FREE VIBRATION ANALYSIS OF MULTIPLE CRACKED UNIFORM AND STEPPED BEAMS USING FINITE ELEMENT ANALYSIS

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Free Vibration Analysis of Multiple Cracked Uniform and Stepped Beams Using Finite Element Analysis

Master of Technology

In

Structural Engineering

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CERTIFICATE

*This is to affirm that the thesis entitled, **"Free vibration analysis of multiple cracked stepped beam using finite element analysis"** submitted by **Boga Sharathdhruthi** in partial fulfilment of the requirements for the award of **Master of Technology** Degree in **Civil Engineering** with specialization in "Structural Engineering" at **National Institute of Technology**, **Rourkela**, is an authentic work carried out by her under my supervision and guidance.*

To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other university/ institute for award of any Degree or Diploma.

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B.SHARATHDHRUTHI

TABLE OF CONTENT

ABSTRACT

Beams with variable cross-section and material properties are frequently used in aeronautical engineering, mechanical engineering and civil engineering (e.g., rotor shafts, beams, columns and functionally graded beams). Stepped beam-like structures are widely used in various engineering fields, such as robot arm and tall building, etc. Beams are a standout amongst the most usually utilized structural components within various structural elements in numerous engineering applications and experience a wide mixed bag of static and element loads. Fracture may create in beam like structures because of such loads. The progressions of cracks can severely decrease the stiffness of an element and further lead to the failure of the complete structure. Fractures or different blotch in a component of structural type impact its conduct dynamically and transform the coagulate and damping properties. Hence, the frequency occurred naturally and mode pattern of the structure hold data about the position and measurements of the defect. The presence of fracture in the structure are been subjected by the local flexibility which damages the behaviour dynamically of entire structure to a notable degree. Any investigation of these progressions makes it conceivable to inspect the fractures.

In this paper the crack considered is transverse open crack it has been analysed that when the crack is present in beam the reduced stiffness matrix can be found using Fracture mechanics theory. Due to the importance of this problem, the FEM formulation is done for cracked uniform and stepped beams. Analysis includes free vibration analysis of the Cantilever Bernoulli-Euler beam of various cross-sections. The effects of various parameters such as natural frequencies for uniform and stepped beams with and without cracks are presented and convergence study is done. Comparisons of the natural frequencies of the beams with the pervious papers in order to understand the accuracy of present study is included. Numerical Analysis is done considering an Aluminium beam (cantilever beam) with transverse open crack in order to obtain the natural frequencies of uniform beam and stepped beams with out and with multiple cracks. The results are obtained by using Finite Element Method (FEM) in MATLAB environment to find out the overall stiffness matrix, natural frequencies and non- dimensional frequencies.

List of Figures

LIST OF TABLES

NOTATIONS

L_C = Distance between the right hand side end node and the crack location

Le = Length of the beam element.

A = Cross-sectional area of the beam

x = Location of the crack from the fixed of the beam

L_1, L_2, L_3 = Location of first, second, third cracks respectively from the fixed end of the beam

d = Depth of the rectangular beam

b = Width of the beam

a/d = Crack-depth ratio

D = Diameter of the circular beam

CHAPTER 1
INTRODUCTION

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INTRODUCTION

1.1 Introduction

Engineering structures are designed to withstand the loads they are expected to be subject to while in service. Among them Beams are a standout amongst the most usually utilized structural components within various structural elements in numerous engineering applications and experience a wide mixed bag of static and element loads. Beams are widely used as structural components in engineering applications and also provide a fundamental model for many engineering applications. Aircraft wings, helicopter rotor blades, spacecraft antennae, and robot arms are all examples of structures that may be modeled with beam-like elements. Beam sort structures are being generally utilized in steel shaped structure and manufacturing of machines.

Beams with variable cross-section and/or material properties are frequently used in aeronautical engineering (e.g., rotor shafts and functionally graded beams), mechanical engineering (e.g., robot arms and crane booms), and civil engineering (e.g., beams, columns, and steel composite floor slabs in the single direction loading case). Stepped beam-like structures are widely used in various engineering fields, such as robot arm and tall building, etc.

Therefore there is a necessary that construction should securely work during its service period. But, wreck initiates a failure span on structure. The instant changes introduced into a structure, either intentionally or unintentionally which leads to adverse effect the current or future performance of that structure is defined as damage. Damage is one of the important aspects in structural analysis because of safety reason as well as economic growth of the industries. The general structural defect is the existence of cracks and is among usual encountered defects in the structures. Cracks are present in structures due to numerous reasons. The effect cause on the mechanical behavior of the entire structure to an amountable extent along with the local variation in the stiffness is due to presence of a crack in the structure. Fatigue under service conditions leads to formation of cracks because of limited fatigue strength. Generally the cracks are of small sizes. Such small cracks subject to propagate throughout the beam. Due to the lack of undetected cracks, A rapid structural breakdown is subjected if they reach crucial size. Therefore, natural frequency measurements are useful to detect cracks. So early crack detection is important because sudden failure due to high load operation leads to serious damage or injury.

It is well known that the cracks appearing in a structure yield an increase of the vibrational level, result in the reduction of their load carrying capacity, and can constitute the cause of catastrophic failures. Vibration measurements offer a non-destructive, inexpensive and fast means to detect and locate cracks. The crack detection has importance for structural health monitoring applications because fracture in a structure can be harmful because static dynamic loadings. The SHM process involves the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors, the extraction of damage-sensitive features from these measurements, and the statistical analysis of these features to determine the current state of system health.In the last two decades a lot of research effort has been devoted to developing an effective method of approach for crack detection in structures. Early crack detection plays a very important role for ensuring safety and reliability of in-service structures. For the reason of veracity and testability, the changes in natural frequencies of the structure before and after damage are often used to indicate the state of the structure. A lot of research efforts have been devoted to developing an effective approach for crack detection in structures.

Numerous courses guide with the structural guarding of beams, especially, fracture found by structural health monitoring. Course related subjected on structural health monitoring for fracture detection gives a idea about change in frequencies occurred naturally and mode pattern of the beam. A Vibrational analysis generally used to find defects on structure like fracture, of any structure offer an effective, inexpensive and fast means of non-destructive testing.The presence of a crack or localized damage in a structure reduces the stiffness and increases the damping in the structure. Vibration theory states, reduction in the stiffness is associated with decrease in the natural frequencies and modification of the modes of vibration of the structure. Fractures or other blotch in a structural component influence its dynamic loading and change its stiffness and damping properties. Consequently, the frequency occurred naturally and mode shape of the structure with data about the position and measurement of the defect. Local flexibilities are existing because the presence of fracture in the structure which harms the behavior of the entire structure dynamically to an amountable degree. It leads to decrease in frequencies occurred naturally and changes in mode patterns of vibrations. Any detection of these differences makes likely to detect fractures.

1.2 Research significance

In numerous engineering applications beams are universally used structural elements which experience a wide variety of static and dynamic loads. During their utilisation various engineering structures subjected to degenerative effects, all these are responsible for the development of cracks. The propagation of these cracks decreases the stiffness of an element and sometimes leads to the failure of the complete structure. Immediate detection of these cracks is an important task of an engineer to determine the effect of crack on stiffness on the beam, all these beams or shafts subjected to these conditions are modelled using either Timoshenko beam or Euler-Bernoulli theories. The characteristic equation involving natural frequency, the crack depth and crack location and other properties of the beam are derived using conventional methods like boundary conditions of the beam along with the stress intensity factors. The change in dynamic characteristics of multiple cracked stepped beams with varying cross sections using FEM. This problem has been a subject of many papers, but only a few papers have been devoted to the changes in the dynamic characteristics of multiple cracked stepped beams with varying cross sections using FEM.

1.3 Objective of the Study

To study the free vibrational analysis of uniform cantilever beams of varying cross sections such as rectangular and circular, stepped beams and study the result of position of cracks, crack depth and number of cracks present in the beams with multiple cracks.

1.4 Organisation of Thesis

The present thesis is divided in to five chapters:

In chapter 1, the general introduction to uniform beams, stepped beams their importance in different engineering fields and damage due to presence of cracks in the beams along with the objective of the present work are outlined.

In Chapter 2, a detailed review of the literature relevant to the previous research works made in this field has been listed. A critical discussion of the earlier investigations is done. The scope of the present study is also outlined in this chapter.

In Chapter 3, a description of the theory and formulation of the problem and the finite element procedure used to analyse the vibration of beams of circular and rectangular cross-sections are explained in detail. The relevant computer program used to implement the formulation is briefly described.

In Chapter 4, contains the test results and discussion. It includes convergence study, comparison with previous results, numerical analysis.

In Chapter 5, the conclusions drawn from the above studies are explicitly described. There is also a brief note on the scope for further investigation in this field.

In Chapter 6, the References include few publications, journals and books referred during the present investigation.

CHAPTER 2
LITERATIVE REVIEW

CHAPTER 2

LITERATURE REVIEW

2.1 Brief Introduction

The cracks present in the structure interrupt the continuity of the assembly in most of the engineering structures like beam, columns in which geometrical properties can also be altered. Cracks caused due to fatigue stresses or stress concentration reduces the natural frequency and change mode of vibration due to local flexibility induced by the crack. All these effects due to concentrated cracks have been exclusively discussed in this literature. A crack is modeled by describing the variation of the stiffness matrix of the member in the vicinity of a crack. The presence of a crack in a structural member introduces a local compliance that affects its response to varying loads. The change in dynamic characteristics can be measured and lead to identification of structural alteration, which at the end finally might lead to the detection of a structural flaw. A lot of analytical, experimental and numerical investigations now exist. The solutions for various conditions of loading such as and their solution for different loading conditions such as axial, shear, torsion, flexure etc., derived have been used in the literature. No unified or general style exists in the governing equations and in the solution of cracked members.

2.2 Uniform Beam with Cracks

Many papers considering the effect of cracks and other defects on the behavior of uniform beam and stepped beam like structures have been published in the last twenty years. Shen and Pierre(1986) have presented a finite element approach which makes it possible to predict the changes in the first few Eigen frequencies, Eigen modes due to cracks is presented in this paper. Eight nodes Isoperimetric element is used to model across the thickness of the beam. Rizos et al.(1989) Measurement of flexural vibrations of a cantilever beam with rectangular cross-section having a transverse surface crack extending uniformly along the width of the beam and analytical results are used to relate the measured vibration modes to the crack location and depth. From the measured amplitudes at two points of the structure vibrating at one of its natural modes, the respective vibration frequency and an analytical solution of the dynamic response, the crack location are found and depth is estimated. Shen and Pierre(1994) have derived the equation of motion and associated boundary conditions for a uniform Bernoulli-Euler beam containing one single-edge crack. The generalised principle used allows for modified stress, strain and displacement fields that satisfy the compatibility requirements in the vicinity of the crack. Kracwczuk and Ostachowicz(1996) have presented a survey of damage indicators which can be used for diagnose of fatigue cracks in constructional elements. The advantages and disadvantages of the presented damage indicators are described. The Hu-Washizu-Barr variational detailing was utilized to create the differential mathematical statement and the limit states of the fractured beam as an one dimensional continuum. Lee and Chung(1999) In this paper, a simple and easy non-destructive evaluation procedure was presented for identifying a crack, the location and size of the crack, in a one-dimensional beam-type structure using the natural frequency data. F.E.M. model is adopted and the crack size was determined by F.E.M. Finally, the actual crack location can be identified by Gudmundson's equation using the determined crack size and the aforementioned natural frequencies. Kisa et al.(1998) In the present paper vibrational characteristics for a cracked Timoshenko beam were analyzed. Each substructure was modeled by Thimoshenko beam finite elements with 2 nodes and 3-degree of freedom (axial, transverse, rotation) at each node. Shifrin and Rutolo(1999) have given a new technique is proposed for calculating natural frequencies of a vibrating beam with an arbitrary finite number of transverse open cracks. The main feature of this method was related to decreasing the dimension of the matrix involved in the calculation, so that reduced computation time was required for evaluating natural frequencies compared to alternative methods which also make use of a continuous model of the beam.

Khiem et al.(2001) The author used transfer matrix method and rotational spring model of crack natural frequency analysis with an arbitrary number of cracks is developed. The resulted frequency equation of a multiple cracked beam is general with respect to the boundary condition. The procedure proposed was advanced by elimination of numerical computation of the high order determinant so that the computer time for calculating natural frequencies was consequence and was significantly reduced. Chondros et al.(2001) considered by a continuous cracked beam vibration theory for the prediction of changes in transverse vibration of a simply supported beam with a breathing crack. The equation of motion and the boundary conditions of the cracked beam considered as a one-dimensional continuum were used. Zeng et al.(2003) found the frequencies occurred naturally and mode pattern of a fractured beam by FEM. An overall additional Flexibility matrix, instead of the local additional Flexibility matrix is mixed-up to the present intact beam element to obtain the total flexibility matrix and also the stiffness matrix. In this paper a pattern function was constructed that will precisely answer the local flexibility positions of the location of the crack, which can tell more precisely vibration pattern. Kisa and Gurel(2006) proposed a numerical model that combines the finite element and component mode synthesis methods for the modal analysis of beams with circular cross section and containing multiple non-propagating open cracks. Three numerical illustrations are explained to examine the effects and depth of crack on the frequency occurred naturally and beams with mode patterns. And examination of proposed model obtained modal data tells us regarding the data of the position and size of defect in the beams.

2.3 Stepped Beam with Cracks

Satho(1980) The non-linear free vibrations of stepped thickness beams were analyzed by assuming sinusoidal responses and using the transfer matrix method. The numerical results for clamped and simply supported, one-stepped thickness beams with rectangular cross-section were presented and the effects of the beam geometry on the non-linear vibration characteristics are discussed. Subramanian and Balasubramanian(1987) Here higher order beam elements were used to study the dynamic behaviour of stepped beams with various end conditions and step ratios. The study has determined that if steps can help tailor the frequencies according to requirements.

Jang and Bert(1989) have presented the exact solutions for fundamental natural frequencies of stepped beams for various boundary conditions were presented, In the present paper, higher mode frequency of a stepped beam along two different cross-sections are sought for various boundary conditions. The effects of steps on frequency of a beam are investigated. Balasubramanian(1990) the vibration analysis of beams in this paper was done by the finite element method higher degree polynomials to describe the deflection functions were desirable to predict higher mode frequencies and mode shapes with good accuracy. Lee(1994) A finite element method was presented for the analysis of free vibration of arbitrarily stepped beams on the basis of a first-order shear deformation beam theory. The effects of shear deformation, step geometry, step eccentricity and multiple stepped sections were investigated.

Saavedra and Cuitino(2001) have presented a theoretical and experimental dynamic behaviour of different multi-beams systems containing a transverse crack. The additional flexibility that the crack generates in its vicinity is evaluated using the strain energy density function were given by linear fracture mechanic theory. Friswell and Penny(2002) have compared different approaches to crack modelling, for demonstrating the structural health monitoring using low frequency vibration, simple models of crack flexibility based on beam elements are adequate. Douka et al.(2003) have given a simple method for crack identification in beam structures based on wavelet analysis were presented. The fundamental vibration mode of a cracked cantilever beam was analyzed using continuous wavelet transform and both the location and size of the crack were estimated. Koplow et.al(2006) have presented an analytical solution for the dynamic response of a discontinuous beam with one step change and an aligned neutral axis. The case of free–free boundary conditions was considered to obtain direct frequency response functions due to harmonic force or couple excitation at either end location. Jaworski and Dowell(2008) has presented the flexural-free vibration of a cantilevered beam with multiple cross-section steps was investigated theoretically and experimentally. Experimental results were compared against Euler–Bernoulli beam theory solutions from Rayleigh–Ritz and component modal analyses, as well as finite element results using ANSYS. Zhang et.al(2009) has illustrated the crack identification method combining wavelet analysis with transform matrix. This method can be used for crack identification in a complex structure. Firstly, the fundamental vibration mode was applied to wavelet analysis was done. The crack location was found by the peaks of the wavelet coefficients. Secondly, based on the identified crack locations a simple transform matrix method requiring only the first two tested natural frequencies was used to further identify the crack depth. The present method is used for crack identification in a complex structure. Huang and Liu(2011) have given a new method is to analyse the free and forced vibrations of beams with either a single step change or multiple step changes using the composite element method (CEM). The principal advantage of this method is that it does not need to partition the stepped beam into uniform beam segments between any two successive discontinuity points and the whole beam can be treated as a uniform beam.Mao(2011) The Adomian decomposition method (ADM) is employed in this paper to investigate the free vibrations of the Euler–Bernoulli beams with multiple cross-section steps. The proposed ADM method is used to analyze the vibration of beams consisting of an arbitrary number of steps through a recursive way. Attar(2012) A new method to solve the inverse problem of determining the location and depth of multiple cracks is also presented. Based on the euler-Bernoulli beam theory, the stepped cracked beam is modeled as an assembly of uniform sub-segments connected by massless rotational springs representing local flexibility induced by the non-propagating edge cracks. Ameneh et.al(2012) In the present paper we brief a simplistic way for finding, localizing and quantifying number of fractures formed in Euler-Bernoulli multi-stepped beams, by measurement of frequencies occurred naturally and evaluating the unfractured mode patterns. The main advantage of this method is that it has the power of detecting the unknown number of cracks.

Wang(2013) The differential quadrature element method (DQEM) is proposed for obtaining highly accurate natural frequencies of multiple-stepped beams with an aligned neutral axis. The proposed method is simple and efficient, and can be used to analyze beams with any step changes in cross-section conveniently. Duan and Wang(2013) In this paper, free vibration of beams with multiple step changes are successfully analyzed by using the modified discrete singular convolution (DSC).

The jump conditions at the steps are used to overcome the difficulty in using ordinary DSC for dealing with ill-posed problems. It is also found that the DSC results are even more accurate than the data obtained by the differential quadrature element method for much higher mode frequencies. In this paper the research extends the application range of the DSC algorithm. Shao et al.(2013) have carried out the dynamic distinguishing(vibration shape and frequency occurred naturally) of the gear body. The gear has tooth type cracks.

2.4 Scope of Study

Most of researchers studied the effect of single crack on the dynamics of structures. Free vibration analysis of one or multiple stepped beams variation in cross-section has drawn attentions of researchers for many years. However in actual practice structural members such as beams are highly susceptible to multiple cracks. Therefore to attempt has been made to investigate the dynamic behavior of basic structures with crack systematically in the following manner:

- Vibrational analysis of uniform and stepped cantilever beams of varying cross sections such as rectangular and circular is studied and compared where the reduced stiffness matrix is found using fracture mechanics theory.
- The effect of crack-depth ratio, crack location in the beam, number of cracks present in uniform and stepped beams on the dynamic behaviour of the beams.
- To develop a computer program to perform all the necessary computations in MATLAB environment.

CHAPTER 3
THEORY AND FORMULATION

CHAPTER 3

THEORY AND FORMULATION

3.1 Introduction

In this paper it has been analysed that when the crack is present in beam the reduced stiffness matrix can be found using Fracture mechanics theory. The existence of a crack results in a reduction of the local stiffness, and this additional flexibility alters also the global dynamic structural response of the system. Due to the importance of this problem, the FEM formulation is done for stepped beams. A relevant computer program is developed in MATLAB to find out the overall stiffness matrix, natural frequencies and non- dimensional frequencies.

3.2 Crack Theory

Physical parameters affecting Dynamic characteristics of cracked structures:

The dynamic response of a structure is normally determined by the physical properties, boundary conditions and the material properties. The changes in dynamic characteristics of structures are caused by their variations. The presence of a crack in structures also modifies its dynamic behavior. The following properties of the crack influence the dynamic response of the structure.

• Depth of crack

• Location of crack

• Orientation of crack

• Number of cracks

Classification of cracks:

On the basis of geometry, cracks can be broadly classified into:

- **Transverse cracks-** cracks are perpendicular to axis of the beam. Transverse cracks reduces the cross-section of the structure and weakens the beam. This reduction introduces local flexibility in the stiffness of the beam due to strain energy concentration in the vicinity of the crack.

- **Longitudinal cracks-** cracks are parallel to axis of the beam. These cracks are dangerous when tensile load is applied perpendicular to beam axis.

- **Slant cracks-** These cracks are at an angle to axis of the beam. The torsional behavior of the beam gets influenced. These cracks have less effect on lateral vibrations than that of transverse cracks of comparable severity.

- **Breathing cracks- T**he cracks that open when the affected part of the material is subjected to tensile stresses and close when the stress is altered. Under tension the stiffness of the component is most effected.

- **Surface cracks-** These are the cracks that are opened on the surface. These can be easily detected by dye-penetrations or visual inspection. Surface cracks have a greater effect than subsurface cracks in the vibration behavior of shafts.

- **Subsurface cracks-** These cracks are not on the surface. Detection of these cracks need special techniques such as ultrasonic, radiography, magnetic particle or shaft voltage drop to detect them.

3.3 Fracture Mechanics Theory

Fracture mechanics is the field of mechanics concerned with the study of the propagation of cracks in materials Fracture mechanics is a field that tells us about the capacity of cracks and the crack propagation law. Its job is to examine the situation of stress and strain near the tip of the crack, master crack propagation law subjected load, and understand the loading capacity of bodies with fractures, thereby, suggesting anti-crack design solutions to ensure the construction is safe. Fracture mechanics was improved from conventional design and also acts as a complement to it. The specific conditions are :

- Its objective is the material containing fractures or defects
- The stress that flexibly guides to crack is generally a tensile stress
- The microstructure of the body itself is delicate to brittle type of cracks, that is to say, its fracture toughness is comparatively less.

A crack is likely to form when joining these three conditions. Fracture extension under outer forces can generally be separated into three patterns of cracks:

Mode 1:- Represents the opening mode or first mode. It has an edge crack and the forces attempt to extend the crack. In this opening mode the crack faces separates in a direction perpendicular to the plane of the crack and the respective displacements of crack walls are symmetric with respect to the crack front. Loading is perpendicular to the crack plane, and it has the tendency to open the crack. Generally Mode I is considered the most dangerous loading condition.

Mode 2:- Represents the in-plane shear loading. In this one crack face tends to slide with respect to another (shearing mode). The displacements stay within the plane of the crack. Here the stress is parallel to the crack growth direction.

Mode 3:- In the below figure mode 3 shows shear loading for out-of-plane is called as tearing mode. This indicates a crack faces are subjected sheared parallel to the crack front.

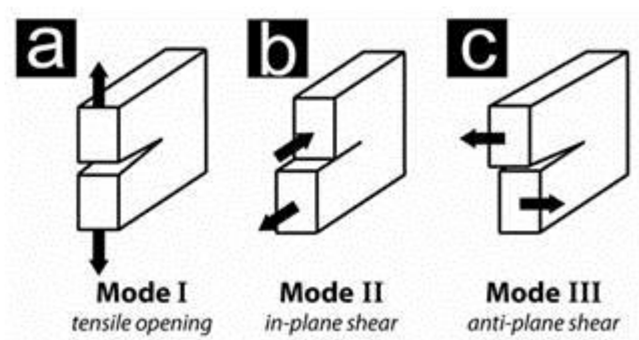


Figure 3.1: Three basic modes of fracture

Major applications of fracture mechanics design are material selection, effect of defects, failure analysis and control/monitoring of components. Fracture analysis includes the usage of mathematical models such as linear elastic fracture mechanics (LEFM), crack opening displacement (COD) and J-integral approaches by using finite element analysis (FEM). The relationship used for estimating stress intensity factor is

$$K = c\sigma\sqrt{a}$$

Where K is the critical fracture toughness value, c a constant that depends on crack and specimen dimensions, σ the applied stress, and the flaw size.

Fracture toughness and SIF:

Usually, the longer the crack in the component, the severer the stress concentration is and the fracture stress of the component is also lower. A large number of analyses and experiments related to fracture accidents of components containing cracks show that:

- Fracture stress σ_c is conversely corresponding to the square root of crack size.
- Crack stress σ_c is also relevant to the form of the crack and the way of loading, for each material in a particular process state. $\sigma_c \sqrt{aY} =$ constant, where Y is a parameter relevant to the crack formation and the way of loading, for example, the center crack from an infinite body, $Y = \sqrt{\pi}$. This constant shows that the ability of a material to resist fracture. Usually this constant is called the fracture toughness, as expressed by K_{IC} . i.e.,

$$K_{IC} = \sigma_c \sqrt{aY} .$$

K_{IC} is the measurement of the material to prevent unstable crack growth capacity, and is the material characteristic parameter, it is irrelevant to the crack size, geometry, and loading, and is only relevant to material composition [32].

3.4 Methodology

Mathematical Formulation for uniform beam of rectangular cross-section:

Considering a typical cracked uniform beam element of rectangular cross-section of breadth '*b'* , depth '*h*' with a depth of crack '*a* '. The left hand side end node '***i'*** is assumed to be fixed, while the right hand side end node '***j'*** is subjected to shearing force P_1 and bending moment P_2. The corresponding generalized displacements are denoted as q_1 and q_2 as shown in Figure 3.2. The equation governed of the vibrated analysis of the uniform beam along an open transverse crack are computed on basis of the model proposed by Zheng D. Y. and Kessissoglou N. J. K. (2004).

L_C = Distance between the right hand side end node *j* and the crack location.

Le = Length of the beam element.

A = Cross-sectional area of the beam.

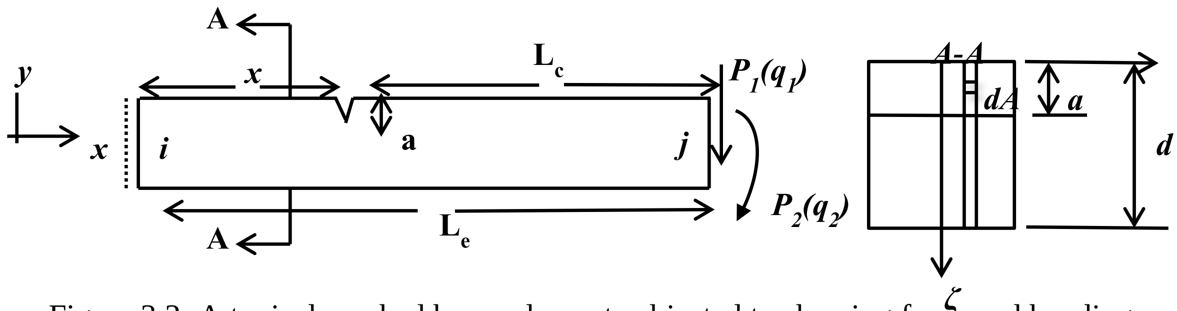


Figure 3.2: A typical cracked beam element subjected to shearing force and bending moment of rectangular cross-section

The governing equations for the vibration analysis of the uniform beam with an open transverse crack are followed as following

According to Zheng [2004], the additional strain energy due to existence of crack can be expressed as

$$\pi = \int_A G\, dA_c$$

Where, G = the strain energy release rate and

A_C = the effective cracked area

$$G = \frac{1}{E}\left[\left(\Sigma_{n=1}^{2} K_{In}\right)^2 + \left(\Sigma_{n=1}^{2} K_{IIn}\right)^2 + \left(\Sigma_{n=1}^{2} K_{IIIn}\right)^2\right]$$

Where, $E' = E$ for plane stress

$= E/1-\upsilon^2$ for plane strain

K_I, K_{II} *and* K_{III} = stress intensity factors for opening, sliding and tearing type cracks respectively. Neglecting effect of axial force and for open cracks, the above equations can be written as

$$G = \frac{1}{E}\left[\left(K_{I1} + K_{I2}\right)^2 + K_{II1}{}^2\right]$$

The expressions for stress intensity factors from earlier studies are given by,

$$K_{I1} = \frac{6P_1L_c^2}{bh^2}\sqrt{\pi\xi}\,F_1\left(\frac{\xi}{h}\right)$$

$$K_{I2} = \frac{6P_2}{bh^2}\sqrt{\pi\xi}\,F_{II}\left(\frac{\xi}{h}\right)$$

$$K_{I2} = \frac{P_2}{bh}\sqrt{\pi\xi}F_{II}\left(\frac{\xi}{h}\right)$$

Where s = ξ/h

$$F_I(s) = \sqrt{\frac{\tan\left(\pi s/2\right)}{\left(\pi s/2\right)}}\left[\frac{0.923 + 0.199\left(1-\sin\left(\pi s/2\right)\right)^4}{\cos\left(\pi s/2\right)}\right]$$

$$F_{II}(s) = \frac{1.122 - 0.561s + 0.085s^2 + 0.180s^3}{\sqrt{1-s}}$$

FI(s) and FII(s) are correction factors for stress intensive factors.

From definition, the elements of the overall additional flexibility matrix C_{ij} can be expressed as

$$C_{ij} = \frac{\partial \delta_i}{\partial P_j} = \frac{\partial^2 \Pi_c}{\partial P_i \partial P_j}$$

Substituting the values,

$$C_{ij} = \frac{b}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\int\left[\left\{\frac{6P_1L_c^2}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right) + \frac{6P_2}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)\right\}^2 + \left\{\frac{P_2}{bh}\sqrt{\pi\xi}F_{II}\left(\frac{\xi}{h}\right)\right\}^2\right]d\xi$$

Substituting i, j $(1, 2)$ values, we get

$$C_{11} = \frac{2\pi}{E'b}\left[\frac{36L_c^2}{h^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx + \int_0^{\frac{a}{h}} xF_{II}^2(x)dx\right]$$

$$C_{12} = \frac{72\pi L_c}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx = C_{21}$$

$$C_{22} = \frac{72\pi}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx$$

Now, the overall flexibility matrix C_{ovl} is given by,

$$C_{ovl} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Mathematical Formulation for Uniform Beam of Circular Cross-section:

Considering a typical cracked uniform beam element of circular cross-section of diameter '*D*' with a crack of depth '*a* '. The left hand side end node '***i'*** is assumed to be fixed, while the right hand side end node '***j'*** is subjected to axial force P_1, shearing force P_2 and bending moment P_3 as shown in Figure 3.3.

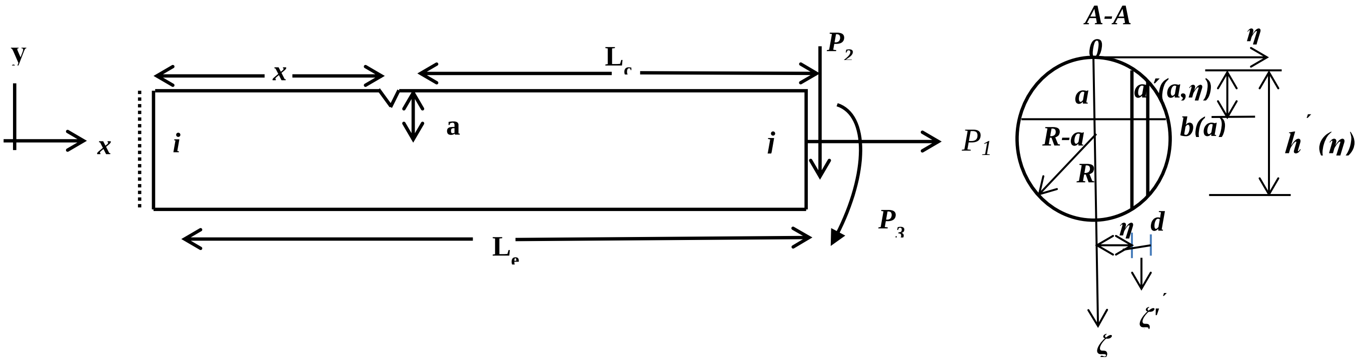


Figure 3.3: A typical cracked beam element subjected to shearing force and bending moment of circular cross-section.

The equations governed for the vibration analysis of uniform beam along an open transverse crack are computed on basis of the model proposed by Zheng D. Y. and Kessissoglou N. J. K. (2004).The geometrical dimensions are as follows:

$$\xi' = \xi + \sqrt{\frac{D^2}{4} - \eta^2} - \frac{D}{2}$$

$$b\,(a) = \sqrt{Da - a^2}$$

$$h'(\eta) = \sqrt{D^2 - 4\eta^2}$$

$$a'\,(a,\eta) = \sqrt{\frac{D^2}{4} - \eta^2} - \left(\frac{D}{2} - a\right)$$

where D is the diameter of the beam. A similar procedure to the rectangular cross-sectional beam is used to derive the overall additional flexibility matrix for a circular cross-sectional beam. The additional strain energy due to the existence of the crack can be expressed as

$$\prod_c = \int_{A_C} G dA = \int_{-b(a)}^{b(a)} \left[\int_0^{a'(a,\eta)} G d\xi' \right]$$

Where, G is the strain energy release rate function and A_c is the effective cracked area. The strain energy release rate function G can be related to the stress intensity factors as mentioned above in which

$$K_{I1} = \frac{4P_1}{\pi D^2}\sqrt{\pi\xi'}F_1\left(\frac{\xi'}{h'}\right),\ K_{I2} = \frac{32P_2L_Ch'}{\pi D^4}\sqrt{\pi\xi'}F_2\left(\frac{\xi'}{h'}\right)$$

$$K_{I3} = \frac{32P_3 h'}{\pi D^4}\sqrt{\pi\xi'}F_2\left(\frac{\xi'}{h'}\right), K_{I I2} = \frac{4P_2}{\pi D^2}\sqrt{\pi\xi'}F_{II}\left(\frac{\xi'}{h'}\right),$$

Where ξ' is the penetrating depth of the strip. Substituting the above equations, the following expression can be derived:

$$C_{ij} = \frac{1}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\int_{-\sqrt{Da-a^2}}^{\sqrt{Da-a^2}}\int_0^{\sqrt{(D^2/4-\eta^2)}-(D/2-a}\left\{\left[\frac{4P_1}{\pi D^2}\sqrt{\pi\xi'}F_1\left(\frac{\xi'}{h'}\right)+\frac{32P_2 L_c h'}{\pi D^4}\sqrt{\pi\xi'}F_2\left(\frac{\xi'}{h'}\right)+\frac{32P_3 h'}{\pi D^4}\sqrt{\pi\xi'}F_2\left(\frac{\xi'}{h'}\right)\right]^2 + \left\{\frac{16P_2^2}{\pi D^4}\xi' F_{II}^2\left(\frac{\xi'}{h'}\right)\right\} d\xi' d\eta$$

(i , j = 1,2,3)

Where A_s is the integration area in a unit circle as shown in Figure3.3. The corresponding least squares best fitted formulas are as follows:

$F(1,1) \approx e^{1(1-x)}$(-0.12323$x^{0.4}$ + 3.156480 $x^{0.8}$ + 34.490509 $x^{1.2}$ + 211.429280 $x^{1.6}$ – 802.944428 x^2 + 1964.215885 $x^{2.8}$ + 3036.531592 $x^{3.2}$ – 1692.594137 $x^{3.6}$ + 411. 505609 x^4) ($0 \le x$ = a / D $\le$ 0.5)

$F(1,2) \approx e^{1/(1-x)}$(0.0525646 $x^{0.4}$-1.694740 $x^{0.8}$ +22.910177 $x^{1.2}$ -171.535649 $x^{1.6}$+789.046673 x^2 -2318.500920 $x^{2.4}$ +4461.869140 $x^{2.8}$ -5337.583060 $x^{3.2}$ +3599.915932 $x^{3.6}$ -1044.227437 x^4) ($0 \le x$ = a / D $\le$ 0.5) = $F(1,3)$

$F(2,2) \approx e^{1/(1-x)}$(-0.0181106$x^{0.4}$+ 0.483199 $x^{0.8}$ – 5.519102 $x^{1.2}$ + 35.485789 $x^{1.6}$-141.871055 x^2 – 367.85339 $x^{2.4}$ +610.901666 $x^{2.8}$ + 639.711620$x^{3.2}$ - 384.398763$x^{3.6}$ – 98.728659 x^4) + (Lc/D)2F(3,3) ($0 \le x$ =a/D $\le$ 0.5)

$F(3,3) \approx e^{1/(1-x)}$(-0.102895$x^{0.4}$+ 3.653566 $x^{0.8}$ – 53.161890$x^{1.2}$ + 423.977411$x^{1.6}$-2072.129084x^2 – 6447.218742$x^{2.4}$ + 13613.390334$x^{2.8}$ -17873.887075$x^{3.2}$ +12985.643127$x^{3.6}$ – 3999.17110x^4)
($0 \le x$ =a/D $\le$ 0.5) = $F(2,3)$.

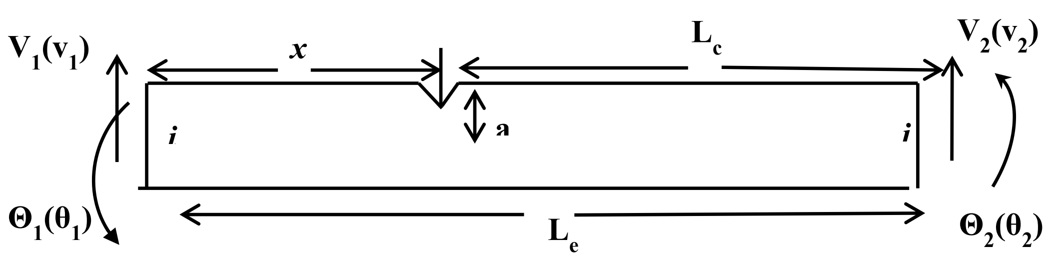


Figure 3.4: Typical cracked beam element subject to shearing force and bending moment under the conventional FEM coordinate system

Flexibility matrix C_{intact} related to intact beam element subjected to shear force and bending Moment

$$C_{intact} = \begin{bmatrix} \frac{L_e^3}{3EI} & \frac{L_e^2}{2EI} \\ \frac{L_e^2}{2EI} & \frac{L_e}{EI} \end{bmatrix}$$

Total flexibility matrix C_{tot} of the cracked beam element

$$C_{total} = C_{intact} + C_{ovl}$$

$$C_{total} = \begin{bmatrix} \frac{L_e^3}{3EI} + C_{11} & \frac{L_e^2}{2EI} + C_{12} \\ \frac{L_e^2}{2EI} + C_{21} & \frac{L_e}{EI} + C_{22} \end{bmatrix}$$

Stiffness matrix K_C of a cracked beam element:

From the equilibrium conditions as shown in Figure.3.4

$$(V_1 \quad \Theta_1 \quad V_2 \quad \Theta_2)^T = [L](V_2 \quad \Theta_2)^T$$

Where the transformation matrix is,

$$L = \begin{bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence the stiffness matrix K_C of a cracked beam element can be obtained as

$$K_c = L C_{tot}^{-1} L^T$$

Where L is the transformation matrix of the beam for equilibrium conditions. Similarly standard procedure is carried out to find mass and geometric matrix.

In this study, we have considered the change in mass matrix and geometric matrix to negligible for cracks.

Hence the mass matrix of a cracked beam for a flexural beam is

$$Me = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$

The equation of motion for an undamped free vibration analysis of a beam is

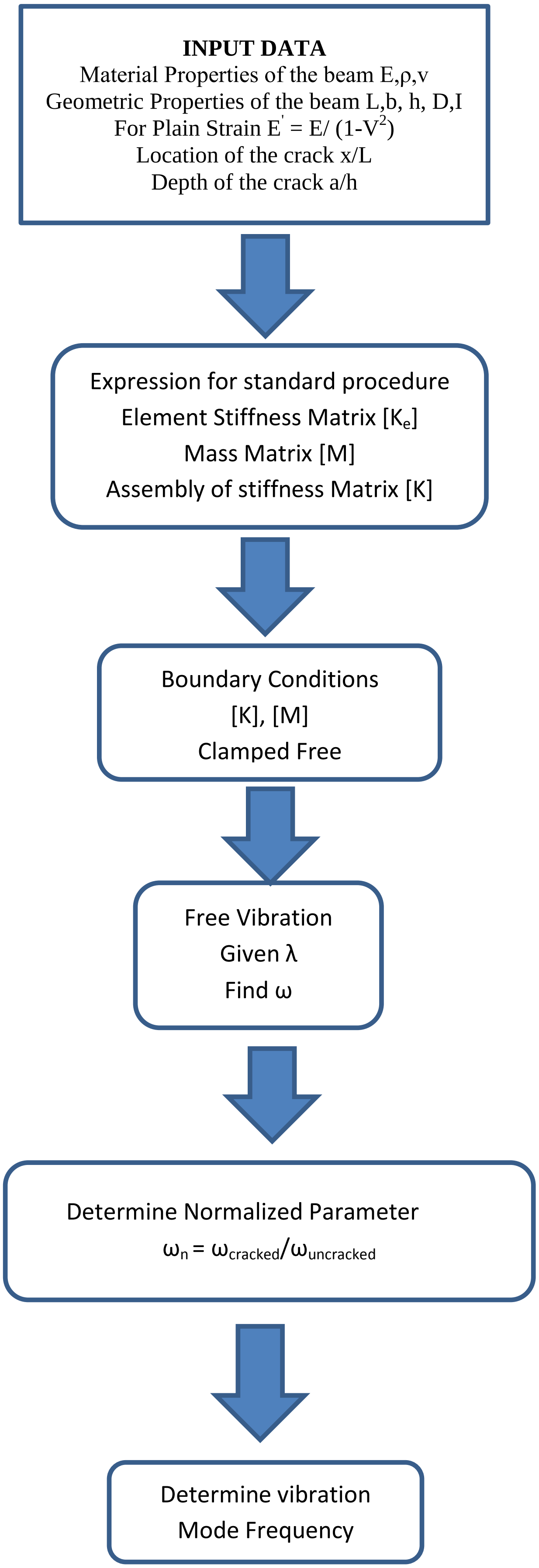
$[\mathrm{M}]\ddot{u} + [\mathrm{K}]u = 0$

The above equation reduces to find the natural frequencies of a beam as

$[K_e] - \omega^2 [M_e] = 0$

A computer program is to be developed to perform all the necessary computations in MATLAB environment. In the initialization phase, geometry and material parameters are specified. For example for a Euler–Bernoulli beam model with localized crack, material parameters like modulus of elasticity, Poisson ratio and the mass density of the beam material and geometric parameters like dimensions of the beam, also the specifications of the damage like size of the crack, location of the crack and extent of crack are supplied as input data to the computer program. The built in MATLAB function “eig” is used to calculate the Eigen values, eigenvectors.

FLOW CHART


INPUT DATA
Material Properties of the beam E,ρ,v
Geometric Properties of the beam L,b, h, D,I
For Plain Strain E' = E/ (1-V^2)
Location of the crack x/L
Depth of the crack a/h
Expression for standard procedure
Element Stiffness Matrix [K_e]
Mass Matrix [M]
Assembly of stiffness Matrix [K]
Boundary Conditions
[K], [M]
Clamped Free
Free Vibration
Given λ
Find ω
Determine Normalized Parameter
ω_n = ω_cracked/ω_uncracked
Determine vibration
Mode Frequency

CHAPTER 4
RESULTS AND DISCUSSION

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

This chapter includes free vibration analysis of the Cantilever Bernoulli-Euler beam of various cross-sections. The effects of various parameters such as natural frequencies for uniform and stepped beams with and without cracks are presented and convergence study is done. Comparisons of the natural frequencies of the beams with the pervious papers in order to understand the accuracy of present study is included. Numerical Analysis is done considering an Aluminium beam (cantilever beam) with transverse crack in order to obtain the natural frequencies of uniform beam and stepped beams with out and with multiple cracks. The results obtained by Finite Element Method (FEM) in MATLAB environment have been discussed and analysed in this chapter. At the end of this chapter a comparative result has been shown between the uniform beams of rectangular and circular cross-sections, effect of step present in beams.

This chapter contains

- Study on Convergence
- Verification with past studies
- Results of numeric.

4.2 Convergence Study

Case 1:- Uniform Cantilever Beam with Cracks of Rectangular Cross-section:

In order to check the accuracy of the present analysis the convergence of beams for corresponding cases are done respectively. Different mesh divisions are considered.

The convergence study for the cantilever uniform beam of rectangular cross-section with single crack is done with the case considered in Shiffrin(1999) in Figure4.1, it is observed that convergence starts when the number of mesh divisions is 14 and convergence up to 20 numbers of elements is shown. Hence for the present study of all uniform cantilever beams of rectangular cross-sections is done considering mesh division of 20 elements.

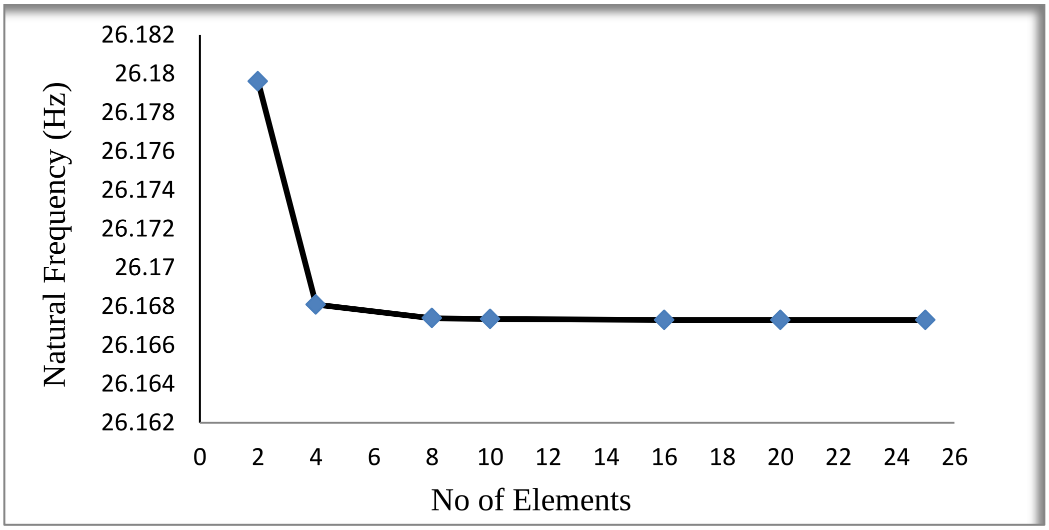


Figure 4.1: Convergence graph of uniform cantilever beam of rectangular cross-section

Case 2:- Uniform Cantilever Beam with Cracks of Circular Cross-section:

In Figures4.1 and 4.2, the convergence study for the simply supported uniform beam of circular cross-section with single crack is done with the case considered in Zheng D.Y (2004), it is observed that convergence starts when the number of mesh divisions is 16 and convergence up to 25 numbers of elements is shown. Hence for the present study of all uniform beams of circular cross-sections is done considering mesh division of 16 elements.

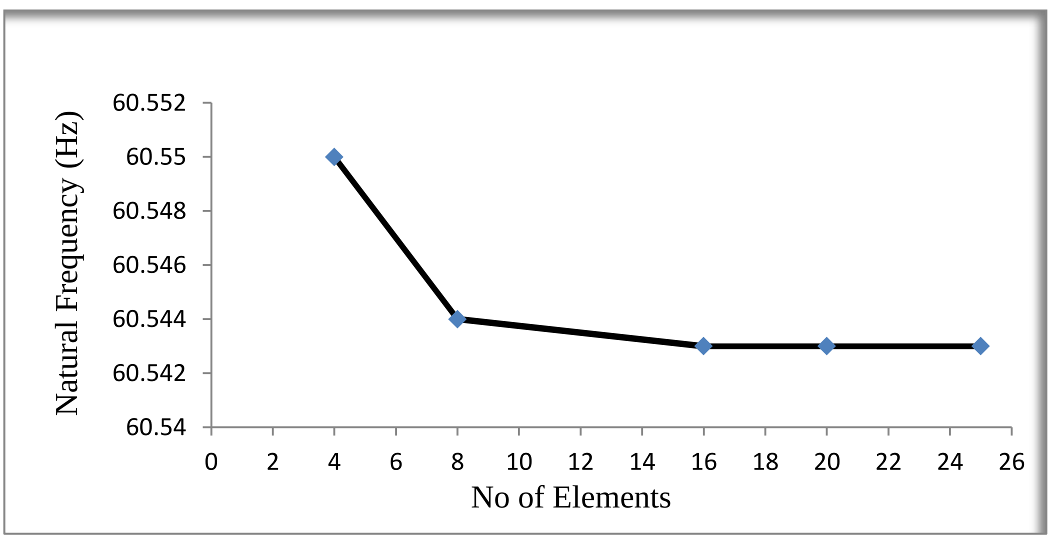


Figure 4.2: Convergence graph of uniform cantilever beam of circular cross-section

Case3:- Stepped Beam with Cracks:

From Figure4.3, The convergence study for the Free-Free stepped beam of rectangular cross-section without crack is done with the case considered in Guohui.D(2013), it is observed that convergence starts when the number of mesh divisions is 10 and convergence up to 20 numbers of elements is shown. Hence for the present study of all stepped beams of rectangular cross-sections is done considering mesh division of 18 elements.

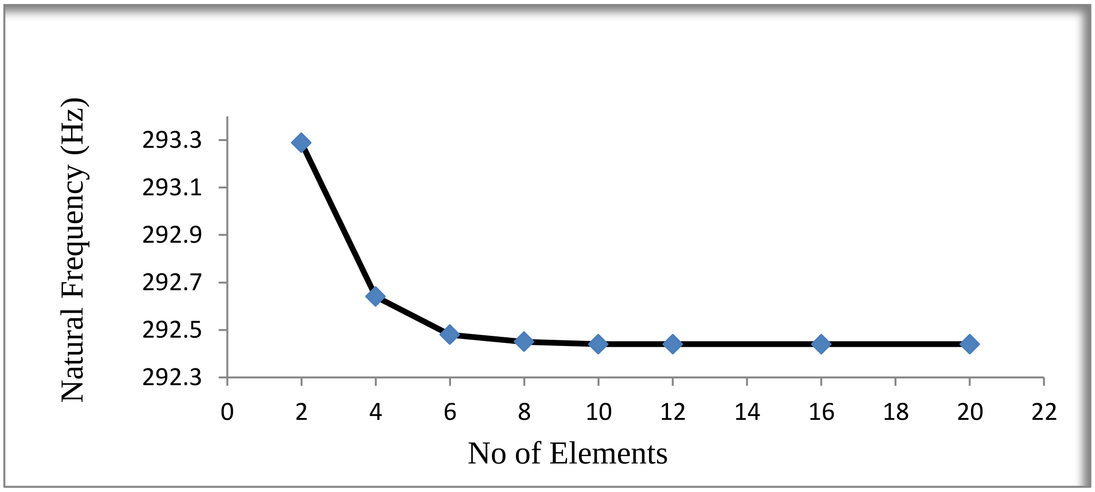


Figure 4.3: Convergence graph of stepped cantilever beam of rectangular cross-section

4.3 Comparison with Previous Studies

In order to check the accuracy of present analysis and to understand the results of the free vibration of the Bernoulli-Euler beam ,the effect of various parameters with multiple cracks are presented .The natural frequencies of the beams are compared with the pervious papers.

This includes

- Comparison of analysis of freely vibrated beam of uniform and even stepped beam of rectangular cross-sections with multiple cracks.
- Comparison of Free Vibrational analysis of uniform beams of circular cross-sections with multiple cracks.

4.3.1 Free Vibration Analysis of Cracked Uniform Cantilever Beam

Case (1):- Comparison of natural frequencies for a cantilever beam with single crack with results of Shiffrin(1999)

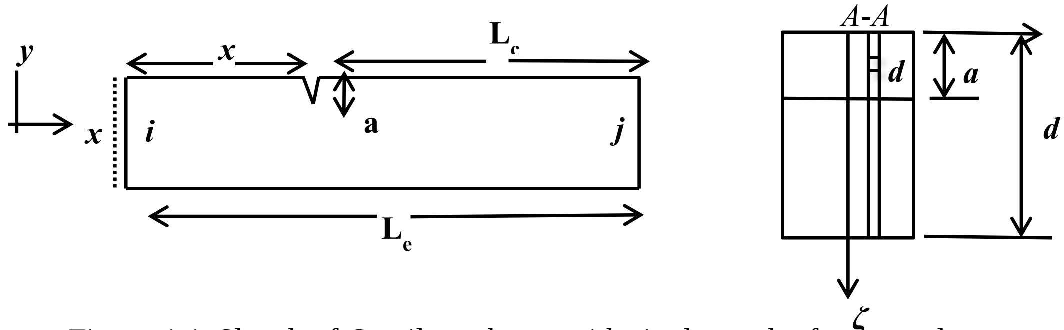


Figure 4.4: Sketch of Cantilever beam with single crack of rectangular cross-section

Table 1: Comparison of natural frequency of single cracked uniform cantilever beam with F.E.M

Elastic modulus of the beam = 210MPa**,** Poisson's Ratio = 0.3**,** Density = 7800 kg/m^3**,** Beam Width = 0.02m**,** Beam depth = 0.02 m**,** Beam length = 0.8m, Position of the crack from clamped end x_1= 0.12m, Crack depth a_1=0.002 m.

Modes	Natural frequency (Hz) Shiffrin	Present analysis using FEM	% Error
Mode 1	26.1231	26.1673	0.168
Mode 2	164.0921	164.1292	0.022
Mode 3	459. 6028	459.620	0.004

Case (2):- Comparison of natural frequencies for a cantilever beam with double crack with results of Shiffrin(1999)

Table 2: Comparison of natural frequency of doubled cracked uniform cantilever beam with F.E.M

Position and crack depth of first crack: a_1=0.002m; x_1=0.12 m

Position and crack depth of second crack: a_2=0.003m; x_2=0.4m

Mode Number	Natural frequency (Hz)Shiffrin	Present analysis using FEM	% Error
1	26.0954	26.1539	0.223
2	164.3221	164.7585	0.266
3	459.6011	459.6173	0.003

From Table 1 and Table 2 it is, observed that natural frequencies of Shiffrin(1999) agrees with the present MATLAB analysis using FEM formulation in case of both single and double cracks.

Case (3):- Comparison of uniform cantilever beam of square cross-section with multiple cracks with results of Mostafa Attar(2012)

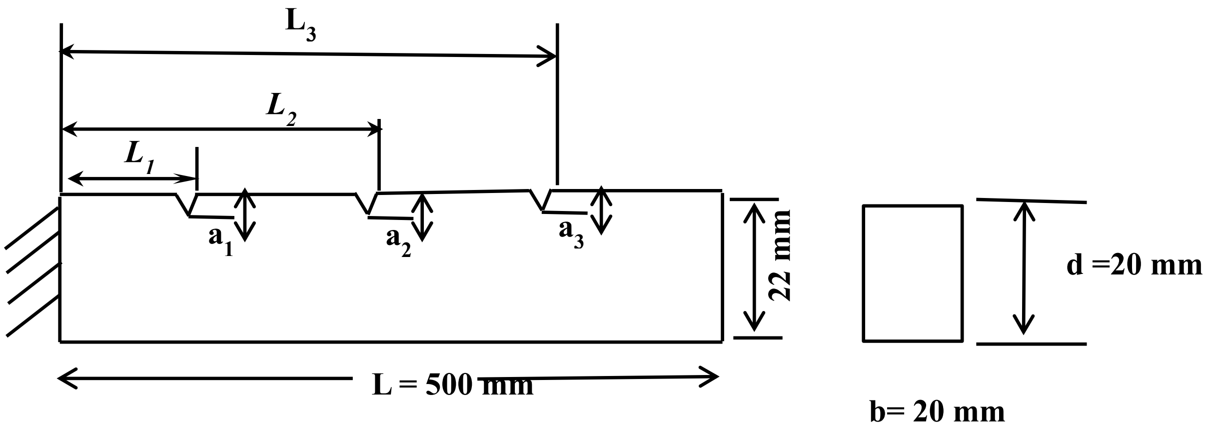


Figure 4.5: Sketch of Triple cracked uniform rectangular beam

Table 3: Comparison of Natural frequency of triple cracked cantilever beam with TMM and FEM

Elastic modulus of the beam = 210MPa, Poisson's Ratio = 0.3, Density = 7860 kg/m3, Beam width = 0.02m, Beam depth = 0.02 m, Beam length = 0.5m

Case	Crack Location			Methods	Natural Frequencies ω_s (rad/s)					
	x_1	x_2	x_3		ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
1	0.2	0.4	0.6	Present study	418.8631	2624.823	7352.854	14411.38	23789.01	35655.29
				FEM*[19]	416.8933	2612.065	7323.879	14356.68	23589.91	35603.94
				% Error FEM	0.470	0.486	0.394	0.379	0.836	0.144
				TMM*[19]	416.9159	2612.213	7324.210	14357.28	23592.02	35604.06
				% Error TMM	0.464	0.480	0.389	0.375	0.828	0.143
2	0.2	0.4	0.8	Present study	418.9152	2627.313	7351.134	14394.25	23790.05	35646.15
				FEM*	417.0652	2620.375	7318.436	14299.97	23600.29	35573.62
				% Error FEM	0.441	0.264	0.444	0.654	0.797	0.203
				TMM*	417.0864	2620.455	7318.811	14301.02	23602.31	35574.00
				% Error TMM	0.436	0.261	0.439	0.647	0.789	0.202
3	0.2	0.6	0.8	Present study	419.0779	2626.284	7350.398	14392.93	23799.17	35654.07
				FEM*	417.6291	2617.683	7315.436	14300.48	23601.47	35573.74
				% Error FEM	0.345	0.327	0.475	0.642	0.830	0.225
				TMM*	417.6464	2617.786	7315.833	14301.53	23603.48	35574.12
				% Error TMM	0.341	0.323	0.470	0.635	0.822	0.224
4	0.4	0.6	0.8	Present study	419.4202	2624.264	7348.95	14405.69	23784.43	35653.36
				FEM*	418.7431	2610.199	7311.806	14337.70	23575.09	35597.99
				% Error FEM	0.161	0.535	0.505	0.471	0.880	0.155
				TMM*	418.7517	2610.361	7311.243	14338.46	23577.32	35598.16
				% Error TMM	0.159	0.529	0.513	0.466	0.870	0.154

In Table 3 it is observed that natural frequencies of Mostafa Attar (2012) agrees with the present MATLAB analysis using FEM formulation for all modes but in mode 5 we observe more percentage error in case of both TMM (Transfer Matrix Method) and FEM (Finite Element Method) compared to other modes.

4.3.2 Free Vibration Analysis of Uncracked Stepped Beams

In this solution it associates the computation of frequencies occurred naturally for Uncracked Bernoulli-Euler beam of cantilever type. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained by using methods Discrete Singular convolution (DSC), Differential Quadrature Element method (DQEM), Finite Element Method (FEM), Composite Element Method (CEM) given by Guohui D and Xinwei W(2013).

Case (1):- Comparison of Natural Frequencies of a Single Stepped Free-Free beam

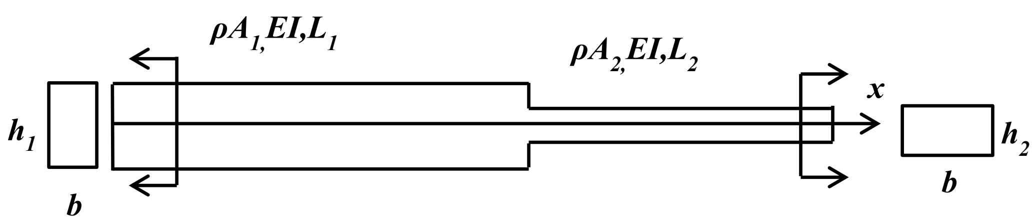


Figure 4.6: Sketch of Single stepped beam Free-free beam

Table 4: Comparison of the natural frequencies of the single stepped F-F beam.

Elastic modulus = 71.7GPa, Density = 2830 kg/m3, Width b = 20mm, Depth of the beam h_1= 19.05 mm, Depth of the stepped beam h_2= 5.49 mm, Length of the beam = 254 mm, Length of the stepped beam = 140 mm

Mode	DSC Discrete Singular convolution (Hz)	% Error DSC	DQEM Differential Quadrature Element method (Hz)	% Error DQEM	FEM Finite Element Method (Hz)	% Error FEM	CEM Composite Element Method (Hz)	% Error CEM	Present analysis using MATLAB (Hz)
1	292.3	0.047	292.44	0	292.44	0	291.44	0.341	292.44
2	1179.3	0.176	1181.3	0.006	1181.3	0.006	1176.2	0.438	1181.38
3	1800.1	0.232	1804.1	0.011	1804.1	0.011	1795.7	0.476	1804.30

From Table 4 it is observed that natural frequencies of Guohui D and Xinwei W(2013) agrees with the present MATLAB analysis using FEM formulation with less percentage error in case of FEM and DQEM but in case of DSC and CEM the percentage error is quite high.

Case (2):- Comparison of Natural Frequencies of a Three Stepped Simply Supported Beam

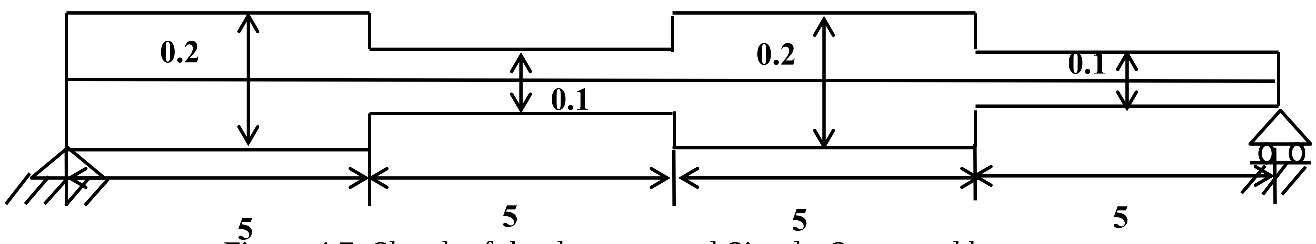


Figure 4.7: Sketch of the three stepped Simply-Supported beam

Table 5: Comparison of natural frequencies of a three stepped simply supported beam.

Elastic modulus =34GPa**,** Density = 2830 kg/m^3**,** Width b = 0.2 m**,** Depth of the beam h_1= 0.2 m**,** Depth of the stepped beam h_2= 0.1 m**,** Total length of the beam =20 m

Mode	DSC		DQEM	FEM	% Error for DSC, DQEM, FEM	CEM	% Error CEM	Present analysis using FEM
	6^{th} order	10^{th} order						
1	0.43369	0.43369	0.43369	0.43369	0	0.433	0.159	0.43369
2	1.80276	1.80276	1.80276	1.80276	0.021	1.799	0.229	1.80314
3	4.41471	4.41470	4.41470	4.41470	0.116	4.411	0.200	4.41985
4	9.54118	9.54133	9.54133	9.54133	0.648	9.522	0.849	9.6036
5	13.26588	13.26609	13.26609	13.26609	1.051	13.246	1.201	13.4071
6	19.35805	19.35885	19.35885	19.35885	2.651	19.301	2.942	19.8861
7	25.76000	25.76031	25.76032	25.76032	3.127	25.721	3.275	26.592
8	34.99914	35.00414	35.00420	35.00420	10.978	34.959	11.093	39.321
9	43.21409	43.21867	43.21882	43.21882	13.049	43.174	13.139	49.705
10	55.63908	55.66238	55.66242	55.66242	14.319	55.473	14.610	64.965

From Table 5 it is observed that natural frequencies of Guohui D and Xinwei W(2013) agrees with the present MATLAB analysis using FEM formulation with less percentage error in case of FEM, DQEM and DSC but in case CEM the percentage error is quite high.

Case (3):- Comparison of Natural Frequencies of Twelve Stepped Cantilever Beam

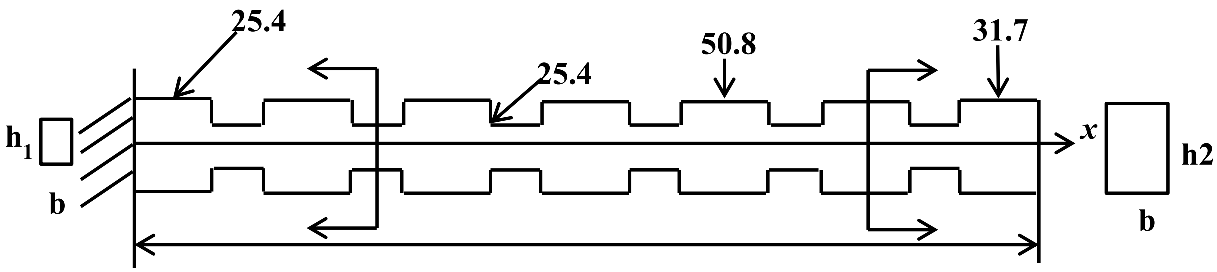


Figure 4.8: Sketch of a twelve stepped Cantilever beam

Table 6: Comparison of natural frequencies of a twelve stepped cantilever beam

Elastic modulus = 60.6GPa, Density = 2664 kg/m^3, Width b = 3.175 mm, Depth of the beam h_1= 12.7 mm, Depth of the stepped beam h_2= 25.4 mm, Total length of the beam = 463.55 mm

DSC	DQEM	CEM	% Error DSC, DQEM	% Error CEM	FEM	Experiment	% Error Experiment	Present analysis using FEM
1	54.496	54.695	0.0009	0.002	54.795	54.985	0.004	54.496
2	344.793	344.808	0.0052	0.0008	-	344.807	0.001	344.811
4	977.740	977.812	0.0170	0.009	-	977.809	0.009	977.906
5	1951.199	1951.409	0.0482	0.037	-	1951.398	0.038	1952.14
10	3301.141	3301.639	0.1098	0.094	-	3301.606	0.095	3304.77

From Table 6 it is observed that natural frequencies of Guohui D and Xinwei W(2013) agrees with the present MATLAB analysis using FEM formulation with less percentage error in case of FEM, DQEM , DSC and CEM.

4.3.3 Free Vibration Analysis of Cracked Stepped Beams of Rectangular Cross-Section

In this solution it associates the computation of frequencies occurred naturally for cracked Bernoulli-Euler beam of cantilever type. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained by Ameneh M (2012) using a novel local flexibility-based damage index method.

Case (1):- Comparison of Single Cracked Two Step Cantilever Beam

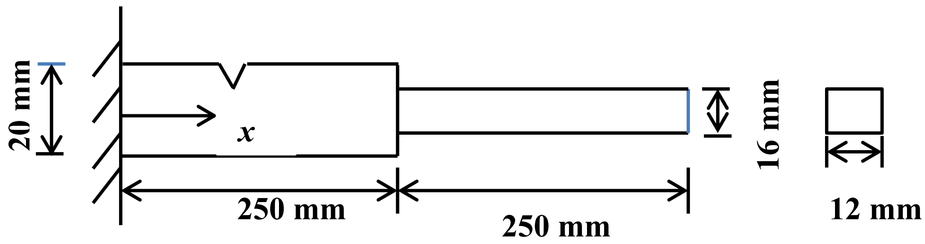


Figure 4.9: Sketch of Single Cracked Two Step Cantilever Beam

Table 7: Comparison of Single Cracked Two Step Cantilever Beam with Novel local Flexibility-based damage index method

Elastic modulus = 210MPa, Density = 7800 kg/m^3, Poisson's ratio =0.3, Width b = 12 mm, Depth of the beam h_1= 20 mm, Depth of the stepped beam h_2= 16 mm, Total length of the beam = 500mm

Case No	Location of Crack	Crack Depth	Natural Frequencies (Hz)			
			ω_1	ω_2	ω_3	ω_4
1	0.05	0.2	71.492	372.24	1041.6	2007.8
Reference[1]			*70.514	366.64	1024.5	1950.4
% Error			1.367	1.504	1.641	2.858
2	0.1	0.5	60.759	343.55	1033.3	2026.8
Reference			*61.379	353.62	1024.1	1925.2
% Error			1.010	2.847	0.890	5.012
3	0.2	0.4	66.456	375.59	1033.0	1921.8
Reference			* 67.529	371.85	1019.2	1875.0
% Error			1.588	0.995	1.335	2.435
4	0.3	0.3	71.185	375.96	1030.1	2003.5
Reference			*70.652	371.94	1005.3	1932.4
% Error			0.748	1.069	2.407	3.548
5	0.4	0.5	63.738	339.40	926.24	2017.6
Reference			*68.711	355.75	968.21	1955.2
% Error			7.237	4.595	4.334	3.092
6	0.45	0.1	72.477	376.53	1053.8	2027.4
Reference			*72.334	372.12	1034.6	1959.9
% Error			0.197	1.171	1.821	3.329
7	0.55	0.1	72.4827	376.33	1054.2	2026.0
Reference			*72.385	371.55	1036.5	1957.0
% Error			0.134	1.270	1.678	3.405
8	0.6	0.3	72.183	367.23	1047.8	1167..9
Reference			*71.996	360.21	1027.7	1039.3
% Error			0.259	1.911	1.918	11.011
9	0.7	0.2	72.472	375.11	1047.3	2025.
Reference			*72.370	369.55	1022.6	1957.5
% Error			0.140	1.482	2.358	3.333
10	0.8	0.4	72.4198	369.88	992.57	1880.9

			*72.383	367.76	989.29	1849.4
% Error			0.050	0.573	0.330	1.674
11	0.2	0.06	72.478	376.8	1054.1	2027.7
Reference			*72.310	372.94	1035.5	1960.2
% Error			0.231	1.024	1.764	3.328
12	0.65	0.085	72.498	376.56	1053.7	2028.2
Reference			*72.426	372.24	1034.4	1963.5
% Error			0.099	1.147	1.831	3.190

In Table 7, the first six cases where crack is present in the first half of the beam and the second six cases are where crack is present in step of the beam we observe that natural frequencies of Ameneh M (2012) agrees with the present MATLAB analysis using FEM formulation. Case 5 and Case 8 show quite high percentage error.

Case (2):- Comparison of Double Cracked Two Step Cantilever Beam

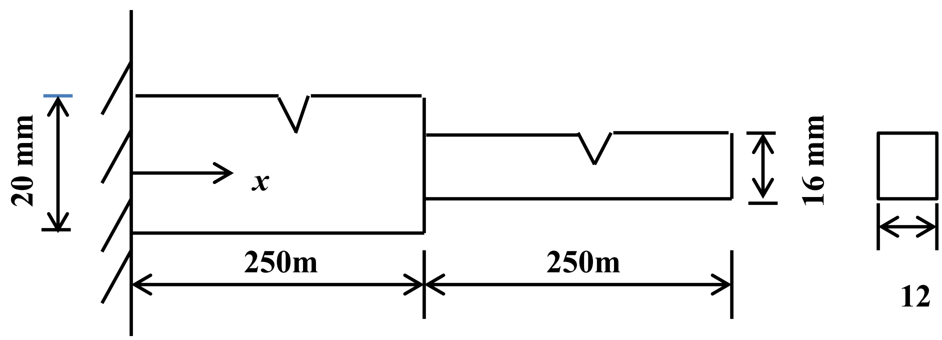


Figure 4.10: Sketch of Double Cracked Two Step Cantilever Beam

Table 8: Comparison of Double Cracked Two Step Cantilever Beam with Novel local Flexibility-based damage index method

Case No	Simulated data				Natural frequencies (Hz)			
	Crack No.1		Crack No.2					
	β	γ	β	γ	ω_1	ω_2	ω_3	ω_4
1	0.1	0.2	0.4	0.2	71.489	373.966	1047.92	1027.55
Ref[1]					* 70.349	367.24	1023.9	1962.0
% Error					1.594	1.798	2.292	3.232
2	0.25	0.1	0.35	0.3	71.385	374.457	1029.71	2021.28
Ref					* 70.762	369.96	1003.5	1950.9
% Error					0.872	1.200	2.545	3.481
3	0.45	0.3	0.2	0.2	69.973	370.669	1038.89	1971.72
Ref					* 70.474	365.46	1021.4	1919.3
% Error					0.710	1.405	1.683	2.658
4	0.55	0.3	0.7	0.4	71.486	339.83	931.50	1858.92
Ref					* 70.918	336.9	980.7	1835.5
% Error					0.795	0.862	5.016	1.259
5	0.6	0.5	0.8	0.4	68.314	286.64	925.93	1623.97
Ref					* 70.833	329.65	972.81	1750.5
% Error					3.556	13.047	4.819	7.228
6	0.75	0.4	0.6	0.3	71.99	355.30	973.71	1896.36
Ref					* 72.024	355.39	974.8	1863.6
% Error					0.047	0.025	0.111	1.727
7	0.1	0.2	0.7	0.3	71.598	369.139	1029.146	2018.126
Ref					* 70.598	361.66	1002.9	1949.5
% Error					1.396	2.026	2.550	3.400
8	0.3	0.2	0.6	0.5	68.28	295.86	1000.207	1890.88
Ref					* 70.148	333.62	995.9	1880.2
% Error					2.662	11.318	0.430	0.564

From Table 8 it is observed that natural frequencies of Ameneh.M (2012) agree with the present MATLAB analysis using FEM formulation in case of double cracks. In Case 5 and Case 6 Show quite high percentage error for Mode 2.

Case (3):- Comparison of Triple Cracked Two Step Cantilever Beam

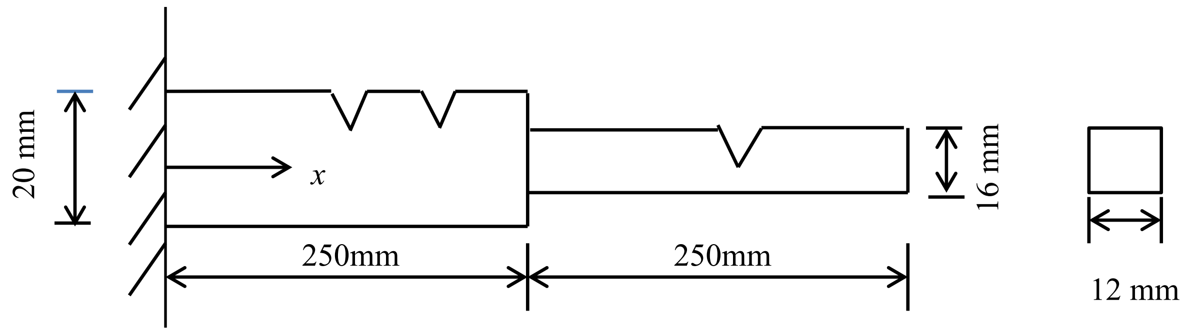


Figure 4.11: Sketch of Triple Cracked Two Step Cantilever Beam

Table 9: Comparison of Triple Cracked Two Step Cantilever Beam with Novel local flexibility-based damage index method.

Case no.	Crack no.1		Crack no.2		Crack no.3		Natural Frequencies (Hz)		Natural Frequencies (Hz)	
	Crack Location	Crack Depth	Crack Location	Crack Depth	Crack Location	Crack Depth	ω_1	ω_2	ω_3	ω_4
1	0.15	0.3	0.3	0.45	0.7	0.35	63.272	359.59	904.08	1898.5
Ref							*65.272	356.98	927.95	1858.2
%Error							3.064	0.725	2.572	2.122
2	0.1	0.15	0.35	0.25	0.6	0.35	71.014	358.22	1028.6	1991.9
Ref							*70.025	352.22	1011.1	1926.7
%Error							1.392	1.674	1.701	3.273

From Table 9, it is observed that natural frequencies of Ameneh M,(2012) agrees with the present MATLAB analysis using FEM formulation in case of both triple cracks. In case 2 we observe quite high percentage error for 6^{th} mode.

4.3.4 Free Vibration Analysis of Uniform Simply Supported Shaft

Case (1):- Comparison of Natural Frequencies of Fixed-Free Circular beam without crack

The problem involves calculation of natural frequencies for un-cracked Bernoulli-Euler Cantilever beam. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained by Zheng D. Y (2004) using Finite Element Method using Gauss quadrature.

Elastic modulus = 206 GPa, Density = 7800 kg/m^3, Poisson's ratio =0.3, Diameter of the beam D = 0.03 m, Total length of the beam L= 1.0 m

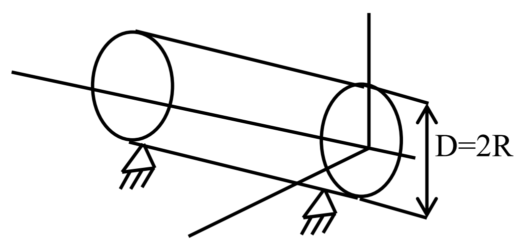


Figure 4.12: Sketch of Uniform Simply supported shaft

Table 10: Comparison of natural frequencies of uniform simply supported shaft

Mode	Natural Frequency (Hz) Present Analysis	Natural Frequency (Hz) (D.Y Zheng)	%Error
Mode1	60.539	60.543	0.006
Mode2	242.204	242.177	0.011
Mode3	544.941	544.936	0.0009

4.3.5 Analysis of freely vibrated Cracked Beams of uniform with Circular Cross-Section

Case (1):- Free Vibration Analysis of Uniform Simply supported shaft with single crack

In this solution it associates the computation of frequencies occurred naturally for cracked Bernoulli-Euler beam of cantilever type. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained by Zheng D. Y (2004) using Finite Element Method using Gauss quadrature.

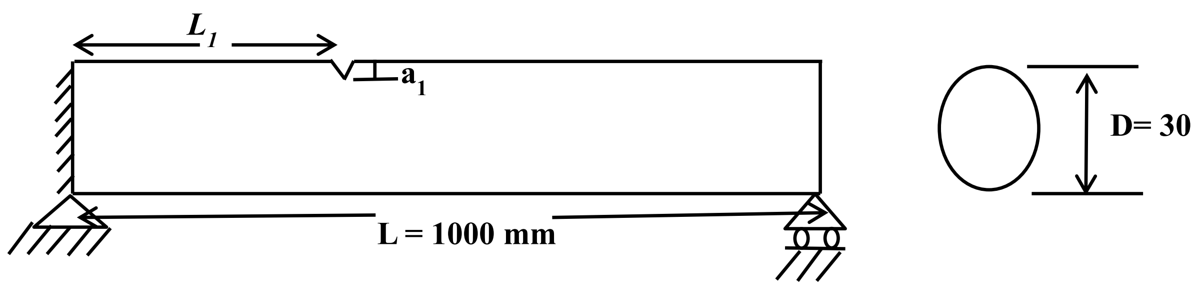


Figure 4.13: Sketch of Single cracked uniform Simply-Supported beam of circular cross-section

Table 11: Comparison of natural frequencies of single cracked uniform cantilever beam of circular cross-section

Mode	Natural Frequency (Hz) Present Analysis	Natural Frequency (Hz) (D.Y Zheng)	%Error
Mode1	56.008	55.92	0.157
Mode2	242.16	242.18	0.008
Mode3	506.90	506.85	0.009

Table 11 shows the percentage error graph, we observe that natural frequencies of Zheng D.Y(2004) agrees with the present MATLAB analysis using FEM formulation in case of both without and with crack.

Case (2):- Single Cracked Uniform Beam of Circular cross-section

In this solution it associates the computation of frequencies occurred naturally for cracked Bernoulli-Euler beam of cantilever type. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained using numerical model by Kisa.M (2006) which adds with finite elemental and structure synthesis mode procedure for analysis of beams with cross section of circular.

Material Parameters:

Elastic modulus = 216 GPa

Density = 7850 kg/m^3

Poisson ratio =0.33

Geometric Parameters:

- Total length of the beam = 2.0 m
- Here three different diameters are considered

1. R/L=0.1 (D=0.2L)
2. R/L=0.06 (D=0.12L)
3. R/L=0.04 (D=0.16L)

- For all the above three cases the crack is located at $L_1/L=0.2$

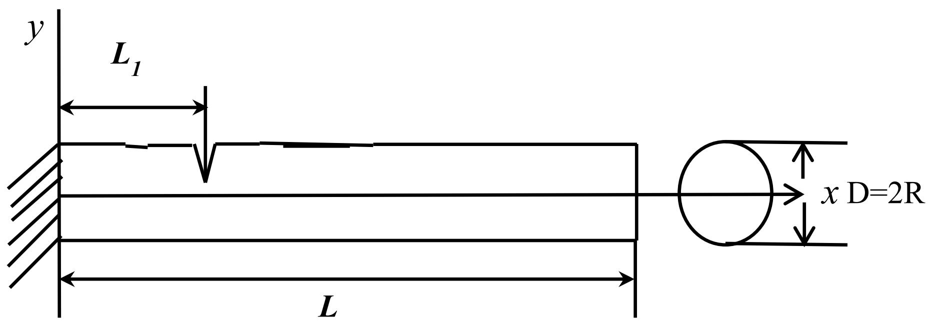
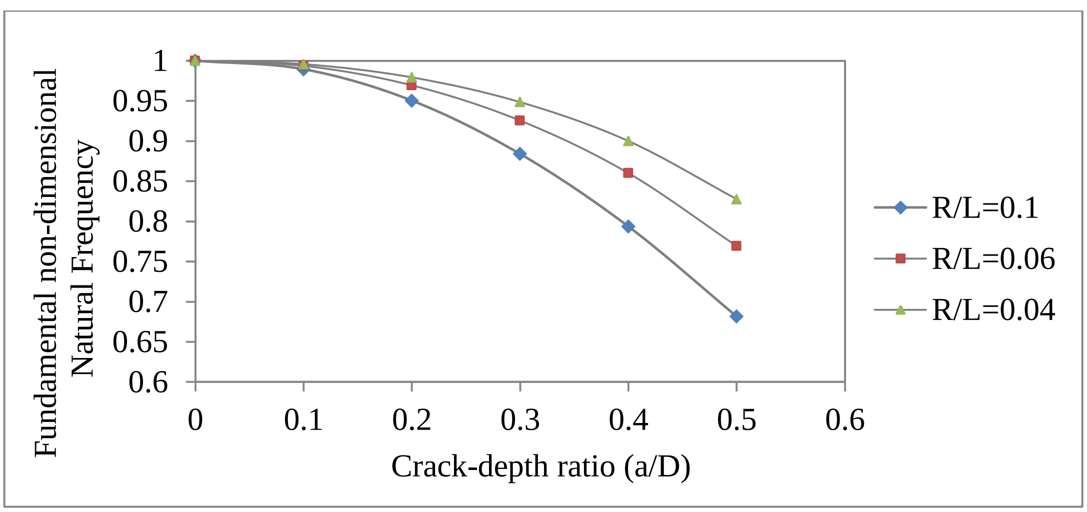


Figure 4.14: Sketch of Single cracked cantilever beam of circular cross-section



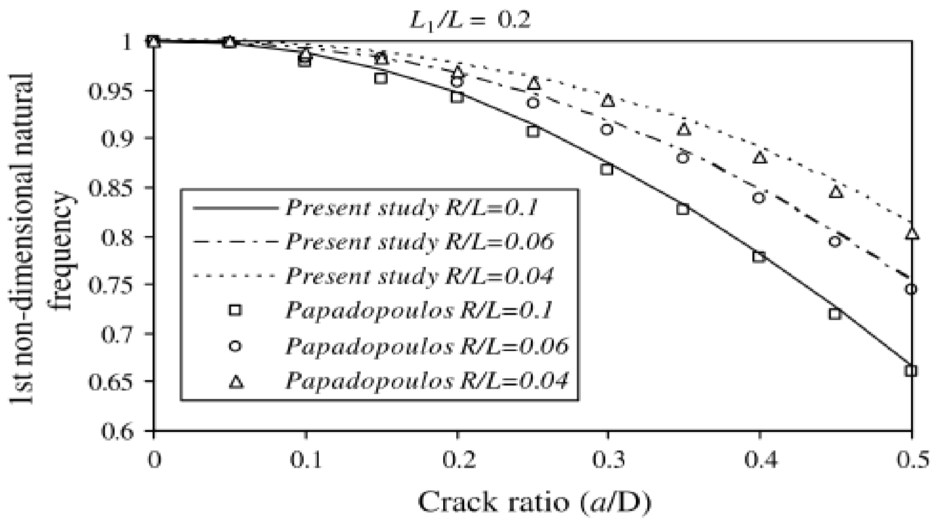


Figure 4.15: Comparison of fundamental Normalized natural frequencies of Single cracked beam of circular cross-section with Component mode Synthesis Method

Case (3):- Multi-cracked Uniform Circular Cantilever Beam

Material Parameters:

Elastic modulus = 216 GPa

Density = 7850 kg/m^3

Poisson ratio =0.33

Geometric Parameters:

Total length of the beam = 2.0 m

R/L ratio = 0.04 (D=0.08L)

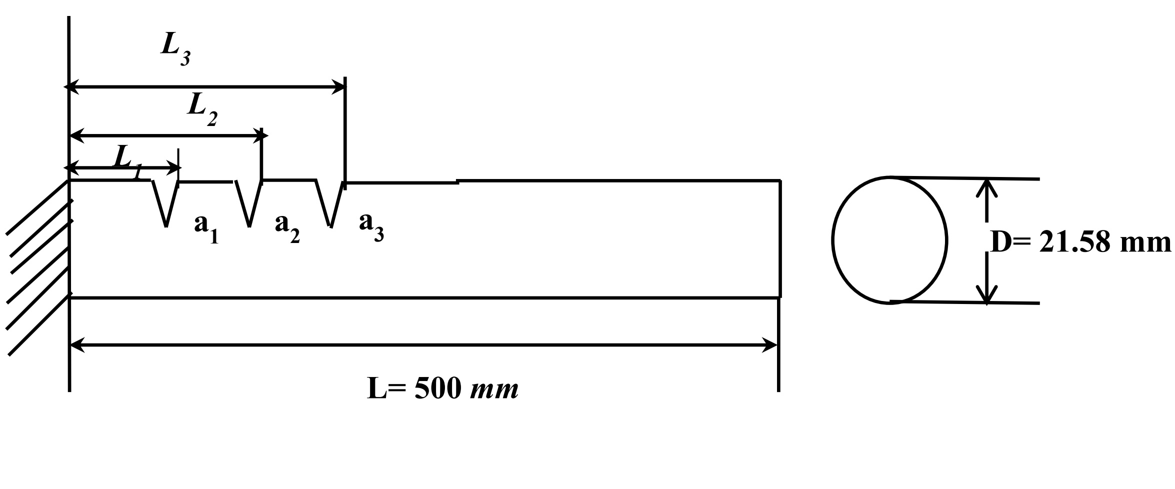


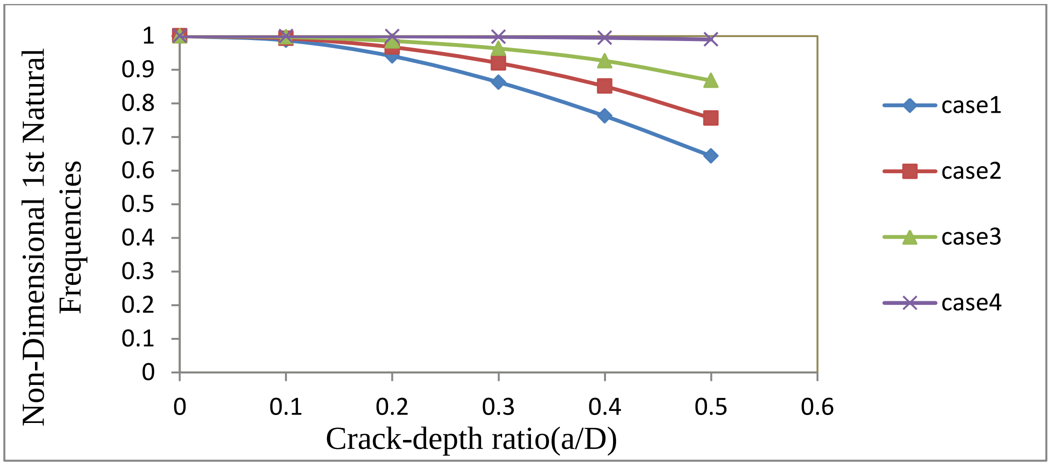
Figure 4.16: Sketch of multi-cracked beam of circular cross-section

Case 1 = $L_1/L=0.1$, $L_2/L=0.2$, $L_3/L=0.3$

Case 2 = $L_1/L=0.1$, $L_2/L=0.5$, $L_3/L=0.9$

Case 3 = $L_1/L=0.4$, $L_2/L=0.7$, $L_3/L=0.6$

Case 4 = $L_1/L=0.7$, $L_2/L=0.8$, $L_3/L=0.9$



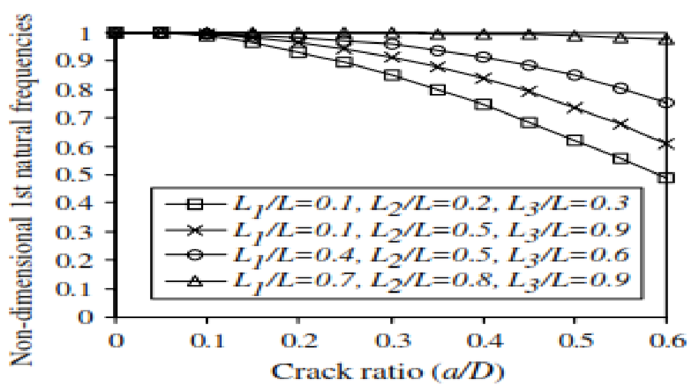
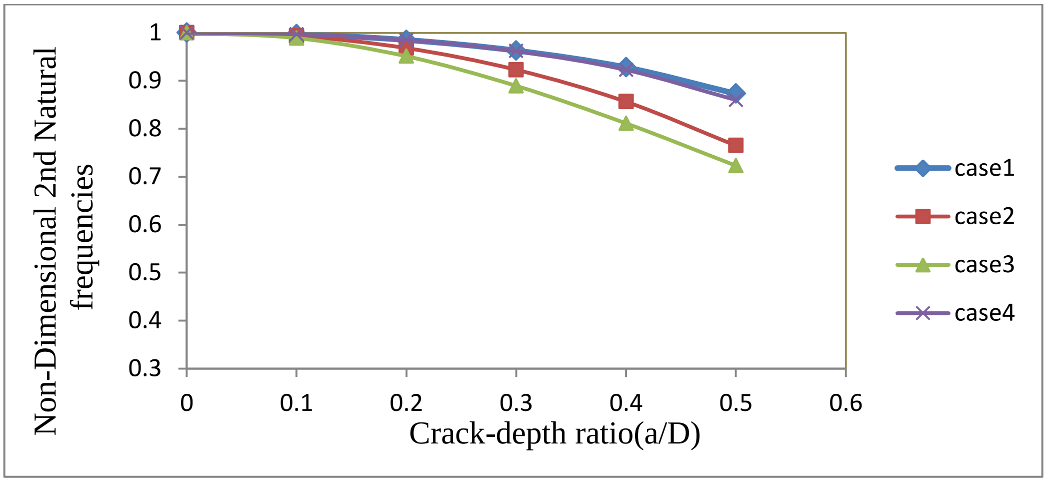


Figure 4.17: Comparison of fundamental non- dimensional natural frequencies of Triple cracked beam of circular cross-section with Component mode Synthesis Method



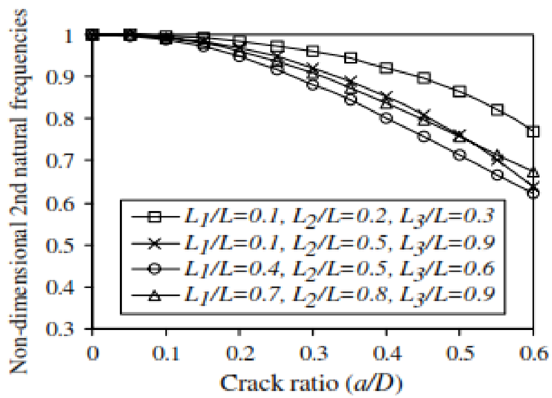
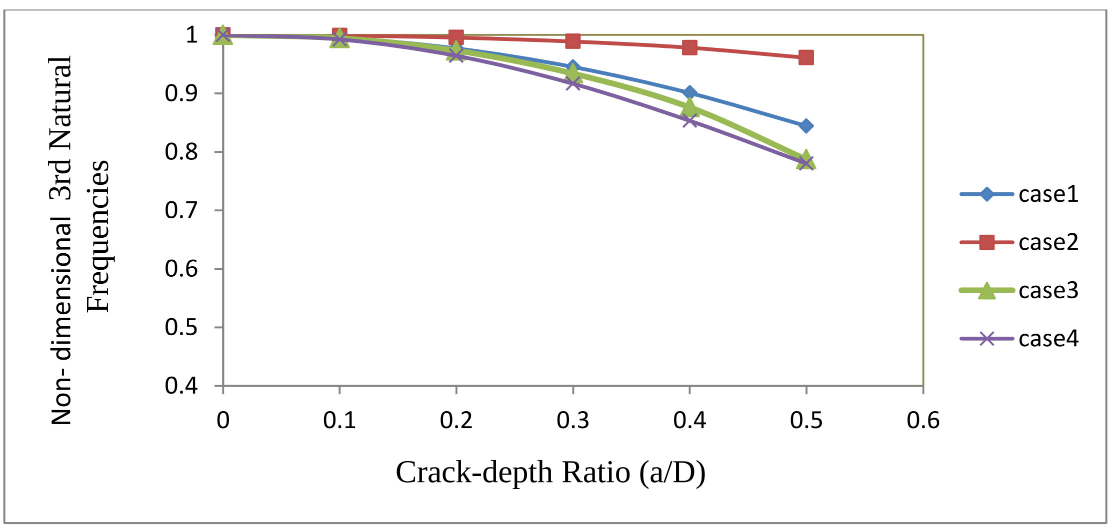


Figure 4.18: Comparison of 2nd non- dimensional natural frequencies of Triple cracked beam of circular cross-section with Component mode Synthesis Method



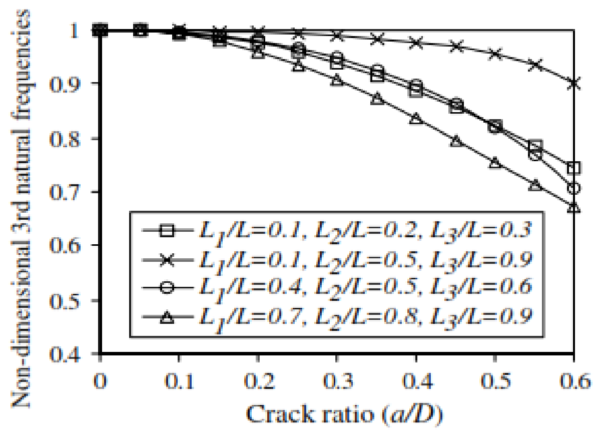


Figure 4.19: Comparison of 3rd non- dimensional natural frequencies of Triple cracked beam of circular cross-section with Component mode Synthesis Method

From Figures 4.17-4.19 it is observed that irrespective of single and multiple cracks of cantilever beam of circular cross-section the comparison of non- dimensional natural frequencies of present analysis using FEM agrees with Kisa.M (2006) which used FEM and Component mode Synthesis Method.

4.4 Numerical Results

Analysis of vibration subjected freely of the Euler-Bernoulli beam of multiple fractures considering the effect by various parameters such as crack location, crack depth ratio, numbers of cracks are presented. The method described has been used to analyse uniform and stepped beams considering Aluminum as the material property of the beam. The Normalized frequencies are found as ratio of frequency occurred naturally for a fractured beam / frequency occurred naturally of the un-fractured beam. The results are obtained by implementing the methodology given in Chapter 3 using Finite Element Method (FEM) in the MATLAB environment.

Following types of beams have been considered for the analysis

Material Properties:

Elastic modulus of the beam = 70 GPa

Poisson’s Ratio = 0.35

Density = 2700 kg/m

Uniform Beam with Multiple Cracks

- Uniform beam of rectangular and circular cross-sections with multiple cracks.

Stepped Beam with Multiple Cracks

- Uniform beam, Single step beam and Two step beam with multiple cracks of rectangular cross-sections.

  The results are analysed in the following manner

- Comparison between uniform beam of rectangular and circular cross-sections without crack and single crack and multiple cracks at respective locations.
- Effect of single step and two steps present in beams of rectangular cross-section without crack and with single crack are compared.
- Variation of frequencies with respect to single, double, multiple cracks in two step cantilever beam.

4.4.1 Uniform Beam with Multiple Cracks

Case (1):- Comparison between Uniform Beam of Rectangular and Circular cross-sections without Crack

Dimensions of the rectangular beam:

Beam width = 0.12m

Beam depth = 0.22 m

Beam length = 0.5 m

Figure 4.20: Sketch of uniform cantilever beam of rectangular cross-section

Dimensions of the circular beam:

Diameter of the beam D = 0.02158 m

Length of the beam = 0.5 m

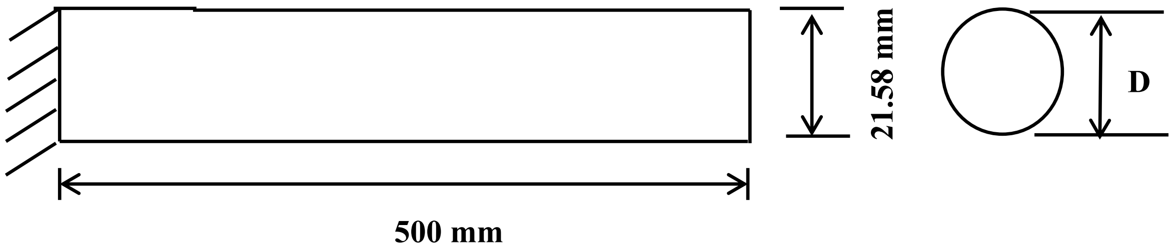


Figure 4.21: Sketch of uniform cantilever beam of circular cross-section

Mode	Natural Frequency of Rectangular beam (Hz)	Natural Frequency of Circular beam (Hz)	% Error
1	59.221	61.489	3.688
2	371.135	385.338	3.685
3	1039.196	1078.964	3.685
4	2036.448	2114.38	3.685

Table 12: Comparison of natural frequencies of uniform beam of rectangular cross-section with circular cross-section

Moment of inertia of the rectangular beam and circular beam is considered equal and comparison is done. From Table 12, it is observed that the % error is almost same for all the modes for uniform rectangular beam and uniform circular beam.

Case (2):- Comparison between Uniform Beam of Rectangular and Circular cross-sections with Single Crack

Case (a):- Location of Single Crack

- Case R1 = $L_1/L=0.1$
- Case R2 = $L_1/L=0.5$
- Case R3 = $L_1/L=0.85$

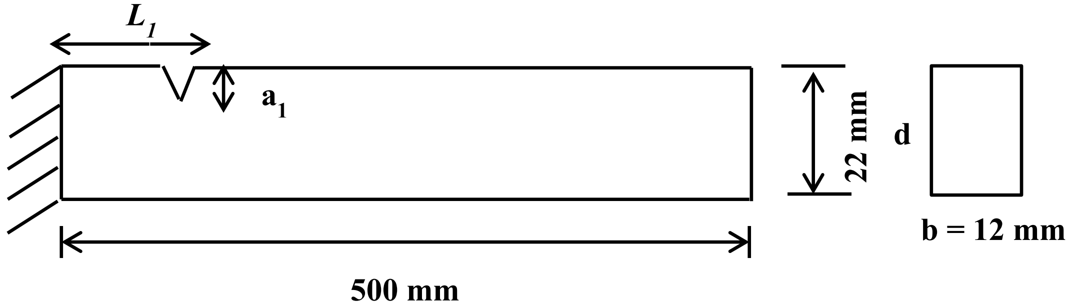


Figure 4.22: Sketch of uniform cantilever beam with single crack of rectangular cross-section

Case (b):- Location of Single Crack

- Case C1 = $L_1/L=0.1$
- Case C2 = $L_1/L=0.5$
- Case C3 = $L_1/L=0.85$

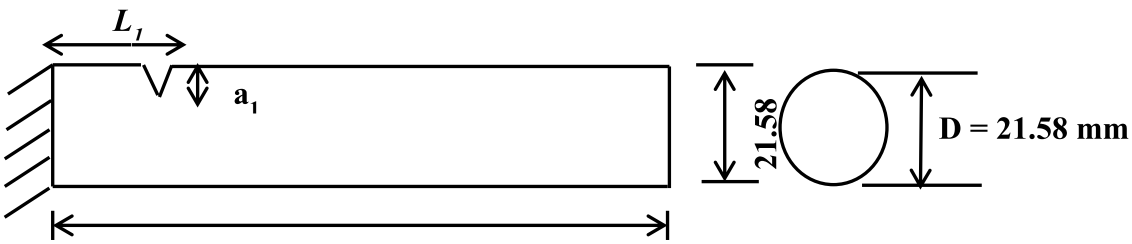


Figure 4.23: Sketch of uniform cantilever beam with single crack of circular cross-section

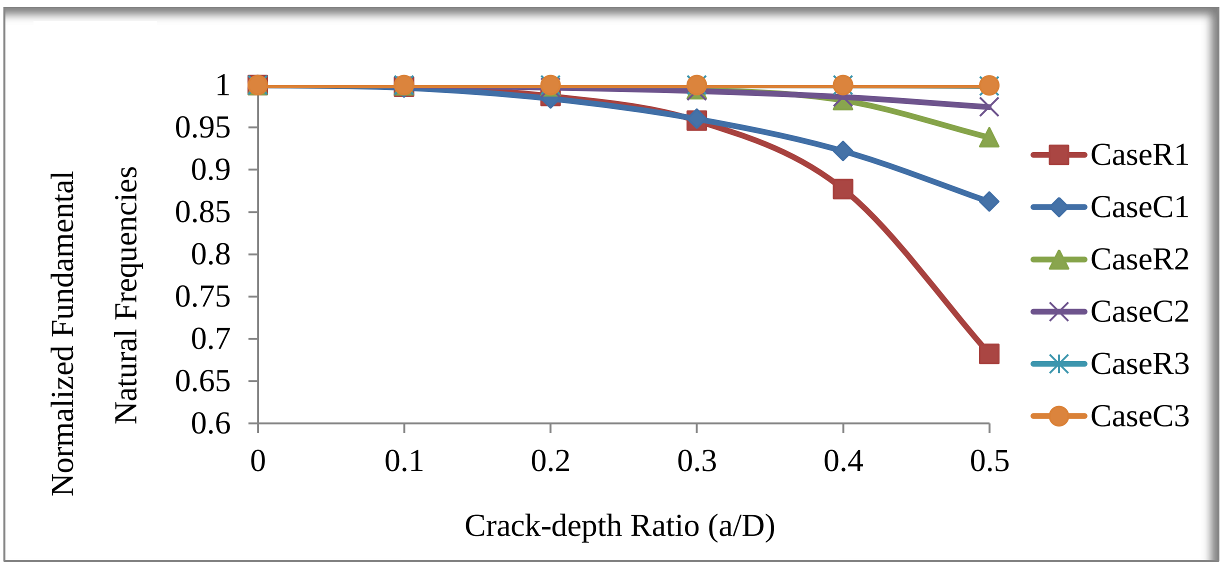


Figure 4.24: Comparison of Normalized fundamental natural frequencies of single cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

From Figure4.24, it is observed that in case of rectangular cross-section the normalized fundamental natural frequencies reduction is high than that of circular cross-section for all the cases respectively. It also shows that when crack is located near the fixed end of the beam the fundamental natural frequencies reduction is higher and when crack is positioned at free end of beam the fundamental natural frequencies are generally unharmed although when the depth of crack is quite more When crack-depth ratio is 0.5 the natural frequency reduction of rectangular beam is 25% more than circular beam when crack is located near to fixed end of the beam.

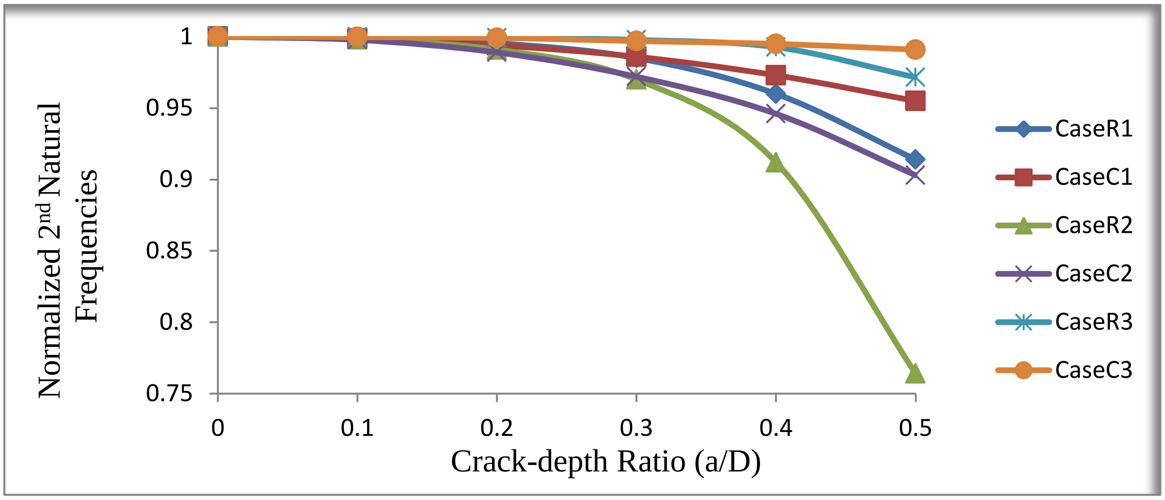


Figure 4.25: Comparison of normalized 2nd natural frequencies of single cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

In Figure4.25 it is observed that in case of rectangular cross-section the normalized 2nd natural frequencies difference is high than that of circular cross-section for all the cases respectively. The Figure shows when crack is located at center of the beam the 2nd natural frequencies reduction is higher and when crack-depth ratio is 0.5 the frequency reduction for rectangular beam in more by 15% than circular beam. It is also observed that when crack is located near free end of the circular beam the 2nd natural frequencies are generally unharmed although when the depth of crack is quite more.

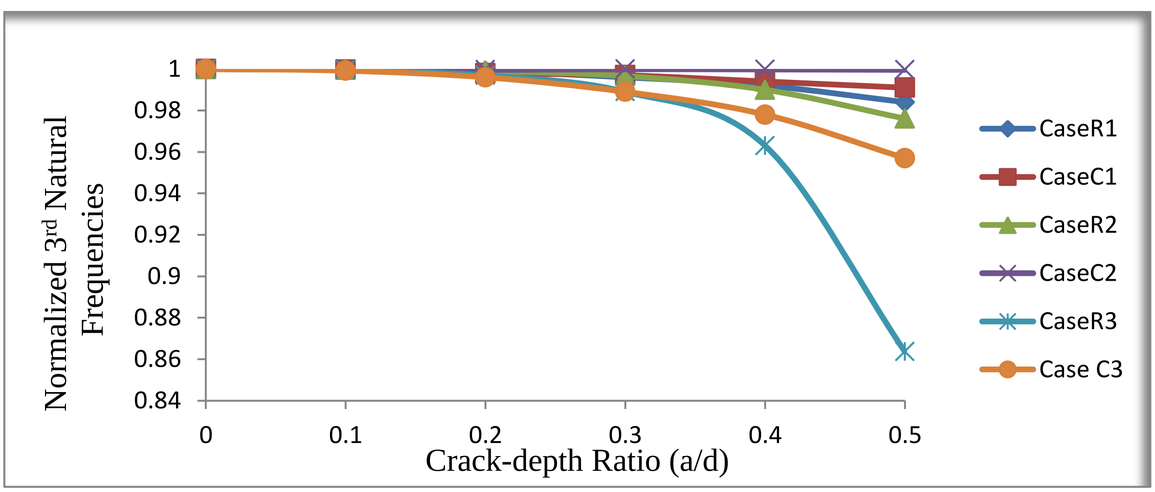


Figure 4.26: Comparison of Normalized 3rd natural frequencies of single cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

From Figure4.26, it is observed that in case of rectangular cross-section the Normalized 3rd natural frequencies difference is high than that of circular cross-section for all the cases respectively. It also shows that when crack is located at the free end of the beam the 3rd natural frequencies reduction is higher and for crack-depth ratio 0.5 the frequency reduction is more by 10% for rectangular beam than circular beam. When crack is located near fixed end of the circular beam the 3rd natural frequencies are generally unharmed although when the depth of crack is quite more.

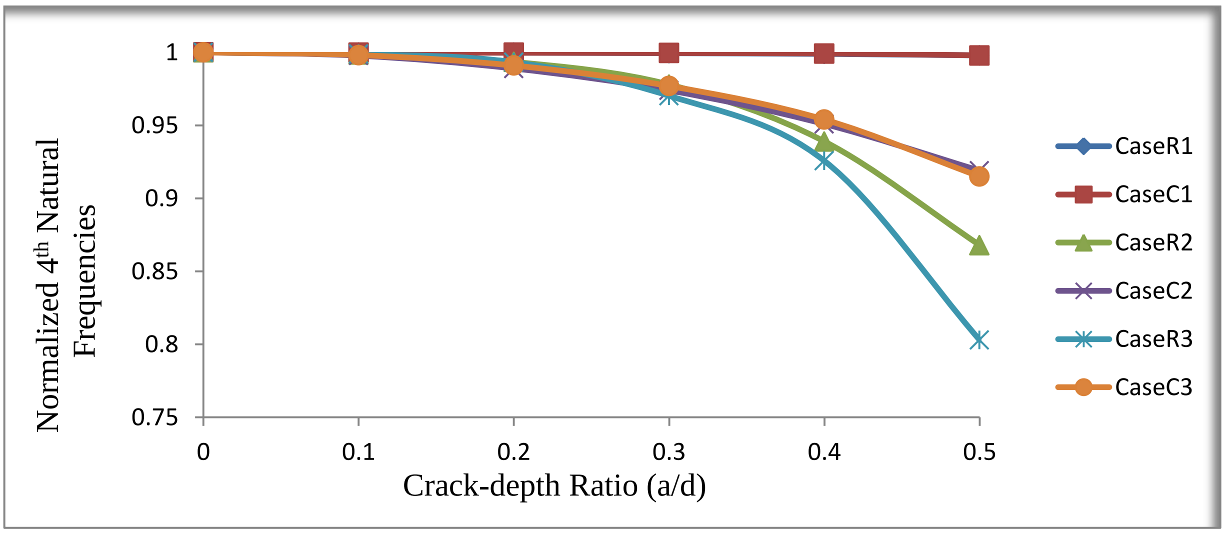


Figure 4.27: Comparison of Normalized 4th natural frequencies of single cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio

In Figure4.27 it is observed that in case of rectangular cross-section the Normalized 4th natural frequencies difference is high than that of circular cross-section for all the cases respectively. It also shows that when crack is located at center and end of the rectangular beam the 4th natural frequencies reduction is higher, it is observed that when crack is located at center and end of the circular beam the 4th natural frequencies reduction are almost same and when crack is located near free end of the beams the 4th natural frequencies are generally unharmed although when the depth of crack is quite more.

Case (3):- Comparison between Uniform Beam of Rectangular and Circular cross-sections with Multiple Cracks

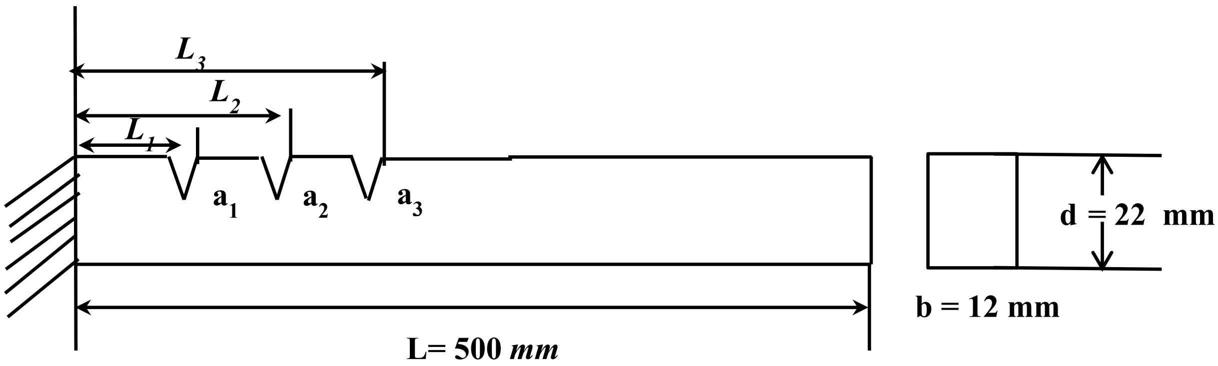


Figure 4.28: Sketch of multiple cracked uniform cantilever beam of rectangular cross-section

Location of Cracks for both rectangular (R) and circular beams (C):

Case 1: $L_1/L=0.1$, $L_2/L=0.2$, $L_3/L=0.3$

Case 2: $L_1/L=0.6$, $L_2/L=0.7$, $L_3/L=0.85$

Case 3: $L_1/L=0.25$, $L_2/L=0.50$, $L_3/L=0.75$

For all the cases**:** a_1/d =0.2

a_2/d =0.3

a_3/d varies

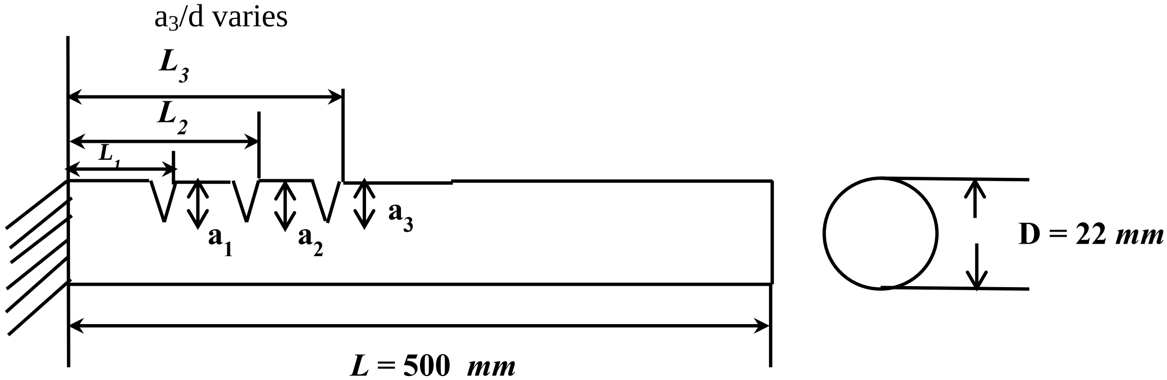


Figure 4.29: Sketch of multiple cracked uniform cantilever beam of circular cross-section

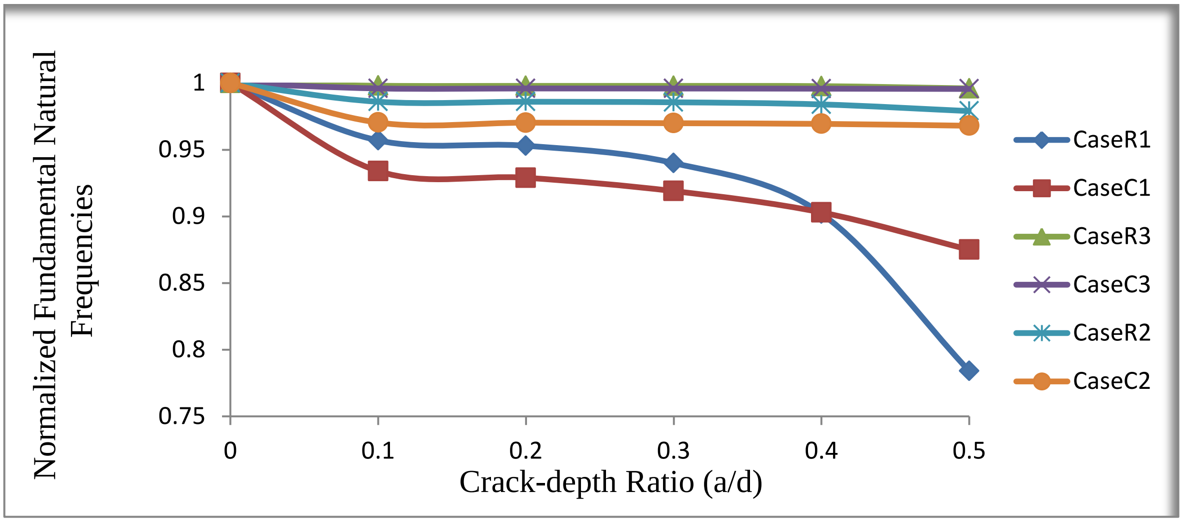


Figure 4.30: Comparison of normalized fundamental natural frequencies of multiple cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

From Fig4.30, it is observed that when cracks are located near the fixed end of the beam the normalized fundamental natural frequencies of multiple cracked beams of both rectangular and circular cross-sections are high to that of cracks located at center and at free end of the beam. When fractures are positioned at free end of the beam the fundamental natural frequencies are generally unharmed although when the depth of crack is quite more as in case of single cracked beams. It is also observed that in case of rectangular cross-section the fundamental normalized natural frequencies reduction is less than that of circular cross-section for all the cases respectively except when crack is located near the fixed end, crack depth ratio is 0.5 where the difference in frequency reduction is 10%.

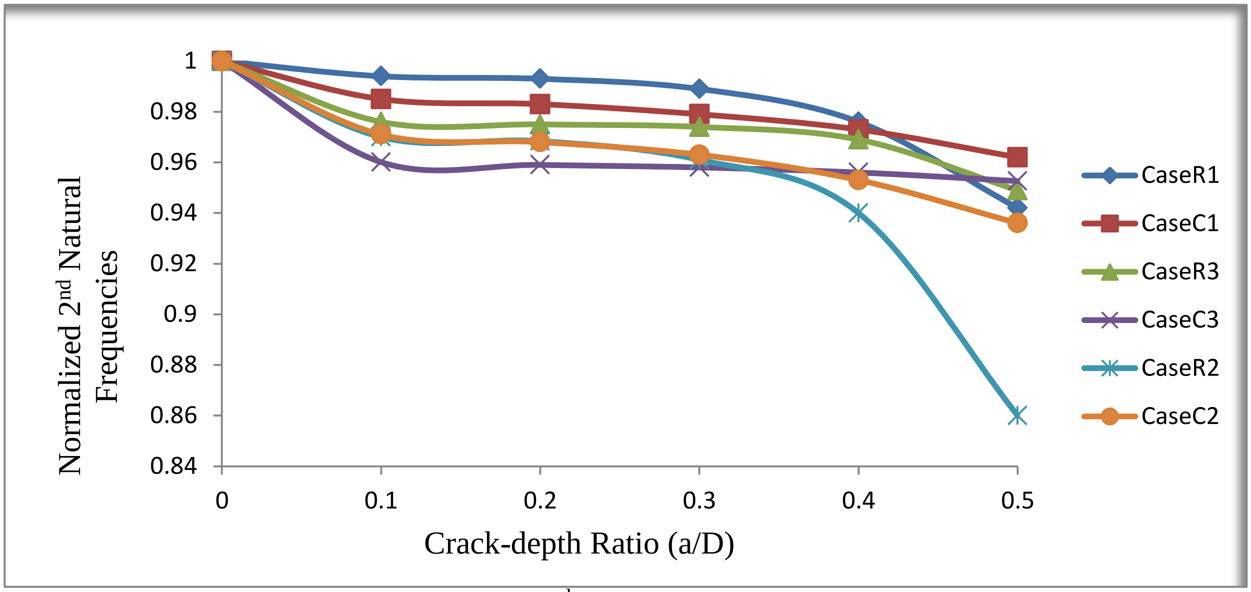


Figure 4.31: Comparison of normalized 2nd natural frequencies of multiple cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

From Figure 4.31, it is observed that when crack is located at center of the beam the 2nd natural frequencies reduction is higher for both rectangular and circular beams, when crack- depth ratio is 0.5 the frequency reduction of rectangular beam is more by 8% than circular beam. When crack is located near free end fixed end of the beams they show similar pattern of variation.

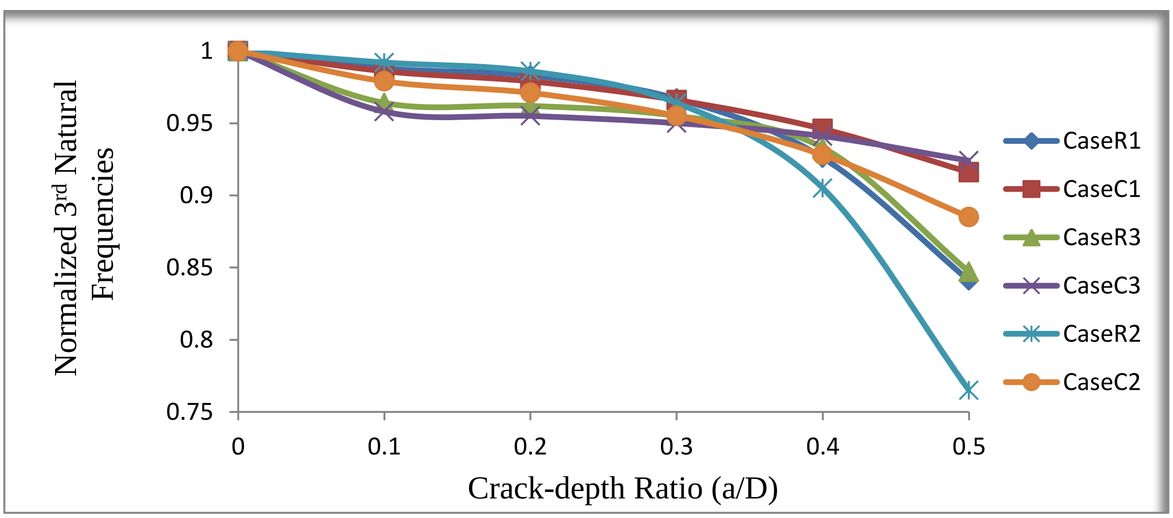


Figure 4.32: Comparison of normalized 3rd natural frequencies of multiple cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

In Figure4.32 it is observed that in case of rectangular cross-section the 3rd natural frequencies difference is high than that of circular cross-section for all the cases respectively. When crack is located at near fixed end or at center or near free end of the beam the rectangular beam will show more variation than circular beam for all the crack positions.

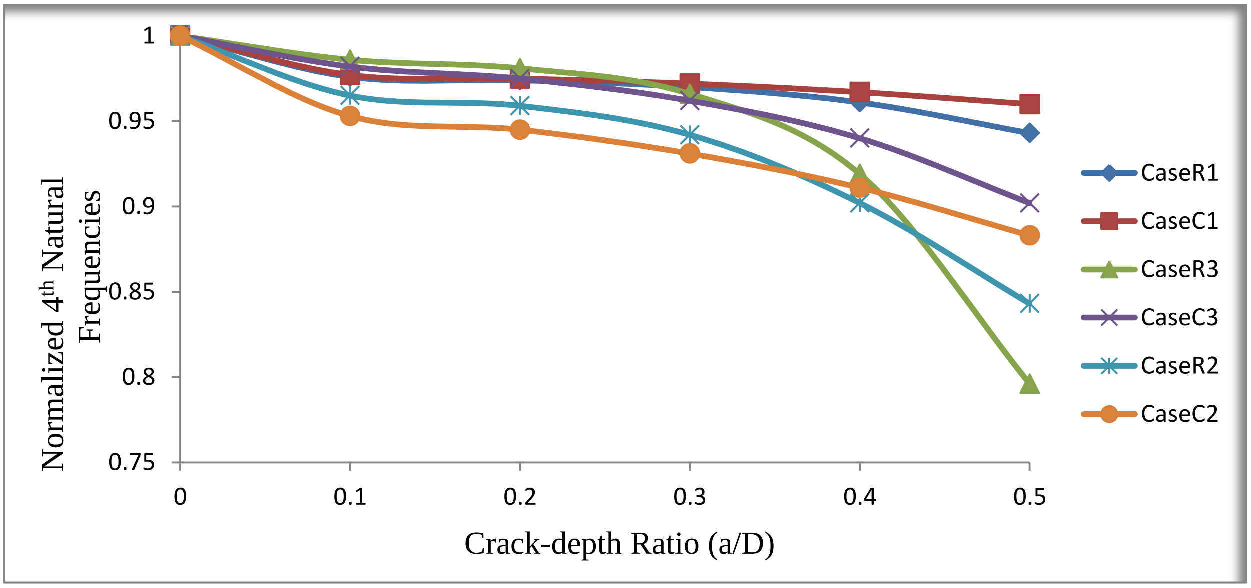


Figure 4.33: Comparison of normalized 4th natural frequencies of multiple cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

From Figure4.33, it is observed that for crack locations at center and near free end of the beam the 4th natural frequency reduction is higher for rectangular section than circular section when the crack depth ratio is 0.5. However we find changes in the frequency reduction when crack-depth ratio is 0.4.

4.4.3 STEPPED BEAM WITH MULTIPLE CRACKS

Case (1):- Effect of Step present in Uniform, Single stepped and Two stepped beams of Rectangular cross-sections

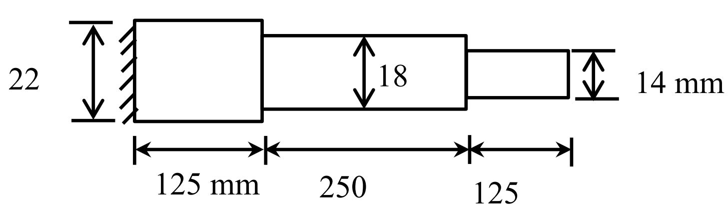


Figure 4.34: Sketch of Two stepped cantilever beam

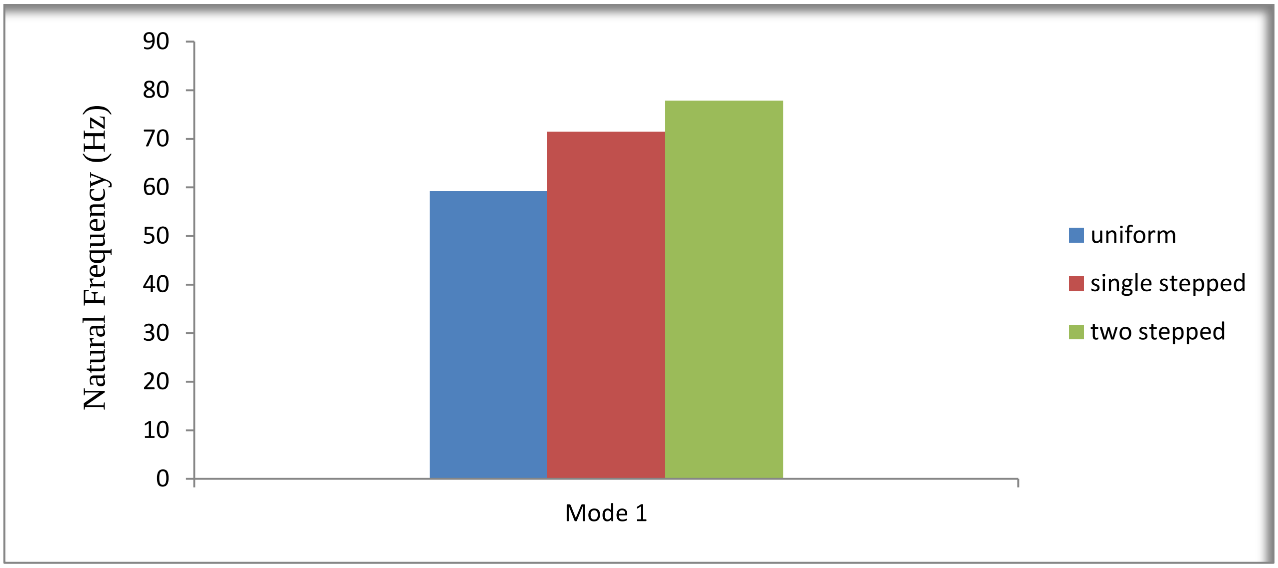


Figure 4.35: Plot of fundamental natural frequency (Hz) variation of uniform, single stepped and two stepped beams with respect to Mode1

From Figure4.35, it is observed that there is step wise increase in the fundamental frequency variation for uniform, single stepped and two stepped beams by 17.08% and 23.97% for single stepped and two stepped beams with respect to uniform beam respectively.

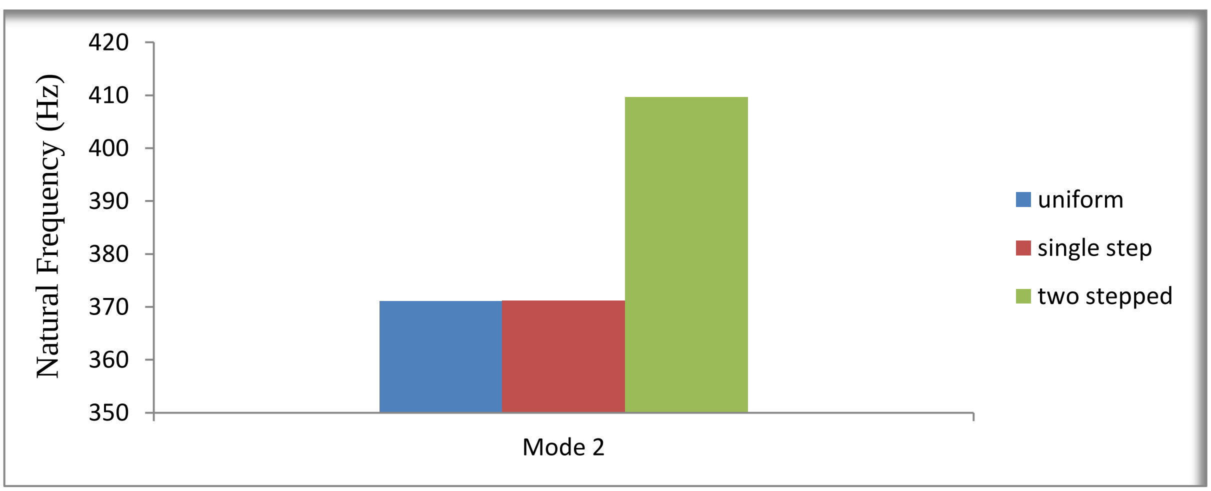


Figure 4.36: Plot of 2nd natural frequency (Hz) variation of uniform, single stepped and two stepped beams with respect to Mode 2

In Figure4.36 it is observed that the 2nd natural frequency for single stepped beam is increased by very less percentage 0.014% whereas for two stepped beam the increase is 9.41% when compared to uniform beam respectively.

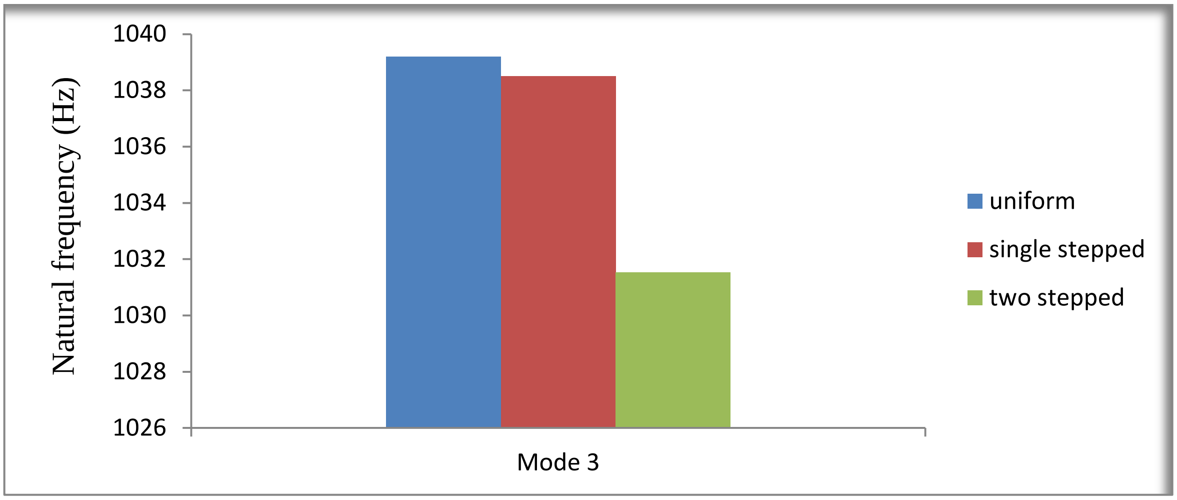


Figure 4.37: Plot of 3rd natural frequency variation (Hz) of uniform, single stepped and two stepped beams with respect to Mode 3

From Figure4.37, it is observed of 3rd natural frequency is high for uniform beam and the difference with respect to single stepped beam is reduced by 0.067%. For two stepped beam frequency reduction is 0.73% compared to uniform beam.

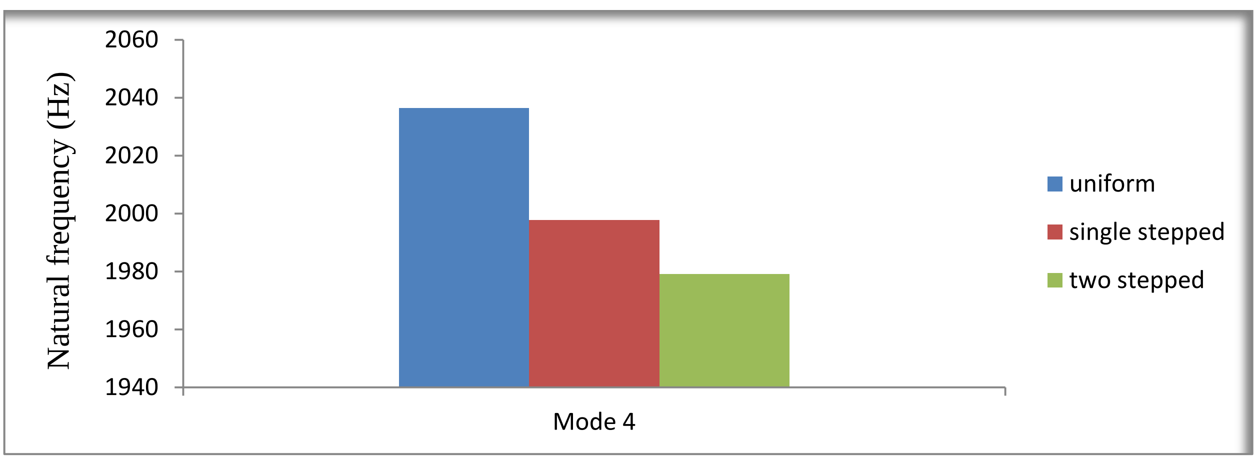


Figure 4.38: Plot of 4th natural frequency (Hz) variation of uniform, single stepped and two stepped beams.

In Figure4.38 it is observed that there is step wise decrease in the 4th natural frequency for uniform, single stepped and two stepped beams. There is decrease by 1.90% and 2.81% for single stepped and two stepped beams with respect to uniform beam respectively.

From Figure 4.35, 4.36 for Mode1 and Mode 2 the natural frequency increases as step is present in the beam whereas from Figure 4.37, 4.38 for Mode3 and Mode 4 the natural frequency decreases.

Case (2):- Effect of Step present in Single Cracked Beam of Rectangular cross-section

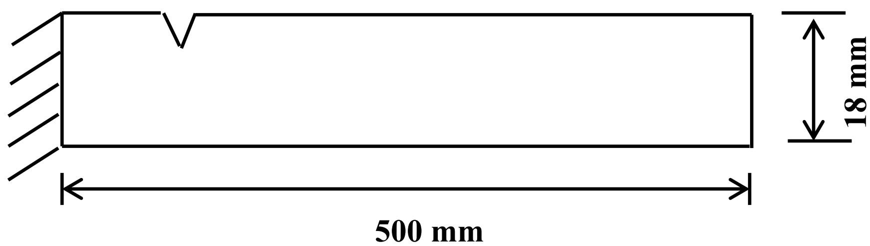


Figure 4.39: Sketch of Uniform cantilever beam with single crack of rectangular cross-section

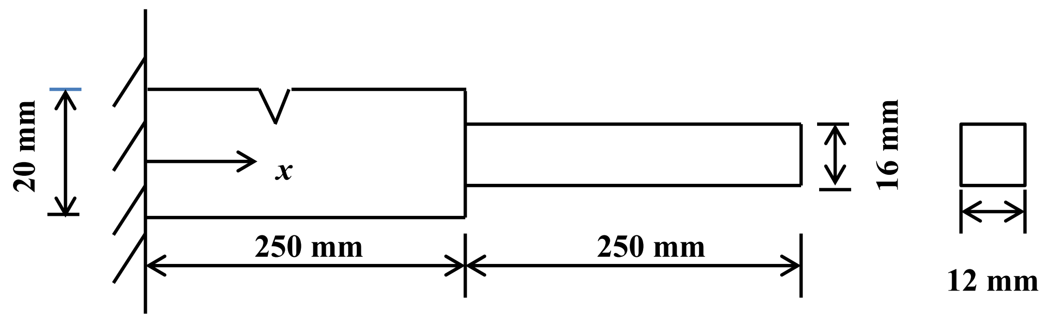


Figure 4.40: Sketch of single step cantilever beam of rectangular cross-section

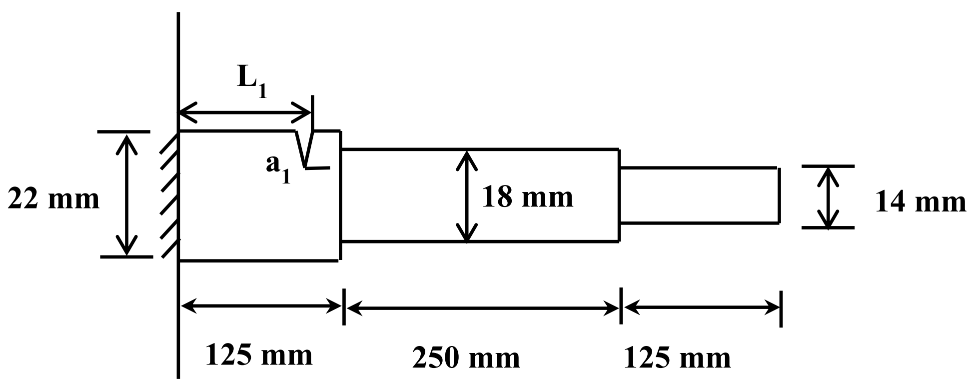


Figure 4.41: Sketch of two stepped cantilever beam of rectangular cross-section with single crack

Case (a): L_1/L=0.2; Crack is located near the free end of the beams

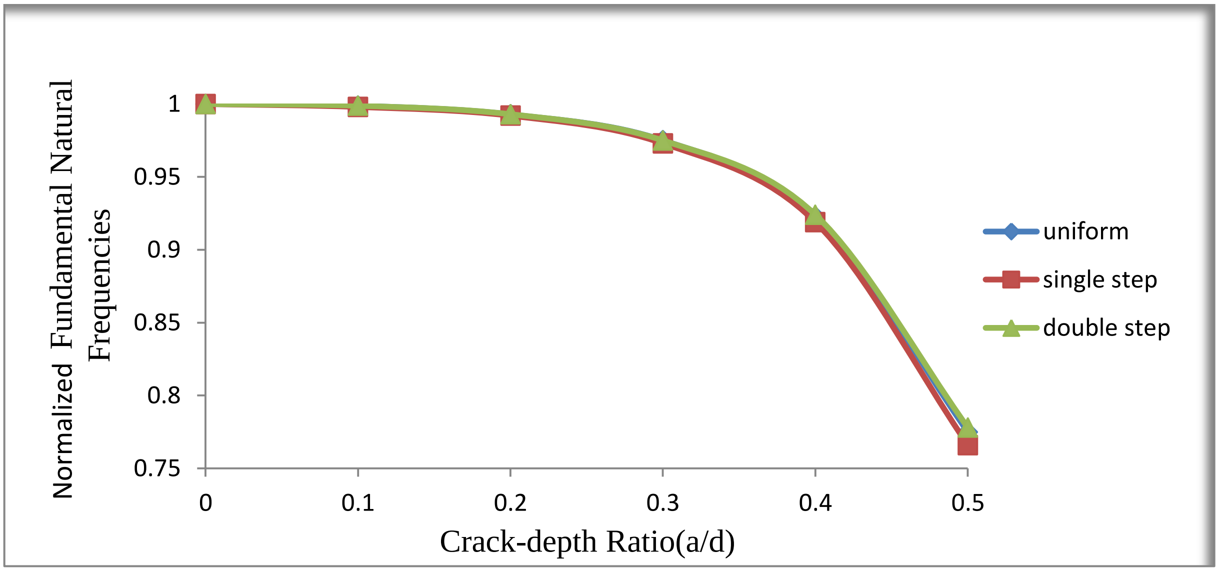


Figure 4.42(a): Comparison of normalized fundamental natural frequencies of uniform, single stepped and two stepped rectangular beams of single crack with respect to crack-depth ratio

From Figure4.42(a) it is observed that there is no much variation in normalized fundamental natural frequencies for uniform, single stepped and double stepped rectangular beams with single crack where all the beams exhibit the same pattern.

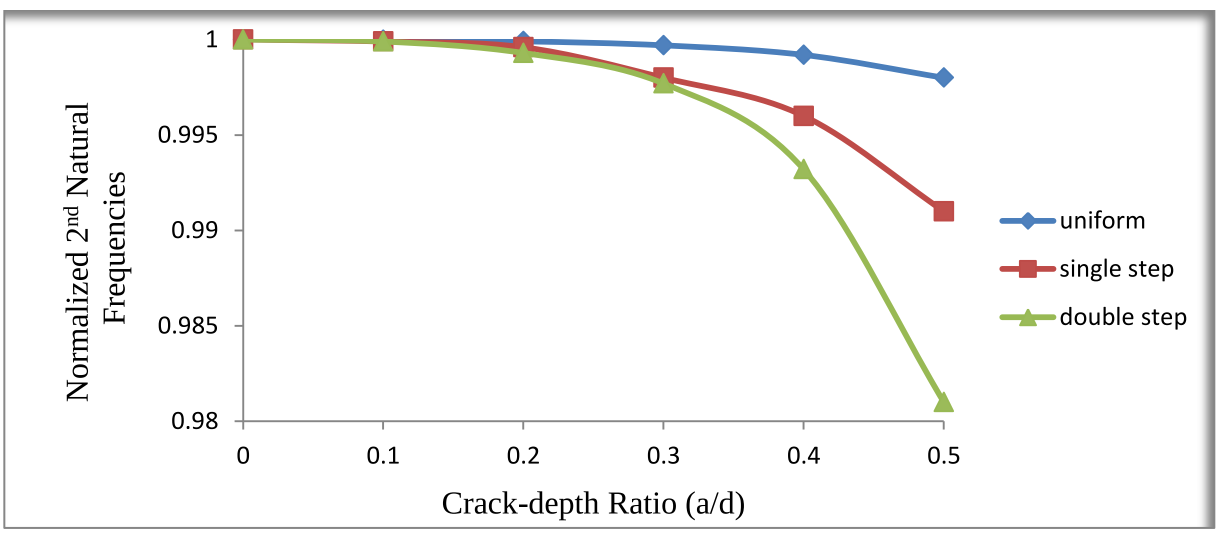


Figure 4.43(a): Comparison of normalized 2nd natural frequencies of uniform, single stepped and two stepped rectangular beams with single crack.

From Figure 4.43(a) it is observed that uniform, single stepped and double stepped rectangular beams with single crack follow ascending pattern however the frequency reduction of normalized 2nd natural frequencies is less for all the beams. The frequency reduction of two stepped beam is more by 1.7% to that of uniform beam.

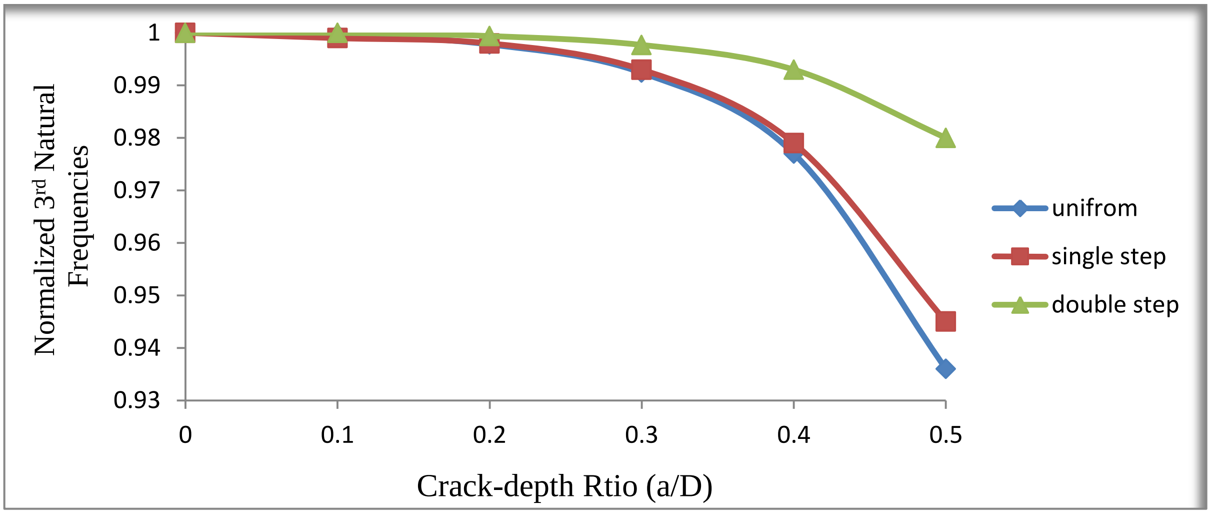


Figure 4.44(a): Comparison of normalized 3rd natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack.

From Figure 4.44(a) it is observed that normalized 3rd natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack follow descending pattern but the variation between uniform and stepped beams is high. The frequency reduction of two stepped beam is less by 5% to that of uniform beam.

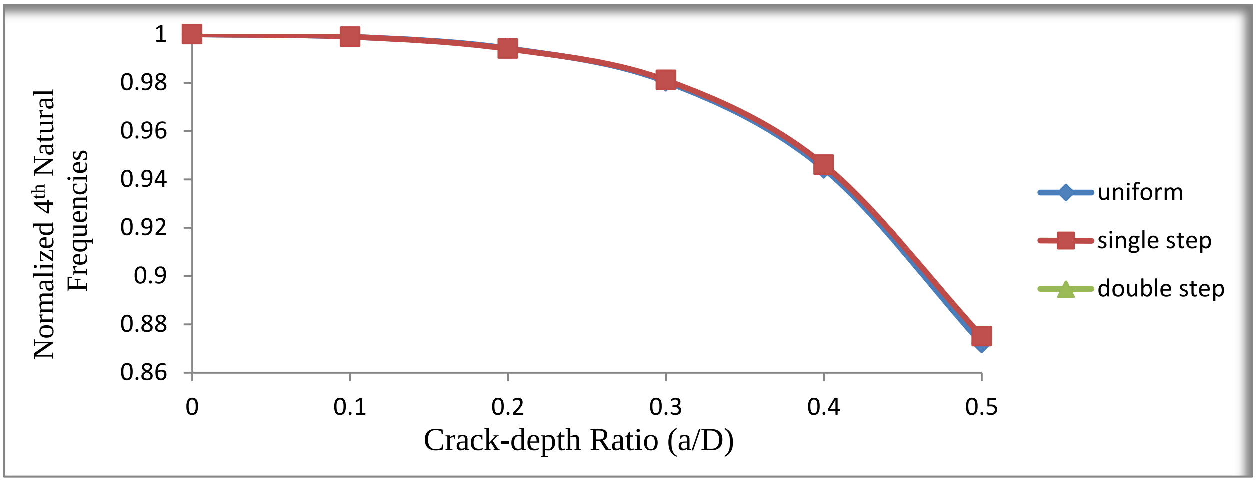


Figure 4.45(a): Comparison of normalized 4th natural frequencies of uniform, single stepped and two stepped rectangular beams with single crack.

From Figure4.45(a) it is observed that there is no variation in normalized 4th natural frequencies for uniform, single stepped and double stepped rectangular beams with single crack where all the beams exhibit the same pattern.

From Figures 4.42(a)-4.45(a) it can be resulted as cracks located near free end of the beam the fundamental frequencies and natural frequencies of higher modes are less affected due to presence of step in the beam in 2nd natural frequency variation shows increasing pattern and 3rd natural frequency shows descending pattern respectively for uniform, single, two stepped beams.

Case (b): $L_1/L=0.5$;

- Crack is located at center of the uniform beam.
- Crack is located at center i.e., at the step location of the single stepped beam.
- Crack is located at center i.e., at the center of the first step present in two stepped beam.

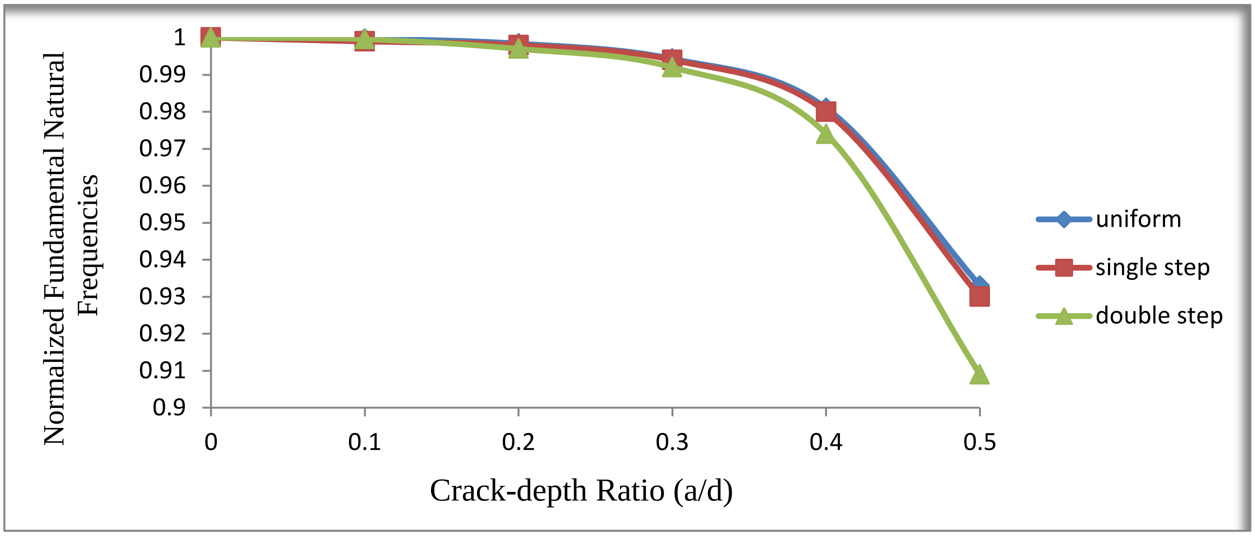


Figure 4.42(b): Comparison of normalized fundamental natural frequencies of uniform, single stepped and two stepped rectangular beams with single crack.

From Figure 4.42(b), it is observed that there is no variation in normalized fundamental natural frequencies for uniform, single stepped rectangular beams with single crack, but the two stepped beam shows high frequency reduction is high by 2.5% than uniform beam.

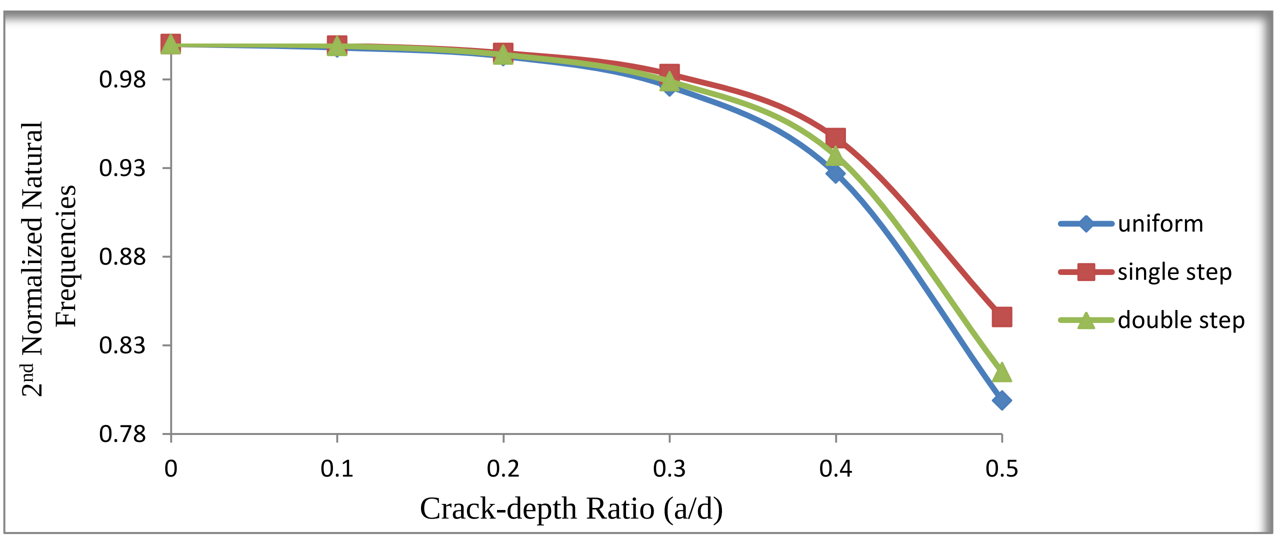


Figure 4.43(b): Comparison of 2nd normalized natural frequencies of uniform, single stepped and two stepped rectangular beams with single crack.

From Figure 4.43(b), it is observed that there is more variation in normalized 2nd natural frequencies for uniform beam than double stepped and single stepped beams with single crack.

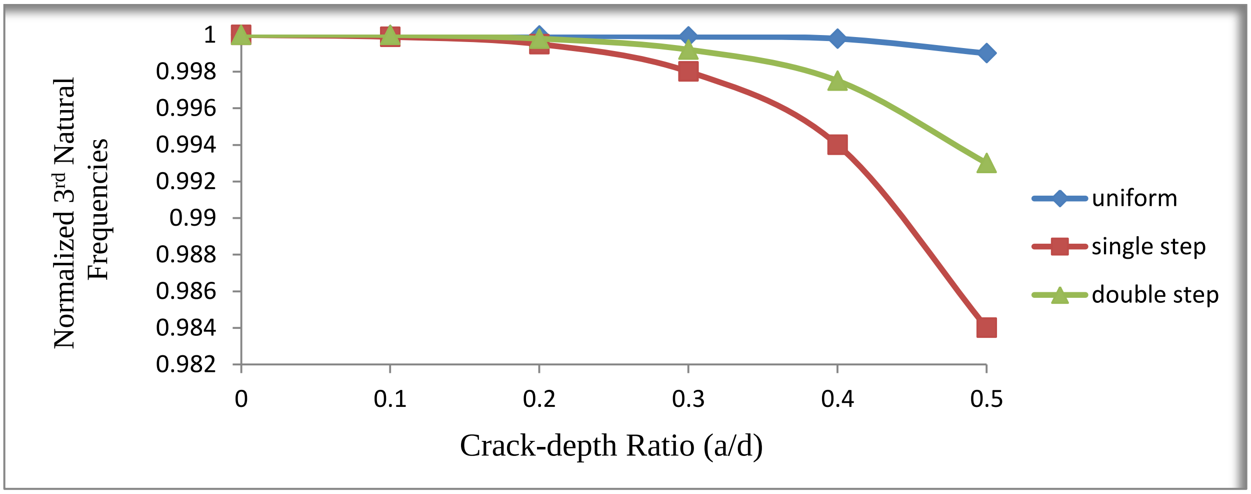


Figure 4.44(b): Comparison of normalized 3rd natural frequencies of uniform, single stepped and two stepped rectangular beams with single crack.

From Figure 4.44(b), it is observed that uniform, double stepped and single stepped rectangular beams with single crack follow ascending pattern however the overall variation of frequency reduction is less for all the beams.

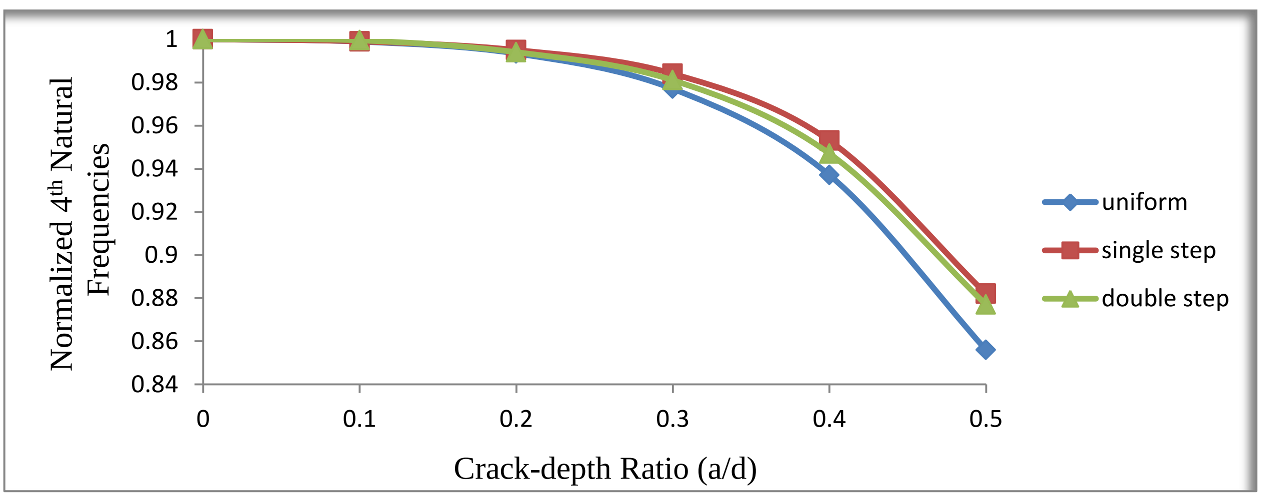


Figure 4.45(b): Comparison of normalized 4th natural frequencies of uniform, single stepped and two stepped rectangular beams with single crack.

In Figure 4.45(b) it is observe that there is less variation in normalized 4th natural frequencies for single stepped, double stepped and uniform rectangular beams with single crack which are following descending pattern.

Case (c): L_1/L=0.75; Crack is located near the free end of the beams

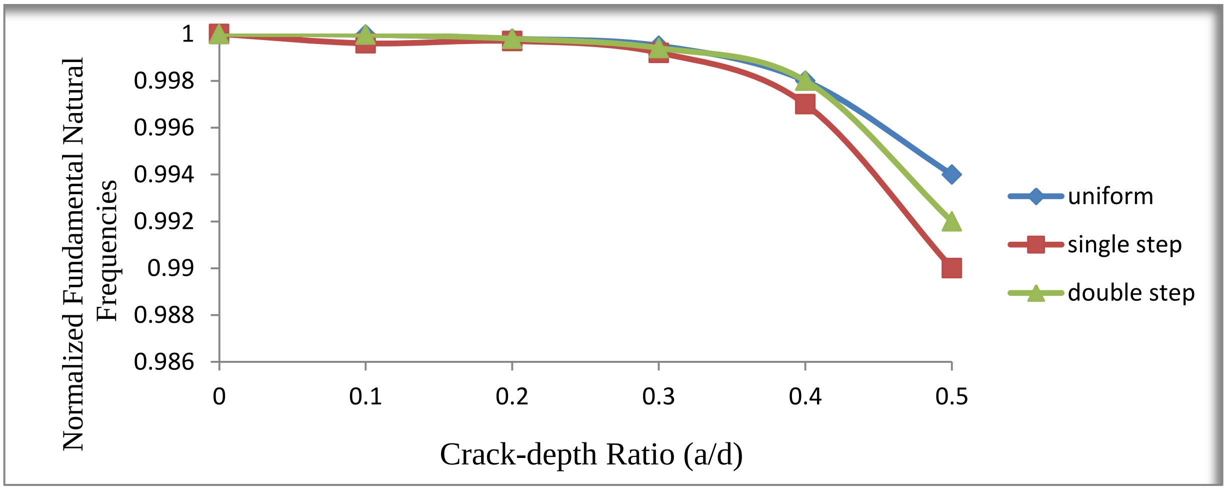


Figure 4.42(c): Comparison of normalized fundamental natural frequencies of uniform, single stepped and two stepped rectangular beams with single crack.

From Figure4.42(c), it is observed that uniform, double stepped and single stepped rectangular beams with single crack follow ascending pattern however the overall variation of frequency reduction is less for all the beams.

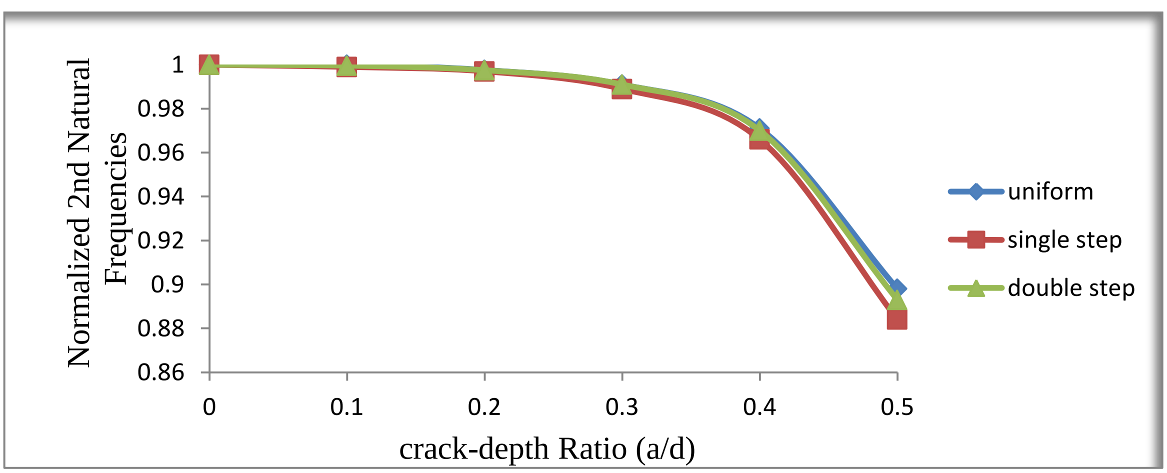


Figure 4.43(c): Comparison of normalized 2nd natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack with respect to crack-depth ratio.

In Figure4.43(c) it is observed that for of normalized 2nd natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack the pattern is precise.

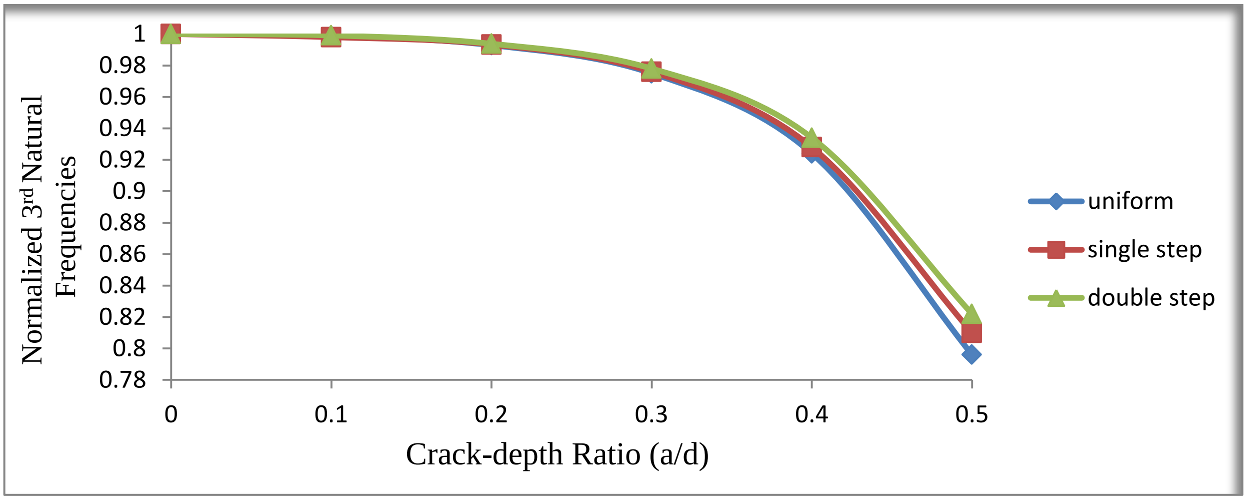


Figure 4.44(c): Comparison of normalized 3rd natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack with respect to crack-depth ratio.

From Figure4.44(c), it is observed that normalized 3rd natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack follows same pattern but the variation of frequency reduction is more than Figure 56.

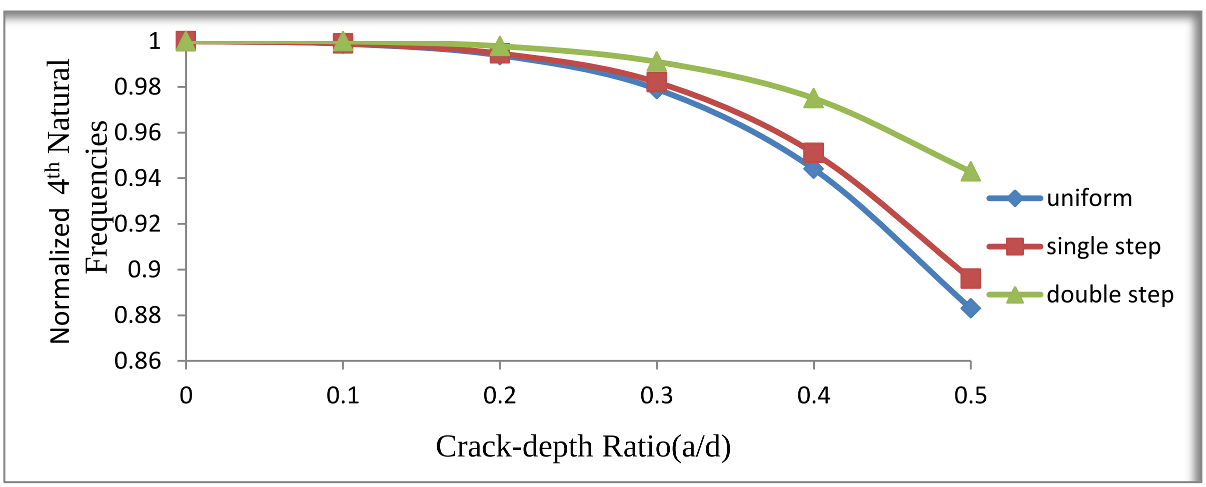


Figure 4.45 (c): Comparison of normalized 4th natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack with respect to crack-depth ratio.

From Figure 4.45(c), it is observed that normalized 4th natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack show increasing variation pattern of the frequency reduction.

When crack is located near the fixed end of the beam it shows that the frequency reduction difference is more for uniform and two-stepped beams and when crack is located at center of the beam it shows the frequency reduction difference more for uniform and single stepped beams since the crack is located at step of the beam. When crack is located near free end of the beam the difference of 4th natural frequency reduction among uniform single and two stepped beams is more than others.

4.4.3 Variation of Natural Frequencies With Respect To Single And Multiple Cracks In Two Stepped Cantilever Beam

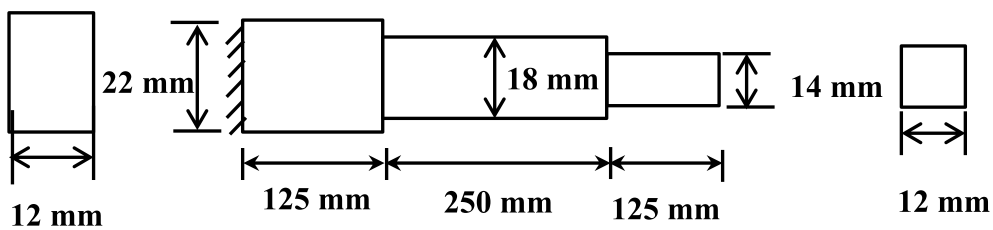


Figure 4.46: Sketch of two step cantilever beam of rectangular cross-section

Case (1):- Natural Frequencies of Two Stepped Cantilever Beam without Crack

Mode	Natural Frequency (Hz)
Mode1	77.900
Mode2	409.711
Mode3	1031.526
Mode4	1979.115

Table 13: Natural frequency of two stepped cantilever beam

Case (2):-Natural Frequencies of Single Cracked Two Stepped Cantilever Beam

Case(a):

a) L1/L=0.2: Crack is located near to the fixed end of the beam
b) L1/L=0.3: Crack is located in the first step of the beam.
c) L1/L=0.85: Crack is located in second step near free end of the beam.

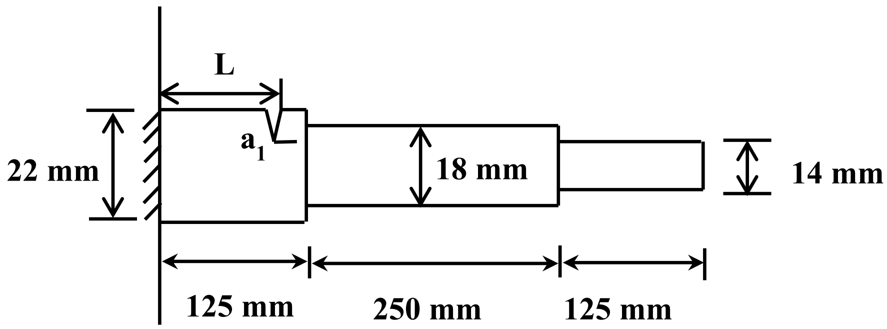


Figure 4.47: Sketch of single cracked two stepped cantilever beam of rectangular cross-section

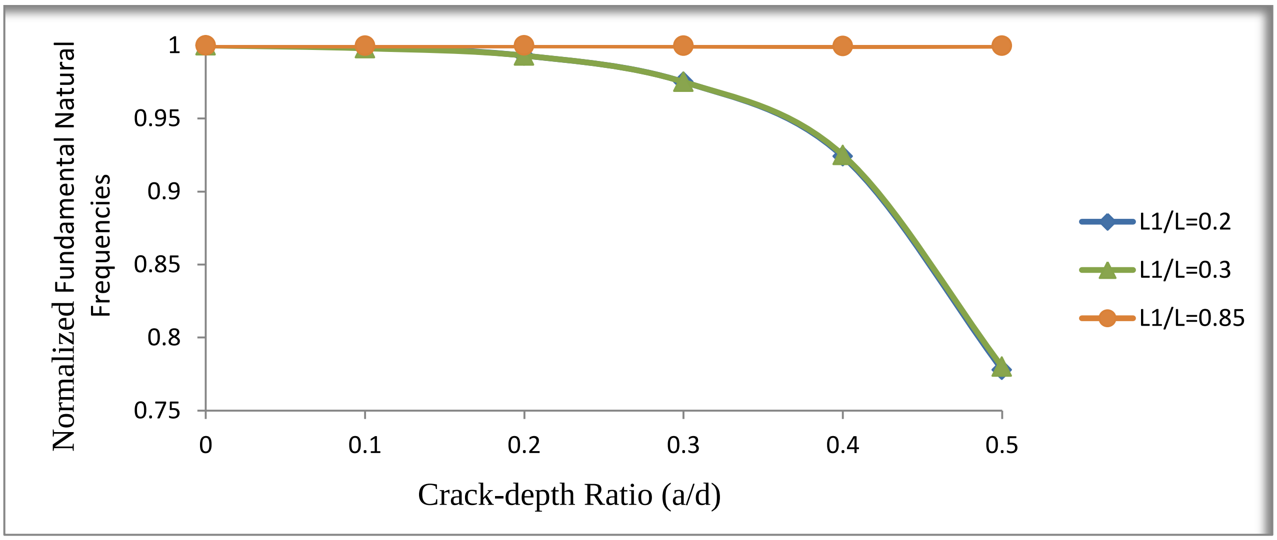


Figure 4.48(a): Comparison of normalized fundamental natural frequencies of single cracked two stepped cantilever beam with respect to crack-depth ratio.

From Figure4.48(a), it is observed that the normalized fundamental natural frequencies of two stepped rectangular beams with single crack show similar pattern of variation as uniform beam with single crack as it indicates when the crack is located at the free end of the beam the frequency reduction is almost unaffected even when the crack-depth ratio is high. It is also observed that when crack depth ratios are 0.2 and 0.3 the variation is same.

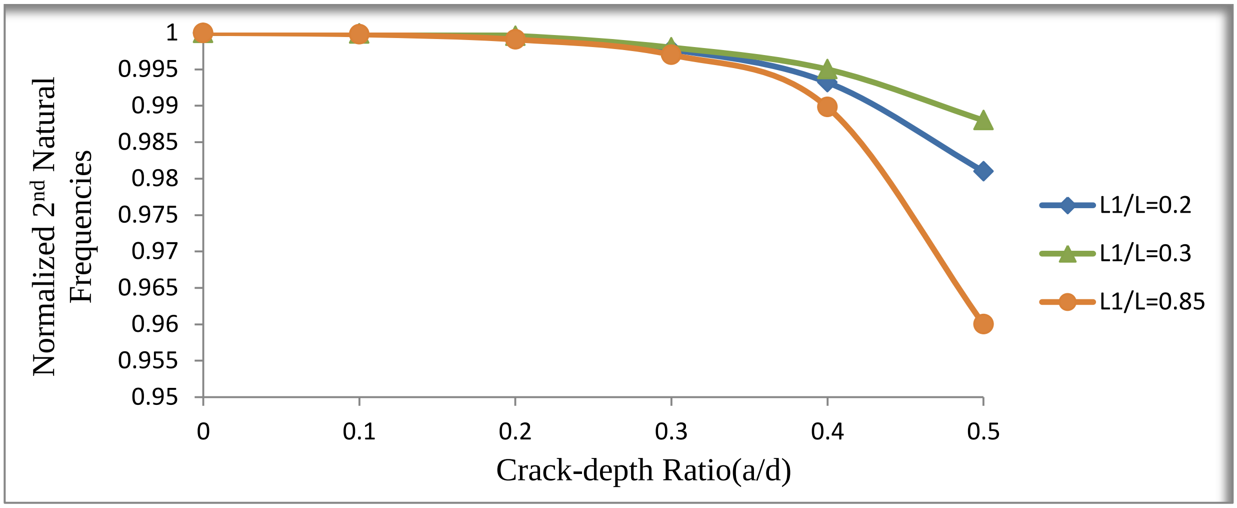


Figure 4.49(a): Comparison of normalized 2nd natural frequencies of two stepped rectangular beams with single crack with respect to crack-depth ratio.

In Figure4.49(a) it is observed that there is relatively high variation in the frequency reduction when crack-depth ratio is higher than 0.3.It shows that when crack is located at free end of the beam the frequency reduction is higher than other two cases.

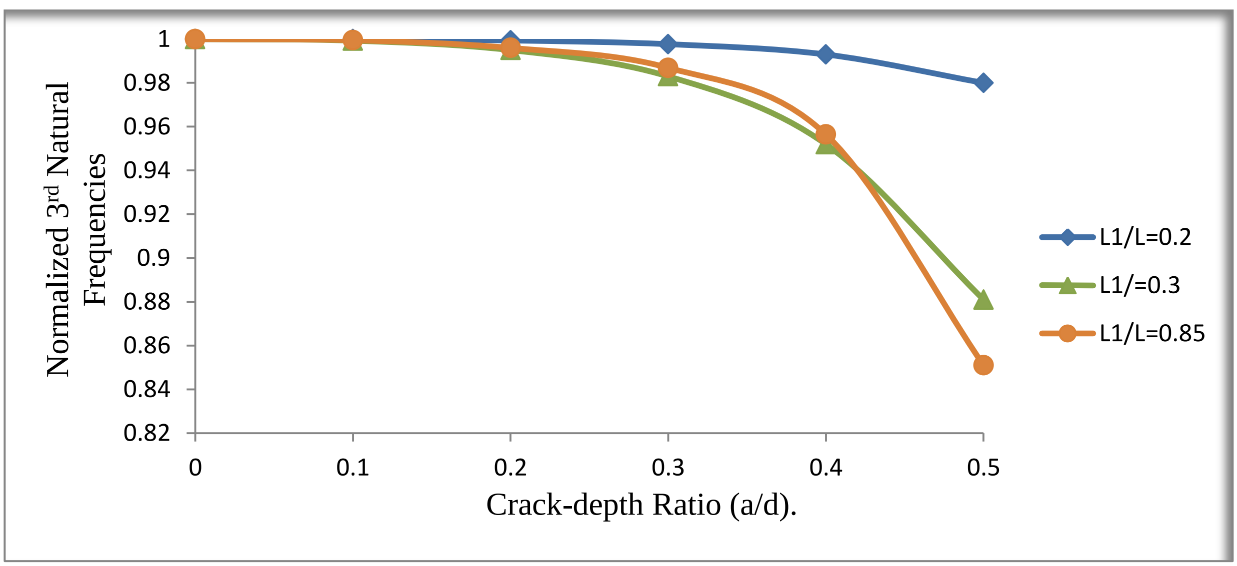


Figure 4.50 (a): Comparison of normalized 3rd natural frequencies of two stepped rectangular beams with single crack with respect to crack-depth ratio.

In Figure4.50(a) it is observed that there is relatively high variation in the frequency reduction when crack location as near free end of the beam for crack-depth ratio 0.5. Till crack-depth ratio 0.4 the frequency variations for all the crack locations follow same pattern.

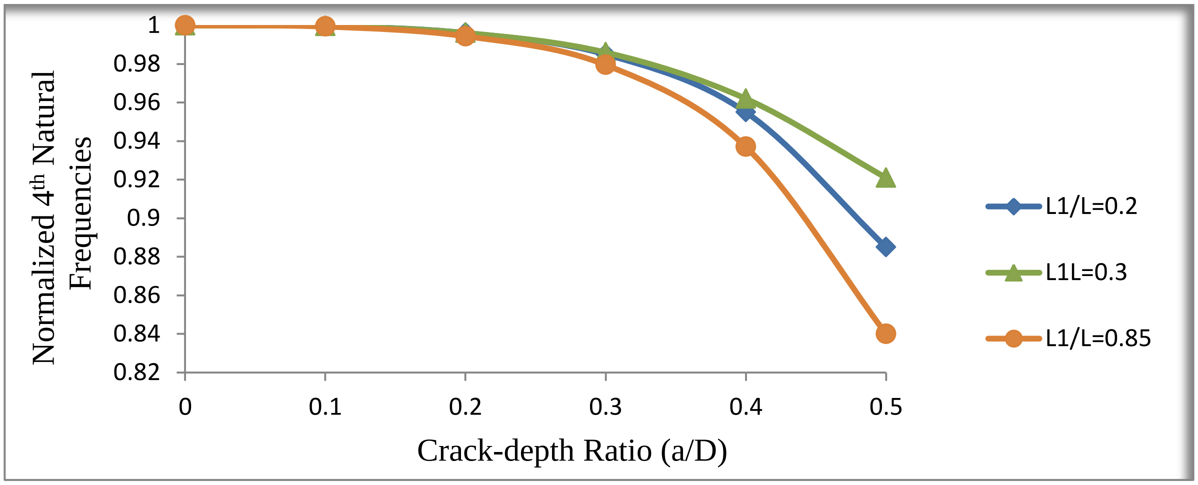


Figure 4.51(a): Comparison of normalized 4th natural frequencies of two stepped rectangular beam with single crack with respect to crack-depth ratio.

From Figure4.51(a), it is observed the variation the normalized 4th frequencies of two stepped rectangular beam follows ascending pattern of natural frequency reduction for crack being present along the length of the beam. It also shows as the crack-depth ratio increases the natural frequency decreases same as uniform beam.

Case(b):

a) L1/L=0.25: Crack is located at the first step of the beam.
b) L1/L=0.5: Crack is located at the centre of the first step of the beam i.e., center of the two stepped cantilever beam.
c) L1/L=0.75: Crack is located at the edge of the first step of the beam.

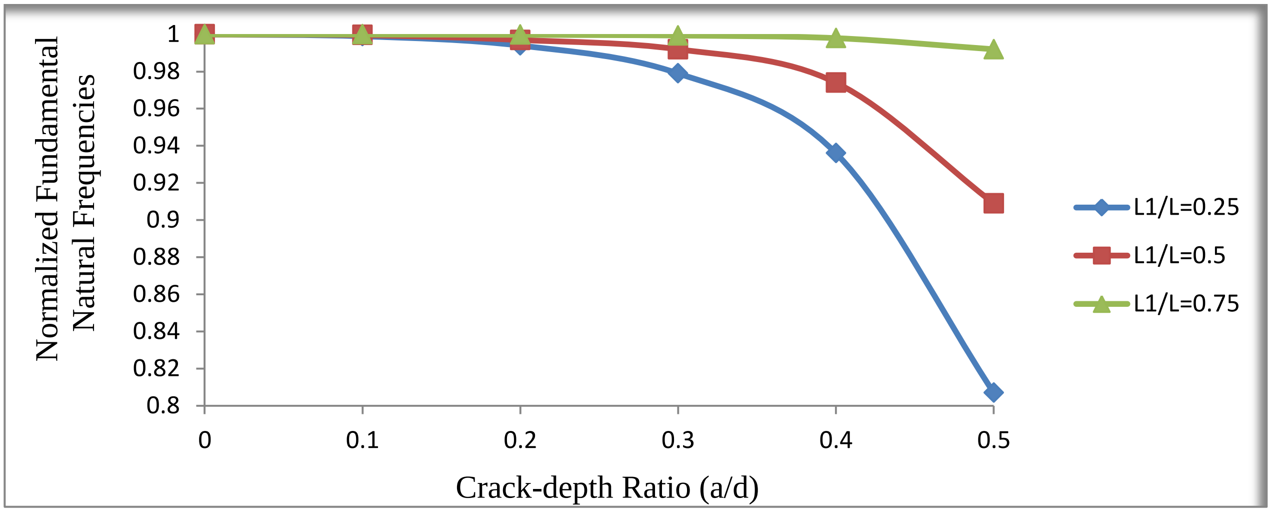


Figure 4.48(b): Comparison of normalized fundamental natural frequencies of two stepped rectangular beams with single crack with respect to crack-depth ratio.

In Figure 4.48(b) it is observed that as the crack proceeds away from the fixed end of two stepped beam of rectangular cross-section the frequency reduction of the fundamental normalized natural frequencies decreases.

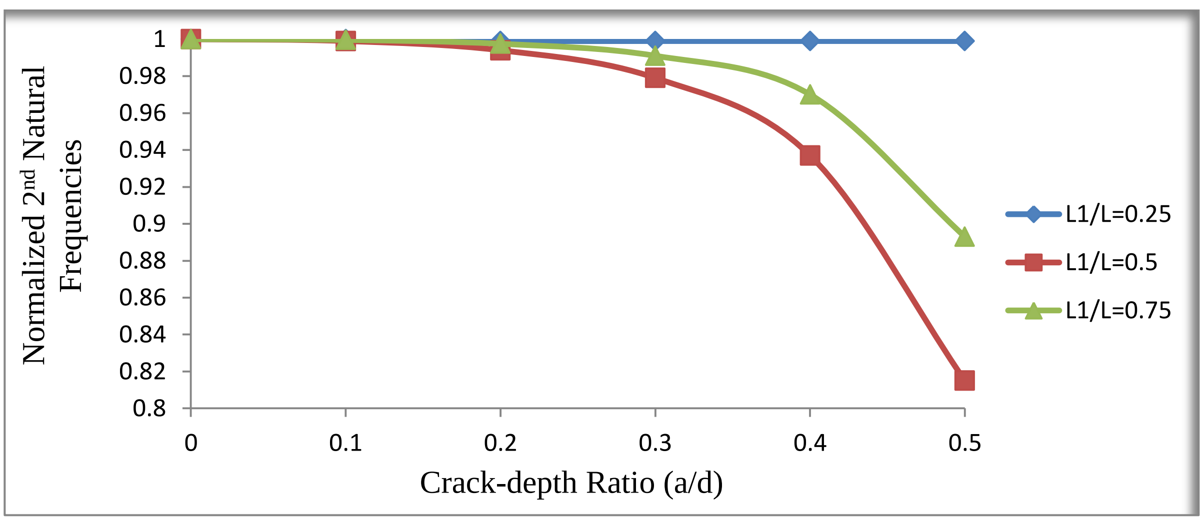


Figure 4.49(b): Comparison of normalized 2^{nd} natural frequencies of two stepped rectangular beams with single crack with respect to crack-depth ratio.

From Figure 4.49(b), it is observed that when crack is located at the center of the beam there is high variation of frequency reduction followed by crack located at edge of second step end. When crack is located near the first step of the two stepped beam the 2^{nd} natural frequencies are almost unaffected.

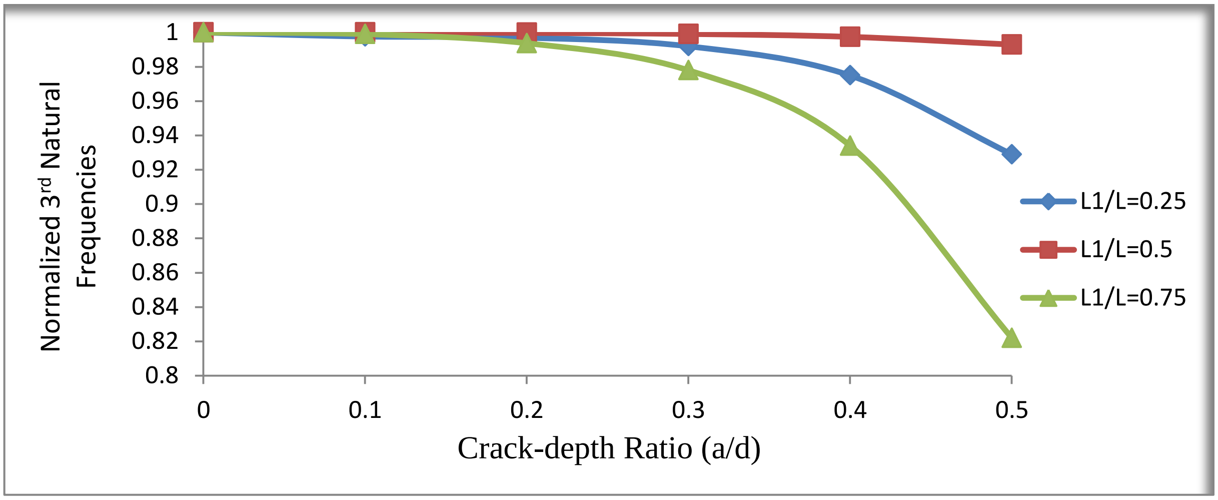


Figure 4.50(b): Comparison of normalized 3rd natural frequencies of two stepped rectangular beams with single crack with respect to crack-depth ratio.

From Figure50(b), it is observed that when crack is located at the center of the beam the normalized 3rd natural frequency reduction is very less affected. When crack is located at edge of second step of the beam the frequency reduction is higher compared to the crack located at edge of first step location of the beam.

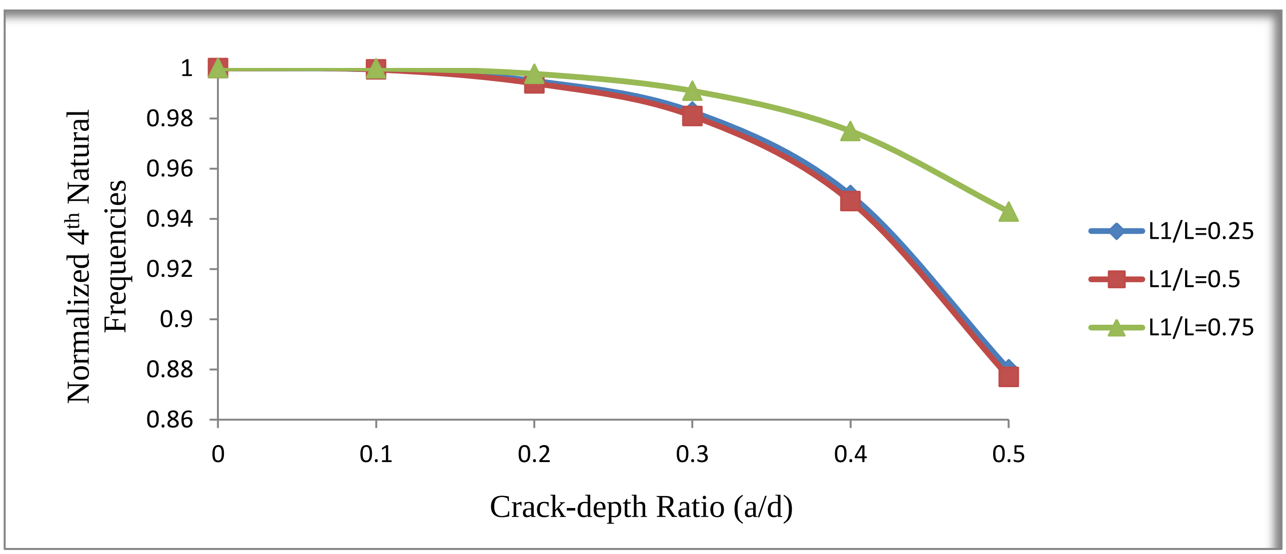


Figure 4.51(b): Comparison of normalized 4th natural frequencies of two stepped rectangular beams with single crack with respect to crack-depth ratio.

From Figure4.51 (b) it is observed that when cracks are located at the edge of the first step location of the beam and center of the beam then the normalized 4th natural frequencies of two stepped rectangular beam is high and shows the same frequency reduction in same pattern .Hence when crack is located at the second step of the beam the frequency reduction is comparatively less due to crack being present near free end of the beam.

From Figure 4.48(b)-4.51(b) it can be concluded as the fundamental frequency variation is more affected when crack is near to the two stepped beam even when the step is present and 2nd natural frequency reduction is high when crack is located at center of the two stepped beam and 3rd natural frequency variation is high when located at free end of the two stepped beam.

Natural Frequencies of Multiple Cracked Two Stepped Cantilever Beam:

Case1: L1/L=0.1; L2/L=0.2; L3/L=0.3

Cracks are located near the free end of the two stepped beam

Case2: L1/L=0.7; L2/L=0.8; L3/L=0.9

Cracks are located near the far end of the two stepped beam

Case3: L1/L=0.1; L2/L=0.5; L3/L=0.9

Each crack is located at center of each step of the two stepped beam

Case4: L1/L=0.25; L2/L=0.75; L3/L=0.95

Cracks are located at the edge of the step being presents in the two stepped beam

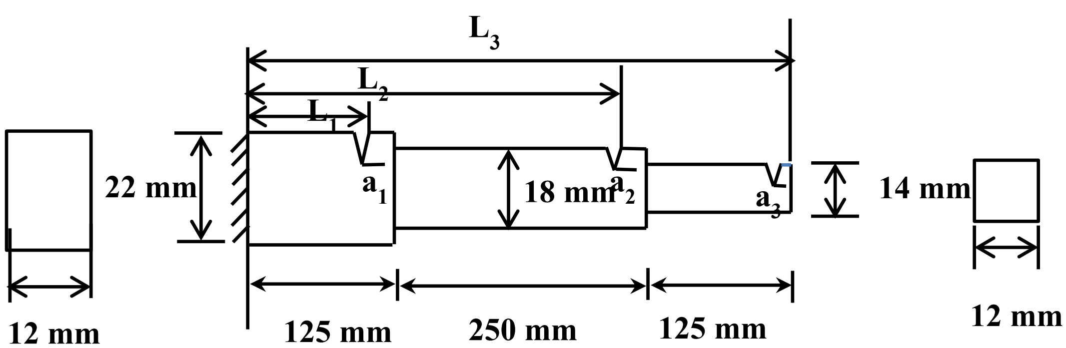


Figure 4.52: Sketch of multiple cracked two stepped cantilever beam of rectangular cross-section

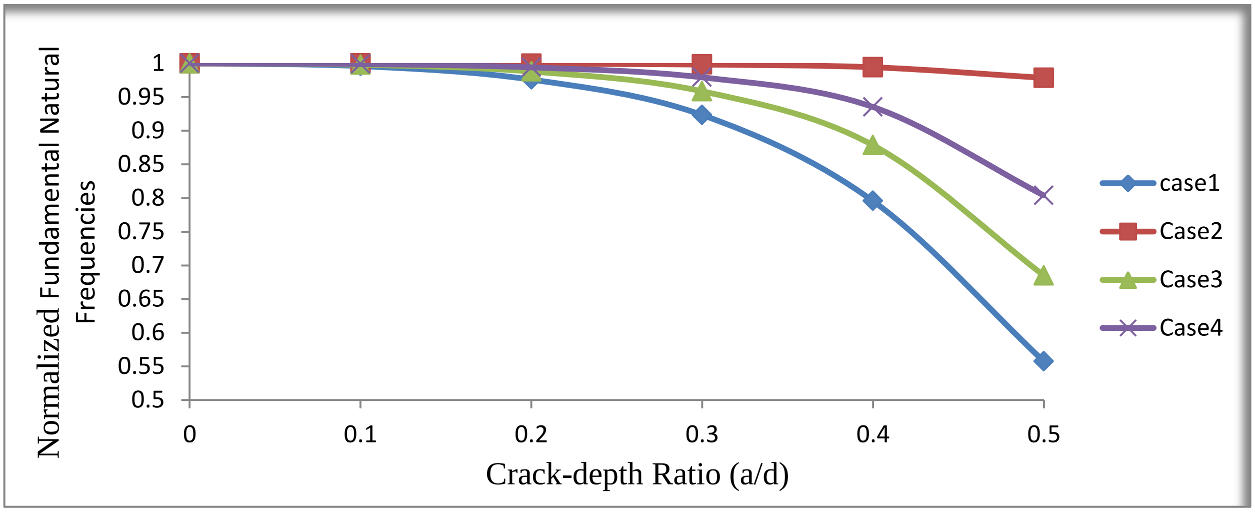


Figure 4.53: Comparison of normalized fundamental natural frequencies of multiple cracked two stepped rectangular beams with respect to crack-depth ratio.

From Figure4.53, it is observed that even when cracks are located at the edge of the step locations of the beam the normalized fundamental frequency reduction is less. This Figure also shows that when cracks are located at free end of the beam the fundamental normalized frequency reduction is less affected and when cracks are located at fixed end of the beam the fundamental normalized frequency reduction is more affected.

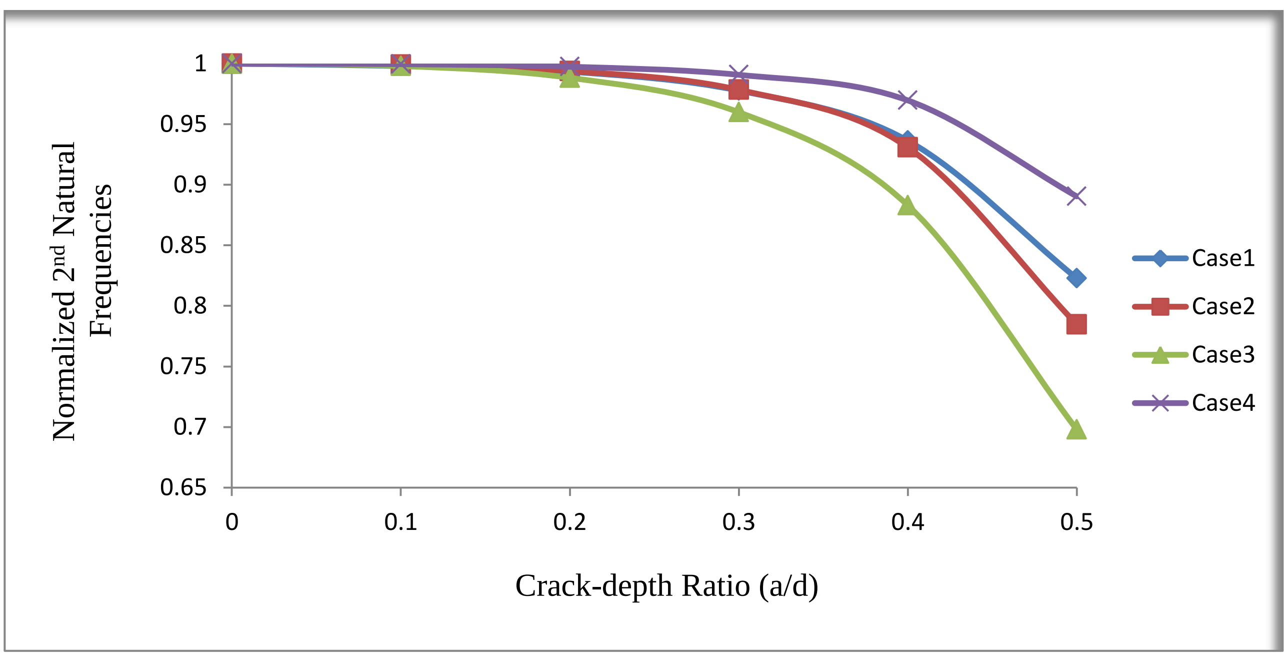


Figure 4.54: Comparison of normalized 2^{nd} natural frequencies of multiple cracked two stepped rectangular beams with respect to crack-depth ratio.

In Figure 4.54 it is observed that when cracks are located either at fixed end or free end of the multiple cracked two stepped rectangular beams the normalized 2nd natural frequencies reduction is more or less same. When cracks are located at the step location of the beam the frequency reduction is less. It is observed when cracks are located at the center of the step locations of the two stepped beam there is high reduction in the 2nd natural frequencies.

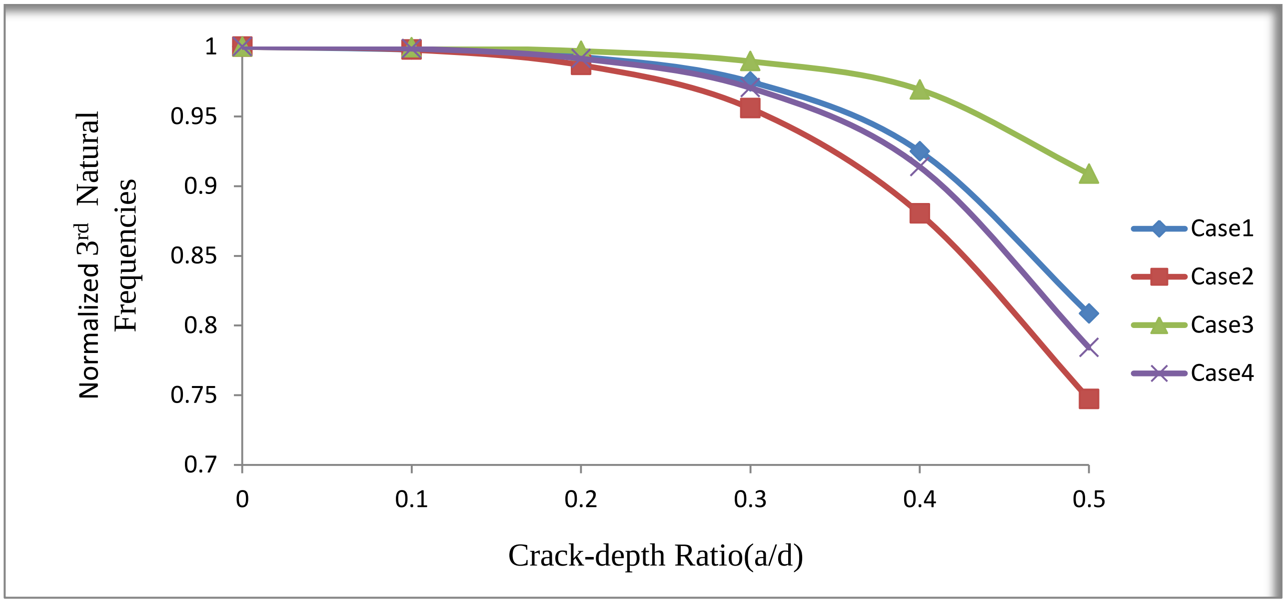


Figure 4.55: Comparison of normalized 3rd natural frequencies of multiple cracked two stepped rectangular beams with respect to crack-depth ratio.

In Figure4.55 it is observed that when cracks are located at free end of the beam the 3rd frequency reduction is higher followed by cracks located at the step locations of the beam and cracks located near to free end of the beam. It is observed when crack is located at the center of the steps of the two stepped beam the 3rd normalized natural frequencies variation is less.

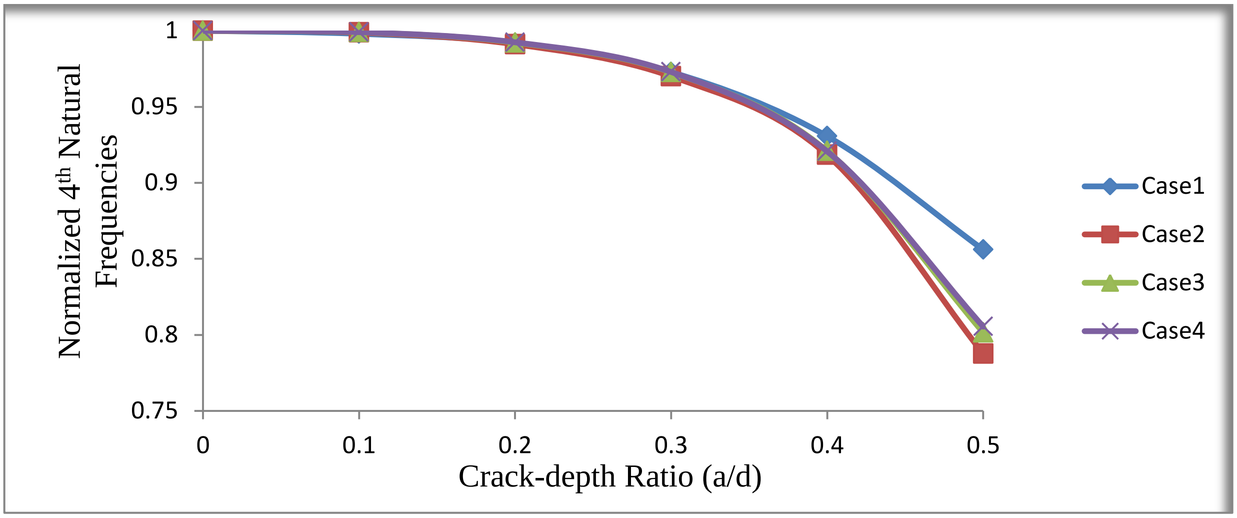


Figure 4.56: Comparison of normalized 4th natural frequencies of multiple cracked two stepped rectangular beams with respect to crack-depth ratio.

In Figure4.56 it is observed that when cracks are located at the free end of the two stepped beam the frequency reduction is less whereas when cracks are located at free end of the two stepped beam, center of the steps of the two stepped beam, step location of the beam the 4th normalized natural frequencies reduction variation is very less among each other.

From Figure4.53-4.56, it is concluded that in case 3 when cracks are located at the center of the steps of the two stepped beam the 2nd and 3rd natural frequency reductions are least affected.

CHAPTER 5
CONCLUSIONS & RECOMMENDATIONS

CHAPTER 5

CONCLUSIONS

The presence of number of cracks, crack location, crack-depth ratio the analysis of dynamic properties of the beam is done by finding the natural frequencies. The following conclusions are drawn from the present investigation of the uniform and stepped beams subjected to vibrate freely with multiple cracks using finite element analysis by using Finite Element Method (FEM) in MATLAB environment.

- A detailed formulation is presented for free vibration of uniform and stepped beam with multiple transverse open cracks.
- The frequency reduction increases as the crack-depth ratio increases for all the modes irrespective of uniform beam or stepped beam.
- Crack located closer of the fixed end of the beams in all cases frequency reduction variation is significant than crack closer to the free end of the beam even when the crack-depth ratio is relatively high.
- Crack located closer of the free end of the beams in all cases will have higher effect on the 4^{th} natural- frequencies than crack closer to the fixed end of the beam.
- Crack located at the center of the uniform beam will have higher 3^{rd} natural frequency reduction.
- For all the beams either uniform beams or stepped beams irrespective of number of steps as the number of cracks present in the beam enhances, frequency occurred naturally of the beams reduces for extact crack-depth ratio due to reduction of stiffness.
- The frequency reduction is higher for uniform rectangular beam than uniform circular beam for both beams having same moment of inertia.
- When crack is located at near fixed end or at center or near free end of the beam i.e., for any crack location along the length of the beam the 3^{rd} natural frequency reduction for rectangular beam is more than circular beam.
- Irrespective of number of cracks present in the beam the uniform beams show similar pattern of variation for all the normalized frequencies.

- Positions of cracks present along the length of the beam across uniform, single step, two stepped beams the major variation in the frequency reduction starts when crack-depth ratio is 0.3 and increases up to crack-depth ratio 0.5.
- For uniform beam or stepped beam with single or multiple cracks when located near the free end of the beam the fundamental frequency reduction is highest.

From the above discussions, it is clear that cracks cause the reduction of natural frequency. The presence of multiple cracks weakens the beam from the point of view of reduction in natural frequency. So cracks play a critical role on the vibration behaviour of the structures. The vibration behaviour of cracked circular and rectangular uniform as well as stepped beams is influenced by the geometry, material, location and size of cracks. The figures dealing with variation of the frequencies are recommended for identification of crack location and intensity for uniform and stepped beams. The above recommendations for design of beams are valid within the range of geometry and material considered in this study. So the designer has to be careful while dealing with structures subjected to cracks. This can be used to the advantage of design of stepped beams. The vibration characteristics of the cracked beams can be used as a tool for structural health monitoring, identification of crack location and extend of damage in beams and also helps in assessment of structural integrity of the structures.

5.2 SCOPE FOR FUTURE STUDY

1. The complete analysis of the current work is carried out based on the Bernoulli-Euler beam structure and it can be extended for Timoshenko beam based structure for hygrothermal effects.
2. Comparison of the analytical results of present analysis can be done with experimental results using FFT Analyser.
3. The present study can be extended to study the effects of various parameters such as natural frequencies and bending modes for multi-cracked stepped beams of circular cross-sections.
4. The study of free vibrational analysis of beams presently done can be extended by studying the buckling analysis of stepped beams.
5. This study can be extended to study the variations in the dynamics parameters of the composite stepped beams with multiple cracks.

CHAPTER 6
REFERENCES

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1. Ameneh M., Amin G., and Hamid R. M. (2012), "Multiple-crack damage detection in multi-step beams by a novel local flexibility-based damage index", *Journal of sound and vibration,* 332, 294-305.

2. Almeida M. T., and Dias J. C. (1984), "Free vibration analysis of beams and shafts by programmable calculators", *journal of sound and vibration*, 92(1), 39-45.

3. Chondros T. G., Dimarogonas A. D., and Yao J. (1998), "A continuous cracked beam vibration theory", *Journal of sound and vibration*, 215(1), 17-34.

4. Chondros T. G., Dimarogonas A. D., and Yao J. (2001), "Vibration of a beam with a breathing crack", *Journal of sound and vibration*, 239(1), 57-67.

5. Douka E., Loutridis S., and Trochidis A. (2003), "Crack identification in beams using wavelet analysis", *International journal of solids and structures*, 40, 3557-3569.

6. Friswell M. I., and Penny J. E. T., (2002), "Crack Modelling for structural health monitoring", *Structural health monitoring,* 1(2), 0139-0148.

7. Guohui D., and Xinwei W. (2013), "Free vibration analysis of multiple-stepped beams by the discrete singular convolution", *Applied Mathematics and Computation*, 219, 11096-11109.

8. H M., and Shen H. (1994), "Free vibrations of beams with a single-edge crack", *Journal of sound and vibration*, 170(2), 237-259.

9. Jang S. K., and Bert C. W. (1988), "Free vibration of stepped beams: Exact and numerical solutions", *Journal of sound and vibration,* 130(2), 342-346.

10. Jaworski J. W., and Dowell D. H. (2008), "Free vibration of a cantilevered beam with multiple steps", *Journal of Sound and Vibration*, 312, 713–725.

11. Jang S. K., and Bert C. W. (1988), "Free vibration of stepped beams: Higher mode frequencies and effects of steps on frequency", *Journal of sound and vibration,* 132(1), 164-168.

12. Kisa M., Brandon J., and Topcu M. (1998), "Free Vibrational Analysis of Cracked beams by combination of finite elements and component mode synthesis method", *Computers and Structures*, 67, 215-223.

13. Kisa M., and Arif G. (2006), "Modal analysis of multi-cracked beams with circular cross section", *Engineering Fracture Mechanics*, 73, 963–977.

14. Kisa M. and Arif G. (2007), "Free vibration analysis of uniform and stepped cracked Beams with circular cross sections", *International Journal of Engineering Science*, 45, 364-380.

15. Krawczuk M., and Ostachowicz W. (1996), "Damage indicators for diagnostic of fatigue cracks in structures by vibration measurements", *Journal of theoretical and applied mechanics*, 34(2), 307-326.

16. Koplow M. A., Bhattacharyya A., and Mann B. P. (2006), "Closed form solutions for the dynamic response of Euler-Bernoulli beams with step changes in cross-section", *Journal of sound and vibration*, 295, 214-225.

17. Krawczuk M., (1993), "A new finite element for static and dynamic analysis of cracked composite beams", *International journal computers and beams*, 43, 1-37.

18. Lu Z. R., Huang M., and Liu J. K. (2011), "Vibration analysis of beams with and without cracks using the composite element model", *Advances in vibration analysis research,* 2, 23-36.

19. Mostafa A. (2012), "A transfer matrix method for free vibration analysis and crack idenfication of stepped beams with multiple edge cracks and different boundary conditions", International *journal of Mechanical sciences*, 57, 19-33.

20. Pandey A. K., Biswas M., and Samman M. (1991), "Damage detection from changes in curvature mode shapes", *Journal of sound and vibration*, 145(2), 321-332.

21. Qibo Mao. (2011), "Free vibration analysis of multiple-stepped beams by using Adomain decomposition method", 54, 756–764.

22. Rizos P. S., Aspragathos T., and Dimarogonas, A. D. (1989). "Identification of crack location and magnitude in a cantilever beam from the vibration modes", *Journal of Sound and Vibration*, 138(3), 381-388.

23. Subramaniam G., and Balasubramanian T. S. (1987), "Beneficial effects of steps on the free vibration characteristics of beams", *Journal of sound and vibration*, 118(3), 555-560.

24. Shifrin E. I., and Rutolo R. (1999). "Natural frequencies of a beam with an arbitrary number of cracks", *Journal of Sound and Vibration*, 222(3), 409-423.

25. Saavedra P. N., and Cuitino L. A. (2001), "Crack detection and vibration behavior of cracked beams", *Computers and structures*, 79, 1451-1459.

26. Shao R., Dong F., Wang W., and Jia P. (2013), "Influence of cracks on dynamic characteristics and stress intensity factor of gears", *Engineering failure analysis*, 32, 63-80.

27. Shen M. H., and Pierre C. (1986), "Modes of free vibrations of cracked beams", 1-46.

28. Xinwei W., and Yongliang W. (2013), "Free vibration analysis of multiple-stepped beams." *Applied Mathematics and Computation*, 219, 5802–5810.

29. Young-Shin., Myung-Jee., and Chung. (1999), "A study on crack detection using eigen frequency test data", *Computers and Structures*, 77 (2000) 327-342.

30. Zheng D. Y., and Kessissoglou N. J. K. (2004), "Free Vibrational Analysis of Cracked beam by finite element method", *Journal of Sound and Vibration*, 273, 457-475.

31. Zhang W., Wang Z., and Hongwei M. (2009), "Crack identification In stepped cantilever beam combining Wavelet Analysis with Transform matrix", *Acta Mechanica Solida Sinica*, 22(4), 405-421.