MHD NANOFLUID FLOW IN A SEMI-POROUS CHANNEL

A PROJECT REPORT

submitted by

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the partial fulfilment for the award of the degree

of

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under the supervision

of

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Declaration

I declare that the topic "**MHD nanofluid flow in a semi-porous channel** " for completion for my master degree has not been submitted in any other institution or university for the award of any other degree or diploma.

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Certificate

This is to certify that the project report entitled **MHD NANOFLUID FLOW IN A SEMI-POROUS CHANNEL** submitted by **Prachismita Samal** to the National Institute of Technology, Rourkela, Odisha for the partial fulfilment of requirements for the degree of master of science in Mathematics and the review work is carried out by her under my supervision and guidance. It has fulfilled all the guidelines required for the submission of her research project paper for MSc. degree. In my opinion, the contents of this project submitted by her is worthy of consideration for MSc. degree and in my knowledge this work has not been submitted to any other institute or university for the award of any degree.

May, 2014

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Abstract

This research project paper, the problem of laminar nanofluid flow in a semi-porous channel is investigated analytically using Homotopy Perturbation Method (HPM), Least Square Method (LSM) and Differential Transformation Method (DTM). This problem is in the presence of transverse magnetic field. Due to existence some shortcomings in each method, a novel and efficient method named LS-DTM is introduced which omitted those defects and has an excellent agreement with numerical solution. Here, it has been attempted to show the capabilities and wide-range applications of the Homotopy Perturbation Method in comparison with the numerical method used for solving problems. Then, we consider the influence of the three dimensionless numbers: the nanofluid volume friction, Hartmann number for the description of the magnetic forces and the Reynolds number for the dynamic forces.

Keywords:- Nanofluid, Laminar Flow, Semi-porous Channel, Magnetohydrodynamics, Homotopy Perturbation Method (HPM), Least Square Method (LSM), Differential Transformation Method (DTM).

Nomenclature

A^*, B^* Constant parameter
P Fluid Pressure
q Mass transfer parameter
X_k General coordinates
f Velocity function
k^- Fluid thermal conductivity
n Power law index in temperature distribution
Re Reynolds number
Ha Hartmann number
u, v Dimensionless components velocity in x and y directions, respectively
u^*, v^* Velocity components in x and y directions, respectively
x, y Dimensionless horizontal, vertical coordinates respectively
x^*, y^* Distance in x,y directions parallel to the plates

Greek symbols

υ Kinematic viscosity
σ Electrical conductivity
ε Aspect ratio h/L_x
ρ Fluid density

subscripts

∞ Condition at infinity
nf Nanofluid
f Base fluid
s Nano-solid-particles

Contents

Chapter 1

1 Introduction

1.1 Magnetohydrodynamics:

Magnetohydrodynamics (MHD) (magneto fluid dynamics or hydromagnetics) is the study of the dynamics of electrically conducting fluids. Examples of such fluids include plasmas, liquid metals, and salt water or electrolytes. The word magnetohydrodynamics (MHD) is derived from magneto- meaning magnetic field, hydro- meaning liquid, and dynamics meaning movement. The field of MHD was initiated by Hannes Alfvn, for which he received the Nobel Prize in Physics in 1970. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and also changes the magnetic field itself. The set of equations which describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations have to be solved simultaneously, either analytically or numerically.

1.2 Nanofluids

Now, there is an increasing interest of the researchers in the analysis of nanofluids. The word nanofluid was introduced by Choi. In fact a **nanofluid** is a dilute suspension of solid nanoparticles with the average size below 100 nm in a base fluid, such as: water, oil and ethylene glycol. Nano comes from the Greek word for **dwarf**. The prefix nano means a factor of one billionth(10^{-9}) and can be applied, e.g., to time(nano second), volume(nano liter), weight(nano gram)or length(nano meter or nm). In its popular use nano refers to length, and the nanoscale usually refers to a length from the atomic level of around 1nm upto 100nm.

- A sheet of paper is about 100,000 nanometers thick.
- A human hair is approximately 80,000-100,000 nanometers wide.
- Your finger nails grow about one nanometer per second.
- A single gold atom is about a third of a nanometer in diameter.
- On a comparativescale, if the diameter of a marble was one nanometer, then diameter of the Earth would be about one meter.

Nanofluids exhibit thermal properties superior to those of the base fluids of the conventional particlefluid suspensions. The nanoparticles can be made of metal, metal oxide, carbide, nitride and even immiscible nanoscale liquid droplets. Some advantages of nanofluids which make them useful are: a tiny size, along with a large specific surface area, high effective thermal conductivity and high stability and less clogging and abrasion. The materials with sizes of nanometers possess unique physical and chemical properties. They can flow smoothly through microchannels without clogging them because it is small enough to behave similar to liquid molecules.

1.3 Laminar flow

The flow of a fluid is said to be streamline if every particle of the fluid follows exactly the path of its preceding particle and has the same velocity as that of its preceding particle when crossing a fixed point of reference. Pipe intake and aerofoil are two good examples of streamline motion. Laminar flow[2] is a type of streamline flow in which a liquid flows over a fixed surface, the layer of molecules in the immediate contact of surface is stationary. The velocity of upper layers increases as the distance of layers from the fixed layer increases. In this flow liquid layers travel in parallel lines, without affecting the flow of each other. So the liquid flows in an orderly manner and thus has a smooth appearance. All layers need not travel with same velocities For example, when honey flows on a fixed surface, then the layer in immediate contact with the surface has the lowest velocity. As we move away from the surface, the velocity of subsequent layers increases. Hence the velocity of the outermost layer is the maximum, while the velocity of innermost layer in contact with the surface is the least.

1.4 Equation of Continuity

In fluid dynamics[3], the continuity equation states that, in any steady state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system. The differential form of the continuity equation is:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1.4.1}$$

where ρ is fluid density, t is time, u is the flow velocity vector field. In this equation is also one of Euler equations (fluid dynamics). If ρ is a constant, as in the case of incompressible flow, the mass continuity equation simplifies to a volume continuity equation:

$$\nabla \cdot \mathbf{u} = 0, \tag{1.4.2}$$

which means that the divergence of velocity field is zero everywhere. Physically, this is equivalent to saying that the local volume dilation rate is zero.

1.5 Reynolds Number

In fluid mechanics[3], the Reynolds number (Re) is a dimensionless quantity that is used to help predict similar flow patterns in different fluid flow situations. The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number is named after Osborne Reynolds (1842−1912), who popularized its use in 1883. The Reynolds number is defined as the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions. Reynolds numbers frequently arise when performing scaling of fluid dynamics problems, and as such can be used to determine dynamic similitude between two different cases of fluid flow. They are also used to characterize different flow regimes within a similar fluid, such as laminar or turbulent flow: laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion; turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces.
The Reynolds number is generally defined as:

$$Re = \frac{hq}{\mu} \times \rho \tag{1.5.1}$$

1.6 Hartmann Number

Hartmann number (Ha) is the ratio of electromagnetic force to the viscous force first introduced by Hartmann[3]. It is defined by:

$$\mathrm{Ha} = Bh\sqrt{\frac{\sigma}{\rho\nu}} \tag{1.6.1}$$

or

$$\mathrm{Ha} = Bh\sqrt{\frac{\sigma}{\mu}} \tag{1.6.2}$$

where
B is the magnetic field
h is the characteristic length scale
σ is the electrical conductivity
μ is the dynamic viscosity
ν is the kinematic viscosity ($\nu = \mu/\rho$)
ρ is the density of the fluid

1.7 Homotopy Perturbation Method

In fluid mechanics, many of the problems end up to a complicated set of nonlinear ordinary differential equations which can be solved using different analytic method, such as

homotopy perturbation method, variational iteration method introduced by **He** [6]. The homotopy perturbation method, proposed first by He in 1998 and was further developed and improved by He [6]. It yields a very rapid convergence of the solution series in most cases. Sheikholeslami et al.[10] applied this method to investigate Hydromagnetic flow between two horizontal plates in a rotating system. They reported that increasing magnetic parameter or viscosity parameter leads to decreasing Nu. By increasing the rotation parameter, blowing velocity parameter and the Nusselt number increases. Sheikholeslami et al. [10] studied the three-dimensional problem of steady fluid deposition on an inclined rotating disk using HPM. They concluded that by increasing normalized thickness, Nusselt number increases. However, this trend is more noticeable in grater Prandtl numbers.

1.8 Least Square Method

Least square method is introduced by A. Aziz and M.N. Bouaziz and is applied for a predicting the performance of a longitudinal fin. They found that least squares method is simple compared with other analytical methods. Shaoqin and Huoyuan [9] developed and analyzed least-squares approximations for the incompressible magneto-hydrodynamic equations.

1.9 Differential Transformation method

The concept of differential transformation method (DTM) was first introduced by Zhou [11] in 1986 and it was used to solve both linear and nonlinear initial value problems in electric circuit analysis. This method can be applied directly for linear and nonlinear differential equation without requiring linearization, discretization, or perturbation and this is the main benefit of this method.

Most scientific problems in fluid mechanics and heat transfer problems are inherently nonlinear. All these problems and phenomena are modelled by ordinary or partial nonlinear differential equations. Most of these described physical and mechanical problems are with a system of coupled nonlinear differential equations. For an example heat transfer by natural convection which frequently occurs in many physical problems and engineering appli cations such as geothermal systems, heat exchangers, chemical catalytic reactors and nanofluid flow in a semi-porous channel has a system of coupled nonlinear differential equations for temperature or velocity distribution equations.

The flow problem in porous tubes or channels has been under considerable attention in recent years because of its various applications in biomedical engineering, for example, in the dialysis of blood in artificial kidney, in the flow of blood in the capillaries, in the flow in blood oxygenators as well as in many other engineering areas such as the design of filters, in transpiration cooling boundary layer control and gaseous diffusion.

In 1953, Berman[2] described an exact solution of the Navier-Stokes equation for steady two-dimensional laminar flow of a viscous, incompressible fluid in a channel with parallel, rigid porous walls driven by uniform, steady suction or injection at the walls. This mass transfer is paramount in some industrial processes. More recently, Sheikholeslami et al. [10] analyzed the effects of a magnetic field on the nanofluid flow in a porous channel through weighted residual methods called Galerkin method. Nanofluid, which is a mixture of nano-sized particles (nanoparticles) suspended in a base fluid, is used to enhance the rate of heat transfer via its higher thermal conductivity compared to the base fluid. Natural convection heat transfer in a semi-annulus enclosure filled with nanofluid using the Control Volume based Finite Element Method. They found that the angle of turn has an important effect on the streamlines, isotherms and maximum or minimum values of local Nusselt number. Natural convection of a non- Newtonian copper-water nanofluid between two infinite parallel vertical flat plate. They conclude that as the nanoparticle volume fraction increases, the momentum boundary layer thickness increases, whereas the thermal boundary layer thickness decreases. Sheikholeslami et al.[10] performed a numerical analysis for natural convection heat transfer of Cu-water nanofluid in a cold outer circular enclosure containing a hot inner sinusoidal circular cylinder in presence of horizontal magnetic field using the Control Volume based Finite Element Method.

Sheikholeslami et al. [10] have investigated the flow of nanofluid and heat transfer characteristics between two horizontal plates in a rotating system. Their results show that for suction and injection, the heat transfer rate at the surface increases by increasing the nanoparticle volume fraction, Reynolds number, and injection/suction parameter and it decreases with power of rotation parameter. Natural convection of a non- Newtonian copper-water nanofluid between two infinite parallel vertical flat plates.

They have concluded that as the volume fraction of nanoparticle increases, the momentum boundary layer thickness increases, while the thermal boundary layer thickness decreases. Sheikholeslami et al. [10] studied the natural convection in a concentric annulus between a cold outer square and heated inner circular cylinders in presence of static radial magnetic field. They have reported that average Nusselt number is an increasing function of nanoparticle volume fraction as well as Rayleigh number, while it is a decreasing function of Hartmann number.

Sheikholeslami et al.[10] performed a numerical analysis for natural convection heat transfer of Cu-water nanofluid in a cold outer circular enclosure containing a hot inner sinusoidal circular cylinder in presence of horizontal magnetic field using the Control Volume based Finite Element Method. They have induced that in absence of magnetic field, enhancement ratio decreases as Rayleigh number increases; while in presence of magnetic field an opposite trend, was observed. Sheikholeslami et al.[10] studied the effects of magnetic field and nanoparticle on the Jeffery-Hamel flow by ADM. They have shown that increasing Hartmann number will lead to backflow reduction. In greater angles or higher Reynolds numbers, high Hartmann number is needed to reduce the backflow. Also, the results show that momentum boundary layer thickness causes increase of nanoparticle volume fraction. The main aim is to investigate the problem of laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field using Homotopy Perturbation Method. The effects of the nanofluid volume friction, Hartmann number and Reynolds number on velocity profile are considered.

The main aim of this research project paper is to investigate the problem of laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field using Homotopy Perturbation Method, Least Square (LSM) and Differential Transformation Methods (DTM). Also a novel and combined method from these two methods is introduced as LS-DTM which is very accurate and efficient. The effects of the nanofluid volume friction, Hartmann number and Reynolds number on velocity profile are considered. Furthermore velocity profiles for different structures of nanofluid.

Chapter 2

2 Problem Issue

Consider the laminar two-dimensional stationary flow of an electrically conducting incompressible viscous fluid in a semi-porous channel made by a long rectangular plate with length of L_x in uniform translation in x^* direction and an infinite porous plate. The distance between the two plates is h . We observe a normal velocity q on the porous wall. A uniform magnetic field B is assumed to be applied towards direction y^*. In the case of a short circuit to neglect the electrical field and perturbations to the basic normal field and without any gravity forces, the governing equations are:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{2.1}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho_{nf}} \frac{\partial P^*}{\partial x^*} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right) - u^* \frac{\sigma_{nf} B^2}{\rho_{nf}}, \tag{2.2}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho_{nf}} \frac{\partial P^*}{\partial x^*} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right), \tag{2.3}$$

The appropriate boundary conditions for the velocity are:

$$y^* = 0 : u^* = u_0^*, v^* = 0, \tag{2.4}$$

$$y^* = h : u^* = 0, v^* = -q, \tag{2.5}$$

Calculating a mean velocity U by the relation:

$$y^* = 0 : u^* = u_0^*, v^* = 0, \tag{2.6}$$

We consider the following transformations:

$$x = \frac{x^*}{L_x}; y = \frac{y^*}{h}, \tag{2.7}$$

$$u = \frac{u^*}{U}; v = \frac{v^*}{q}; P_y = \frac{P^*}{\rho_f q^2} \tag{2.8}$$

Then, we can consider two dimensionless numbers: the Hartman number Ha for the description of magnetic forces and the Reynolds number Re for dynamic forces:

$$Ha = Bh\sqrt{\frac{\sigma_f}{\rho_f \upsilon_f}} \tag{2.9}$$

$$Re = \frac{hq}{\mu_{nf}}\rho_{nf}. \tag{2.10}$$

where the effective density(ρ_{nf}) is defined as :

$$\rho_{nf} = \rho_f(1-\phi) + \rho_s\phi \tag{2.11}$$

Where ϕ is the solid volume fraction of nanoparticles. The dynamic viscosity of the nanofluids given by Brinkman is

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{2.12}$$

The effective thermal conductivity of the nanofluid can be approximated by the Maxwell-Garnetts (MG) model as:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \tag{2.13}$$

The effective electrical conductivity of nanofluid was presented by Maxwell as

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \tag{2.14}$$

Introducing Eqs.(2.6) and (2.10) into Eqs. (2.1) and (2.3) leads to the dimensionless equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.15}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\epsilon^2\frac{\partial P_y}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\frac{1}{hq}\left(\epsilon^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - u\frac{Ha^2}{Re}\frac{B^*}{A^*}, \tag{2.16}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P_y}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\frac{1}{hq}\left(\epsilon^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{2.17}$$

where A^* and B^* are constant parameters:

$$A^* = (1-\phi) + \frac{\rho_s}{\rho_f}\phi, \;\; B^* = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \tag{2.18}$$

Quantity of ε is defined as the aspect ratio between distance h and a characteristic length L_x of the slider.

This ratio is normally small. Bermans similarity transformation is used to be free from the aspect ratio of ε :

$$v = -V(y); u = \frac{u^*}{U} = u_0U(y) + x\frac{dV}{dy} \tag{2.19}$$

Introducing Eq.(2.19) in the second momentum equation (2.17) shows that quantity $\frac{\partial P_y}{\partial y}$ does not depend on the longitudinal variable x . With the first momentum equation, we also observe that $\frac{\partial^2 P_y}{\partial x^2}$ is independent of x.
We omit asterisks for simplicity. Then a separation of variables leads to :

$$V'^2 - VV' - \frac{1}{Re}\frac{1}{A^*(1-\phi)^{2.5}}V''' + \frac{Ha^2}{Re}\frac{B^*}{A^*}V' = \epsilon^2\frac{\partial^2 P_y}{\partial x^2} = \epsilon^2\frac{1}{x}\frac{\partial P_y}{\partial x}, \tag{2.20}$$

$$UV' - VU' = \frac{1}{Re}\frac{1}{A^*(1-\phi)^{2.5}} \times \left[U'' - Ha^2B^*(1-\phi)^{2.5}U\right]. \tag{2.21}$$

The right-hand side of equation. (2.20) is constant. So, we derive this equation with respect to x. This gives:

$$V^{iv} = Ha^2B^*(1-\phi)^{2.5}V'' + ReA^*(1-\phi)^{2.5}[V'V'' - VV'''], \tag{2.22}$$

Where primes denote differentiation with respect to y and asterisks have been omitted for simplicity. The dynamic boundary conditions are:

$$y = 0 : U = 1; V = 0; V = 0, \tag{2.23}$$

$$y = 1 : U = 0; V = 1; V = 0 \tag{2.24}$$

CHAPTER 3

3 Analysis And Interpretation

3.1 Homotopy Perturbation Method

3.1.1 Analysis of HPM

To illustrate the basic ideas of this method, we consider the following equation:

$$A(u) - f(r) = 0 \quad r \in \Omega \tag{3.1.1.1}$$

With the boundary condition of:

$$B(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \Gamma \tag{3.1.1.2}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and $\Gamma$is the boundary of the domain Ω. A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Equation (3.1.1.1) can therefore be rewritten as follows:

$$L(u) + N(u) - f(r) = 0 \quad r \in \Omega \tag{3.1.1.3}$$

Homotopy perturbation structure is:

$$H(v,p) = (1-p)(L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{3.1.1.4}$$

$$v(r,p) : \Omega \times [0,1] \to R \tag{3.1.1.5}$$

Where $p \in [0,1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that a power series in p, as following:

$$v = v_0 + pv_1 + p^2 v_2 + ... \tag{3.1.1.6}$$

and the best approximation for solution is:

$$u = lim_{p \to 1} v = v_0 + v_1 + v_2 ... \tag{3.1.1.7}$$

3.1.2 Implemention of the method

According to HPM, we construct a homotopy. Suppose the solution of Equation (3.1.1.4) has the form:

$$H(V,p) = (1-p)(V^{iv} - V_0^{iv}) + p(-V^{iv} + Ha^2 B^*(1-\phi)^{2.5} V'' + ReA^*(1-\phi)^{2.5}[V'V'' - VV''']) = 0 \tag{3.1.2.1}$$

$$H(U,p) = (1-p)(U'' - U_0'') + p\left(-UV' + VU' + \frac{1}{ReA^*(1-\phi)^{2.5}}[U'' - Ha^2B^*(1-\phi)^{2.5}U]\right) \tag{3.1.2.2}$$

We consider V and U as follows:

$$V(y) = V_0(y) + V_1(y) + ... = \sum_{i=0}^{n} V_i(y) \tag{3.1.2.3}$$

$$U(y) = U_0(y) + U_1(y) + ... = \sum_{i=0}^{n} U_i(y) \tag{3.1.2.4}$$

By substituting F from Equations (3.1.2.3) and (3.1.2.4) Equations (3.1.2.1) and (3.1.2.2) into and some simplification and rearranging based on powers of p terms, according to the boundary conditions, we have:

$$p^0 : V_0^{iv} = 0, U_0'' = 0 \tag{3.1.2.5}$$

$$p^1 : -Ha^2(1-\phi)^{-2.5}B^*V_0'' + V_1^{iv} + ReA^*(1-\phi)^{-2.5}V_0'''V_0 - ReA^*(1-\phi)^{-2.5}V_0''V_0' = 0 \tag{3.1.2.6}$$

$$-Ha^2(1-\phi)^{-2.5}B^*U_0'' + U_1'' - ReA^*(1-\phi)^{-2.5}V_0'U_0 + ReA^*(1-\phi)^{-2.5}V_0''U_0' = 0 \tag{3.1.2.7}$$

Solving Equations (3.1.2.5),(3.1.2.6) and (3.1.2.7) with boundary conditions, we have:

$$V_0(y) = -2y^3 + 3y^2, U_0(y) = -y + 1. \tag{3.1.2.8}$$

$$\begin{aligned} V_1(y) = {} & 0.0571428ReA^*(1-\phi)^{-2.5}y^7 \\ & - 0.2ReA^*(1-\phi)^{-2.5}y^6 \\ & - 0.1Ha^2(1-\phi)^{-2.5}B^*y^5 \\ & + 0.3ReA^*1-\phi^{-2.5}y^4 \\ & - 0.385714ReA^*(1-\phi)y^3 \\ & - 0.2Ha^2(1-\phi)^{-2.5}B^*y^3 \\ & + 0.22857142ReA^*(1-\phi)^{-2.5}y^2 \\ & + 0.5Ha^2(1-\phi)^{-2.5}B^*y^2 \end{aligned} \tag{3.1.2.9}$$

$$
\begin{aligned}
U_1(y) = &-0.2ReA^*(1-\phi)^{-2.5}y^5 \\
&-0.7ReA^*1-\phi^{-2.5}y^4 \\
&-0.16667Ha^2(1-\phi)^{-2.5}B^*y^3 \\
&+0.1ReA^*(1-\phi)^{-2.5}y^3 \\
&+0.5Ha^2(1-\phi)^{-2.5}B^*y^2 \\
&-0.45ReA^*(1-\phi)^{-2.5}y \\
&0.3333Ha^2(1-\phi)^{-2.5}B^*y
\end{aligned}
\tag{3.1.2.10}
$$

when $i \geq 2$ the terms $V_i(y), U_i(y)$ are too large; that is graphically mentioned. When $p \to 1$, we have the following relations:

$$V(y) = V_0(y) + V_1(y) + ... = \sum_{i=0}^{n} V_i(y) \tag{3.1.2.11}$$

$$U(y) = U_0(y) + U_1(y) + ... = \sum_{i=0}^{n} U_i(y) \tag{3.1.2.12}$$

3.2 Least Square method

3.2.1 Analysis of LSM

Suppose a differential operator D is acted on a function u to produce a function p:

$$D(u(x)) = p(x) \tag{3.2.1.1}$$

It is considered that u is approximated by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is,

$$u \cong \tilde{u} = \sum_{i=1}^{n} c_i \phi_i \tag{3.2.1.2}$$

Now, when substituted into the differential operator,D the result of the operations generally isn't $p(x)$ Hence an error or residual will exist:

$$R(x) = D(\tilde{u}x) - p(x) \neq 0 \tag{3.2.1.3}$$

The notion in WRMs is to force the residual to zero in some average sense over the domain. That is:

$$\int_x R(x)W_i(x) = 0 \quad i = 1, 2, 3.... \tag{3.2.1.4}$$

Where the number of weight functions W_i is exactly equal the number of unknown constants c_i in $u^{\sim}$. The result is a set of n algebraic equations for the unknown constants c_i. If the continuous summation of all the squared residuals is minimized, the rationale the name can be seen. In other words, a minimum of

$$S = \int_x R(x)R(x)dx = \int_x R^2(x)dx \tag{3.2.1.5}$$

In order to achieve a minimum of this scalar function, the derivatives of S with respect to all the unknown parameters must be zero. That is,

$$\frac{\partial S}{\partial c_i} = 2\int_x R(x)\frac{\partial R}{\partial c_i}dx = 0 \tag{3.2.1.6}$$

Comparing with Eq.(3.2.1.4), the weight functions are seen to be

$$W_i = 2\frac{\partial R}{\partial c_i} \tag{3.2.1.7}$$

However, the "2" coefficient can be dropped, since it cancels out in the equation. Therefore the weight functions for the Least Squares Method are just the derivatives of the residual with respect to the unknown constants

$$W_i = \frac{\partial R}{\partial c_i} \tag{3.2.1.8}$$

3.2.2 Implemention of the method

Because trial functions must satisfy the boundary conditions in Eq. (6.23) and (6.24) so they will be considered as,

$$U(y) = 1 - y + c_1(y - y^2) + c_2(y - y^3) \tag{3.2.2.1}$$

$$V(y) = c_3(\frac{y^2}{2} - \frac{y^3}{3}) + c_4(\frac{y^2}{2} - \frac{y^4}{4}) + c_5(\frac{y^2}{2} - \frac{y^5}{5}) \tag{3.2.2.2}$$

In this problem, we have two coupled equations (Eqs.(6.21) and (6.22)), so two residual functions will be appeared as,

$$\begin{aligned} R_1(c_1, c_2, c_3, c_4, c_5, y) = {} & (1-y+c_1(y-y^2)+c_2(y-y^3))(c_3(y-y^2)+c_4(y-y^3)+c_5(y-y^4)) \\ & - (c_3(\frac{y^2}{2} - \frac{y^3}{3}) + c_4(\frac{y^2}{2} - \frac{y^4}{4}) + c_5(\frac{y^2}{2} - \frac{y^5}{5})(-1 + c_1(1-2y) + c_2(1-3y^2) \\ & - \frac{-2c_1 - 6c_2y - Ha^2(1 + \frac{3(\frac{\sigma_s}{\sigma_f})\phi}{((\frac{\sigma_s}{\sigma f}+2)-(\frac{\sigma_s}{\sigma f}-1)\phi})(1-\phi)^{2.5}(1 - y + c_1(y-y^2) + c_2(y-y^3))}{Re(1 - \phi + \frac{\rho_s\phi}{\rho_f})(1-\phi)^{2.5}} \end{aligned} \tag{3.2.2.3}$$

$$R_2(c_1,c_2,c_3,c_4,c_5,y) = -6c_4 - 24c_5y - Ha^2(1+\frac{3(\frac{\sigma_s}{\sigma_f}-1)\phi}{((\frac{\sigma_s}{\sigma f}+2)-(\frac{\sigma_s}{\sigma f}-1)\phi})(1-\phi)^{2.5}$$
$$(c_3(1-2y)+c_4(1-3y^2)+c_5(1-4y^3)) + Re(1-\phi+\frac{\rho_s\phi}{\rho_f})(1-\phi)^{2.5}$$
$$(c_3(y-y^2)+c_4(y-y^3)+c_5(y-y^4))(c_3(1-2y)+c_4(1-3y^2)+c_5(1-4y^3))$$
$$-(c_3(\frac{y^2}{2}-\frac{y^3}{3})+c_4(\frac{y^2}{2}-\frac{y^4}{4})+c_5(\frac{y^2}{2}-\frac{y^5}{5})(-2c_3-6c_4y-12c_5y^2) \tag{3.2.2.4}$$

By substituting the residual functions, $R_1(c_1,c_2,c_3,c_4,c_5,y)$ and $R_2(c_1,c_2,c_3,c_4,c_5,y)$ into Eq. (7.2.1.6), a set of equation with five equations will appear and by solving this system of equations, co-efficients $c_1 - c_5$ will be determined. For example, Using Least Square Method for a water-copper nanofluid with $Re = 0.5, Ha = 0.5\ and\ \phi = 0.05.U(y)\ and\ V(y)$ are as follows:

$$U(y) = 1 - 1.334953917y + .3461783819y^2 - .01122446534y^3 \tag{3.2.2.5}$$

$$V(y) = 1.8703229y^2 + 3.1584693y^3 - 6.9279074y^4 + 2.8991125y^5 \tag{3.2.2.6}$$

3.3 Differential transformation Method

3.3.1 Analysis of DTM

In this section the fundamental basic of the Differential Transformation Method is introduced. For understanding method′s concept, suppose that $x(t)$ is an analytic function in domain D, and $t = t_i$ represents any point in the domain. The function $x(t)$ is then represented by one power series whose center is located at t_i. The Taylor series expansion function of $x(t)$ is in form of:

$$x(t) = \sum_{k=0}^{\infty} \frac{(t-t_i^k)}{k!}[\frac{d^k x(t)}{dt^k}]_{t=i} \quad \forall t \in D \tag{3.3.1.1}$$

The Maclaurin series of $x(t)$ can be obtained by taking $t_i = 0$ in Eq. (3.3.1.1) expressed as:

$$x(t) = \sum_{k=0}^{\infty} \frac{(t^k)}{k!}[\frac{d^k x(t)}{dt^k}]_{t=0} \quad \forall t \in D \tag{3.3.1.2}$$

As explained in [3] the differential transformation of the function $x(t)$ is defined as follows:

$$X(k) = \sum_{k=0}^{\infty} \frac{(H^k)}{k!}[\frac{d^k x(t)}{dt^k}]_{t=0} \tag{3.3.1.3}$$

Where $X(k)$ represents the transformed function and $x(t)$ is the original function. The differential spectrum of $X(k)$ is confined within the interval $t \in [0, H]$, where H is a constant value. The differential inverse transform of $X(k)$ is defined as follows:

$$x(t) = \sum_{k=0}^{\infty} (\frac{t}{H})^k X(k) \tag{3.3.1.4}$$

Theorem 1.
If $f(\eta) = g(\eta) \pm h(\eta)$, then $F(k) = G(k) \pm H(k)$.

Theorem 2.
If $f(\eta) = cg(\eta)$, then $F(k) = cG(k)$,where c is a constant.

Theorem 3.
If $f(\eta) = \frac{d^n g(\eta)}{d\eta^n}$ then $F(k) = \frac{(k+n)!}{k!} G(k+n)$.

Theorem 4.
If $f(\eta) = g(\eta) \times h(\eta)$, then $F(k) = \sum_{l=0}^{k} G(l)H(k-l)$.

3.3.2 Implemention of the Method

From above, it is clear that the concept of differential transformation is based upon the Taylor series expansion. The values of function X(k) at values of argument k are referred to as discrete, i.e. X(0) is known as the zero discrete, X(1) as the first discrete, etc. The more discrete available, the more precise it is possible to restore the unknown function. The function x(t) consists of the T-function X(k), and its value is given by the sum of the T-function with (t/H)k as its coefficient. In real applications, at the right choice of constant H, the larger values of argument k the discrete of spectrum reduce rapidly. The function x(t) is expressed by a finite series and Eq. (3.3.1.4) can be written as:

$$x(t) = \sum_{k=0}^{\infty} (\frac{t}{H})^k X(k) \tag{3.3.2.1}$$

Some important mathematical operations performed by differential transform method are in above theorem. Now we apply Differential Transformation Method (DTM) from above theorem in to Eqs. (2.21) and (2.22) for finding $U(y)$ and $V(y)$.

$$\sum_{l=0}^{k}(k+1-l).\tilde{U}(l).\bar{V}(k+1-l) - \sum_{l=0}^{k}(k+1-l).\bar{V}(l).\bar{U}(k+1-l) -$$
$$\frac{1}{Re}.\frac{1}{A(1-\phi)^{2.5}} \times \big((k=1).(k+2).\bar{U}(k+2) - Ha^2.B.(1-\phi)^{2.5}.\bar{U}(k)\big) = 0 \tag{3.3.2.2}$$

$$\begin{aligned}(k+1)(k+2)(k+3)(k+4)\bar{V}(k+4) - Ha^2.B.(1-\phi)^{2.5}(k+1)(k+2)\bar{V}(k+2)\\ - Re.A.(1-\phi)^{2.5} \times (\sum_{l=0}^{k}(l+1)\bar{V}(l+1)(k-1)(k+1-l)\\ - \sum_{l=0}^{k}\bar{V}(k)(k+1-l)(k+2-l)(k+3-l)) = 0\end{aligned} \tag{3.3.2.3}$$

Where $\bar{U}$ and $\bar{V}$ represent the DTM transformed form of U and V respectively. The transformed form of boundary conditions can be written as:

$$\bar{V}(0) = 0,\ \ \bar{V}(1) = 0,\ \ \bar{V}(2) = a,\ \ \bar{V}(3) = b,\ \bar{U}(0) = 1,\ \ \bar{U}(1) = 0, \tag{3.3.2.4}$$

Using transformed boundary condition and Eq. we have,

$$\begin{aligned}\bar{U}(2) = 0.5Ha^2B\sqrt{1-\phi} - Ha^2B\sqrt{1-\phi}\phi + Ha^2B\sqrt{1-\phi}\phi^2\\ \bar{V}(4) = 0.0833Ha^2B\sqrt{1-\phi}a - 0.1667Ha^2B\sqrt{1-\phi}a\phi + 0.0833Ha^2B\sqrt{1-\phi}a\phi^2\\ \bar{U}(3) = 0.333aReA\sqrt{1-\phi} - 0.667aReA\sqrt{1-\phi}\phi + 0.333aReA\sqrt{1-\phi}\phi^2 +\\ 0.1667Ha^2B\sqrt{1-\phi}c - 0.0333Ha^2B\sqrt{1-\phi}c\phi + 0.1667Ha^2B\sqrt{1-\phi}c\phi^2\\ \bar{V}(5) = 0.05Ha^2B\sqrt{1-\phi}b - .1Ha^2B\sqrt{1-\phi}b\phi + .05Ha^2B\sqrt{1-\phi}b\phi^2\end{aligned} \tag{3.3.2.5}$$

Where a, b, c are unknown coefficients that after specifying $U(y)$ and $V(y)$ and applying boundary condition (Eq. (3.3.2.4)) into it, will be determined. For water-copper nanofluid with $Re = 0.5$, $Ha = 0.5$ and $f = 0.05$ following values were determined for a, b and c coefficients.

$$a = 3.01179150, b = -2.049532443, c = -1.673547080 \quad (3.3.2.6)$$

Finally, $U(y)$ and $V(y)$ are as follows,

$$U(y) = 1 - 1.673547080y - .01273175011y^2 + 0.5462295787y^3 \quad (3.3.2.7)$$

$$V(y) = 3.011719150y^2 - 2.049532443y^3 + 0.06390742601y^4 - 0.02609413491y^5 \quad (3.3.2.8)$$

3.4 LS−DTM Combined Method

Since LSM and DTM have a little shortcoming in some areas for predicting the V(y) and U(y) (See results section), we combined these two methods as LS-DTM combined method which eliminated those defects and for all areas has an excellent agreement with numerical procedure. For this purpose we selected U(y) from Eq. (3.2.2.5),(3.2.2.6) and V(y) from Eq. (3.3.2.3). By using these two equations four unknown coefficients will be existed: a, b, c_1,c_2. For finding these coefficients, four equations are needed; two of them are obtained from Eq. (3.2.1.6) for c_1 and c_2 and other two equations are selected from boundary condition for V(y) in Eq. (2.23),(6.24). For water-copper nanofluid with $Re = 0.5$, $Ha = 0.5$ and $f = 0.05$ following formula are calculated for U(y) and V(y) by this efficient and novel method,

$$U(y) = 1 - 1.332674596y - .3491297634y^2 - 0.01645516732y^3 \quad (3.4.1)$$

$$V(y) = 3.011719150y^2 - 2.049532443y^3 + 0.06390742601y^4 - 0.02609413491y^5 \quad (3.4.2)$$

Chapter 4

4 Conclusion

In research project paper, Least Square and Differential Transformation Methods are combined to eliminate the shortcoming of each method for solving the problem of laminar nanofluid flow in a semi-porous channel in the presence of uniform magnetic field. The above indicate that velocity boundary layer thickness decrease with increase of Reynolds number and nanoparticles volume fraction and it increases as Hartmann number increases.

Also it was found that HPM is a powerful approach. The velocity boundary layer thickness decreases with increasing Reynolds number and nanoparticle volume friction and it increases while Hartmann number increases. Furthermore, it can be seen that for low Reynolds numbers, as Hartmann number increases.

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