A project report on

NUMERICAL STUDY OF AXIAL WALL CONDUCTION IN FULLY HEATED MICROTUBES FOR CRYOGENIC FLUID

By

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June-2014

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CERTIFICATE

This is to certify that the thesis entitled**, "NUMERICAL STUDY OF AXIAL WALL CONDUCTION IN FULLY HEATED MICROTUBES FOR CRYOGENIC FLUID"** submitted by **Mr. Abhimanyu Yadav** in partial fulfillment of the requirements for the award of Master of Technology Degree in **Mechanical Engineering** with specialization in **Cryogenic and Vacuum Technology** at the National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university/institute for the award of any degree or diploma.

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SELF DECLARATION

I, Mr Abhimanyu Yadav, Roll No. 212ME5412, student of M. Tech (2012-14), Cryogenic and Vacuum Technology at Department of Mechanical Engineering, National Institute of Technology Rourkela do hereby declare that I have not adopted any kind of unfair means and carried out the research work reported in this thesis work ethically to the best of my knowledge. If adoption of any kind of unfair means is found in this thesis work at a later stage, then appropriate action can be taken against me including withdrawal of this thesis work.

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02 June 2014

ACKNOWLEDGEMENT

I would like to thank and express my gratitude towards my supervisor **Dr. Manoj K Moharana** for his extensive support throughout this project work. I am greatly indebted to him for giving me the opportunity to work with him and for his belief in me during the hard time in the course of this work. His valuable suggestions and constant encouragement helped me to complete the project work successfully. Working under him has indeed been a great experience and inspiration for me.

I would also like to thank **Mechanical Department** for providing the **CFD Lab** where I completed the maximum part of my project work.

Date: **Abhimanyu Yadav**

Place:

ABSTRACT

In this rapidly progressing era everything is going in small size or which we called miniaturization of technology. This is the time of miniature things due to compactness. There is a huge industry growing on parallel to the high temperature which we called cryogenic world. In cryogenic field there are a lot of gases which we use for different purposes. Helium is one of these cryogenic gases which liquefy at 4.2K. Helium is very costly gas so we use this in any industry in closed cycle manner. Helium goes down to the 4.2K so we can use it in different processes. The thermo-physical properties of Helium change with temperature appreciably, so we cannot treat it as a constant property fluid. In this age the micro-tube heat exchanger are used for heating or cooling of cryogenic gases. In this work we tried to find out the most suitable material, suitable thickness of microtube with the help of change in different parameters. In this work, a two dimension numerical study is carried out to study the effect of axial wall conduction in fully heated circular microtube (in conjugate heat transfer mode) subjected to constant wall heat flux at the outer surface. A microtube of inner diameter of 0.4 mm and total length of 60 mm is considered in the numerical modeling. The cross sectional surfaces of the microtube are keeps adiabatic. Simulation have been done for the change in parameter like flow rate (Re =100-500), wall to fluid conductivity ratio (k_{sf} =1.71-2822.3684) and wall thickness to inner radius ratio (δ_{sf} =1-5). The result shows that conductivity ratio and wall thickness play dominant role in conjugate heat transfer process. It is found that there exist an optimum k_{sf} at which Nu_{avg} is maximum when other parameters are kept constant. Nu_{avg} is found to be lower for higher wall thickness (δ_{sf}). When Helium flow rate is increased, it is found that Nu_{avg} increases.

Keywords: microtube, conjugate heat transfer, axial wall conduction

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Nomenclature

c_p	Specific heat of fluid, J/kg-K
T_w	Wall temperature, K
T_f	Bulk fluid Temperature, K
q_w	Wall heat flux, W/m^2
$\bar{u}$	Average velocity at inlet, m/s
ks	Solid thermal conductivity, W/m-K
k_f	Fluid thermal conductivity, W/m-K
k_{sf}	Ratio of k_s and k_f
L	Total length of tube, m
P	Parameter for axial conduction
D_i	Inner diameter of micro-tube, m
r_i	Inner radius of micro-tube, m
h_z	Local heat transfer coefficient, W/m^2-K
Nu_z	Local Nusselt number
Nu_{fd}	Nusselt number for fully developed flow
Nu	Average Nusselt number
Pr	Prandtl number
Re	Reynolds number
Pe	Peclet number
u	Velocity in the axial direction, m/s
q"	Heat flux experienced at the solid-fluid interface of the micro-tube, W/m^2
q''_w	Heat flux experienced on the outer surface of the micro-tube, W/m^2
Z	Axial coordinate, m
Z*	Non dimensional axial coordinate

Greek symbols

δ_f	Inner radius of the tube, m
δ_s	Thickness of the tube wall, m
δ_{sf}	Ratio of δ_s and δ_f
∇	Differential parameter

μ	Dynamic viscosity, Pa-s
ρ	Density, kg/m^3
Φ	Non-dimensional local heat flux
Θ	Non-dimensional temperature

Subscripts

f	Fluid
i	Inner surface of tube
o	Outer surface of tube
s	Solid
w	Outer wall surface of tube

Chapter 1

Introduction

Background

In 21st century we are seeing the wide range of miniaturized power sources. Many different technologies are incorporated into the design of these systems including electrochemistry, turbo machinery, high performance insulation, micro heat exchanger, micro reactors, and miniaturized fluidic components. The developing utilization of gadgets in military and citizen requisitions has led to the widespread realization for need of thermal packaging and management. The utilization of greater densities and frequencies in microelectronic circuits for machines and computers are expanding step by step. It requires active cooling due to heat generated that is to be degenerate from a comparatively low surface area. So the progress of effective cooling techniques for integrated circuit chips is one of the most important existing requests of micro scale heat transfer which has accepted much consideration for cooling of high power electronics and applications in biomechanical and aerospace field. With the modern improvements in micro-fabrication, numerous equipment containing dimensions in the range of microns e.g. micro heat sinks, microreactors etc. have been developed (Khandekar and Moharana, 2014). These are widely used in thermal management of electronic devices, spacecraft thermal management, microfluidics applications, MEMS systems. Due to overheating of ICs, the use of micro heat sink gained momentum. Innovative thermal packaging systems dealing with active thermal management which are compulsory for such application are being developed.

Overview

The need of efficient cooling methods for microchips has started wide research attention in microscale heat transfer. Microchannels have been suggested to be the eventual solution for eliminating high heat removal mode in microchannel systems. A micro heat sink is a structure that consists of many microchannels engraved on the electric insulated surface of the microchip. The main advantages of microchannel heat sinks are their exceptionally surface area per unit volume through which heat transfer takes place. We can divide the miniaturize systems in two different categories.

(a) Microscale energy systems

(b) Mesoscale energy systems

Microscale energy systems: - According to length scale is characterized by the 1μm to 0.1 mm size lower range of this length scale is too small for maintaining a practical temperature difference for energy production, however, process heating in MEMS scale devices is feasible. Thus power producing concepts relying on ambient temperature processes will be the key to constructing practical microscale devices. Thin film structure will be important in this size range.

Mesoscale energy systems: - The mesoscale regime will have the greatest opportunity for constructing miniature power sources. From about the 100μm level up to a few centimeters, this size regime allows all major power producing concepts to be fundamentally realized. This includes thermally based systems. The systems include are elevated temperature fuel cells, standard electrochemical batteries, nuclear batteries, thermal engines, and harvesting energy from environmental sources.

In a landmark talk given in 1959 entitled “There’s plenty of room at the bottom” Richard P. Feynman (1959) introduced the field of microscale and nanoscale engineering by describing a number of different scenarios and approaches to making things very small. He linked the potential of this new technology to early physics research at low temperature, or at high pressures, where discovery led to important advances in both science and technology. For cooling of microelectronic chips we use Micro (MHP) and miniature heat pipes (MHP). MHP contains microchannels for fluid flow which have extremely smaller hydraulic diameters normally in the range of 10 to 500 μm. Smaller channels used due to (a) higher heat transfer coefficient (b) higher heat transfer surface area per unit volume. In real applications emerging cooling methods are being used to remove high heat fluxes from electronic devices in the range of 100 - 1000 W/cm^2. MHPs are thus capable to supply and extract heat to some biological micro-entities. So there is need to increase heat pipe parameters.

Objective

The challenge in the field of microchannel heat transfer are thermophysical properties of working fluid, and their flow Re, bulk fluid and channel wall temperatures, passage cross sectional dimensions (D_h) and the number of parallel channels in the microchannel array. The objective of this project is to achieve all these parameter in optimize condition. The main parameters of interest are listed below.

1:- Dimensionless wall and fluid temperature

2:- Local Nusselt number

3:- Local surface heat flux

This work contains the application of heat transfer in lesser space at low temperature. The recent development toward miniaturization of equipment helps to better understanding the heat transfer phenomena in small dimensions. The cooling of superconducting and other electronic equipment in micro electro mechanical systems (MEMS) needs a better understanding of heat transfer problems in very small sizes and low temperature. We use water or air for cooling. Water and air remove heat from the surface of component but it not too much effective when heat generation is much higher due to their physical properties. We can use cryogenic fluid for remove of heat from the surface of component. In this work we are trying to find the efficient parameter for cryogenic fluid (Helium) flow.

Axial back conduction

Consider laminar fluid flow over a circular micro-channel exposed to "constant heat flux" boundary condition on its outer surface. In this condition it is considered that the heat applied on the outer surface flows in radial direction along the solid wall of the micro-channel by means of conduction. Once it reaches the solid-fluid interface, the heat flows into water and gets carried along with the flow of the fluid. The surface area (solid-fluid interface) increases linearly in the flow direction. So, heat is added to fluid continuously as the fluid moves in the direction of flow, thus bulk fluid temperature increases in a linear manner. The micro-channel wall (at the solid-fluid interface) temperature also increases linearly beyond the thermally developing zone. The applied heat flux (which is uniformly applied over the outer surface of the micro-channel) is given by

$$q'' = h\left(T_w - T_f\right) \tag{1.1}$$

The channel wall temperature can be calculated from the above heat balance equation

$$T_w = \frac{q''}{h} + T_f \tag{1.2}$$

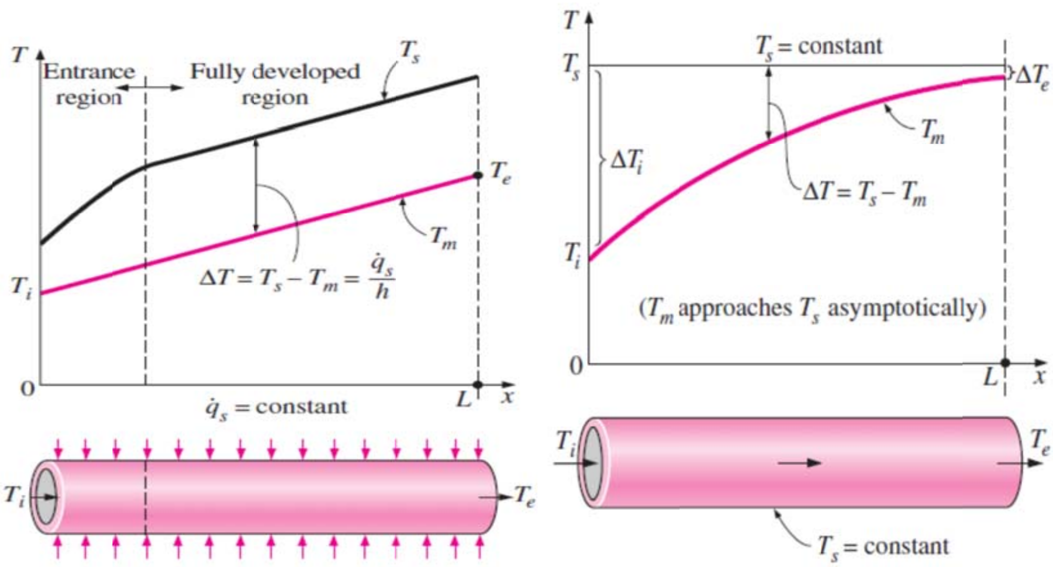

Figure 1.1: Axial variation of bulk fluid and local wall temperature in the flow direction of a circular micro-channel subjected to (a) constant wall heat flux (b) constant wall temp (Cengel (2003))

In the fully developed region, the wall temperature will also increase linearly in the direction of flow as the heat transfer coefficient h is constant in this region. This analysis is established on the postulation that the fluid properties remain constant during flow through the channel. The typical representation of such phenomena is shown in Fig. 1.1 where the bulk fluid temperature and channel wall temperature at the solid-fluid interface is varying in the axial direction.

Here it is to mention that the constant heat flux is applied on the outer surface of the circular channel and the channel wall temperature used in Eq. (1.1) or (1.2) indicate the temperature of the inner surface of the channel i.e. the temperature of the solid-fluid interface. It is important that the channel wall is finite in thickness. Therefore when the heat flux is applied on the outer surface, it conducted radially towards the center of the channel. Once the heat reaches the inner surface of the channel, it is carried by the working fluid in the direction of flow. From Fig. 1.1 it can be observed that there is a temperature difference between any two points along the axial direction (both in the solid and the fluid medium) and the maximum temperature difference between the inlet and the outlet of the heated channel. This axial temperature difference causes potential for heat conduction axially along the solid, and also along the fluid towards inlet of the channel i.e. in a direction opposite to the direction of fluid flow. Such a situation is called "axial back conduction" and leads to conjugate heat transfer. It is interesting to observe in Fig. 1.1(b) that there is no axial temperature gradient when constant wall temperature boundary condition is applied. As of now it is expected that under such condition there will not be any axial heat conduction.

Fluid flow and heat transfer modelling

The Navier-Stokes equation is valid when the mean free path is much smaller than the characteristics dimension of the channel. Generally, continuum approach is valid of the Knudsen number, Kn < 0.1.

Knudsen number is represented, ratio of mean free molecular path (λ) to the characteristic flow dimension of the channel (L or D_h). The value of λ for an ideal gas model considering as a rigid sphere, is given by (Kakac et al., 2004)

$$\lambda = \frac{\bar{k}}{\sqrt{2}\pi P \sigma^2} T \tag{1.3}$$

Governing equation for cylindrical coordinates

For steady two dimensional and incompressible fluid flows, the continuity, momentum and energy equations can be written as

Momentum equations:

x-component:

$$u\frac{\partial u}{dx} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial x^2}\right] \tag{1.4}$$

y-component:

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\upsilon\left[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}(rv)\right\}+\frac{\partial^2 v}{\partial x^2}\right] \tag{1.5}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\alpha\frac{\partial^2 T}{\partial x^2}+\phi \tag{1.6}$$

where

$$\phi=2\mu\left\{\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{v}{r}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2\right\} \tag{1.7}$$

Chapter 2

Literature review

Many researchers had studied on axial wall conduction in conventional size channels (Bahnke and Howard (1964); Cotton and Jackson (1985); Faghri and Sparrow (1980); Barozzi and Pagliarin (1985); Davis and Gill (1970)). Studies on conventional size channels were less important in that rapid progressing age so research on this field was saturated as effect of axial wall conduction is less prominent. Due to advantages of microchannel systems again it gained momentum in study of axial wall conduction in microchannels (Peterson (1999); Maranzana et al. (2004)).

Harley et al. (1995) had done an experimental and theoretical examination of low Reynolds number, high subsonic Mach number Compressible gas stream in channel. Helium, Nitrogen, and argon gasses were utilized. The Knudsen number extended from 0.4 to 10.3. The measured friction factor was in great concurrence with theoretical expectations considering isothermal, local fully developed, first- order, slip stream

Yang et al. (1998) studied water flow through micro-tubes with diameters across going from 50 to 254 µm. Micro-tubes of fused silica (FS) and stainless steel (SS) were utilized. Pressure drop and flow rates were measured to examine the flow parameters. The test results demonstrate critical takeoff of flow parameters from the forecasts of the tried and true hypothesis for micro-tubes with smaller diameters. For micro-tubes with larger diameters, the trial results are in harsh concurrence with the conventional theory. For lower Re, the obliged pressure drop the same as anticipated by the Poiseuille flow theory. At the same time, as Re increments, there is a huge increment in pressure gradient contrasted with that anticipated by the Poiseuille flow theory. The friction factor hence is higher than that given in the conventional theory. The results additionally demonstrate material reliance of the flow behavior.

Oubarra et al. (2013) displayed hydrodynamic and thermal aspects of laminar incompressible slip flow over an isothermal semi-infinite even plate at a generally low Mach number are acknowledged and reconsidered. The non-similar and nearby comparability results of the boundary layer mathematical statements with velocity-slip and temperature-jump boundary conditions are acquired numerically for the vaporous slip flow. The numerical calculations are made by accepting no thermal jump for the fluid flow. Moreover, the inexact expository result of the boundary layer comparisons for high slip parameter is exhibited. Results from nonsimilar solution, nearby closeness approach, and inexact explanatory result are analyzed. We indicate that the likeness methodology utilized by a few researchers as a part of the most recent decades produces generous blunders in hydrodynamic and thermal aspects of the flow. Besides, faultless associations of these attributes are proposed for gaseous and fluid slip flows. The aftereffects of no similar result show, not at all like the past studies, that the general skin friction coefficient introduces an exceptionally slight reduction (vague) in the interim of the slip flow regime, though it diminishes essentially as the flow gets more rarefied. In addition, with expanding slip condition, the after effects of general Nusselt number, for gaseous flow, indicate that the heat transfer at the plate diminishes somewhat in the interim of slip flow regime while it expands on account of fluids flow. This study affirms that for the exact forecast of qualities of slip flow, the slip parameter must be dealt with as a variable instead of a consistent in the boundary layer.

Faghri et al. (1980) displayed an exploratory examination of laminar gas flow through micro-channels. The free variables: relative surface roughness, Knudsen number and Mach number were methodically shifted to focus their effect on the friction factor. The micro-channels were made into silicon wafers, topped with glass, and have hydraulic diameters between 5 and 96 mm. The pressure was measured at seven areas along the channel length to focus nearby values of Knudsen number, Mach number and friction factor. All estimations were made in the laminar flow regime with Reynolds numbers running from 0.1 to 1000. The results show close understanding for the erosion consider in the restricting instance of low M and low Kn with the incompressible continuum flow theory. The impact of compressibility is seen to have a gentle (8 percent) expansion in the friction factor as the Mach number approaches 0.35. A 50 percent diminish in the friction factor was seen as the Knudsen number was expanded to 0.15. At long last, the impact of surface roughness on the friction factor was indicated to be immaterial for both continuum and slip flow regime.

Hsieh et al. (2004) both theoretically and experimentally studied gas flow in a microchannel using nitrogen as working fluid (Kn 0.001 – 0.02). In analytical method a 2-D continuous flow model is used where first slip boundary conditions is used and solved using a perturbation method. They found that the results are in good agreement with analytical solutions

Satapathy (2010) studied steady state heat transfer in laminar 2D thin gas flows in a limitless micro-tube subjected to mixed boundary condition analytically. In this paper he principally concentrated on velocity slip and temperature bounce limit condition on the wall. The energy comparison is settled diagnostically by separation of variable strategy. For the hot liquid and cold wall circumstance the local bulk mean temperature increments with increment in Peclet number however diminishes with increment in Knudsen number. As Peclet number of Knudsen number increases thermal entrance length also increases. Local Nusselt number increments with increment in Peclet number yet diminish with increment in Knudsen number.

Zhang et al. (2010) studied effect of axial wall conduction in conventional thick wall of circular tube with constant outside wall temperature and found that axil wall conduction unifies the inner wall surface heat flux.

Moharana et al. (2011) both experimentally and numerically studied effect of conjugate heat transfer in mini channel array. Based on their study they concluded that axial wall conduction causes distortion in boundary condition at the solid fluid interface and thus influence heat transfer process.

Moharana et al. (2012) had numerically investigated effect of axial wall conduction in a square microchannel engraved on a solid substrate whose bottom face is subjected to constant wall heat flux. The parametric variations include solid to fluid conductivity ratio (k_{sf}), wall thickness to inner radius ratio (δ_{sf}), and flow Re. They found that there exists an optimum k_{sf} at which average Nu is maximum. They also found similar observation in circular microtube.

Moharana and Khandekar (2012) numerically studied the effect of axial wall conduction in microtube subjected to constant wall temperature on outer surface. The parametric variations considered in their study are solid to fluid conductivity ratio (k_{sf}), wall thickness to inner radius ratio (δ_{sf}), and flow Re. They found that with decreasing value of k_{sf}, the average Nusselt number over the full channel length increases. When k_{sf} approaches to a very smaller value near zero, the average Nu starts to increase sharply.

Moharana and Khandekar (2013) studied effect of rectangular microchannel aspect ratio on axial wall conduction in solid substrate and found that average Nu is minimum corresponding to channel aspect ratio slightly lower than 2.0

Kumar and Moharana (2013) numerically studied axial wall conduction in a partially heated microtube subjected to constant wall temperature in microtube and found that wall to fluid conductivity ratio (k_{sf}) and wall thickness to inner radius ratio (δ_{sf}) plays a prevailing role in the conjugate heat transfer process.

Tiwari et al. (2013) numerically studied axial wall conduction in a partially heated microtube subjected to constant wall heat flux in microtube and found that for both fully and partially heated microtube, there exists an optimum k_{sf} at which Nu_{avg} is maximum.

Mishra and Moharana (2014) numerically studied axial wall conduction in a microtube where flow is sinusoidally varying with time i.e. pulsatile flow in nature. Based on the numerical simulation, they concluded that for a particular pulsation frequency (Wo) there exists an optimum value of k_{sf} at which overall Nusselt number (Nu) is maximum, similar to the observation by Moharana et al. (2012).

In recent years some researchers (Rostami et al., 2002; Shen et al., 2003; Baek et al., 2012; Lahjomri and Oubarra, 2013) had seen some rarefied gases can be used as a working fluid for refrigeration and cooling systems. They had been carried out their investigation on the thermo-physical properties and other physical parameters of gases which affect the heat transfer rate. They had tried to find optimal conditions for cryogenic gases like Nitrogen, organ, and oxygen to use as a working fluid.

Jiao et al. (2004) studied heat transfer in a copper tube with helium gas as the working fluid. They considered temperature dependent thermo-physical properties. They predicted temperature distribution and velocity profile in the miniature tube and presented correlation for Nusselt number.

Still the conjugate heat transfer under temperature dependent cryogenic fluid in microchannel is not explored. Therefore this study is undertaken to explore it in a systematic manner.

Chapter 3

Numerical simulation

Introduction

In this work a two-dimensional numerical study has been undertaken to study the effects of axial back conduction in conjugate heat transfer situation in simultaneously developing laminar flow and heat transfer in a fully heated micro-tube subjected to constant heat flux boundary condition on its outer surface. Microtube of total length 60 mm is considered for the numerical study. In this work, simultaneously developing single-phase, steady-state laminar fluid flow with constant thermo physical properties for solid and temperature dependent thermo-physical properties of liquid are considered. Microtube geometry considered in the present study is schematically shown in Fig. 3.1.

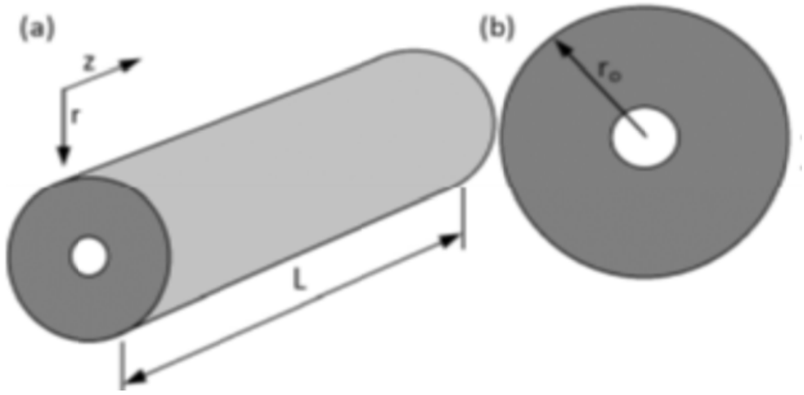

Fig. 3.1: Microtube and its cross section view

Heat transfer through natural convection and radiation mode is neglected. The inner radius (r_i) of the tube is represented as δ_f and the thickness of the micro-tube (r_o - r_i) is represented as δ_s. Thus, a parameter called δ_{sf} is defined as the ratio of tube thickness to the tube inner radius. The inner radius (δ_f) and the length (L) of the micro-tube is maintained constant in the computational domain at 0.2 mm and 60 mm respectively. The

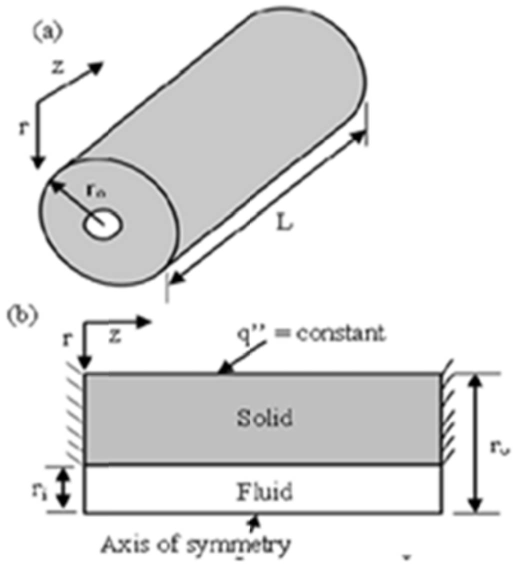

Fig. 3.2: (a) Micro-tube, and its computational domain, (b) Fully heated

microtube wall thickness (δ_s) is varied such that the value of δ_{sf} = 1 and 5. Helium gas (inlet temperature of 100 K) with conductivity k_f is used as the working fluid. The wall conductivity (k_s) is varied such that k_{sf} (= k_s/k_f) varies in the range of 1.7105 – 2822. The flow Re is varied in the range of 100 – 500.

Table 3.1: Different materials used for the micro-channel

Metal	**ρ (kg/m³)**	**Cp (J/kg-K)**	**k_s (w/m-K)**	**k_f (w/m-K)**	**$k_{sf} = k_s/k_f$**
Sulfur	2070	708	.206	0.152	1.7105
Sio_2	2220	745	1.38	0.152	9.03
Bismuth	9780	122	7.86	0.152	51.7105
Nichrome	8400	420	12	0.152	78.947368
SS316	8238	468	13.4	0.152	88.1578
Constantan	8920	384	23	0.152	151.3158
Cr-steel	7822	444	37.7	0.152	248.0263
Bronze	8780	355	54	0.152	355.2632
Zinc	7140	389	116	0.152	763.1579
Alloy-195	2790	883	168	0.152	1105.2632
Silver	10500	235	429	0.152	2822.3684

The cross-sectional solid faces of the micro-tube are assumed to be insulated. Considering angular symmetry, two dimensions Cartesian coordinate system (r-z) is used in our computational domain. Only one half of the transverse section of the micro-tube was included in the computational domain, in view of the symmetry conditions. Constant heat flux boundary condition is applied on the outer surface of the micro-tube

The governing differential equations i.e. continuity, Navier-Stokes, and Energy equations are for fluid domain

$$\nabla.\vec{u} = 0 \tag{3.1}$$

$$\vec{u}.\nabla\vec{u} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2\vec{u} \tag{3.2}$$

$$\vec{u}.\nabla T = \left(\frac{k}{\rho}c_p\right).\nabla^2 T \tag{3.3}$$

For solid domain

$$\nabla^2 T = 0 \tag{3.4}$$

The boundary conditions are

At, z = 0 to z = L and y = 0, symmetric axis

At, z = 0 and y = 0 to y = δ_f,

At, z = L and y =0 to y = δ_f, gauge pressure

At, z = 0 and y = δ_f to y = $\delta_s + \delta_f$, $\frac{\partial T}{\partial z} = 0$

At, z = L and y = δ_f to y = $\delta_s + \delta_f$, $\frac{\partial T}{\partial z} = 0$

The governing differential equations are solved using commercial platform Ansys-Fluent®. The “standard” scheme was used for pressure discretization. For velocity-pressure coupling the SIMPLE algorithm was used in the multi-grid solution procedure. “second-order upwind” scheme was used for solving the momentum and energy equations. An absolute convergence criterion for continuity and momentum equations is taken as 10^{-6} and for energy equation it is 10^{-9}. Helium enters the micro-tube with a slug velocity profile. Thus, the flow is hydro dynamically and thermally developing in nature at the tube inlet. Rectangular elements were used for meshing the computational domain and the grid independence test was ensured for all geometry included in the study.

Cryogenic fluid properties

In a cryogenic fluid the thermo-physical properties are very sensitive to temperature; thus they change appreciably for smaller change in temperature. Therefore it is very important to consider this in conjugate heat transfer situation. The thermo-physical properties as a function of temperature used as UDF in the simulation process using the following mathematical equations for viscosity, density, and thermal conductivity (Yaws, 1999).

$$\mu_f(T) = -4.25462\times10^{-7} + 8.25786\times10^{-8}T - 9.43838\times10^{-11}T^2 + 7.6085\times10^{-14}T^3, \qquad (3.5)$$

$$\rho_f(T) = 0.1186 + 3.38334e^{\left(-\frac{T}{20.41}\right)} + 0.93142e^{\left(-\frac{T}{99.36854}\right)} \qquad (3.6)$$

$$k_f(T) = 0.00793 + 0.000878621T - 2.50172\times10^{-6}T^2 + 3.92\times10^{-10}T^3 \qquad (3.7)$$

Grid independence test:

Rectangular elements were used for meshing the computational domain and the grid independence test was ensured for all geometry considered in this study. Local Nusselt number obtained for a tube with negligible wall thickness for three different grids of 32×4800, 40×6000, and 50×7500, for Re = 100 and $q'' = 88000$ w/m^2, is shown in fig.3.2. The local Nusselt number at the fully developed flow regime (near the micro-tube outlet) changed by 0.68% from the mesh size of 32×4800 to 40×6000, and changed by less than 0.55% on further refinement to mesh size of 50×7500. Moving from first to the third mesh, no appreciable change is observed. So, the middle grid (40×6000) is selected, It can also be observed in Fig. 3.2 that the local Nusselt number values in the fully developed region are nearly equal to the theoretical value of $Nu_z = 4.36$ where Nu_z is the Nusselt number for fully developed flow in a circular tube subjected to constant heat flux.

Data reduction

The interesting parameters are (a) peripheral averaged local heat flux (b) local bulk fluid temperature and (c) peripheral averaged local wall temperature. These parameters allow us to determine the effect of axial conduction on the local Nusselt number. The conductivity ratio (k_{sf}) is defined as the ratio of thermal conductivity of the microtube wall (k_s) to that of the working fluid (k_f). The axial coordinate, z, in non-dimensional form is as follows

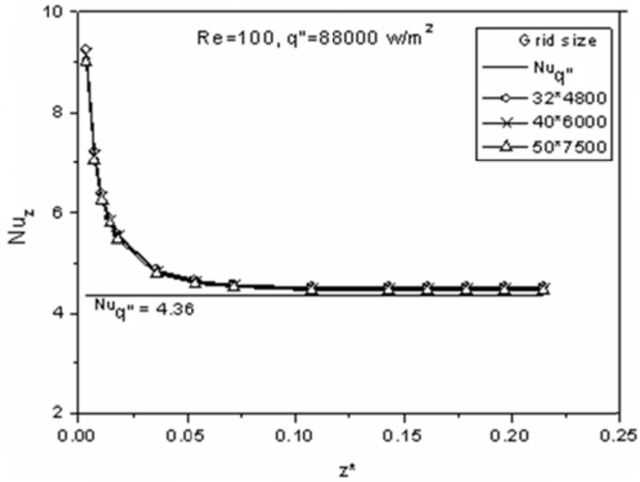

Fig. 3.3: Axial variation of local Nusselt number for $\delta_s = 0$ for three different mesh sizes

$$z^* = \frac{z}{\mathrm{Re}.\mathrm{Pr}.D_h} \tag{3.8}$$

The non-dimensional local heat flux at the fluid-solid interface is given by

$$\phi = \frac{q''}{q''_w} \tag{3.9}$$

where, q" is the local heat flux transferred at the solid-fluid interface along the micro-tube length and q''_w is heat flux on the outer surface of the micro-tube due to constant wall heat flux boundary condition at outer surface along the micro-tube length. The dimensionless bulk fluid and tube inner wall temperatures are represented

$$\theta_w = \frac{(T_w - T_{fi})}{(T_{fo} - T_{fi})} \tag{3.10}$$

$$\theta_f = \frac{(T_f - T_{fi})}{(T_{fo} - T_{fi})} \tag{3.11}$$

where, T_{fi} and T_{fo} are the bulk fluid temperature (averaged over the cross-section) at the channel inlet and outlet. T_f is the average bulk fluid temperature at any location and T_w is the wall temperature at that position. The Nusselt number (local) is given by

$$Nu_z = \frac{h_z.D}{k_f} \tag{3.12}$$

where, the local heat transfer coefficient is given as

$$h_z = \frac{q''_z}{(T_w - T_f)} \tag{3.13}$$

For calculating he average Nusselt number over the full length of the microtube the following equation used

$$\overline{Nu} = \frac{1}{L}\int_0^L Nu_z dz \tag{3.14}$$

Chapter 4

Results and discussion

As we said in previous chapter, while the inner radius of the micro-tube is keep constant, the thickness of the micro-tube wall in the radial direction (see Fig. (3.2)) is varied. This will help to understand the effect of wall thickness on the conjugate heat transfer. When thickness of the solid wall increases, the surface on which constant wall heat flux applied moves away from the solid-fluid interface.

For flow through a microtube subjected to constant wall heat flux, heat transfer coefficient will be maximum when constant heat flux is experienced at the solid-fluid interface of the micro-tube. Under ideal conditions (zero wall thickness) if constant wall heat flux is applied to a circular tube, it will lead to maximum value of Nusselt number for fully developed laminar flow i.e. Nu = 4.36. For similar condition and constant wall temperature boundary condition, the fully developed Nusselt number will be equal to Nu = 3.66. Practically every channel will have some finite wall thickness and because of conjugate heat transfer conditions it is not guaranteed to have the same boundary condition at the solid-fluid interface which is applied on the outer surface of the micro-channel. The objective of this work is to find the actual boundary condition experienced at the solid-fluid interface of a micro-channel subjected to fully heating by constant wall heat flux at its outer surface.

The important parameters are axial variation of wall temperature, bulk fluid temperature, and local Nusselt number. We consider the micro-channel is heated over its full length. The axial variation of bulk fluid and wall temperature under ideal condition was presented in Fig. 1.1. Fig. 4.1 shows the axial variation of dimensionless wall and bulk fluid temperature as a function of, k_{sf}, δ_{sf} and Re. The dotted line represents linear variation of bulk fluid temperature between the inlet and the outlet of the micro-tube, its limiting values in dimensionless form are 0 and 1. Under ideal conditions, the bulk fluid temperature varies linearly between the inlet and the outlet if the tube is subjected to constant heat flux boundary condition. At lower flow rate Re = 100 and higher value of

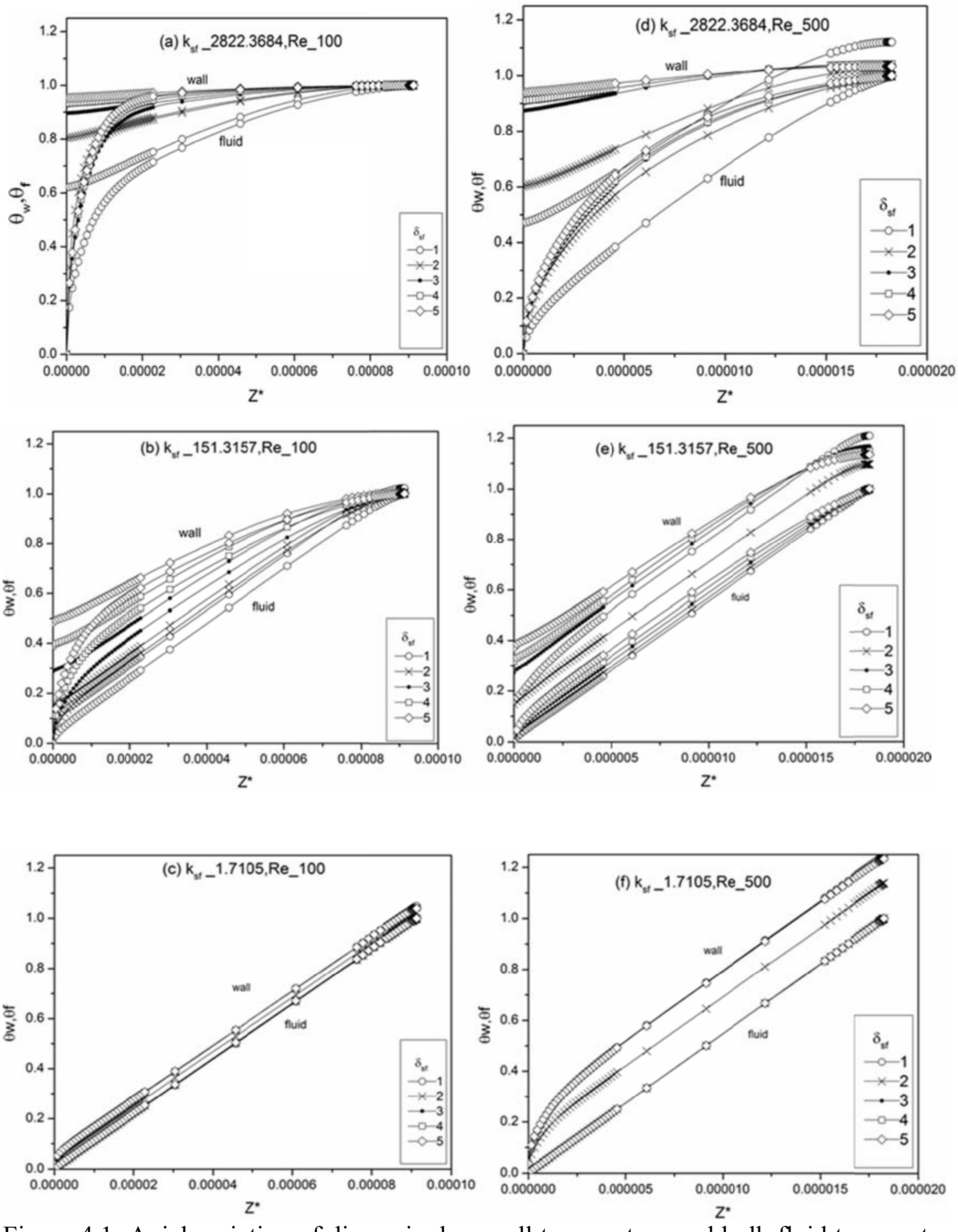

Figure 4.1: Axial variation of dimensionless wall temperature and bulk fluid temperature as a function of δ_{sf}, k_{sf} and Re.

k_{sf} = 2822.3684 the axial variation of dimensionless bulk fluid temperature can be seen to be away from linear variation as shown in Fig. 4.1(a). It is higher at the entrance developing region and become linear in developed region of the micro-tube which is clearly shown in Fig. 4.1(a). It is quite similar to the result of Fig. 1.1(a). When we goes at higher value of δ_{sf} = 5, kept all other parameters constant we have seen it become linear throughout the micro-channel length Fig. 4.1(a).

When we goes down to the value of k_{sf} = 1.7105 at Re = 100, we have seen the dimensionless bulk fluid temperature and solid-fluid interface wall temperature varies linearly which is similar to the ideal condition. This result we can see in the Fig. 4.1(b, c). As we have seen in Fig. 4.1(c) this is the case in which the difference temperature between the solid-fluid interface wall temperature and bulk fluid temperature is very less so we can say this is the isothermal condition.

Now when we kept all other parameters constant and move towards higher flow rate Re = 500, we have seen the results again come closer to the ideal condition for lower value of micro-tube and wall thickness ratio (δ_{sf} = 1), while it going away from ideal condition for higher value of wall thickness and micro-tube diameter ratio (δ_{sf} = 5).

At higher value of flow rate Re = 500 and lower value of conducting ratio k_{sf} = 1.7105, it can be clearly observed that as shown in Fig. 4.1(f), the constant temperature difference between the dimensionless bulk fluid temperature and solid-fluid interface wall temperature.

From this study, it can be seen that the heat flux experienced at the solid-fluid interface is almost axially constant irrespective of the conductivity ratio (k_{sf}), thickness ratio (δ_{sf}). At higher thickness ratio (δ_{sf}), the actual heat flux experienced at the solid-fluid interface is much more than for lower thickness ratio (δ_{sf}). This overall axial variation in heat flux is due to the fact that low thermal conductivity ratio and lower thickness ratio (δ_{sf}) causes higher axial thermal resistance in the wall and vice versa. Accordingly, at higher δ_{sf} low axial thermal resistance of the wall leads to significant back conduction; this effect becomes more noticeable with increase in solid to fluid conductivity ratio (k_{sf}). As we can see in Fig. 4.2 (a) and Fig. 4.2 (b). We can see that for some distance from inlet in the direction of axial location the value of ratio of heat flux is approximately constant for lower value of k_{sf} = 1.7105 with value nearly equal to 2 and

6 for the case of δ_{sf} = 1 and 5 respectively. As well as this result is also same for higher Reynolds number as shown in Fig. 4.2 (c) and Fig. 4.2 (f). So from these graphs we can

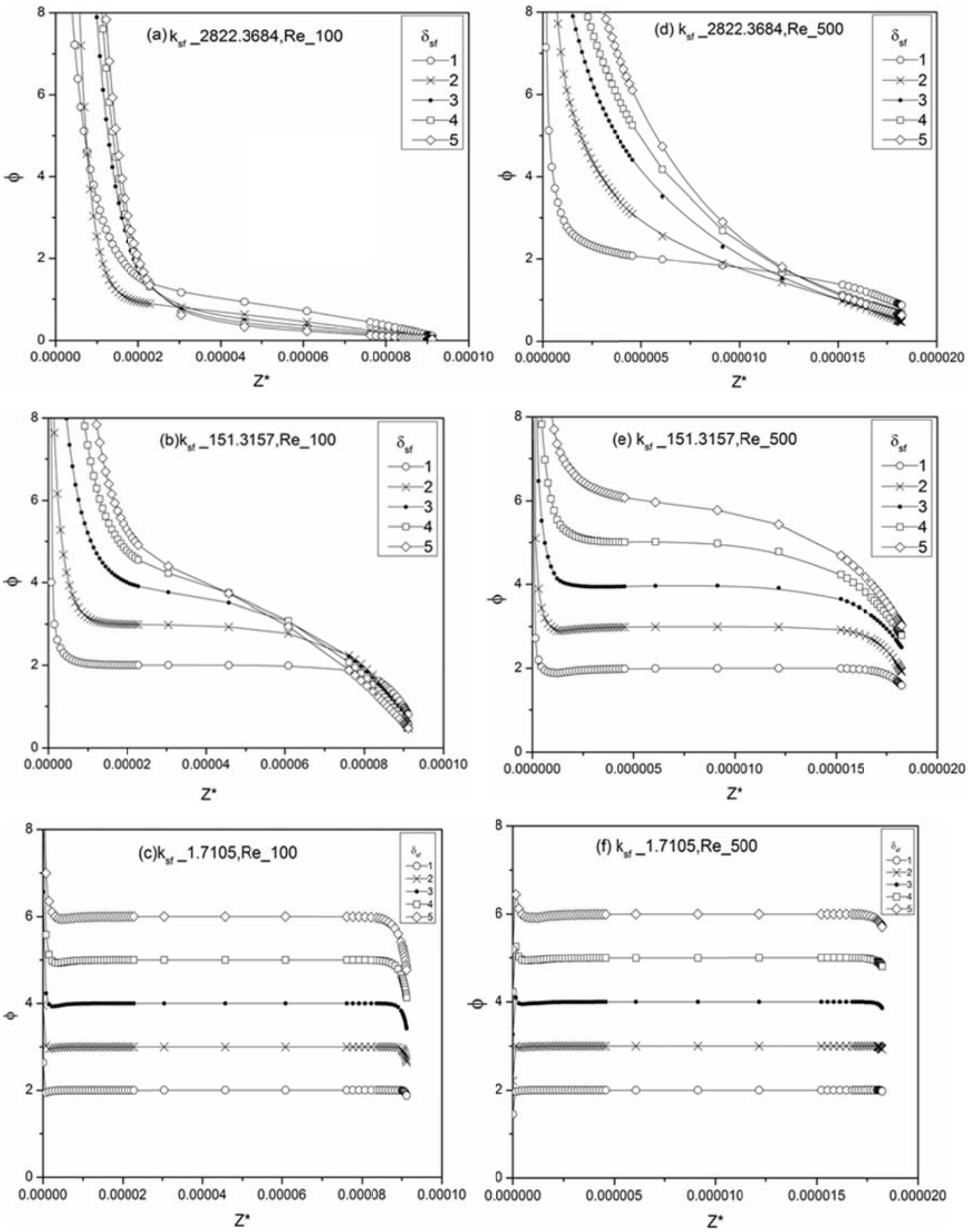

Fig.4.2: Axial variation of dimensionless heat flux as a function of δ_{sf}, k_{sf} and Re

say that value of heat flux at solid-fluid interface is mainly depend on wall thickness and conductivity ratio when at outer periphery constant wall heat flux is applied.

Zhang et al. (2010) had also reported similar observations. They had applied a constant temperature boundary condition at the outer surface of a circular tube and found that the dimensionless heat flux at the solid-fluid boundary tends to become constant when axial conduction in the tube wall dominates. That the dimensionless heat flux at the solid-fluid boundary tends to become constant when axial conduction in the tube wall dominates. From above Fig. 4.2 it is clear that for all conditions the heat flux is strong function of k_{sf} and δ_{sf}, but it is a weakly dependent on the flow rate Re due to axial back conduction.

The axial variation in dimensionless wall and bulk fluid temperature (as in Fig. 4.1) and dimensionless wall heat flux (as in Fig. 4.2) will be decides the value of local Nusselt number. Axial variation of local Nusselt number for micro-tube subjected to boundary condition constant wall heat flux is represented in the Fig. 4.3.

As we discussed previously if the if the boundary condition experienced at the solid-fluid interface is close to constant wall heat flux, then the local Nusselt number in the fully developed zone will converge close to $Nu_z = 4.36$. And if the boundary condition experienced at the solid-fluid interface is close to constant wall temperature, then the local Nusselt number in the fully developed zone will converge close to $Nu_T = 3.66$. For low flow rate Re = 100 and lower wall thickness, the fully developed Nusselt numbers (Nu_z) are slightly lower than Nu_z and the value of Nu_z decreasing with increasing value of δ_{sf}, which can be observed in Fig. 4.3(a). Secondly, when we go down to the value of k_{sf}, kept all other parameters constant, the value of local Nusselt number become constant along the axial direction of micro-tube only except entrance region as shown in Fig. 4.3 (a, b and c). When we increase the flow rate, kept all other parameters constant, local Nusselt number increase and it decreases, increases the wall thickness. It goes down near the ideal value of $Nu_z = 4.36$.

As shown in above Fig. 4.3 (a and b) the value of local Nusselt number is very less than the ideal value ($Nu_z = 4.36$) at the entrance region. These lobes are formed due to the large temperature difference between solid-fluid wall and bulk fluid temperature. This effect is firstly described by the Brinkman and it’s shown by the Brinkman number.

When the value of Brinkman number is high the value of average Nusselt number is goes down and it can decrease this value up to 40%.

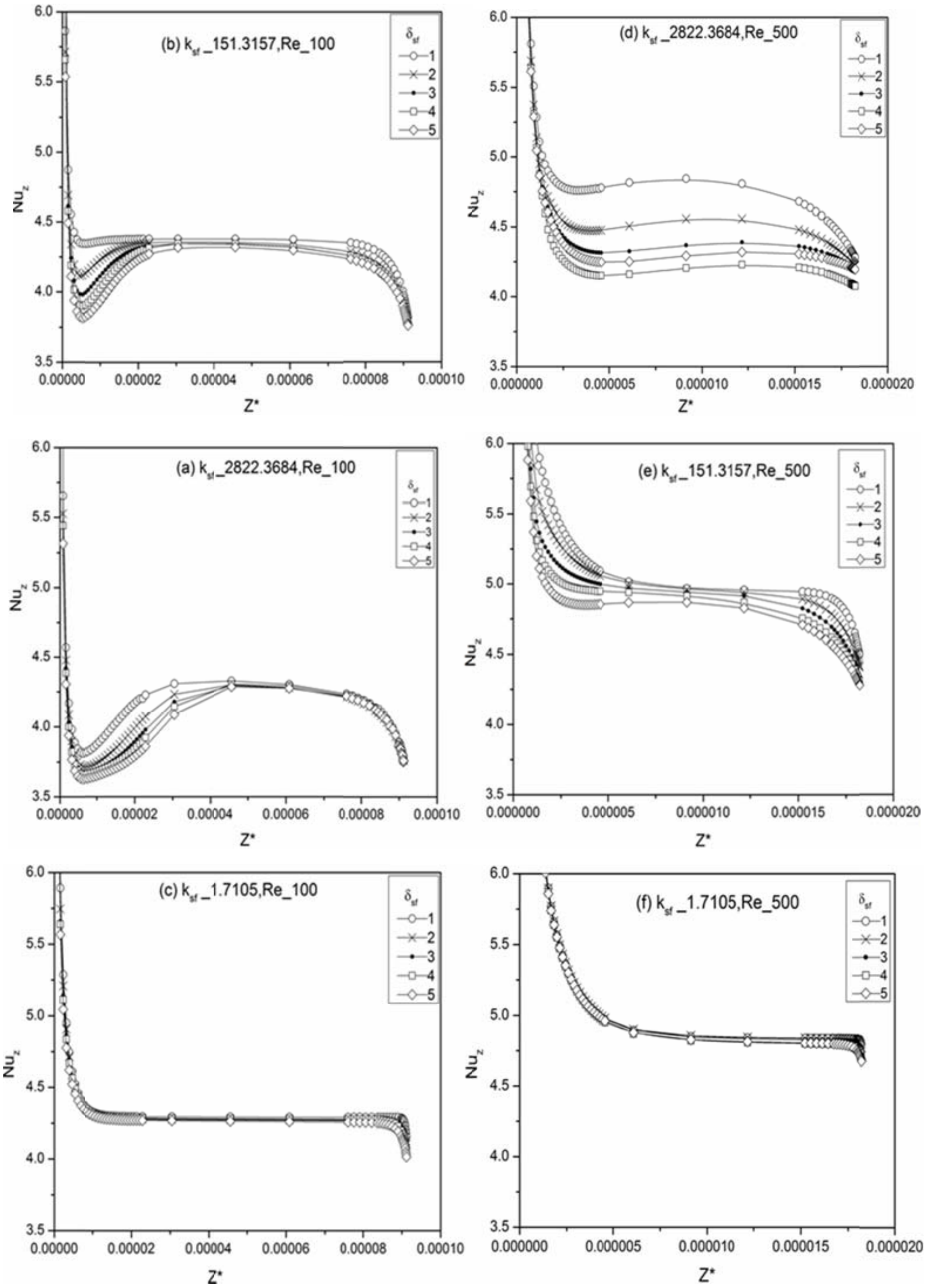

Fig. 4.3: Axial variation of local Nusselt number as a function of δ_{sf}, k_{sf} and Re

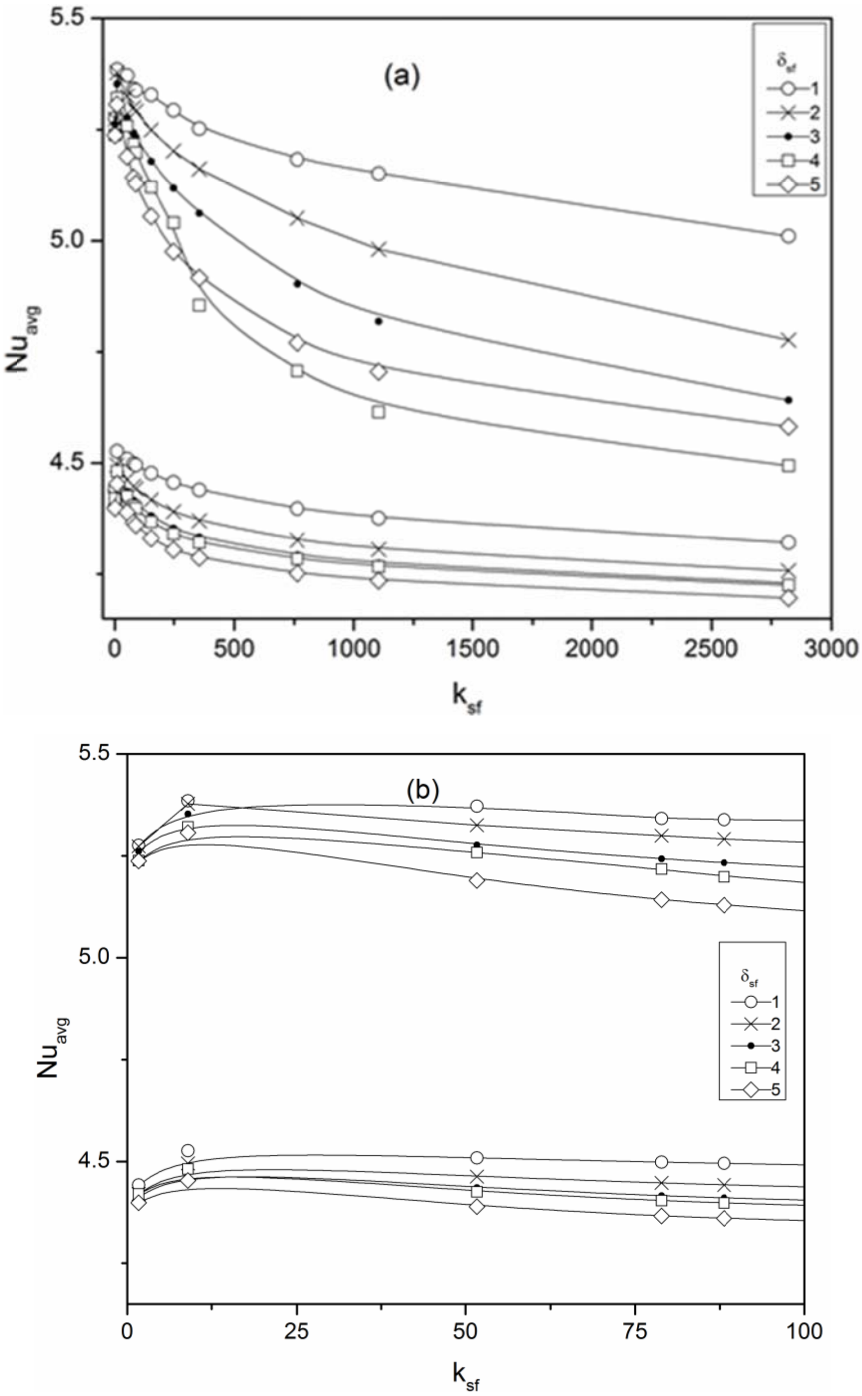

Fig. 4.4: Variation of average Nusselt number with k_{sf}, for (Re = 100-500), & (δ_{sf} = 1-5)

From above discussion we can say the value of local Nusselt number is function of Re, k_{sf}, and δ_{sf}. Brinkman number is also play a measure role to decide the local Nusselt number in the case of cryogenic fluid flow through a micro-tube.

The axial variation of wall temperature at the solid-fluid interface drifts more towards the trend of constant heat flux in the thicker wall. This generates higher Nusselt number compared to thinner tube wall thickness. It is very clear from Fig.4.4 that the value of average Nusselt number is maximum at very low k_{sf} and decreases when move towards higher value of k_{sf}. The value of average Nusselt number is maximum at an optimum value of k_{sf}. So we can say, for an optimum value of Nusselt number heat transfer will not vary with thickness ratio (δ_{sf}).

Chapter 5

Conclusion

A numerical study has been carried out for internal convective cryogenic fluid flows in a micro-tube subjected to conjugate heat transfer situation. This study has been carried out to understand the effect of axial wall conduction in cryogenic fluid for developing laminar flow and heat transfer in a circular micro--tube subjected to constant heat flux boundary condition imposed on its outer surface. Simulations have been carried out for a wide range of pipe wall to fluid conductivity ratio (k_{sf}: 1.7105-2822.3684), pipe wall thickness to inner radius ratio (δ_{sf}: 1- 5), and flow Re (100 - 500). The main outcomes of this study are, for fully heated micro-tube, it is found that the value of Nu_{avg} is increasing with decreasing value of k_{sf} and the rate of increase of Nu_{avg} is higher for smaller values of k_{sf} ($k_{sf} < 30$). Secondly, when other parameters are remaining same, for lower δ_{sf}, Nu_{avg} is higher than higher δ_{sf}. The difference between the Nu_{avg} values (corresponding to δ_{sf} = 1-5) at lower k_{sf} is higher compared to higher k_{sf} values. Finally, the value of Nu_{avg} increases with increasing fluid flow Re while other parameters are constant. Therefore, depending on situation, axial conduction enhance/reduce overall heat transfer process.

References

ANSYS-FLUENT V.13.0 Users Guide, Ansys Inc., USA.

Bahnke G.D., and Howard C.P., 1964, Discussion: "The Effect of Longitudinal Heat Conduction on Periodic-Flow Heat Exchanger Performance" Journal of Enginering Power, 86(2), pp. 105–117.

Barozzi G.S., Pagliarin G., 1985, A Method to Solve Conjugate Heat Transfer Problems: The Case of Fully Developed Laminar Flow in a Pipe, Journal of Heat Transfer, 107, pp. 77-83.

Cengel U.A., 2003, Heat Transfer: A Practical Approach, 2 edition, McGraw-Hill, New York, USA.

Choi SB., Barron RF., Warrington RO., 1991, Fluid flow and heat transfer in microchannels. In: Winter annual meeting of the American Society Mechanical Engineers, pp. 123–34.

Cotton M.A., Jackson J.D., 1985, The Effect of heat conduction in a tube wall upon forced convection heat transfer in the thermal entry region. In: Numerical methods in thermal problems, Vol. IV. Pineridge Press, Swansea, pp. 504–515.

Cotton Mark, Jackson J.D. 1985, The effect of heat conduction in a tube wall upon forced convection heat transfer in the thermal entry region. 4th Int. Conf. on Numerical Methods in Thermal Problems, pp 504-515.

Davis E.J., Gill N.W., 1970, The effect of axial conduction in the wall on heat transfer with laminar flow, International Journal of Heat and Mass Transfer, 13(3), pp. 459–470.

Faghri M., Sparrow E.M., 1980, Simultaneous wall and fluid axial conduction in laminar pipe-flow heat transfer, Journal Heat Transfer, 102(1), pp. 58-63.

Fenyman R.P., 1959, Plenty of Room at the Bottom, American Physical Society, Pasadena, December.

Harley J.C., Hung Y., Bau H.H., Jaemel J. N., 1995, Gas flow in microchannel, Journal of Fluid Mechanics, 284, pp. 257-274.

Hsieh S.S., Tsai H.H., Lin C.Y., Huang C.F., Chien C.M., 2004, Gas flow in a long microchannel, International Journal of Heat and Mass Transfer, 47(17-18), pp. 3877–3887.

Jiao A., Jeong S., Ma H.B., 2004, Heat transfer characteristics of cryogenic helium gas through a miniature tube with a large temperature difference, Cryogenics, 44(12), pp. 859–866.

Kakac S., Vasiliev L.L., Yener Y., 2004, Microscale heat transfer fundamentals and application-vol. 193, Nato Science Series.

Khandekar S., Moharana, M. K., 2014, Some Applications of Micromachining in Thermal-Fluid Engineering, Book chapter in: Introduction to Micromachining, Edited by V.K. Jain, Narosa Publishing House, New Delhi.

Kumar M, Moharana MK, Axial wall conduction in partially heated microtubes, 22nd National and 11th International ISHMT-ASME Heat and Mass Transfer Conference, 28-31 December 2013, Kharagpur, India.

Lahjomri J., Oubarra A., 2013, Hydrodynamic and thermal characteristics of laminar slip flow over a horizontal isothermal flat plate, Journal of Heat Transfer, 135(2), pp. 1-9.

Maranzana, G., Perry, I., and Maillet, D., 2004, Mini- and micro-channels:Influence of axial conduction in the walls, International Journal of Heat and Mass Transfer, 47(17), pp. 3993–4004.

Mishra P, Moharana MK, Axial wall conduction in pulsating laminar flow in a microtube, 12th International Conference on Nanochannels, Microchannels, and Minichannels, 3-7 August 2014, Chicago, USA (Accepted for presentation).

Moharana MK, Agarwal G, Khandekar S, 2011, Axial conduction in single-phase simultaneously developing flow in a rectangular mini-channel array, International Journal of Thermal Sciences, 50(6), pp 1001-1012.

Moharana MK, Khandekar S, 2013, Effect of aspect ratio of rectangular microchannels on the axial back-conduction in its solid substrate, International Journal of Microscale and Nanoscale Thermal and Fluid Transport Phenomena, 4(3-4), pp. 1-19.

Moharana MK, Khandekar S, Numerical study of axial back conduction in microtubes, 39th National Conference on Fluid Mechanics and Fluid Power (FMFP2012), 13-15 December 2012, Surat, India.

Moharana MK, Singh PK, Khandekar S, 2012, Optimum Nusselt number for simultaneously developing internal flow under conjugate conditions in a square microchannel, Journal of Heat Transfer, 134(7), pp. 1-10.

Peterson R.B., 1999, Numerical modeling of conduction effects in microscale counter flow heat exchangers, Microscale Thermophysics Engineering, 3(1), pp. 17-30.

Satapathy A. K., 2010, Slip flow heat transfer in a semi-infinite microchannel with axial conduction, Journal of Mechanical Engineering Science, 224(2), pp. 357-361.

Tiwari N, Moharana MK, Sarangi SK, Influence of axial wall conduction in partially heated microtubes, 40th National Conference on Fluid Mechanics and Fluid Power (FMFP2012), 12-14 December 2013, Hamirpur, India.

Turner S. E., Lam L.C., Faghri M., Gregory O.J., 2004, Experimental investigation of gas Flow in microchannels, Journal of Heat Transfer, 126, pp. 753-763.

Yaws C.L., 1999Chemical Properties Handbook, McGraw-Hill, New York.

Zhang S.X., He Y.L., Lauriat, G., Tao, W.Q., 2010, Numerical studies of simultaneously developing laminar flow and heat transfer in microtubes with thick wall and constant outside wall temperature, International Journal of Heat and Mass Transfer, 53(19-20), pp. 3977–3989.