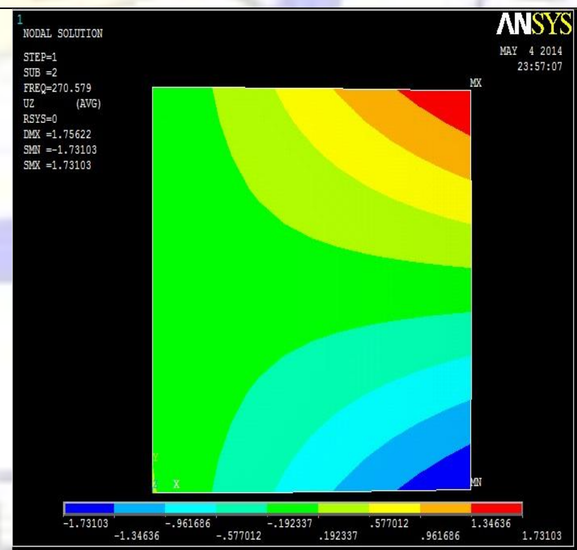
<table>
<tr><th>Mode1</th><th>Mode2</th></tr>
<tr><td>b/h=100</td><td></td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>b/h=250</td><td></td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Fig 5.6 Mode shapes for twisted plate by varying the width to thickness ratio

Comparison through graphs:

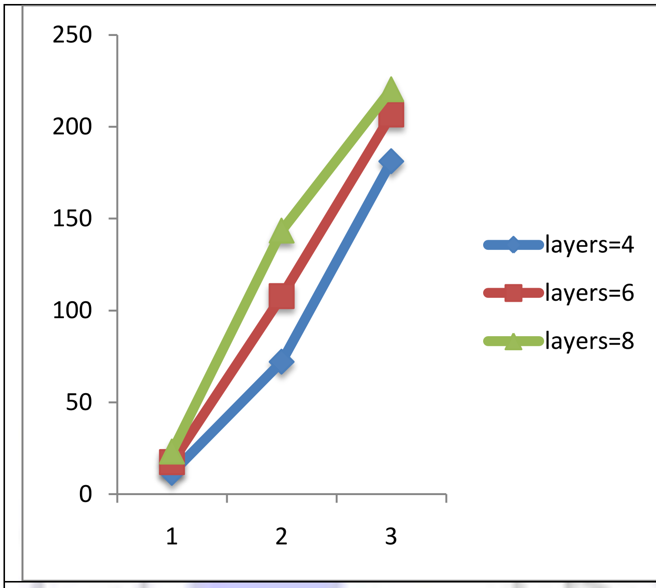


Fig 5.7(a): Graph showing comparison of frequencies due to different number of layers

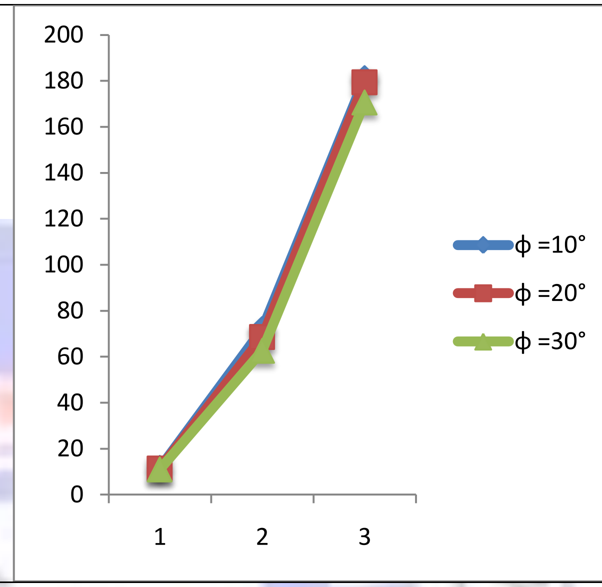


Fig 5.7(b): Graph showing comparison of frequencies due to different angles of twist

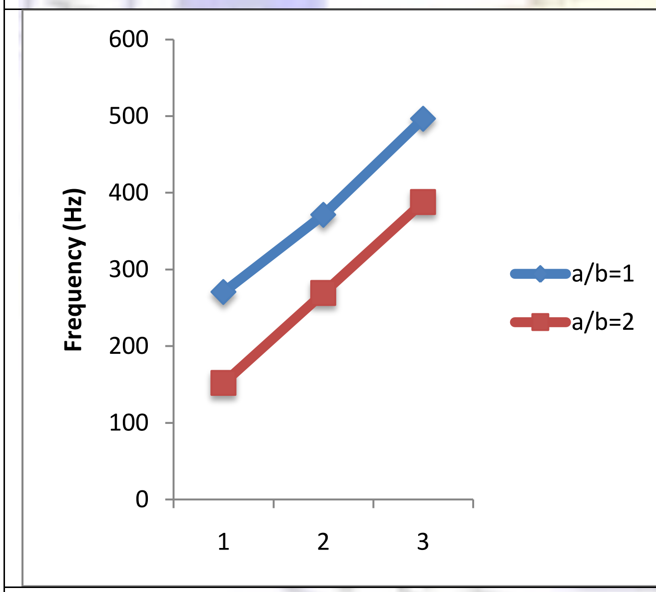


Fig 5.7(c) Graph showing Comparison of frequencies for different values of aspect ratio

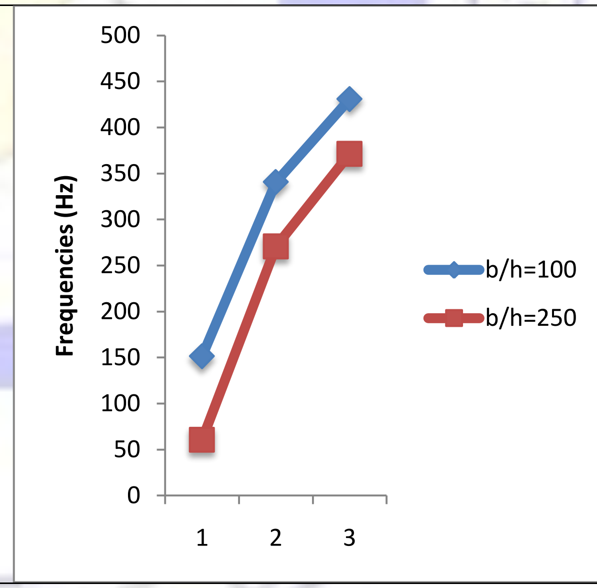


Fig5.7(d) : Graph showing Comparison of frequencies for different values of (b/h) ratios

Fig 5.7 Results in graphical form

6

CONCLUSION

The present experimental study on "Vibration Analysis of Laminated Composite Turbomachinery Blade" has analyzed the effect of various parameters on the vibration characteristics of a laminated composite twisted panel subjected to free vibration, using ANSYS. Based on the results obtained following conclusions can be drawn:

- The frequency values obtained for plate are slightly higher than those of shell.
- The frequency values obtained experimentally are found to be higher than the values found numerically. The difference may result due to the assumptions made in the coding such as homogeneity of the material, purely cantilever condition ,uniform thickness etc which are otherwise found to deviate in laboratory conditions.
- The values obtained for a mesh size of 8×8 are found to be most satisfactory.
- From the above results showing the effect of twist angles on the plate, we conclude that vibration decreases on increasing twist angle.
- From the above results showing the effect of number of layers on the plate, we conclude that vibration increases on increasing number of layers.
- From the above results showing the effect of aspect ratios on the plate, we conclude that frequency decreases on increasing aspect ratios.

- From the above results showing the effect of b/h ratios on the plate, we conclude that frequency decreases on increasing b/h ratio.
- From the mode shapes obtained, one can conclude that for mode 1 the maximum and minimum displacement almost remains same in case of variation in number of layers and angles of twist. But in mode 2 the displacement is maximum at the mid surface, and the maximum value decreases as the number of layers increases and increases as the angle of twist increases. It is inferred from the shape of the red zone in the mode shape in mode 2 in both the cases.

7

FUTURE SCOPE OF RESEARCH

In the present work, though a sound number of parameters have been included, still there are identified areas where the project can be extended to.

The current works deals basically with the case of an isotropic laminated composite pre-twisted plate. The study may be further carried to analyze anisotropic laminated plate.

In addition to free-free vibrations, work may also be done to include the influence of various parametric conditions for forced vibrations. The effects of the harmonic loading may also be included.

The work can preferably extend to functionally graded material which are vehemently used in the current scenario.

8

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