Economic Design of X-bar Control Chart using Gravitational Search Algorithm

A thesis submitted in partial fulfilment of the requirements for the degree of

Master of Technology

in

Mechanical Engineering

by

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CERTIFICATE

This is to certify that thesis entitled **"Economic Design of X-bar Control Chart using Gravitational Search Algorithm"** submitted by **RAMBABU JUVVALA** bearing **Roll No 213ME2406** in partial fulfillment of the requirements for the award of **Master of Technology in Mechanical Engineering** with **"Production Engineering"** Specialization during session 2014-2015 in the Department of Mechanical Engineering, National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other University/Institute for award of any Degree or Diploma.

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Abstract

Control chart is a major and one of most widely used statistical process control (SPC) tools. It is used to statistically monitor the process through sampling inspection. Control chart tells us when to allow the process to continue or avoid unnecessary adjustments with machine and when to take the corrective action. On to same problem either on the material side or from the operator side it is quite possible that either targeted value X-bar has changed or process dispersion has changed. These changes must be reflected on the control chart so that the corrective action can be taken. The use of control chart requires selection of three parameters namely sample size n, sampling interval h, and width of control limits k for the chart. Duncan developed a loss cost function for X-bar control chart with single assignable cause. The function has to be optimized using metaheuristic optimization technique. In the present project, the economic design of the X-bar control chart using Gravitational Search Algorithm (GSA) has been developed MATLAB software to determine the three parameters i.e. n , h and k such that the expected total cost per hour is minimized. The results obtained are found to be better than that reported in literature.

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Chapter 1

Introduction

1.1 Statistical process control

Statistical process control (SPC) is a powerful collection of statistical methods to the monitor and control of a process to ensure that it operates as its full potential to produce conforming product. Under SPC, a process desired to produce as much conforming product as possible with the least possible waste. A major objective of statistical process control is to quickly detect the occurrence of assignable causes of process shifts so that investigation of the process and corrective action may be undertaken before many nonconforming units are manufactured.

1.2 Control chart

Control chart is one of the widely used statistical process control (SPC) tools. It is used to statistically monitor the process through sampling inspection. It indicates whether the process is in-control or out-of-control. If any point falls within the upper control limit and lower control limit, the process is referred to as “in-control” where if the point falls outside the control limits, the process is referred to as “out-of-control”. The major function of control chart is to detect the occurrence of assignable causes so that the necessary corrective action may be taken before a large number of nonconforming products are manufactured.

Control chart tells us when to allow the process to continue or avoid unnecessary adjustments with machine and when to take the corrective action. On to same problem either on the material side or from the operator side it is quite possible that either targeted value X-bar has changed, or process dispersion has changed. These changes must be reflected on the control chart so that the corrective action can be taken.

The following are the factors that change the mean of the process:

- Tool and die wear,
- Machine vibrations,
- Wear or minor failure of machine parts change of machine or process, and
- Change of machine or process.

The following are the factors that change the process dispersion:

- Deviation of depression,
- Carelessness of operator,
- New or inexperienced work, and
- Frequent resetting of machine.

In any production process, regardless of how well designed or carefully maintained certain amount of inherent or natural variability will always exist. This natural variability or "background noises the cumulative effect of many small" essentially unavoidable causes. In the framework of statistical quality control, this natural variability is often called a "Stable system of change causes". A process that is operating with only change causes of variation present is said to be in statistical control .in other words, the chance causes are an inherent part of the process.

Other kinds of variability may occasionally be present in the output of a process. This variability in key quality characteristics usually arises from three sources: improperly adjusted or control machines, operator errors, or defective raw material. Such variability is generally large when compared to be background noise, and it usually represents an unacceptable level of process performance. Refer to these sources of variability that are not part of the chance cause pattern as assignable causes or variation. A process that is operating in the presence of assignable cause is said to be an out of control process.

As illustrated in Fig. 1.1 a control chart consists of the following lines:

i) a centre line ,

ii) a upper control limit, and

iii) a lower control limit.

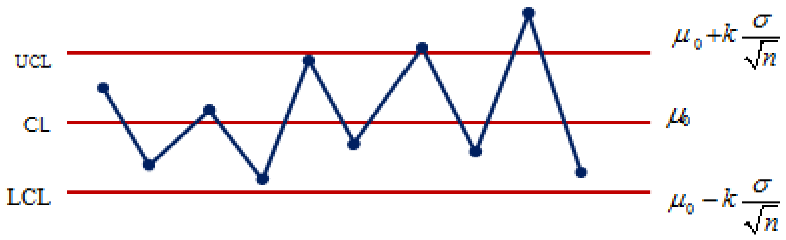

Fig. 1.1: X-bar control chart

1.3 Types of control chart

The types of charts are often classified according to the type quality characteristics that they are supposed to monitor. Control charts can classify for variables chart and attributes charts.

1.3.1 Variable charts

The classical type of control chart is constructed by collecting data periodically and plotting it versus time. If more than one data value is collected at the same time, statistics such as the mean, range, median, or standard deviation are plotted. Control limits are added to the plot to signal unusually large deviations from the centreline, and run rules are employed to detect other unusual patterns. Variable control charts are used for those quality characteristics which follow normal distribution, e.g. length, diameter etc.

- **X-bar chart:** The X-bar chart monitors the process location over time, based on the average of a series of observations, called a subgroup. This chart is used to plot the sample means in order to control the mean value of a variable.
- **R chart**: This chart is used to plot the sample ranges in order to control the variability of a variable. The range chart monitors the variation between observations in the subgroup over time.
- **S chart:** This chart is used to plot the sample standard deviations in order to control the variability of a variable.
- **S^2 chart:** This chart is used to plot the sample variances in order to control the variability of a variable.

Advantages of attribute control charts:

Attribute control charts have the advantage of allowing for quick summaries of various aspects of the quality of a product, that is, the engineer may simply classify products as acceptable or unacceptable, based on various quality criteria. Thus, attribute charts sometimes the need for expensive, precise devices and time consuming measurement procedures. Also, this type of chart tends to be more easily understood by persons those are unfamiliar with quality control procedures: therefore, it may provide more persuasive (to management) evidence of quality problems.

1.3.2 Attribute charts

For attribute data, such as arise from pass or fail testing, the charts used most often plot either rates or proportions. When the sample sizes vary, the control limits depend on the size of the samples.

Attribute charts are a set of control charts specifically designed for attributes data. Attribute charts monitor the process location and variation over time in a single chart.

- **np chart**: For monitoring the number of times a condition occurs, relative to a constant sample size, when each sample can either have this condition, or not have this condition.
- **p chart:** For monitoring the percent of samples having the condition, relative to either a fixed or varying sample size, when each sample can either have this condition, or not have this condition.
- **c chart:** For monitoring the number of times a condition occurs, relative to a constant sample size, when each sample can have more than one instance of the condition.
- **u chart:** For monitoring the percent of samples having the condition, relative to either a fixed or varying sample size, when each sample can have more than one instance of the condition.

Advantages of variable control charts:

Variable control charts are more sensitive than attribute control charts. Therefore, variable control charts may alert us to quality problems before any actual "unacceptable" (as detected by the attribute chart) will occur. The variable control charts are leading indicators of trouble that will sound an alarm before the number of scraps increases in the production process.

1.3.3 Other specialized control charts

Some of the specialized control charts are listed below:

1. Time–weighted charts,
2. Cumulative sum (CUSUM) control chart,
3. Moving average (MA) control chart, and
4. Exponential weighted moving average (EMWA) control.

1. Time-Weighted Charts

When data is collected one sample at a time and plotted on an individual's chart, the control limits are usually quite wide, causing the chart to have poor power in detecting out-of- control situations. This can be remedied by plotting a weighted average or cumulative sum of the data, not just the most recent observation. The average run length of such charts is usually much less than that of a simple X chart.

2. CUSUM Charts

A CUSUM Chart is a control chart for variables data which plots the cumulative sum of the deviations from a target. A V-mask is used as control limits. Because each plotted point on the CUSUM Chart uses information from all prior samples, it detects much smaller process shifts than a normal control chart would. CUSUM Charts are especially effective with a subgroup size of one. Run tests should not be used since each plotted point is dependent on prior points as they contain common data values.

3. Moving Average (MA) Chart

To return to the piston ring example, suppose we are mostly interested in detecting small trends across successive sample means. For example, we may be particularly concerned about machine wear, leading to a slow but constant deterioration of quality (i.e., deviation from specification). The CUSUM chart described above is one way to monitor such trends, and to detect small permanent shifts in the process average. Another way is to use some weighting 8 schemes that summarize the means of several successive samples moving such a weighted mean across the samples will produce a moving average chart.

4. Exponentially-Weighted Moving Average (EWMA) Chart

The idea of moving averages of successive (adjacent) samples can be generalized. In principle, in order to detect a trend we need to weight successive samples to form a moving average; however, instead of a simple arithmetic moving average, it could compute a geometric moving average.

1.4 Design of control chart

Design of a control chart involves the selection of three parameters, namely the sample size (n), the sampling interval (h) and the width of control limits (k). The selection of these three parameters is called the design of control chart.

Basically design of control chart is of three types explained below:

1. **Statistical design of control chart:** Since control chart is based on sampling inspection, it is always associated with two types of statistical errors namely Type-I error and Type-II error. These two errors cannot be completely eliminated since 100% inspection is not carried out. However, these two errors can be minimized which serves as the basic principle of statistical design of control chart.
2. **Economic design of control chart:** In economic design of control chart the objective is to reduce the total cost of maintaining the control chart as minimum as possible. It is used to determine the values of various design parameters i.e. sample size (n), sampling interval (h), and control limit coefficient (k) that minimizes total economic cost.
3. **Statistical economical design of control chart:** Statistical-economic design is basically a combination of statistical and economic design of control chart. In this type of design, the total cost of maintaining the control chart need to be minimized and at the same time Type-I and Type-II errors are not allowed to exceed their permissible level.

1.5 Gravitational search algorithm

Gravitational Search Algorithm (GSA) has been proposed by Rashedi and Nezamabadi (2009). Gravitational search algorithm (GSA) based on the law of gravity and the concept of mass interactions. Using the gravitational force, each mass in the framework can see the circumstance of different masses. The gravitational force is in this way a method for exchanging data between distinctive masses. In GSA, masses are considered as objects and their performance is calculated by their masses. Every one of these objects attracts in one another by a gravity force, and this force causes a development of all objects universally towards the object with heavier masses. Hence, heaviest mass it will give the solution of the problem. The position of the agent corresponds to a solution of the problem, and its mass is determined using a fitness function. By lapse of iteration, masses are attracted by the heaviest mass, this agent present an optimum solution in the search domain. Every particle attract to other particle with a gravitational force. The force is directly proportion to product of two masses and inversely proportional to the square to of Euclidian distance between them.

$$F = G \frac{M_1 * M_2}{R^2} \tag{1.1}$$

where

F = gravitation force,

G = proportional constant,

M_1 = active gravitational mass,

M_2 = passive gravitational mass, and

R = Euclidian distance between the two particles.

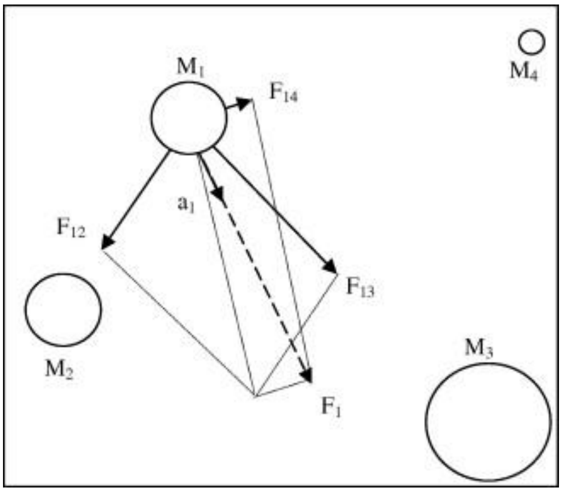

Fig. 1.2: Mass interactions in GSA

The steps of the Gravitational search algorithm are as follows:

1. Search domain identification,
2. Initialize the random numbers,
3. Evaluate the fitness for each agent,
4. Update *G(θ),* best(θ), worst(θ) and *Mi(θ)*, for i =1, 2, 3,… N
5. Calculate the total force in different directions,
6. Calculate the acceleration *a(θ)* and velocity *V(θ)*,
7. Update the agents position,
8. Repeat steps 3 to 7 until the stop criterion is reached, and
9. End.

Chapter 2

Literature Review

2.1 Design of control chart

Duncan (1956) proposed the first economic model for determining the three test parameters i.e. sample size (*n*), sample intervals (*h*) and width of control limit (*k*). For the X-bar control chart that minimizes the average cost when a single out-of-control state (assignable cause) exists. Duncan's cost model includes the cost of sampling and inspection, the cost of defective products, the cost of a false alarm, the cost of searching for an assignable cause, and the cost of process correction. Since then, considerable attention has been devoted to the optimal economic determination of the three parameters of X-bar charts.

Rahim et al. (1993) has presented an integrated model for the joint optimization of the maintenance level and the economic design of X-bar control chart. In this model, preventive maintenance reduces the shift rate to an out-of-control state by an amount proportional to the preventive maintenance level. A numerical experiment is conducted to investigate the Weibull shock model with increasing hazard rate is used to illustrate the effect of the maintenance level on economic design of control chart.

Alexander et al. (1995) have applied Duncan's cost model with Taguchi's loss function to incorporate losses that result from both inherent variability due to assignable causes. Whereas Duncan applies a penalty cost for operating out-of-control, he does not show this cost can be obtained or quantified. They are test the behaviour of this model through sensitivity analysis. They analysis is indicated that the design parameters for the x-bar chart are fairly robust when the cost of finding assignable cause and the frequency of occurrence of an assignable cause are not too high.

Goel et al. (1968) introduced an algorithm to find the exact solution to the Duncan's single assignable cause model. Some manufacturing situations may not permit stopping of the process after the control chart signals an out-of-control situation and in some cases it may be advantageous to stop the process and take a remedial action. The quality characteristic observed can be a variable or an attribute. Different models have been developed to accommodate such different situations encountered in the manufacturing.

Prabhu et al. (1994) have designed X-bar control charts with adaptive parameters and showed that they are superior to conventional control chart designs. In general, the complexity of the models has grown from single assignable cause models to multiple assignable cause models and exponential failures to Weibull and gamma distributions. The quality characteristics considered in the models have grown from monitoring a single quality characteristic (univariate) to multiple quality characteristics (multivariate). Control charts that can use present and past information effectively have been introduced. Use of adaptive parameters has been studied. In spite of many theoretical developments in the area of control chart designs, it is observed that very little has been implemented in practice. Researchers have attributed many reasons for this situation.

Niaki et al. (2011) presented an economic model using particle swarm optimization technique for the economic-statistic model of MEWMA (multivariate exponentially weighted moving average) control charts. MEWMA control chart is used to monitor several correlated quality characteristics simultaneously and where we have to detect small deviations of the characteristics. Particle swarm approach has been used for both economic model and economic-statistical model. The comparative tests between the economic and the economic-statistical models show better statistical performances of the economic-statistical design with negligible increase in cost.

Pignatiello and Tsai (1988) have introduced the robust economic design of control charts when the cost and process parameters are precisely not known. This type of design induces confidence in the user since the design procedure is based on a range of values for every parameter instead of point estimates. Even though the process parameters are not known accurately, the losses in operating the control chart can be controlled by robust designs.

Cai et al. (2002) have proposed an economic model for the design of a control chart for a trended process. Traditional applications of the control charts are based on the assumption of process stability. But this is violated in many cases. The authors opine that the trended output resulting from a deteriorating factor like tool wear, material consumption, power consumption have to be interpreted differently. The researchers developed an economic model and tested the results.

Rashedi et al. (2009) have proposed a new metaheuristic optimization algorithm based on law of gravity and mass interactions is introduced, namely gravitational search algorithm. To evaluate the GSA, they have examined it on a set of various standard benchmark functions.

Dowlatshahi et al. (2014) have introduced GGSA i.e. A Grouping Gravitational Search Algorithm for data clustering. They adapt the structure of GSA for solving the data clustering problem, the problem of grouping data into clusters such that the data in each cluster share a high degree of similarity while being very dissimilar to data from other clusters.

Vijaya et al. (2007) has provided a simple approach to the robust economic design of control charts. Robust economic designs are capable of incorporating in them robustness corresponding to the ambiguity of cost in the cost and process parameters. Robust economic designs are of two types. One type considers the uncertainty in the estimation of cost and makes the design suitable for any scenario. The second type considers different discreet scenarios for a single process and makes the design robust for all possible scenarios. The researchers have introduced a simple statistic for the robust economic design process with many different scenarios. Simple genetic algorithm has been employed to optimize the test parameters.

Gupta and Patel (2011) presented an economic design of X-bar control chart using particle swarm optimization technique. In this work a computer programme in C language based on non-traditional optimization technique PSO have been developed for economic design of the X-bar control chart giving the optimum of n, h and k such that the expected total cost per hour is minimized.

Ganguly and Patel (2012) have proposed an economic design of X-bar control chart using stimulated annealing optimization technique. The mat lab codes are generated to minimize the loss cost by optimizing the design parameters like sample size n, sample frequency h, and control width k.

Chen and Yang (2002) have presented an economic design of X-bar control chart with a Weibull distributed process failure mechanism when there is a possibility of multiple assignable causes. A cost model based upon variable sampling intervals was developed and analyzed. Optimal values of the design parameters including the sample size, the sampling intervals, and control limit coefficient were solved by minimizing the expected total cost per unit time, based on the varieties of combinations of Weibull parameters. The comparative tests performed on a multiple cause model and single cause model show that the former provides a lower loss-cost than the latter when the process has an increasing hazard rate.

Taisir and Qasim (2013) have implemented on The Performance of the gravitational search algorithm (GSA) are heuristic optimization algorithm based on Newton’s law of gravity and mass interactions. In this work by fine tuning the algorithm parameters and transition functions towards better balance between exploration and exploitation. To assess its performance and robustness, compare it with that of genetic algorithms (GA), using the standard cell placement problem as benchmark to evaluate the solution quality, and a set of artificial instances to evaluate the capability and possibility of finding an optimal solution.

Yu et al. (2010) have proposed an economic design of X-bar control chart with multiple assignable causes. An economic design does not consider the statistical properties, for example, Type-I or Type-II error and average time to signal (ATC). To improve these issues, an economic statistical design of control charts has been developed under the consideration of one assignable cause. However, there are multiple assignable causes in the real practice such as machine problem, material deviation, human errors; etc. In order to have a real application, this research will extend the original research from single to multiple assignable causes to establish an economic statistical model of X-bar control chart.

2.2 Objective of the present work

From the literature survey, it is observed that lot of work has been done on design of X-bar control chart of various types following various approaches and optimization techniques. The gravitational search algorithm technique has been observed to have application in variety of fields. However, this technique has not been tried in the economic design of control chart. With this motivation, the objectives of the present work are as follows:

- To use the GSA technique in economic design of X-bar control chart where quality characteristic is normally distributed and compares the results with reported in literature.
- To develop a MATLAB computer program based on GSA algorithm to minimize the loss cost by optimizing the design parameters like sample size *n*, sample frequency *h*, and control width *k*.

Chapter 3

Economic model for the design of control chart

The loss cost function has been formulated by Montgomery (1980) based on Duncan's (1956) economic model where the process is initially assumed to be in-control and normally distributed with mean μ_0 and variance σ^2. If sample size is n, for the X-bar chart the centre line is at mean μ_0 and the two control limits are at

$$\mu_0 \pm k * \frac{\sigma}{\sqrt{n}} \tag{3.1}$$

Type-I error committed when control chart indicates that the process is out-of-control when it is actually in-control. It is also called as false alarm or producer's risk.

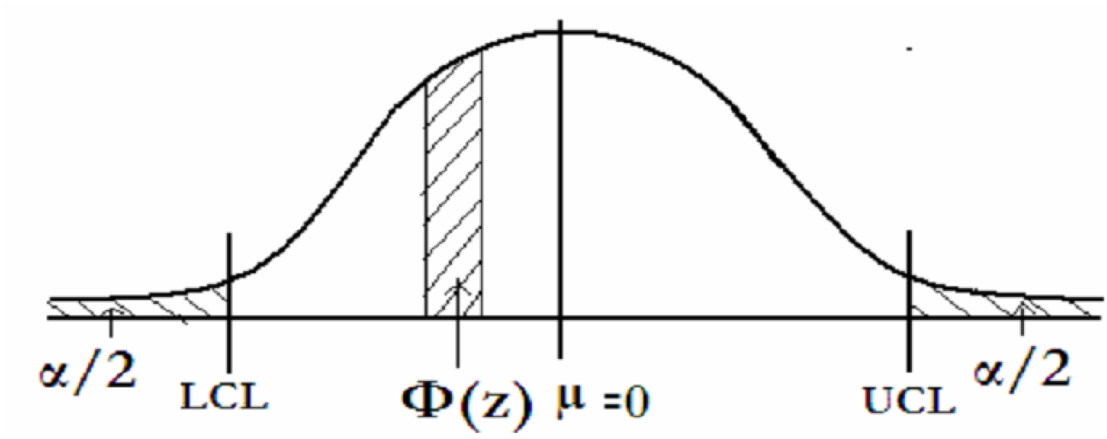

Fig. 3.1: Type-I Error

As illustrated in Fig. 3.1, the probability of committing Type-I error or the rate of false alarm (α) is given by

$$\alpha = 2\int_{k}^{\infty} \psi(z) dZ \tag{3.2}$$

If the process mean has shifted by δ, but the area which is falling under the control limits is called as the probability of Type-II error or β error. It is also as called misdetection or consumer's risk.

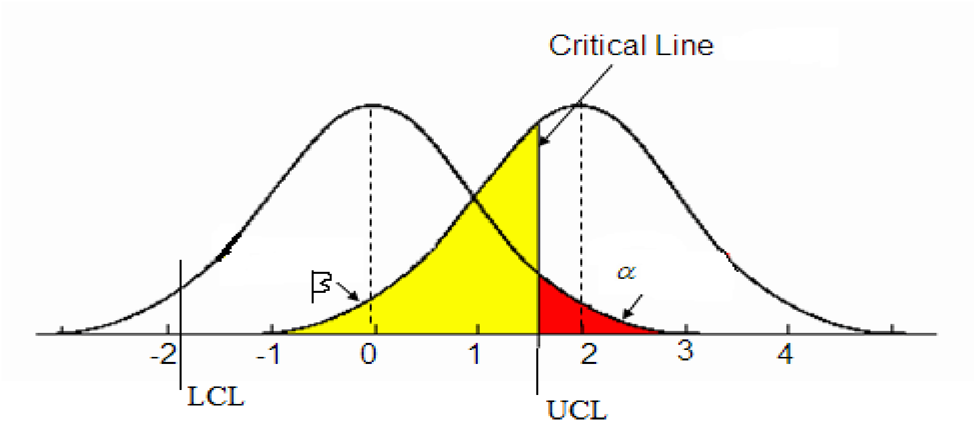

Fig. 3.2: Type II- Error

As illustrated in Fig. 3.2, when the assignable cause occurs, the probability that it will be detected on any subsequent sample is power is calculated as follows

$$1\text{-}\beta = \int_{-\infty}^{(-k-\delta\sqrt{n}} \psi(z)dz + \int_{(k-\delta\sqrt{n}}^{\infty} \psi(z)dz \tag{3.3}$$

Notations:

a_1 = Fixed component of sampling cost

a_2 = Variable component of sampling cost

a_3 = Cost of finding an assignable cause

a_3^1 = Cost of investigating a false alarm

a_4 = Hourly penalty cost associated with production in out-of-control state

λ = Rate of occurrences of assignable cause per hour

g = Time to test and interpret the result per sample unit

τ = Expected time of occurrence of assignable cause since immediate past sample

D = Time required to find an assignable cause and its elimination

n = Sample size

h = Sampling interval in hour

k = Width of control limits

α = Type – I error

β = Type – II error

μ_0 = Process mean for in-control process

σ = Standard deviation

δ = Shift in process mean in multiple of σ

V_0 = the net income per hour of operation in the in-control state

V_1 = the net income per hour of operation in the out-of-control state

E (L) = Expected loss per hour incurred by the process.

A production cycle consists of following four phases:

- The in-control phase,
- The out-of-control ,
- The time to take a sample and interpret the results i.e., gn, and
- The time to find the assignable cause i.e., D.

In this work, the continuous model is assumed. The entire cycle is represented in Fig. 3.3.

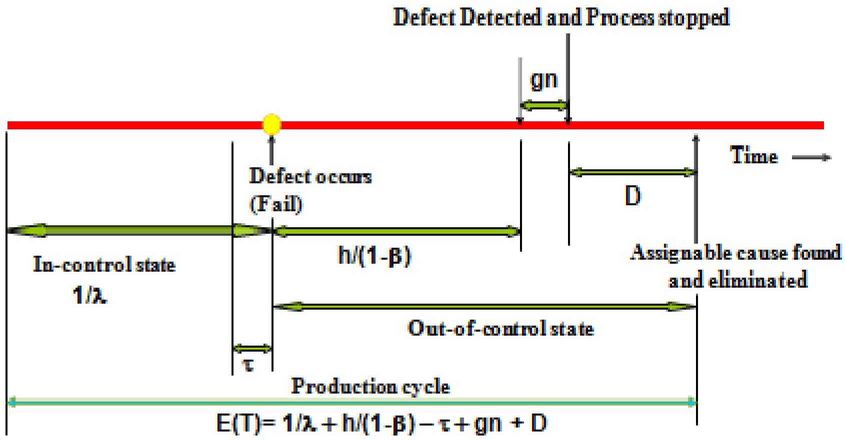

Fig. 3.3: Stages of a Production Cycle.

The four phases of a production cycle are as follows:

i. Assuming that the process begins in the in-control state and the assignable cause occurs at a rate of λ times per hour as per Poisson distribution, the time interval that the process remains in-control is an exponential random variable with a mean of $1/\lambda$. Thus, the expected in-control period is $1/\lambda$.

ii. If the number of samples required to produce an out-of-control signal when the process actually out-of-control is a geometric random variable with mean $1/(1-\beta)$, then the expected length of out-of-control period can be given by $[h/(1-\beta)] - \tau$.

iii. If g is the time required to take a sample of size 1 and interpret the results, then the total time for a sample size *n* will be gn.

iv. The time required to identify and remove the assignable cause following the signal is assumed to be a constant D.

Thus, the expected length of a cycle is

$$E(T) = \frac{1}{\lambda} + \left[\frac{h}{1-\beta} - \tau + gn + D \right] \tag{3.4}$$

When the process mean shifts, and the probability of its detection will be 1− β where β is Type-II error. Thus, the expected number of samples taken before the detection of shift will be 1/ (1− β). If the assignable cause occurs in a sample, then the expected initial time lag in that sample will be

$$\tau = \frac{\left(1-(1+\lambda h)e^{-\lambda h}\right)}{\lambda(1-e^{-\lambda h})} \tag{3.5}$$

The expected number of false alarms generated during a cycle is α times the expected numbers of samples taken before the shift, or

$$s = \frac{\alpha e^{-\lambda h}}{1-e^{-\lambda h}} \tag{3.6}$$

If a_3' is cost of investigating a false alarm, total cost due to false alarm will be

$$\frac{a_3^1 \alpha e^{-\lambda h}}{1-e^{-\lambda h}} \tag{3.7}$$

Let a_1 and a_2 are fixed and variable components of sampling cost. Expected number of samples per cycle is E(T)/h. Thus, total cost of taking samples will be (a_1 + a_2 n) * E(T)/h .If V_0 and V_1 are the net income per hour of operation while the process is in-control and out-of-control respectively and a_3 is cost of finding an assignable cause, the expected total net income per cycle will be

$$E(C) = V_0(\frac{1}{\lambda}) + V_1(\frac{h}{1-\beta} - \tau + gn + D) - a_3 - \frac{a_3^1 \alpha e^{-\lambda h}}{1-e^{-\lambda h}} - \frac{(a_1 + a_2 n)}{h} \tag{3.8}$$

Expected net income per hour is E (A) = E(L)/E(T)

$$E(A) = V_0 - \frac{a_1 + a_2 n}{h} - \frac{\left(a_4 \left(\frac{h}{(1-\beta)} - \tau + gn + D \right) + a_3 + a_3^1 \alpha \frac{e^{-\lambda h}}{1 - e^{-\lambda h}} \right)}{\frac{1}{\lambda} + \left(\frac{h}{(1-\beta)} - \tau + gn + D \right)} \tag{3.9}$$

Let the hourly penalty cost of producing with out-of-control process = $V_0 - V_1 = a_4$

$$E(A) = V_0 - \frac{a_1 + a_2 n}{h} - \frac{\left(a_4 \left(\frac{h}{(1-\beta)} - \tau + gn + D \right) + a_3 + a_3^1 \alpha \frac{e^{-\lambda h}}{1 - e^{-\lambda h}} \right)}{\frac{1}{\lambda} + \left(\frac{h}{(1-\beta)} - \tau + gn + D \right)}$$

(3.10)

Or $$E(A) = V_0 - E(L) \tag{3.11}$$

Where $$E(L) = \frac{a_1 + a_2 n}{h} - \frac{\left(a_4 \left(\frac{h}{(1-\beta)} - \tau + gn + D \right) + a_3 + a_3^1 \alpha \frac{e^{-\lambda h}}{1 - e^{-\lambda h}} \right)}{\frac{1}{\lambda} + \left(\frac{h}{(1-\beta)} - \tau + gn + D \right)} \tag{3.12}$$

The expression E(L) represents the expected loss per hour incurred by the process and it is a function of three variables n, h and k. Maximizing the expected net income per hour E(A) is equivalent to minimizing E(L).

Chapter 4

Optimization technique

4.1 Introduction

Optimization algorithms are becoming increasingly popular in multi-engineering design activities, primarily because of the availability and affordability of high speed computers. They are extensively used in those engineering design problems where the emphasis is on maximizing or minimizing a certain goal. For example, optimization algorithms are routinely used in aerospace design activities to minimize the overall weight, simply because every element or component adds to the overall weight of the aircraft. Thus, the minimization of the weight of aircraft components is of major concern to aerospace designers. Mechanical engineers design mechanical components for the purpose of achieving either a minimum manufacturing cost or a maximum component life. Production engineers are interested in designing optimum schedules of various machining operations to minimize the idle time of machines and the overall job complete time.

In first step, the formulation of optimization problems begins with identifying the underlying design variables, which are primarily varied during the optimization process. Generally while formulating an optimization problem few design variables are chosen. To increase or decrease the design variables depends on the outcome of that optimization procedure.

In second step it is necessary to identify the constraints that are associated to the optimization problem. The constraints represent some functional relationships among the design variables and other design parameters satisfying certain physical phenomenon and certain resource limitations. There are usually two types of constraints under consideration such as; i) in-equality type and ii) equality type.

The third step in the formulation procedure is to find the objective function in terms of the design variables and other problem parameters. The objective involves either minimization or maximization. For example minimization of overall manufacturing cost, minimization of overall component weight, or maximization of net profit, maximization total life of a product etc.

In fourth and final step of the formulation procedure is to set the minimum and the maximum bounds on each design variable.

4.2 Metaheuristic algorithms

A metaheuristic is a set of algorithmic concepts that can be used to define heuristic methods applicable to a wide set of different problems. in other words, a metaheuristic is a general purpose heuristic method designed to guide an underlying problem-specific (e.g. a local search algorithm or a construction heuristic) toward promising regions of the search space containing high-quality solutions. Metaheuristic concern the techniques employed to avoid getting stuck in suboptimal solutions and the type of trajectory followed in the space of either partial or full solutions. Some of the examples of the metaheuristic algorithm as given below:

Table 4.2.Some of the metaheuristic algorithm

Sr.no.	Name of technique	Author/s	Year
1	Gravitational search algorithm	Rashedi et al	2009
2	Genetic algorithm	Holland	1975
3	Simulated annealing	Kirkpatrick et al	1983
4	Ant colony algorithms	Marco Dorigo	1992
5	Particle swarm optimization	Kennedy and Eberhart	1995

One can define two common aspects in the population-based heuristic algorithms: exploration and exploitation. The exploration is the capacity of expanding search domain, where the exploitation is the capacity of discovering the optima around a best solution. In the initial iterations, a heuristic search algorithm explores the search domain to find new local optimum solutions. To avoid trapping in a local optimum, the algorithm must use the exploration in the initial few iteration. Hence, the exploration is an essential issue in a population-based heuristic algorithm.

By lapse of iterations, we expect that agents be attracted by biggest agent. This agent will present an optimum solution in the search domain. To have a high performance search, an essential key is suitable tradeoffs between exploration and exploitation. However, all the population-based heuristic algorithms employ the exploration and exploitation aspects but they use different approaches and researchers. Simply, all search algorithms have a common framework.

From an alternate perspective, the agents of a population-based search algorithm pass three stages in each and every iteration to realize the subject of exploration and exploitation: self-adaptation, cooperation and competition. In the self-adaptation stage, each agent improves its performance. In the cooperation stage, agents collaborate with each other by in data transferring. Finally, in the competition stage, agents compete to survive. These stages have generally stochastic forms, and could be realized in distinctive ways. These stages, inspired from nature, are the principle ideas of the population-based heuristic algorithms and used to find a global optimum.

4.3 Gravitational search algorithm

In GSA, every mass (agent) has four items: position, inertial mass, active gravitational mass, and passive gravitational mass. The position of the mass corresponds to a solution of the problem, and its gravitational and inertial masses are determined using a fitness function. The positions of the N number of agents are initialized randomly.

$$X_i = (x_i^1, \ldots x_i^d, \ldots x_i^n) \quad \text{for i =1, 2, 3…N} \tag{4.1}$$

where x_i^d represents the position of ith agent in the dth dimension.

At a specific interval time 'θ', the gravitational force acting on particle mass I from j as follows,

$$F_{ij}^d(\theta) = G(\theta)\frac{M_{pi}(\theta) * M_{aj}(\theta)}{R_{ij}(\theta) + \in}(x_j^d(\theta) - x_i^d(\theta)) \tag{4.2}$$

where M_{aj} is the active mass related to mass j. M_{pj} is the passive mass related to mass i. G(θ) is proportional constant at time 'θ', ε is a small constant, and R_{ij} is the Euclidian distance between two masses i and j.

$$R_{ij}^d = \| x_i(\theta), x_j(\theta) \|_Z \tag{4.3}$$

To give a stochastic characteristic to gravitational search algorithm, the total force that acts on mass i in a dimension d be a randomly weighted sum of d^{th} components of the forces exerted from other masses.

$$F_i^d(\theta) = \sum_{j=1, j \neq i}^{N} rand_j F_{ij}^d(\theta) \tag{4.4}$$

where $rand_j$ is a uniformly distributed random number within the interval [0, 1]

Hence, by Newton law of motion, the acceleration of the mass i at specific time θ, and in direction dth $a_i^d(\theta)$, is given follows,

$$a_i^d(\theta) = \frac{F_i^d(\theta)}{M_{ii}(\theta)}, \tag{4.5}$$

where $M_{ii}(\theta)$ is the inertia l mass of i^{th} agent.

Furthermore, the next velocity of an agent is considered as sum of the present velocity and its acceleration.

Then position and velocity could be calculated as follows:

$$v_i^d(\theta+1) = rand_i * v_i^d(\theta) + a_i^d(\theta) \tag{4.6}$$

$$x_i^d(\theta+1) = x_i^d(\theta) + v_i^d(\theta+1) \tag{4.7}$$

where $rand_i$ is a uniformly distributed random number within the interval [0, 1]. By using random number to give a randomized characteristic to the search domain.

The proportional constant $G(\theta)$ is initialized at the beginning and will be reduced with time to control the search accuracy. G is a function of the initial value $G_0(\theta)$ and time (θ):

$$G(\theta)=f(G_0(\theta)\ ,\ \theta) \tag{4.8}$$

$$G(\theta) = G_0\, e^{-\alpha(t/T)} \tag{4.9}$$

Where G_0 is initial proportional constant =100, α =20, t= current iteration and T= total number of iterations. Gravitational and inertia masses are simply calculated by the fitness evaluation. Update the gravitational and inertial masses by using the below equations:

$$M_{ai} = M_{pi} = M_{ii} = M_i\ ,\ \text{where i = 1, 2, 3,...N}$$

$$m_i(\theta) = \frac{fit_i(\theta) - worst(\theta)}{best(\theta) - worst(\theta)} \tag{4.10}$$

$$M_i(\theta) = \frac{m_i(\theta)}{\sum_{j=1}^{N} m_j(\theta)} \tag{4.11}$$

where $fit_i(\theta)$ is the fitness value of the agent i at specific time θ ,worst(θ) and best(θ) are defined as

For minimization problem:

best (θ) = min value of fitness

worst(θ) = max value of fitness

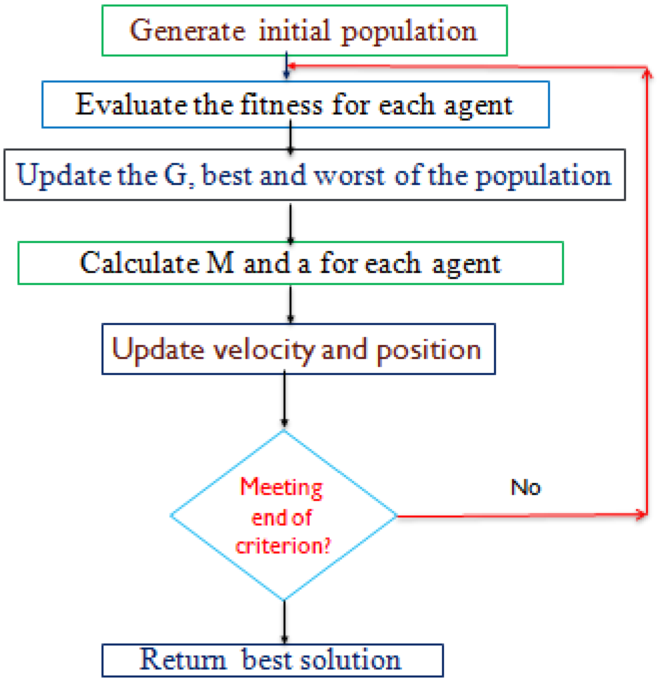

Fig. 4.1: Flow chart of GSA

4.4 GSA versus PSO

In both GSA and PSO the optimization is obtained by agent's movement in the search space, however the movement strategy is different. Some important differences are as follows:

- In PSO the direction of an agent is calculated using only two best positions, *pbesti* and *gbest*. But in GSA, the agent direction is calculated based on the overall force obtained by all other agents.
- In PSO, updating is performed without considering the quality of the solutions, and the fitness values are not important in the updating procedure while in GSA the force is proportional to the fitness value and so the agents see the search space around themselves in the influence of force.
- PSO uses a kind of memory for updating the velocity (due to *pbesti* and *gbest*). However, GSA is memory-less and only the current position of the agents plays a role in the updating procedure.
- In PSO, updating is performed without considering the distance between solutions while in GSA the force is reversely proportional to the distance between solutions.
- Finally, note that the search ideas of these algorithms are different. PSO simulates the social behaviour of birds and GSA inspires by a physical phenomena.

4.5 Summary of the gravitational search algorithm

1. Since every agent could identify the performance of the other agents, the gravitational force is a data transferring tool.
2. Due to the gravitational force that acts on an agent from its vicinity agents, it can indentify search domain around itself.
3. A large weight mass has highly attraction radius and hence a great intensity of attraction. Therefore, agents with a higher performance have a greater gravitational mass. As a result, the agents have tendency to move towards the best agent.
4. The inertia mass is against the movement and make move slow. Hence, agents with heavy inertia mass move slowly and hence search the space more locally. So, it can be considered as an adaptive learning rate.
5. Proportional constant controls the accuracy of the search, so it will be decreases with time .similar to the temperature in a simulated annealing.
6. Here, we assume that the gravitational mass and inertia mass are same. However, for some applications different values can be used. A heavier inertia mass gives a slower motion of agents in the search domain and hence a more precise search. Conversely, a heavier gravitational mass causes more attraction of agents. This allows a faster convergence.

Chapter 5

Results and Comparison

5.1 Numerical Example

For testing the effectiveness of gravitational search algorithm in economic design of control chart, a numerical example earlier solved by Montgomery (1980) was considered.

A manufacturer produces a nonreturnable glass bottles for packaging a carbonated soft drink beverage. The wall thickness of the bottles is an important quality characteristic. If the wall is too thin, internal pressure generated during filling will cause the bottle to burst. The manufacturer has used X-bar control charts for process surveillance for some time. These control charts have been designed with respect to statistical criteria. However, in an effort to reduce costs, the manufacturer wishes to design an economically optimum X-bar control chart for the process.

Based on an analysis of quality control technicians' salaries and the costs of the test equipment, it is estimated that the fixed cost of taking a sample is $1. The variable cost of taking a sample is estimated to be $0.01 per bottle. It takes approximately 1 minute (0.0167 h) to measure and record the wall thickness of the bottle. The process is subject to several different types of assignable causes. However when the process goes out-of-control, the magnitude of shift is approximately two standard divisions. Process shift occurs at random with a frequency of about one every 20h operation. Thus the exponential distribution with parameter $\lambda = 0.05$ is a reasonable model of the run length in control. The average time required to investigate an out-of-control signal is 1h. The cost of investigating an action signal that results in the elimination of the assignable cause is $25, while the cost of investigating a false alarm is $50. The manufacturer estimates that the penalty cost of operating in the out-of-control state for one hour is $100. An economic model for the X- bar control chart had to be designed.

Data given: a_1=$1, a_2=$0.10, a_3=$25, a_3^1= $50, a_4=$100, λ=0.05, δ=2.0, g=0.0167, D=1.0 and σ=2.

In the present work, the loss cost function has been minimized by running the MATLAB computer program which was developed based on gravitational search algorithm. Out of three variables, sample size (*n*) is interfere where as other i.e. sampling interval (*h*) and width of control limits (*k*) take real values on continuous scale. The search domain selected for searching the optimum solution are 1 to 15 for *n*, 0.1h to 1.0 *h* for *h* and 0.1 to 5 for *k*. The optimum values of *h* and *k* along with corresponding minimum values of expected loss cost E(L) obtained for various integer values of *n* varying from 1 to 15 have been listed in Table 5.1.

Table 5.1. Optimum design of X-bar control chart

Sample size (n)	Width of control limit (k)	Sampling interval in hours(h)	α	Power 1-β	Loss cost E(L)	Std. deviation σ_{n-1}
1	2.2958	0.4988	0.0217	0.3837	14.6562	0.00002
2	2.5122	0.6177	0.012	0.6241	11.8763	0.00001
3	2.6793	0.7061	0.0074	0.7837	10.8815	0.00002
4	2.8336	0.7684	0.0046	0.8783	10.4895	0.00002
5	2.9815	0.8147	0.0029	0.9320	10.3601	0.00000
6	3.1245	0.8517	0.0018	0.9620	10.3802	0.00002
7	3.2632	0.8837	0.0011	0.9787	10.4654	0.00001
8	3.3979	0.9128	0.0006	0.9881	10.5895	0.00002
9	3.5289	0.9401	0.0004	0.9933	10.7345	0.00000
10	3.6564	0.9662	0.0002	0.9962	10.8903	0.00001
11	3.7805	0.9915	0.0001	0.9978	11.0513	0.00003
12	3.9015	1.0161	0.0000	0.9988	11.2143	0.00001
13	4.0196	1.0402	0.0000	0.9993	11.3775	0.00000
14	4.1378	1.0638	0.0000	0.9996	11.5399	0.00002
15	4.2474	1.0870	0.0000	0.9998	11.7010	0.00001

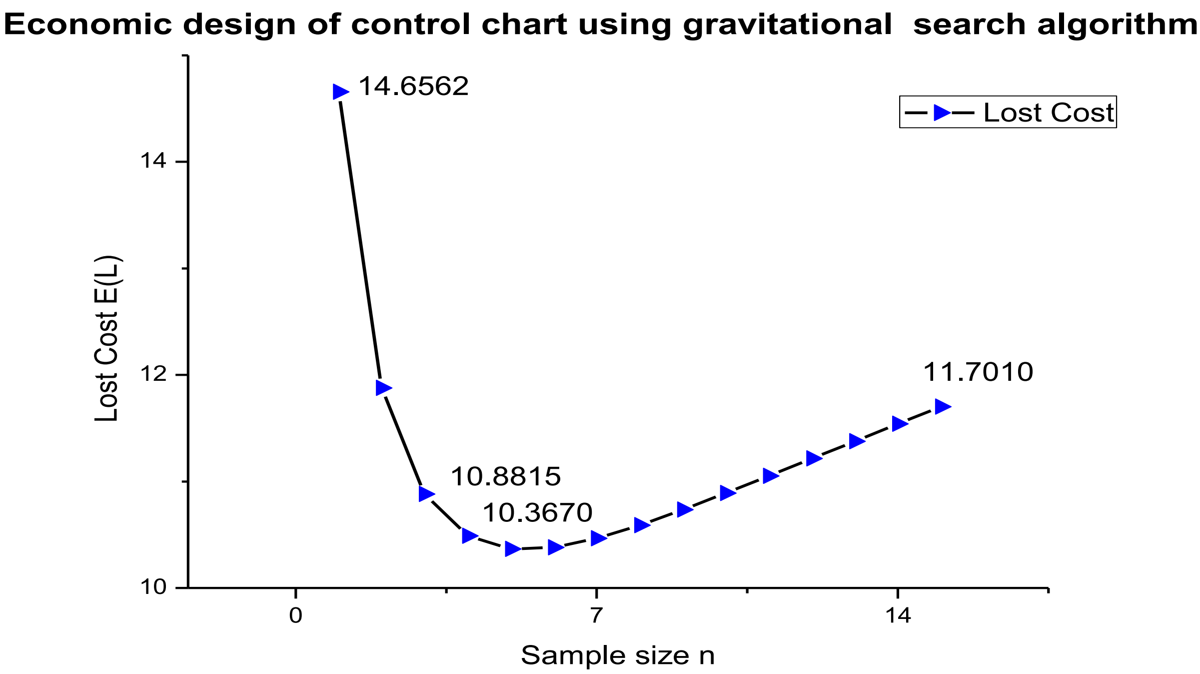

Fig. 5.1: Variation of loss cost with sample size using GSA

From Table, the optimum value of cost E(L) decreases an *n* value increases from 1 to 5 and then E(L) increases at higher values of *n*. This trend is also graphically shown in Fig. 5.1. Thus, the minimum possible cost is found to be E(L) =10.3601 and this occurs at n=5. The corresponding values of *h* and *k* at optimum solution are 0.8147 *h* and 2.9815 *k* respectively.

Table 5.2. Comparison of optimum design of X-bar control chart

	Montgomery (1980)			Simulated annealing (Ganguly and Patel,2012)			Gravitational search algorithm		
n	**h**	**k**	**E(L)**	**h**	**k**	**E(L)**	**k**	**h**	**E(L)**
1	0.45	2.3	14.71	0.49	2.30	14.66	2.2958	0.4988	14.6562
2	0.57	2.52	11.91	0.62	2.51	11.88	2.5122	0.6177	11.8763
3	0.66	2.68	10.90	0.72	2.68	10.88	2.6793	0.7061	10.8815
4	0.71	2.84	10.51	0.77	2.84	10.49	2.8336	0.7684	10.4895
5	**0.76**	**2.99**	**10.38**	**0.81**	**2.98**	**10.37**	**2.9815**	**0.8147**	**10.3601**
6	0.79	3.13	10.39	0.85	3.12	10.38	3.1245	0.8517	10.3802
7	0.82	3.27	10.48	0.89	3.26	10.47	3.2632	0.8837	10.4654
8	0.85	3.40	10.60	0.92	3.40	10.59	3.3979	0.9128	10.5895
9	0.87	3.53	10.75	0.95	3.51	10.74	3.5289	0.9401	10.7345
10	0.89	3.66	10.90	0.97	3.67	10.89	3.6564	0.9662	10.8903
11	0.92	3.78	11.06	0.98	3.82	11.05	3.7805	0.9915	11.0513
12	0.94	3.90	11.23	0.99	3.94	11.22	3.9015	1.0161	11.2143
13	0.96	4.02	11.39	0.99	4.15	11.38	4.0196	1.0402	11.3775
14	0.98	4.14	11.56	0.99	4.19	11.55	4.1378	1.0638	11.5399
15	1.00	4.25	11.72	1.00	4.26	11.72	4.2474	1.0870	11.7010

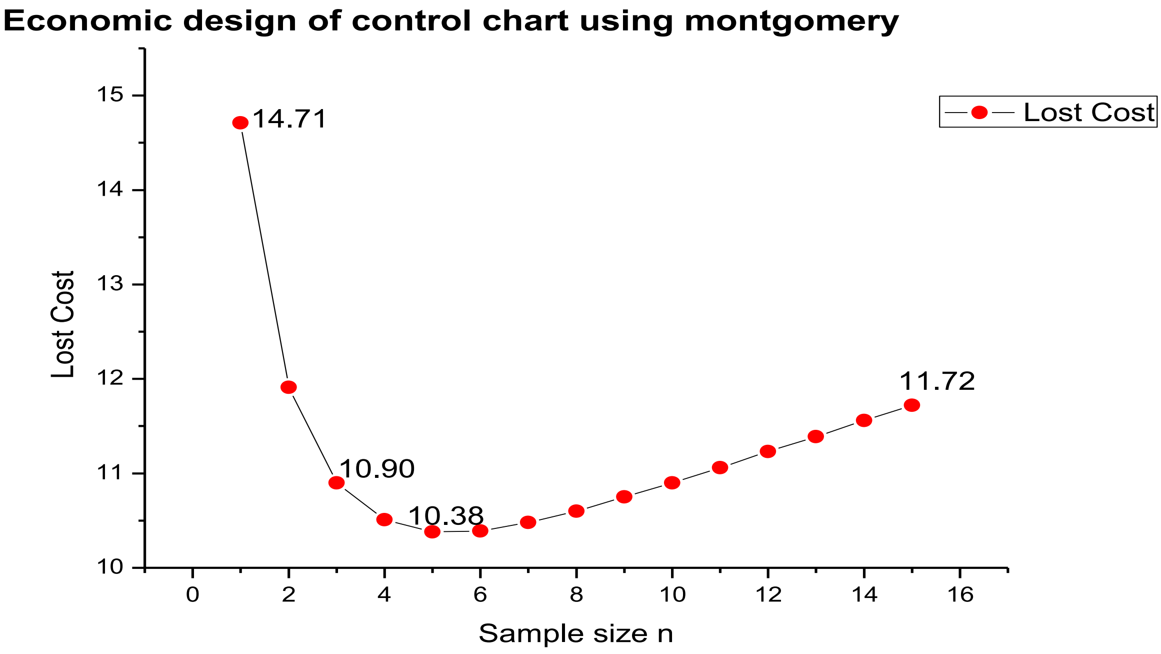

Fig. 5.2: Variation of loss cost with sample size using Montgomery (1980)

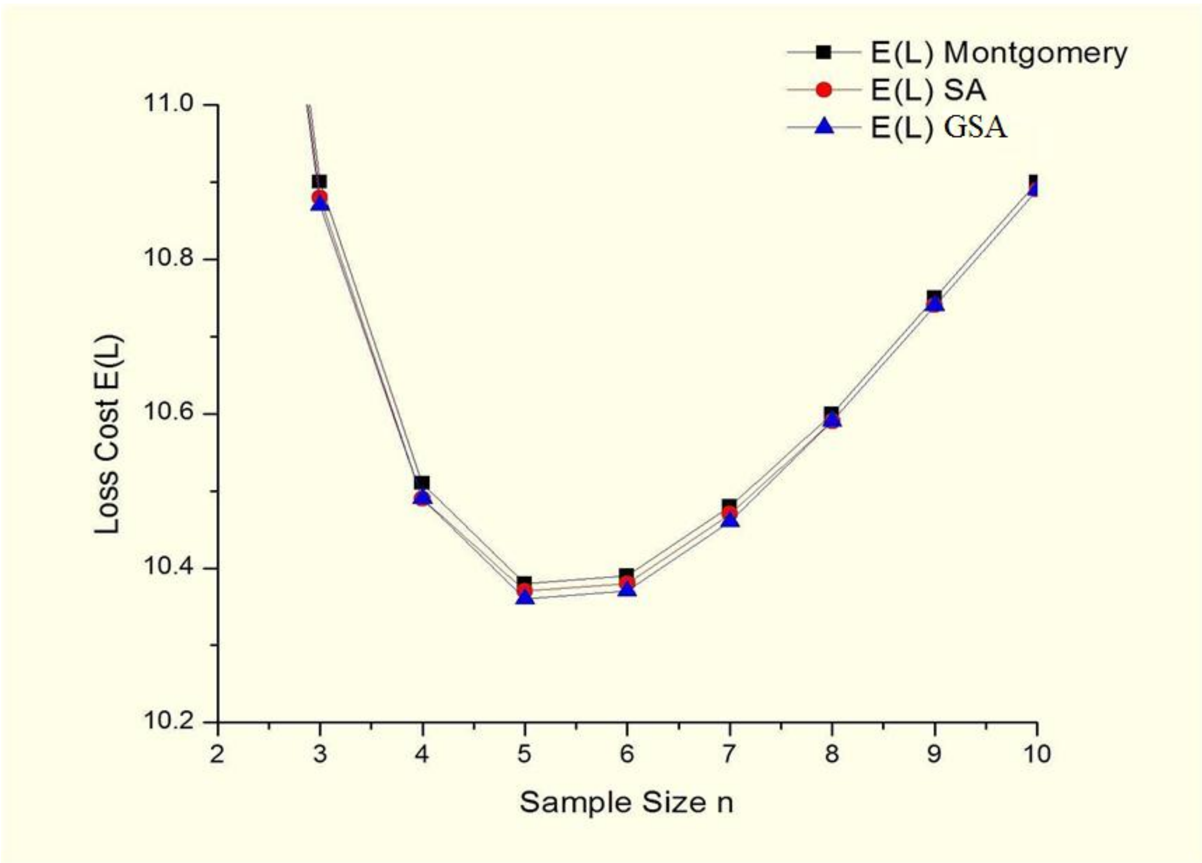

Fig. 5.3: Graphical representation of Loss Cost

Furthermore, the results are compared with the optimum design of X-bar control chart for the same example reported in literature as shown in Table 5.2. The same trend of relationship between E(L) and *n* has also been graphically represented by Montgomery(1980) from Fig 5.2. On comparison of results, it is clear that at all values of sample size the optimum cost at n=5 obtained by GSA are lower than simulated annealing also lower than that of Montgomery (1980).

Chapter 6

Conclusions

So here it is observed that the result obtained from GSA is better than that of literature by Montgomery (1980) and also it is better than that of Ganguly and Patel (2012) on simulated annealing.

- From Gravitational Search Algorithm (GSA) :

  Sample size (n) = 5, sampling interval (h) = 0.8147hr and (k) = 2.9815σ

  Loss Cost E (L) =10.3601$.

- From simulated annealing (SA) :

  Sample size (n) = 5, sampling interval (h) = 0.81hr and (k) = 2.98σ

  Loss Cost E (L) =10.37$.

- From Montgomery (1980) :

  Sample size (n) = 5, sampling interval (h) = 0.76hr and (k) = 2.99σ

  Loss Cost E (L) =10.37$.

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