

# **Vibration and Acoustic Analysis of Laminated Composite Plate**

*Thesis Submitted to*

*National Institute of Technology, Rourkela  
for the award of the degree  
of*

**Master of Technology  
In Mechanical Engineering with Specialization  
“Machine Design and Analysis”**

*by*

**Suraj Sitaram Bankar  
Roll No. 213ME1375**

*Under the Supervision of*

**Prof. Subrata Kumar Panda**



**Department of Mechanical Engineering  
National Institute of Technology Rourkela  
Odisha (India) -769 008**

**May 2015**

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**NATIONAL INSTITUTE OF TECHNOLOGY  
ROURKELA**

**CERTIFICATE**

This is to certify that the work in this thesis entitled “**Vibration and Acoustic Analysis of Laminated Composite Plate**” by **Mr. Suraj Sitaram Bankar** (213ME1375) has been carried out under my supervision in partial fulfilment of the requirements for the degree of **Master of Technology** in Mechanical Engineering with **Machine Design and Analysis** specialization during session 2013 - 2015 in the Department of Mechanical Engineering, National Institute of Technology, Rourkela.

To the best of my knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.

Date:

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# **SELF DECLARATION**

I, Mr. Suraj Sitaram Bankar, Roll No. 213ME1375, student of M. Tech (2013-15), Machine Design and Analysis at Department of Mechanical Engineering, National Institute of Technology Rourkela do hereby declare that I have not adopted any kind of unfair means and carried out the research work reported in this thesis work ethically to the best of my knowledge. If adoption of any kind of unfair means is found in this thesis work at a later stage, then appropriate action can be taken against me including withdrawal of this thesis work.

NIT Rourkela

Suraj Sitaram Bankar

Date

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My first thanks are to the Almighty God, without whose blessings, I wouldn't have been writing this “acknowledgments”. I am extremely fortunate to be involved in an exciting and challenging research project work on “**Vibration and Acoustic Analysis of Laminated Composite Plate**”. It has enriched my life, giving me an opportunity to work in a new environment of ANSYS. This project increased my thinking and understanding capability as I started the project from scratch.

I would like to express my greatest gratitude to my supervisor **Prof. S. K. Panda**, for his excellent guidance, valuable suggestions and endless support. He has not only been a wonderful supervisor but also an honest person. I consider myself extremely lucky to be able to work under guidance of such a dynamic personality. He is one of such genuine person for whom my words will not be enough to express.

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Finally, I express my sincere gratitude to my parents for their constant encouragement and support at all phases of my life.

**Suraj Sitaram Bankar**

## ABSTRACT

Conventional materials such as steel, aluminium etc. are used in industries because of their high strength and stiffness. But composite materials have taken their places because they are giving excellent strength and stiffness with low weight. Currently, many industries such as automobile, aerospace, trains, buildings are using sandwich materials to reduce noise level. These sandwich materials consist of sheets of conventional materials which are bonded by polymers, plastics to reduce vibration and noise. In this study, vibration and acoustic analysis of laminated composite plate are carried out experimentally. Carbon fibre reinforced polymer and glass fibre reinforced polymer plates are used to study low frequency vibration and their effect of surrounding air medium. Combined modes shapes are formed because of resonance of natural frequencies of the structure and acoustic cavity. These combined mode shapes generally occur in low frequency region and possesses both high-order displacement and high-order pressure amplitude. The effect of number of plies and ply angle are investigated on the natural frequency and the pressure amplitude. The finite element simulation model is developed to validate the results obtained from experiment.

*Keywords:* Laminated composite plate, vibration analysis, acoustic analysis, sound waves, FEM, ANSYS.

# CONTENTS

<b>Title Page</b>	<b>(I)</b>
<b>Certificate of Approval</b>	<b>(IV)</b>
<b>Self-Declaration</b>	<b>(V)</b>
<b>Acknowledgement</b>	<b>(VI)</b>
<b>Abstract</b>	<b>(VII)</b>
<b>Contents</b>	<b>(VIII)</b>
<b>List of Symbols</b>	<b>(XI)</b>
<b>List of Tables</b>	<b>(XIII)</b>
<b>List of Figures</b>	<b>(XIV)</b>
<b>Chapter 1 Introduction</b>	<b>(1-8)</b>
1.1 Overview	(1)
1.2 Introduction of Finite Element Method and ANSYS	(4)
1.3 Motivation of the Present Work	(6)
1.4 Objectives and Scope of the Present Thesis	(7)
1.5 Organisation of the Thesis	(7)
<b>Chapter 2 Literature Review</b>	<b>(9-13)</b>
2.1 Introduction	(9)
2.2 Structural acoustic Analysis	(10)
2.3 Structural acoustic analysis for laminated composite structures	(12)
<b>Chapter 3 General Mathematical Formulation</b>	<b>(14-20)</b>
3.1 Introduction	(14)



3.2	Assumptions	(14)
3.3	Basics of acoustic	(15)
3.4	Uncoupled acoustic analysis of fluid medium	(17)
3.5	Coupled fluid-structural interaction acoustic analysis	(18)
<b>Chapter 4 Results and Discussions</b>		<b>(21-34)</b>
4.1	Introduction	(21)
4.2	Governing Equation and Solution	(23)
4.3	Results and Discussions	
4.3.1.	Convergence study for natural frequency parameter	(24)
4.3.2.	Comparison study of natural frequency obtained from experiments and finite element simulation	(25)
4.3.3.	Comparison of pressure wave obtained from experiment and finite element simulation	(27)
4.3.4.	Comparison of structural and acoustic frequencies of CFRP ( $\pm 45$ ) at low frequency region	(29)
4.3.5.	Effect of fibre orientation on the frequency and the pressure for laminated plate	(30)
4.3.6.	Free vibration analysis of vehicle compartment	(32)
4.4	Conclusions	(34)
<b>Chapter 5 Closure</b>		<b>(35-37)</b>
5.1	Concluding Remarks	(35)
5.2	Significant Contribution of the Thesis	(36)
5.3	Future Scope of the Research	(37)
<b>References</b>		<b>(38-40)</b>

## List of Symbols

Most of the symbols are defined as they occur in the thesis. Some of the most common symbols, which are repeatedly used, are listed below:

$x, y, z$	Co-ordinate axis
$u, v$ and $w$	Displacements corresponding to $x, y$ and $z$ directions, respectively
$E_1, E_2$ and $E_3$	Young's modulus
$G_{12}, G_{23}$ and $G_{13}$	Shear modulus
$\nu_{12}, \nu_{23}$ and $\nu_{13}$	Poisson's ratios
$a, b$ and $h$	Length, width and thickness of the shell panel
$F$	Global force vector
$\rho$	Density of the material
$p$	Pressure
$C$	Velocity of sound
$t$	Time
$K$	Bulk modulus
$k$	Wave number
$V$	Acoustic velocity
$I$	Acoustic intensity
$[M_f]$	Mass matrix for fluid
$[C_f]$	Damping matrix for fluid

$[K_f]$	Stiffness matrix for fluid
$\{F_f\}$	Acoustic load vector
$[N_f]$	Shape function related to fluid
$\omega$	Angular frequency
$[R]$	Coupling matrix
$\{F_s\}$	Structural load vector
$[M_s]$	Mass matrix for structure
$[C_s]$	Damping matrix for structure
$[K_s]$	Stiffness matrix for structure

### Subscript

$f$	Fluid
$s$	Structure

### Abbreviation

CFRP	Carbon fibre reinforced polymer
GFRP	Glass fibre reinforced polymer
FSDT	First order shear deformation theory
APDL	ANSYS parametric design language
Eq.	Equation
GPa	Giga Pascal
SPL	Sound pressure level

## **List of Tables**

<b>Table No.</b>		<b>Page No.</b>
4.1	Material properties of CFRP and GFRP plates	(23)
4.2	Comparison study for natural frequency of cantilever CFRP plates	(28)
4.3	Comparison study for natural frequency of cantilever GFRP plates	(28)
4.4	Comparison of natural frequency of vehicle compartment	(34)

## List of Figures

Figure No.		Page No.
1.1	Propagation of sound wave	(1)
1.2	Shell181 element geometry	(6)
1.3	FLUID29 element geometry	(6)
4.1	Experimental setup	(24)
4.2	NI cDAQ-9178 instrument	(25)
4.3	Convergence of first natural frequency with different mesh divisions of CFRP plate	(26)
4.4	Convergence of first natural frequency with different mesh divisions of GFRP plate	(27)
4.5	Pressure variations for CFRP $0^0/90^0$ experimentally and ANSYS	(29)
4.6	Pressure variations for GFRP $0^0/90^0$ experimentally and ANSYS	(30)
4.7	FFT curves for CFRP ( $\pm 45$ ) of displacement and pressure respectively	(31)
4.8	Effect of ply angle on first natural frequency for CFRP plate	(32)
4.9	Effect of ply angle on pressure amplitude for CFRP plate	(33)
4.10	Mode shapes of vehicle compartment	(35)

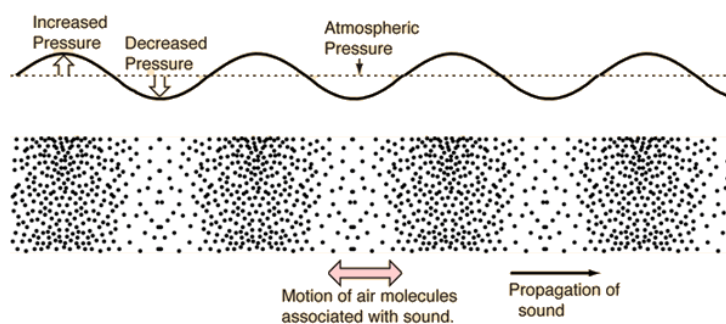
# CHAPTER 1

## INTRODUCTION

### 1.1 Overview:

Acoustics is the study of the sound waves including its generation, propagation, and effects. Acoustic or sound pressure is the difference between the average local pressure of medium and pressure within sound wave at same point and time. The sound is a travelling wave created by a vibrating object and propagated through a medium (gas, liquid, or solid) due to particle interactions. Thus, sound waves cannot travel through a vacuum.

A sound wave propagates in the form of longitudinal waves comprising of successive compression and rarefaction of the elastic medium as shown in the Fig. 1.1 (courtesy: <http://hyperphysics.phy-astr.gsu.edu>). Sound pressure is nothing but pressure fluctuations about ambient atmospheric pressure.



**Fig. 1.1** Propagation of sound wave

The entire acoustic spectrum can be divided into three sections: audio, ultrasonic, and infrasonic. In general, the range of acoustic wave is audible to human is in between 20 Hz to 20,000 Hz. The ultrasonic region consists high frequency waves i.e. greater than 20,000 Hz. Application of ultrasonic waves is in imaging technologies, sonography. Infrasonic has very frequency waves and they are used to geological study like earthquakes.

Devices called electroacoustic transducers converts sound energy into electric energy or vice versa. Loudspeakers, headphones, microphones, hydrophones, sonar projectors etc. are electroacoustic transducers. Nowadays microphones are vastly used in sound measurement process in air medium while hydrophones are used in a water medium.

The sound propagation can be classified into following categories

- Free field: Sound is propagated in a free region from any form of obstructions.
- Near field: Near field region is very close to a source where sound pressure may vary significantly for a small change in position. Generally it is less than a wavelength of a sound wave.
- Far field: The far field begins where the near field ends and extends to infinity. It can be divided into a free field or reverberant field.
- Reverberant field: This field is a result of direct waves as well as reflected waves from walls or other obstacles.

### ***Structural acoustic:***

Structural Acoustics is the study of the interaction of vibrating structures with adjacent fluid along with the accompanying radiated or scattered sound. Nowadays Structural acoustics became an important field in almost every industry. Any structure which consist relative motion tends to vibrate from low frequency to high frequency. These vibrations

produce pressure fluctuations in surrounding medium. In some cases, pressure fluctuations cause vibration in adjacent structures. These fluctuations are nothing but acoustic waves. Structures typically plates, shells, membranes under transient or oscillatory loads are the common sound source. Vibration characteristics of structure can significantly change because of a presence of a fluid medium. These effects can adversely affect the dynamics of a system can be studied. Especially for coupled analysis of systems interaction between fluid and structure also called as FSI is of great concern.

The acoustic wave is driven by velocity not by structural displacement. Acoustics assumes ideal non-viscous fluid without shear layer's effect. An only normal component of structural surface velocity is important. Generally sound wave propagating through a solid medium is called structural-borne sound and if it is propagating through the air it is called air-borne sound. Air-borne sound often originates from external sources and propagates into the structures like vehicles compartment panels, planes interior through the floor, wall panels etc. Whereas, a structure-borne sound is the result of mechanical vibrations propagating through the vehicle structure and eventually causing localized displacements in an air.

The structural acoustic can be divided into two types: internal and external problems. In an internal acoustic analysis, an acoustic cavity is enclosed in structure and corresponding loading conditions like FSI layer, impedance etc. need to apply. But for external problems unbounded fluid domain needs to be considered otherwise due to reflections of propagating acoustic waves the frequency response of the system will differ significantly.

### ***Noise and vibration control:***

Noise can be defined as disagreeable or undesired sound. Many industries like transportation, construction are involved in a continuous effort to optimize noise and vibration characteristics. The noise inside the vehicle compartment is mainly structural borne



and it is low frequency generally less than 400 Hz. Internal sound field of the compartment is affected by the acoustic behaviour of cavity, dynamics of surround structure and fluid-structure coupling. Resonant frequencies and acoustic mode shape primarily depend on the design of enclosed cavity.

Sound and vibration are generally controlled by active and passive methods. In the active vibration control method, piezoelectric materials are used as sensors and actuators in flexible structures. Whereas, in the passive control method different materials used such as barriers, absorption materials, damping materials and vibration isolation. Barriers and absorption materials are used to attenuate sound which is already propagating in medium while damping materials and vibration isolation used to reduce structural borne vibrations subsequently reducing noise. For effective noise control, we can use different combinations of above materials.

Most of the vehicle companies use laminated steel to improve the acoustic characteristic of a system which consists of two layers of steel bonded by polymer core.

## **1.2 Finite Element Method and ANSYS:**

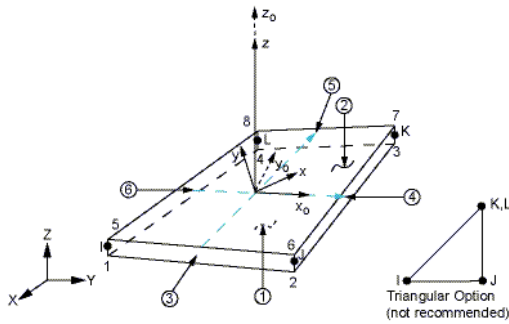
With the advancement in technology, the design process is too close to precision, so the finite element method (FEM) is used widely and capable to draw complicated structure and this is very trusted tool for designing of any shape and structure. It plays an important role in predicting the responses of various products, parts, assemblies and subassemblies. Nowadays, FEM is extensively used by all advanced industries which save their huge time of prototyping with reducing the cost due to physical test and increases the innovation at a faster and more

accurate way. There are many optimized finite element analysis (FEA) tools available in the market and ANSYS is one of them which is acceptable to many industries and analysts.

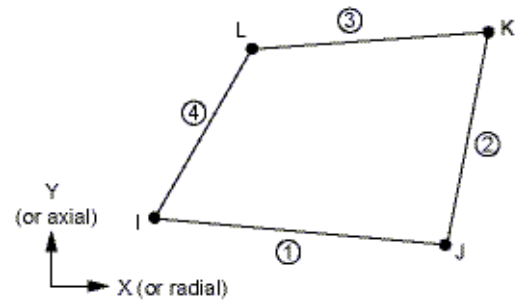
Structural acoustic analysis both; coupled and uncoupled can be done in ANSYS using ANSYS/ Multiphysics and ANSYS/Mechanical programmes. The acoustic analysis in ANSYS is generally done by modelling structure domain, modelling fluid domain depending on external or internal problem with appropriate fluid elements. Then apply boundary conditions and loads, solve using a valid method and review the results.

Acoustic model generally consists of structural domain fluid domain, FSI interfaces, and infinite acoustic domain. ANSYS uses elements FLUID129 for 2D and FLUID130 for 3D which are the infinite acoustic elements while FLUID29 for 2D and FLUID30 for 3D are finite acoustic elements. Infinite acoustic element modelled as a circle in 2D and sphere in 3D. It absorbs all pressure waves generated by a source without any reflections. The infinite element cannot be used with structural elements.

Elements used for modelling of structure and acoustic domain are shown in Fig. 1.2 and Fig. 1.3. Shell elements are used for thin and moderately thick plate or shell structures. Shell181 element is used for structure modelling. It has 4 nodes with each node has 6 degrees of freedom; 3 translations and 3 rotations. FLUID29 is used to model fluid element. It is a 2D element which has 4 nodes. It can be used for both coupled and uncoupled fluid elements. Coupled element has three degrees of freedom UX, UY and pressure while an uncoupled element has only one i.e. pressure.



**Fig 1.2** Shell181 element geometry



**Fig 1.3** FLUID29 element geometry

Pressures, impedance, rigid wall, free surface, PML are some boundary conditions used for acoustic analysis can be applied to both fluid and solid entities. Acoustic domain can be excited using normal velocity (or acceleration), wave sources, mass sources commands. The output from the acoustic analysis is generally pressure, Sound pressure limit, acoustic cavity modes shapes and frequencies.

### 1.3 Motivation of the present work:

The basic requirement of dynamically loaded structures was high material damping with low minimum weight and adequate stiffness. But for nowadays these lightweight structures will have to meet not only these dynamic demands but also improved acoustic standards. Materials like aluminium, magnesium, titanium etc. which are mainly used for excellent dynamic properties such as strength and stiffness, have relatively low damping which leads to intense acoustic radiation.

Laminated composite materials are widely used in different industries like transportation, buildings, naval and space projects. They possess outstanding properties such as low coefficient of thermal expansion, high elastic properties, corrosion and chemical

resistance and they are very lightweight as compared to conventional materials such as metal, concrete and wood. By investigating different combinations for composite materials offers the possibility of fulfilling the requirements concerning strength, stiffness, damping and sound reduction.

## **1.4 Objective and Scope of the Present Work:**

This study aims to develop a finite element formulation for structural acoustic analysis for laminated composite plates surrounded by air medium. Carbon fibre reinforced polymer (CFRP) and Glass fibre reinforced polymer (GFRP) plate with various fibre orientations are used for this study.

- Mathematical model is developed based on first order deformation theory for laminated composite structures
- Investigate vibration behaviour of CFRP and GFRP plates experimentally.
- Study the various properties of sound waves generated by plate vibration.
- Study the effect of various fibre orientations on vibration and acoustic parameters.
- Validate these results with finite element software package ANSYS.

## **1.5 Organization of the thesis:**

The overview and motivation of the present work followed by the objectives and scope of the present thesis are discussed in this chapter. This chapter is divided into five sections. The first section contains the background of acoustic waves, structural acoustics and methods to control noise and vibration. In section two, a brief introduction of finite element method

and finite element analysis software ANSYS used for structural acoustic analysis is presented. The motivation of the present work is discussed in fourth and in fifth objective and scope of present work is incorporated. The remaining part of the thesis are organised in the following fashion.

In chapter two, a brief introduction of the previous publishes literature has been presented along with their theory and method adopted for the analysis by the authors. The chapter is subdivided in two parts consisting of structural acoustic analysis and structural acoustic analysis of laminated composites.

In chapter three, general mathematical formulation for the acoustic wave is discussed. Finite element formulation for uncoupled and coupled acoustic analysis is investigated. First order shear deformation theory is used for analysis of laminated composite plate.

Chapter four illustrates the vibration and acoustic responses of laminated composite plates for different fibre orientation. Effect of different laminate parameters on acoustic wave is investigated.

Chapter five summarizes the whole work and it contains the concluding remarks based on the present study and the future scope of the work.

Some important books and publications referred during the present study have been listed in the References section.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 Introduction:**

Commonly sound sources are structures consisting of plates, membranes, shells which are generally surrounded by air or water and excited by oscillatory or transient loads. The vibration of structure causes compression and expansion of surrounding fluid which subsequently applies oscillatory pressure on the surface of a structure, modifying the response of a structure. The analysis of vibrating elastic structures and their interactions with surrounding acoustic fluid has developed for past few years. Historically, the main inspiration behind the analytical study of acoustics was a development of underwater sound sources required for echo-ranging submerged targets.

There are various literature presented on structural acoustics of different parts of an automobile, aircrafts, and underwater structures. Most of the literature concern about noise and vibration control. Nowadays laminated composites are being used in many fields because of their excellent properties. There is much literature available on the vibration of laminated composite structures using various classical and shear deformation theories such as classical plate theory, first-order shear deformation theory and higher order shear deformation theories.

In the following section of this chapter existing literature based on the vibro-acoustic analysis are discussed. The review of literature is carried out for different analytical methods

and finite element modelling for solving different fluid-structure interaction problems. Most of structural acoustic analysis is carried out to control noise and vibration.

## **2.2 Structural acoustic analysis:**

There are several finite element formulations to solve structural acoustics and fluid structural interactions. The acoustic fluid can be modelled by finite element formulations based on fluid pressure, displacement, displacement potential and velocity potential, each of them has advantages in different situations. Everstine (1997) developed the finite element formulations for fluid domain based on pressure and displacement, radiation and scattering from elastic structures, fluid structure eigenvalue problem for added mass and interior fluid problems. Sandberg and Göransson (1988) proposed the symmetric finite element formulation for coupled acoustic vibration in which pressure and displacement potential used to define the fluid. Olson and Bathe (1985) proposed model for the symmetric finite element analysis of fluid-structure interaction problem using pressure and velocity potential as degrees of freedom in the fluid region and displacement in the solid region.

For external problems, it is must take into account unbounded fluid domain, otherwise due to a reflection of propagating acoustic waves frequency response of the system will differ considerably. In the analysis done by Vogel and Grandhi (2012) when the thickness of structure was changed from 1mm to 10mm, pressure was reduced by nearly 20dB. This thickness change also affects the frequency at which maximum pressure is observed. Ding and Chen (2001) carried out coupled acoustic analysis of elastic thin-walled cavity excited by both exterior structural loading and interior acoustic sources using the symmetric finite element formulation. Foin, Nicolas *et al.* (1999) proposed a new tool that describes the vibro-acoustics of baffled, simply supported, multi-layered plate in both light and heavy fluids. 2D extension of the Ungar's model was used for evaluating equation of motion of the plate.

Which found considerably accurate when compared to simplified discrete layer theory used for multi-layered plated. This method reduces the size of mass and stiffness matrix resulting in less computing time without decreasing in the precision of results.

Modal analysis of these systems shows vibration modes which involve both structural and fluid domain. Kruntcheva (2007) called these combined mode shapes as acoustic structural resonances and they possess both high order displacement and high order pressure amplitude. Lim (2000) proposed the spectral formulation to evaluate radiated noise contributions of automotive body panels to interior sound pressure levels. He introduced a function called acoustic sensitivity which was the base of model. Ding and Chen (2002) put theoretical algorithm to compute interior noise contributed from a local structural panel of an elastic thin-walled cavity. This approach can be used to identify noise source in the vehicle compartment. It is suitable for low-frequency range but it is not suitable for investigation where the maximal dimension of a local panel is larger than the minimal acoustical wavelength of a frequency range. Nefske, Wolf *et al.* (1982) done finite element formulation to compute acoustic modes and resonant frequencies of vehicle compartment. Identification of critical panels around the compartment and its noise level was done by forced vibration analysis. Kim, Lee *et al.* (1999) used the practical method to reduce noise in the vehicle passenger compartment. This method uses interior pressure in terms of modal parameters and structural acoustic modal coupling coefficients of vehicle body and compartment. Assaf and Guerich (2008) proposed the numerical prediction of noise transmission loss through sandwiched plate. Sandwiched plate made up of viscoelastic core sandwiched between two elastic faces and subjected to acoustic plane wave or diffuse sound field excitation. Narayanan and Shanbhag (1981) studied the sound transmission loss through viscoelastic sandwich panels into the rectangular enclosure. Song, Hwang *et al.* (2003) investigated structural vibration control for coupled acoustic system using modal testing, finite element



method, piezoelectric material and robust LQG controller. The paper used structural vibration control instead of fully structural acoustic coupling control. The robust LQG controller can reduce interior noise as well as structural vibrations. Li and Zhao (2004) proposed the finite element formulation for modelling of the dynamic behaviour of laminated plate incorporated with piezoelectric layers and viscoelastic layer based on FSDT.

In the upcoming section the literature survey on noise and vibration control of various structures mainly vehicle compartments, transmission loss through body panels presented.

### **2.3 Structural acoustic analysis for laminated composite structures:**

Thai and Choi (2013) investigated laminated composite plates using simple first order shear deformation theory which had only four unknown and had strong resemblance with classical plate theory. Yin, Gu *et al.* (2007) investigated acoustic radiation from laminated composite plate reinforced by doubly periodic parallel stiffeners. Anders, Rogers *et al.* (1992) proposed analytical modelling technique to find out modal and structural acoustic behaviour of locally activated SMA hybrid composite panel using Ritz method, classical laminated plate theory, and finite panel acoustic radiation theory. Chandra, Raja *et al.* (2014) presented analytical solutions for determining transmission loss and vibro-acoustic response of FGM plate using simple FSDT. Nilsson (1990) determined the loss factors at different frequency ranges for a sandwich plate. Hufenbach, Kroll *et al.* (2001) studied vibro-acoustic characteristics of laminates composites by performing numerical solution using FEM and BEM. Assaf, Guerich *et al.* (2010) proposed finite element modelling to analyse vibro-acoustic response of sandwich plate under constrained layer damping treatment.

## CHAPTER 3

# MATHEMATICAL FORMULATION

### 3.1 Introduction:

The solution of a real life problem involving an arbitrary plate geometry and complicated loading and boundary conditions cannot be easily realized using analytical methods. A numerical analysis technique, especially finite element analysis method, is suited most to solve such problems. This chapter includes basics of acoustic waves, fundamental wave equation. Finite element formulation for uncoupled, coupled acoustic analysis investigated. Also simple first order shear deformation theory is studied for analysis of laminated composite plates.

### 3.2 Assumptions:

In acoustic fluid-structure interaction problems, both acoustic wave equation and the structural equation need to be coupled to each other. In deriving the discretized acoustic wave equation, there are some necessary assumptions (Hosseini-Toudeshky, Karimi *et al.* 2011) are made.

1. The fluid is compressible except only relative small pressure changes with respect to mean pressure are allowed.
2. The fluid is inviscid.
3. There is no mean flow of the flow.
4. The mean density and mean ambient pressure are uniform throughout the fluid.

5. The analysis is linear.
6. The damping has not been considered in this study.

### 3.3 Basics of acoustic:

The equation of a sound wave can be described in pressure or displacement. Generally acoustic equation is in the form of pressure instead of velocity. The main reason is pressure is scalar quantity so practically easier to work than velocity. Also, pressure is nothing but the sound we hear and can be easily measure using a microphone.

The fundamental wave equation of acoustic (Cook 2007) is given as:

$$\nabla^2 p = \frac{1}{C^2} \frac{\partial^2 p}{\partial t^2} \quad (3.1)$$

The equation of a wave is in a form of pressure which described with both space and time.

Simple one-dimensional equation of sound wave is,

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 p}{\partial t^2} \quad (3.2)$$

where, C is the acoustic wave speed given as,

$$C = \sqrt{\frac{K}{\rho}} \quad (3.3)$$

$\rho$  is the fluid density, and K is the fluid bulk modulus.

For harmonic loading, we can write,

$$p = P e^{-j\omega t} \quad (3.4)$$

where,  $\omega$  is the radian frequency, the wave equation becomes the Helmholtz equation which is denoted as:

$$\nabla^2 P + k^2 P = 0 \quad (3.5)$$

Note that  $t$  has disappeared, reducing the order of the equation by one.

The wavenumber  $k = \omega/c$

Spherical waves more closely approximate true source waves but approximate to plane waves at large distances from a source. It may be shown that the wave equation in spherical coordinates (Jacobsen and Polack 2007) is given as:

$$\frac{\partial^2(rp)}{\partial t^2} = c^2 \frac{\partial^2(rp)}{\partial r^2} \quad (3.6)$$

To describe the amplitude of a sound we usually use RMS pressure. Sound pressure level is given by,

$$SPL = 20 \log_{10} \frac{\bar{p}}{\bar{p}_{ref}} \quad (3.7)$$

The reference pressure is  $20\mu Pa$  and unit of SPL is dB.

Some cases we need quantities like velocity, intensity.

Acoustic velocity  $V$ ,

$$V = \frac{P}{\rho_0 C} \quad (3.8)$$

Acoustic mean intensity  $I$ ,

$$I = \frac{\bar{p}^2}{\rho C} \quad (3.9)$$

### 3.4 Uncoupled acoustic analysis:

The acoustic wave equation is given by equation (3.1),

$$\nabla^2 p = \frac{1}{C^2} \frac{\partial^2 p}{\partial t^2}$$

The finite element formulation for acoustic wave is given by (Cook 2007),

$$[M_f]\{\ddot{p}\} + [C_f]\{\dot{p}\} + [K_f]\{p\} = -\{F_f\} \quad (3.11)$$

where,

$$[M_f] = \frac{1}{C^2} \int_{vol} [N_f]^T [N_f] dV$$

$$[C_f] = \frac{\beta}{c} \int_{surface} [N_f]^T [N_f] dS$$

$$[K_f] = \int_{vol} ([N_{f,x}]^T [N_{f,x}] + [N_{f,y}]^T [N_{f,y}] + [N_{f,z}]^T [N_{f,z}]) dV$$

$$\{F_f\} = \rho \int_{vol} [N_f]^T \ddot{u}_n ds$$

where, n is an outward normal direction and  $\ddot{u}_n$  is the acceleration of the boundary in direction n.

The essential boundary conditions are:

On the free surface of cavity,

$$P = 0 \quad (3.12)$$

On solid boundary of the cavity,

$$\frac{\partial p}{\partial n} = -\rho \ddot{u}_n \quad (3.13)$$

For rigid boundary,

$$\ddot{u}_n = 0 \quad (3.14)$$

### ***Acoustic modes:***

Let the walls of cavity be rigid and stationary, so that  $\ddot{u}_n = 0$  and the pressure for harmonic loading is taken as,

$$p = \bar{p} \sin \omega t \quad (3.15)$$

Putting these values into equation we get (Cook 2007),

$$([K_f] - \omega^2 [M_f])\{\bar{p}\} = 0 \quad (3.16)$$

The solution of this eigenvalue problem yields natural frequencies of a cavity and corresponding pressure modes.

## **3.5 Coupled acoustic analysis:**

Load acting on surface of fluid cavity by motion of structure is given by (Cook 2007),

$$[F_f] = \rho [R] \{\ddot{u}\} \quad (3.17)$$

where, [R] is the coupling matrix denoted as,

$$[R] = \int_{surface} [N_f]^T [N_s] dS$$

where,  $N_f$  &  $N_s$  are shape functions for fluid and a structural surface of coupled interface.

Thus, coupled acoustic equation becomes,

$$[M_f]\{\ddot{p}\} + [C_f]\{\dot{p}\} + [K_f]\{p\} = -\rho[R]\{\ddot{u}\} \quad (3.18)$$

Similarly, load applied to structural surface by fluid pressure is given as,

$$[F_s] = [R]^T\{p\} \quad (3.19)$$

Dynamic equation of structure becomes,

$$[M_s]\{\ddot{u}\} + [C_s]\{\dot{u}\} + [K_s]\{u\} = \{F_s\} + [R]^T\{p\} \quad (3.20)$$

Thus coupled equation is (Cook 2007),

$$\begin{bmatrix} [M_s] & 0 \\ \rho[R] & [M_f] \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} [C_s] & 0 \\ 0 & [C_f] \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} + \begin{bmatrix} [K_s] & -[R]^T \\ 0 & [K_f] \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix} \quad (3.21)$$

Due to the presence of  $[R]$  matrix equation is a non-symmetric matrix. To solve this equation we convert it to symmetric form using displacement potential function (Everstine 1981). To do so first integrate pressure equation with time and put,

$$p = \dot{q} \quad (3.22)$$

Rearranging the terms we will get,

$$\begin{bmatrix} [M_s] & 0 \\ 0 & \frac{1}{\rho}[M_f] \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} [C_s] & -[R]^T \\ [R] & \frac{1}{\rho}[C_f] \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} [K_s] & 0 \\ 0 & [K_f] \end{bmatrix} \begin{Bmatrix} u \\ q \end{Bmatrix} = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix} \quad (3.23)$$

The coupling matrix [R] transferred from mass and stiffness parts to the damping part when the equation is switched to velocity potential as unknown.

For low-frequency eigenvalue calculations, the fluid is frequently assumed to be incompressible and also neglecting damping effect,,

$$M_f = 0 \text{ \& } C_f = 0 \quad (3.24)$$

Giving simpler eigenvalue problem (Cook 2007),

$$(M_s + M_a)\ddot{u} + K_s u = F_s \quad (3.25)$$

Where  $M_a$  is added mass of fluid loading ,

$$M_a = \rho R K_f^{-1} R^T \quad (3.26)$$



## CHAPTER 4

### RESULTS AND DISCUSSIONS

#### 4.1 Introduction:

In the present investigation, natural frequencies of rectangular laminated composite plates for various ply angles are studied by means of experiments. These vibrations generate acoustic waves in surrounding medium. To study the acoustic behaviour of plates, pressure variations are measured at specified location. The results obtained by the experiments are compared with the finite element package (ANSYS).

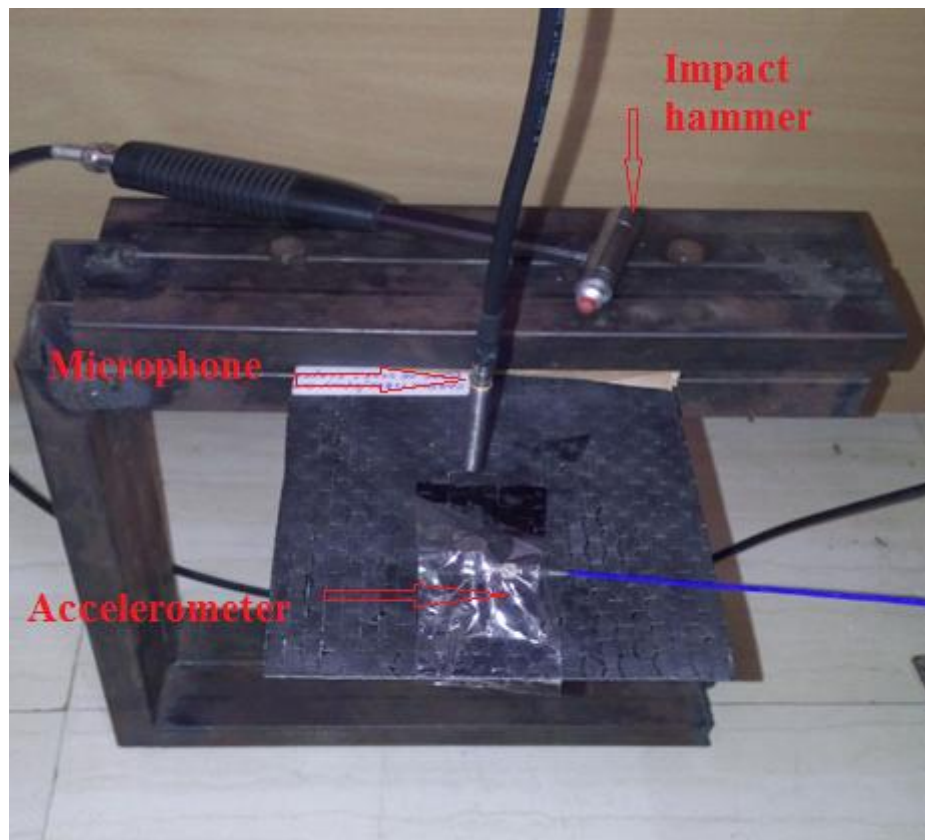
Glass fibre reinforced polymer (GFRP) and carbon fibre reinforced polymer (CFRP) plates are used for present study and their properties are given in the following table.

**Table 4.1** Material properties of CFRP and GFRP plates.

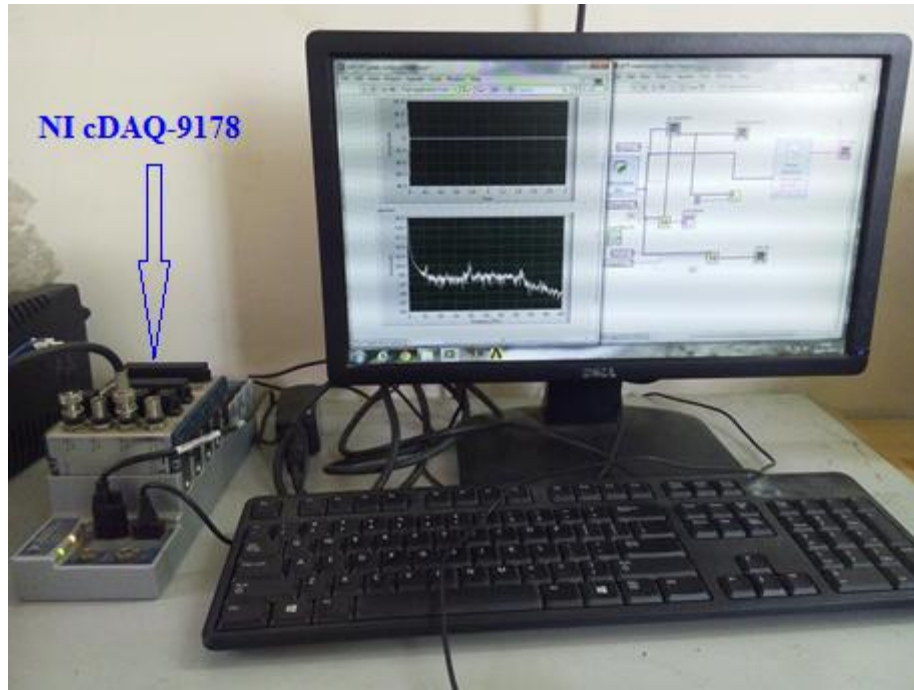
Properties	CFRP	GFRP
Young's modulus ( $E_1$ ), GPa	85.45	12.73
Young's modulus ( $E_2$ ), GPa	6.49	6.476
Shear modulus ( $G_{12}$ ), GPa	2.26	1.73
Poisson's ratio ( $\mu_{12}$ )	0.32	0.17
Density ( $\rho$ ), Kg/m <sup>3</sup>	1500	1900
Ply angles, °	0/90	0 <sup>0</sup> /90 <sup>0</sup>
	0 <sup>0</sup> /90 <sup>0</sup> /90 <sup>0</sup> /0 <sup>0</sup>	0 <sup>0</sup> /90 <sup>0</sup> /90 <sup>0</sup> /0 <sup>0</sup>
	45 <sup>0</sup> /-45 <sup>0</sup>	45 <sup>0</sup> /-45 <sup>0</sup>
	45 <sup>0</sup> /-45 <sup>0</sup> /-45 <sup>0</sup> /45 <sup>0</sup>	45 <sup>0</sup> /-45 <sup>0</sup> /-45 <sup>0</sup> /45 <sup>0</sup>

### ***Experimental Setup:***

The experimental setup is shown in Fig. 4.1 consists of the plate clamped in one end of the frame forming cantilever position. The accelerometer is attached at the free end to measure acceleration. The Microphone is held at a distance of approximately 1cm above the plate. It will give pressure variation at that point in Pascal. Experimental data was collected by tapping the centre of the plate with an impact hammer. Dimensions of plates were 180mm×150mm and thicknesses of the plate were 1.5mm or 3mm depending on number of plies. NI cDAQ-9178 instrument is shown in Fig. 4.2 was used to collect data from microphone, accelerometer and impact hammer. This data was processed for further study using LabVIEW software.



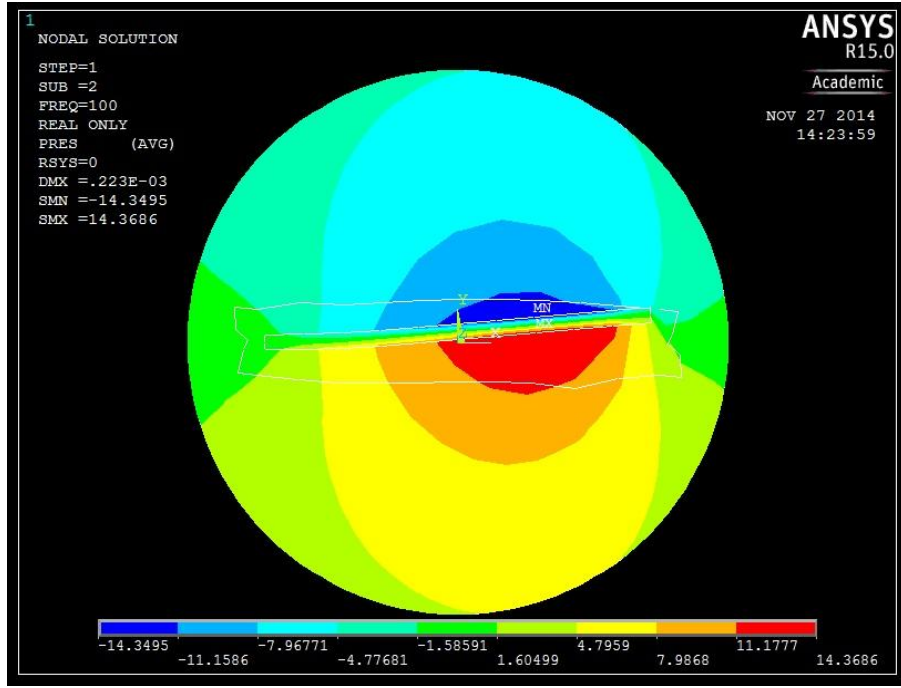
**Fig. 4.1** Experimental setup



**Fig. 4.2** NI cDAQ-9178 instrument

***Finite element simulation model:***

Vibration and acoustic analysis of laminated composite plate was carried out in finite element simulation. Simulation model consist of structure domain, fluid domain and fluid-structure interface. Structure domain was modelled using SHELL181 element while fluid domain is modelled using FLUID29. Fluid-structure interface was specified on common nodes fo structure and fluid domain. Fig. 4.3 shows simulation model in which Cantilever plate is surrounded by fluid domain. Figure shows pressure distribution due to plate vibration.



**Fig. 4.3** Simulation model

## 4.2 Governing equations:

Coupled acoustic equation is given by equation (3.21) is

$$\begin{bmatrix} [M_s] & 0 \\ \rho[R] & [M_f] \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} [C_s] & 0 \\ 0 & [C_f] \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} + \begin{bmatrix} [K_s] & -[R]^T \\ 0 & [K_f] \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix}$$

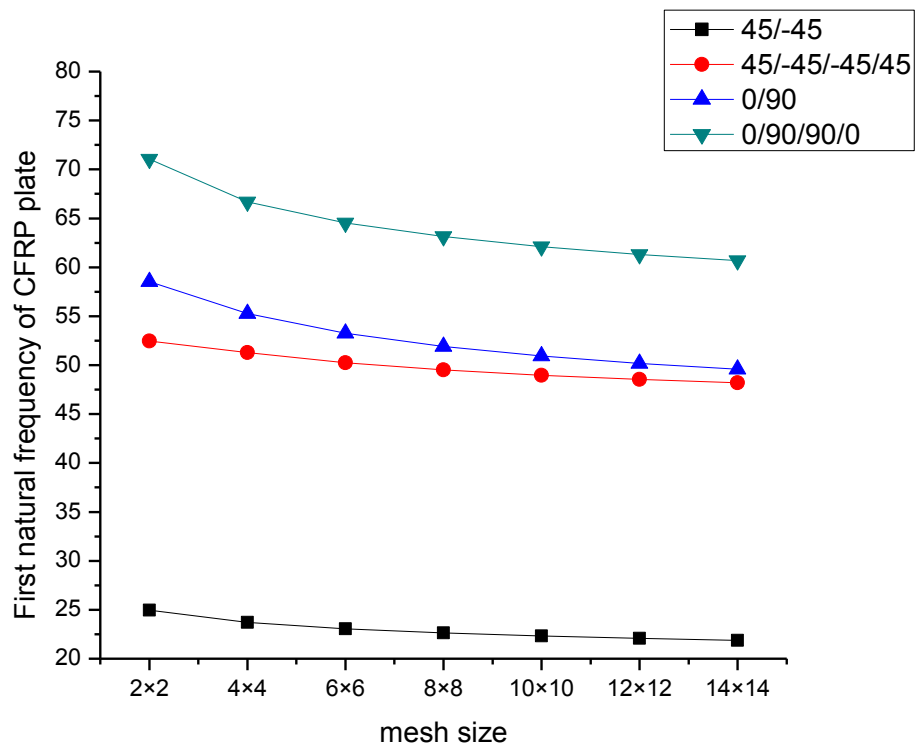
This equation is used to find the displacement of nodes of structure and pressure variation at nodes of the fluid domain.

## 4.3 Results and discussions:

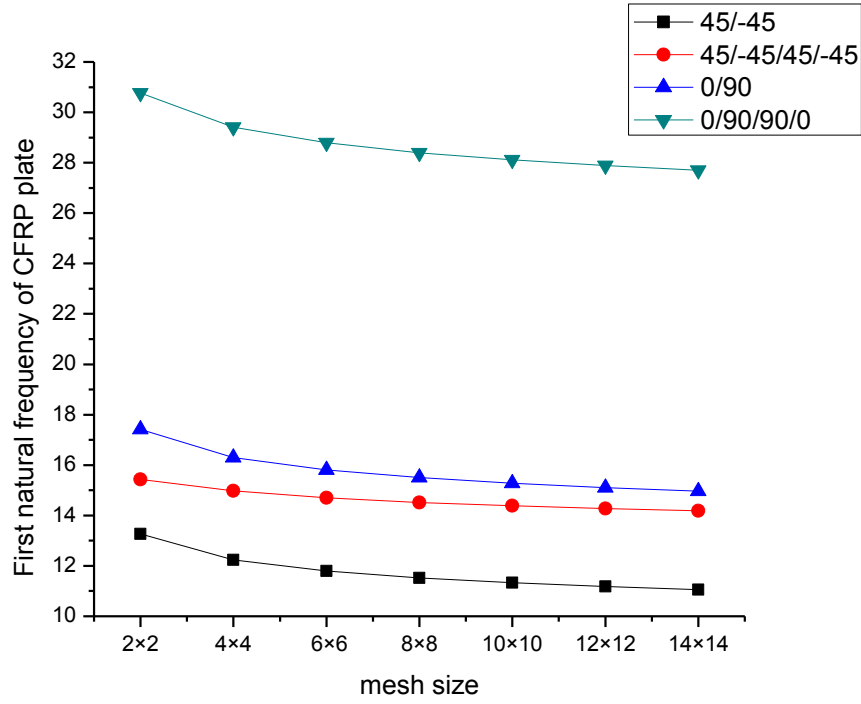
### 4.3.1 Convergence study for natural frequency parameter:

The convergence of presently developed model is carried out for various laminated plates. Structural and acoustic domains are modelled in finite element simulation ANSYS. Fig. 4.3 and Fig. 4.4 shows convergence study of CFRP and GFRP plates with first natural frequency taken as variable. Following graphs shows results obtained for different mesh divisions. As convergence started approximately at mesh division of  $12 \times 12$ , further results are computed for mesh division of  $12 \times 12$ .

From the figure, it is shown that as number of plies increases the natural frequency of both materials increases because of increase in bending stiffness.



**Fig. 4.4** Convergence for CFRP



**Fig. 4.5** Convergence for GFRP

#### ***4.3.2 Comparison study of natural frequency obtained from experiments and finite element simulation:***

Natural frequencies of CFRP and GFRP plates obtained from the experiment are compared with frequencies computed from finite element simulation, ANSYS. Table 4.2 and 4.3 shows matching frequency responses for experimental and ANSYS analyses. As shown in the table the frequencies are consistent with both experimental and ANSYS results with negligible variations. Thus, the present analysis yields accurate results.

**Table 4.2** Comparison study for natural frequency of cantilever CFRP plates

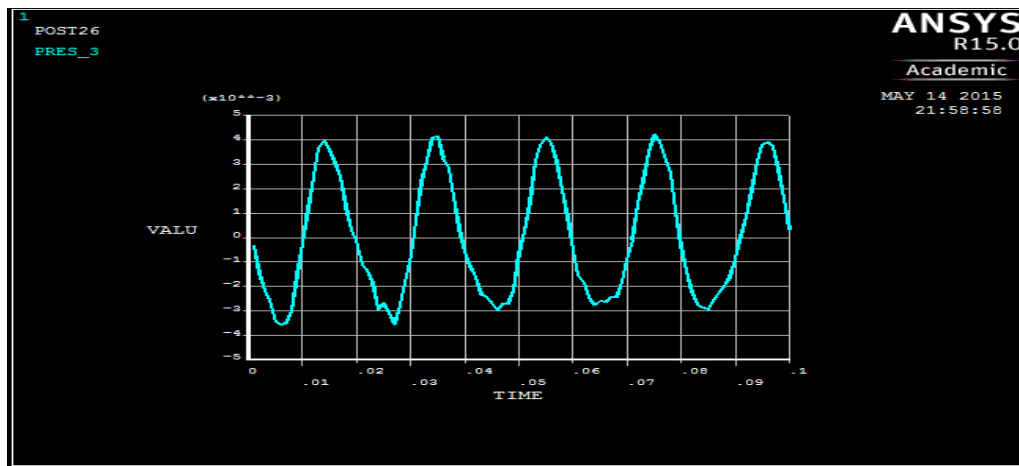
Natural frequency				
Mode no.	45 <sup>0</sup> /-45 <sup>0</sup>		0 <sup>0</sup> /90 <sup>0</sup>	
	Experimental	Simulation model	Experimental	Simulation model
1	23	22.073	50	50.2
2	70	67.551	100	98.7
3	162	161.14	310	309
4	260	257.27	515	514
5	325	320.87	567	567

**Table 4.3** Comparison study for natural frequency of cantilever GFRP plates

Natural frequency				
Mode no.	45 <sup>0</sup> /-45 <sup>0</sup>		0 <sup>0</sup> /90 <sup>0</sup>	
	Experimental	Simulation model	Experimental	Simulation model
1	12	11.179	15	15.109
2	34	32.149	31	33.411
3	100	98.297	120	120.55
4	120	136.01	161	158.35
5	155	156.79	175	175.38

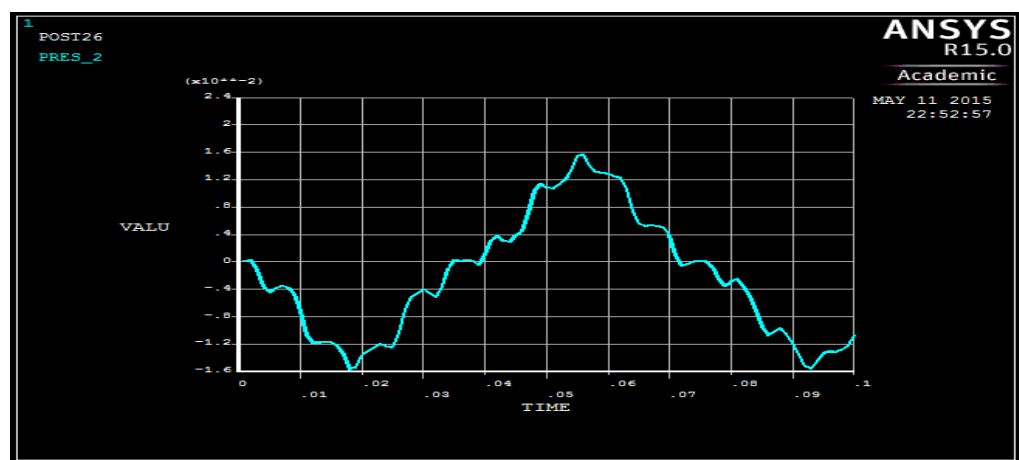
### 4.3.3 Comparison of pressure wave obtained from experiment and finite element simulation:

The acoustic pressure was measured at a specially located point in air medium surrounding vibrating plate. The point was located around 1cm above the plate. Time interval set for analysis was 2 seconds. Coupled analysis was performed in ANSYS for same laminated composites and boundary conditions. Fig. 4.5 shows pressure variation for CFRP  $0^0/90^0$  obtained by experiment and simulation respectively.



**Fig. 4.6** Pressure variations for CFRP  $0^0/90^0$  in ANSYS

Fig. 4.6 shows pressure variation for GFRP  $0^0/90^0$  obtained by experiment and simulation respectively.



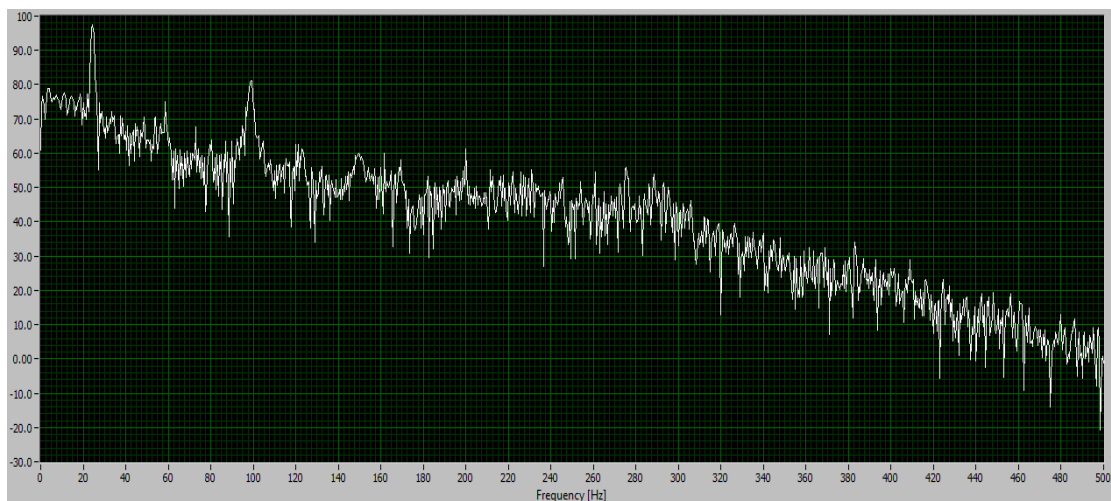
**Fig. 4.7** Pressure variations for GFRP  $0^0/90^0$  in ANSYS



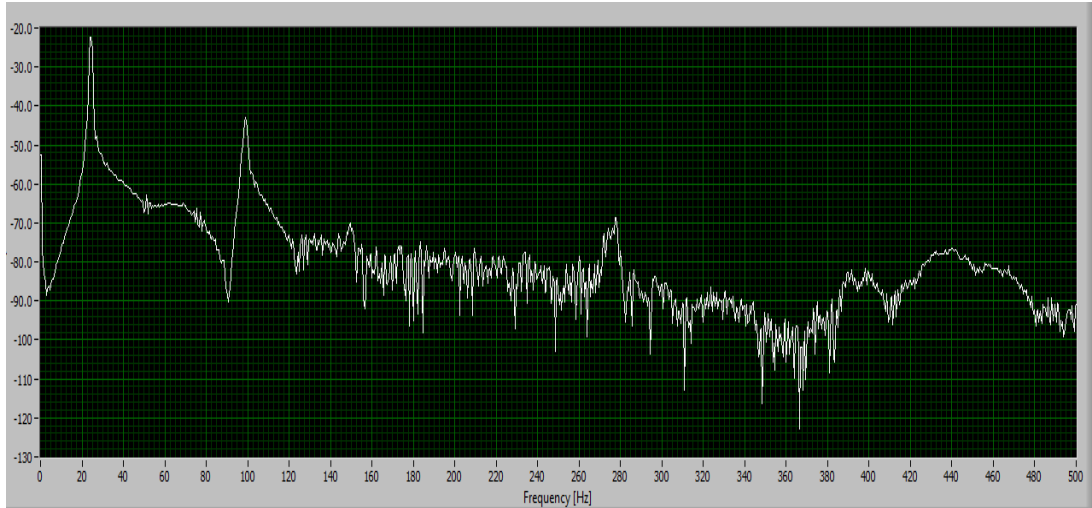
Figures show pressure variations for a 0.1-second interval for CFRP and GFRP plates for ply angle  $0^0/90^0$ . Both experimental and ANSYS results show same amplitude and frequency of an acoustic wave. Pressure responses obtained from experiments of other plates are also in good agreement with ANSYS simulation. These graphs show that pressure amplitude of GFRP plate is much higher than CFRP plate. So CFRP material is more effective for noise reduction than GFRP.

#### ***4.3.4 Comparison of structural and acoustic frequencies of CFRP ( $\pm 45^\circ$ ) at low-frequency region:***

Vibration and acoustic output data obtained from the experiment were further processed for finding FFT curves for displacement and pressure. Fig. 4.7 shows FFT curves for CFRP plate with ply angle  $45^0/-45^0$ .



**Fig. 4.8(a)** FFT curves for CFRP ( $\pm 45^\circ$ ) of displacement

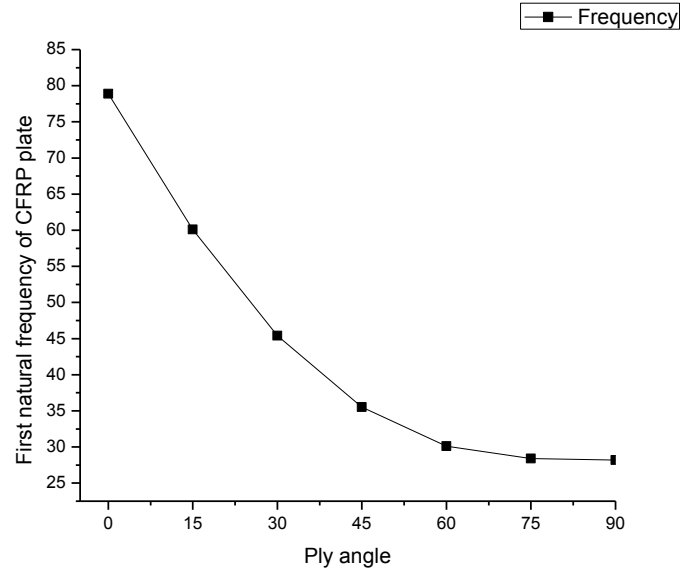


**Fig. 4.8(b)** FFT curves for CFRP ( $\pm 45$ ) of pressure

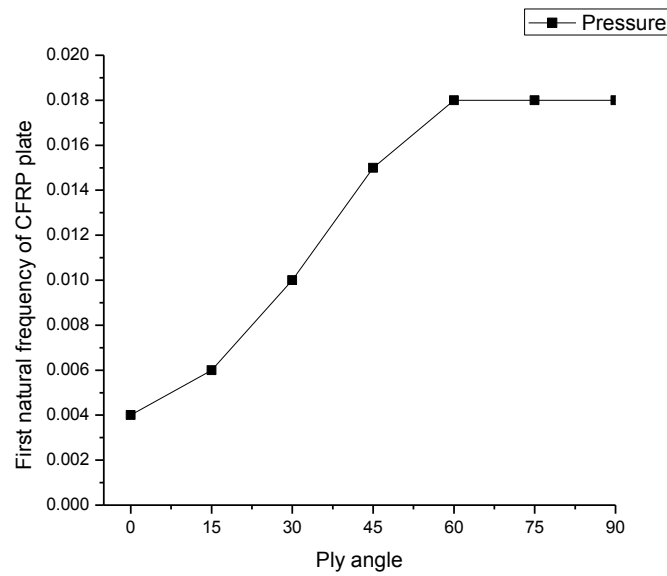
The graph shows that three types of modes coupled structural modes, coupled cavity modes and coupled combined resonances. In the low-frequency region, structural natural frequencies are sufficiently close forming coupled combined resonances. This phenomenon generally occurs for the frequency range of 0-250 Hz. Due to these resonances amplitude of displacement of structure and pressure are at maximum limit causing intense vibration and noise.

#### ***4.3.5 Effect of fiber orientation on the frequency and the pressure for laminated plate:***

Vibration and acoustic analysis of laminated composite plate were done to study the effect of different ply angles and number of plies on the natural frequency of plate and pressure variations. Fig. 4.8 and 4.9 shows the variation of natural frequency and pressure amplitude for CFRP  $0^0/90^0/90^0/0^0$ .



**Fig. 4.9** Effect of ply angle on first natural frequency for CFRP plate



**Fig. 4.10** Effect of ply angle on pressure amplitude for CFRP plate

The graph shows by increasing ply angle from  $0^0$  to  $90^0$ , natural frequency decreases. This is explained as when ply angle changes from  $0^0$  to  $90^0$ , transverse elasticity modulus accounts more for stiffness of the beam, and elasticity modulus of transverse direction is much smaller than elasticity modulus of longitudinal direction, and stiffness of the material is

directly proportional to elasticity modulus. So when stiffness decreases with the same weight and geometry, correspondingly it decreases the natural frequency of the material.

Whereas an increase in ply angle pressure amplitude increases. As sound waves are longitudinal wave amplitude increases as frequency decreases. Thus, laminated composites with higher ply angle can be used to reduce noise.

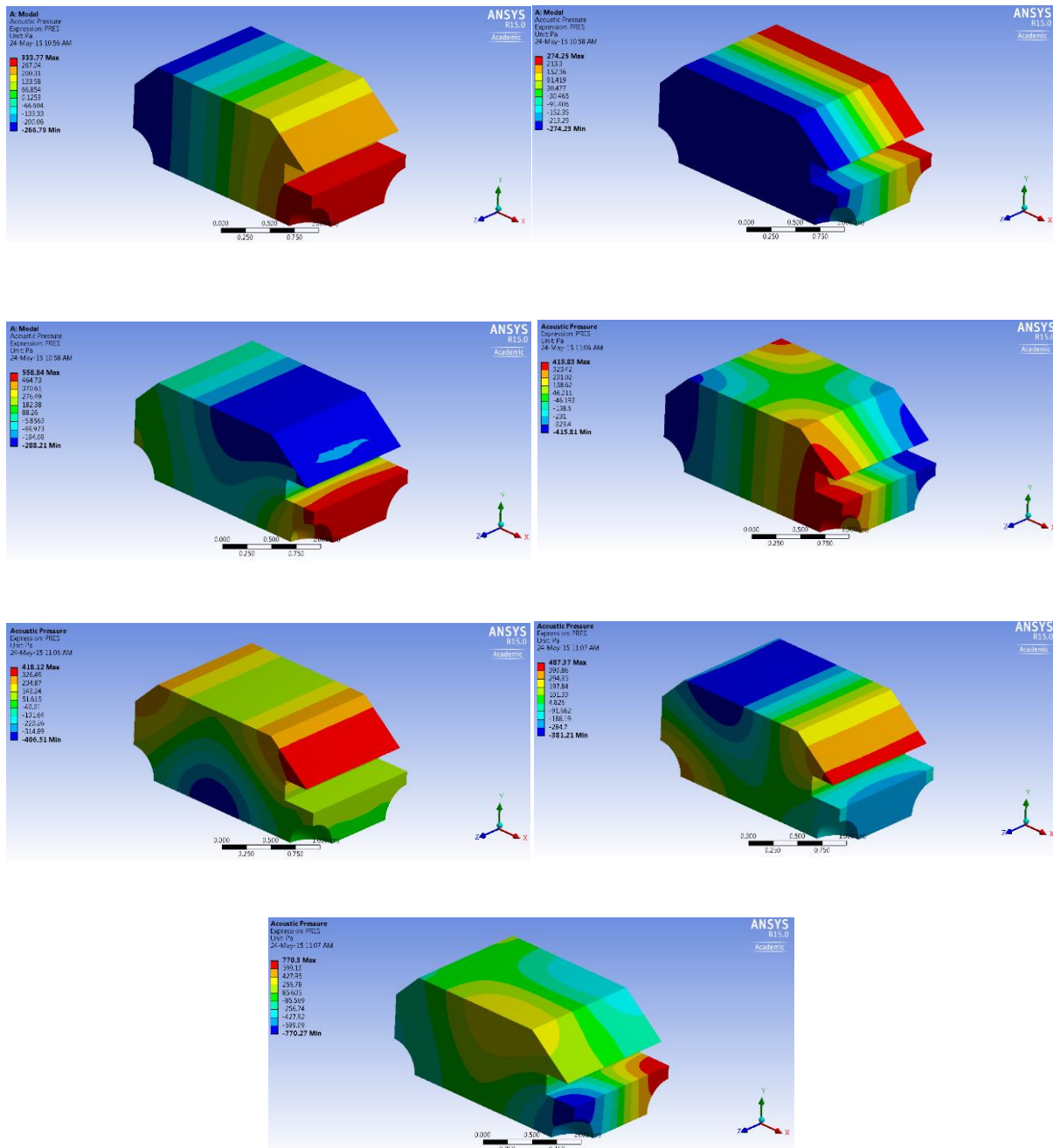
#### ***4.3.6 Free vibration analysis of vehicle compartment:***

Natural frequencies of vehicle compartment are computed using finite element simulation and it compared with published literature (Utsuno, Tanaka et al. 1989). Table 4.4 shows the comparison of natural frequency taken from literature and computed from the simulation.

**Table 4.4** Comparison of natural frequency of vehicle compartment

Mode number	Natural frequency	
	ANSYS	Utsuno and Tanaka (1989)
1	76.342	76
2	123.99	122
3	133.31	126
4	147.97	147
5	162.16	175
6	172.6	178
7	203.2	202

Results obtained from finite element simulation and literature is in excellent agreement and the maximum difference is 8%. Fig. 4.10 shows modes shapes for an acoustic cavity of the vehicle compartment.



**Fig. 4.11** Mode shapes of vehicle compartment

#### 4.4 Conclusions:

Vibration and acoustic analysis of CFRP and GFRP plates were carried out experimentally. The effect of ply orientation and number of layers on coupled vibro-acoustic behaviour of plates has been analysed for various combinations. Results obtained from experiments were validated with ANSYS results and found to be in excellent agreement.

1. In the present work, it is found that combined modes shapes are formed because of resonance of natural frequencies of the structure and acoustic cavity. These combined mode shapes generally occur in the low-frequency region and possess both high-order displacement and high-order pressure amplitude.
2. A fundamental frequency decreases when ply angle increases and increases when number of ply increases in the considered interval ( $0^\circ$ - $90^\circ$ ). Whereas pressure amplitude increases with increase in ply angle.
3. Laminated composite materials can be used in many applications instead of conventional materials where noise reduction is one of the essential parameters.

## CHAPTER 5

### CLOSURES

#### 5.1 Concluding Remarks:

In this present work, the vibration and acoustic behaviour of the laminated composite plate are examined through experimentally and finite element simulation. A suitable finite element model is proposed and developed using ANSYS for the analysis of vibration and acoustic. A parametric study has been carried out for the frequency and the pressure amplitude variations of the laminated composite plates. The most specific conclusions as a result of the present investigation are stated below:

- A linear finite element model is proposed and implemented for the discretisation of the plate model by using a four noded SHELL181 element having six degrees of freedom per node. Discretisation of a fluid domain is done with the FLUID29 element having four nodes and three degrees of freedom per node.
- Convergence study is performed by refining the mesh density. The comparison study for different cases indicates the necessity and requirement of the present mathematical model for an accurate prediction of the structural behaviour.
- The carbon fibre reinforced polymer plates has been examined by taking the different ply orientation and number of layers. Effects of the ply angle, number of ply on the frequency of vibration and pressure amplitude are studied in detail.
- A fundamental frequency decreases when the ply angle increases and with an increase in the number of ply the frequency increases whereas, the pressure amplitude increases with an increase in ply angle.

## 5.2 Significant Contribution of the Thesis:

The contributions of the present research work are as follows:

- The vibro-acoustic behaviour of a laminated composite plate is investigated experimentally as well as using finite element simulation model.
- The experimental analysis is carried out on carbon fibre reinforced polymer and glass fibre reinforced polymer plates in cantilever position.
- The panel model has been developed in the commercial FE software ANSYS by using APDL code. Four noded shell element (SHELL181) is employed to discretise the simulation model.
- The convergence and comparison study for the vibro-acoustic behaviour of a laminated composite plate is presented. The results obtained shows that the good efficiency of the experimental model and developed simulation model.
- The effects of various ply orientation and number of ply on frequency and pressure responses are studied.

Finally, it is understood from the previous discussions that the developed experimental model for vibro-acoustic analysis would be useful for analysis of laminated composite structures under different boundary and loading conditions. It is observed that the present developed FE model in ANSYS environment is also capable to solve any fluid-structure interaction problem easily and with less computational time. And hence, the present analysis would be useful for the practical design of the structure.



### **5.3 Future Scope of the Research:**

- An analytical study on vibration and acoustic analysis of laminated composite plates will give better understanding about the present developed experimental and finite element simulation model.
- The present study has been done by using the linear vibration which can be extended for the nonlinear analysis of laminated composite plate.
- The present study can be extended for the analysis of laminated composite panels also.
- The present study can be extended to investigate the vibration and acoustic analysis of laminated composite curved and/or flat panel under thermal and/or hygro-thermal environmental conditions.

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