Multiobjective Output Feedback Controller Compare with IMC-Based PID Controller

Thesis submitted in partial fulfilment of the requirements for the degree of

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In

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(Control & Automation)

By

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Dedicated to my family,

Supervisor and my friends

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Certificate

This is to certify that the work in the thesis entitled ***Multiobjective output feedback controller compare with IMC-Based PID Controller*** by ***Maddela Chinna Obaiah*** is a record of an original research work carried out by him under my supervision and guidance in partial fulfilment of the requirements for the award of the degree of Master of Technology with the specialization of Control & Automation in the department of Electrical Engineering, National Institute of Technology Rourkela. Neither this thesis nor any part of it has been submitted for any degree or academic award elsewhere.

Place: NIT Rourkela
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Last, but not the least, I would like to dedicate this thesis to my family, for their love, patience, and understanding.

Maddela Chinna Obaiah

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ABSTRACT

The present work aims at comparison between Internal Model Control (IMC) and Multiobjective Output Feedback Controller. The inter model control (IMC) based tuning principle is straightforward, simple to use, and easy to implement which is exceptionally appealing to professionals in the real practice. The most essential reality is IMC-PI/PID tuning guideline has one and only characterized tuning parameter, which is straightforwardly identified with the closed loop time constant. Internal Model Control selecting among the other conventional PID Controllers by considering values of the Integral of the squared value of the error (ISE) and Integral of the absolute value of the error (IAE). IMC is comparing with Direct Synthesis Method (DSM) this method is based on the desired closed loop characteristic equation.

In Multiobjective the design objectives are H-infinity and Pole Placement Constraints. These design objectives are formulated in terms of the common lyapunov function. A complete Linear Matrix Inequality (LMI) of the output feedback synthesis with H-infinity control with pole placement is presented. By change of controller variables the output feedback control would be linearized due to the nonlinear terms include in the objectives constraints. The Linear Matrix Inequality (LMI) constraints of the design objectives i.e. H-infinity and Pole Placement Constraints are derived, and these LMI constraints are solving by using LMI Solvers. The comparison of the methods is illustrated by a realistic design example and the simulation results are presents.

Contents

Certificate iv

Acknowledgement v

Abstract vi

List of Figures vii

List of Tables viii

Nomenclature ix

1 Introduction 1

1.1 Review on PID Controllers 1

1.2 Review on Robust Control 3

1.3 Motivation 4

1.4 Organization of the Thesis 4

2 Preliminaries 5

2.1 Linear Matrix Inequality (LMI) 5

2.1.1 Definition of LMIs 5

2.1.2 Generic LMI Problems 6

2.2 Schur Complement 6

2.3 Bounded Real Lemma 7

2.4 Mathematical Preliminaries and Notations 7

2.4.1 Norms of Systems and Signals 7

2.5 Hamiltonian Matrix Notation 8

2.6 Two-port Block Diagram Representation 8

2.7 H-infinity Design Problem 9

3 Internal Model Control & IMC-Based PID Controller 10

3.1 Internal Model Control 10

3.1.1 IMC strategy 11

3.1.2 IMC Design Procedure 14

3.2 IMC-Based PID Controller 15

3.2.1 Standard feedback form to IMC 15

3.2.2 Procedure of IMC-Based PID control Design 17

3.3 Comparison of Internal Model Control and Direct Synthesis Method 17

3.3.1 Direct Synthesis Method 18
3.3.2 Example 18
4 H-infinity Control & Robust Pole Placement 20
4.1 H-infinity Control 20
4.1.1 H-infinity Description 20
4.1.2 H-infinity Control Problem with Solution 21
4.1.3 Properties of H-infinity Controller 23
4.2 Robust Pole Placement 24
4.2.1 Kronecker Product 24
4.2.2 Lyapunov theorem for Pole Placing 25
4.2.3 Pole Placement in LMI Regions 25
4.2.3.1 Left half plane 26
4.2.3.2 ∝-Stability 26
4.2.3.3 Disk 27
4.2.3.4 Conical Sector 27
4.2.4 Robust $\mathcal{D}$-Stability Quadratic $\mathcal{D}$-Stability 28
4.2.5 Output Feedback Synthesis 29
5 Comparing H-infinity with pole placement and IMC based PID Control, Simulation & Results 31
5.1 Comparing H-infinity with pole placement and IMC based PID Control 31
5.1.1 H-infinity 32
5.1.2 Linearizing Change of Variables 32
5.1.3 LMI region 34
5.2 Simulation & Results 35
5.2.1 H-infinity Control 36
5.2.2 IMC-Based PID Controller 38
6 Conclusion & Future 40
6.1 Conclusion 40
6.2 Future Scope 41
References 42

List of figures

Fig No.	Figure Description	Page No.
1.1	Closed loop control system	3
2.1	Generalized Two port block diagram	8
3.1	Open loop control strategy	10
3.2	Schematic of the IMC scheme	11
3.3	IMC structure	15
3.4	Change in IMC structure	16
3.5	Rearrangement of IMC structure	16
3.6	IMC-Based PID Controller	16
3.7	Closed loop feedback control	18
3.8	Comparison results of IMC and DSM	19
4.1	Two port block diagram	20
4.2	Open left half plane	26
4.3	Semi left half plane	26
4.4	LMI region (Disk)	27
4.5	LMI region (Conic sector)	28
5.1	Set point tracking for Step input	39

List of Tables

Table No.	Table Description	Page No.
3.1	IAE, ISE performances of Direct Synthesis Method and IMC-Based PID Controller	19
5.1	IAE, ISE performances of H-infinity with Pole Placement and IMC-Based PID Controller	38

Nomenclature

d_i	Input disturbance
d_o	Output disturbance
$e(s)$	Error
$u(s)$	Manipulated input
$r(s)$	Reference signal
$G_p(s)$	Process transfer function
$G_c(s)$	IMC Controller transfer function
$\tilde{G}_p(s)$	Model transfer function
$y(s)$	Output
$G_f(s)$	Filter transfer function
λ	Tuning parameter
IAE	Integral of the absolute value of the error
ISE	Integral of the squared value of the error
LMI	Linear Matrix Inequalities
FOPTD	First order plus time delay
z	Vector of performances output of the system
w	Vector of exogenous inputs
K	Controller transfer function
$V(x)$	Weighting function of system
P	Symmetric matrix
$\mathbb{R}^n$	The set of n component real vectors
$\mathbb{R}^{n \times m}$	The set of n by m real matrices
ϵ	Belongs to
$\otimes$	Kronecker Product
Δ	Uncertainty
$\|u\|$	Norm of signal
sup	Supremum

Chapter 1

Introduction

Chapter 1

Introduction

1.1 Reviews on PID Controllers:

In control applications, it is not possible to attain the properties of an ideal feedback controller because they include inherent conflicts and trade-offs. The trade-offs must adjust two essential objectives robustness and performance [9]. A control system displays a high degree of performance on the off chance that it gives quick and better responses to set-point changes and disturbances with oscillation. A control system is robust if it provides satisfactory performance for a reasonable degree of model inaccuracy and for a wide range of process conditions (parameter variations). Robustness might be accomplished by picking correct controller settings (typically, small value of $K_c$and large value ofτ_I), however this decision has a tendency to bring about poor performance. Hence, moderate controller settings present performance keeping in mind the end goal to accomplish robustness over performance.

Then again, if the controller settings are specified to give superb set-point following, the disturbance responses could be sluggish. Hence, a trade-off between disturbance dismissal and set-point following happens for conventional PID controllers [26]. Luckily, this trade-off could be evaded by utilizing a controller with two degree of flexibility (2-DOF).

Different methods are used in PID controller setting:

- Frequency response techniques
- Direct Synthesis (DS) method
- Controller tuning relations
- Internal Model Control (IMC) method
- On-line tuning after the control system is installed.
- Computer simulation

Direct synthesis method

In principle, a feedback controller can be designed by using a process model transfer function and specifying the desired close-loop response, and it utilizes the set point changes and disturbance transfer function. The direct synthesis approaches is valuable because it provides insight about the relation between the process and resulting controller. A disadvantage of this approach is that the resulting controller may not have a PID structure. In spite of the fact that these feedback controllers don't generally have a PID structure, it is based on the common process models the DS strategy does produce PI or PID. The direct synthesis method (DSM), however, the controller design is depends on a desired closed-loop transfer function. Based on this the controller produces the control action that trying to match the closed loop set point responses with the desire response. The advantage of this method is the performance requirements are the part of the specification of the closed-loop transfer function [6].

PI or PID controller can be design for the first or second order i.e. simple models is done by selecting the desired closed loop transfer function. The λ-tuning method is generally utilized within the process industries. However, IMC plan technique is nearly identified with the DS strategy and design conventional PID controllers for an extensive variety of problems.

IMC control strategy

Internal Model Control (IMC) is a powerful framework for design and implementation of control systems. It is a plays an important role in control design strategy for linear system. It uses the process model as the internal model to predict the process output. In this method there is only one tuning parameter to tune the control and the design is trade-off between robust and performance is easily understood, due to this properties it's attracted to many users [25]. When the model is perfect, the IMC system becomes an open-loop system and controller design and stability analysis issue become trivial. When a model mismatch exists, by appropriately modifying the difference, robustness can be obtained. The IMC enables the transient response and robustness to be addressed independently. Single–loop control and most of the existing advanced controller such as the linear quadratic optimal controller and Smith predictor can equivalently be put into the general IMC form. The advantages of IMC are exploited in many industrial applications [5].

1.2 Robust control

Robust control manages with control design and system analysis for such defectively known plant transfer functions. One of the principle objectives of the robust control is to attain system performance and stability at the condition of the plant has uncertainties.

Robust control definition expressed as

"Robust control aims at designing a fixed (non–adaptive) controller such that some defined level of performance5 of the controlled system is guaranteed, irrespective of changes in plant dynamics within a predefined class"[25].

It is important to remind that the inherent trade-offs that is when the robustness increases the controller may "less aggressive" and there is possibility to system performance become decreases. These Robust control gives the better system performances under plant uncertainty's and predict the trade-off among closed loop performances and robustness.

"Robust control alludes to the control of unknown plants with unknown dynamics subject to unknown disturbances (Rollins 1999)". Fig.1.1 shows the closed loop control system with different disturbances [20]. There is three possible ways to get uncertainty's in plant they

- Input disturbance (d_i);
- Output disturbance (d_o);
- Measuring noise (n).

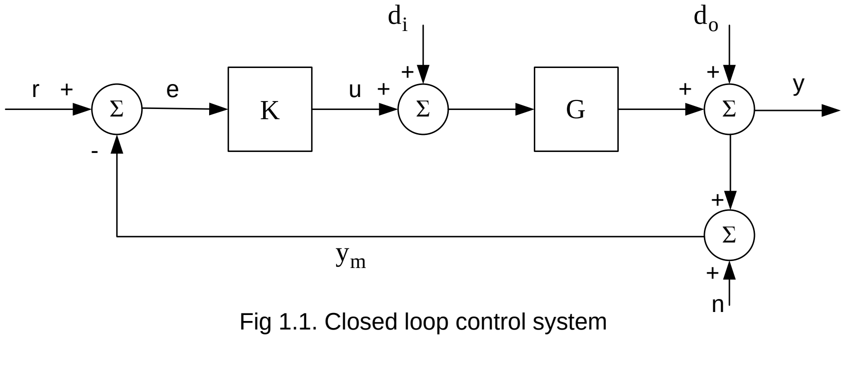

Fig 1.1. Closed loop control system

1.3 Motivation

PID controllers are most likely the most generally utilized industrial controller. PID controller an imperative control apparatus for three reasons: wide accessibility, past response, and easy to utilize. PID controllers have the property that the robustness that is it gives the better result in presence of the parameter variation. In this project we are considering robust control that overcomes the above problem. Here IMC-based PID control and H-infinity with pole placement two robust controls are considering. Comparing these two methods to find which one is gives the better performance.

1.4 Organization of the Thesis

The thesis is organized as follows:

- **Chapter 2:** This chapter presents the basics that are useful for the project they are basics of LMI and some mathematical notations and basics of H-infinity.
- **Chapter 3:** This chapter presents the Internal Model Control, IMC-based PID controller and also comparison result between IMC-based PID controllers and Direct Synthesis Method.
- **Chapter 4:** This chapter presents the basics and LMI formulation of the H-infinity and pole placement.
- **Chapter 5:** This chapter presents the problem statement and simulation results of the comparison of the Multiobjective output feedback controller and IMC-based PID controller.
- **Chapter 6:** This chapter presents the conclusion.

Chapter 2

Preliminaries

Chapter 2
Preliminaries

2.1 Linear Matrix Inequality (LMI)

"Linear Matrix Inequalities (LMIs) and LMI techniques have emerged as powerful design tools in areas ranging from control engineering to system identification and structural design" [3].

Three elements that make LMI technique alluring are as takes after

- A mixture of design details and constraints might be communicated as LMIs.
- Once detailed as far as LMIs, an issue could be settled precisely by proficient convex optimization algorithms.
- While most issues with numerous constraints or objectives need analytical results regarding grid mathematical statements, they frequently stay tractable in the LMI schema. This makes LMI-based design a significant elective to established "analytical" methods.

2.1.1 Definition of LMIs:

A linear matrix inequality (LMI) is a statement of the structure [3]

$$F(x) = F_0 + x_1 F_1 + \cdots + x_s F_s = F_0 + \sum_{i=1}^{s} x_i F_i < 0 \tag{2.1}$$

where the $F_i(i = 1,2, \dots\dots, s)$are given true symmetric matrices and the $x_i$are they looked for scalar decision variables.The inequality < 0)signifies 'negative definite', i.e. $u^T F(x) u < 0$for all $u \in R^n$, $u \neq 0$. Proportionally, the greatest eigenvalue of F(x) is negative.In most requisitions, LMIs don't regularly emerge in the canonical form (1), but instead in the form

$$L(X_1, \dots\dots, X_s) < R(X_1, \dots\dots, X_s) \tag{2.2}$$

where L(■) and R(■) are affine functions of some organized matrix variable $X_1, \dots\dots X_s$. In Case

$$AX + XA^T < 0 \tag{2.3}$$

where$X$ is a symmetric positive definite matrix.

A LMI characterizes a convex demand on a decision variable. The set $\Phi = \{x: F(x) < 0\}$ is convex. Without a doubt, if $x_{1,} x_2 \in \Phi$ and $\alpha \in (0,1)$ then

$$F(\alpha x_1 + (1-\alpha)x_2) = \alpha F(x_1) + (1-\alpha)F(x_2) < 0 \tag{2.4}$$

where in the first equality we utilized the way that F is affine. The last inequality takes after from the way that $\alpha \geq 0$ and $(1-\alpha) \geq 0$. This is a vital property since compelling numerical result techniques are accessible for the issues including convex result.

2.1.2 Generic LMI Problems [3]:

- Feasibility problem: Finding an answer x to a LMI is known as a feasibility problem.
- Minimization problem (or eigenvalue problem): Minimizing a convex goal capacity under some LMI stipulation. This linear goal minimization problem

  $$\min c^T x, \text{ subjected to } F(x) < 0 \tag{2.5}$$

  assumes a vital part in LMI based design.
- Generalized eigenvalue problem: This adds up to minimizing a scalar $\lambda \epsilon R$ subject to

  $$\begin{Bmatrix} \lambda F(x) - G(x) < 0 \\ F(x) < 0 \end{Bmatrix} \tag{2.6}$$

2.2 Schur Complement:

Schur complements are used to convert nonlinear convex inequalities into LMI form [1].

- $\begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} > 0$

  If and only if

  $R > 0$and$Q - SR^{-1}S^T > 0$
- $\begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} > 0$

If and only if

$Q > 0$and$R - S^T Q^{-1} S > 0$

2.3 Bounded Real Lemma:

Consider a dynamical system [4]

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{2.7}$$

The H_∞ norm of the transfer function of the system is less than γ , γ is positive scalar, if $\gamma^2 I - D^T D > 0$ and there exists a matrix $P = P^T > 0$ such that

$$(A^T P + PA + C^T C) + (PB + C^T D)(\gamma^2 I - D^T D)^{-1}(B^T P + D^T C) < 0 \tag{2.8}$$

Apply the Schur complement lemma to that Algebraic Riccati Inequality equation, if there existence of $P = P^T > 0$ such that

$$\begin{bmatrix} A^T P + PA & PB & C^T \\ B^T P & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0 \tag{2.9}$$

2.4 Mathematical Preliminaries and Notations

2.4.1 Norms of Systems and Signals:

Consider LTI causal and finite dimensional system. The convolution function of the input output model of system in time domain as [17]

$$y(t) = \int_{-\infty}^{\infty} g(t - \tau) u(\tau) d\tau \tag{2.10}$$

State space model of the above time domain system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{2.11}$$

The system transfer matrix$G(s)$ is

$$G(s) = D + C(sI - A)^{-1} B \tag{2.12}$$

The block-matrix notation given by

$$G(s) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{2.13}$$

One approach to depict the performance control system is regarding the measure of specific signs of investment. There are a few methods for characterizing norms of a scalar signal.

L_p-norm, $1 \leq p < \infty$: TheL_p norm of a signal $u(t)$ is

$$\|u\|_p = \left(\int_{-\infty}^{\infty} |u(t)|^p dt\right)^{\frac{1}{p}} \tag{2.14}$$

L_∞-norm

$$\|u\|_\infty = \sup_t |u(t)| \tag{2.15}$$

H_p-norm, $1 \leq p < \infty$:

$$\|G\|_p = \left(\int_{-\infty}^{\infty} |G(j\omega)|^p d\omega\right)^{\frac{1}{p}} \tag{2.16}$$

H_∞-norm

$$\|G\|_\infty = \sup_\omega |G(j\omega)| \tag{2.17}$$

2.5 Hamiltonian Matrix Notation

"The H-Infinity Control issue holds Algebraic Riccati Equations; the accompanying Hamiltonian matrix notation (Knobolch et al., 1993) is acquainted with improves result representation". Consider the following Riccati equation for above system

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{2.18}$$

The stabilizing result of this mathematical statement is meant by$P = Ric(H)$ where H is

$H = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}$and $(A - BR^{-1}B^T P)$ is stable.

2.6 Two-Port Block Diagram Representation

Generalized plant (G) having two inputs; manipulated variable u, the exogenous input w. The exogenous input includes reference signal, sensor noise and disturbances. The manipulated variable is a control input to the process which controls the system characteristics. There are two outputs; the performance outputs z and measured output$y$. Here performance outputs are we have to minimize and the measured outputs are used to control the system [4].

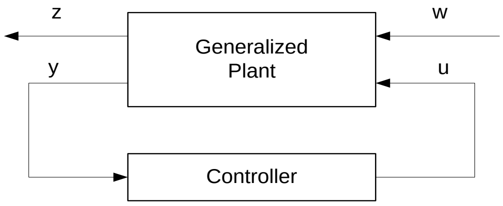

Fig. 2.1. Generalized Two port block diagram

In the above two port block diagram measured output is the input to the controller and the manipulated variable is the output of the controller which as forced to the system to meet requirements. Here z, y, w and u are generally vectors and the process G and controller K are matrices.

The generalized system is represented as

$$\begin{bmatrix} z \\ y \end{bmatrix} = G(s) \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{2.19}$$

The output feedback control law is

$$u = K(s)y \tag{2.20}$$

The performance output z depends on exogenous input w as:

$$z = F(G, K)w \tag{2.21}$$

it is called LFT (linear fractional transformation), and F is

$$F(G, K) = G_{11} + G_{12}(I - G_{22}K)^{-1}G_{21} \tag{2.22}$$

The aim of the H_∞ control is to design a controller K which minimizes the$F(G, K)$ according to the H_∞ norm. The infinity norm is defined as

$$\|F(G, K)\|_\infty = \sup_{\omega} \sigma_{max} \left(F(G, K)(j\omega)\right) \tag{2.23}$$

where σ is the highest eigenvalue of the matrix $F(G, K)(j\omega)$).

2.7 H-Infinity Design Problems

"H-Infinity control issues might be formulated from multiple points of view, here is the most improved translation of the issue is to discover controller for the generalized plant such that Infinity norm of the transfer function relating exogenous input $w(t)$to performance output $z(t)$is least (consider the generalized two port diagram in Figure 1.1). The minimum gain is signified by γ^*. On the off chance that the norm for a subjective settling controller is $\gamma > \gamma^*$ then system is L_2gain bounded. To settle the H-Infinity issue we begin with a worth of γ and lessen it until γ^* is achieved".

Chapter 3

INTERNAL MODEL CONTROL & IMC-Based PID Controller

Chapter 3

INTERNAL MODEL CONTROL& IMC-Based PID Controller

3.1 Internal Model Control

Internal Model Control (IMC) is plays a very important role in control system, it was invented by Morari and his co-workers. Internal Model Control (IMC) is model based procedure to synthesize a controller that yields a desired closed-loop response trajectory. Internal Model control is depends on Internal Model Principle. Internal Model Principle states that "*control can be achieved only if the control system encapsulates, either implicitly or explicitly (includes model uncertainties, delay, RHP zeros etc.,), some representation of the process to be controlled*". Advantages of the internal model control (IMC) as follows [27]:

- Model uncertainty, delays, and RHP zeros are consider the explicitly part of the system.
- Trade-off the control system performance between the robustness and process parameter changes and modelling errors.

In IMC control scheme the controller is depends on a perfect model of the process. Consider, for example, the open loop control system shown in the fig below:

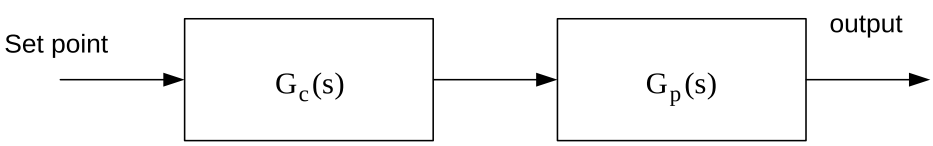

Fig 3.1 : open loop control strategy

Here $G_c(s)$ is controller transfer function and $G_p(s)$ is process transfer function. A Controller $G_c(s)$ is used to force the process$G_p(s)$ to meet specifications. Consider $\widetilde{G}_p(s)$

is the model of the $G_p(s)$. From IMC method the controller $G_c(s)$ to be the inverse of the model of the process, that is

$$G_c(s) = \tilde{G}_p^{-1}(s) \tag{3.1}$$

Consider $G_p(s) = \tilde{G}_p(s)$, that is model of the process is a same as the process. In IMC method the output of the process and model are compare and given to the feedback, by considering the above case the process and the model representation is equal so the feedback signal is zero and the output is same as the set point or reference signal. We can design a perfect controller only if we have knowledge the plant in case of without feedback. But in practical complete knowledge of the plant is incomplete and inaccurate, so it is necessary to use feedback control.

3.1.1 IMC Strategy

In practical model of the process is different from the. The open loop control may not reach the requirements due to unknown. The effect of unknown disturbances on the system is avoided by using the closed loop control arrangement, and to achieve the perfect control a new control strategy is proposed. It is known as Internal Model Control, and the schematic diagram of the IMC method as shown in Fig.3.2 [26].

In the diagram, $u(s)$ is manipulated input, $y(s)$ is the output of the process, $y'(s)$ is the output of the model and $d(s)$is an unknown. In IMC strategy the control input is apply to the plant and model. The output of the process and model is compare and the difference signal is feedback to the system. The resulting signal is

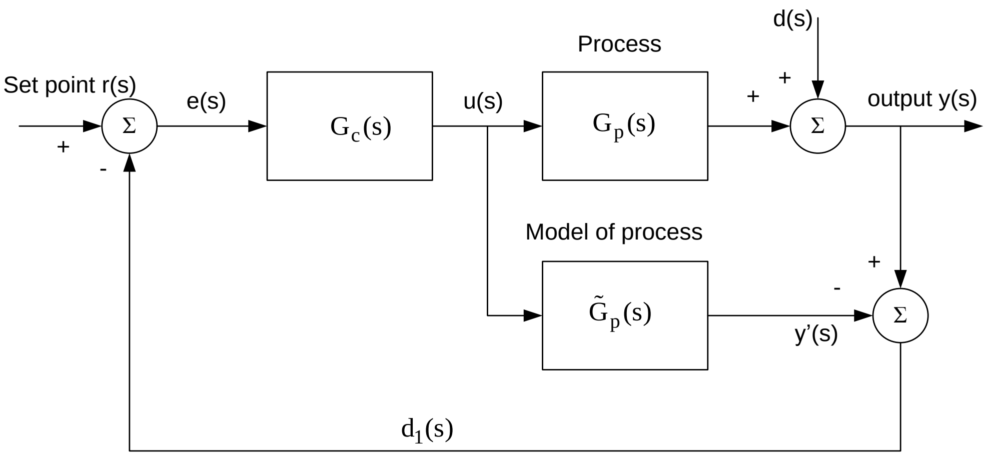

Fig 3.2 : Schematic of the IMC scheme

$$d_1(s) = [G_p(s) - \tilde{G}_p(s)]u(s) + d(s) \tag{3.2}$$

Considering the set point tracking problem the disturbance is considering zero that is $d(s) = 0$. When considering the disturbance rejection problem the set point$r(s) = 0$. Consider $G_p(s) = \tilde{G}_p(s)$ then $d1(s)$ is equal to the disturbance, and feedback to the system to compare with the reference signal. The error signal from comparison is fed to the controller. The controller generates the control signal to reduce the error. The control signal is given by,

$$u(s) = [r(s) - d1(s)]G_c(s) \tag{3.3}$$

$$u(s) = \left\{r(s) - \left[[G_p(s) - \tilde{G}_p(s)]u(s) + d(s)\right]\right\} G_c(s) \tag{3.4}$$

Thus,$u(s) = \frac{[r(s)-d(s)]G_c(s)}{1+[G_p(s)-\tilde{G}_p(s)]G_c(s)}$since $y(s) = G_p(s)u(s) + d(s)$

The closed loop transfer function of the system for the IMC given by

$$y(s) = \frac{G_c(s)G_p(s)r(s) + [1 - G_c(s)\tilde{G}_p(s)]d(s)}{1 + [G_p(s) - \tilde{G}_p(s)]G_c(s)} \tag{3.5}$$

In closed loop transfer function by principle of the IMC method the controller is $G_c(s) = (G_p(s))^{-1}$, and if $G_p(s) = \tilde{G}_p(s)$, then perfect disturbance rejection and set point tracking is achieved. But theoretically, $G_p(s) \neq \tilde{G}_p(s)$ then the disturbance rejection can realised provided$G_c(s) = (G_p(s))^{-1}$.

Additionally, we have to improve robustness and minimise the effects of process model mismatch. At the high frequency of the system there occurs the mismatch of process and model [19]. To overcome this problem a low-pass filter $G_f(s)$is added to remove the process-model mismatch. Therefore in internal model control method the controller is usually designed as the inverse of the process model in series with a low-pass filter, i.e. $G_{IMC}(s) = G_c(s)G_f(s)$. The filter order is chosen such that $G_c(s)G_f(s)$is proper, to reduce the more differential control action.

Consider some limiting cases [7].

Perfect Model, No Disturbances

If the model is perfect $\left(G_p(s) = \tilde{G}_p(s) \right)$ and there are no disturbances $(d(s) = 0)$, then the feedback signal is zero. The relation between $r(s)$ and $y(s)$ is then

$$y(s) = G_p(s)G_c(s)r(s) \tag{3.6}$$

This is the same relationship as the open loop control system design.

Perfect Model, Disturbance Effect

If the model is perfect $\left(G_p(s) = \tilde{G}_p(s) \right)$ and there is a disturbance, then the feedback signal is $d_1(s) = d(s)$. this illustrates that feedback is needed because of unmeasured disturbances entering a process.

Model Uncertainty, No Disturbances

If there is no disturbances$(d(s) = 0)$but there ismodel uncertainty$\left(G_p(s) \neq \tilde{G}_p(s) \right)$, which is always the case in the real world, then the feedback signal is

$$d_1(s) = \left[G_p(s) - \tilde{G}_p(s)\right]U(s) \tag{3.7}$$

This illustrates that feedback is needed because of model uncertainty.
The closed loop relationship is

$$Y(s) = \frac{G_{IMC}(s)G_p(s)R(s) + \left[1 - G_{IMC}(s)\tilde{G}_p(s)\right]d(s)}{1 + \left[G_p(s) - \tilde{G}_p(s)\right]G_{IMC}(s)} \tag{3.8}$$

Recapitulating, the reason for the feedback control includes the following:

- Unmeasured disturbance
- Model uncertainty
- Faster response than the open loop system
- Open loop unstable systems have the problem of closed loop stability.

The primary disadvantage of IMC is that it does not guarantee stability of open loop unstable systems. IMC based PID control handle these systems.

3.1.2IMC Design Procedure

Procedure of the Internal Model Control design consists of the following four steps [7].

1. Factorize the process model in to noninvertible (time delays and RHP zeros) and invertible element (generally, an all-pass factorization will be used).

$$\tilde{G}_p(s) = \tilde{G}_{p-}(s)\tilde{G}_{p+}(s) \tag{3.9}$$

The factorization is performed so that the resulting controller will be stable.

2. Form the idealized Internal Model Control, the ideal internal model controller includes the inverse of the invertible portion of the process model.

(3.10)

3. To make the controller proper add a low pass filter.

$$G_{IMC}(s) = G_c(s)G_f(s) = \tilde{G}_p^{-1}(s)\, G_f(s) \tag{3.11}$$

If it is most desirable to track step set point changes, the filter transfer function is

$$f(s) = \frac{1}{(\lambda s + 1)^n} \tag{3.12}$$

For improved disturbance rejection we use filter with the form

$$f(s) = \frac{\gamma s + 1}{(\lambda s + 1)^n} \tag{3.13}$$

where n- is number that makes the controller proper.

γ- is selected to achieve good disturbance rejection. In practice γ will be selected to cancel a slow disturbance time constant.

λ- filter tuning parameter.

To improve the speed of the response of closed loop system adjusts the filter tuning parameter.

3.2 IMC-Based PID Controller

In IMC procedure there is only single tuning parameter λ to change the controller performance. For minimum phase system the tuning parameter λ is equal to a closed loop constant. Although the Internal Model Control (IMC) procedure is simple and easily implemented, but the most industries still uses the PID controller. So the IMC structure can be modified and rearranged to the form of a standard feedback control diagram or Conventional PID structure. In the IMC the controller $G_c(s)$ is based directly on the invertible portion of the process transfer function. The IMC-based PID controller the tuning parameters are a function of closed loop time constant. The tuning parameter i.e. closed loop time constant is related to the robustness and sensitivity to model error of the closed loop system. Also, for open loop unstable process, it is necessary to implement the IMC strategy in standard feedback (PID) form, because the IMC suffers internal stability problem. The IMC-based PID procedure uses an approximation for dead time, but the IMC strategy uses in operation the exact representation for dead time [9].

3.2.1Standard feedback form to IMC

The step by step rearrange the IMC block diagram to standard feedback form as shown below [7].

Step: 1

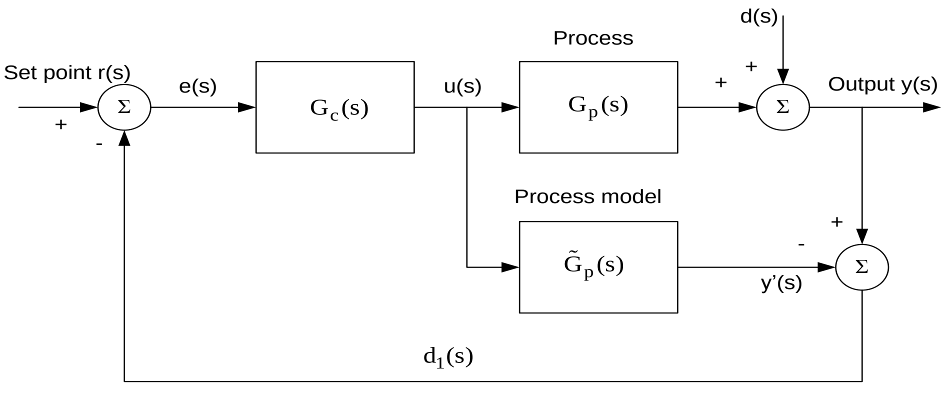

Fig 3.3 : IMC structure

Step: 2

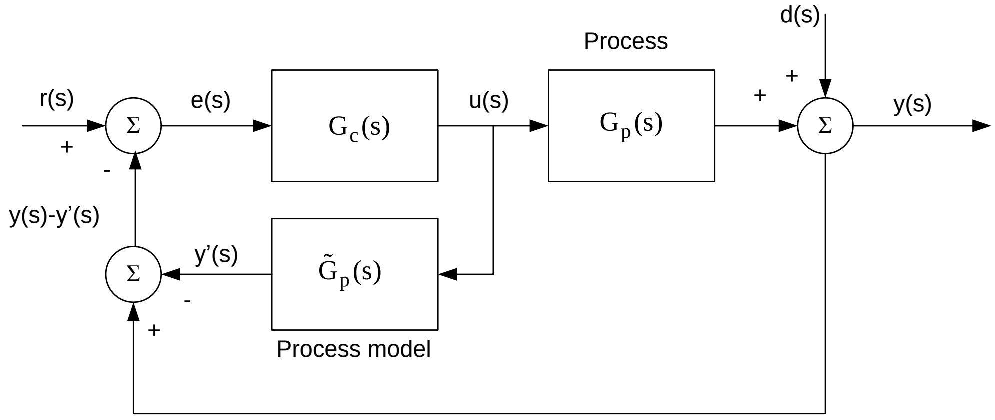

Fig 3.4 : change in IMC structure

Step: 3

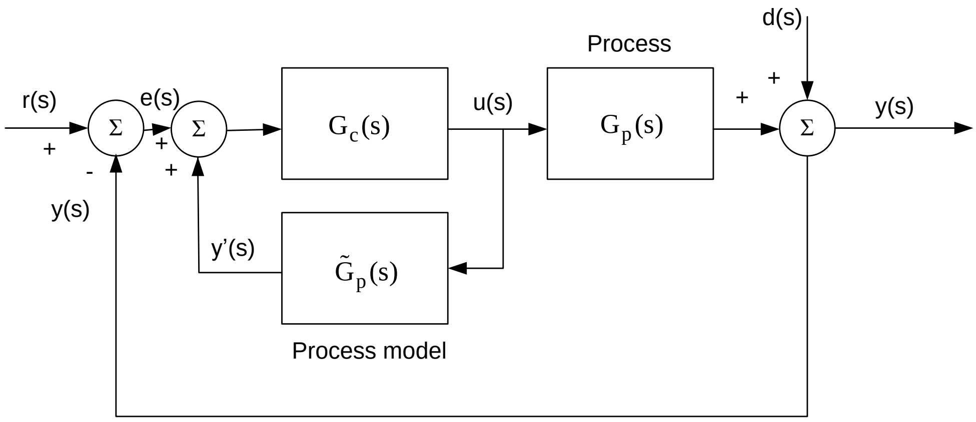

Fig 3.5 : Rearrangement of IMC structure

Step: 4

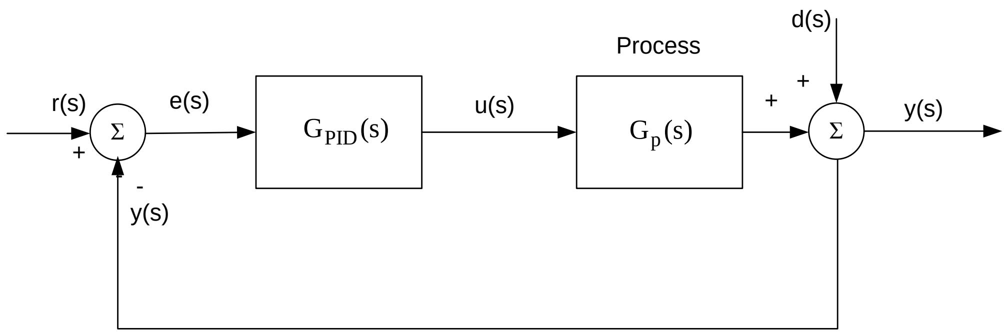

Fig 3.6 : IMC-Based PID Controller

Here $G_{PID}(s) = \frac{G_c(s)}{1-G_c(s)\tilde{G}_p(s)}$.

3.2.2Procedure of IMC-Based PID Control Design

Procedure of IMC-based PID controls system design as follows [7].

1. Design the IMC controller $G_c(s)$ which series with a filter $G_f(s)$ to make controller semi proper. For integrating or unstable processes, or for better disturbance rejection, a filter with the following form will often be used

$$G_f(s) = \frac{\gamma s + 1}{(\lambda s + 1)^n} \tag{3.14}$$

2. By using transformation design the equivalent standard feedback controller.

$$G_{PID}(s) = \frac{G_c(s)G_f(s)}{1 - G_c(s)G_f(s)\tilde{G}_p(s)} \tag{3.15}$$

3. Write this in conventional PID form and calculate PID parameters. The PID form is

$$PID = K_c \left[\frac{\tau_I \tau_D s^2 + \tau_I s + 1}{\tau_I s}\right]\left[\frac{1}{\tau_f s + 1}\right] \tag{3.16}$$

4. The trade-off between performance and robustness depending on the choosing of tuning parameter λ .

3.3 Comparison of Internal Model Control and Direct Synthesis Method:

In this project we are considering the comparison of the Internal Model Control (IMC) with Direct Synthesis Method (DSM). Integral of the absolute value of the error (IAE) and Integral of the squared value of the error (ISE) has been used as the criterion for comparison. Here we are considering the examples are Unstable FOPTD with positive zero for set point tracking to comparison between IMC and DSM.

3.3.1 Direct Synthesis Method:

Consider the feedback control system [6]

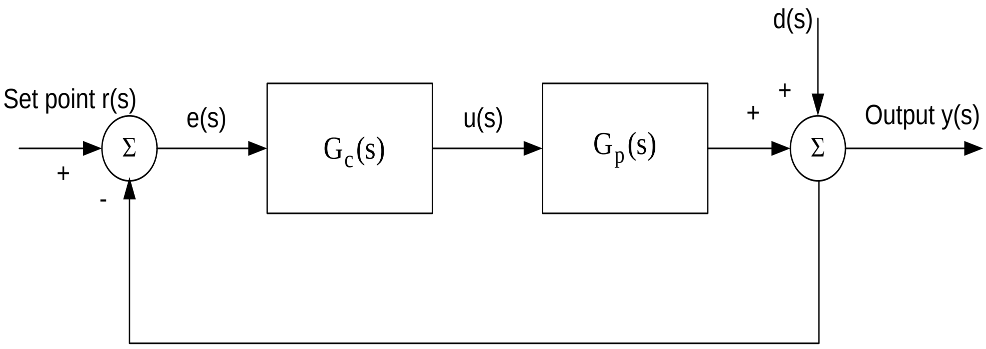

Fig 3.7 : Closed loop feedback control

The transfer function of the closed loop system for set point tracking problem

$$G_{sp} = \frac{y(s)}{r(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} \tag{3.17}$$

Let consider the desired closed-loop transfer function of system for set point changes are chosen as$\left(\frac{y}{r}\right)_d$. Rearranging the design equation by replacing the unknown $\left(\frac{y}{r}\right)$ by $\left(\frac{y}{r}\right)_d$.

$$G_c = \frac{1}{G_p}\left[\frac{\left(\frac{y}{r}\right)_d}{1 - \left(\frac{y}{r}\right)_d}\right] = \frac{1}{G_p}\left[\frac{G_{sp}}{1 - G_{sp}}\right] \tag{3.18}$$

3.3.2 Example:

Consider Unstable FOPTD system with positive zero for set point tracking. The system transfer function as given below [8]

$$G_p(s) = \frac{(1 - 0.25s)}{(s - 1)} e^{-0.25s} \tag{3.19}$$

Controller transfer function by Internal Model Control Method

$$G_c(s) = \frac{2.03s^2 + 16.42s + 1.45}{0.94s^2 + 11.325s} \quad (3.20)$$

and Direct synthesis Method

$$G_c(s) = \frac{18s + 1.36}{0.235s^2 + 13.22s} \quad (3.21)$$

Simulation results of the comparison as show in below fig.

Table 3.1: IAE, ISE performances of Direct Synthesis Method and IMC-Based PID Controller

Performances	Direct Synthesis Method	IMC based PID Controller
IAE	11.50	9.396
ISE	23.81	15.38

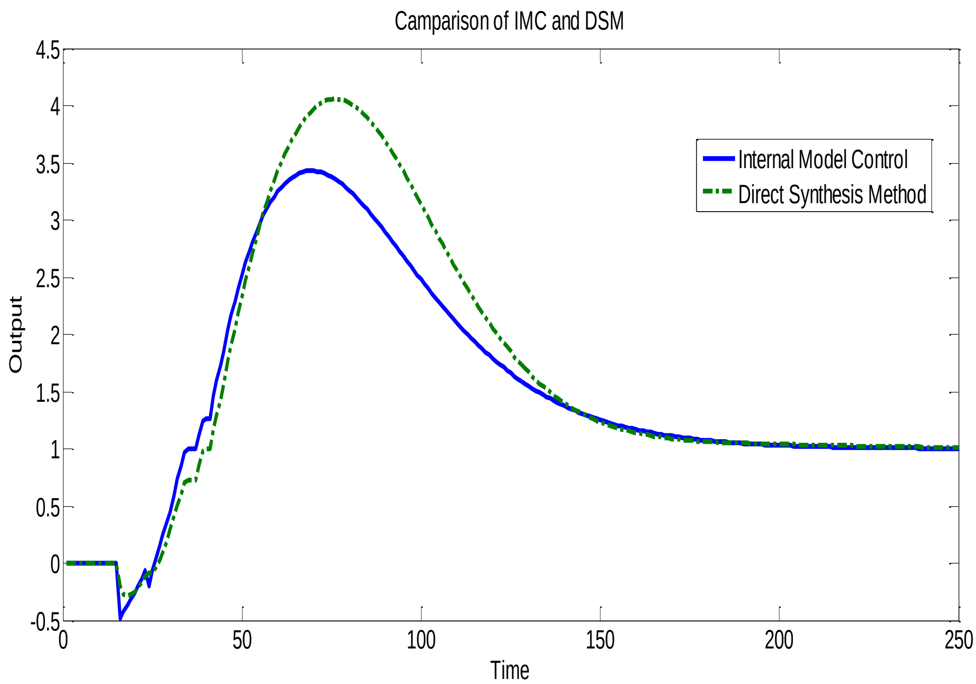

Fig 3.8. Comparison results of IMC and DSM

Chapter 4

H_∞ Control & Robust Pole Placement

Chapter 4

H_∞Control& Robust Pole Placement

4.1 H_∞Control

H_∞Control plays an important role in the control theory; it is first invented by Zames in 1981. H_∞gives an better response in presence of disturbance compare with H_2optimal technique. Doyle *et al*. states in 1989 that the state space H_∞ solutions are derived from the linear time invariant case by solving the Riccati equations associated with it. Later Basar *et al.*, 1991 gives more insight into the problem was given after the H_∞ linear control problem was present as a two player zero-sum differential game. The case of Output Feedback Control problem with dynamic feedback of H_∞ design is given in Knoblauch *et al.*, (1993) [20].

4.1.1 H_∞Description:

The process is represented by the following Two-Port Block Diagram Representation [4]:

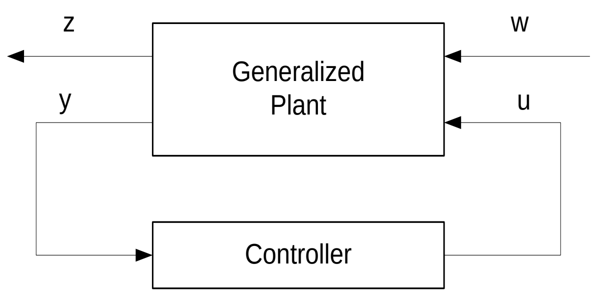

Fig. 4.1. Two port block diagram

Generalized plant (G) having two inputs; manipulated variable u the exogenous input w. The exogenous input includes reference signal, sensor noise and disturbances. The manipulated variable is a control input to the process which controls the system characteristics. There are two outputs; the performance outputs z and measured output$y$. Here performance outputs are we have to minimize and the measured outputs are used to control the system. In the above two port block diagram measured output is the input to the controller and the manipulated variable is the output of the controller which as forced to the system to meet requirements. Here z, y, w and u are generally vectors and the process G and controller K are matrices.

The generalized system is represented as

$$\begin{bmatrix} z \\ y \end{bmatrix} = G(s)\begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}\begin{bmatrix} w \\ u \end{bmatrix} \tag{4.1}$$

The output feedback control law is

$$u = K(s)y \tag{4.2}$$

The performance output z depends on exogenous input w as:

$$z = F(G, K)w \tag{4.3}$$

it is called LFT (linear fractional transformation), and F is

$$F(G, K) = G_{11} + G_{12}(I - G_{22}K)^{-1}G_{21} \tag{4.4}$$

The aim of the H_∞ control is to design a controller K which minimizes the$F(G, K)$ according to the H_∞ norm. The infinity norm is defined as

$$\|F(G, K)\|_\infty = \sup_{\omega} \sigma_{max}\,(F(G, K)(j\omega)) \tag{4.5}$$

whereσ is the highest eigenvalue of the matrix $F(G, K)(j\omega)$).

4.1.2 H_∞ Control Problem with Solution:

Consider the LTI plant with State Space equations [16]

$$\begin{gathered} \dot{x} = Ax + B_w w + Bu \\ z = C_z x + D_{zw} w + D_z u \\ y = Cx + D_w w + D_{yu} u \end{gathered} \tag{4.6}$$

and can be express in the matrix form

$$G(s) = \begin{bmatrix} A & B_w & B \\ C_z & D_{zw} & D_z \\ C & D_w & D_{yu} \end{bmatrix} \tag{4.7}$$

Where $y \in \mathbb{R}^{n_y}$ measured output, w is an exogenous inputs vector (such as sensor noise, disturbance signals, and reference signals,), $u \in \mathbb{R}^{n_u}$ manipulated variable or control input and $z$is a vector of performance output of the system.

And that state space data satisfies the following assumptions [17]

- The pair $(A, B)$and (C, A)must be stabilizable and detectable respectively.
- The size or the dimensions of dim of $x = n$,$w = m_1$, $u = m_2$,$z = p_1$ and dim $y = p_2$, then the Rank of $D_z = m_2$and Rank of $D_w = p_2$to guarantee the controllers are proper.
- For all frequencies the Rank$\begin{bmatrix} A - j\omega I & B \\ C_z & D_z \end{bmatrix} = n + m_2$.
- For all frequencies the Rank$\begin{bmatrix} A - j\omega I & B_w \\ C & D_w \end{bmatrix} = n + p_2$.
- $D_{zw} = 0$and $D_{yu} = 0$.

So our modified problem is

$$G(s) = \begin{bmatrix} A & B_w & B \\ C_z & 0 & D_z \\ C & D_w & 0 \end{bmatrix} \tag{4.8}$$

The control law is given by

$$u = Ky \tag{6.9}$$

where$K \in \mathbb{R}^{m_2 \times n}$ is the output feedback gain. The closed loop system admits the realization

$$\begin{aligned} \dot{x} &= (A + BK)x + B_w w \\ z &= (C_z + D_z K)x + D_{zw} w \end{aligned} \tag{4.10}$$

To obtain the H_∞constraint the Bounded Real Lemma plays a central role, and must admits a quadratic Lyapunov function $V(x) = x^T P x,\ P > 0$, and $\gamma > 0$ such that for all t

$$\frac{d}{dx} V(x) + z^T z - \gamma^2 w^T w < 0 \tag{4.11}$$

From Eq. (5) and (6),

$$\begin{aligned} &[(A + BK)x + B_w w]^T P x + x^T P[(A + BK)x + B_w w] \\ &\quad + [(C_z + D_z K)x + D_{zw} w]^T [(C_z + D_z K)x + D_{zw} w] - \gamma^2 w^T w \\ &\quad < 0 \end{aligned} \tag{4.12}$$

Rearrange the inequality (7) and written as

$$\begin{bmatrix} \begin{pmatrix} (A+BK)^T P + P(A+BK) \\ +(C_z+D_zK)^T(C_z+D_zK) \end{pmatrix} & PB_w + (C_z+D_zK)^T D_{zw} \\ B_w^T P + D_{zw}^T(C_z+D_zK) & -\gamma^2 + D_{zw}^T D_{zw} \end{bmatrix} < 0 \qquad (4.13)$$

Suppose P and R symmetric matrices by considering the Schur compliment to the condition of below

$$\begin{bmatrix} P & S \\ S^T & R \end{bmatrix} > 0 \qquad (4.14)$$

is equivalent to

$$R > 0, \;\; P - SR^{-1}S^T > 0 \qquad (4.15)$$

Apply the Schur complement for the Eq. (8) and multiply by P^{-1} from left and right

$$P^{-1}(A+BK)^T + (A+BK)P^{-1} + P^{-1}(C_z+D_zK)^T(C_z+D_zK)P^{-1} - (B_w + P^{-1}(C_z+D_zK)^T D_{zw})(-\gamma^2 + D_{zw}^T D_{zw})^{-1}(B_w^T + D_{zw}^T(C_z+D_zK)P^{-1}) < 0 \qquad (4.16)$$

Consider the variable$X_\infty = P^{-1}$and assigns in LMI form of Eq. (8)

$$\begin{bmatrix} \begin{pmatrix} X_\infty(A+BK)^T + (A+BK)X_\infty \\ +X_\infty(C_z+D_zK)^T(C_z+D_zK)X_\infty \end{pmatrix} & B_w + X_\infty(C_z+D_zK)^T D_{zw} \\ B_w^T + D_{zw}^T(C_z+D_zK)X_\infty & -\gamma^2 + D_{zw}^T D_{zw} \end{bmatrix} < 0 \qquad (4.17)$$

This inequity can also be expressed by

$$\begin{bmatrix} (A+BK)X_\infty + X_\infty(A+BK)^T & B_w \\ B_w^T & -\gamma I \end{bmatrix} + \frac{1}{\gamma}\begin{bmatrix} X_\infty(C_z+D_zK)^T \\ D_{zw}^T \end{bmatrix}\begin{bmatrix} (C_z+D_zK)X_\infty & D_{zw} \end{bmatrix} < 0 \qquad (4.18)$$

Apply Schur complement for the above equation, then we can be get the H_∞ constraint i.e. Eq. (5), for symmetric matrix $X_\infty > 0$,

$$\begin{bmatrix} (A+BK)X_\infty + X_\infty(A+BK)^T & B_w & X_\infty(C_z+D_zK)^T \\ B_w^T & -\gamma I & D_{zw}^T \\ (C_z+D_zK)X_\infty & D_{zw} & -\gamma I \end{bmatrix} < 0 \qquad (4.19)$$

4.1.3 Properties of H_∞ controller:

Following important properties of H_∞controller as shown in below [4]

- The stabilising feedback law $u = Ky$ minimizes the $F(G,K)$ i.e.

$$F(G,K) = G_{11} + G_{12}(I - G_{22}K)^{-1}G_{21}$$

- In H_∞ controller the cost function is all pass. ($\sigma = 1$ *for all values of* ω).
- A n state augmented plant has at most $n-1$ states in the H_∞optimal controller.
- A n state augmented plant has exactly n states in the H_∞sub optimal controller.

4.2 Robust Pole Placement

Pole placement important tool for the design of control system. The controllers used in control system to achieve the desired specification of the process. In this Pole placement method achieving the desired performance by putting the closed loop pole in a desired region. Thus, the designer can modifies the system characteristics to meet the desired specifications by obtaining a feedback control such that the closed loop poles approach the desired poles [14].

4.2.1 Kronecker Product:

Leopold Kronecker is invented Kronecker product [22]. The **Kronecker product**, denoted by ⊗, is a product operation of two matrixes resulting in a block matrix. This is an entirely different operation with the usual matrix multiplication. The block matrix C with block$C_{ij} = A_{ij}B$ is the Kronecker product of A and B matrices, i.e.

$$A \otimes B = \langle A_{ij}B \rangle_{ij}. \tag{4.20}$$

Some properties of the Product:

$$1 \otimes A = A$$

$$[A + B] \otimes C = A \otimes C + B \otimes C$$

$$[A \otimes B][C \otimes D] = AC \otimes BD$$

$$[A \otimes B]^T = A^T \otimes B^T$$

$$[A \otimes B]^{-1} = A^{-1} \otimes B^{-1}.$$

The product of eigenvalues of the A and B i.e, $\lambda_i(A)\lambda_j(B)$ is equal to the eigenvalues of $A \otimes B$. The Results of the Product of two positive definite matrices is also a positive definite matrix. The singular values of Kronecker product of the A and B correspond of all pairwise multiplication$\sigma_i(A)\sigma_j(B)$of singular values of A and B.

If A is a $m \times n$ matric and B is a $p \times q$ matric, then the Kronecker product $A \otimes B$ is the $mp \times nq$ block matrix:

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}$$

Example:

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} \otimes \begin{bmatrix}0 & 5\\6 & 7\end{bmatrix} = \begin{bmatrix}1\times 0 & 1\times 5 & 2\times 0 & 2\times 5\\1\times 6 & 1\times 7 & 2\times 6 & 2\times 7\\3\times 0 & 3\times 5 & 4\times 0 & 4\times 5\\3\times 6 & 3\times 7 & 4\times 6 & 4\times 7\end{bmatrix} = \begin{bmatrix}0 & 5 & 0 & 10\\6 & 7 & 12 & 14\\0 & 15 & 0 & 20\\18 & 21 & 24 & 28\end{bmatrix}.$$

4.2.2 Lyapunov theorem for Pole Placing:

"Let $\mathcal{D}$ be a region of the complex left-half s-plane [11]. A LTI system $\dot{x} = Ax$is called $\mathcal{D}$ stable if all its poles lie in $\mathcal{D}$" i.e., all eigenvalues of the matrix A or closed loop poles lies in $\mathcal{D}$ region [11]. Then matrix A is called $\mathcal{D}$-stable. Consider$\mathcal{D}$ is the entireLeft-half plane, this implies to asymptotic stability, by the Lyapunov theorem which is characterised in LMI. Consider A is stable only if there subsists a positive symmetric matrix Xwhich satisfying

$$AX + XA^T < 0, \;\; X > 0. \tag{4.21}$$

This Lyapunov theorem characterization of stability has been extended to a different of regions for example disk, half planes etc., by Gutman. The pole placement regions are considered as polynomial form

$$\mathcal{D} = \left\{ z \epsilon \mathbb{C} \colon \sum_{0 \le k,l \le m} c_{kl} z^k z^{-l} < 0 \right\} \tag{4.22}$$

where $C_{kl}$are real, positive and satisfy $C_{kl} = C_{lk}$.For polynomial form regions, states that "a matrix A is $\mathcal{D}$-stable if and only if there exists a positive symmetric matrix X " such that

$$\sum_{k,l} C_{kl} A^k X (A^T)^l < 0, \quad X > 0. \tag{4.23}$$

Replace $z^k z^{-l}$ with $A^k X (A^T)^l$.

4.2.3 Pole Placement in LMI Regions:

An LMI region is any subset $\mathcal{D}$ of the complex s-plane that can be expressed as [2]

$$\mathcal{D} = \{ z \epsilon \mathbb{C} \colon L + zM + \bar{z} M^T < 0 \} \tag{4.24}$$

Where L and M are real matrices and$L^T = L$, and $M = M_1^T M_2$ the matrix function

$$f_{\mathcal{D}}(z) = \; L + zM + \bar{z} M^T \tag{4.25}$$

is called the characteristic function of $\mathcal{D}$.

LMI regions include different regions such as half s-plane, disks, conics, strips, and any intersection of the above. For such different LMI regions "Lyapunov theorem" is available. Specifically, if $(\lambda_{ij})_{1\le i,j\le m}$ and $(\mu_{ij})_{1\le i,j\le m}$ are the entries of the matrices L and M. A matrix A has all its eigenvalues or closed loop poles in $\mathcal{D}$-region if and only if there exists a positive definite matrix P such that

$$\{\lambda_{ij}P + \mu_{ij}AP + \mu_{ji}PA^T\}_{1\le i,j\le m} < 0. \tag{4.26}$$

Some LMI regions with characteristic equation described below [14]

- Half-plane region $Re(z) < -\propto$: $f_{\mathcal{D}}(z) = z + \bar{z} + 2 \propto < 0$
- disk region centred at $(-q, 0)$ with radius r :

$$f_{\mathcal{D}}(z) = \begin{bmatrix} -r & q+z \\ q+\bar{z} & -r \end{bmatrix} < 0 \tag{4.27}$$

- conic sector region with inner angle 2θ and apex at the origin:

$$f_{\mathcal{D}}(z) = \begin{bmatrix} sin\theta(z+\bar{z}) & cos\theta(z-\bar{z}) \\ cos\theta(\bar{z}-z) & sin\theta(z+\bar{z}) \end{bmatrix} < 0. \tag{4.28}$$

4.2.3.1 Left half-plane

$$f_{\mathcal{D}}(z) < 0 \Leftrightarrow z + z^T < 0 \tag{4.29}$$

It is sufficient to take $L = 0$ and $M = 1$. The following LMI is derived from expression (6):

$$AX + XA^T < 0 \tag{4.30}$$

4.2.3.2 $\propto -$Stability

$$f_{\mathcal{D}}(z) < -\propto \Leftrightarrow 2 \propto + z + z^T < 0 \tag{4.31}$$

It is sufficient to take $L = 2 \propto$ and $M = 1$, which gives the following LMI for $\propto -$Stability:

$$2 \propto X + AX + XA^T < 0 \tag{4.32}$$

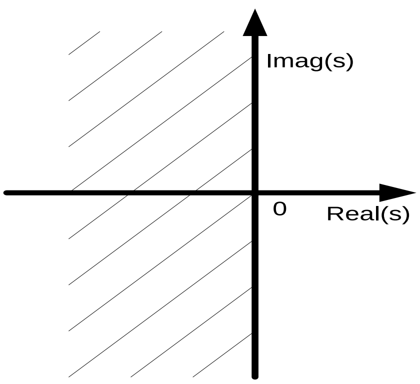

Fig 4.2 : open left half plane

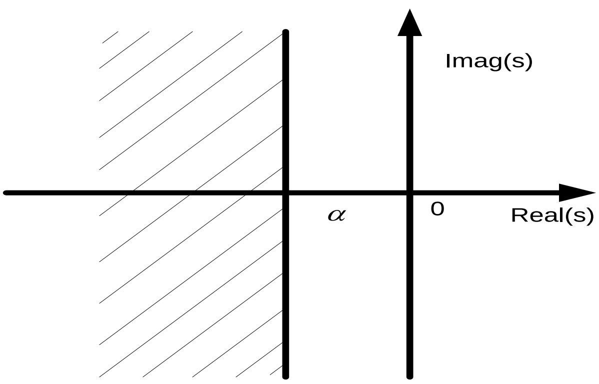

Fig 4.3 : Semi left half plane

4.2.3.3 Disk

Disk of Radius r, Centred at $(q, 0)$ [24], [12]

$$|z-q| < r \iff \begin{bmatrix} -r & z-q \\ \bar{z}-q & -r \end{bmatrix} < 0 \qquad (4.33)$$

It is sufficient to take the matrices:

$$L = \begin{bmatrix} -r & -q \\ -q & -r \end{bmatrix}, M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

which gives the following LMI for disk region:

$$\begin{bmatrix} -rX & -qX + AX \\ -qX + XA^T & -rX \end{bmatrix} < 0 \qquad (4.34)$$

For example take $r = 1$ and $q = 0$ we obtain

$$A^T XA - X < 0.$$

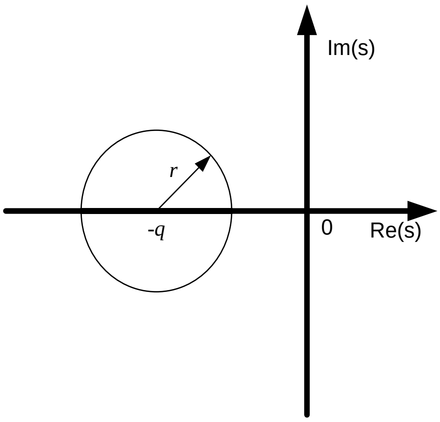

Fig 4.4: LMI region (Disk)

4.2.3.4 Conical Sector

$$a.Re(z) + |b.Im(z)| < 0 \iff \begin{bmatrix} a(z+\bar{z}) & -b(z-\bar{z}) \\ b(z-\bar{z}) & a(z+\bar{z}) \end{bmatrix} < 0 \qquad (4.35)$$

It is sufficient to take the matrices [2]:

$$L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad M = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

which gives the following LMI for conic sector region:

$$\begin{bmatrix} a(AX + XA^T) & -b(AX - XA^T) \\ b(AX - XA^T) & a(AX + XA^T) \end{bmatrix} < 0 \qquad (4.36)$$

We have:

$$0 < \theta < \frac{\pi}{2}, \cos(\theta) = \frac{-b}{\sqrt{a^2+b^2}}, \sin(\theta) = \frac{a}{\sqrt{a^2+b^2}}$$

Thus:

$$\begin{bmatrix} \sin(\theta)(AX + XA^T) & \cos(\theta)(AX - XA^T) \\ -\cos(\theta)(AX - XA^T) & \sin(\theta)\,(AX + XA^T) \end{bmatrix} < 0 \tag{4.37}$$

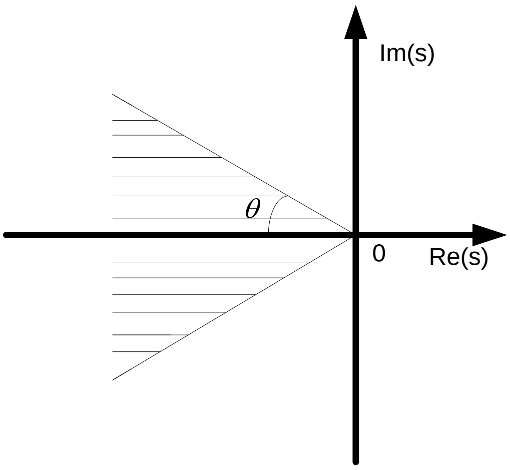

Fig 4.5: LMI region (Conic sector)

4.2.4 Robust $\mathcal{D}$-Stability Quadratic $\mathcal{D}$-Stability:

Consider the uncertain system [11]

$$\begin{gathered} \dot{x} = A(\Delta)x \\ A(\Delta) = A + B(I - \Delta D)^{-1}\Delta C,\;\; A \in R^{n\times n} \end{gathered} \tag{4.38}$$

where matrix $A(\Delta)$ depends on the norm-bounded uncertainty state matrix

$$\Delta \in E^{m\times m}, \qquad \sigma_{max}(\Delta) \le \gamma^{-1} \tag{4.39}$$

with $E = R, \mathrm{C}$.

Let

$$\mathcal{D} = \{z\epsilon \mathrm{C}\colon L + zM + \bar{z}M^T < 0\} \tag{4.40}$$

be any LMI region, and state matrix A is $\mathcal{D}$-stable, i.e., all its closed loop poles in $\mathcal{D}$.

*Robust $\mathcal{D}$-Stability: "*Consider the uncertain system (4.38)–(4.39) is robustly $\mathcal{D}$-stable if the eigenvalues of matrix of $A(\Delta)$lie in $\mathcal{D}$-region for all permissible uncertaintiesΔ".

Quadratic $\mathcal{D}$-Stability: "The LMI $\mathcal{D}$ region expressed by (4.40), the uncertain linear system (15), (16) is said to be quadratic$\mathcal{D}$-stable if a positive real symmetric matrix $X > 0$ exists such that

$$M_{\mathcal{D}}(A(\Delta), X) = L \otimes X + Herm(M \otimes \{X(A + B(I - \Delta D)^{-1}\Delta C)\}) < 0 \tag{4.41}$$

For all matrices Δ such that$||\Delta|| \leq \gamma^{-1}$".

"Matrix Ais said to the $\mathcal{D}$-stable if $X > 0$exists and $M_{\mathcal{D}}(A, X) < 0$. Hence, quadratic $\mathcal{D}$-stability express robust $\mathcal{D}$-stability, but the discourse is generally false because quadratic $\mathcal{D}$-stability is requires a single satisfies for all admissible uncertaintiesΔ's".

4.2.5 OUTPUT-FEEDBACK SYNTHESIS:

Consider output feedback controller for the system that robustly put the poles in a desired LMI region$\mathcal{D}$ [11].

Consider LTI plant by the state-space equations with

$$\acute{P}\begin{cases} \dot{x} = Ax + B_w w + Bu \\ z = C_z x + D_{zw} w + D_z u \\ y = Cx + D_w w + D_{yu} u \end{cases} \tag{4.42}$$

where $A \in \mathbb{R}^{n\times n}$. Given the LMI region

$$\mathcal{D} = \{z\epsilon\mathbb{C}: L + zM + \bar{z}M^T < 0\}$$

Our goal is to compute a dynamical full-order output feedback controller

$$K\begin{cases} \dot{x}_K = A_K x_K + B_K y \\ u = C_K x_K + D_K y \end{cases} \tag{4.43}$$

that robustly puts the closed-loop poles in $\mathcal{D}$.

Some dynamical output feedbacks with control law $= Ky$ for the closed-loop transfer function from w to z. The closed loop system described as

$$\dot{x}_{cl} = Ax_{cl} + Bw$$
$$z = Cx_{cl} + Dw$$

where

$$A = \begin{bmatrix} A + BD_K C & BC_K \\ B_K C & A_K \end{bmatrix}$$

$$B = \begin{bmatrix} B_w + BD_K D_w \\ B_K D_w \end{bmatrix}$$

$$C = [C_z + D_z D_K C \quad D_z C_K]$$

$$and\ D = [D_{zw} + D_z D_K D_w].$$

A sufficient condition for robust $\mathcal{D}$-stability of system is existence of symmetric matrix $X > 0$ such that

$$\begin{bmatrix} \mathrm{M}_{\mathcal{D}}(\mathrm{A},\mathrm{X}) & M_1^T \otimes (XB) & M_2^T \otimes C^T \\ M_1 \otimes (B^T X) & -\gamma I & I \otimes D^T \\ M_2 \otimes C & I \otimes D & -\gamma I \end{bmatrix} < 0 \qquad (4.44)$$

where $M_1^T M_2 = M$ is a factorization of M.

*Theorem: O*utput feedback controller $K(s)$and a symmetric matrix $X > 0$ exist such that (21) holds if and only if two $n \times n$positive symmetric matrices R and S and matrices A_K, B_K, C_K, D_K exists such that

$$\Lambda(R,S) = \begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0 \qquad (4.45)$$

and

$$\begin{bmatrix} \mathrm{L} \otimes \Lambda(R,S) + M \otimes \Phi_A + M^T \otimes \Phi_A^T & M_1^T \otimes \Phi_B & M_2^T \otimes \Phi_C^T \\ M_1 \otimes \Phi_B^T & -\gamma I & I \otimes \Phi_D^T \\ M_2 \otimes \Phi_C & I \otimes \Phi_D & -\gamma I \end{bmatrix} < 0 \qquad (4.45)$$

Where$\Phi_A = \begin{bmatrix} AR + B\hat{C} & A + B\widehat{D}C \\ \hat{A} & SA + \hat{B}C \end{bmatrix}$

$$\Phi_B = \begin{bmatrix} B_w + B\widehat{D}D_w \\ SB_w + \hat{B}D_w \end{bmatrix}$$

$$\Phi_C = [C_z R + D_z \hat{C} \quad C_z + D_z \widehat{D} C]$$

$$\Phi_D = [D_{zw} + D_z \widehat{D} D_w]$$

The controller that robustly put the closed-loop poles of a system in $\mathcal{D}$ is

$$K(s) = D_K + C_K (sI - A_K)^{-1} B_K$$

The matrices A_K, B_K, C_K are derived as follows.

- Compute square matrices $N$and M such that $MN^T = I - RS$.
- Solve the change of controller variables:

$$\begin{cases} \hat{C} = C_K M^T + D_K CR \\ \hat{B} = NB_K + SBD_K \\ \hat{A} = NA_K M^T + NB_K CR + SBC_K M^T + S(A + BD_K C)R. \end{cases} \qquad (4.46)$$

Chapter 5

Comparing H-infinity with pole placement and IMC based PID Control, Simulation & Results

Chapter 5
Comparing H-infinity with pole placement and IMC based PID Control, Simulation & Results

5.1Comparing H-infinity with pole placement and IMC based PID Control

Consider the LTI plant with State Space equations [16]

$$\acute{P}\begin{cases} \dot{x} = Ax + B_w w + Bu \\ z = C_z x + D_{zw} w + D_z u \\ y = Cx + D_w w \end{cases} \tag{5.1}$$

“Where $y \in \mathbb{R}^{n_y}$is the measured output,, $w$is a vector of exogenous inputs (such as disturbance signals, reference signals, sensor noise), $u \in \mathbb{R}^{n_u}$the manipulated variable or control input, $z$is a vector of performance output of the controlled system. The closed-loop transfer functions from w to z for output-feedback system with control law$u = Ky$”. The objective is to design a dynamic output feedback controller

$$K\begin{cases} \dot{x}_K = A_K x_K + B_K y \\ u = C_K x_K + D_K y \end{cases} \tag{5.2}$$

The closed-loop system of a system described as

$$T\begin{cases} \dot{x}_{cl} = Ax_{cl} + Bw \\ z = Cx_{cl} + Dw \end{cases} \tag{5.3}$$

Where

$$A = \begin{bmatrix} A + BD_K C & BC_K \\ B_K C & A_K \end{bmatrix}$$

$$B = \begin{bmatrix} B_w + BD_K D_w \\ B_K D_w \end{bmatrix}$$

$$C = [C_z + D_z D_K C \quad D_z C_K]$$
$$and\ D = [D_{zw} + D_z D_K D_w].$$

5.1.1 H_∞ Criteria:

The L_p norm of a signal $u(t)$ is [13]

$$\|u\|_p = \left(\int_{-\infty}^{\infty} |u(t)|^p dt\right)^{\frac{1}{p}} \tag{5.4}$$

L_∞-norm

$$\|u\|_\infty = \sup_{\omega} \sigma_{max} |u(t)| < \gamma \tag{5.5}$$

That is the maximum value of the eigenvalue of the system is less than the prescribed value.

From Bounded Real Lemma [13], the state matrix A is stable and the H-infinity norm is less than γ that is possible only if there exists a symmetric positive definite matrix P with

$$\begin{bmatrix} A^T P + PA & PB_w & {C_z}^T \\ {B_w}^T P & -\gamma I & {D_{zw}}^T \\ C_z & D_{zw} & -\gamma I \end{bmatrix} < 0 \tag{5.6}$$

In output feedback case there is a difficulty that is it involves nonlinear terms in the above constraint. By congruence transformation these nonlinearities can be eliminated by change of controller variables.

5.1.2 Linearizing Change of Variable:

Partition the symmetric matrix into P and P^{-1} [16]

$$P = \begin{pmatrix} S & N \\ N^T & * \end{pmatrix}, \qquad P^{-1} = \begin{pmatrix} R & M \\ M^T & * \end{pmatrix} \tag{5.7}$$

where$S$ and R are $n \times n$ and symmetric matrices. From $PP^{-1} = I$ refer $P\begin{pmatrix} R \\ M^T \end{pmatrix} = \begin{pmatrix} I \\ 0 \end{pmatrix}$ which proceeds to

$$P\pi_1 = \pi_2 \text{ with } \pi_1 = \begin{pmatrix} R & I \\ M^T & 0 \end{pmatrix},\ \pi_2 = \begin{pmatrix} I & S \\ 0 & N^T \end{pmatrix}. \tag{5.8}$$

The change of controller variables defined as follows:

$$\begin{cases} \hat{A} = NA_K M^T + NB_K CR + SBC_K M^T + S(A + BD_K C)R \\ \hat{B} = NB_K + SBD_K \\ \hat{C} = C_K M^T + D_K CR \\ \hat{D} = D_K \end{cases} \tag{5.9}$$

By performing congruence transformation with $diag(\pi_1, I, I)$ on H_∞ and converts the nonlinear matrix inequalities into LMIs.

After some short calculation of Eq. (21) and (22) the following identities are derived

$$\pi_1^T PA\pi_1 = \pi_2^T A\pi_1 = \begin{pmatrix} AR + B\hat{C} & A + B\hat{D}C \\ \hat{A} & SA + \hat{B}C \end{pmatrix} \tag{5.10}$$

$$\pi_1^T PB_w = \pi_2^T B_w = \begin{pmatrix} B_w + B\hat{D}D_w \\ SB_w + \hat{B}D_w \end{pmatrix} \tag{5.11}$$

$$C_z\pi_1 = (C_z R + D_z\hat{C} \quad C_z + D_z\hat{D}C) \tag{5.12}$$

$$\pi_1^T P\pi_1 = \pi_1^T \pi_2 = \begin{pmatrix} R & I \\ I & S \end{pmatrix} \tag{5.13}$$

After performing the congruence transformation with $diag(\pi_1, I, I)$ on Eq. (19), we obtain

$$\begin{bmatrix} AR + RA^T + B\hat{C} + \hat{C}^T B^T & \hat{A}^T + A + B\hat{D}C & B_w + B\hat{D}D_w & RC_z^T + \hat{C}^T D_z^T \\ * & A^T S + SA + \hat{B}C + C^T\hat{B}^T & SB_w + \hat{B}D_w & C_z^T + C^T\hat{D}D_z^T \\ * & * & -\gamma I & D_{zw}^T + D_w^T\hat{D}D_z^T \\ * & * & * & -\gamma I \end{bmatrix} < 0 \tag{5.14}$$

In order for (26) to be true the following relationship must hold

$$MN^T = I - RS. \tag{5.15}$$

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} > 0 \tag{5.16}$$

This relationship can be solved by utilizing the singular value decomposition (SVD).

5.1.3 LMI Region:

Let $\mathcal{D}$is any subset of the LMI region that can be expressed as [11]

$$\mathcal{D} = \{z\epsilon\mathbb{C}\colon L + zM + \bar{z}M^T < 0\} \tag{5.17}$$

Where M and L are matrices such that$L^T = L$. The matrix valued function

$$f_{\mathcal{D}}(z) = L + zM + \bar{z}M^T$$

is called the characteristic function of $\mathcal{D}$. The regions include half planes, conic sectors, strips, disks, ellipses etc. Specifically, if $\{\mu_{ij}\}_{1\le i,j\le m}$ and $\{\lambda_{ij}\}_{1\le i,j\le m}$represents the entries of the matrices M and L, and a matrix A has all its eigenvalues or closed loop poles lies in $\mathcal{D}$is possible only ifthere exists a symmetric positive definite matrix $P$from that

$$\{\lambda_{ij}P + \mu_{ij}AP + \mu_{ji}PA^T\}_{1\le i,j\le m} < 0. \tag{5.18}$$

In output feedback controller for LMI region we have to include the change of variable because of the nonlinearity appears in the closed loop system. Then we obtain the LMI for the regional pole placement is

$$\begin{bmatrix} L\otimes\Lambda(R,S) + M\otimes\Phi_A + M^T\otimes\Phi_A^T & M_1^T\otimes\Phi_B & M_2^T\otimes\Phi_C^T \\ M_1\otimes\Phi_B^T & -\gamma I & I\otimes\Phi_D^T \\ M_2\otimes\Phi_C & I\otimes\Phi_D & -\gamma I \end{bmatrix} < 0 \tag{5.19}$$

Where

$$\Phi_A = \begin{bmatrix} AR + B\hat{C} & A + B\hat{D}C \\ \hat{A} & SA + \hat{B}C \end{bmatrix}$$

$$\Phi_B = \begin{bmatrix} B_w + B\widehat{D}D_w \\ SB_w + \hat{B}D_w \end{bmatrix}$$

$$\Phi_C = [C_z R + D_z \hat{C} \quad C_z + D_z \widehat{D} C]$$

$$\Phi_D = [D_{zw} + D_z \widehat{D} D_w]$$

In our project we define the desired region as conic sector as shown in fig, with apex at the $\beta = 1$ and inner angle 30°. This determines the region

$$-2\beta cos\theta + ze^{i\theta} + \bar{z}e^{-i\theta} < 0$$

From this we can find that the matrices $L$and M have the following form

$$L = \begin{pmatrix} -2\beta cos\theta & 0 \\ 0 & -2\beta cos\theta \end{pmatrix}, \qquad M = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \tag{5.20}$$

Finally, the controller matrices can be found by the following relationship

$$\begin{cases} D_K = \widehat{D} \\ C_K = (\hat{C} - D_K C R) M^{-T} \\ B_K = N^{-1}(\hat{B} - SBD_K) \\ A_K = N^{-1}\big(\hat{A} - NB_K CR - SBC_K M^T - S(A + BD_K C)R\big) M^{-T}. \end{cases} \tag{5.21}$$

5.2 SIMULATION AND RESULTS:

Consider the LTI unstable plant with state space equations [16]

$$\begin{gathered} \dot{x} = \begin{bmatrix} 0 & 10 & 2 \\ -1 & 1 & 0 \\ 0 & 2 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} w + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \\ y = x_2 \\ z_\infty = [x_2]. \end{gathered} \tag{5.22}$$

From the given example the state space matrices are

$$A = \begin{bmatrix} 0 & 10 & 2 \\ -1 & 1 & 0 \\ 0 & 2 & -5 \end{bmatrix}, \qquad B_w = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$C = [0\ 1\ 0], \qquad C_z = [0\ 1\ 0], \qquad D_{zw} = D_z = D_w = 0$$

and the matrix form

$$G(s) = \begin{bmatrix} A & B_w & B \\ C_z & D_{zw} & D_z \\ C & D_w & D_{yu} \end{bmatrix}$$

5.2.1 H_∞ Control:

Here we have to find a performance of t H_∞ from w to z_∞ by using LMI approach. In LMI approach we have to solve the following LMI constraints to find the output feedback controller. The LMI constraints are [1], [3]

Minimizing γ and satisfying:

$$\begin{bmatrix} AR + RA^T + B\hat{C} + \hat{C}^T B^T & \hat{A}^T + A + B\hat{D}C & B_w + B\hat{D}D_w & RC_z^T + \hat{C}^T D_z^T \\ * & A^T S + SA + \hat{B}C + C^T \hat{B}^T & SB_w + \hat{B}D_w & C_z^T + C^T \hat{D} D_z^T \\ * & * & -\gamma I & D_{zw}^T + D_w^T \hat{D} D_z^T \\ * & * & * & -\gamma I \end{bmatrix} < 0 \qquad (5.23)$$

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} > 0 \qquad (5.24)$$

$$\begin{bmatrix} L \otimes \Lambda(R,S) + M \otimes \Phi_A + M^T \otimes \Phi_A^T & M_1^T \otimes \Phi_B & M_2^T \otimes \Phi_C^T \\ M_1 \otimes \Phi_B^T & -\gamma I & I \otimes \Phi_D^T \\ M_2 \otimes \Phi_C & I \otimes \Phi_D & -\gamma I \end{bmatrix} < 0 \qquad (5.25)$$

Where

$$\Phi_A = \begin{bmatrix} AR + B\hat{C} & A + B\widehat{D}C \\ \hat{A} & SA + \hat{B}C \end{bmatrix}$$

$$\Phi_B = \begin{bmatrix} B_w + B\widehat{D}D_w \\ SB_w + \hat{B}D_w \end{bmatrix}$$

$$\Phi_C = [C_z R + D_z\hat{C} \quad C_z + D_z\widehat{D}C]$$

$$\Phi_D = [D_{zw} + D_z\widehat{D}D_w]$$

Solve the Eq. (34), (35), and (36) by using LMI solvers we will get the change of controller variables i.e. $\hat{A}, \hat{B},\ \hat{C}$ and $\widehat{D}$. And find the non-singular matrices M, N by satisfying below relationship

$$MN^T = I - RS$$

And the output feedback controller is define by

$$\begin{cases} D_K = \widehat{D} \\ C_K = (\hat{C} - D_K CR)M^{-T} \\ B_K = N^{-1}(\hat{B} - SBD_K) \\ A_K = N^{-1}\big(\hat{A} - NB_K CR - SBC_K M^T - S(A + BD_K C)R\big)M^{-T}. \end{cases} \tag{5.26}$$

By writing MATLAB program to solve the LMI constraints by using LMI Solver *"mincx"* we will get the controller transfer function and the γ value is

$$K(s) = \frac{-1075s^3 - 2.394 \times 10^6 s^2 - 2.326 \times 10^8 s - 1.511 \times 10^9}{s^3 + 4738s^2 + 2.083 \times 10^4 s + 2246}. \tag{5.27}$$

$$\gamma = 0.1296 \tag{5.28}$$

Compare the H_∞ performance with IMC based PID Controller.

5.2.2 IMC-based PID Controller:

The transfer function of the system is [28]

$$G_p(s) = \frac{s^2 + 5s}{s^3 + 4s^2 + 5s + 54} \tag{5.29}$$

The controller $G_c(s) = \left(G_p(s)\right)^{-1} \times G_f(s)$

where$G_f(s)$ is filter transfer function, that is chosen as

$$G_f(s) = \frac{1}{(1 + \lambda s)^2} \tag{5.30}$$

whereλ- is a tuning parameter, and in this project we are considering the tuning parameter λ as one (i.e. $\lambda = 1$).

The controller transfer function from IMC based PID controller is

$$G_c(s) = \frac{s^3 + 4s^2 + 5s + 54}{s^4 + 7s^3 + 10s^2}. \tag{5.31}$$

Simulation results of the comparison of H_∞ performance with IMC based PID Controller for set point tracking as shown below, and IAE, ISE performances are show in table 2.

Table 2: IAE, ISE performances of H-infinity with pole placement and IMC-Based PID Controller

Performances	H-infinity with pole placement	IMC based PID Controller
IAE	0.004337	1.999
ISE	0.0009103	1.25

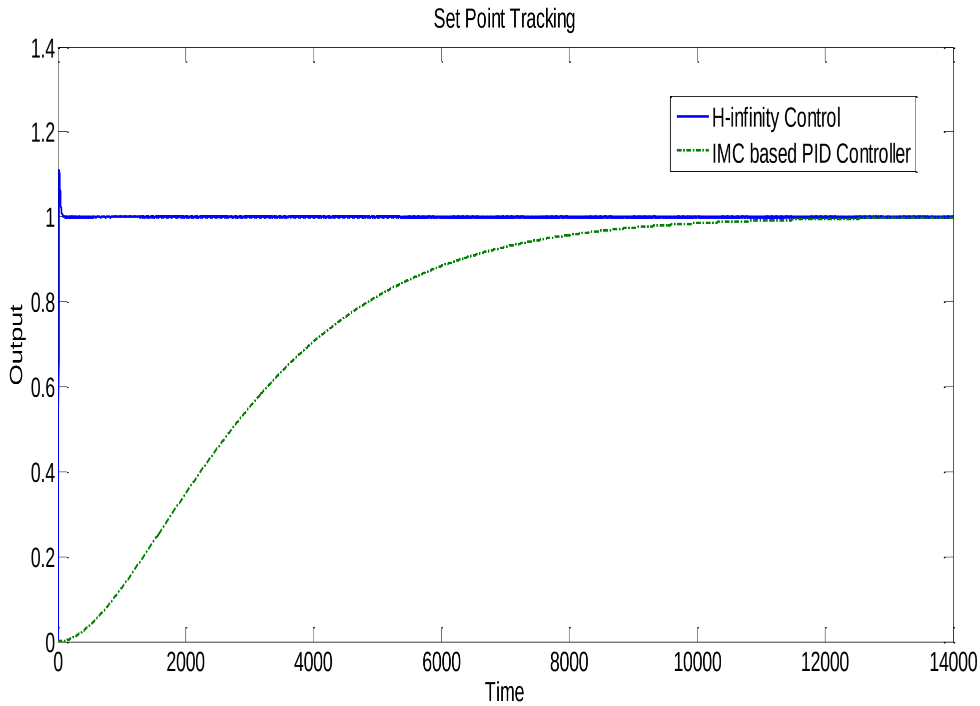

Fig 5.1: Set point tracking for Step input

Chapter 6

Conclusion & Future Scope

Chapter 6

Conclusion & Future Scope

6.1 Conclusion

In this work we compare Multiobjective Output feedback controller with IMC-Based PID controller. The Internal model control is compared with the Direct Synthesis Method. From the IAE, ISE performance values we can conclude that the IMC gives the better results than the DSM. The well-known IMC-PID tuning rule uses a single tuning parameter to get the trade-off between robustness to model inaccuracies and closed-loop performance. Stability analysis of the IMC-PID controller is simple to use, and trade-off between robustness and performance is clearly understood. Internal Model Control is robust control. This controller is compare with one of the robust control approaches. In this project work we are considering H-infinity synthesis with additional pole placement constraints. These objectives are formulated by using the common lyapunov function. A complete Linear Matrix Inequality (LMI) of the output feedback synthesis with H-infinity control with pole placement is presented. In the above problem we get the nonlinearities these nonlinearities are linearism by using congruence transformation and a change of variable technique. This formulation had formerly been applied to the three state unstable plants. From the results the H-infinity control with pole placement minimizes γ the prescribed value and kept the closed loop poles inside the LMI region. The above results compare with the IMC-Based PID controller. The result shows that the Multiobjective output feedback controller gives the better performances than the IMC-Based PID controller. The IAE, ISE performances are show in the table.

6.2 Future Scope

In this present work we are consider the objectives are only the H-infinity and Pole Placement. We can also refer to the H-infinity performance and asymptotic tracking and settling time or H_2 performance or regulation or saturation constraints with mix of time domain and frequency domain specifications. We design here Full order controller but when the order of the process increases the design controller become very difficult. So Reduced order controller easy to implement.

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