STAGNATION POINT FLOW OF A NON-NEWTONIAN FLUID

A Dissertation

Submitted in partial fulfilment of the requirements for the award of the degree of

MASTER OF SCIENCE

IN

MATHEMATICS

by

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DECLARATION

I, the undersigned, declare that the work contained in this thesis entitled "Stagnation point flow of a non-Newtonian fluid" in partial fulfilment of the requirement for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology, Rourkela, under the supervision of Prof. Bikash Sahoo, Department of Mathematics, NIT Rourkela, is entirely my own work and has not previously in its entirely or quoted have been indicated and approximately acknowledged by complete references.

Place : Rourkela

Date : 11th may 2015

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CERTIFICATE

This is to certify that the thesis entitled “Stagnation point flow of a non-Newtonian fluid” submitted by Mr. M. Sai Ravi Teja Varma, Roll No.410MA5053, for the award of the degree of Master of Science from National Institute of Technology, Rourkela, is absolutely best upon his own review work under the guidance of Prof. Bikash Sahoo. The results embodied in this dissertation has not been submitted for any degree/diploma or any academic award anywhere before.

Place :

Date:

Prof. Bikash Sahoo

Department of Mathematics

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ABSTRACT:

Both Hiemenz flow and Homann flow are two classical problems in the field of fluid dynamics. In this project, both the flows are re-considered and the numerical solutions are obtained. Homann flow is studied with and without the presence of partial slip. These partial differential equations are reduced to ordinary differential equations using the similarity transformations. The obtained highly nonlinear ordinary differential equations with the relevant boundary conditions are solved using *4^{th} order Runge-kutta method*. The effects of flow parameters on the momentum boundary layer are studied in detail. It is observed that slip has significant effects on the velocity profiles.

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INTRODUCTION:

In fluid dynamics, stagnation point is a point where the local velocity of the fluid is zero. The stagnation point flow problem has attracted many researchers from the past decade due to its high industrial applications. The two-dimensional and three-dimensional stagnation point flows are considered as the fundamental problems in fluid dynamics.

It was the beginning of twentieth century when the solution for Two-dimensional stagnation point flow was studied. The solution was first investigated by Hiemenz (1911). He demonstrated that the Navier Stokes equations governing the flow can be reduced to ordinary differential equation using similarity transformations. The reduced differential equation was subjected to a numerical approach using the two boundary conditions, one of which is at infinity. Some care is needed in the solution of the boundary value problem (BVP) because of the asymptotic boundary condition. Hiemenz's solution can also be obtained without making the simplifications of boundary layer theory, which was first noted by Goldstein. Recently, for the planar and axisymmetric stagnation point flows of a viscous fluid with surface slip, Wang has obtained an exact similarity solution of the Navier–Stokes equations even without the boundary layer approximations. However, his report reveals that the flows have boundary layer character, although they are also exact solutions of the Navier–Stokes equations. The problem of stagnation point flow has been extended in various ways. Howarth has also done similar work to that of that of Hiemenz and found out solutions using the Finite scheme method. Yang was the first to work on unsteady two-dimensional stagnation flow. Wang and Libby worked on three-dimensional stagnation flow for a moving plate. The axisymmetric flow was studied by Homann . Howarth and Davey extended the axisymmetric flow to three-dimensional flow.

All the studied mentioned above were considered for a Newtonian fluid. Non-Newtonian fluids have attracted many researchers because of its wide applications in the industry. Srivastava and Sharma have researched the axisymmetric flow of a Reiner-Rivlin fluid and found an approximate solution, adopting the Kármán–Pohlhausen method. Jain solved the axisymmetric stagnation point flow of a Reiner–Rivlin fluid with and without suction. Rajeswari and Rathna used an extension of the Kármán–Pohlhausen integral approach to find the solution of the BVP of the two-dimensional and axisymmetric stagnation point flow of a second order fluid. Santra et al. have solved the resulting equation due the stagnation point flow of a Newtonian fluid over a lubricated surface by the Runge-Kutta method. Like the stretching sheet problems, a typical characteristic of the stagnation point flow of the non-Newtonian second grade and third grade fluids is that their constitutive equations generate momentum equations, which have terms of derivatives whose order exceeds the number of available boundary conditions. Davies, Serth , Teipel , and Ariel solved the original BVP using various methods, without making any restriction on the size of the non-Newtonian parameters arising due to the two-dimensional and axisymmetric stagnation point flows.

The inclusion of magnetic field in the study of stagnation point flow has many practical applications, for example, the cooling of turbine blades, where the leading edge is a stagnation point, or cooling the nose cone of the rocket during re-entry. Magneto hydro-dynamics (MHD) may provide a means of cooling the turbine blade and keeping the structural integrity of the nose cone. Hence, the boundary layer MHD flows of Newtonian and different non-Newtonian fluids have drawn the attention of many researchers since the past few decades. Attia has studied the effects of uniform suction and injection on the Homann flow of an electrically conducting fluid.

In hydro magnetics, Na studied the two-dimensional stagnation point flow of a Newtonian fluid. For various values of the Hartmann number (M_n), Na has listed the values of $\varphi''(0)$ at each iteration. From the theoretical point of view, his most important result is that as M_n increases from its value = zero, $\varphi''(0)$ first decreases up to a certain value of Mn, and then it increases monotonically. In other flow problems of hydro magnetics, the stress at the wall increases steadily as the Hartmann number is increased, which makes the result deduced by Na a interesting one. However, the values of $\varphi''(0)$ obtained by Na correspond to another solution of the BVP which does not satisfy the asymptotic boundary condition at infinity and this was reported by Ariel. He has reexamined the Hiemenz flow of an incompressible viscous fluid in hydro magnetics and reported that. Nachtsheim and Swigert reported these peripheral solutions for the nonmagnetic case. Ariel suggested a simple modification in Na's procedure, which eliminated the problem mentioned above. His reported numerical results show that $\varphi''(0)$ increases steadily as Mn is increased from zero. A detailed report on the above problem is given by B. Sahoo.

NEWTONIAN FLUID:

Newtonian fluids are in which the viscous stresses arising from the flow, at every point, are linearly proportional to the local strain rate. Local strain rate is nothing but the rate of change of deformation over time. More accurately we can say that a fluid is Newtonian if the strain rate and the tensors that describe viscous stress are related by a constant viscosity tensor that does not depend on the velocity of the flow and stress state.

$$T = \mu \frac{\partial u}{\partial y}$$

Where T is the viscous stress, μ is the constant scalar viscosity tensor and $\frac{\partial u}{\partial y}$ is the derivative of the velocity component that is parallel to the course of shear, in respect to displacement in the perpendicular direction.

Non-Newtonian REINER-RIVLIN FLUID:

Reiner-Rivlin fluids are incompressible fluids with constitutive equation

$$T = \alpha I + \varphi_1 D + \varphi_2 D^2$$

Here, we assume that T is the stress tensor only dependable on D for an incompressible fluid, α is the Lagrange multiplier, which can be found by solving the governing equations subject to boundary conditions. And φ_1, φ_2 are functions of the three principal invariants of D as well as the mass density ρ.

HIEMENZ FLOW:

The two-dimensional stagnation point flow was first studied and investigated by Hiemenz and thus it was named after the researcher as Hiemenz flow. He demonstrated that the Navier-Stokes equations governing the flow can be reduced to an ordinary differential equation. In the absence of analytical solution, the equation are solved using numerical approach. This research has helped and brought the applications of Hiemenz flow noticeable.

In this paper, a steady, laminar, incompressible, Newtonian fluid is impenged perpendicularly on a infinite flat surface. No body forces are included. Boundary layer assumptions are applied. Equations are written in terms of u and v as the velocity components of x and y respectively. No heat transfer, No diffusion.

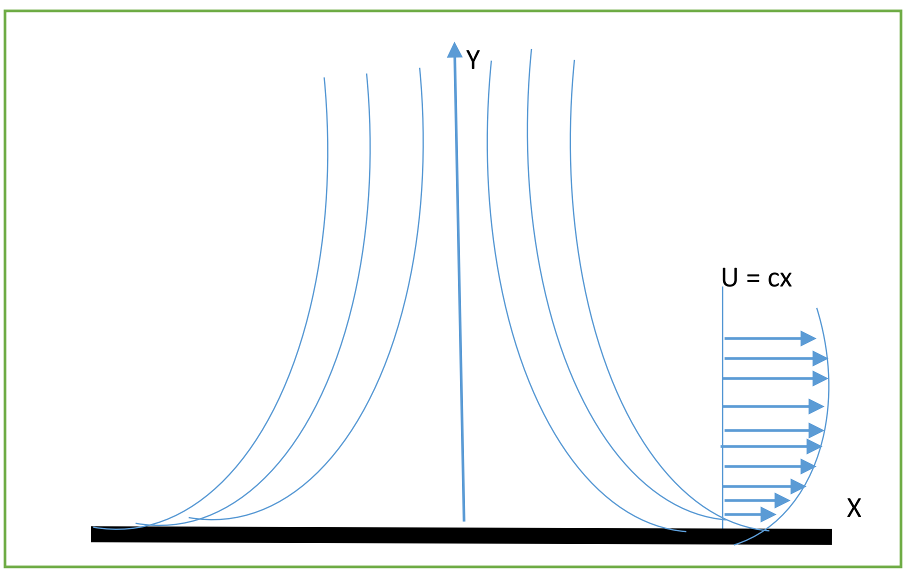

Fig.1 : Hiemenz flow

HOMANN FLOW:

The axisymmetric is known as Homann flow. Homann flow is also named after the first researcher Homann, who demonstrated the solutions for the axisymmetric flow. In this paper, we considered the Homann flow of Reiner-Rivlin fluid in presence of external magnetic fluid.

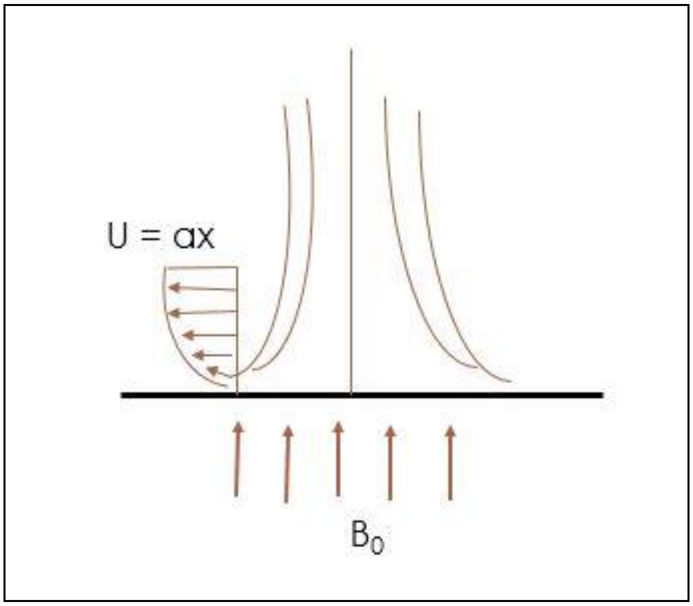

Fig. 2 : Homann Flow

MATHEMATICAL MODELLING:

HIEMENZ FLOW:

GOVERNING EQUATIONS:

From Fig. 1 we can see the type of Hiemenz flow. Now we consider the Navier-Stokes Equations and Boundary conditions of the flow. We take the velocity components as u and v for x and y axis respectively. Therefore, the corresponding flow equations would be

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Navier-Stokes Equations:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+u(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+u(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})$$

BOUNDARY CONDITIONS:

$$u(x,0)=0$$

$$v(x,0)=0$$

$$u(x,y)=cx \text{ (away from the surface that is} y\ \rightarrow \infty)$$

$$v(x,y)=-cy \text{(away from the surface that is} y\ \rightarrow \infty)$$

SIMLARITY TRANSFORMATIONS:

Now, let us take that $v(x,y)=f(y)$, since there is no x in $v(x,y)$.

From the equation of continuity which is $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$, we get $u(x,y)=-xf'(y)$

Substituting the respective values of u and v in Navier-Stokes equations we get

$$f'^2-ff''+vf'''=-\frac{\partial p}{\rho x\partial x}$$

$$ff'-vf''=-\frac{\partial p}{\rho\partial y}$$

After integrating the latter equation and puuting the value of p in the former, we get

$$f'^2-ff''+vf'''=c^2$$

And we have boundary conditions transformed to $f(0)=0=f'(0)$ and $f'(\infty)=-c$

With the use of characteristic scales we change the above equation to a dimensionless form, the characteristic scale for length is $\sqrt{\frac{v}{c}}$. Thus

$$\eta = \frac{y}{\sqrt{\frac{v}{c}}} = \sqrt{\frac{c}{v}}y$$

The characteristic scale for velocity is $\sqrt{vc}$. Thus

$$\varphi(\eta) = \frac{-f(y)}{\sqrt{vc}}$$

From which we get

$$f' = -c\varphi', f'' = -c\varphi''\sqrt{\frac{c}{v}} \text{ and } f''' = -\frac{c^2}{v}\varphi'''$$

Substituting these values we have

$$\varphi''' + \varphi\varphi'' - {\varphi'}^2 + 1 = 0$$

and the corresponding boundary conditions change to

$$\varphi(0) = 0, \varphi'(0) = 0, \varphi'(\infty) = 1$$

HOMANN FLOW:

GOVERNING EQUATIONS:

Fig. 2 in this paper, corresponds to the Homann flow. We here consider the Homann flow of a Reiner-Rivlin fluid. The work is done is done both, with and without partial wall slip. First let's consider the work corresponding to Homann flow without slip.

1.WITHOUT SLIP:

$$T = \alpha I + \varphi_1 D + \varphi_2 D^2$$

Since a Reiner-Rivlin fluid is used for this study.

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Navier-Stokes Equations:

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \nabla^2 u + \frac{\partial}{\partial z}\left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial z} + 2\frac{\partial w}{\partial x}\frac{\partial w}{\partial z}\right] + \lambda\frac{\partial}{\partial x}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2\right] - Mu$$

$$u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \nabla^2 v + \frac{\partial}{\partial z}\left[2u\frac{\partial^2 z}{\partial x \partial z} + 4\left(\frac{\partial w}{\partial z}\right)^2 + 2v\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial u}{\partial z}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right] + \lambda\frac{\partial}{\partial z}\left[4\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2\right]$$

BOUNDARY CONDITIONS:

$$z = 0: u = 0, v = 0$$

$$z \to \infty: u \to U$$

$$U = ax, V = 0, W = -2az$$

SIMILARITY TRANSFORMATIONS:

Now, the Navier-Stokes equations are subjected to similarity transformations such that we obtain a dimensionless ordinary differential equation of 3rd order. It has been shown by Jain[13] that the similarity transformations for a Reiner-Rivlin fluid are

$$u = ax\varphi'(\xi), w = -2\sqrt{\frac{a\mu}{\rho}}\varphi(\xi), \xi = \sqrt{\frac{a\rho}{\mu}} z$$

Using this similarity transformation, the Navier-Stokes equation is converted to

$$\varphi''' + 2\varphi\varphi'' + 1 - \varphi'^2 - L\left(2\varphi'\varphi''' + \varphi''^2\right) + M(1 - \varphi') = 0$$

And the corresponding boundary conditions are changed to

$$\varphi = 0, \varphi' = 0 \; for \; \xi = 0$$

$$and \; \varphi \to 1 \; as \; \xi \to \infty$$

2.WITH SLIP:

The equations of continuity and the Navier-Stokes Equations are same as that of without slip condition, the only thing that changes is the boundary condition. The slip condition is

$$u_y = A_p\frac{\partial u_t}{\partial n}$$

Where u_t is the tangential velocity component, n is normal to the plate, and A_p is almost =

$$\frac{2(mean\ free\ path)}{\sqrt{\Pi}}$$

This condition was proposed by Navier, nearly two hundred years ago.

Now, with the same similarity transformations as that of the without slip condition, the boundary conditions are changed to

$$\varphi = 0, \varphi'(0) = \varphi''(0)\ for\ \xi = 0$$

$$and\ \varphi \rightarrow 1\ as\ \xi \rightarrow \infty$$

NUMERICAL APPROACH:

For both Hiemenz flow and Homann flow, the Runge-Kutta method has been adopted.

RUNGE-KUTTA METHOD:

The Runge-Kutta method for the first order differential equation is given as

$$\frac{dx}{dy} = f(x, y), f(x_0) = y_0$$

$$y_{j+1} = y_j + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where,

$$k_1 = f(x_j, y_j)$$

$$k_2 = f(x_j + \frac{h}{2}, y_j + \frac{k_1}{2})$$

$$k_3 = f(x_j + \frac{h}{2}, y_j + \frac{k_2}{2})$$

$$k_4 = f(x_j + h, y_j + k_3)$$

HIEMENZ FLOW:

Here we have 3rd ordered differential equation

$$\varphi''' + \varphi\varphi'' - {\varphi'}^2 + 1 = 0$$

$$\Rightarrow \varphi''' = -\varphi\varphi'' + {\varphi'}^2 - 1$$

We adopt Runge-Kutta method to this differential equation with the boundary conditions

$$\varphi(0) = 0, \varphi'(0) = 0, \varphi''(0) = 1.232586$$

For C++ code of this particular problem refer Appendix-1

HOMANN FLOW:

For the Homann flow, we have a 3rd order differential equation which can also be solved by Runge-Kutta Method.

$$\varphi''' + 2\varphi\varphi'' + 1 - \varphi'^2 - L(2\varphi'\varphi''' + \varphi''^2) + M(1 - \varphi') = 0$$

$$\Rightarrow \ \varphi''' = \frac{-2\varphi\varphi'' - 1 + \varphi'^2 + L\varphi''^2 - \ M(1 - \varphi')}{1 - 2L\varphi'}$$

With and Without slip, the C++ code is same except for the initial condition. Refer Appendix-2

RESULTS AND DISCUSSIONS:

HIEMENZ FLOW:

The graphs have been plotted for φ and φ'.

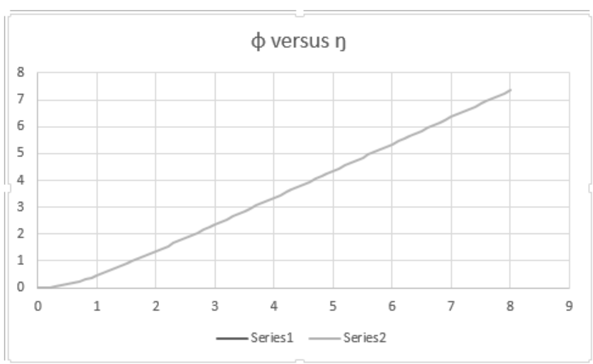

Fig. 3: Plot of φ versus η of the hiemenz flow

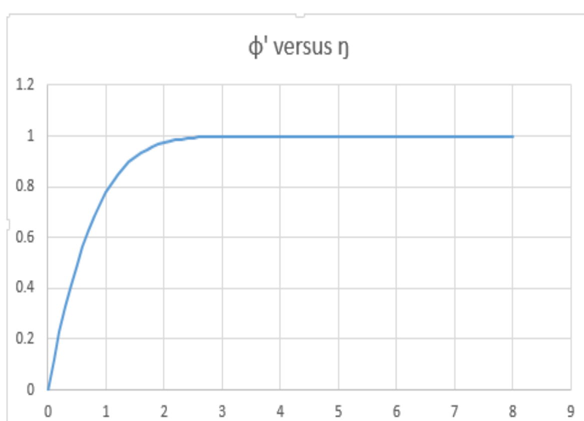

Fig. 4: Plot of φ' versus η of the hiemenz flow

HOMANN FLOW:

WITHOUT SLIP:

For both slip and without slip conditions, as L is arbitrary, we have chosen it to be < 0.

M_n	L	ϕ''(0)
0.5	-1	1.29903
	-3	1.29785
	-5	1.32215
0	-1	1.29903
5		2.5857693
10		3.4360084

Table 1: Values of $\varphi''(0)$ varying with different values of L and M_n

We have also plotted the grpahs for Homann flow without slip condition, the variation of the velocity with different values of Magnetic and non-dimensional parameter L have been verified. It has been noticed that there is either continuous increase or decrease with the change in the values of the parameters.

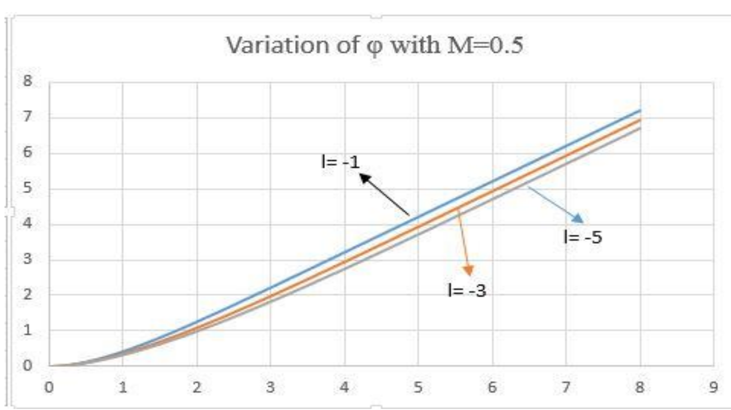

Fig.5: Variation of φ with a change in $L, and\ M_n = 0.5$

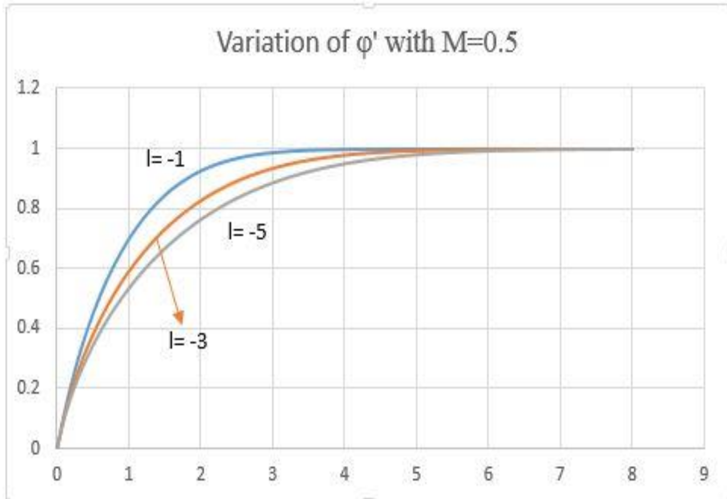

Fig.6: Variation of φ' with change in values of L and $M_n = 0.5$

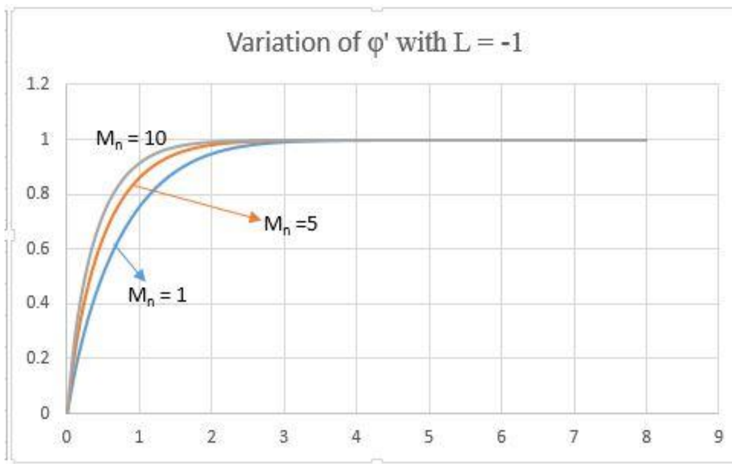

Fig.7: Variation of φ' with change in values of $M_n$and $L = -1$

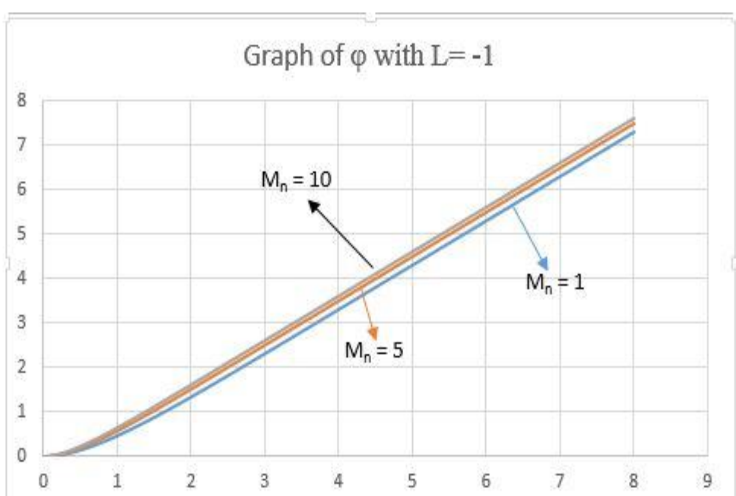

Fig.8: Variation of φ with change in values of $M_n$and $L = -1$

WITH SLIP:

L	M_n	γ	ϕ''(0)
-1			0.521666
-3	0.5	1	0.442406
-5			0.40055
	0		0.521666
-1	1	1	0.560989
	2		0.589448
		0	1.29903
-1	0.5	0.5	0.721849
		2	0.339982

Table 2: Variation of values of $\varphi''(0)$ with different values of all the three parameters.

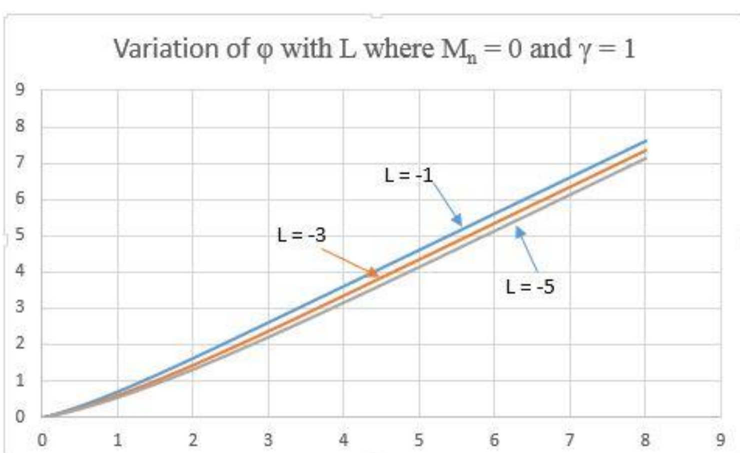

Fig.9: Variation of φ with L, where $M_n = 0$ and $\gamma = 1$

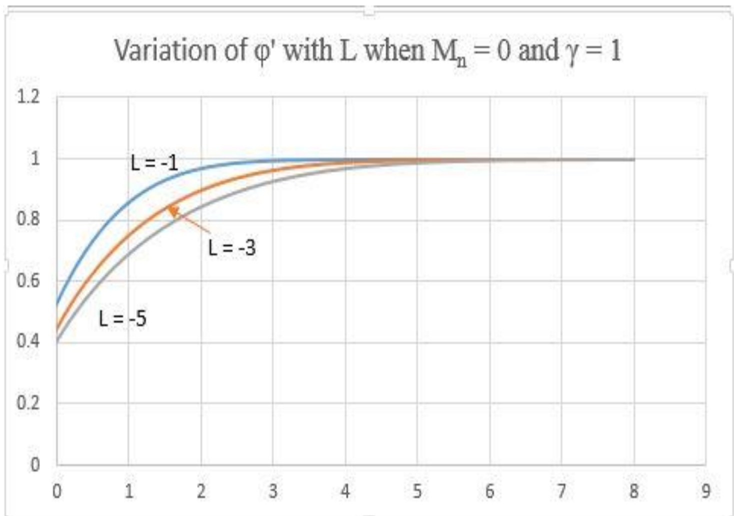

Fig.9: Variation of φ' with L, where $M_n = 0\ and\ \gamma = 1$

CONCLUSIONS:

The steady Hiemenz flow and Homann flow are examined. It is observed that the value of φ' increases with the increase in the value of M_n in the no slip case. The skin friction decreases with an increase in slip and the flow behaves as inviscid for very high value of slip parameter. On the other hand the skin friction increases with an increase in the magnetic parameter Mn.

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APPENDIX 1

The C++ code for the Hiemenz flow

```
#include<iostream>

#include<stdio.h>

#include<conio.h>

#include<math.h>


using namespace std;

double fun1(double t, double u1, double v1, double w1)

{

	return (v1);

}

double fun2(double t, double u1, double v1, double w1)

{

	return (w1);

}

double fun3(double t, double u1, double v1,double w1)

{

	return ((-1)+(v1*v1)-(u1*w1));

}


int main()

{

	double h=0.1,t=0,k[4][3],u[3],t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11;

	u[0]=0;

	u[1]=0;

	u[2]=1.232586;

	int i,j;

	cout<<"R.K. 4th order method to solve a 2nd order diff eqn"<<endl<<endl;

	cout<<"t \t\tu1 \t\tu2 \t\t"<<endl<<endl;
```

```
for(i=0;i<81;i++)
{

    cout<<t<<"\t\t"<<u[0]<<"\t\t"<<u[1]<<"\t\t"<<u[2]<<endl;

    k[0][0]=h*fun1(t,u[0],u[1],u[2]);
    k[0][1]=h*fun2(t,u[0],u[1],u[2]);
    k[0][2]=h*fun3(t,u[0],u[1],u[2]);

    t1=t+(h/2);
    t2=u[0]+((k[0][0])/2);
    t3=u[1]+((k[0][1])/2);
    t4=u[2]+((k[0][2])/2);
    k[1][0]=h*fun1(t1,t2,t3,t4);
    k[1][1]=h*fun2(t1,t2,t3,t4);
    k[1][2]=h*fun3(t1,t2,t3,t4);

    t5=u[0]+((k[1][0])/2);
    t6=u[1]+((k[1][1])/2);
    t7=u[2]+((k[1][2])/2);
    k[2][0]=h*fun1(t1,t5,t6,t7);
    k[2][1]=h*fun2(t1,t5,t6,t7);
    k[2][2]=h*fun3(t1,t5,t6,t7);

    t8=t+h;
    t9=u[0]+(k[2][0]);
    t10=u[1]+(k[2][1]);
    t11=u[2]+(k[2][2]);
    k[3][0]=h*fun1(t8,t9,t10,t11);
    k[3][1]=h*fun2(t8,t9,t10,t11);
    k[3][2]=h*fun3(t8,t9,t10,t11);
```

```
	for(j=0;j<=3;j++)
	{
		u[j]=u[j]+((k[0][j]+(2*(k[1][j]))+(2*(k[2][j]))+k[3][j])/6);
	}
	t=t+h;
}
getch();
return 0;
}
```

APPENDIX 2:

The C++ code for Homann flow is also made for both the cases of with slip and without slip conditions.

```
#include<iostream>
#include<stdio.h>
#include<conio.h>
#include<math.h>

using namespace std;
double fun1(double t, double u1, double v1, double w1)
{
	return (v1);
}
double fun2(double t, double u1, double v1, double w1)
{
	return (w1);
}
double fun3(double t, double u1, double v1,double w1)
{
	int m=2, l=-1;	/*change the values according to the problem*/
```

```
        return (((-1-m)+(v1*v1)-(2*u1*w1)+(l*w1*w1)+(m*v1))/(1-(2*l*v1)));
}

int main()
{
        double h=0.1,t=0,k[4][3],u[3],t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11;
        u[0]=0;
        u[2]=0.367032;          /*Change the intial value according to the Table 2 and Table 3*/
        u[1]= 2*u[2];           /*Replace 2 with the γ value*/
        int i,j;
        cout<<"R.K. 4th order method to solve a 2nd order diff eqn"<<endl<<endl;
        cout<<"t \t\tu1 \t\tu2 \t\t"<<endl<<endl;
        for(i=0;i<81;i++)
        {
                cout<<t<<"\t\t"<<u[0]<<"\t\t"<<u[1]<<"\t\t"<<u[2]<<endl;

                k[0][0]=h*fun1(t,u[0],u[1],u[2]);
                k[0][1]=h*fun2(t,u[0],u[1],u[2]);
                k[0][2]=h*fun3(t,u[0],u[1],u[2]);

                t1=t+(h/2);
                t2=u[0]+((k[0][0])/2);
                t3=u[1]+((k[0][1])/2);
                t4=u[2]+((k[0][2])/2);
                k[1][0]=h*fun1(t1,t2,t3,t4);
                k[1][1]=h*fun2(t1,t2,t3,t4);
                k[1][2]=h*fun3(t1,t2,t3,t4);

                t5=u[0]+((k[1][0])/2);
                t6=u[1]+((k[1][1])/2);
                t7=u[2]+((k[1][2])/2);
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            k[2][0]=h*fun1(t1,t5,t6,t7);
            k[2][1]=h*fun2(t1,t5,t6,t7);
            k[2][2]=h*fun3(t1,t5,t6,t7);

            t8=t+h;
            t9=u[0]+(k[2][0]);
            t10=u[1]+(k[2][1]);
            t11=u[2]+(k[2][2]);
            k[3][0]=h*fun1(t8,t9,t10,t11);
            k[3][1]=h*fun2(t8,t9,t10,t11);
            k[3][2]=h*fun3(t8,t9,t10,t11);
        for(j=0;j<=3;j++)
        {
            u[j]=u[j]+((k[0][j]+(2*(k[1][j]))+(2*(k[2][j]))+k[3][j])/6);
        }
        t=t+h;
    }
    getch();
    return 0;
}
```