# EFFECTS OF HIGHWAY GEOMETRIC ELEMENTS ON ACCIDENT MODELLING

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# EFFECTS OF HIGHWAY GEOMETRIC ELEMENTS ON ACCIDENT MODELLING

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### CERTIFICATE

This is to certify that the thesis entitled, "EFFECTS OF HIGHWAY GEOMETRIC ELEMENTS ON ACCIDENT MODELLING" submitted by Mohita Mohan Garnaik bearing roll no. 611CE305 in partial fulfilment of the requirements for the award of Master of Technology (Research) degree in Civil Engineering with specialization in "Transportation Engineering" during 2012-2014 session at the National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

January, 2014

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#### ABSTRACT

The tremendous traffic growth generally observed in road transportation has led to a lot of negative consequences in the form of road accidents both in developed and developing countries. This observation calls for considerable attention towards development a system for the road safety mechanism of rural highway. Road accident prediction plays an important role in accessing and improving the road safety. Fuzzy logic is one of the popular techniques in the broad field of artificial intelligence and ability to improve performance similar to human reasoning and describe complex systems in linguistic terms instead of numerical values. In this thesis, a system was established based on Fuzzy Inference System (FIS) in which output data such as traffic Accident Rate (AR) and input data such as various highway geometric elements. The study was conducted on two road segments from plain & rolling terrain highway and two road segments from mountainous & steep terrain highway within the rural area of the Indian Territory. Two Highway Accident Rate Prediction Models (HARPMPRT and  $HARPM_{MST}$ ) were developed due to the complexity of geometric elements of rural highway on different terrain conditions which take horizontal radius, superelevation, K-value, vertical gradient and visibility as input variables and Accident Rate (AR) as output variables. The findings show that the proposed model can be effectively applied as a useful Road Safety tool capable of identifying risk factors related to the characteristics of the road and great support to the decision making of incident management in Intelligent Transportation Systems. Significant positive relationships were also identified between the geometric elements and accident rate. A simulation study and real life data analysis are performed to demonstrate model fitting performances of the proposed model.

Keywords: AADT, FIS, Fuzzy Logic, FCM, AR, DTM, TIN and SCF

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### Abbreviations and Notations

AADT	Average Annual Daily Traffic
ANOVA	Analysis of Variances
AR	Accident Rate
CoG	Center of Gravity
FIS	Fuzzy Inference System
F	F-test (F statistical hypothesis)
ITS	Intelligent Transportation Systems
NH	National Highway
$R^2$	Correlation Coefficient
SCF	Seasonal Correction Factor
MS	Mean of Square
SS	Sum of Square
VCL	Vertical Curve Length
А	A notation of set
U	A notation of set
Х	Element of set
Х	Universal set
У	Output for fuzzy inference system
α	Parameter of fuzzy set
β	Parameter of fuzzy set
σ	Parameter of fuzzy set
μ	Membership function of fuzzy set

#### Introduction

#### 1.1 Background

Motor vehicle accidents kill about 1.2 million people a year world-wide and the number will grow to more than 2 million in 2020 unless steps are taken, a study released by the World Health Organisation (WHO) and the World Bank has found [*Washington: Article-Traffic accidents becoming one of world's great killers, By Matthew Wald, April 8, 2004*]. WHO has revealed in its first Global Status Report on Road Safety that more people die in India due to road accidents than anywhere else in the world, including the more populous China [*New Delhi:Articles.timesofindia.indiatimes.com/2009-08-17*].

Road transport plays vital role in economic development, trade and social integration, which rely on the conveyance of both people and goods. Vehicular traffic carrying goods and people increases with the increasing economy resulting in an increase of traffic accidents. Three major factors causing traffic accidents are human, road and vehicles. The human factor has the most significant effect on accident. However, this factor is governed by an individual thought process and cannot be studied empirically. Moreover, any design solution mitigating this kind of individual human behaviour cannot be predicted only some safety rules can be enforced. Also, different mechanical behaviour of vehicles factors are not the scope of civil engineering study. Hence, road factors are only considered as a part of this study. It is very important for the highway to establish a harmony between the all the three factors at the design stage of a highway. With a geometrically good design, it is possible to compensate for the other factors and thus decrease the number of traffic accidents (A.F. Iyinam et al., 1997).

#### 1.2 Objectives

The high socio-economic cost of the injuries and fatalities, occurring due to road accidents and the need for effective policies for curbing road accidents make it imperative to study the causes of road accidents. The present study aims to detect and identify the role of alignment geometric elements on accident and prediction of accident rate through artificial intelligence system modelling.

#### **1.3** Basic Parameters of Highway Geometric

#### **1.3.1** Terrain/Topography

The classification of the terrain is done by means of cross slope of the country, i.e., slope approximately perpendicular to the center line of the highway location. To characterize variations in topography, engineers separate it into four classifications according to terrain as listed in Table 1.1.

Terrain Classification	Cross slope of country (%)
Plain	Less than 10
Rolling	Greater than 10 up to 25
Mountainous	Greater than 25 up to 60
Steep	Greater than 60

 Table 1.1: Terrain Classifications

#### 1.3.2 Speed

Speed is defined as the distance covered per unit time. Since speed of every vehicle is impossible to track on a roadway; therefore, in practice, average speed is based on the sampling of vehicles over a period of time on a particular section of road. Speed is one of the most important factors considered by travellers in selecting alternative routes. The speed of a vehicle on a road depends upon five conditions: physical characteristics of the highway, amount of roadside interference, weather, presence of other vehicles, and the speed limitations in addition to the capabilities of the driver and their vehicles. It is the basic parameter which determines all other geometric features of the highway.

#### 1.3.3 Horizontal Alignment

The horizontal alignment is the route of the highway, defined as a series of horizontal tangents and curves. Horizontal curve is the curve in plan to change the direction of the center line of the highway. The geometries of horizontal alignment are based on an appropriate relationship between design speed and curvature and on their joint relationship with superelevation and side friction. Typical horizontal curve furnished in figure 1.1as per Indian Road Congress (IRC) guidelines (IRC: 38-1988 & IRC: 73-1980).

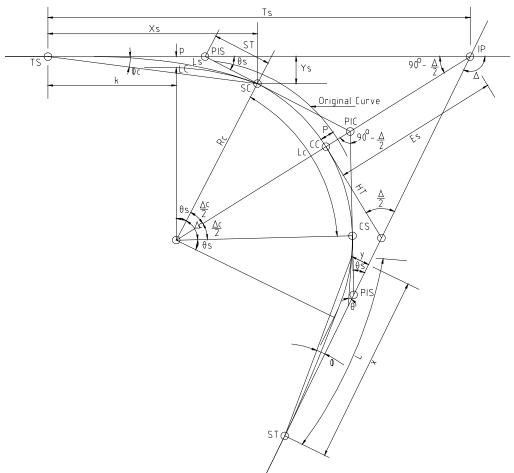


Figure 1.1: Typical Horizontal Curve

#### Where

- Δ : Total deflection angle L
- IP Intersection point of tangents :
- RC Radius of circular curve :
- LS : Length of spiral curve
- LC : Length of circular curve

- Total length of curve :
- ST : Short tangent
- LT Long tangent :
- Ts : Total tangent distance
- External distance Es •

#### **1.3.4** Vertical Alignment

Vertical alignment is the longitudinal section of a roadway to provide easy and safe change of gradient. It is defined as a series of gradients and vertical curves. Gradient is the rate of rise or fall with respect to the horizontal along the length of a road expressed as a percentage or as a ratio or in degrees. Vertical curves to effect gradual changes between gradients with any one of the crest or sag types and result is safe and comfortable in operation, pleasing in appearance, and adequate for drainage. The typical vertical curve in crest condition is furnished in figure 1.2 as per IRC: 23-1989.

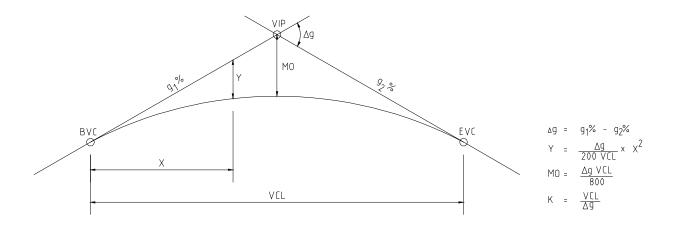


Figure 1.2: Typical Vertical Curve

#### Where

VIP	:	Vertical point of intersection.
g	:	Gradient
MO	:	Mid-ordinate
$\Delta g$	:	Algebraic difference in grades (percent) of the grades tangents.
VCL	:	Vertical curve length measured horizontally.
BVC	:	Beginning of vertical curve
EVC	:	End of vertical curve
Κ	:	Horizontal distance required to effect a one percent change in gradient.

#### 1.3.5 Cross Section

Cross section is defined as the number of lanes and lane width including cross fall, shoulder, sidewalk, earth slope and drainage features in the transverse direction of the roadway. The cross section shows the total formation of the road.

#### 1.3.6 Superelevation

Superelevation is tilting or banking the roadway to counteract the centripetal force developed as the vehicle moves around the horizontal curve. When a vehicle moves in circular path, it undergoes a centripetal acceleration that acts towards the centre of curvature. This acceleration is sustained by a component of the vehicle's weight related to the roadway superelevation, by the side friction developed between the vehicle's tires and or the pavement surface.

#### 1.3.7 Sight Distance

Sight distance is the distance along the road surface at which a driver has visibility of object at a specified height above the carriageway. This is the adequate length along the highway in the different situations to permit drivers enough time and distance to control their vehicles so as to avoid unforeseen accidents.

#### **1.3.8 Traffic Volume**

#### 1.3.8.1 Average Daily Traffic (ADT)

Traffic volume is defined as total traffic movement on the highway in both directions at a particular point in terms of average daily traffic (ADT) volume. The ADT is defined as the total volume during a given period, greater than one day and less than one year, divided by the number of days in that time period.

#### **1.3.8.2** Annual Average Daily Traffic (AADT)

Annual Average Daily Traffic (AADT) is defined as the total traffic volume passing a point or segment of a highway facility in both directions for one year divided by the number of days in the year. It is one of the important traffic variables required for analysis of traffic crash rates.

AADT is calculated by incorporating the seasonal variations in traffic movement on the study road. Seasonality aspect can be captured through direct variables such as month-wise classified traffic count data for past one or two years at different road locations. Other methods involve identifying surrogate variables such as monthly sales of petroleum products, monthly tourist data, monthly traffic record at check-posts, etc.

#### **1.4** Accident Statistics

#### 1.4.1 Cost of Road Accident

Road accidents carry high economic and social costs, which are not easy to measure. The cost of road related injuries and accidents can be viewed in terms of (a) medical costs (b) other cost related to administrative legal and police expenditure (c) collateral damage in terms of damage to property and motor vehicle and (d) loss due to income. In addition, accident survivors often live a poor quality of life and have to live with pain and suffering which are difficult to estimate. In economic terms, the cost of road crash injuries creates direct impact to gross domestic product (GDP) of the country.

#### 1.4.2 Profile of Road Accident

The total numbers of accidents reported by all the States/ Union Territories (UTs) in the year 2012 were 4.90 lakhs of which 1.38 lakh people were killed and more than 5 lakh persons

injured, many of whom are disabled for rest of their lives (*Source: Ministry of Road Transport & Highways*). These numbers translate into one road accident every minute, and one road accident death in less than four minutes. Occurrence of accidents is an outcome of factors which include type of road users, colliding vehicles, environmental/road related factors (road geometry, design, visibility etc), vehicle related, nature of traffic management, composition and flow of road traffic and adherence/enforcement of road safety regulations. The main thrust of accident prevention and control across the world has been on Education, Enforcement, Engineering and Environment & Emergency care of accident victims.

#### 1.4.3 Spatial Distribution of Road Accidents (Urban vis-à-vis Rural)

In 2012, the total number of accidents that occurred in rural areas was at 54.3 per cent while the rest occurred in urban areas. The number of persons injured in rural areas was also higher at 60.2 per cent as compared to urban areas.

#### 1.4.4 Time of Occurrence of Road Accidents

For framing strategies for prevention and organization of care of accident victims, information on timing of accidents is a prerequisite. During 2012, high rates of road accidents were observed between 3pm–6pm, 9am–12am and 6pm–9pm. The distribution of the total accidents during night time (6pm to 6am) and day time (6am to 6pm) is approximately in the ratio of 2:3 i.e. about 40 per cent during night time and 60 per cent during daytime.

Motor vehicle population has recorded significant growth over the year. The motor vehicle growth is higher than the rate of road network developed in India. Different elements of highway geometric as well as surface condition of highway have great influence in occurrence of traffic accidents. To minimize traffic accidents, great attention has to be paid in achieving consistency in highway design, minimize the frequency and extent of violations of driver expectancy and emphasis on concerns of three-dimensional (3D) highway design to achieve a "safe-by-design" (George Kanellaidis et al.,2011).

The current chapter, contains the motivation, aim and scope of the study and the main hypothesis of the study as an introduction.

*In next chapter*, traffic accident prediction models in the literature were examined, and it was recognized that numbers of researcher widely used Linear and Negative Binomial Models. As a result, the use of fuzzy logic method was decided to establish accident rate prediction model.

#### **Literature Review**

The study is an effective traffic accident modelling in minimizing the accident rates depending on road factors and finding the impact of highway geometric elements. Hence, a literature survey was carried out in the field of accident causative factors and accident prediction and optimisation modelling and presented as below.

#### 2.1 Accident Causative Factors Overview

Feng-Bor Lin (1990) studied on flattening of horizontal curve on rural two lane highways and found that horizontal curves on highways are on average more hazardous than tangent sections. As their curvatures increase, horizontal curves tend to have higher accident rates. He suggests that the differences between the 85<sup>th</sup> percentile speeds and the safe speeds have no statistically significant relationships with the accident rates. In contrast, the magnitudes of speed reduction, when vehicle moves from a tangent section to a curve, have a significant impact on traffic safety. Such speed reductions on horizontal curve with gentle grades are strongly correlated with the curvatures of the curves. Therefore, curvatures can be used as a safety indicator of the curves.

Y. Hassan et al. (2003) studied on effect of vertical alignment on driver perception of horizontal curves and found that perception of the driver of the road features ahead is an important human factor and should be addressed in road design. An erroneous perception of the road can lead to actions that may compromise traffic safety and poor coordination of horizontal and vertical alignments is believed to cause such wrong perceptions. Through statistical analysis, they suggested that the horizontal curvature looked consistently sharper when it overlapped with a crest curve and consistently flatter when it overlaps with a sag curve.

Zhang Yingxue (2009) analysed the relation between highway horizontal curve and traffic safety and found that curve radius, super-elevation, widening, transition curve and sight distance have the important effect on traffic accidents,

Ali Aram (2010) studied on effective safety factors on horizontal curves of two-lane highway and observed that several traffic volumes and mix, geometric features of the curve, cross section, roadside hazards, stopping sight distance, curve coordination, pavement friction and traffic control devices affect the safety performance of horizontal curve. He found that degree of horizontal curve, length of curve, superelevation, transition length, shoulder width and ADT responses are the important independent effective variables. He also suggested that horizontal curves have higher crash rates than straight section of similar length and traffic composition.

Kay Fitzpatrick et al. (2010) studied on horizontal curve accident modification factor with consideration of driveway density on rural four-lane highways in Texas. They developed horizontal curve accident modification factor (AMF) for rural four-lane divided and undivided highways and determined the effect of driveway density is different for horizontal curves as compared to tangent sections. Negative binomial regression models were used to determine the effects of independent variables on crashes.

George Kanellaidis et al. (2011) studied highway geometric design from the perspective of recent safety developments and suggested emphasis on concerns of three-dimensional (3D) highway design to achieve a "safe-by-design".

#### 2.2 Accident Prediction Model

Eric T. Donnell et al. (2009) studied on appraisal of the interactive highway safety design model's crash prediction and design consistency modules and evaluated the safety and operational effects of geometric on two lane rural highways through interactive highway safety design model (IHSDM). The design consistency module can evaluate the alignment complexity and thus predict the accident.

Jaisung Choi et al. (2011) studied on the safety effects of highway terrain types in a crash model and suggested that when the design speed is changed, the terrain types will have some safety effects using regression analysis. The statistical analysis was performed with an ordinal logistic regression model in order to relate several independent variables of highway geometric elements such as terrain type, tangent length, curve length, radius of curvature and vertical grade to actual crash occurrences. Through this investigation, terrain type was found to be a significant independent variable that explains crash occurrences for rural arterial roads in South Korea.

O. F. Cansiz et al. (2011) studied artificial neural network to predict collisions on horizontal tangents of 3D two-lane highways and explored the safety effects of horizontal tangents combined with vertical curves using artificial neural network (ANN) models. The collision prediction models were established using artificial neural network for these horizontal tangents and were compared with the existing regression models. The ANN method provided better results for predicting collision frequency on horizontal tangents. They identified the variables which are related to vertical curves, horizontal tangents, and cross-sections. The regression models were estimated using the significant variables for all combinations.

Fajaruddin Mustakim et al. (2011) were developed accident predictive models for rural roadway in Malaysia using multiple non-linear regression method with the road and traffic flow explanatory variable and concluded that, the existing number of major access points, without traffic light, rise in speed, increasing number of Annual Average Daily Traffic (AADT), growing number of motorcycle and motorcar and reducing the time gap are the potential contributors of increment accident rates on multiple rural roadway.

Iljoon Chang et al. (2012) were developed model for identifying accident-prone spots based on the total number of accidents. They considered a mixture of the zero-inflated Poisson and the Poisson regression models to analyze zero-inflated data sets drawn from traffic accident studies.

Miloud Driss et al. (2013) studied on traffic accident prediction system based on fuzzy logic which allows to identify "the degree of exposure to road accidents' risk", and to analyze the level of complexity of the factors involved. A Geographic Information System (GIS) was integrated into the analysis process to enable a spatial visualization of the degrees of exposure to road accidents' risk, provided a cartographically measurable solution to establish and attenuate accident risk. The developed system can be effectively applied to identifying risk factors related to the characteristics of the road.

#### 2.3 Accident Optimisation Model

A.F. Iyinam et al. (1997) studied relationship between highway safety and road geometric design elements and observed that the relationship between safety and road geometric has meaningful relationships through regression analysis. They suggested that the control of the road factor is much easier than the human factor and by making a geometrically good design, it was even possible to compensate for the other factors and thus decrease the number of

traffic accidents through a regression analysis is made between the geometric parameters and accident rates.

#### 2.4 Summary of the Literature Review

Highway geometric elements have great influence in traffic accidents and also effective factors on highway safety. As the relationships between highway accidents and highway geometric elements are considered some relationships are seen intuitively at a first approach. However, the important point is to determine the intensity of these relationships quantitatively.

Apart from the above study, many researchers recommended other promising methodologies like artificial neural networks, fuzzy methods and genetic algorithms for development of crash model.

#### **Empirical Data Collection and Extraction**

#### 3.1 Background

For this study, two roads in plain & rolling terrain National Highway (NH) 23 &200 and two roads in mountainous & steep terrain National Highway 22 & 87 were selected. Various field data such as 3D Topographic features, Accident records and Traffic volume were collected for these roads. Careful observation and collection of such data with accuracy were carried out.

National Highway 22 (NH-22) is a 459 km National Highway in Northern India that runs from Ambala through Chandigarh and Himachal Pradesh up to Khab on the Chinese border. The study corridor takes off from km 230/0 near village Narkanda, traverse in south-northeast direction and end at km 330/0 near village Wangtu. This part of the road falls in the state of Himachal Pradesh and situated between 31° 18.78' N & 31° 44.48' N latitude, 77° 27.28' E & 78° 44.14' E longitude. Total length of the study corridor is about 100km and study area is given under Figure 3.1a.



Figure 3.1a: Location Map of NH-22

National Highway 87 (NH-87) is a National Highway in Northern India. NH-87, which runs for a distance of 316 km, links Rampur in Uttar Pradesh with Karna Prayag in Uttarakhand. The study corridor takes off from km 10/0 at village Jeolikote, traverse in south-north direction and end at km 80/0 near village Kharkuna. This part of the road falls in the state of Uttarakhand and situated between 29° 20.65' N & 29° 38.33' N latitude, 79° 28.98' E & 79° 34.56' E longitude. Total length of the study corridor is about 70km and the study area is given under Figure 3.1b.

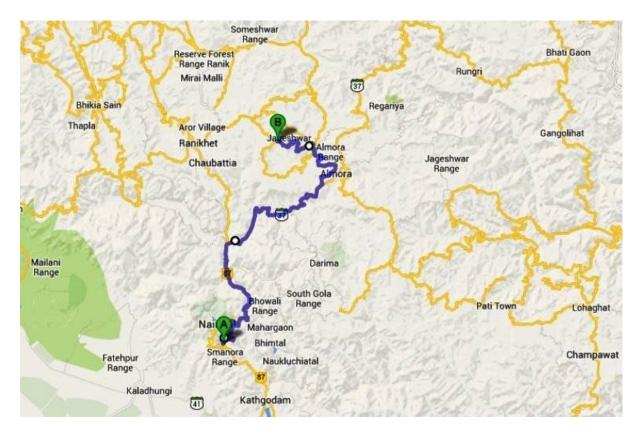


Figure 3.1b: Location Map of NH-87

National Highway 23 (NH-23) is a 459 km National Highway in central-eastern India that connects Chas in Jharkhand with the intersection of National Highway 42 at Banarpal in Odisha. The study corridor takes off from km 338/0 at village Pallahara, traverse in north-south direction and end at km 405/0 near Talcher. This part of the road falls in the state of Odisha and situated between 21° 25.92' N & 20° 56.84' N latitude, 85° 11.21'E & 85° 16.30'E longitude. Total length of the study corridor is about 67km and study area is given under Figure 3.1c.

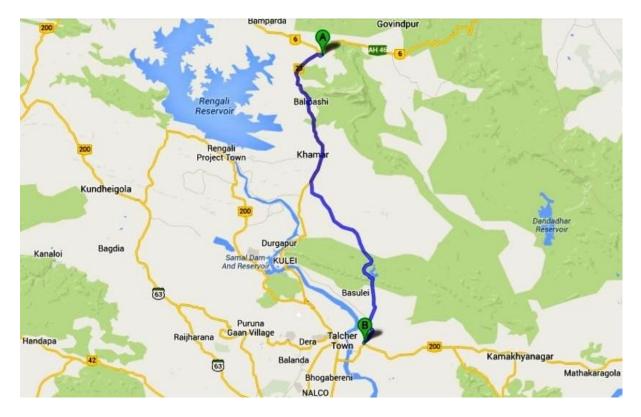


Figure 3.1c: Location Map of NH-23

National Highway 200 (NH-200) is a 740 km National Highway in central-eastern India that connects Raipur, the state capital of Chhattisgarh to Chandikhole in Odisha. The study corridor takes off from 130/0 at village Bhojpur, traverse in north-south direction and end at km 190/0 near village Chhatabar. This part of the road falls in the state of Odisha and situated between 21° 25.92' N & 20° 56.84' N latitude, 85° 11.21' E & 85° 16.30' E longitude. Total length of the study corridor is about 60km and the study area is given under Figure 3.1d.



Figure 3.1d: Location Map of NH-200

#### **3.2 Data Collection**

#### 3.2.1 Topographic Survey

The topographic survey has been carried out with Total Station survey equipment at accident locations. Total Station is a high precision surveying equipment to carry out 3-dimensional feature of the existing road. This survey equipment can measure distance, angle, and coordinates with relative to the known position and it calculates using coordinate geometry and triangulation. All the measurements are controlled by an internal programme & interfaced via computer. The captured digital data has been downloaded into a CAD programme (AutoCAD) to visualize the surveying data as vector entities. Finally, this data has been analysed later with the design application software (MX Road) which is extensively used for highway design.

#### 3.2.1.1 Topographic Survey Methodology

In order to have the accurate topographical survey work, a network of horizontal control has been established using differential GPS techniques and levelling network using Digital Auto Levels. The fixing of major control stations, the station points are embedded in the ground, in pair with inter visibility with a distance of around 200m. The horizontal control coordinates were observed and worked out by use of GPS instrument for each pair and the elevation were provided by an independent levelling survey by Digital Auto Level. These Major Control Stations were kept at a distance of 500m at a safe location.

The control traverse is the base framework for all the further survey work. This provides a coordinated horizontal grid and a level reference system to ensure accuracy. Thus the measured coordinates of these survey grids (Northing and Easting) and the levels are to be tied to GTS benchmark wherever available, to verify the accuracy of survey. The GPS/ Benchmark points and Reference Benchmark points established acted as both horizontal and vertical control points.

Levels along centre line of the existing road are taken at every 10m interval and at all intermediate breaks in ground using Total Station. The said spacing has suitably reduced at horizontal curves to 5m. Cross sections are taken at every 20 m intervals and at each cross section the survey normally extends beyond 20m on each side of the existing road center line with survey points at 5-10 m apart and at all variations in the natural ground or breaks in level. The topographic survey thus carried out contains the details of all physical and topographical features within the survey corridor. The survey data were processed and converted to graphic files using Highway Design software called "MX Road".

The 3D topographic points have been carried out along the road as given under Figure 3.2 and Figure 3.3.

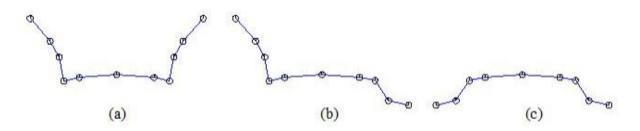


Figure 3.2: Topographic Points on Cross Sectional View

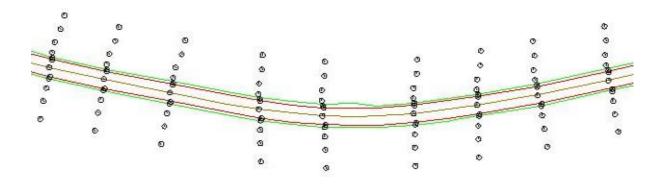


Figure 3.3: Topographic Points on Plan View

#### 3.2.2 Accident Record

Accident records have been collected from concerned police station from their accident record books. All these accident points have been verified with policemen as well as local villagers at the site.

#### 3.2.3 Traffic Volume

The primary objectives of traffic surveys are to determine the characteristics of traffic movement in the study corridor. To find out the annual average daily traffic (AADT), three

days traffic survey has been carried out at the respective location of the road. Also, the traffic volume has been collected from concerned department (Public Works Department) and verified with three day traffic survey to compare with the existing data.

### 3.2.3.1 Traffic Survey Methodology

The homogeneous traffic sections have been identified based on the locations of major intersection along the study corridor. The directional classified traffic volume count has been carried out for each traffic homogeneous section for 24 hours a day continuously for 3 days. The survey stations have been located, away from urban area and villages to minimize interference of local traffic. The traffic count data analysis would present realistic forecast of traffic volume. Classified manual traffic counts have been recorded in 15 minutes intervals, using Tally marks on a standard format. The vehicle classification has been prepared as per IRC: SP 19-1991 and IRC: 9-1972 code requirements as given in Table 3.1.

Motorized Vehicle	Non Motorised Vehicle
• 2-Wheeler & 3-Wheeler	• Cycle
Passenger Car	Cycle Rickshaw
• Utility Vehicle (Jeep, Van, etc.)	Animal Drawn Vehicle
• Bus (mini bus, standard bus)	Other Non-Motorized Vehicle
Light Commercial Vehicle (freight)	
• Truck	
<ul> <li>MCV (2-axle rigid chassis)</li> </ul>	
HCV (Multi-axle Rigid)	
HCV (Multi-axle articulated)	
Agricultural Tractors (with trailer/without trailer)	

 Table 3.1: Vehicles Classification System

The purpose of the survey was to calculate the Average Daily Traffic (ADT) on the traffic homogeneous sections and to convert the traffic into Annual Average Daily Traffic (AADT)

with the multiplication of seasonal correction factor. The seasonal correction factors (SCF) adopted for the calculation of AADT has been furnished in Table 3.2.

Sl. No.	Study Corridor	Seasonal Correction Factors (SCF)
1	NH-22	0.96
2	NH-23	1.15
3	NH-87	0.85
4	NH-200	1.10

Table 3.2: Seasonal Correction Factor (SCF)

The summary of daily traffic count present in Appendix-3.1 to Appendix-3.4 and AADT in all study corridors have been furnished in Table 3.3.

Sl. No.	National Highway No.	AADT
1	22	2108
2	23	5039
3	87	2300
4	200	2417

Table 3.3: Summary of Annual Average Daily Traffic

## **3.3 Data Extraction**

To find out the geometric parameters of the existing road, topographic survey points (X, Y and Z / Easting, Northing and Elevation), road center line, carriageway edge and shoulder edge line has been imported to MX Road software and a 3-dimensional digital terrain model (DTM) was developed. Then Triangulation Integrated Network (TIN) was modelled as 3-dimensional surface. After this, geometric elements such as horizontal curve radius and Horizontal curve length, deflection angle, superelevation / crossfall, vertical gradient, vertical

curve length and sight distance have been extracted. The Triangulation Integrated Network (3D Model) developed along the road is given under Figure 3.4.

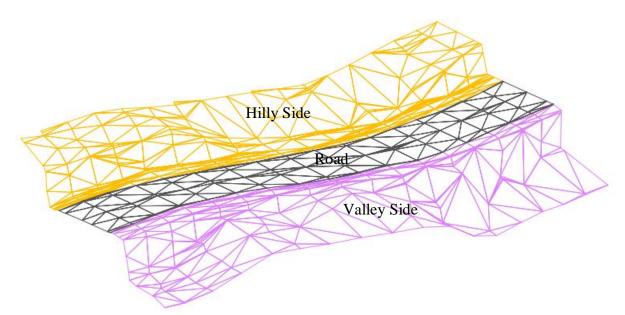


Figure 3.4: Typical Triangulation Integrated Network (TIN) of Existing Alignment

## 3.3.1 Data Extraction Methodology

The geometric elements of existing road at accident locations have been extracted as described under sub sections.

## 3.3.1.1 Horizontal Radius

The existing radius has been measured from the best fitted radius with the existing road alignment. In this process, the two tangents are fixed with the existing center line and then, radius fitted with the two tangents, which is the best fit with existing alignment. The unit of the radius is metre. Horizontal parameters extracted from existing alignment on an accident location have been furnished in Figure 3.5 (using AutoCAD) as expressive sample.

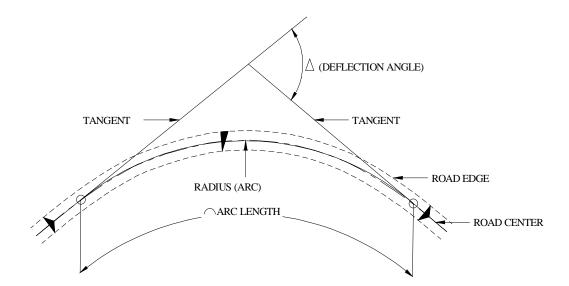


Figure 3.5: Data Extraction on Horizontal Alignment

### **3.3.1.2 Deflection Angle**

The tangents have been fixed on the existing road alignment on curve approaches. The total angle between the two tangents is called deflection angle and measured the same. The unit of the deflection angle is degree-minutes-seconds (DMS) or decimal degree.

#### 3.3.1.3 Horizontal Arc Length

The arc length has been measured from the best fit radius with the two tangents in existing road alignment. The unit of the arc length is metre.

#### 3.3.1.4 Superelevation / Cross Fall

The cross section of the existing road alignment has been developed from Digital Terrain Model (DTM) in every 10m interval through software on accident location. The maximum superelevation has been considered on outer side edge of carriageway in the horizontal curvature section. The unit of the superelevation is %.

#### **3.3.1.5** Rate of change of Superelevation

The cross section of the existing road alignment has been developed from Digital Terrain Model (DTM) in every 10m interval through software on accident location. The rate of change of superelevation is calculated from the average of the rate of change of cross fall of consecutive cross section in the superelevation runoff section. The unit of the rate of change of superelevation is 1 in n meter.

#### 3.3.1.6 Vertical Gradient

From the digital terrain model, the longitudinal section of the existing road center line has been developed with the software, and then, the maximum vertical gradient measured from the existing longitudinal section / profile of the road on accident location.

#### 3.3.1.7 Vertical Curve Length

From the digital terrain model, the longitudinal section of the existing road center line has been developed with the software and then the vertical gradient was fitted on existing longitudinal section on grade part and then the best fitted parabolic curve was fixed in between two grades. The best fitted vertical curve length has been measured. The unit of the vertical curve length is metre.

#### **3.3.1.8** K-value of Vertical Curve (K)

The K-value or equivalent radius of vertical curve defines the sharpness or flatness of the vertical curve. This is a ratio of parabolic curve length and change of vertical grade. Also, this is horizontal distance required to effect a one percent change in gradient. This is

expressed as:  $K = \frac{VCL}{\Delta g}$  Where, VCL = Vertical curve length and  $\Delta g$  = Change of grade

## 3.3.1.9 Visibility /Sight Distance

Sight distance has been measured with function of the horizontal and vertical alignment, in a 3D coordinate system. The actual sight distance has been generated along the road alignment through software considering with eye height and object height as 1.2m and 0.15m respectively. The achieved sight distance has been calculated in every 10m interval and the minimum sight distance is considered at the accident location. The figure 3.6 shows an example of measurement of sight distance along the existing roadway.

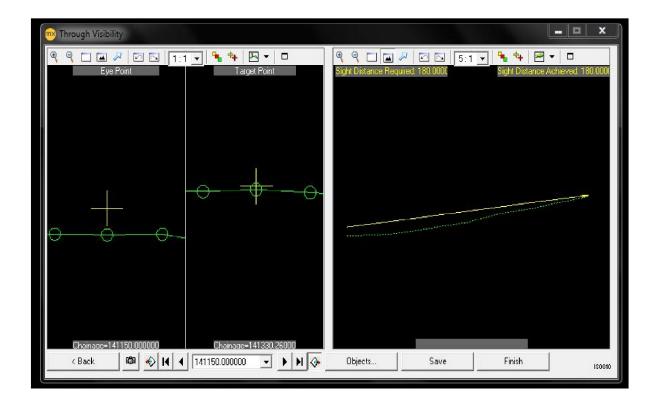


Figure 3.6a: Sight Distance Achieved on Road Alignment

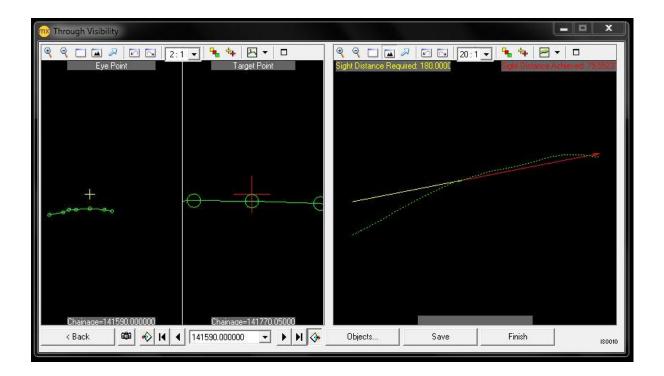


Figure 3.6b: Sight Distance Not Achieved on Road Alignment

#### 3.4 Summary

Total 339 numbers of accident locations have been considered on the 297km segment of road length in the plain & rolling and mountainous & steep terrain in rural areas. Data collected include: accident counts, traffic volume count (Average Daily Traffic), 3-D topographic survey of horizontal and vertical alignments.

The details of geometric elements of highway have been extracted through software with high accuracy level. These geometric elements are horizontal radius, deflection angle, horizontal arc length, superelevation, rate of change of superelevation, vertical gradient, vertical curve length, K-value and visibility/sight distance.

The collected data provide a comparatively varied and comprehensive basis for analysis and modeling. Driver and vehicle characteristics were neither collected, nor considered for accident modeling. The highway geometric parameters have been analysed with accident rate in the next chapter.

# Analysis of Empirical Data and Results

### 4.1 Background

Accident analysis has been carried out in order to determine the effects of different geometric elements of the highway with accident rate of the same highway. These geometric elements are horizontal radius, deflection angle, horizontal arc length, superelevation, rate of change of superelevation, vertical gradient, vertical curve length, K-value and visibility/sight distance. Finally, these geometric elements are statistically analysed and considered for model development which are statistically significant.

### 4.2 Accident Rate

The accident rate is defined as the ratio between the number of accidents which happened in a given year and the number of vehicles with kilometres of travels length during that same year. It is generally expressed in crashes per million vehicle-kilometres of travel.

$$AR = \frac{C \times 100,000,000}{V \times 365 \times N \times L}$$

The variables in this equation are:

AR = Accident Rate expressed as crashes per 100 million vehicle-kms of travel (100mvkm)

- C = Total number of crashes in the study period
- V = Traffic volumes using Annual Average Daily Traffic (AADT)
- N = Number of years of data
- L = Length of the roadway in km

The summary of Accident Rate in all study corridors have been furnished in Table 4.1.

		5	e	5
Variables	NH-22	NH-23	NH-87	NH-200
С	58	165	62	54
V	2108	5039	2300	2417
N	3	3	3	3
L	100	66.5	70	60
AR	25.13	44.97	35.17	34.01

Table 4.1: Summary of Accident Rate of Highway

## 4.3 Analysis of Geometric Variables

The existing geometric elements of highway has been analysed with accident rate of the same highway. Also, the variables are grouped with same manner as per highway terrain condition.

## 4.3.1 Analysis of Accident Rate versus Horizontal Radius

Total number of accidents has been counted within appropriate range of radius and then, the accident rate has been calculated as illustrated in *Appendix-4.1* and same has been plotted in Figure 4.1.

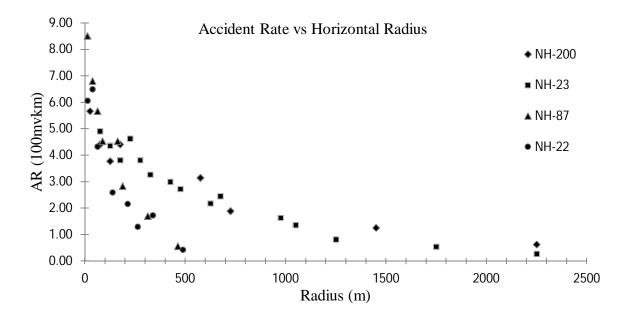


Figure 4.1: Accident Rate versus Horizontal Radius

### 4.3.2 Analysis of Accident Rate versus Deflection Angle

Total number of accidents has been counted in every  $10^0$  interval of deflection angle and then, the accident rate has been calculated as illustrated in *Appendix-4.2* and same has been plotted in Figure 4.2.

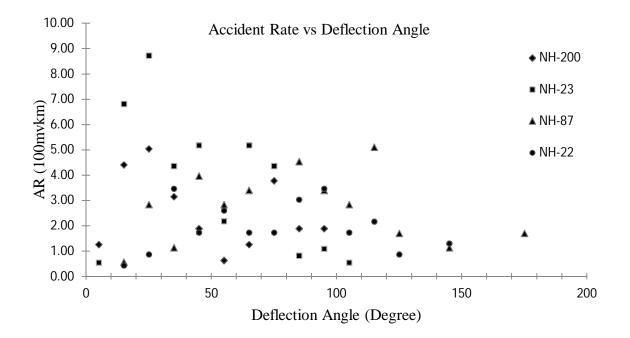


Figure 4.2: Accident Rate versus Deflection Angle

## 4.3.3 Analysis of Accident Rate versus Horizontal Arc Length

Total number of accidents has been counted within appropriate range of arc length and then, the accident rate has been calculated as illustrated in *Appendix-4.3* and same has been plotted in Figure 4.3.

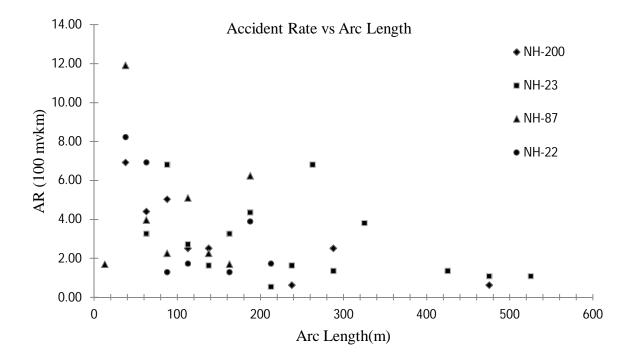


Figure 4.3: Accident Rate versus Horizontal Arc Length

# 4.3.4 Analysis of Accident Rate versus Superelevation

Total number of accidents has been counted in every 1% interval of superelevation and then, the accident rate has been calculated as illustrated in *Appendix-4.4* and same has been plotted in Figure 4.4.

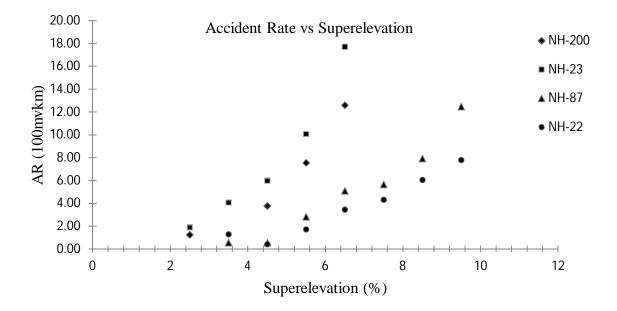


Figure 4.4: Accident Rate versus Superelevation

## 4.3.5 Analysis of Accident Rate versus Rate of change of Superelevation

Total number of accidents has been counted within appropriate range of rate of change of superelevation and then, the accident rate has been calculated as illustrated in *Appendix-4.5*and same has been plotted in Figure 4.5.

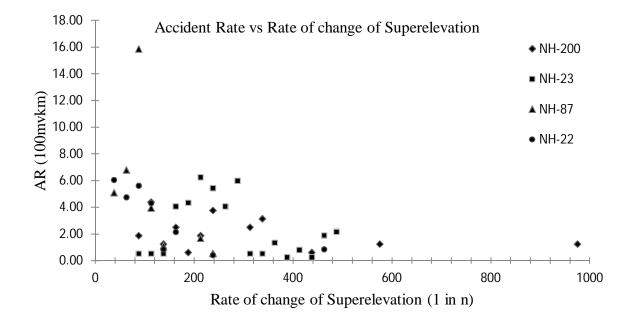


Figure 4.5: Accident Rate versus Rate of Change of Superelevation

## 4.3.6 Analysis of Accident Rate versus Vertical Gradient

Total number of accidents has been counted in every 1% interval of vertical gradient and then, the accident rate has been calculated as illustrated in *Appendix-4.6* and same has been plotted in Figure 4.6.

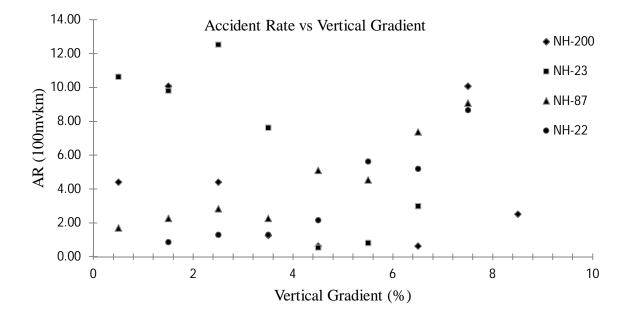


Figure 4.6: Accident Rate versus Vertical Gradient

### 4.3.7 Analysis of Accident Rate versus Vertical Curve Length

Total number of accidents has been counted in every 25m interval of vertical curve length and then, the accident rate has been calculated as illustrated in *Appendix-4.7* and same has been plotted in Figure 4.7.

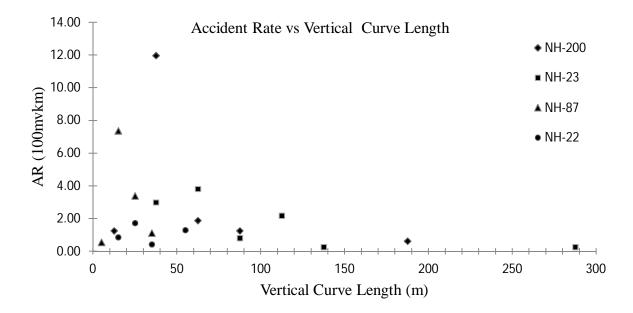
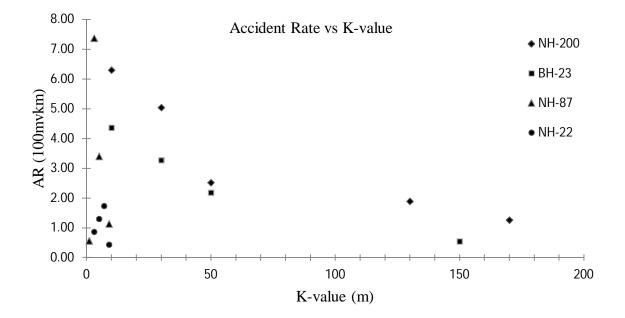
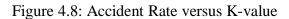


Figure 4.7: Accident Rate versus Vertical Curve Length

## 4.3.8 Analysis of Accident Rate versus K-value

Total number of accidents has been counted in every 25m interval of vertical curve length and then, the accident rate has been calculated as illustrated in *Appendix-4.8* and same has been plotted in Figure 4.8.





#### 4.3.9 Analysis of Accident Rate versus Visibility

Total number of accidents has been counted in every 10m interval of sight distance/ visibility and then, the accident rate has been calculated as illustrated in *Appendix-4.9* and same has been plotted in Figure 4.9.

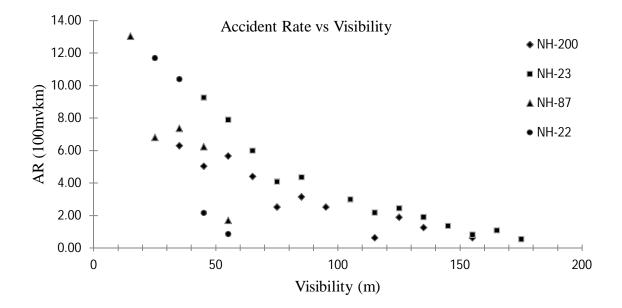


Figure 4.9: Accident Rate versus Visibility

### 4.4 Statistical Analysis of Variance

Statistical process or methods can summarize or describe a collection of data. Analysis of variance (ANOVA) is a collection of statistical models used to analyze the differences between group means and their association. In ANOVA setting, the observed variance in a particular variable is partitioned into components attributable to different sources of variation. ANOVAs are useful in comparing for statistical significance.

All geometric elements has been analysed independently with accident rate of the highway as previously shown and the details are illustrated in *Appendix-4.1 to Appendix-4.9*. Microsoft

Office Excel has been used in the regression analysis and the result of ANOVA has been furnished in Table 4.2.

Variables	$\mathbf{R}^2$	Adjusted R <sup>2</sup>	Standard Error	SS	MS	F
Radius	0.83	0.80	0.78	17.74	17.74	28.96
Deflection Angle	0.10	-0.02	1.50	1.92	1.92	0.86
Arc Length	0.61	0.54	1.48	20.23	20.23	9.28
Superelevation	0.90	0.85	1.94	65.44	65.44	17.40
Rate of Change of Superelevation	0.12	0.04	1.19	2.01	2.01	1.43
Vertical Gradient	0.02	-0.14	4.16	2.50	2.50	0.14
Vertical Curve Length	0.15	-0.13	5.11	14.08	14.08	0.54
K-value	0.78	0.71	1.16	14.68	14.68	10.91
Visibility	0.87	0.85	0.77	34.67	34.67	58.31

Table 4.2a: Summary of Analysis of Variance of NH-200

SS=Sum of Square, MS= Mean of Square

Variables	$\mathbf{R}^2$	Adjusted R <sup>2</sup>	Standard Error	SS	MS	F
Radius	0.86	0.84	0.59	26.99	26.99	76.69
Deflection Angle	0.26	0.17	2.53	19.74	19.74	3.08
Arc Length	0.19	0.12	1.90	10.35	10.35	2.85
Superelevation	0.91	0.88	2.14	141.45	141.45	30.98
Rate of Change of Superelevation	0.05	-0.01	2.20	3.88	3.88	0.80
Vertical Gradient	0.69	0.62	3.03	99.84	99.84	10.88
Vertical Curve Length	0.49	0.36	1.19	5.53	5.53	3.87
K-value	0.91	0.86	0.61	7.19	7.19	19.11
Visibility	0.88	0.87	0.99	81.03	81.03	82.33

Table 4.2b: Summary of Analysis of Variance of NH-23

Variables	$\mathbf{R}^2$	Adjusted R <sup>2</sup>	Standard Error	SS	MS	F
Radius	0.86	0.84	1.06	42.05	42.05	37.66
Deflection Angle	0.00	-0.09	1.45	0.00	0.00	0.00
Arc Length	0.03	-0.13	3.68	2.77	2.77	0.20
Superelevation	0.93	0.92	1.22	101.54	101.54	67.76
Rate of Change of Superelevation	0.32	0.19	4.77	54.55	54.55	2.40
Vertical Gradient	0.86	0.84	1.08	43.09	43.09	37.25
Vertical Curve Length	0.01	-0.49	3.77	0.26	0.26	0.02
K-value	0.04	-0.44	3.71	1.11	1.11	0.08
Visibility	0.83	0.77	1.94	54.09	54.09	14.37

Table 4.2c: Summary of Analysis of Variance of NH-87

Table 4.2d: Summary of Analysis of Variance of NH-22

Variables	$\mathbf{R}^2$	Adjusted R <sup>2</sup>	Standard Error	SS	MS	F
Radius	0.81	0.78	1.06	28.45	28.45	25.34
Deflection Angle	0.00	-0.09	1.02	0.03	0.03	0.03
Arc Length	0.41	0.30	2.43	20.72	20.72	3.51
Superelevation	0.92	0.91	0.81	39.74	39.74	59.89
Rate of Change of Superelevation	0.53	0.45	1.72	19.76	19.76	6.71
Vertical Gradient	0.86	0.83	1.21	45.07	45.07	30.56
Vertical Curve Length	0.01	-0.49	0.68	0.01	0.01	0.01
K-value	0.04	-0.44	0.67	0.04	0.04	0.08
Visibility	0.90	0.84	2.19	82.92	82.92	17.26

### 4.5 Regression Analysis

Analysis of Variance (ANOVA) shows that the highway alignment geometric elements like, radius, superelevation, K-value and visibility are significant to cause accident on NH-200 & NH-23 in plain & rolling terrain and geometric elements like, radius, superelevation, vertical gradient and visibility are significant to cause accident on NH-87 & NH-22 in mountainous & steep terrain. The group effect of highway geometric element on accident rate has been calculated through regression model as below and same has been furnished in Figure 4.10.

Accident Rate (NH-200) = -0.002(RA) + 2.7349(SE) - 0.0279(K) - 0.0476(VB) + 10.7396Accident Rate (NH-23) = -0.0022(RA) + 3.7610(SE) - 0.0249(K) - 0.0600(VB) + 9.4498Accident Rate (NH-87) = -0.0159(RA) + 1.9043(SE) + 1.0129(G) - 0.2326(VB) + 15.1894Accident Rate (NH-22) = -0.0122(RA) + 1.1914(SE) + 1.2687(G) - 0.4072(VB) + 21.8108

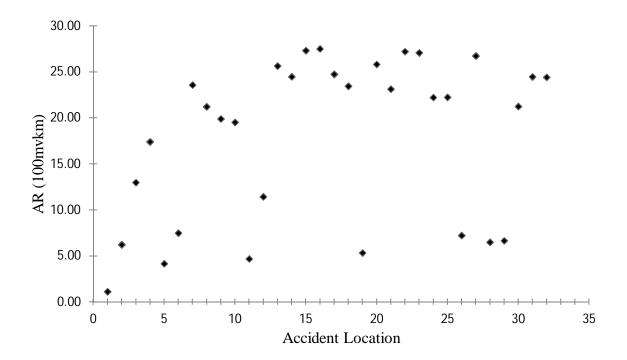


Figure 4.10a: Accident Rate on NH-200

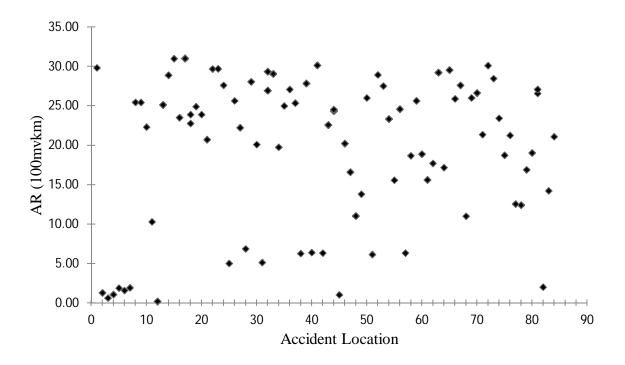


Figure 4.10b: Accident Rate on NH-23

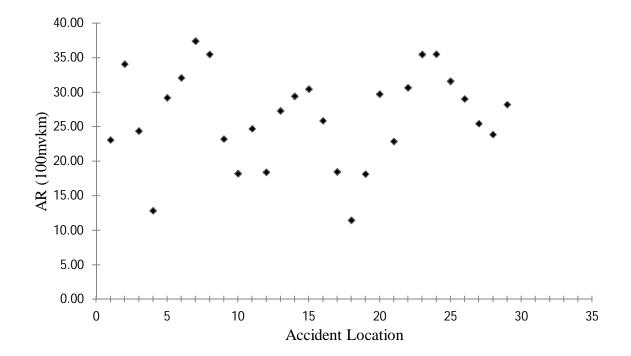


Figure 4.10c: Accident Rate on NH-87

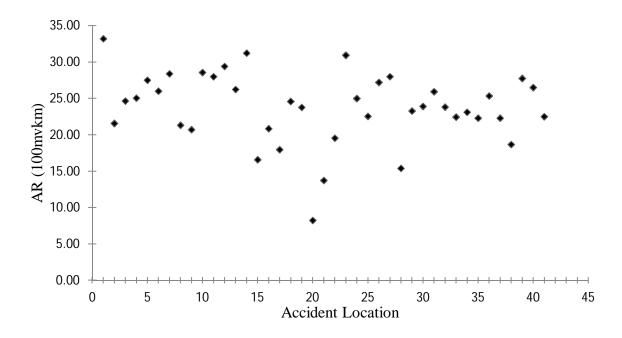


Figure 4.10d: Accident Rate on NH-22

### 4.6 **Result and Discussion**

From the statistical analysis, it has been observed that several variables are very significant to cause the accident in the highway. Highway alignment geometric elements like, radius, superelevation and sight distance/visibility play a very significant role in causing accident both in plain & rolling and mountainous & steep terrain highway. However, K-value is very significant in plain & rolling terrain highway and vertical gradient is very significant in mountainous & steep terrain highway to cause accident.

The above studies show that the higher accident rate occurs with decreasing horizontal radius, higher superelevation, poor visibility, steep gradient and lesser K-value of highway alignment. In view of complexity of highway geometric parameters, two accident models have been proposed which provide the accident rate of the existing highway. For plain & rolling terrain highway, accident cause variables are radius, superelevation, K-value and visibility. For mountainous & steep terrain highway accident cause variables are radius, superelevation, vertical gradient and visibility.

# **Proposed Model**

### 5.1 Background

In the literature studies, generally the traffic accident models were developed as statistical prediction model with limited parameters. The nature of the traffic accidents required a flexible model that can accept imprecise data. For more complex issues, fuzzy logic is very convenient in explaining traffic accidents, in which uncertainty is principal.

This chapter is about the proposed fuzzy logic model, where an attempt has been made to predict the Accident Rate (AR) with respect to the various highway geometric elements mentioned in chapter 4. Two models have been developed due to the complexity of the geometric parameters of rural highway on different terrain conditions. First one is Highway Accident Rate Prediction Model for Plain & Rolling Terrain (HARPM<sub>PRT</sub>) and second one is Highway Accident Rate Prediction Model for Mountainous & Steep Terrain (HARPM<sub>MST</sub>).

HARPM<sub>PRT</sub> has been proposed which provide the accident rate of the highway as output variables considering radius, superelevation, K-value and visibility as input variables. HARPM<sub>MST</sub> has been proposed which provide the accident rate of the highway as output variables considering radius, superelevation, vertical gradient and visibility as input variables.

Fuzzy logic is very powerful mathematical tool for modeling the common-sense reasoning in decision making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution.

This study aims to contribute all related numeric or linguistic parameters of traffic accidents to the accident rate prediction model. Hence, for all the stated reasons Fuzzy Inference System (FIS) of fuzzy logic modeling approach has been ideal for modeling the accident rate on rural highway.

#### 5.2 Introduction to Fuzzy Logic

Lotfi A. Zadeh (1965) introduced the mathematical expression of an infinite-valued logic by his Fuzzy Sets and defined the concept of Fuzzy Sets as "A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that fuzzy sets to be disjoint".

### 5.2.1 Basics of Fuzzy Sets

A fuzzy set is defined as the extension of a crisp (classical) set which allows only full membership or no membership to its elements (Zadeh, 1965). A set is a collection of similar elements having common group properties. When the belonging to the group is complete

without any doubt, the set is called a classical or crisp set.

A crisp set *A* can be defined like this:

 $A = \{ x \mid x \in X \}$ 

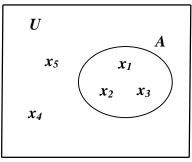


Figure 5.1: Representation of a crisp set

Where, x is an element of the set and X is the common property of the set.

A fuzzy set is such kind of set, where belonging to that group may not be complete. In a fuzzy set an element can belong to any group either completely or partially and can also belong to any other group partially. The difference between a crisp set and a fuzzy set lies in the nature of their boundary. In a crisp set, the boundary is crisp, i.e., well defined. Whereas, in a fuzzy set, the boundary is a vague region. The degree of belonging to a set is defined by membership value, which is obtained using some membership function. For a crisp set, if an element belongs to it, the membership value is 1 and if does not it is 0. For a fuzzy set, it is any value between 0 to 1.

So, a fuzzy set  $\tilde{A}$  can be defined as:

 $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) \mid x \in X\}$ 

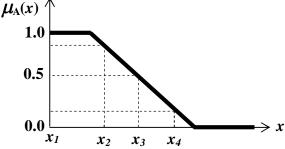


Figure 5.2: Representation of a fuzzy set

Where,  $\mu_{\tilde{A}}$  is called the membership function of x in set  $\tilde{A}$ , value of  $\mu_{\tilde{A}}$  is in between 0 to 1.

Linguistic variables are used in conjunction with fuzzy membership function for fuzzy analysis. While variables in mathematics usually take numerical values, in fuzzy logic application, the non-numeric linguistic variables are often used to facilitate the expression of rules and facts.

### 5.2.2 Basics of Fuzzy Membership Function

The membership function of a fuzzy set is a generalization of the indicator function in classical sets. In fuzzy logic, it represents the degree of truth as an extension of valuation. Degrees of truth are often confused with probabilities, although they are conceptually distinct because fuzzy truth represents membership in vaguely defined sets, not likelihood of some event or condition. Fuzzy membership functions may take on many forms according to the

experts. However, in practical applications triangular and trapezoidal functions are preferred as simple linear functions.

For any set X, a membership function on X is any function from X to the real unit interval [0,1].

Membership functions on X represent fuzzy subsets of X. The membership function which represents a fuzzy set  $\tilde{A}$  is usually denoted by  $\mu_A$ . For an element x of X, the value  $\mu_A(x)$  is called the *membership degree* of x in the fuzzy set  $\tilde{A}$ . The membership degree  $\mu_A(x)$ quantifies the grade of membership of the element x to the fuzzy set A. The value 0 means that x is not a member of the fuzzy set; the value 1 means that x is fully a member of the fuzzy set. The values between 0 and 1 characterize fuzzy members, which belong to the fuzzy set only partially. The membership functions, both linear and non-linear, and most commonly used in engineering can be classified into four types as follows:

The function S:  $x \rightarrow [0,1]$  defined as

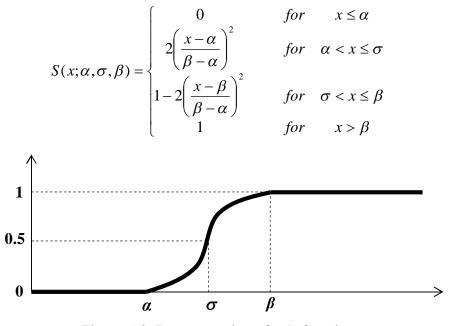


Figure 5.3: Representation of a S -function

The function  $\Gamma: x \to [0,1]$  defined as

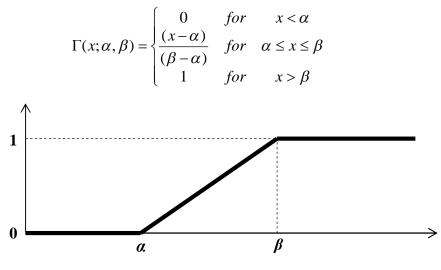


Figure 5.4: Representation of a  $\Gamma$  -function

The function L:  $x \rightarrow [0,1]$  defined as

$$L(x;\alpha,\beta) = \begin{cases} 1 & \text{for} & x < \alpha \\ \frac{(\beta - x)}{(\beta - \alpha)} & \text{for} & \alpha \le x \le \beta \\ 0 & \text{for} & x > \beta \end{cases}$$

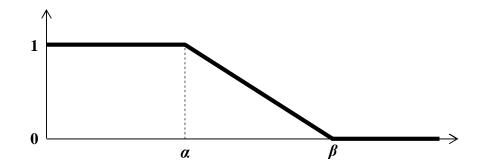


Figure 5.5: Representation of a L -function

The function A:  $x \rightarrow [0,1]$  defined as

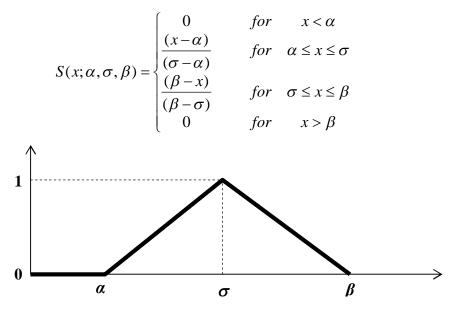


Figure 5.6: Representation of a A –function

### 5.2.3 Basics of Fuzzy Set Operations

The three basic set operations namely union, intersection and complement for fuzzy set are as follows:

**Union:** If there are two fuzzy sets  $\tilde{A}$  and  $\tilde{B}$ , then their union  $\tilde{C}$ , is such that the membership function of  $\tilde{C}$  is maximum between the membership functions of  $\tilde{A}$  and  $\tilde{B}$ , i.e.,

$$\mu_{\tilde{C}}(x) = Max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$$

**Intersection:** If there are two fuzzy sets  $\tilde{A}$  and  $\tilde{B}$ , then there intersection  $\tilde{D}$ , is such that the membership function of  $\tilde{D}$  is the minimum of the membership functions of  $\tilde{A}$  and  $\tilde{B}$ , i.e.,

$$\mu_{\tilde{D}}(x) = Min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$$

**Complement:** For a fuzzy set  $\tilde{A}$ , its complement  $\overline{\tilde{A}}$  is such that,

$$\mu_{\overline{A}}(x) = 1 - \mu_{\widetilde{A}}(x)$$

As stated before boundary of  $\tilde{A}$  is not crisp, so boundary of  $\tilde{A}$  is also not crisp. Thus,  $\tilde{A}$  and  $\overline{\tilde{A}}$  are not mutually exclusive to each other. So,

$$\widetilde{A} \cap \overline{\widetilde{A}} = \phi$$

Where,  $\phi$  is the Null set.

### 5.2.4 Basics of Fuzzy Arithmetic

The arithmetic application to fuzzy number is referred to as Fuzzy arithmetic. Arithmetic namely addition, subtraction, multiplication and division are used in general. One fuzzy arithmetic namely weighted average is described here. If there are N number of fuzzy numbers,  $\tilde{M}_i$  (i = 1 to N), in R, having weight  $w_i$  (say); then the weighted average is given by

$$\overline{M} = \frac{\sum_{i=1}^{N} w_i . \widetilde{M}_i}{\sum_{i=1}^{N} w_i}$$

### 5.2.5 Basics of Fuzzy Logical Operation

The fuzzy logical operations for multi-valued membership functions are AND, OR, and NOT. In more general terms, these definitions are known as the fuzzy intersection or conjunction (AND), fuzzy union or disjunction (OR) and fuzzy complement (NOT). Zadeh (1965) proposed that operators for these functions can be defined as follows: AND = minimum, OR = maximum and NOT = additive complement.

The rules of these functions then are as follows:

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$
$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

$$A = \{x \in X : x \notin A\}$$

### 5.2.6 Premise Variable

In fuzzy logic, the proposition/s representing the prevailing condition/s is represented as a linguistic variable, named as premise variables. It certainly can carry a value with it. But, that value does not always guarantee its exact grouping. Variables whose values are words or sentences in natural or artificial languages are called linguistic variables. It carries a value but they need to be grouped according to their membership values.

#### 5.2.7 Consequence Variable

This represents the course of action corresponding to a particular combination of premise variables. It is a fuzzy number representing the approximate value of the course of action. This fuzzy number is approximately equal to a crisp value. This process also called defuzzification.

### 5.2.8 Fuzzy Inference System (FIS)

Fuzzy logic deals with linguistic variables through approximate reasoning. Fuzzy inference is based on approximate reasoning. According to Zadeh, fuzzy inference is "the process or process by which a possibly imprecise conclusion is deduced from a collection of imprecise premises". The fuzzy inference system is the process of formulating the mapping from a given input to an output.

The statement of if-then (or rules) is the main mechanism in the fuzzy inference system. This fuzzy inference system makes the system natural and beneficial to model a complex humanistic in the loop system. The components of a fuzzy inference system are the fuzzification, rules, aggregation and defuzzification as illustrated in Figure 5.7.

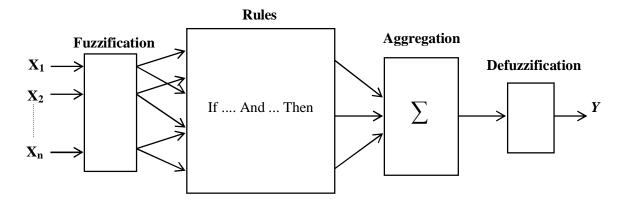


Figure 5.7: Schematic diagram of a Fuzzy Inference System (FIS)

#### **Fuzzification**

The function of the fuzzification is to convert a crisp numerical value from the universe of discourse of the input variable into a linguistic variable and corresponding level of belief. This step takes the current value of a process state variable and gives levels of belief in input fuzzy sets, in order to make it compatible with the fuzzy set representation of the process state variable in the rule-antecedent. The level of belief is equal to the degree of membership in the qualifying linguistic set which can take any value from the closed interval [0,1].

## Rules

Fuzzy sets and fuzzy operators are the subjects and verbs of fuzzy logic. These if-then rule statements are used to formulate the conditional statements that comprise fuzzy logic. A single fuzzy if-then rule assumes the form as "if x is A then y is B". Where A and B are linguistic values defined by fuzzy sets on the ranges (universes of discourse) X and Y,

respectively. The if-part of the rule "x is A" is called the *antecedent* or premise, while the then-part of the rule "y is B" is called the *consequent* or conclusion.

### Aggregation

In the aggregation of the consequents across the rules phase, the outputs of all rules are joined. Thus, a further reduction method is necessary for this phase, such as the maximum, the algebraic sum and the sum method. The maximum method takes the maximum of the degree membership function for the output. The algebraic sum method computes the algebraic sum of the outputs and the sum method is to add the output degrees. The results of these methods produce slightly different results and the most appropriate one depending on the purpose of the application. The aggregation produces one fuzzy set as an output of the fuzzy system.

### Defuzzification

Defuzzification is the process of producing a quantifiable result in fuzzy logic, given fuzzy sets and corresponding membership degrees. Defuzzification is interpreting the membership degrees of the fuzzy sets into a specific decision or real value. A common and useful defuzzification technique is center of gravity. This calculation is to determine the centre of gravity within the area under the membership curve as follows:

$$y_{output} = \frac{\int_{y_{min}}^{y_{max}} y \mu_{agg}(y) dy}{\int_{y_{min}}^{y_{max}} \mu_{agg}(y) dy}$$

#### 5.3 Fuzzy Clustering

Data clustering is the process of dividing data elements into classes or clusters so that items in the same class are as similar as possible and items in different classes are as dissimilar as possible. Depending on the nature of the data and the purpose for which clustering is being used, different measures of similarity may be used to place items into classes, where the similarity measure controls how the clusters are formed.

In fuzzy clustering, data elements can belong to more than one cluster and associated with each element is a set of membership levels. These indicate the strength of the association between that data element and a particular cluster. Fuzzy clustering is a process of assigning these membership levels and then using them to assign data elements to one or more clusters.

Fuzzy C-Means (FCM) is the one of the most widely used fuzzy clustering algorithms (Bezdek 1981). The FCM algorithm attempts to partition a finite collection of n elements  $X = \{x_1, ..., x_n\}$  into a collection of c fuzzy clusters with respect to some given criterion. Given a finite set of data, the algorithm returns a list of c cluster centres  $C = \{c_1, ..., c_c\}$  and a partition matrix  $W = w_{i,j} \in [0,1], \quad i = 1, ..., n, \quad j = 1, ..., c$ , where each element  $w_{ij}$  tells the degree to which element  $x_i$  belongs to cluster  $c_i$ . The standard function is:

$$w_k(x) = \frac{1}{\sum_{j} \left(\frac{d(center_k, x)}{d(center_j, x)}\right)^{2/(m-1)}}$$

In fuzzy clustering, every point has a degree of belonging to clusters as in fuzzy logic, rather than belonging completely to just one cluster. Thus, points on the edge of a cluster may be in the cluster to a lesser degree than points in the center of cluster. Any point x has a set of coefficients giving the degree of being in the  $k^{th}$  cluster  $w_k(x)$ . With fuzzy c-means, the centroid of a cluster is the mean of all points, weighted by their degree of belonging to the cluster:

$$c_k = \frac{\sum_x w_k(x)x}{\sum_x w_k(x)}$$

The degree of belonging,  $w_k(x)$ , is related inversely to the distance from x to the cluster center.

The cluster centers coordinates give the initial estimate for the membership function parameters, then to be fine-tune for high level of accuracy.

## 5.4 Model Development

The proposed model is the quantifying the accident rate considering various geometric elements of alignment of the highway as premise variables. As mentioned earlier, two types of model (HARPM<sub>PRT</sub> & HARPM<sub>MST</sub>) are proposed for different magnitude of parameters of highway alignment geometrics as considering terrain condition. For the complexity of the formulation, fuzzy logic has been applied in the model. The components of proposed model using fuzzy inference system (FIS) are the fuzzification, rules, aggregation and defuzzification as illustrated in Figure 5.8.

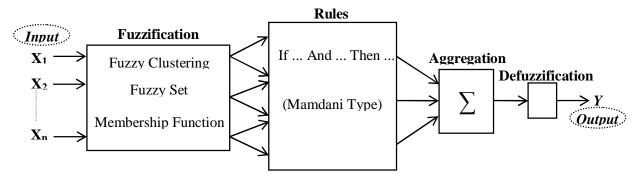


Figure 5.8: Schematic Diagram of Proposed Model Structure

The proposed model developed as Mamdani type fuzzy inference system for quantifying the accident rate with the help of fuzzy logic toolbox in MATLAB release R2012b as illustrated in Figure 5.9.

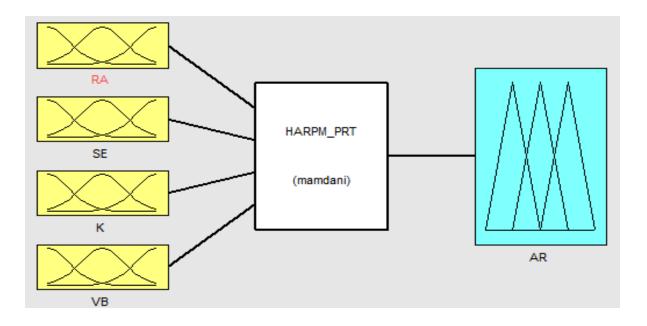


Figure 5.9: Mamdani Type Fuzzy Inference System (MATLAB R2012b)

The accident data of NH-200 are selected for the calibration of model and accident data of NH-23 are selected for validation of the model (HARPM<sub>PRT</sub>) for plain & rolling terrain highway. The accident data of NH-87 are selected for the calibration of model and accident data of NH-22 are selected for validation of the model (HARPM<sub>MST</sub>) for mountainous & steep terrain highway.

Table 5.1 illustrates the descriptive statistics of the variables of the calibration set of data and Table 5.2 illustrates the descriptive statistics of the variables of the validation set of data.

Variables	Min	Max	Mean	Std. Deviation	Std. Error
Radius	27	2080	324.80	460.16	72.76
Superelevation	2.85	7.00	5.97	1.12	0.18
K-value	4.88	166.67	47.60	49.02	9.43
Visibility	32	158	68.81	29.47	4.05
Accident Rate	1.11	27.50	18.36	8.81	1.20

Table 5.1a: Descriptive Statistics of the Calibration set of Data of NH-200

Table 5.1b: Descriptive Statistics of the Calibration set of Data of NH-87

Variables	Min	Max	Mean	Std. Deviation	Std. Error
Radius	7.5	475	91.11	92.25	11.72
Superelevation	3.20	10.00	8.17	1.68	0.21
Vertical Gradient	1.00	8.00	5.37	2.10	0.27
Visibility	15	55	29.95	12.36	1.57
Accident Rate	11.46	37.42	27.77	6.81	0.86

Table 5.2a: Descriptive Statistics of the Validation set of Data of NH-23

Variables	Min	Max	Mean	Std. Deviation	Std. Error
Radius	55	2150	405.81	366.40	30.32
Superelevation	2.60	7.00	5.48	1.19	0.10
K-value	6.15	142.86	29.20	29.80	4.83
Visibility	41	180	82.24	35.62	2.77
Accident Rate	0.21	31.00	21.78	8.29	0.65

Table 5.2b: Descriptive Statistics of the Validation set of Data of NH-22

Variables	Min	Max	Mean	Std. Deviation	Std. Error
Radius	15	480	103.36	110.50	14.51
Superelevation	3.10	10.00	7.98	1.81	0.24
Vertical Gradient	1.10	8.00	6.02	1.73	0.23
Visibility	23	60	33.36	7.68	1.01
Accident Rate	8.25	33.21	24.12	4.66	0.61

### 5.4.1 Fuzzy Clustering of Variables

Fuzzy c-mean clustering method is applied in this model for defining the parameter of the membership function of input variables and grouped according to their membership values with the help of fuzzy logic toolbox in MATLAB. The output result of fuzzy c-mean clustering has been take advantage of to develop of membership function on modeling. The Figure 5.10 shows an example of a clustering fuzzification method.

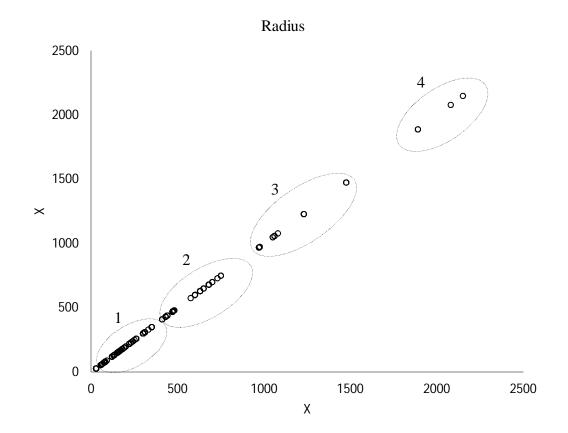


Figure 5.10: Sample of Fuzzy Clustering

The summary of fuzzy clustering has been furnished in Table 5.3a and Table 5.3b.

Innut Variables	Fuzzy Clustering						
Input Variables	Group	Group Center	Group Range				
	1	152	27 - 330				
Radius	2	531	350 - 750				
Kadius	3	1055	970 - 1475				
	4	1963	1890 - 2150				
	1	4	2.6 - 4.4				
Superelevation	2	5	4.6 - 6.0				
	3	7	6.1 - 7.0				
	1	14	5 -23				
K-value	2	41	30 - 60				
	3	142	125 - 167				
	1	51	32 - 67				
Visibility	2	84	68 - 109				
	3	137	113 - 180				

Table 5.3a: Summary of Fuzzy Clustering of NH200 (HARPM<sub>PRT</sub>)

Table 5.3b: Summary of Fuzzy Clustering of NH87 (HARPM<sub>MST</sub>)

	Fuzzy Clustering						
Input Variables	Group	Group Center	Group Range				
	1	25	7.5 - 48				
Radius	2	73	58-100				
Kaulus	3	178	130 - 220				
	4	339	260 - 480				
	1	5	3.1 - 6.4				
Superelevation	2	8	7.0 - 8.7				
	3	10	8.8 - 10.0				
	1	2	1.0 - 3.5				
Gradient	2	5	3.9 - 6.1				
	3	7	6.3 - 8.0				
	1	19	15 - 25				
Visibility	2	32	28 - 39				
	3	47	40 - 60				

#### 5.4.2 Fuzzification of Variables

Fuzzification is the initial process of a fuzzy model where fuzzy subsets of universal set of fuzzy variable are constructed. Fuzzification needs two main stages; derivation of the membership functions for both input and output variables and the linguistic representation of these functions. Four input variables and one output variable are considered for the fuzzy modelling study and same as illustrated in Table 5.4.

Туре	Fuzzy sets	Fuzzy subset		
		Very Sharp (VS)		
	Doding (DA)	Sharp (SH)		
	Radius (RA)	Mild (MI)		
		Flat (FL)		
		Low (LO)		
	Superelevation (SE)	Average (AV)		
		High (HI)		
Transat		Small (SM)		
Input	K-value (K)	Medium (ME)		
		Large (LA)		
		Flatter (FT)		
	Vertical Gradient (G)	Mild (MD)		
		Steeper (ST)		
		Poor (PO)		
	Visibility (VB)	Average (AG)		
		Good (GO)		
		Very High (VH)		
		High (HG)		
Output	Accident Rate (AR)	Medium (MO)		
		Low (LW)		
		Very Low (VL)		

Table 5.4: Linguistic variables and labels of the Fuzzy set and Fuzzy subset

## 5.4.2.1 Fuzzification of Input Variables of HARPMPRT

The deterministic values of the input variables are turned in membership degree to fuzzy sets. These sets are labelled with commonly used linguistic values. Triangular, trapezoidal types of membership function have been used for this model.

The variable RA is divided into two triangular and two trapezoidal fuzzy subsets due to the distribution of the data. Figure 5.11 shows the data distribution for RA for the calibration set. Thus, four fuzzy subsets are defined for the variable RA.

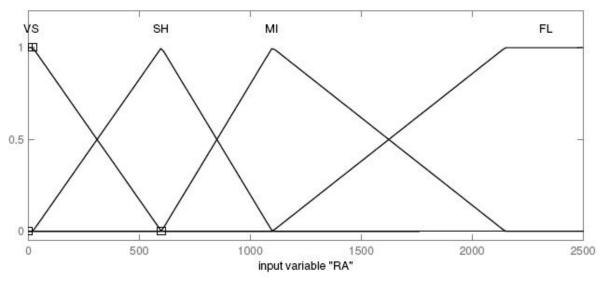


Figure 5.11: Fuzzification of the Input Variable RA

Mathematical expression of the variable Radius (RA) has been described in below section. The membership function  $\mu_{VS}(RA)$ , describing the set "Very Sharp" is described as

$$\mu_{VS}(RA) = \begin{cases} 1 & for & x < 20\\ \frac{(600 - x)}{(600 - 20)} & for & 20 \le x \le 600\\ 0 & for & x > 600 \end{cases}$$

Similarly, the membership function  $\mu_{SH}(RA)$ , describing the set "Sharp" is

$$\mu_{SH}(RA) = \begin{cases} 0 & for \quad x < 20 \\ \frac{(x-20)}{(600-20)} & for \quad 20 \le x \le 600 \\ \frac{(1100-x)}{(1100-600)} & for \quad 600 \le x \le 1100 \\ 0 & for \quad x > 1100 \end{cases}$$

Similarly, the membership function  $\mu_{\scriptscriptstyle MI}(\it RA)$  , describing the set "Mild" is

$$\mu_{MI}(RA) = \begin{cases} 0 & for & x < 600\\ \frac{(x - 600)}{(1100 - 600)} & for & 600 \le x \le 1100\\ \frac{(2150 - x)}{(2150 - 1100)} & for & 1100 \le x \le 2150\\ 0 & for & x > 2150 \end{cases}$$

The membership function  $\mu_{\scriptscriptstyle FL}(RA)$  , describing the set "Flat" is

$$\mu_{FL}(RA) = \begin{cases} 0 & for & x < 1100\\ \frac{(x - 1100)}{(2150 - 1100)} & for & 1100 \le x \le 2150\\ 1 & for & x > 2150 \end{cases}$$

The variable SE is divided into one triangular and two trapezoidal fuzzy subsets due to the distribution of the data. Figure 5.12 shows the data distribution for SE for the calibration set. Thus, three fuzzy subsets are defined for the variable SE.

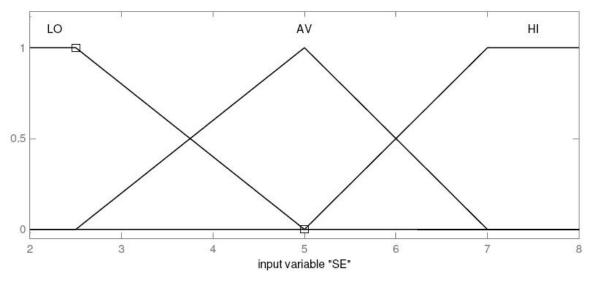


Figure 5.12: Fuzzification of the Input Variable SE

Mathematical expression of the variable Superelevation (SE) has been described in below section. The membership function  $\mu_{LO}(SE)$ , describing the set "Low" is described as

$$\mu_{LO}(SE) = \begin{cases} 1 & for & x < 2.5\\ (5.0 - x) & for & 2.5 \le x \le 5.0\\ 0 & for & x > 5.0 \end{cases}$$

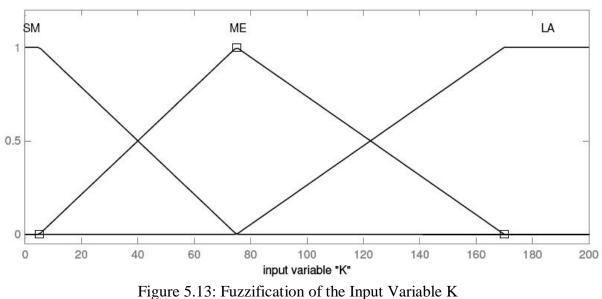
Similarly, the membership function  $\mu_{AV}(SE)$ , describing the set "Average" is

$$\mu_{AV}(SE) = \begin{cases} 0 & for \quad x < 2.5\\ \frac{(x-2.5)}{(5.0-2.5)} & for \quad 2.5 \le x \le 5.0\\ \frac{(7.0-x)}{(7.0-5.0)} & for \quad 5.0 \le x \le 7.0\\ 0 & for \quad x > 7.0 \end{cases}$$

The membership function  $\mu_{\rm HI}(SE)$ , describing the set "High" is

$$\mu_{HI}(SE) = \begin{cases} 0 & for \quad x < 5.0\\ \frac{(x-5.0)}{(7.0-5.0)} & for \quad 5.0 \le x \le 7.0\\ 1 & for \quad x > 7.0 \end{cases}$$

The variable K is divided into one triangular and two trapezoidal fuzzy subsets due to the distribution of the data. Figure 5.13 shows the data distribution for K for the calibration set. Thus, three fuzzy subsets are defined for the variable K.



rigure 5.15. ruzzmeation of the input variable K

Mathematical expression of the variable K-value (K) has been described in below section. The membership function  $\mu_{SM}(K)$ , describing the set "Small" is described as

$$\mu_{SM}(K) = \begin{cases} 1 & \text{for} & x < 5\\ \frac{(75-x)}{(75-5)} & \text{for} & 5 \le x \le 75\\ 0 & \text{for} & x > 75 \end{cases}$$

Similarly, the membership function  $\mu_{\scriptscriptstyle ME}(K)$ , describing the set "Medium" is

$$\mu_{ME}(K) = \begin{cases} 0 & for \quad x < 5\\ \frac{(x-5)}{(75-5)} & for \quad 5 \le x \le 75\\ \frac{(170-x)}{(170-75)} & for \quad 75 \le x \le 170\\ 0 & for \quad x > 170 \end{cases}$$

The membership function  $\mu_{LA}(K)$ , describing the set "large" is

$$\mu_{LA}(K) = \begin{cases} 0 & for & x < 75\\ \frac{(x - 75)}{(170 - 75)} & for & 75 \le x \le 170\\ 1 & for & x > 170 \end{cases}$$

The variable VB is divided into one triangular and two trapezoidal fuzzy subsets due to the distribution of the data. Figure 5.14 shows the data distribution for VB for the calibration set. Thus, three fuzzy subsets are defined for the variable VB.

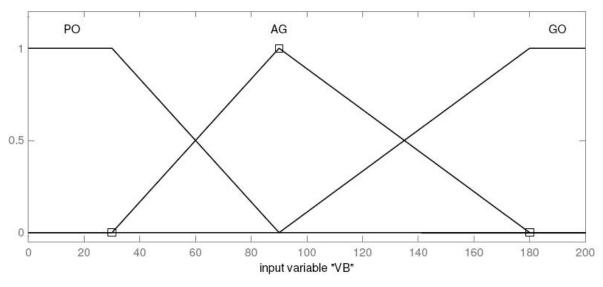


Figure 5.14: Fuzzification of the Input Variable VB

Mathematical expression of the variable Visibility (VB) has been described in below section. The membership function  $\mu_{PO}(VB)$ , describing the set "Poor" is described as

$$\mu_{PO}(VB) = \begin{cases} 1 & for & x < 30\\ \frac{(30-x)}{(90-30)} & for & 30 \le x \le 90\\ 0 & for & x > 90 \end{cases}$$

Similarly, the membership function  $\mu_{AG}(VB)$ , describing the set "Average" is

$$\mu_{AG}(VB) = \begin{cases} 0 & \text{for} & x < 30\\ \frac{(x-30)}{(90-30)} & \text{for} & 30 \le x \le 90\\ \frac{(180-x)}{(180-90)} & \text{for} & 90 \le x \le 180\\ 0 & \text{for} & x > 180 \end{cases}$$

The membership function  $\mu_{GO}(VB)$ , describing the set "Good" is

$$\mu_{GO}(VB) = \begin{cases} 0 & for & x < 90\\ \frac{(x-90)}{(180-90)} & for & 90 \le x \le 180\\ 1 & for & x > 180 \end{cases}$$

## 5.4.2.2 Fuzzification of Input Variables of HARPM<sub>MST</sub>

The deterministic values of the input variables are turned in membership degree to fuzzy sets. These sets are labelled with commonly used linguistic values. Triangular, trapezoidal types of membership function have been used for this model.

The variable RA is divided into two triangular and two trapezoidal fuzzy subsets due to the distribution of the data. Figure 5.15 shows the data distribution for RA for the calibration set. Thus, four fuzzy subsets are defined for the variable RA.

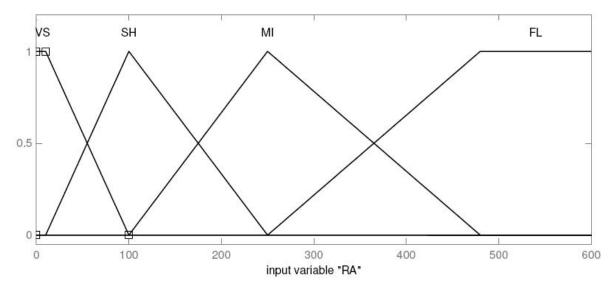


Figure 5.15: Fuzzification of the Input Variable RA

Mathematical expression of the variable Radius (RA) has been described in below section. The membership function  $\mu_{VS}(RA)$ , describing the set "Very Sharp" is described as

$$\mu_{VS}(RA) = \begin{cases} 1 & for & x < 10\\ \frac{(100 - x)}{(100 - 10)} & for & 10 \le x \le 100\\ 0 & for & x > 100 \end{cases}$$

Similarly, the membership function  $\mu_{SH}(RA)$ , describing the set "Sharp" is

$$\mu_{SH}(RA) = \begin{cases} 0 & for & x < 10\\ \frac{(x-10)}{(100-10)} & for & 10 \le x \le 100\\ \frac{(250-x)}{(250-100)} & for & 100 \le x \le 250\\ 0 & for & x > 250 \end{cases}$$

Similarly, the membership function  $\mu_{MI}(RA)$ , describing the set "Mild" is

$$\mu_{MI}(RA) = \begin{cases} 0 & for \quad x < 100\\ \frac{(x-100)}{(250-100)} & for \quad 100 \le x \le 250\\ \frac{(480-x)}{(480-250)} & for \quad 250 \le x \le 480\\ 0 & for \quad x > 480 \end{cases}$$

The membership function  $\mu_{FL}(RA)$ , describing the set "Flat" is

$$\mu_{FL}(RA) = \begin{cases} 0 & for & x < 250\\ \frac{(x - 250)}{(480 - 250)} & for & 250 \le x \le 480\\ 1 & for & x > 480 \end{cases}$$

The variable SE is divided into one triangular and two trapezoidal fuzzy subsets due to the distribution of the data. Figure 5.16 shows the data distribution for SE for the calibration set. Thus, three fuzzy subsets are defined for the variable SE.

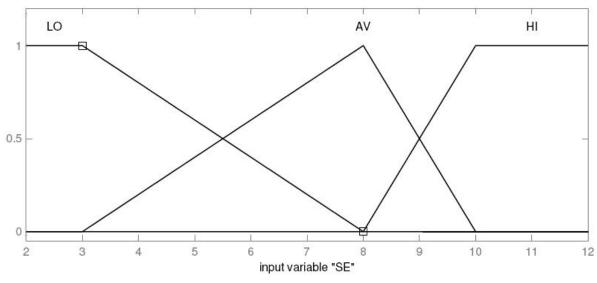


Figure 5.16: Fuzzification of the Input Variable SE

Mathematical expression of the variable Superelevation (SE) has been described in below section. The membership function  $\mu_{LO}(SE)$ , describing the set "Low" is described as

$$\mu_{LO}(SE) = \begin{cases} 1 & for & x < 3.0\\ \frac{(8.0 - x)}{(8.0 - 3.0)} & for & 3.0 \le x \le 8.0\\ 0 & for & x > 8.0 \end{cases}$$

Similarly, the membership function  $\mu_{AV}(SE)$ , describing the set "Average" is

$$\mu_{AV}(SE) = \begin{cases} 0 & for & x < 3.0\\ \frac{(x-3.0)}{(8.0-3.0)} & for & 3.0 \le x \le 8.0\\ \frac{(10.0-x)}{(10.0-8.0)} & for & 8.0 \le x \le 10.0\\ 0 & for & x > 10.0 \end{cases}$$

The membership function  $\mu_{HI}(SE)$ , describing the set "High" is

$$\mu_{HI}(SE) = \begin{cases} 0 & for & x < 8.0\\ \frac{(x-8.0)}{(10.0-8.0)} & for & 8.0 \le x \le 10.0\\ 1 & for & x > 10.0 \end{cases}$$

The variable G is divided into one triangular and two trapezoidal fuzzy subsets due to the distribution of the data. Figure 5.17 shows the data distribution for G for the calibration set. Thus, three fuzzy subsets are defined for the variable G.

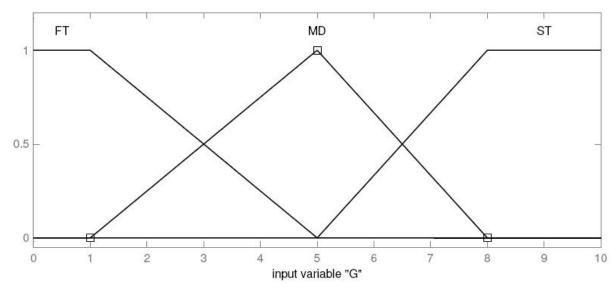


Figure 5.17: Fuzzification of the Input Variable G

Mathematical expression of the variable Vertical Gradient (G) has been described in below section. The membership function  $\mu_{FT}(G)$ , describing the set "Flatter" is described as

$$\mu_{FT}(G) = \begin{cases} 1 & for & x < 1.0\\ \frac{(5.0 - x)}{(5.0 - 1.0)} & for & 1.0 \le x \le 5.0\\ 0 & for & x > 5.0 \end{cases}$$

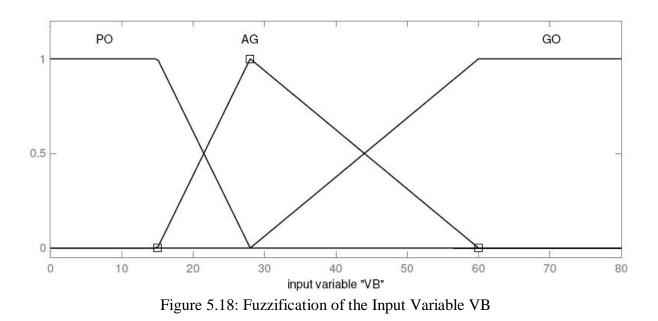
Similarly, the membership function  $\mu_{\rm MD}(G)$  , describing the set "Mild" is

$$\mu_{MD}(G) = \begin{cases} 0 & for \quad x < 1.0\\ \frac{(x-1.0)}{(5.0-1.0)} & for \quad 1.0 \le x \le 5.0\\ \frac{(8.0-x)}{(8.0-5.0)} & for \quad 5.0 \le x \le 8.0\\ 0 & for \quad x > 8.0 \end{cases}$$

The membership function  $\mu_{ST}(G)$ , describing the set "Steeper" is

$$\mu_{ST}(G) = \begin{cases} 0 & for & x < 5.0\\ \frac{(x-5.0)}{(8.0-5.0)} & for & 5.0 \le x \le 8.0\\ 1 & for & x > 8.0 \end{cases}$$

The variable VB is divided into one triangular and two trapezoidal fuzzy subsets due to the distribution of the data. Figure 5.18 shows the data distribution for VB for the calibration set. Thus, three fuzzy subsets are defined for the variable VB.



Mathematical expression of the variable Visibility (VB) has been described in below section.

The membership function  $\mu_{PO}(VB)$ , describing the set "Poor" is described as

$$\mu_{PO}(VB) = \begin{cases} 1 & \text{for} & x < 15\\ \frac{(28-x)}{(28-15)} & \text{for} & 15 \le x \le 28\\ 0 & \text{for} & x > 28 \end{cases}$$

Similarly, the membership function  $\mu_{AG}(VB)$ , describing the set "Average" is

$$\mu_{AG}(VB) = \begin{cases} 0 & for \quad x < 15\\ \frac{(x-15)}{(28-15)} & for \quad 15 \le x \le 28\\ \frac{(60-x)}{(60-28)} & for \quad 28 \le x \le 60\\ 0 & for \quad x > 60 \end{cases}$$

The membership function  $\mu_{GO}(VB)$ , describing the set "Good" is

$$\mu_{GO}(VB) = \begin{cases} 0 & for \quad x < 28\\ \frac{(x-28)}{(60-28)} & for \quad 28 \le x \le 60\\ 1 & for \quad x > 60 \end{cases}$$

#### 5.4.2.3 Fuzzification of Output Variable

The variable AR is divided into five triangular fuzzy subsets due to the distribution of the data in both fuzzy model (HARPM<sub>PRT</sub> & HARPM<sub>MST</sub>). Figure 5.19 shows the data distribution for AR for the calibration set.

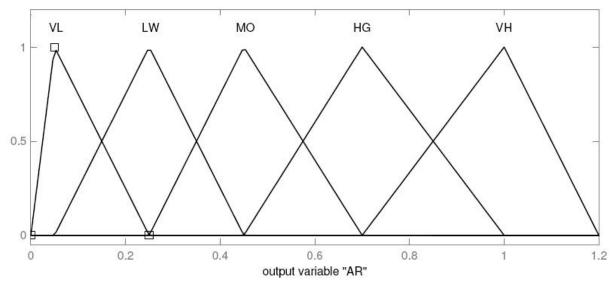


Figure 5.19: Fuzzification of the Output Variable AR

Mathematical expression of the variable Accident Rate (AR) has been described in below section. The membership function  $\mu_{VL}(AR)$ , describing the set "Very Low" is described as

$$\mu_{VL}(AR) = \begin{cases} 0 & \text{for} & x < 0.0\\ \frac{(x-0.0)}{(0.05-0.0)} & \text{for} & 0.0 \le x \le 0.05\\ \frac{(0.25-x)}{(0.25-0.05)} & \text{for} & 0.05 \le x \le 0.25\\ 0 & \text{for} & x > 0.25 \end{cases}$$

Similarly, the membership function  $\mu_{LW}(AR)$ , describing the set "Low" is

$$\mu_{LW}(AR) = \begin{cases} 0 & for \quad x < 0.05 \\ \frac{(x - 0.05)}{(0.25 - 0.05)} & for \quad 0.05 \le x \le 0.25 \\ \frac{(0.45 - x)}{(0.45 - 0.25)} & for \quad 0.25 \le x \le 0.45 \\ 0 & for \quad x > 0.45 \end{cases}$$

Similarly, the membership function  $\mu_{MO}(AR)$ , describing the set "Moderate" is

$$\mu_{MO}(AR) = \begin{cases} 0 & for & x < .25 \\ \frac{(x - 0.25)}{(0.45 - 0.25)} & for & 0.25 \le x \le 0.45 \\ \frac{(0.79 - x)}{(0.70 - 0.45)} & for & 0.45 \le x \le 0.70 \\ 0 & for & x > 070. \end{cases}$$

Similarly, the membership function  $\mu_{\rm HG}(AR)$  , describing the set "High" is

$$\mu_{HG}(AR) = \begin{cases} 0 & for & x < 0.45 \\ \frac{(x - 0.45)}{(7.0 - 0.45)} & for & 0.45 \le x \le 0.70 \\ \frac{(1.0 - x)}{(1.0 - 0.70)} & for & 0.70 \le x \le 1.0 \\ 0 & for & x > 1.0 \end{cases}$$

The membership function  $\mu_{\rm VH}(AR)$  , describing the set "Very High" is

$$\mu_{VH}(AR) = \begin{cases} 0 & for \quad x < 0.7\\ \frac{(x - 0.7)}{(1.0 - 0.7)} & for \quad 0.7 \le x \le 1.0\\ \frac{(1.2 - x)}{(1.2 - 1.0)} & for \quad 1.0 \le x \le 1.2\\ 0 & for \quad x > 1.2 \end{cases}$$

#### **5.4.3** Production of the Rule Base

In this model study, fuzzy rules relating input variables to output variable has been constructed from the calibration data set.

Four input variables and four fuzzy subsets of one variable & three fuzzy subsets of three variables has been identified and evaluate all the possibilities that variables form with each other, and subsequently, 108 rules has been established as a result of  $(4 \times 3 \times 3 \times 3 =)$  108 relations. Mandani type of rule system is employed for the study.

In this process, the truth value of each rule is computed, and then applied to the corresponding part of each rule. Fuzzy Rule Base contains all the possible fuzzy relations between input variables and the output variable.

Interpreting an If-Then rule production is a three part process. These are as below;

- (i) *Fuzzify inputs*: Resolve all fuzzy statements in the antecedent to a degree of membership between 0 and 1.
- (ii) Apply fuzzy operator to multiple part antecedents: If there are multiple parts to the antecedent, apply fuzzy logic operators and resolve the antecedent to a single number between 0 and 1, is the degree of support for the rule.
- (iii) Apply the implication method: Using the degree of support for the entire rule to shape the output fuzzy set. If the rule has more than one antecedent, the fuzzy operator is applied to obtain one number that represents the result of applying that rule.

Following rules are constituted for proposed fuzzy model.

Rule 1: If (RA is VS) and (SE is HI) and (K is SM) and (VB is PO) then (AR is VH) Rule 2: If (RA is VS) and (SE is HI) and (K is SM) and (VB is AG) then (AR is VH) Rule 3: If (RA is VS) and (SE is HI) and (K is SM) and (VB is GO) then (AR is HG)

Rule 106: If (RA is FL) and (SE is LO) and (K is LA) and (VB is PO) then (AR is LW) Rule 107: If (RA is FL) and (SE is LO) and (K is LA) and (VB is AG) then (AR is VL) Rule 108: If (RA is FL) and (SE is LO) and (K is LA) and (VB is GO) then (AR is VL)

#### 5.4.4 Aggregation Process

Each fuzzy rule gives a single number that represents the truth value of that rule. The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set. The summation operation method has been used for aggregation process. MATLAB coding of the Fuzzy Models of **HARPM**<sub>PRT</sub> and **HARPM**<sub>MST</sub> have been given in *Appendix-5.1*. Next stage is the defuzzification process to get crisp output from the aggregated fuzzy output.

#### 5.4.5 Defuzzification Process

In this process each aggregated fuzzy output converting into a single crisp value through the developed fuzzy rules. Center of gravity (CoG) defuzzification method has been applied for the fuzzy model. The following equation is the mathematical expression of the CoG defuzzification method for the discrete fuzzy systems.

$$y^{*} = \frac{\sum_{i=1}^{n} y_{i} \cdot \mu_{U} \cdot (y_{i})}{\sum_{i=1}^{n} \mu_{U} \cdot (y_{i})}$$

Where y\* is the output variable of one set of input variables.

The model applies a defuzzification process for each data point one by one, as given in Figure 5.20.

Rule Viewer: HARPN File Edit View Option	03.6	1		
RA = 20	SE = 7		VB = 20	AR = 0.967
Input: [20 7 5 20]		Plot points: 101	Move: left ri	ght down up
Opened system HARPM	I_PRT, 108 rules		Help	Close

Figure 5.20a: Defuzzification of the Data Point of NH-200 (HARPM<sub>PRT</sub>)

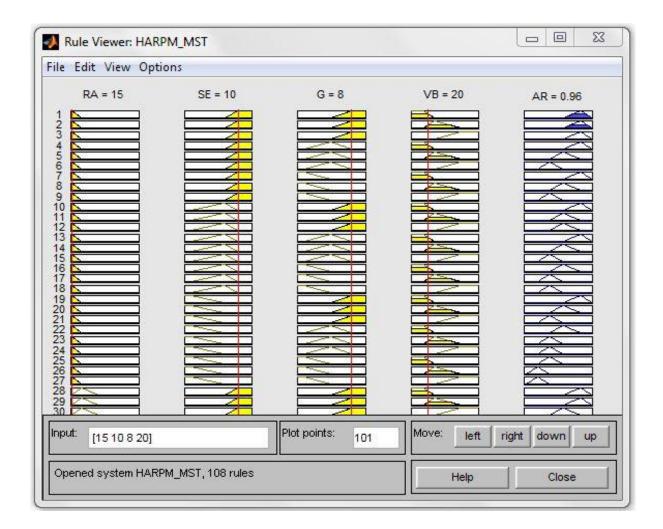


Figure 5.20b: Defuzzification of the Data Point of NH-87 (HARPM<sub>MST</sub>)

## 5.5 Model Results and Discussions

Each set of input data has been entered to the Fuzzy Inference System (FIS) and each output result has been taken. The each crisp output result obtains from model in the range of 0 to 1, and the same has been multiplied with highest value of accident rate at critical conditions. The complete procedure is presented in form of a flow chart in Figure 5.21. The *HARPM*<sub>PRT</sub> model results and the observed data for calibration set group is expressed as scatter diagram in Figure 5.22 and the *HARPM*<sub>MST</sub> model results and the observed data for calibration set group is expressed as scatter diagram in Figure 5.23.

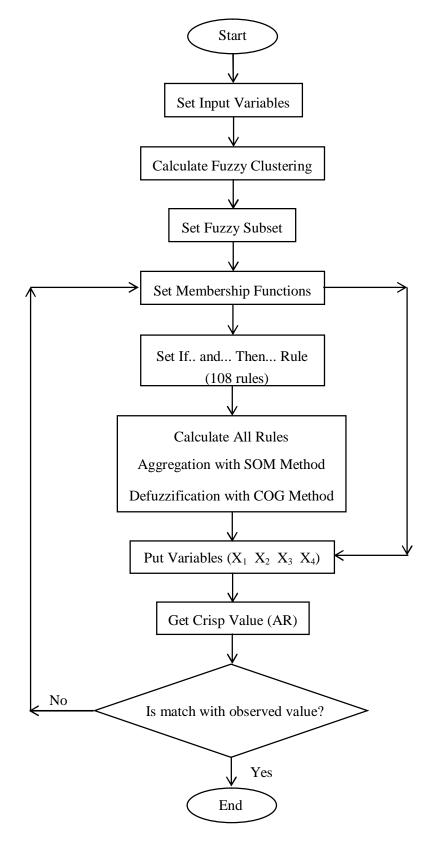


Figure 5.21: Flow Chart of the Proposed Model

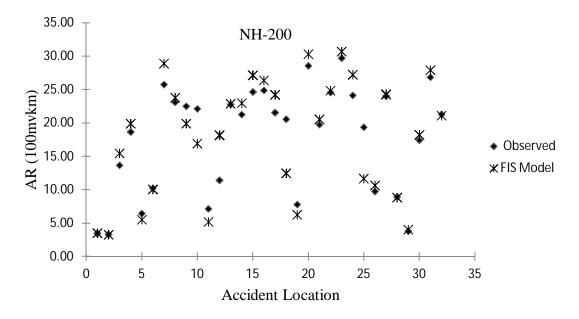


Figure 5.22: Results of Calibration Set Data of NH-200

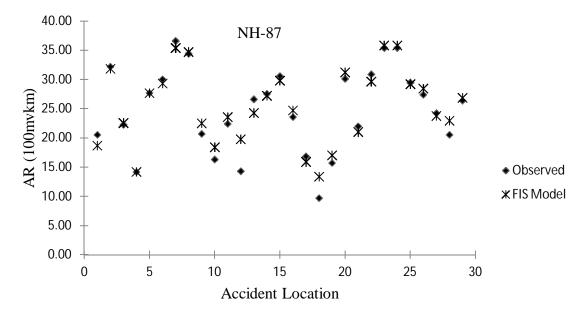


Figure 5.23: Results of Calibration Set Data of NH-87

From the above scatter diagram, it has been seen that as far as the calibration results are concerned, it is seen that Accident Rate (AR) value obtained from statistical analysis and obtained from the model are almost same. So the model can be tested with the validation data set in the next chapter.

# Validation

## 6.1 Background

In this chapter, each set of input data of validation set group has been entered to the Fuzzy Inference System (FIS) and each crisp output result has been taken using both *Highway Accident Rate Prediction Model (HARPM<sub>PRT</sub> & HARPM<sub>MST</sub>)*. Also, simulation results using the proposed model are presented.

#### 6.2 Validation of HARPM<sub>PRT</sub>: Comparison with Observation Results

Simulation results are produced corresponding with each set of input data of validation set group and compared with the combined linear regression analysis results as described in Chapter-4. Simulated results and the observed result of validation data set are expressed as scatter diagram in Figure 6.1.

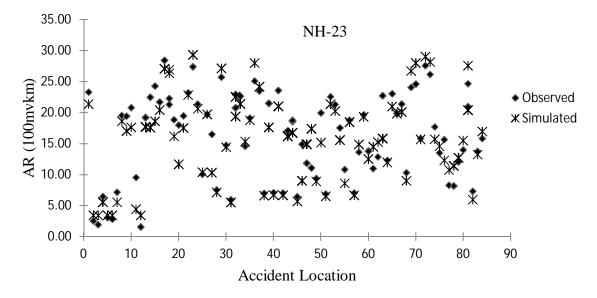


Figure 6.1: Simulated Accident Rate (AR) versus Accident Location of NH-23

#### 6.3 Validation of HARPM<sub>MST</sub>: Comparison with Observation Results

Simulation results are produced corresponding with each set of input data of validation set group and compared with the combined linear regression analysis results as described in Chapter-4. Simulated results and the observed result of validation data set are expressed as scatter diagram in Figure 6.2.

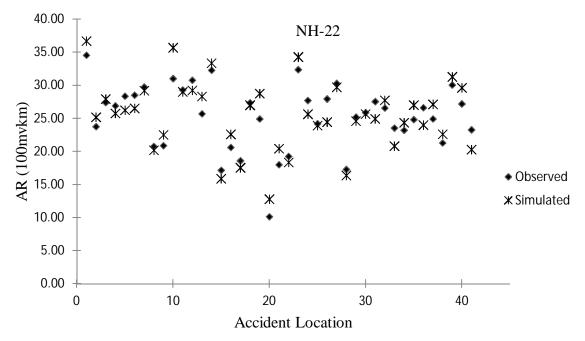


Figure 6.2: Simulated Accident Rate (AR) versus Accident Location of NH-22

#### 6.4 Sensitivity Analysis of Variables

Sensitivity analysis is a technique applies to determine how different values of an independent variable will impact on a particular dependent variable under a given set of assumptions. One of the very simplest and most common approaches is that of changing one-factor-at-a-time (keeping other factors constant) to see what effect this produces on the output.

#### $6.4.1 \quad HARPM_{PRT}$

Each input variable has been entered into the proposed model for plain & rolling terrain highway ( $HARPM_{PRT}$ ) and each output result has been taken. Also same has been expressed as scatter diagram in Figure 6.3.

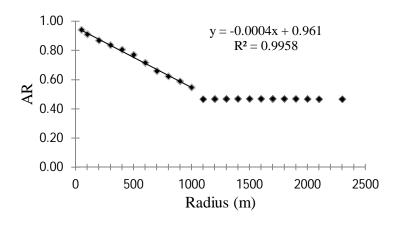


Figure 6.3a: Accident Rate (AR) versus Horizontal Radius

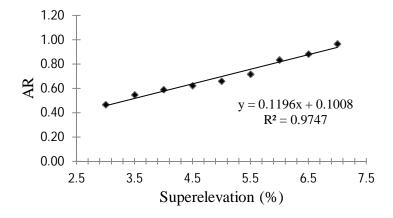


Figure 6.3b: Accident Rate (AR) versus Superelevation

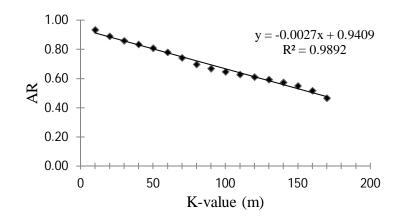


Figure 6.3c: Accident Rate (AR) versus K-value

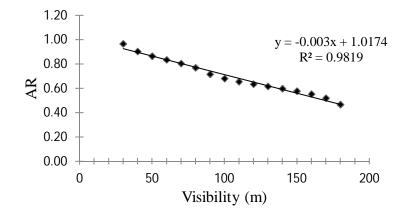


Figure 6.3d: Accident Rate (AR) versus Visibility

From the above analysis on model simulation results, it can be observed that superelevation and visibility have greater impact on accident rate in highway in plain and rolling terrain, i.e., accident rate is very sensitive to these geometric alignment factors.

## 6.4.2 HARPM<sub>MST</sub>

Each input variable has been entered into the proposed model for mountainous & steep terrain highway ( $HARPM_{MST}$ ) and each output result has been taken. Also same has been expressed as scatter diagram in Figure 6.4.

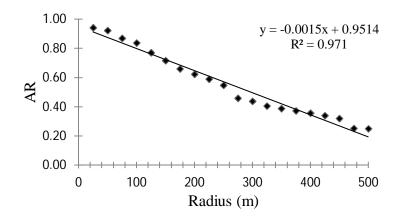


Figure 6.4a: Accident Rate (AR) versus Horizontal Radius

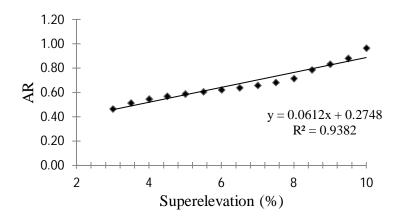


Figure 6.4b: Accident Rate (AR) versus Superelevation

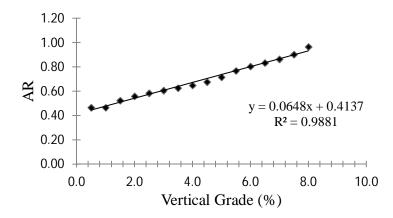


Figure 6.4c: Accident Rate (AR) versus Vertical gradient

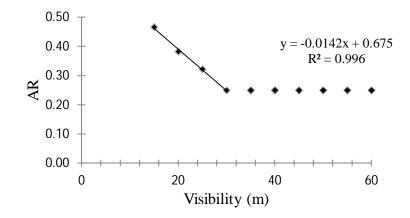


Figure 6.4d: Accident Rate (AR) versus Visibility

From the above analysis on model simulation results, it can be observed that superelevation and vertical gradient have greater impact on accident rate in highway in mountainous and steep terrain, i.e., accident rate is very sensitive to these geometric alignment factors.

#### 6.5 Model Results and Discussions

When the model results are examined in details, it was observed that as far as the simulation results are concerned, the Accident Rate (AR) value obtained from statistical analysis and obtained from the model are almost same with prediction error as 13.2% in  $HARPM_{PRT}$  and 6.8% in  $HARPM_{MST}$ . Hence,  $HARPM_{PRT}$  models can be applied to predict the Accident Rate of plain & rolling terrain highway and, also,  $HARPM_{MST}$  model can be applied to predict the Accident Rate of mountainous & steep terrain highway.

#### **Summary and Conclusions**

The goal of the research was to present an expression of a model that can be used to predict accident rate on existing highway and motivation to implement on highway safety projects in throughout Indian territory. On the other hand, this study produced traffic accident prediction model for the road safety mechanism of rural highway. In this model, a system was established in which output data such as traffic accident rate (AR) and input data such as various highway geometric elements i.e., horizontal radius, superelevation, K-value, vertical gradient and visibility.

In view of complexity of highway geometric elements, Fuzzy Inference System (FIS) based traffic accident prediction algorithm for rural highway was proposed. Comparing to the traditional algorithms, the proposed algorithm has many advantages such as use of linguistic data set variables and apply of the expertise decisions. Two accident models were proposed which provide the accident rate of the existing highway such as *HARPM<sub>PRT</sub>* (*Highway Accident Rate Prediction Model for plain & rolling terrain highway*) and *HARPM<sub>MST</sub>* (*Highway Accident Rate Prediction Model for mountainous & steep terrain highway*). Simulation test shows that the detection results of the algorithm are encouraging and thus get the whole picture of traffic safety improvement based on the condition of the contributing factors.

Statistical analysis indicated that, several highway geometric parameters are very significant to cause accident in the highway. Highway alignment geometric elements such as radius, superelevation, K-value, vertical gradient and sight distance/visibility are very significant in causing accident both in plain & rolling and mountainous & steep terrain highway. However, deflection angle, horizontal arc length, rate of change of superelevation and vertical curve length are insignificant to cause accident in both plain & rolling and mountainous & steep terrain highway.

Analysis indicated that, the variables such as horizontal radius, superelevation, K-value, vertical gradient and visibility indicate very strong correlation with the frequency of accidents. The combined effects of sharp horizontal radius, higher superelevation, lesser K-value and poor visibility tends to increase the accident frequency and provide very high accident rate on the model. On the other hand, the combination effect of flatter horizontal radius, lesser superelevation, higher K-value and more visibility tends to decrease the accident frequency and provide very less accident rate on the model.

Sensitivity analysis demonstrate that superelevation and visibility have greater impact on accident rate in plain and rolling terrain highways; whereas, superelevation and vertical gradient have greater impact on accident rate in mountainous and steep terrain highways.

Generally an accident not take place is caused by one factor but several reasons in any specific location. As the highway alignment concern, horizontal & vertical curve design is one of the important aspects involving highway safety. The design value of each curve factor and its range need to be decided with design consistency.

Further studies are needed to find the effects of geometrics parameter using long-term data and larger sample size on the accident analysis. And also, another subject that should be studied on is the relation between traffic speed, traffic volume, road surface conditions and environment aspect with traffic accidents, which this accident model could not emphasize. It is to conclude that the Fuzzy Logic applied in this accident model is a powerful predictive tool and one should continue further on accident analysis research and can develop a proactive accident mitigation programs.

Many developed nations started a campaign with the motto of "vision zero" that was predicted zero deaths on roads. Thus, there is so much research made on traffic accidents in developed countries. Moreover, developing countries like India need to give emphasis to research on traffic accidents. It is suggested that more importance should be given to the Road Safety issue considering all accident causing factors and a highway safety system should be developed.

## References

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# Appendices

# Appendix-3.1

Fast Moving Vehicles							Slow Moving Vehicles			
Car	2- Wh.	3- Wh.	2- Axle Truck	3- Axle Truck	Multi Axle Truck	Tractor	Bus	Cycle	Cycle Rickshaw	Animal/ Hand Cart
Narka	Narkanda to Rampur at km 231+000 (From 03.06.2013 to 05.06.2013)									
1547	273	0	1003	55	12	13	160	2	0	0
Ramp	Rampur to Narkanda at km 231+000 (From 03.06.2013 to 05.06.2013)									
1687	316	0	1053	58	11	13	182	2	0	0
Ramp	Rampur to Wangtu at km 280+000 (From 06.06.2013 to 08.06.2013)									
1729	269	137	777	41	10	10	340	4	0	0
Wang	Wangtu to Rampur at km 280+000 (From 06.06.2013 to 08.06.2013)									
1772	260	184	765	46	52	21	366	3	0	0

# **Traffic Volume Count on NH-22**

## Appendix-3.2

## **Traffic Volume Count on NH-23**

Fast Moving Vehicles							Slow Moving Vehicles			
Car	2- Wh.	3- Wh.	2- Axle Truck	3- Axle Truck	Multi Axle Truck	Tractor	Bus	Cycle	Cycle Rickshaw	Animal/ Hand Cart
Pallah	Pallahara to Pitiri at Km 369+000 (From 18.02.2013 to 20.02.2013)									
557	453	31	1678	2110	124	6	104	314	0	0
Pitiri (	Pitiri to Pallahara at Km 369+000 (From 18.02.2013 to 20.02.2013)									
446	396	28	1652	2144	127	3	105	274	0	0
Pallah	Pallahara to Pitiri at Km 341+400 (From 21.02.2013 to 23.02.2013)									
1201	1223	22	1904	2432	35	0	181	930	0	1
Pitiri	Pitiri to Pallahara at Km 341+400 (From 21.02.2013 to 23.02.2013)									
972	1147	25	2097	2417	38	4	167	939	0	4

## Appendix-3.3

## **Traffic Volume Count on NH-87**

		Fa	st Movi	ng Vehi	cles			Slow Moving Vehicles		
Car	2- Wh.	3- Wh.	2- Axle Truck	3- Axle Truck	Multi Axle Truck	Tractor	Bus	Cycle	Cycle Rickshaw	Animal/ Hand Cart
Jeolik	Jeolikote to Almora at Km 29+000 (From 17.06.2013 to 19.06.2013)									
1913	873	0	1153	0	0	0	280	6	0	0
Almo	Almora to Jeolikote at Km 29+000 (From 17.06.2013 to 19.06.2013)									
1953	885	1	1243	0	0	1	321	5	0	0
Almo	ra to Site	oli at Kr	n 49+00	0 (Fron	n 20.06.2	013 to 22	.06.20	13)		
1629	830	0	1218	0	0	3	298	6	0	0
Sitoli	Sitoli to Almora at Km 49+000 (From 20.06.2013 to 22.06.2013)									
1669	746	2	1030	0	0	0	163	4	0	0

## Appendix-3.4

### **Traffic Volume Count on NH200**

	Fast Moving Vehicles								Slow Moving Vehicles		
Car	2- Wh.	3- Wh.	2- Axle Truck	3- Axle Truck	Multi Axle Truck	Tractor	Bus	Cycle	Cycle Rickshaw	Animal/ Hand Cart	
Chha	Chhatabar to Deogarh at km 182+000 (From 01.02.2013 to 03.02.2013)										
155	1211	57	123	17	22	44	101	1220	2	8	
Deog	Deogarh to Chhatabar at km 182+000 (From 01.02.2013 to 03.02.2013)										
129	735	44	128	36	38	27	98	1300	1	15	
Deog	arh to B	hojpur a	ut km 13	1+500 (	From 04	.02.2013	to 06.0	02.2013	)		
205	1207	15	246	265	265	54	50	1580	0	0	
Bhojp	Bhojpur to Deogarh at km 131+500 (From 04.02.2013 to 06.02.2013)										
188	1240	8	187	202	202	49	44	1653	6	5	

Appendix-4.1(a)

### Details of Horizontal Radius on NH-200 and NH-23

Ra	dius		NH	-200	NH	[-23
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 50	25	9	5.668	0	0.000
50	- 100	75	7	4.408	18	4.906
100	- 150	125	6	3.778	16	4.361
150	- 200	175	7	4.408	14	3.815
200	- 250	225	0	0.000	17	4.633
250	- 300	275	0	0.000	14	3.815
300	- 350	325	0	0.000	12	3.270
350	- 400	375	0	0.000	0	0.000
400	- 450	425	0	0.000	11	2.998
450	- 500	475	0	0.000	10	2.725
500	- 550	525	0	0.000	0	0.000
550	- 600	575	5	3.149	0	0.000
600	- 650	625	0	0.000	8	2.180
650	- 700	675	0	0.000	9	2.453
700	- 750	725	3	1.889	0	0.000
750	- 800	775	0	0.000	0	0.000
800	- 850	825	0	0.000	0	0.000
850	- 900	875	0	0.000	0	0.000
900	- 950	925	0	0.000	0	0.000
950	- 1000	975	0	0.000	6	1.635
1000	- 1100	1050	0	0.000	5	1.363
1100	- 1200	1150	0	0.000	0	0.000
1200	- 1300	1250	0	0.000	3	0.818
1300	- 1400	1350	0	0.000	0	0.000
1400	- 1500	1450	2	1.259	0	0.000
1500	- 2000	1750	0	0.000	2	0.545
2000	- 2500	2250	1	0.630	1	0.273

Appendix-4.1(b)

Radius			NH	[-87	NH-22		
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate	
0	- 25	12.5	15	8.508	14	6.065	
25	- 50	37.5	12	6.807	15	6.498	
50	- 75	62.5	10	5.672	10	4.332	
75	- 100	87.5	8	4.538	0	0.000	
100	- 125	112.5	0	0.000	0	0.000	
125	- 150	137.5	0	0.000	6	2.599	
150	- 175	162.5	8	4.538	0	0.000	
175	- 200	187.5	5	2.836	0	0.000	
200	- 225	212.5	0	0.000	5	2.166	
225	- 250	237.5	0	0.000	0	0.000	
250	- 275	262.5	0	0.000	3	1.300	
275	- 300	287.5	0	0.000	0	0.000	
300	- 325	312.5	3	1.702	0	0.000	
325	- 350	337.5	0	0.000	4	1.733	
350	- 375	362.5	0	0.000	0	0.000	
375	- 400	387.5	0	0.000	0	0.000	
400	- 425	412.5	0	0.000	0	0.000	
425	- 450	437.5	0	0.000	0	0.000	
450	- 475	462.5	1	0.567	0	0.000	
475	- 500	487.5	0	0.000	1	0.433	

### Details of Horizontal Radius on NH-87 and NH-22

Deflecti	Deflection Angle		NH	-200	NH-23	
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 10	5	2	1.259	2	0.545
10	- 20	15	7	4.408	25	6.813
20	- 30	25	8	5.038	32	8.721
30	- 40	35	5	3.149	16	4.361
40	- 50	45	3	1.889	19	5.178
50	- 60	55	1	0.630	8	2.180
60	- 70	65	2	1.259	19	5.178
70	- 80	75	6	3.778	16	4.361
80	- 90	85	3	1.889	3	0.818
90	- 100	95	3	1.889	4	1.090
100	- 110	105	0	0.000	2	0.545

**Details of Deflection Angle on NH-200 and NH-23** 

Appendix-4.2(b)

## **Details of Deflection Angle on NH-87 and NH-22**

Deflecti	on Angle		NH	[-87	NH	[-22
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 10	5	0	0.000	0	0.000
10	- 20	15	1	0.567	1	0.433
20	- 30	25	5	2.836	2	0.866
30	- 40	35	2	1.134	8	3.466
40	- 50	45	7	3.971	4	1.733
50	- 60	55	5	2.836	6	2.599
60	- 70	65	6	3.403	4	1.733
70	- 80	75	0	0.000	4	1.733
80	- 90	85	8	4.538	7	3.033
90	- 100	95	6	3.403	8	3.466
100	- 110	105	5	2.836	4	1.733
110	- 120	115	9	5.105	5	2.166
120	- 130	125	3	1.702	2	0.866
130	- 140	135	0	0.000	0	0.000
140	- 150	145	2	1.134	3	1.300
150	- 160	155	0	0.000	0	0.000
160	- 170	165	0	0.000	0	0.000
170	- 180	175	3	1.702	0	0.000

Horizontal Arc Length			NH	-200	NH-23	
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 25	12.5	0	0.000	0	0.000
25	- 50	37.5	11	6.927	0	0.000
50	- 75	62.5	7	4.408	12	3.270
75	- 100	87.5	8	5.038	25	6.813
100	- 125	112.5	4	2.519	10	2.725
125	- 150	137.5	4	2.519	6	1.635
150	- 175	162.5	0	0.000	12	3.270
175	- 200	187.5	0	0.000	16	4.361
200	- 225	212.5	0	0.000	2	0.545
225	- 250	237.5	1	0.630	6	1.635
250	- 275	262.5	0	0.000	25	6.813
275	- 300	287.5	4	2.519	5	1.363
300	- 350	325	0	0.000	14	3.815
350	- 400	375	0	0.000	0	0.000
400	- 450	425	0	0.000	5	1.363
450	- 500	475	1	0.630	4	1.090
500	- 550	525	0	0.000	4	1.090

## Details of Horizontal Arc Length on NH-200 and NH-23

Appendix-4.3(b)

### **Details of Horizontal Arc Length on NH-87 and NH-22**

	Horizontal Arc Length		NH	[-87	NH-22	
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 25	12.5	3	1.702	0	0.000
25	- 50	37.5	21	11.912	19	8.231
50	- 75	62.5	7	3.971	16	6.932
75	- 100	87.5	4	2.269	3	1.300
100	- 125	112.5	9	5.105	4	1.733
125	- 150	137.5	4	2.269	0	0.000
150	- 175	162.5	3	1.702	3	1.300
175	- 200	187.5	11	6.240	9	3.899
200	- 225	212.5	0	0.000	4	1.733
225	- 250	237.5	0	0.000	0	0.000

Appendix-4.4(a)

Supere	Superelevation		NH	-200	NH-23	
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 1	0.5	0	0.000	0	0.000
1	- 2	1.5	0	0.000	0	0.000
2	- 3	2.5	2	1.259	7	1.908
3	- 4	3.5	0	0.000	15	4.088
4	- 5	4.5	6	3.778	22	5.996
5	- 6	5.5	12	7.557	37	10.084
6	- 7	6.5	20	12.595	65	17.715

## Details of Superelevation on NH-200 and NH-23

Appendix-4.4(b)

# Details of Superelevation on NH-87 and NH-22

Supere	Superelevation		NH	[-87	NH-22	
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 1	0.5	0	0.000	0	0.000
1	- 2	1.5	0	0.000	0	0.000
2	- 3	2.5	0	0.000	0	0.000
3	- 4	3.5	1	0.567	3	1.300
4	- 5	4.5	1	0.567	1	0.433
5	- 6	5.5	5	2.836	4	1.733
6	- 7	6.5	9	5.105	8	3.466
7	- 8	7.5	10	5.672	10	4.332
8	- 9	8.5	14	7.941	14	6.065
9	- 10	9.5	22	12.479	18	7.798

Rate of Supere	change (	1	NH	-200	NH	[-23
From	Тс	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 25	12.5	0	0.000	0	0.000
25	- 50	37.5	0	0.000	0	0.000
50	- 75	62.5	0	0.000	0	0.000
75	- 100	87.5	3	1.889	2	0.545
100	- 125	112.5	7	4.408	2	0.545
125	- 150	137.5	2	1.259	2	0.545
150	- 175	162.5	4	2.519	15	4.088
175	- 200	187.5	1	0.630	16	4.361
200	- 225	212.5	3	1.889	23	6.268
225	- 250	237.5	6	3.778	20	5.451
250	- 275	262.5	0	0.000	15	4.088
275	- 300	287.5	0	0.000	22	5.996
300	- 325	312.5	4	2.519	2	0.545
325	- 350	337.5	5	3.149	2	0.545
350	- 375	362.5	0	0.000	5	1.363
375	- 400	387.5	0	0.000	1	0.273
400	- 425	412.5	0	0.000	3	0.818
425	- 450	437.5	1	0.630	1	0.273
450	- 475	462.5	0	0.000	7	1.908
475	- 500	487.5	0	0.000	8	2.180
500	- 550	525	0	0.000	0	0.000
550	- 600	575	2	1.259	0	0.000
600	- 650	625	0	0.000	0	0.000
650	- 700	675	0	0.000	0	0.000
700	- 750	725	0	0.000	0	0.000
750	- 800	775	0	0.000	0	0.000
800	- 850	825	0	0.000	0	0.000
850	- 900	875	0	0.000	0	0.000
900	- 950	925	0	0.000	0	0.000
950	- 100	975	2	1.259	0	0.000

Details of Rate of change of Superelevation on NH-200 and NH-23

Appendix-4.5(	<b>b</b> )
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	Rate of change of Superelevation		NH	[-87	NH	-22
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 25	12.5	0	0.000	0	0.000
25	- 50	37.5	9	5.105	14	6.065
50	- 75	62.5	12	6.807	11	4.765
75	- 100	87.5	28	15.882	13	5.632
100	- 125	112.5	7	3.971	10	4.332
125	- 150	137.5	2	1.134	2	0.866
150	- 175	162.5	0	0.000	5	2.166
175	- 200	187.5	0	0.000	0	0.000
200	- 225	212.5	3	1.702	0	0.000
225	- 250	237.5	1	0.567	1	0.433
250	- 275	262.5	0	0.000	0	0.000
275	- 300	287.5	0	0.000	0	0.000
300	- 325	312.5	0	0.000	0	0.000
325	- 350	337.5	0	0.000	0	0.000
350	- 375	362.5	0	0.000	0	0.000
375	- 400	387.5	0	0.000	0	0.000
400	- 425	412.5	0	0.000	0	0.000
425	- 450	437.5	0	0.000	0	0.000
450	- 475	462.5	0	0.000	2	0.866
475	- 500	487.5	0	0.000	0	0.000

Details of Rate of change of Superelevation on NH-87 and NH-22

Appendix-4.6(a)

Vertical	Vertical Gradient		NH	-200	200 NH-23		
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate	
0	- 1	0.5	7	4.408	39	10.629	
1	- 2	1.5	16	10.076	36	9.811	
2	- 3	2.5	7	4.408	46	12.537	
3	- 4	3.5	2	1.259	28	7.631	
4	- 5	4.5	1	0.630	2	0.545	
5	- 6	5.5	0	0.000	3	0.818	
6	- 7	6.5	1	0.630	11	2.998	
7	- 8	7.5	16	10.076	0	0.000	
8	- 9	8.5	4	2.519	0	0.000	

**Details of Vertical Gradient on NH-87 and NH-22** 

Appendix-4.6(b)

**Details of Vertical Gradient on NH-87 and NH-22** 

Vertical	Vertical Gradient		NH	-87	NH-22		
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate	
0	- 1	0.5	3	1.702	0	0.000	
1	- 2	1.5	4	2.269	2	0.866	
2	- 3	2.5	5	2.836	3	1.300	
3	- 4	3.5	4	2.269	3	1.300	
4	- 5	4.5	9	5.105	5	2.166	
5	- 6	5.5	8	4.538	13	5.632	
6	- 7	6.5	13	7.374	12	5.199	
7	- 8	7.5	16	9.076	20	8.665	

Appendix-4.7(a)

Vertical Curve Length			NH	-200	NH-23		
From		То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 2	5	12.5	2	1.259	0	0.000
25	- 5	0	37.5	19	11.965	11	2.998
50	- 7	5	62.5	3	1.889	14	3.815
75	- 1	00	87.5	2	1.259	3	0.818
100	- 11	25	112.5	0	0.000	8	2.180
125	- 1	50	137.5	0	0.000	1	0.273
150	- 1'	75	162.5	0	0.000	0	0.000
175	- 2	00	187.5	1	0.630	0	0.000
200	- 2	25	212.5	0	0.000	0	0.000
225	- 2	50	237.5	0	0.000	0	0.000
250	- 2	75	262.5	0	0.000	0	0.000
275	- 3	00	287.5	0	0.000	1	0.273

**Details of Vertical Curve Length on NH-200 and NH-23** 

## Appendix-4.7(b)

Details of Vertical Curve Length on NH-87 and NH-22

Vertical Curve Length			NH	[-87	NH-22	
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 10	5	1	0.567	0	0.000
10	- 20	15	13	7.374	2	0.866
20	- 30	25	6	3.403	4	1.733
30	- 40	35	2	1.134	1	0.433
40	- 50	45	0 0.000		0	0.000
50	- 60	55	0	0.000	3	1.300

K-1	value		NH-200			NH-23		
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate		
0	- 20	10	10	6.297	16	4.361		
20	- 40	30	8	5.038	12	3.270		
40	- 60	50	4	2.519	8	2.180		
60	- 80	70	0	0.000	0	0.000		
80	- 100	90	0	0.000	0	0.000		
100	- 120	110	0	0.000	0	0.000		
120	- 140	130	3	1.889	0	0.000		
140	- 160	150	0	0.000	2	0.545		
160	- 180	170	2	1.259	0	0.000		

Details of K-value on NH-200 and NH-23

Appendix-4.8(b)

Details of K-value on NH-87 and NH-22

K-value			NH	[-87	NH-22	
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
0	- 2	1	1	0.567	0	0.000
2	- 4	3	13	7.374	2	0.866
4	- 6	5	6	3.403	3	1.300
6	- 8	7	0	0.000	4	1.733
8	- 10	9	2	1.134	1	0.433

Visi	Visibility			NH	-200	NH	[-23
From		То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate
10	-	20	15	0	0.000	0	0.000
20	-	30	25	0	0.000	0	0.000
30	-	40	35	10	6.297	0	0.000
40	-	50	45	8	5.038	34	9.266
50	-	60	55	9	5.668	29	7.903
60	-	70	65	7	4.408	22	5.996
70	-	80	75	4	2.519	15	4.088
80	-	90	85	5	3.149	16	4.361
90	-	100	95	4	2.519	0	0.000
100	-	110	105	0	0.000	11	2.998
110	-	120	115	1	0.630	8	2.180
120	-	130	125	3	1.889	9	2.453
130	-	140	135	2	1.259	7	1.908
140	-	150	145	0	0.000	5	1.363
150	-	160	155	1	0.630 3 0.818		0.818
160	-	170	165	0	0.000 4 1.090		1.090
170	-	180	175	0	0.000	2	0.545

## Details of Visibility/Sight Distance on NH-200 and NH-23

Appendix-4.9(b)

# Details of Visibility/Sight Distance on NH-87 and NH-22

Visi	Visibility		NH	-87	NH-22		
From	То	Median	No. of Accident	Accident Rate	No. of Accident	Accident Rate	
0	- 10	5	0	0.000	0	0.000	
10	- 20	15	23	13.046	0	0.000	
20	- 30	25	12	6.807	27	11.697	
30	- 40	35	13	7.374	24	10.397	
40	- 50	45	11	6.240	5	2.166	
50	- 60	55	3	1.702	2	0.866	

[1]	[System]	[1]	[System]
[2]	Name='HARPM_PRT'	[2]	Name='HARPM_MST
[3]	Type='mamdani'	[3]	Type='mamdani'
[4]	Version=2.0	[4]	Version=2.0
[5]	NumInputs=4	[5]	NumInputs=4
[6]	NumOutputs=1	[6]	NumOutputs=1
[7]	NumRules=108	[7]	NumRules=108
[8]	AndMethod='min'	[8]	AndMethod='min'
[9]	OrMethod='max'	[9]	OrMethod='max'
[10]	ImpMethod='min'	[10]	ImpMethod='min'
[11]	AggMethod='sum'	[11]	AggMethod='sum'
[12]	DefuzzMethod='centroid'	[12]	DefuzzMethod='centroid'
[13]		[13]	
[14]	[Input1]	[14]	[Input1]
[15]	Name='RA'	[15]	Name='RA'
[16]	Range=[0 2500]	[16]	Range=[0 600]
[17]	NumMFs=4	[17]	NumMFs=4
[18]	MF1='VS':'trapmf',[0 0 20 600]	[18]	MF1='VS':'trapmf',[0 0 10 100]
[19]	MF2='SH':'trimf',[20 600 1100]	[19]	MF2='SH':'trimf',[10 100 250]
[20]	MF3='MI':'trimf',[600 1100 2150]	[20]	MF3='MI':'trimf',[100 250 480]
[21]	MF4='FL':'trapmf',[1100 2150 2500 3000]	[21]	MF4='FL':'trapmf',[250 480 600 650]
[22]		[22]	
[23]	[Input2]	[23]	[Input2]
[24]	Name='SE'	[24]	Name='SE'
[25]	Range=[2 8]	[25]	Range=[2 12]
[26]	NumMFs=3	[26]	NumMFs=3
[27]	MF1='LO':'trapmf',[0 0 2.5 5]	[27]	MF1='LO':'trapmf',[0 0 3 8]
[28]	MF2='AV':'trimf',[2.5 5 7]	[28]	MF2='AV':'trimf',[3 8 10]
[29]	MF3='HI':'trapmf',[5 7 8 10]	[29]	MF3='HI':'trapmf',[8 10 12 15.3]
[30]		[30]	
[31]	[Input3]	[31]	[Input3]
[32]	Name='K'	[32]	Name='G'
[33]	Range=[0 200]	[33]	Range=[0 10]
	106		

# MATLAB Coding of the Fuzzy Models (HARPM\_{PRT} and HARPM\_{MST})

[34]	NumMFs=3	[34]	NumMFs=3
[35]	MF1='SM':'trapmf',[0 0 5 75]	[35]	MF1='FT':'trapmf',[0 0 1 5]
[36]	MF2='ME':'trimf',[5 75 170]	[36]	MF2='MD':'trimf',[1 5 8]
[37]	MF3='LA': 'trapmf', [75 170 200 250]	[37]	MF3='ST':'trapmf',[5 8 10 12.5]
[38]		[38]	
[39]	[Input4]	[39]	[Input4]
[40]	Name='VB'	[40]	Name='VB'
[41]	Range=[0 200]	[41]	Range=[0 80]
[42]	NumMFs=3	[42]	NumMFs=3
[43]	MF1='PO':'trapmf',[0 0 30 90]	[43]	MF1='PO':'trapmf',[0 0 15 28]
[44]	MF2='AG':'trimf',[30 90 180]	[44]	MF2='AG':'trimf',[15 28 60]
[45]	MF3='GO':'trapmf',[90 180 200 250]	[45]	MF3='GO':'trapmf',[28 60 80 100]
[46]		[46]	
[47]	[Output1]	[47]	[Output1]
[48]	Name='AR'	[48]	Name='AR'
[49]	Range=[0 1.2]	[49]	Range=[0 1.2]
[50]	NumMFs=5	[50]	NumMFs=5
[51]	MF1='VL':'trimf',[0 0.05 0.25]	[51]	MF1='VL':'trimf',[0 0.05 0.25]
[52]	MF2='LW':'trimf',[0.05 0.25 0.45]	[52]	MF2='LW':'trimf',[0.05 0.25 0.45]
[53]	MF3='MO':'trimf',[0.25 0.45 0.7]	[53]	MF3='MO':'trimf',[0.25 0.45 0.7]
[54]	MF4='HG':'trimf',[0.45 0.7 1]	[54]	MF4='HG':'trimf',[0.45 0.7 1]
[55]	MF5='VH':'trimf',[0.7 1 1.2]	[55]	MF5='VH':'trimf',[0.7 1 1.2]
[56]		[56]	
[57]	[Rules]	[57]	[Rules]
[58]	1 3 1 1, 5 (1) : 1	[58]	1 3 3 1, 5 (1) : 1
[59]	1 3 1 2, 5 (1) : 1	[59]	1 3 3 2, 5 (1) : 1
[60]	1 3 1 3, 5 (1) : 1	[60]	1 3 3 3, 5 (1) : 1
[61]	1 3 2 1, 5 (1) : 1	[61]	1 3 2 1, 5 (1) : 1
[62]	1 3 2 2, 5 (1) : 1	[62]	1 3 2 2, 4 (1) : 1
[63]	1 3 2 3, 4 (1) : 1	[63]	1 3 2 3, 3 (1) : 1
[64]	1 3 3 1, 5 (1) : 1	[64]	1 3 1 1, 5 (1) : 1
[65]	1 3 3 2, 4 (1) : 1	[65]	1 3 1 2, 4 (1) : 1
[66]	1 3 3 3, 3 (1) : 1	[66]	1 3 1 3, 3 (1) : 1
[67]	1 2 1 1, 5 (1) : 1	[67]	1 2 3 1, 5 (1) : 1
[68]	1 2 1 2, 5 (1) : 1	[68]	1 2 3 2, 4 (1) : 1

[69] $1 2 1 3, 4(1) : 1$ [69] $1 2 3 3, 4(1) : 1$ [70] $1 2 2 1, 5(1) : 1$ [70] $1 2 2 1, 5(1) : 1$ [71] $1 2 2 2, 4(1) : 1$ [71] $1 2 2 2, 4(1) : 1$ [72] $1 2 2 3, 3(1) : 1$ [72] $1 2 2 3, 3(1) : 1$ [73] $1 2 3 1, 4(1) : 1$ [73] $1 2 1 1, 4(1) : 1$ [74] $1 2 3 2, 3(1) : 1$ [75] $1 2 1 3, 3(1) : 1$ [75] $1 2 3 3, 3(1) : 1$ [75] $1 2 1 3, 3(1) : 1$ [76] $1 1 1 1, 5(1) : 1$ [76] $1 1 3 1, 5(1) : 1$ [77] $1 1 1 2, 4(1) : 1$ [77] $1 1 3 2, 4(1) : 1$ [78] $1 1 1 3, 3(1) : 1$ [78] $1 1 3 3, 3(1) : 1$ [79] $1 1 2 1, 4(1) : 1$ [79] $1 1 2 2, 3(1) : 1$ [80] $1 1 2 2, 3(1) : 1$ [80] $1 1 2 2, 3(1) : 1$ [81] $1 1 2 3, 3(1) : 1$ [81] $1 1 2 3, 3(1) : 1$ [82] $1 1 1 1, 3(1) : 1$ [83] $1 1 1 2, 2(1) : 1$ [83] $1 1 3 2, 3(1) : 1$ [84] $1 1 1 3, 2(1) : 1$ [84] $1 1 3, 2(1) : 1$ [85] $2 3 1 1, 5(1) : 1$ [85] $2 3 1 1, 5(1) : 1$ [86] $2 3 2 2, 5(1) : 1$ [86] $2 3 2 2, 5(1) : 1$ [87] $2 3 2 3, 3(1) : 1$ [90] $2 3 2 2, 3(1) : 1$ [91] $2 3 1 1, 5(1) : 1$ [91] $2 3 1, 4(1) : 1$ [91] $2 3 1 1, 5(1) : 1$ [92] $2 3 3 2, 3(1) : 1$ [93] $2 3 1 2, 5(1) : 1$ [93] $2 3 3 3, 4(1) : 1$ [94] $2 2 2 3, 3(1) : 1$ [94] $2 2 1 1, 4(1) : 1$				
[71] $1 2 2 2 4 (1) : 1$ $[71]$ $1 2 2 2 4 (1) : 1$ $[72]$ $1 2 2 3 3 (1) : 1$ $[72]$ $1 2 2 3 3 (1) : 1$ $[73]$ $1 2 3 1 4 (1) : 1$ $[73]$ $1 2 1 1 4 (1) : 1$ $[74]$ $1 2 3 2 3 (1) : 1$ $[74]$ $1 2 1 2 3 (1) : 1$ $[75]$ $1 2 3 3 3 (1) : 1$ $[76]$ $1 1 3 1 5 (1) : 1$ $[76]$ $1 1 1 1 5 (1) : 1$ $[76]$ $1 1 3 1 5 (1) : 1$ $[77]$ $1 1 1 2 4 (1) : 1$ $[77]$ $1 1 3 2 4 (1) : 1$ $[78]$ $1 1 1 3 3 (1) : 1$ $[78]$ $1 1 3 3 3 (1) : 1$ $[79]$ $1 1 2 2 4 (1) : 1$ $[79]$ $1 1 2 2 3 (1) : 1$ $[80]$ $1 1 2 2 3 (1) : 1$ $[80]$ $1 1 2 2 3 (1) : 1$ $[80]$ $1 1 2 2 3 (1) : 1$ $[81]$ $1 1 2 3 3 (1) : 1$ $[81]$ $1 1 2 3 3 (1) : 1$ $[81]$ $1 1 2 3 3 (1) : 1$ $[82]$ $1 1 3 1 5 (1) : 1$ $[81]$ $1 1 2 3 3 (1) : 1$ $[83]$ $1 1 3 2 3 (1) : 1$ $[81]$ $1 1 2 2 (1) : 1$ $[84]$ $1 1 3 3 2 (1) : 1$ $[81]$ $1 1 2 3 3 (1) : 1$ $[83]$ $1 3 2 2 3 (1) : 1$ $[85]$ $2 3 3 1 5 (1) : 1$ $[84]$ $1 1 3 3 2 (1) : 1$ $[85]$ $2 3 2 3 1 5 (1) : 1$ $[84]$ $2 3 2 1 5 (1) : 1$ $[85]$ $2 3 2 2 5 (1) : 1$ $[85]$ $2 3 1 2 5 (1) : 1$ $[89]$ $2 3 2 2 5 (1) : 1$ $[86]$ $2 3 2 2 3 3 (1) : 1$ $[91]$ $2 3 1 3 5 (1) : 1$ $[91]$ $2 3 1 3 3 (1) : 1$ $[91]$ $2 3 1 3 5 (1) : 1$ $[92]$ $2 3 2 3 3 3 (1)$	[69]	1 2 1 3, 4 (1) : 1	[69]	1 2 3 3, 4 (1) : 1
[72] $1 2 2 3, 3 (1) : 1$ $[72]$ $1 2 2 3, 3 (1) : 1$ $[73]$ $1 2 3 1, 4 (1) : 1$ $[73]$ $1 2 1 1, 4 (1) : 1$ $[74]$ $1 2 3 2, 3 (1) : 1$ $[74]$ $1 2 1 2, 3 (1) : 1$ $[75]$ $1 2 3 3, 3 (1) : 1$ $[75]$ $1 2 1, 3 (1) : 1$ $[76]$ $1 1 1 1, 5 (1) : 1$ $[76]$ $1 1 3 1, 5 (1) : 1$ $[77]$ $1 1 1 2, 4 (1) : 1$ $[77]$ $1 1 3 2, 4 (1) : 1$ $[77]$ $1 1 2 1, 4 (1) : 1$ $[79]$ $1 1 2 1, 4 (1) : 1$ $[79]$ $1 1 2 1, 4 (1) : 1$ $[79]$ $1 1 2 2, 3 (1) : 1$ $[80]$ $1 1 2 2, 3 (1) : 1$ $[80]$ $1 1 2 2, 3 (1) : 1$ $[81]$ $1 1 2 3, 3 (1) : 1$ $[81]$ $1 1 2 3, 3 (1) : 1$ $[82]$ $1 1 1 1, 3 (1) : 1$ $[81]$ $1 1 2 3, 3 (1) : 1$ $[83]$ $1 1 3 2, 3 (1) : 1$ $[81]$ $1 1 2, 3 (1) : 1$ $[84]$ $1 1 3 3, 2 (1) : 1$ $[83]$ $1 1 1 2, 2 (1) : 1$ $[84]$ $1 1 3, 3 (1) : 1$ $[83]$ $1 1 1 2, 3 (1) : 1$ $[84]$ $1 1 3, 3 (1) : 1$ $[83]$ $1 1 1 2, 2 (1) : 1$ $[84]$ $1 1 3, 3 (1) : 1$ $[84]$ $1 1 1 3, 2 (1) : 1$ $[85]$ $2 3 1 1, 5 (1) : 1$ $[86]$ $2 3 3 2, 5 (1) : 1$ $[86]$ $2 3 1 2, 5 (1) : 1$ $[87]$ $2 3 3, 3 , 4 (1) : 1$ $[87]$ $2 3 1, 4 (1) : 1$ $[89]$ $2 3 2 2, 5 (1) : 1$ $[89]$ $2 3 2 2, 4 (1) : 1$ $[90]$ $2 3 3 2, 3 (1) : 1$ $[91]$ $2 3 1, 4 (1) : 1$ $[91]$ $2 3 1, 5 (1) : 1$ $[92]$	[70]	1 2 2 1, 5 (1) : 1	[70]	1 2 2 1, 5 (1) : 1
[73] $1 \ 2 \ 3 \ 1 \ 4 \ 1 ) : 1$ [73] $1 \ 2 \ 1 \ 1 \ 4 \ 1 ) : 1$ [74] $1 \ 2 \ 3 \ 2 \ 3 \ 3 \ 1 ) : 1$ [74] $1 \ 2 \ 1 \ 2 \ 3 \ 3 \ 1 ) : 1$ [75] $1 \ 2 \ 3 \ 3 \ 3 \ 3 \ 1 ) : 1$ [75] $1 \ 2 \ 1 \ 3 \ 3 \ 1 \ 1 \ 1 \ 1 \ 5 \ 1 \ 1 \ 1 \ 1 \ 5 \ 1 \ 1$	[71]	1 2 2 2, 4 (1) : 1	[71]	1 2 2 2, 4 (1) : 1
$[74]$ $1 \ 2 \ 3 \ 2 \ 3 \ 3 \ 1) : 1$ $[74]$ $1 \ 2 \ 1 \ 2 \ 1 \ 3 \ 3 \ 1) : 1$ $[75]$ $1 \ 2 \ 1 \ 3 \ 3 \ 3 \ 1) : 1$ $[75]$ $1 \ 2 \ 1 \ 3 \ 3 \ 1) : 1$ $[76]$ $1 \ 1 \ 1 \ 1 \ 5 \ 1) : 1$ $[77]$ $1 \ 1 \ 3 \ 3 \ 1) : 1$ $[77]$ $1 \ 1 \ 2 \ 4 \ 1) : 1$ $[77]$ $1 \ 1 \ 3 \ 2 \ 4 \ 1) : 1$ $[78]$ $1 \ 1 \ 3 \ 3 \ 3 \ 1) : 1$ $[78]$ $1 \ 1 \ 3 \ 3 \ 3 \ 1) : 1$ $[79]$ $1 \ 1 \ 2 \ 4 \ 1) : 1$ $[79]$ $1 \ 1 \ 2 \ 3 \ 3 \ 1) : 1$ $[80]$ $1 \ 1 \ 2 \ 3 \ 3 \ 1) : 1$ $[80]$ $1 \ 1 \ 2 \ 3 \ 3 \ 1) : 1$ $[81]$ $1 \ 1 \ 2 \ 3 \ 3 \ 1) : 1$ $[81]$ $1 \ 1 \ 2 \ 3 \ 3 \ 1) : 1$ $[81]$ $1 \ 1 \ 2 \ 3 \ 3 \ 1) : 1$ $[81]$ $1 \ 1 \ 2 \ 3 \ 3 \ 1) : 1$ $[82]$ $1 \ 1 \ 1 \ 3 \ 3 \ 1) : 1$ $[83]$ $1 \ 1 \ 2 \ 3 \ 3 \ 1) : 1$ $[83]$ $1 \ 1 \ 3 \ 2 \ 3 \ 1) : 1$ $[84]$ $1 \ 1 \ 3 \ 2 \ 3 \ 1) : 1$ $[84]$ $1 \ 1 \ 3 \ 3 \ 2 \ 3 \ 1) : 1$ $[85]$ $2 \ 3 \ 3 \ 1 \ 5 \ 1) : 1$ $[85]$ $2 \ 3 \ 1 \ 5 \ 1) : 1$ $[86]$ $2 \ 3 \ 2 \ 3 \ 1, 5 \ 1) : 1$ $[86]$ $2 \ 3 \ 2 \ 3 \ 1, 5 \ 1) : 1$ $[87]$ $2 \ 3 \ 3 \ 3, 4 \ 1) : 1$ $[89]$ $2 \ 3 \ 2 \ 2 \ 3 \ 3, 3 \ 1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3 \ 3, 3 \ 1) : 1$ $[91]$ $2 \ 3 \ 3 \ 3, 3 \ 1) : 1$ $[92]$ $2 \ 3 \ 1, 5 \ 1) : 1$ $[92]$ $2 \ 3 \ 3, 3 \ 3, 1) : 1$ $[93]$ $2 \ 3 \ 3, 3 \ 1) : 1$ $[94]$ $2 \ 2 \ 1, 5 \ 1) : 1$ $[94]$ $2 \ 2 \ 1, 5 \ 1) : 1$ <td< td=""><td>[72]</td><td>1 2 2 3, 3 (1) : 1</td><td>[72]</td><td>1 2 2 3, 3 (1) : 1</td></td<>	[72]	1 2 2 3, 3 (1) : 1	[72]	1 2 2 3, 3 (1) : 1
[75] $1 \ 2 \ 3 \ 3 \ 3 \ 0 \ : 1$ [75] $1 \ 2 \ 1 \ 3 \ 3 \ 0 \ : 1$ [76] $1 \ 1 \ 1 \ 1 \ 5 \ 0 \ : 1$ [76] $1 \ 1 \ 3 \ 1 \ 5 \ 0 \ : 1$ [77] $1 \ 1 \ 2 \ 4 \ 0 \ : 1$ [77] $1 \ 1 \ 3 \ 2 \ 0 \ : 1$ [78] $1 \ 1 \ 3 \ 3 \ 0 \ : 1$ [79] $1 \ 1 \ 2 \ 4 \ 0 \ : 1$ [79] $1 \ 1 \ 2 \ 4 \ 0 \ : 1$ [79] $1 \ 1 \ 2 \ 4 \ 0 \ : 1$ [79] $1 \ 1 \ 2 \ 4 \ 0 \ : 1$ [79] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [80] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [80] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [81] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [81] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [81] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [81] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [82] $1 \ 1 \ 1 \ 3 \ 3 \ 0 \ : 1$ [81] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [83] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [83] $1 \ 1 \ 2 \ 3 \ 0 \ : 1$ [84] $1 \ 1 \ 3 \ 3 \ 0 \ : 1$ [84] $1 \ 1 \ 3 \ 3 \ 0 \ : 1$ [85] $2 \ 3 \ 1 \ 5 \ 0 \ : 1$ [86] $2 \ 3 \ 3 \ 2 \ 0 \ : 1$ [86] $2 \ 3 \ 2 \ 3 \ 1 \ 5 \ 0 \ : 1$ [86] $2 \ 3 \ 2 \ 3 \ 1 \ 5 \ 0 \ : 1$ [87] $2 \ 3 \ 1 \ 5 \ 0 \ : 1$ [88] $2 \ 3 \ 2 \ 5 \ 0 \ : 1$ [88] $2 \ 3 \ 2 \ 2 \ 1 \ 5 \ 0 \ : 1$ [89] $2 \ 3 \ 2 \ 3 \ 1 \ 5 \ 0 \ : 1$ [90] $2 \ 3 \ 2 \ 3 \ 3 \ 0 \ : 1$ [91] $2 \ 3 \ 1 \ 5 \ 0 \ : 1$ [91] $2 \ 3 \ 3 \ 3 \ 0 \ : 1$ [92] $2 \ 3 \ 1 \ 5 \ 0 \ : 1$ [92] $2 \ 3 \ 3 \ 3 \ 0 \ : 1$ [93] $2 \ 3 \ 3 \ 0 \ : 1$ [94] $2 \ 2 $	[73]	1 2 3 1, 4 (1) : 1	[73]	1 2 1 1, 4 (1) : 1
1761 1 1 1, 5 (1): 1[76]1 1 3 1, 5 (1): 1[77]1 1 1 2, 4 (1): 1[77]1 1 3 2, 4 (1): 1[78]1 1 3, 3 (1): 1[78]1 1 3 3, 3 (1): 1[79]1 1 2 1, 4 (1): 1[79]1 1 2 1, 4 (1): 1[80]1 1 2 2, 3 (1): 1[80]1 1 2 2, 3 (1): 1[81]1 1 2 3, 3 (1): 1[81]1 1 2 3, 3 (1): 1[82]1 1 3 1, 3 (1): 1[81]1 1 2 3, 3 (1): 1[83]1 1 3 2, 3 (1): 1[83]1 1 1 2, 2 (1): 1[84]1 1 3, 2, 0 (1): 1[84]1 1 1 3, 2 (1): 1[85]2 3 1 1, 5 (1): 1[85]2 3 3 1, 5 (1): 1[86]2 3 1 2, 5 (1): 1[86]2 3 3 2, 5 (1): 1[87]2 3 1 3, 4 (1): 1[87]2 3 3, 4 (1): 1[88]2 3 2 1, 5 (1): 1[88]2 3 2 1, 5 (1): 1[89]2 3 2 2, 4 (1): 1[89]2 3 2 2, 5 (1): 1[90]2 3 2 3, 3 (1): 1[90]2 3 2 3, 3 (1): 1[91]2 3 3 1, 4 (1): 1[91]2 3 1 1, 5 (1): 1[92]2 3 3 2, 3 (1): 1[93]2 3 1 3, 4 (1): 1[94]2 2 1 1, 4 (1): 1[94]2 2 3 1, 5 (1): 1[95]2 2 1 2, 4 (1): 1[96]2 2 3 3, 3 (1): 1[96]2 2 1 3, 3 (1): 1[97]2 2 2 1, 5 (1): 1[97]2 2 2 1, 4 (1): 1[97]2 2 2 1, 5 (1): 1[98]2 2 2 2, 3 (1): 1[99]2 2 2 3, 3 (1): 1[99]2 2 2 3, 3 (1): 1[100]2 1 1, 3 (1): 1 <tr <td="">[101]2 2 1 2, 3 (1):</tr>	[74]	1 2 3 2, 3 (1) : 1	[74]	1 2 1 2, 3 (1) : 1
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[84] $1 1 3 3, 2 (1) : 1$ $[84]$ $1 1 1 3, 2 (1) : 1$ $[85]$ $2 3 1 1, 5 (1) : 1$ $[85]$ $2 3 3 1, 5 (1) : 1$ $[86]$ $2 3 1 2, 5 (1) : 1$ $[86]$ $2 3 3 2, 5 (1) : 1$ $[87]$ $2 3 1 3, 4 (1) : 1$ $[87]$ $2 3 3 3, 4 (1) : 1$ $[88]$ $2 3 2 1, 5 (1) : 1$ $[88]$ $2 3 2 1, 5 (1) : 1$ $[89]$ $2 3 2 2, 4 (1) : 1$ $[89]$ $2 3 2 2, 5 (1) : 1$ $[90]$ $2 3 2 3, 3 (1) : 1$ $[90]$ $2 3 2 3, 3 (1) : 1$ $[91]$ $2 3 3 1, 4 (1) : 1$ $[91]$ $2 3 1, 5 (1) : 1$ $[92]$ $2 3 3 2, 3 (1) : 1$ $[92]$ $2 3 1, 5 (1) : 1$ $[93]$ $2 3 3, 3 (1) : 1$ $[92]$ $2 3 1, 5 (1) : 1$ $[93]$ $2 3 3, 3 (1) : 1$ $[93]$ $2 3 1, 3, 4 (1) : 1$ $[94]$ $2 2 1 1, 4 (1) : 1$ $[94]$ $2 2 3 1, 5 (1) : 1$ $[95]$ $2 2 1 2, 4 (1) : 1$ $[95]$ $2 2 2 3, 3 (1) : 1$ $[96]$ $2 2 1 3, 3 (1) : 1$ $[96]$ $2 2 3 3, 3 (1) : 1$ $[97]$ $2 2 2 1, 4 (1) : 1$ $[97]$ $2 2 2 1, 5 (1) : 1$ $[98]$ $2 2 2 2, 3 (1) : 1$ $[98]$ $2 2 2 2, 4 (1) : 1$ $[99]$ $2 2 2 3, 3 (1) : 1$ $[99]$ $2 2 2 3, 3 (1) : 1$ $[100]$ $2 2 3 1, 3 (1) : 1$ $[100]$ $2 2 1 1, 3 (1) : 1$ $[101]$ $2 2 3 3, 2 (1) : 1$ $[102]$ $2 2 1 3, 2 (1) : 1$	[82]	1 1 3 1, 3 (1) : 1	[82]	1 1 1 1, 3 (1) : 1
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$[86]$ $2 \ 3 \ 1 \ 2 \ 5 \ (1) : 1$ $[86]$ $2 \ 3 \ 3 \ 2 \ 5 \ (1) : 1$ $[87]$ $2 \ 3 \ 1 \ 3 \ 4 \ (1) : 1$ $[87]$ $2 \ 3 \ 3 \ 3 \ 4 \ (1) : 1$ $[88]$ $2 \ 3 \ 2 \ 1 \ 5 \ (1) : 1$ $[88]$ $2 \ 3 \ 2 \ 1 \ 5 \ (1) : 1$ $[89]$ $2 \ 3 \ 2 \ 2 \ 4 \ (1) : 1$ $[89]$ $2 \ 3 \ 2 \ 2 \ 5 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[91]$ $2 \ 3 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 1 \ 2 \ 5 \ (1) : 1$ $[93]$ $2 \ 3 \ 3 \ 3 \ (1) : 1$ $[93]$ $2 \ 3 \ 1 \ 3 \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 1 \ 1 \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 3 \ 1 \ 3 \ (1) : 1$ $[95]$ $2 \ 2 \ 1 \ 2 \ 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1 \ 4 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 3 \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3 \ 3 \ (1) : 1$ $[102]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$	[84]	1 1 3 3, 2 (1) : 1	[84]	1 1 1 3, 2 (1) : 1
$[87]$ $2 \ 3 \ 1 \ 3, 4 \ (1) : 1$ $[87]$ $2 \ 3 \ 3 \ 3, 4 \ (1) : 1$ $[88]$ $2 \ 3 \ 2 \ 1, 5 \ (1) : 1$ $[88]$ $2 \ 3 \ 2 \ 1, 5 \ (1) : 1$ $[89]$ $2 \ 3 \ 2 \ 2, 4 \ (1) : 1$ $[89]$ $2 \ 3 \ 2 \ 2, 5 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3, 3 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3, 3 \ (1) : 1$ $[91]$ $2 \ 3 \ 1, 4 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3, 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 2, 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 1, 5 \ (1) : 1$ $[93]$ $2 \ 3 \ 3, 3 \ (1) : 1$ $[93]$ $2 \ 3 \ 1, 3 \ (1) : 1$ $[94]$ $2 \ 2 \ 1, 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 3 \ 1, 5 \ (1) : 1$ $[95]$ $2 \ 2 \ 1, 2 \ 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 3, 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 1, 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3, 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1, 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 2 \ 3, 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, 4 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3, 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1, 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 3, 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1, 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3, 2 \ 3, 2 \ (1) : 1$ $[101]$ $2 \ 2 \ 1, 3 \ (1) : 1$	[85]	2 3 1 1, 5 (1) : 1	[85]	2 3 3 1, 5 (1) : 1
$[88]$ $2 \ 3 \ 2 \ 1, 5 \ (1) : 1$ $[88]$ $2 \ 3 \ 2 \ 1, 5 \ (1) : 1$ $[89]$ $2 \ 3 \ 2 \ 2, 4 \ (1) : 1$ $[89]$ $2 \ 3 \ 2 \ 2, 5 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3, 3 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3, 3 \ (1) : 1$ $[91]$ $2 \ 3 \ 3, 3 \ (1) : 1$ $[91]$ $2 \ 3 \ 1, 5 \ (1) : 1$ $[92]$ $2 \ 3 \ 2, 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 1, 5 \ (1) : 1$ $[93]$ $2 \ 3 \ 3, 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 1, 5 \ (1) : 1$ $[94]$ $2 \ 2 \ 1, 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 3, 1, 5 \ (1) : 1$ $[95]$ $2 \ 2 \ 1, 2, 4 \ (1) : 1$ $[95]$ $2 \ 2 \ 3, 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3, 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3, 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2, 1, 4 \ (1) : 1$ $[97]$ $2 \ 2 \ 2, 1, 5 \ (1) : 1$ $[98]$ $2 \ 2 \ 2, 2, 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2, 2, 4 \ (1) : 1$ $[99]$ $2 \ 2 \ 2, 3, 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1, 3 \ (1) : 1$ $[100]$ $2 \ 3 \ 1, 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1, 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3, 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 1, 3 \ (1) : 1$	[86]	2 3 1 2, 5 (1) : 1	[86]	2 3 3 2, 5 (1) : 1
$[89]$ $2 \ 3 \ 2 \ 2 \ 4 \ (1) : 1$ $[89]$ $2 \ 3 \ 2 \ 2 \ 5 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[91]$ $2 \ 3 \ 1 \ 4 \ (1) : 1$ $[91]$ $2 \ 3 \ 1 \ 5 \ (1) : 1$ $[92]$ $2 \ 3 \ 3 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 1 \ 2 \ 5 \ (1) : 1$ $[93]$ $2 \ 3 \ 3 \ 3 \ 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 1 \ 3 \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 1 \ 1 \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 3 \ 1 \ 5 \ (1) : 1$ $[95]$ $2 \ 2 \ 1 \ 2 \ 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 1 \ 3 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 3 \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3 \ 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[102]$ $2 \ 2 \ 3 \ 3 \ (1) : 1$ $[102]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$	[87]	2 3 1 3, 4 (1) : 1	[87]	2 3 3 3, 4 (1) : 1
$[90]$ $2 \ 3 \ 2 \ 3 \ 3 \ (1) : 1$ $[90]$ $2 \ 3 \ 2 \ 3 \ 3 \ (1) : 1$ $[91]$ $2 \ 3 \ 1 \ 4 \ (1) : 1$ $[91]$ $2 \ 3 \ 1 \ 5 \ (1) : 1$ $[92]$ $2 \ 3 \ 3 \ 2 \ 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 1 \ 2 \ 5 \ (1) : 1$ $[93]$ $2 \ 3 \ 3 \ 3 \ 3 \ (1) : 1$ $[93]$ $2 \ 3 \ 1 \ 3 \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 1 \ 1 \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 3 \ 1 \ 3 \ 4 \ (1) : 1$ $[95]$ $2 \ 2 \ 1 \ 2 \ 4 \ (1) : 1$ $[95]$ $2 \ 2 \ 3 \ 2 \ 3 \ 2 \ 3 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1 \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1 \ 5 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[100]$ $2 \ 3 \ 1 \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3 \ 3 \ (1) : 1$ $[102]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$	[88]	2 3 2 1, 5 (1) : 1	[88]	2 3 2 1, 5 (1) : 1
$[91]$ $2 \ 3 \ 3 \ 1, 4 \ (1) : 1$ $[91]$ $2 \ 3 \ 1 \ 1, 5 \ (1) : 1$ $[92]$ $2 \ 3 \ 3 \ 2, 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 1 \ 2, 5 \ (1) : 1$ $[93]$ $2 \ 3 \ 3, 3 \ (1) : 1$ $[93]$ $2 \ 3 \ 1 \ 3, 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 1 \ 1, 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 3 \ 1, 5 \ (1) : 1$ $[95]$ $2 \ 2 \ 1 \ 2, 4 \ (1) : 1$ $[95]$ $2 \ 2 \ 3 \ 2, 5 \ (1) : 1$ $[96]$ $2 \ 2 \ 1 \ 3, 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3, 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1, 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3, 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1, 4 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 3, 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, 4 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3, 3 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3, 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 3 \ 1, 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1, 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3 \ 3, 2 \ (1) : 1$ $[102]$ $2 \ 2 \ 1, 3 \ (1) : 1$	[89]	2 3 2 2, 4 (1) : 1	[89]	2 3 2 2, 5 (1) : 1
$[92]$ $2 \ 3 \ 3 \ 2, \ 3 \ (1) : 1$ $[92]$ $2 \ 3 \ 1 \ 2, \ 5 \ (1) : 1$ $[93]$ $2 \ 3 \ 3 \ 3, \ 3 \ (1) : 1$ $[93]$ $2 \ 3 \ 1 \ 3, \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 1 \ 1, \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 3 \ 1, \ 5 \ (1) : 1$ $[95]$ $2 \ 2 \ 1 \ 2, \ 4 \ (1) : 1$ $[95]$ $2 \ 2 \ 3 \ 2, \ 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 1 \ 3, \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3, \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1, \ 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3, \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1, \ 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3, \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 3, \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 3, \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, \ 4 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3, \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1, \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 3 \ 3, \ 2 \ (1) : 1$ $[100]$ $2 \ 2 \ 1, \ 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3 \ 3, \ 2 \ (1) : 1$ $[102]$ $2 \ 2 \ 1 \ 3, \ 2 \ (1) : 1$	[90]	2 3 2 3, 3 (1) : 1	[90]	2 3 2 3, 3 (1) : 1
$[93]$ $2 \ 3 \ 3 \ 3 \ 3 \ (1) : 1$ $[93]$ $2 \ 3 \ 1 \ 3 \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 1 \ 1 \ 4 \ (1) : 1$ $[94]$ $2 \ 2 \ 3 \ 1 \ 5 \ (1) : 1$ $[95]$ $2 \ 2 \ 1 \ 2 \ 4 \ (1) : 1$ $[95]$ $2 \ 2 \ 3 \ 2 \ 3 \ 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 1 \ 3 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3 \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3 \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 3 \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3 \ 3 \ 2 \ (1) : 1$ $[101]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$ $[102]$ $2 \ 2 \ 3 \ 3 \ 2 \ (1) : 1$ $[102]$ $2 \ 2 \ 1 \ 3 \ (1) : 1$	[91]	2 3 3 1, 4 (1) : 1	[91]	2 3 1 1, 5 (1) : 1
$[94]$ $2\ 2\ 1\ 1,\ 4\ (1)\ :\ 1$ $[94]$ $2\ 2\ 3\ 1,\ 5\ (1)\ :\ 1$ $[95]$ $2\ 2\ 1\ 2,\ 4\ (1)\ :\ 1$ $[95]$ $2\ 2\ 3\ 2,\ 4\ (1)\ :\ 1$ $[96]$ $2\ 2\ 1\ 3,\ 3\ (1)\ :\ 1$ $[96]$ $2\ 2\ 3\ 3,\ 3\ (1)\ :\ 1$ $[97]$ $2\ 2\ 2\ 1,\ 4\ (1)\ :\ 1$ $[97]$ $2\ 2\ 2\ 1,\ 5\ (1)\ :\ 1$ $[98]$ $2\ 2\ 2\ 2,\ 3\ (1)\ :\ 1$ $[98]$ $2\ 2\ 2\ 2,\ 4\ (1)\ :\ 1$ $[99]$ $2\ 2\ 2\ 3,\ 3\ (1)\ :\ 1$ $[99]$ $2\ 2\ 2\ 3,\ 3\ (1)\ :\ 1$ $[100]$ $2\ 2\ 3\ 1,\ 3\ (1)\ :\ 1$ $[100]$ $2\ 2\ 1\ 1,\ 3\ (1)\ :\ 1$ $[101]$ $2\ 2\ 3\ 3,\ 2\ (1)\ :\ 1$ $[102]$ $2\ 2\ 1\ 3,\ 2\ (1)\ :\ 1$	[92]	2 3 3 2, 3 (1) : 1	[92]	2 3 1 2, 5 (1) : 1
$[95]$ $2 \ 2 \ 1 \ 2, 4 \ (1) : 1$ $[95]$ $2 \ 2 \ 3 \ 2, 4 \ (1) : 1$ $[96]$ $2 \ 2 \ 1 \ 3, 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3, 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1, 4 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1, 5 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, 4 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3, 3 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3, 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 3 \ 1, 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1 \ 1, 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3 \ 3, 2 \ (1) : 1$ $[102]$ $2 \ 2 \ 1 \ 3, 2 \ (1) : 1$	[93]	2 3 3 3, 3 (1) : 1	[93]	2 3 1 3, 4 (1) : 1
$[96]$ $2 \ 2 \ 1 \ 3, \ 3 \ (1) : 1$ $[96]$ $2 \ 2 \ 3 \ 3, \ 3 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1, \ 4 \ (1) : 1$ $[97]$ $2 \ 2 \ 2 \ 1, \ 5 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, \ 3 \ (1) : 1$ $[98]$ $2 \ 2 \ 2 \ 2, \ 4 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3, \ 3 \ (1) : 1$ $[99]$ $2 \ 2 \ 2 \ 3, \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 3 \ 1, \ 3 \ (1) : 1$ $[100]$ $2 \ 2 \ 1 \ 1, \ 3 \ (1) : 1$ $[101]$ $2 \ 2 \ 3 \ 3, \ 2 \ (1) : 1$ $[102]$ $2 \ 2 \ 1 \ 3, \ 2 \ (1) : 1$	[94]	2 2 1 1, 4 (1) : 1	[94]	2 2 3 1, 5 (1) : 1
[97] $2 2 2 1, 4 (1) : 1$ $[97]$ $2 2 2 1, 5 (1) : 1$ $[98]$ $2 2 2 2, 3 (1) : 1$ $[98]$ $2 2 2 2, 4 (1) : 1$ $[99]$ $2 2 2 3, 3 (1) : 1$ $[99]$ $2 2 2 3, 3 (1) : 1$ $[100]$ $2 2 3 1, 3 (1) : 1$ $[100]$ $2 2 1 1, 3 (1) : 1$ $[101]$ $2 2 3 2, 3 (1) : 1$ $[101]$ $2 2 1 2, 3 (1) : 1$ $[102]$ $2 2 3 3, 2 (1) : 1$ $[102]$ $2 2 1 3, 2 (1) : 1$	[95]	2 2 1 2, 4 (1) : 1	[95]	2 2 3 2, 4 (1) : 1
[98]       2 2 2 2, 3 (1) : 1       [98]       2 2 2 2, 4 (1) : 1         [99]       2 2 2 3, 3 (1) : 1       [99]       2 2 2 3, 3 (1) : 1         [100]       2 2 3 1, 3 (1) : 1       [100]       2 2 1 1, 3 (1) : 1         [101]       2 2 3 2, 3 (1) : 1       [101]       2 2 1 2, 3 (1) : 1         [102]       2 2 3 3, 2 (1) : 1       [102]       2 2 1 3, 2 (1) : 1	[96]	2 2 1 3, 3 (1) : 1	[96]	2 2 3 3, 3 (1) : 1
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[101]2 2 3 2, 3 (1) : 1[101]2 2 1 2, 3 (1) : 1[102]2 2 3 3, 2 (1) : 1[102]2 2 1 3, 2 (1) : 1	[99]	2 2 2 3, 3 (1) : 1	[99]	2 2 2 3, 3 (1) : 1
[102] 2 2 3 3, 2 (1) : 1 [102] 2 2 1 3, 2 (1) : 1	[100]	2 2 3 1, 3 (1) : 1	[100]	2 2 1 1, 3 (1) : 1
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[103] 2 1 1 1, 4 (1) : 1 [103] 2 1 3 1, 4 (1) : 1	[102]	2 2 3 3, 2 (1) : 1	[102]	2 2 1 3, 2 (1) : 1
	[103]	2 1 1 1, 4 (1) : 1	[103]	2 1 3 1, 4 (1) : 1

[104]	2 1 1 2, 3 (1) : 1	[104]	2 1 3 2, 3 (1) : 1
[105]	2 1 1 3, 3 (1) : 1	[105]	2 1 3 3, 3 (1) : 1
[106]	2 1 2 1, 3 (1) : 1	[106]	2 1 2 1, 3 (1) : 1
[107]	2 1 2 2, 3 (1) : 1	[107]	2 1 2 2, 3 (1) : 1
[108]	2 1 2 3, 2 (1) : 1	[108]	2 1 2 3, 2 (1) : 1
[109]	2 1 3 1, 3 (1) : 1	[109]	2 1 1 1, 3 (1) : 1
[110]	2 1 3 2, 2 (1) : 1	[110]	2 1 1 2, 2 (1) : 1
[111]	2 1 3 3, 1 (1) : 1	[111]	2 1 1 3, 1 (1) : 1
[112]	3 3 1 1, 5 (1) : 1	[112]	3 3 3 1, 5 (1) : 1
[113]	3 3 1 2, 4 (1) : 1	[113]	3 3 3 2, 4 (1) : 1
[114]	3 3 1 3, 3 (1) : 1	[114]	3 3 3 3, 3 (1) : 1
[115]	3 3 2 1, 4 (1) : 1	[115]	3 3 2 1, 5 (1) : 1
[116]	3 3 2 2, 3 (1) : 1	[116]	3 3 2 2, 4 (1) : 1
[117]	3 3 2 3, 3 (1) : 1	[117]	3 3 2 3, 3 (1) : 1
[118]	3 3 3 1, 3 (1) : 1	[118]	3 3 1 1, 3 (1) : 1
[119]	3 3 3 2, 3 (1) : 1	[119]	3 3 1 2, 3 (1) : 1
[120]	3 3 3 3, 2 (1) : 1	[120]	3 3 1 3, 2 (1) : 1
[121]	3 2 1 1, 4 (1) : 1	[121]	3 2 3 1, 4 (1) : 1
[122]	3 2 1 2, 3 (1) : 1	[122]	3 2 3 2, 4 (1) : 1
[123]	3 2 1 3, 3 (1) : 1	[123]	3 2 3 3, 3 (1) : 1
[124]	3 2 2 1, 3 (1) : 1	[124]	3 2 2 1, 5 (1) : 1
[125]	3 2 2 2, 3 (1) : 1	[125]	3 2 2 2, 5 (1) : 1
[126]	3 2 2 3, 2 (1) : 1	[126]	3 2 2 3, 2 (1) : 1
[127]	3 2 3 1, 3 (1) : 1	[127]	3 2 1 1, 4 (1) : 1
[128]	3 2 3 2, 2 (1) : 1	[128]	3 2 1 2, 3 (1) : 1
[129]	3 2 3 3, 1 (1) : 1	[129]	3 2 1 3, 1 (1) : 1
[130]	3 1 1 1, 3 (1) : 1	[130]	3 1 3 1, 3 (1) : 1
[131]	3 1 1 2, 3 (1) : 1	[131]	3 1 3 2, 3 (1) : 1
[132]	3 1 1 3, 2 (1) : 1	[132]	3 1 3 3, 2 (1) : 1
[133]	3 1 2 1, 3 (1) : 1	[133]	3 1 2 1, 3 (1) : 1
[134]	3 1 2 2, 2 (1) : 1	[134]	3 1 2 2, 2 (1) : 1
[135]	3 1 2 3, 1 (1) : 1	[135]	3 1 2 3, 1 (1) : 1
[136]	3 1 3 1, 2 (1) : 1	[136]	3 1 1 1, 2 (1) : 1
[137]	3 1 3 2, 1 (1) : 1	[137]	3 1 1 2, 1 (1) : 1
[138]	3 1 3 3, 1 (1) : 1	[138]	3 1 1 3, 1 (1) : 1

[139]	4 3 1 1, 4 (1) : 1	[139]	4 3 3 1, 5 (1) : 1
[140]	4 3 1 2, 3 (1) : 1	[140]	4 3 3 2, 4 (1) : 1
[141]	4 3 1 3, 3 (1) : 1	[141]	4 3 3 3, 3 (1) : 1
[142]	4 3 2 1, 3 (1) : 1	[142]	4 3 2 1, 3 (1) : 1
[143]	4 3 2 2, 3 (1) : 1	[143]	4 3 2 2, 3 (1) : 1
[144]	4 3 2 3, 2 (1) : 1	[144]	4 3 2 3, 2 (1) : 1
[145]	4 3 3 1, 3 (1) : 1	[145]	4 3 1 1, 3 (1) : 1
[146]	4 3 3 2, 2 (1) : 1	[146]	4 3 1 2, 2 (1) : 1
[147]	4 3 3 3, 1 (1) : 1	[147]	4 3 1 3, 1 (1) : 1
[148]	4 2 1 1, 3 (1) : 1	[148]	4 2 3 1, 4 (1) : 1
[149]	4 2 1 2, 3 (1) : 1	[149]	4 2 3 2, 4 (1) : 1
[150]	4 2 1 3, 2 (1) : 1	[150]	4 2 3 3, 2 (1) : 1
[151]	4 2 2 1, 3 (1) : 1	[151]	4 2 2 1, 3 (1) : 1
[152]	4 2 2 2, 2 (1) : 1	[152]	4 2 2 2, 2 (1) : 1
[153]	4 2 2 3, 1 (1) : 1	[153]	4 2 2 3, 1 (1) : 1
[154]	4 2 3 1, 2 (1) : 1	[154]	4 2 1 1, 2 (1) : 1
[155]	4 2 3 2, 1 (1) : 1	[155]	4 2 1 2, 1 (1) : 1
[156]	4 2 3 3, 1 (1) : 1	[156]	4 2 1 3, 1 (1) : 1
[157]	4 1 1 1, 3 (1) : 1	[157]	4 1 3 1, 3 (1) : 1
[158]	4 1 1 2, 2 (1) : 1	[158]	4 1 3 2, 2 (1) : 1
[159]	4 1 1 3, 1 (1) : 1	[159]	4 1 3 3, 1 (1) : 1
[160]	4 1 2 1, 2 (1) : 1	[160]	4 1 2 1, 3 (1) : 1
[161]	4 1 2 2, 1 (1) : 1	[161]	4 1 2 2, 2 (1) : 1
[162]	4 1 2 3, 1 (1) : 1	[162]	4 1 2 3, 1 (1) : 1
[163]	4 1 3 1, 2 (1) : 1	[163]	4 1 1 1, 3 (1) : 1
[164]	4 1 3 2, 1 (1) : 1	[164]	4 1 1 2, 2 (1) : 1
[165]	4 1 3 3, 1 (1) : 1	[165]	4 1 1 3, 1 (1) : 1